The double logarithms in the cross section of $e^+e^- \to J/\psi + \eta_c$ at NNLO

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Motivation

Heavy quarkonium production is interesting in understanding both perturbative and non-perturbative QCD.

Liu, He, Chao, 2003

It plays a crucial role in investigating various phenomena such as measuring the parton distribution, detecting the Quark-Gluon-Plasma signal and even new physics.

Hagiwara, Kou, Qiao, 2003

The $e^+e^- \to J/\psi + \eta_c$ cross section measured by Belle is about 25.6fb.

Belle, 2002

The LO cross section is much smaller than the experimental measurements.

Liu, He, Chao, 2003

Motivation

The NLO contribution is about 100%.

Zhang, Gao, Chao, 2006

The bulk of the NLO correction arises from double logarithm of Q^2/m_c^2, which already constitutes 72% of the full result at $Q = 10.58$ GeV.

Jia, He, Chao 2011

The NNLO correction to total cross section is sizable.

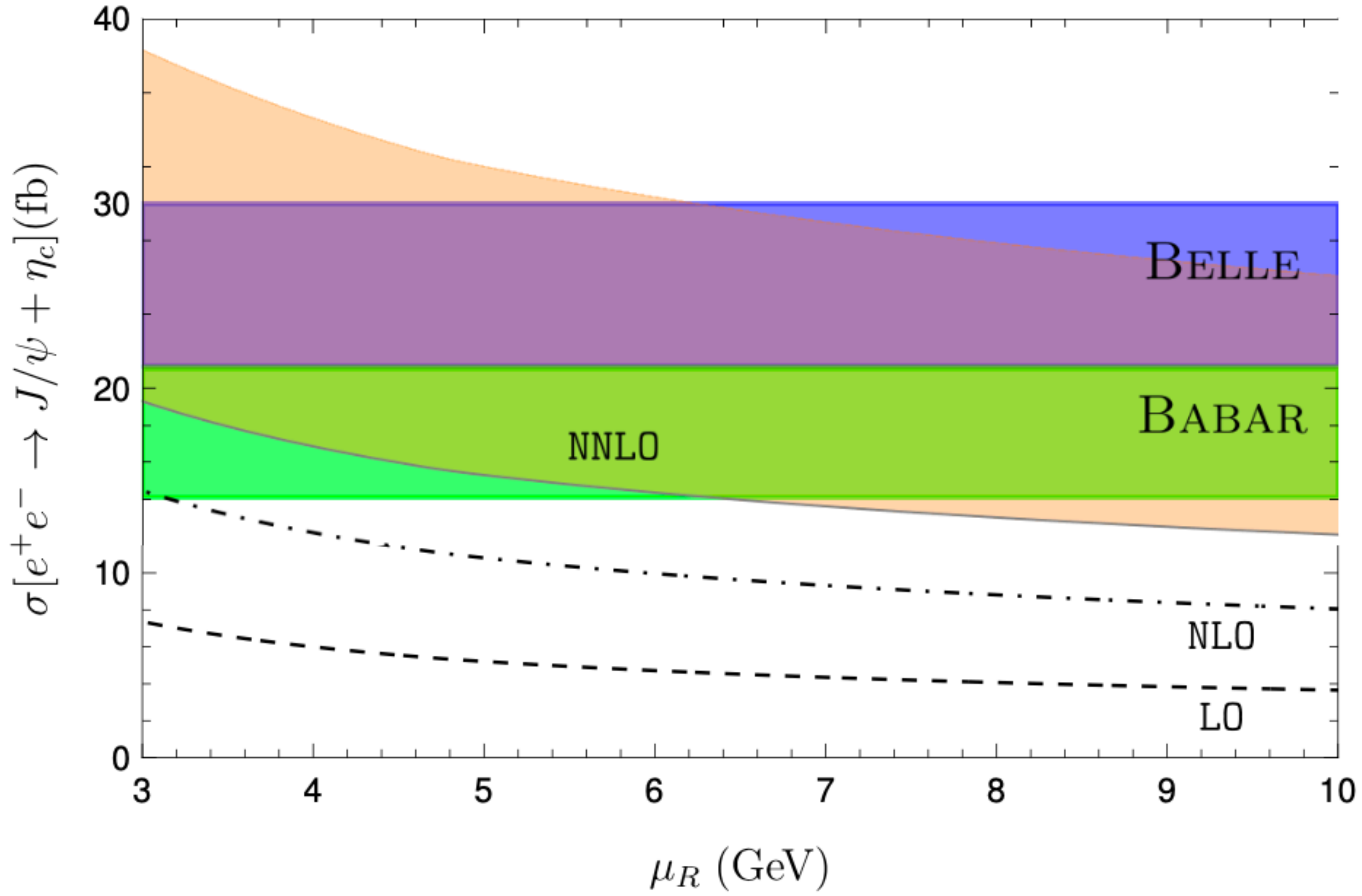
Feng, Jia, Mo, Sang, Zhang, 2019

Huang, Gong, Wang, 2023

Also see Chuan Qi's talk

The Sudakov double logarithms cancel in the sum over Feynman diagrams, but the endpoint double logarithm needs to be resumed.

Bodwin, Chung, 2014



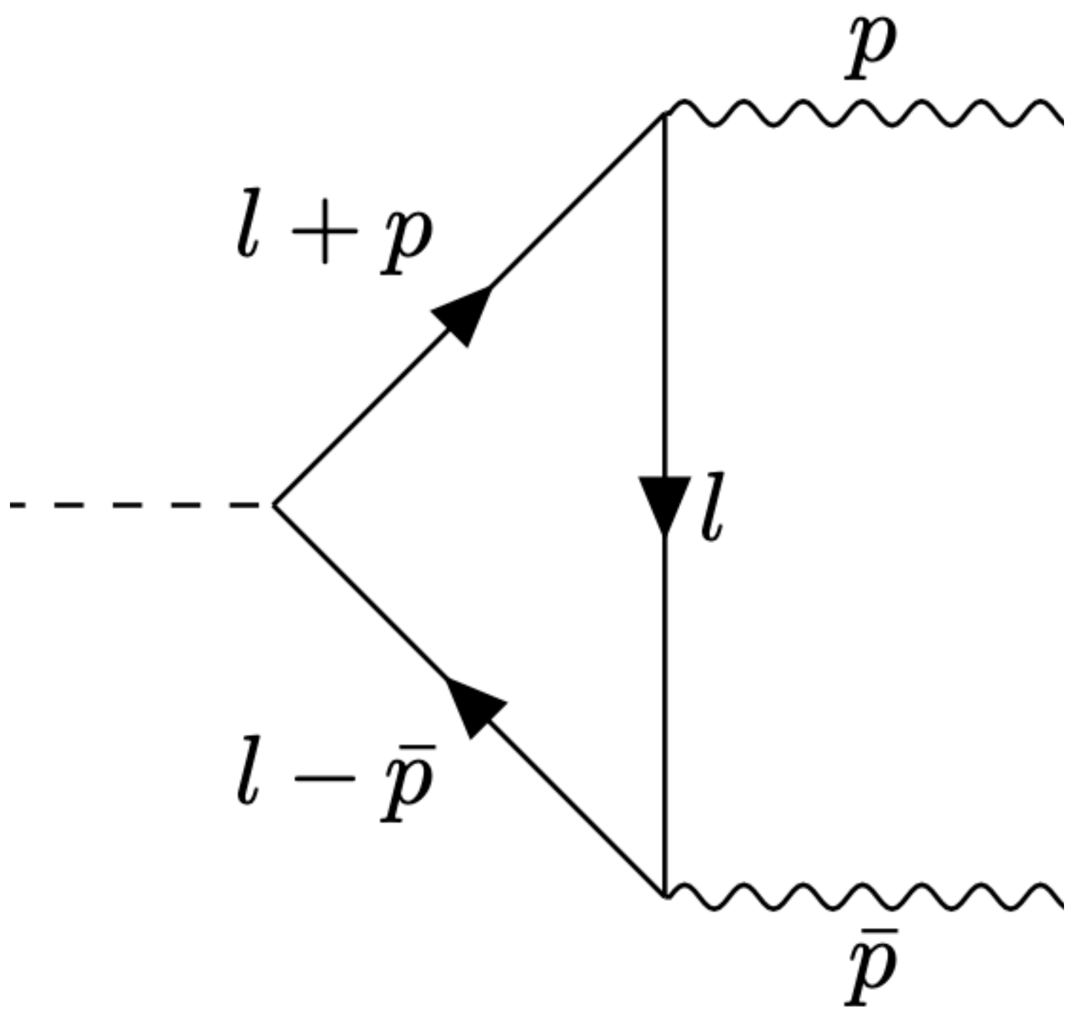
"We attempt to fit the coefficient of the anticipated endpoint logarithm $\alpha_s^2 \ln^4 r$. Pitifully, perhaps because the maximum value of $\sqrt{s}$(500 GeV) is still not asymptotically high, we fail to determine this coefficient in an unambiguous manner. " —Feng, Jia, Mo, Sang, Zhang, 2019

