New Physics Investigations with Triangular Singularity

Yugen Lin[1]

[1]Institute of High Energy Physics, Chinese Academy of Sciences E-mail: linyugen@ihep.ac.cn

Introduction

Loop-induced processes involving new physics particles can readily satisfy Landau Equation and trigger triangular singularities at high energy colliders, leading to fully visible Standard Model final states. Four-particle vertices in new physics allow triangular singularity diagrams to evade large virtuality suppression. We discuss several typical scenarios in supersymmetric models, and three types of final-state kinematic features at the collider. We identify an 'everything on shell' triangular singularity diagram only involving bosonic couplings, which has the potential to completely avoid large virtuality suppression. Such a virtuality-free diagram is missing in the Standard Model at the electroweak scale, and it becomes available in new physics models.

Triangle Singularity (TS)

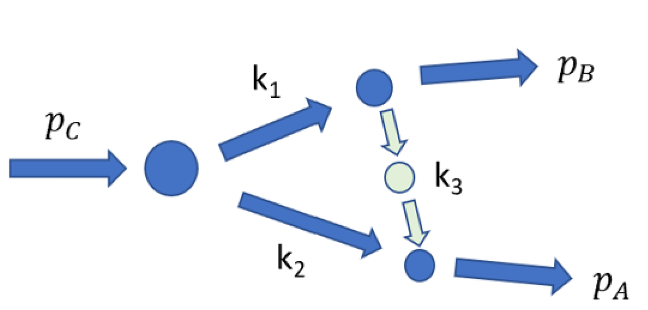
Triangle Singularity, the kinematic situation when all three intermediate particles in a triangle loop diagram for a $1 \to 2$ process become on-shell. Triggering TS requires the internal momenta k_i^μ in a triangle diagram to satisfy the Landau Equation,

$$\sum_i \alpha_i k_i^\mu = 0 \ \text{ and } \ k_i^2 - m_i^2 = 0, \quad (i = 1, 2, 3), \tag{1}$$

where $\sum_i \alpha_i = 1$ and $0 < \alpha_i < 1$ should be satisfied at leading Landau singularity. These conditions can be equivalently expressed in relation with the external invariant momenta p_k^2 that connect to k_i, k_j,

$$\beta_i + \sum_j^{j \neq i} \beta_j y_{ij} = 0, \qquad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j}, \tag{2}$$

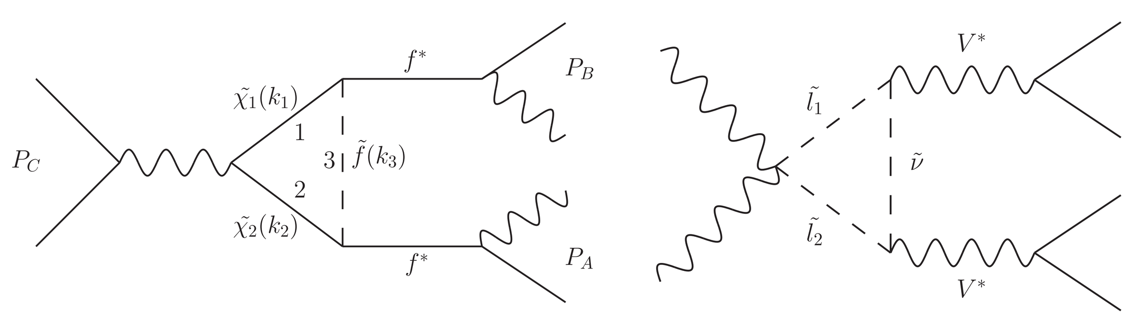
in which $i \neq j \neq k$ and $\beta_i \equiv \alpha_i m_i$. TS typically describes a heavy state splitting into lighter states with three internal on-shell particles with various masses. The corresponding classical processes are described below. Consider the rest frame of particle C. Particle C first decays into particles 1 and 2 flying back to back, then particle 1 decays into particles 3 and B. When particle 3 moves in the same direction as particle 2 with a larger velocity so that it can catch up with particle 2, and then particles 2 and 3 collide to form A in the final state. During this process, all intermediate particles are on their mass shell and this process corresponds to the leading Landau singularity.



A copious number of physical states, like hadrons, make it easier to satisfy singularity conditions. In this work, we explore several TS scenarios with the kaleidoscopic BSM particle content, such as in supersymmetry models, to discuss the kinematic observables and their physical case. Kinematics at TS derive from Landau Equation's requirements on momenta. For clarity, Latin subscripts denote the three external momenta p_i as $\{p_A, p_B, p_C\}$. In order for Eq. 2 to have physical solutions, their invariant self-products $\{p_A^2, p_B^2, p_C^2\}$ satisfy the relations

$$\begin{aligned} p_C^2 &\in \left[(m_1+m_2)^2, m_1^2 + m_2^2 + m_2 m_3 + \frac{m_2}{m_3}\left(m_1^2 - p_B^2\right)\right] \\ p_A^2 &\in \left[(m_2+m_3)^2, m_2^2 + m_3^2 + m_1 m_2 + \frac{m_2}{m_1}\left(m_3^2 - p_B^2\right)\right], \end{aligned} \tag{3}$$

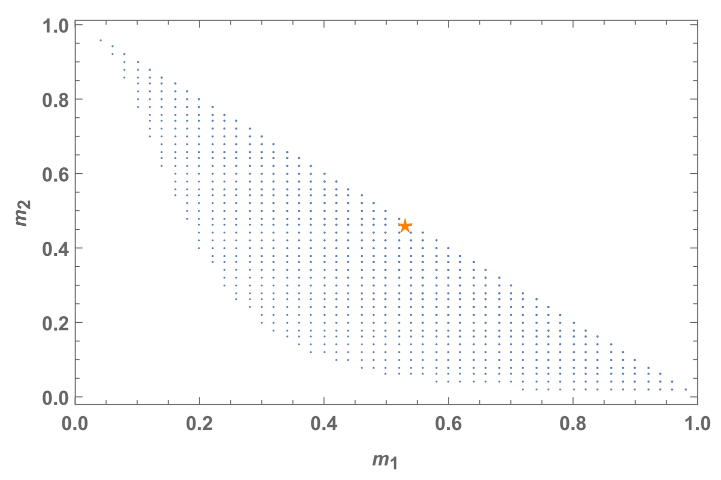
where the internal resonant masses need to be positive and at least two internal particles have non-identical values. We consider some typical processes include the Drell-Yan and Vector Boson fusion (VBF) diagrams, as shown below, in which charged BSM particles can occupy the internal line(s) k_i inside the triangle loop.



Taking an example with the minimal supersymmetric standard model (MSSM), a Drell-Yan process with fermionic p_A, p_B realizes with neutralino/chargino(s) as k_1, k_2, a sfermion as k_3, and two virtual SM fermions as p_A, p_B. In a VBF process, two initial-state radiated bosons can directly couple to k_1, k_2 with a four-particle vertex, in addition to the diagram of two boson first fusing into an s-channel propagator. Renormalizable four-particle couplings would require BSM bosons as k_i. In the MSSM, possible examples include slepton/squarks completing the triangle loop, e.g. with $\{\tilde{l}, \tilde{l}, \tilde{\nu}\}$, $\{\tilde{q}, \tilde{q}, \tilde{q}'\}$.

Parameter Space for Triggering TS

Collider reach at TS can cover a large range of BSM particle masses. In terms of m_1 and m_2, their physical solutions satisfies Eq. 2, and for a given collision center-of-mass energy $\sqrt{p_C^2}$ and free m_3, the valid parameter space is shown by the shaded area.



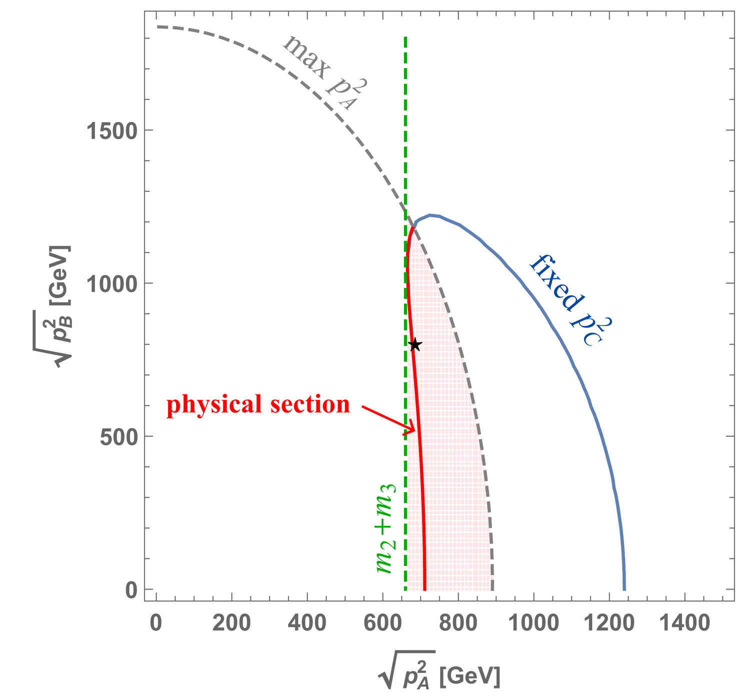
The upper edge of the shaded region, $m_1 + m_2 \leq \sqrt{p_C^2} \leq \sqrt{\hat{s}}$, corresponds to Eq. 3 lower boundary, and the lower edge corresponds to the Eq. 3 upper boundary. Since the beam particle's parton distribution allows $\sqrt{p_C^2}$ to be anywhere below the maximal collision energy $\sqrt{\hat{s}}$, the subspace where m_1, m_2 are too low to hit TS at a given $\sqrt{p_C^2}$ can still trigger TS at a lower value. This allows a high energy collider to sweep through the relevant TS parameter space in an efficient way similar to that for m_1, m_2 pair-production.

The Correlation of External Momentum

It shall be interesting if the TS loop provides extra phenomenon besides conventional BSM pair production, in particular, if the relevant observables are constructed from visible SM final-state particles. So next we will illustrate the relation of internal m_i and external p^2 at TS with given internal BSM masses. Solvable Eq. 2 requires a zero determinant of its β_i's coefficient matrix:

$$\begin{vmatrix} 1 & y_{12} & y_{13} \\ y_{12} & 1 & y_{23} \\ y_{13} & y_{23} & 1 \end{vmatrix} = 1 + 2 y_{12} y_{23} y_{13} - y_{12}^2 - y_{23}^2 - y_{13}^2 = 0, \tag{4}$$

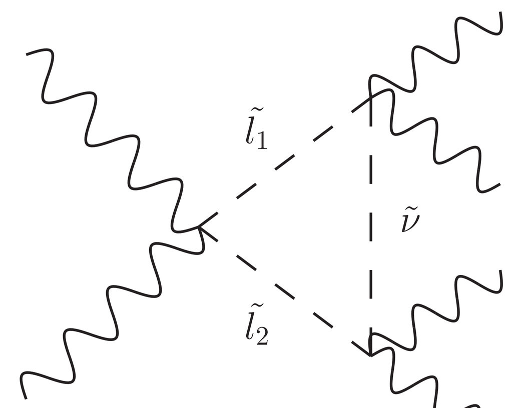
which contain 6 kinematic parameters. Fixing all three internal BSM masses and one external momentum, The relation between the two remaining external invariant momenta is a Dalitz curve, as shown by the blue solid trajectories in below figure. Its 'physical' section satisfies Eq. 2 and is shown in red. Varying the value of the third (fixed) invariant momentum let the physical Dalitz curve sweep across the parameter plane and covers a TS phase space enclosed by the maximal p_A^2 envelope and the minimal p_C^2 from Eq. 3.



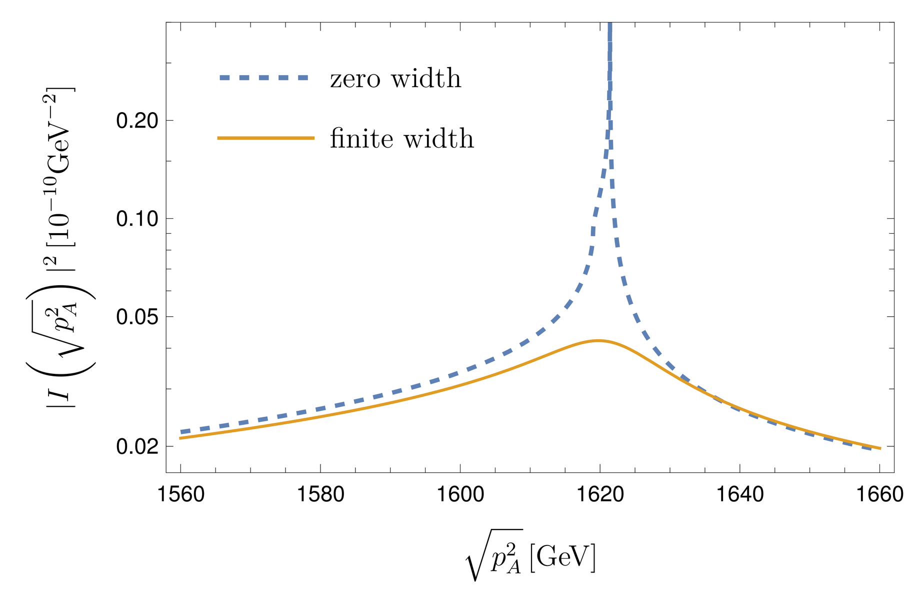
There are two kinds of kinematical features: (i) Peak(s) in distribution in the third invariant momentum when the other two become on-shell. A $\sqrt{\hat{s}}$ peak emerges if both $\sqrt{p_A^2}$, $\sqrt{p_B^2}$ identify with physical particles whose masses can locate inside the shaded region in the above figure. (ii) Correlation between two invariant masses when one external momentum is on-shell, i.e. the 2D phase space of the remaining invariant momenta reduces to a Dalitz curve. In practice, the correlation shape will be smeared by both internal/external particle width and detector resolutions, yet it can be looked for with advanced analysis software.

Less Suppressed Scenario

At a large collision energy, $\sqrt{p_A^2}$, $\sqrt{p_C^2}$ can be much higher than the weak scale. Thus identifying the external line with one (intermediate) single particle can suffer major propagator virtuality suppression $\propto (p_{A,C}^2 - m_{\rm SM}^2)^{-1} \sim p_{A,C}^{-2}$ in the amplitude. However, there are ways to avoid the large virtuality suppression completely in the case of bosons. Some bosonic vertices that carry derivatives (e.g. gauge couplings of bosons) can yield $\partial^\mu \propto p_{A,C}^\mu$. When the propagator further splits to lighter bosons with another derivative coupling, the $k^\mu . k_\mu$ can lift the $p_{A,C}^{-2}$ suppression in the large $p_{A,C}$ limit. A more straight-forward solution is to involve 4-particle gauge-coupling vertices and quartic couplings arising from the scalar potential, which replaces the external momentum with a pair of bosons that can be both put on-shell.



TS features an external momenta dependence in the loop amplitude that becomes singular when Eq. 2 and Eq. 3 are met. When the masses of internal particles and p_B^2, p_C^2 are within the range for the TS to be in the physical region, the enhancement due to TS effects will appear when p_A^2 satisfies the Landau Equation.



In practice, the internal particles' width will provide a small imaginary part to the propagators. Therefore the singularity collapses to a finite peak that broadens with the particle widths.

Summary and Discussion

We discussed several specific cases in the MSSM that satisfy TS with Drell-Yan and vector boson fusion processes. The diverse particle spectrum in BSM theories can provide candidate particles to fill in a triangle loop diagram and satisfy triangular singularity at a high energy collider. TS manifestation with BSM loops can lead to a fully identifiable SM final state, offering a unique alternative for new physics. Although TS is still a near-threshold phenomenon, TS can produce a final state purely composed of visible SM particle-systems that carry BSM scale energies. This would help reconstruct and identify these particles at the collider and reveal the so-called compressed BSM particle spectrum.