Motivation

Our goal is to establish a factorization formula and perform resummation of the endpoint double logarithms.

As a first step, we study the origin of these endpoint double logarithms by using the method of regions.

Endpoint divergence



$H \to \gamma\gamma$

$$n_\mu = (1,0,0,1), \quad \bar{n}_\mu = (1,0,0,-1)$$

l is collinear with p (n)

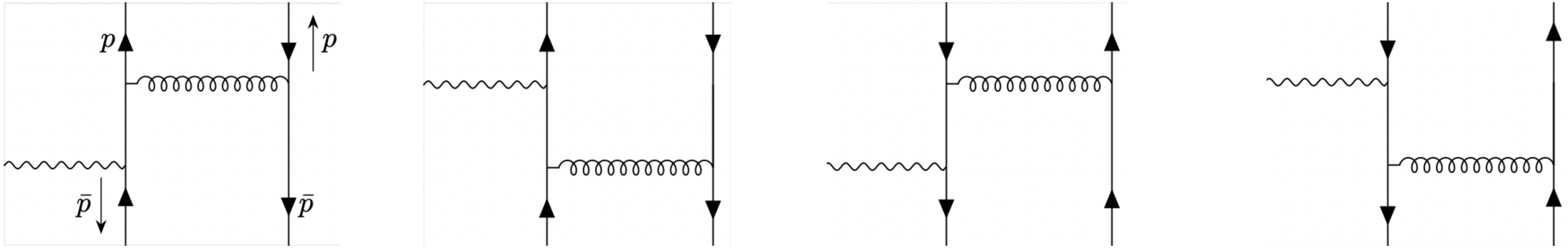
$$p_\mu = \frac{m_H}{2}(1,0,0,1), \quad \bar{p}_\mu = \frac{m_H}{2}(1,0,0,-1)$$

$$\int_0^\infty dl_T^2 (l_T)^{-2\epsilon} \int d(n\cdot l) \int d(\bar{n}\cdot l)$$
$$\times \frac{1}{(l_T^2 + (\bar{n}\cdot l)(n\cdot l) - m^2)\,[l_T^2 + (\bar{n}\cdot l)(n\cdot l) + m_H(n\cdot l)](\bar{n}\cdot l)}$$
$$= \int_0^\infty dl_T^2 (l_T)^{-2\epsilon} \int_{-m_H}^{0} d(\bar{n}\cdot l) \frac{2\pi i}{[m_H(m^2 - l_T^2) + m^2(\bar{n}\cdot l)]\,(\bar{n}\cdot l)}$$

Induce endpoint-divergent integrals

$e^+e^- \to J/\psi + \eta_c$ is very similar to $H \to \gamma\gamma$, but more complicated.

LO



$$\gamma^* \to J/\psi(2p) + \eta_c(2\bar{p})$$

LO

Kinematics

$$p^2 = m_c^2, \quad \bar{p}^2 = m_c^2, \quad (2p + 2\bar{p})^2 = Q^2$$

Spin and color projectors

$$\eta_c : \Pi_1(\bar{p}, \bar{p}) = -\frac{1}{2\sqrt{2}m_c}\gamma_5(\not{\bar{p}} + m_c) \otimes \frac{\mathbf{1}}{\sqrt{N_c}}$$

$$J/\psi : \Pi_3(p, p, \lambda) = -\frac{1}{2\sqrt{2}m_c}\not{\epsilon}^*(\lambda)(\not{p} + m_c) \otimes \frac{\mathbf{1}}{\sqrt{N_c}}$$

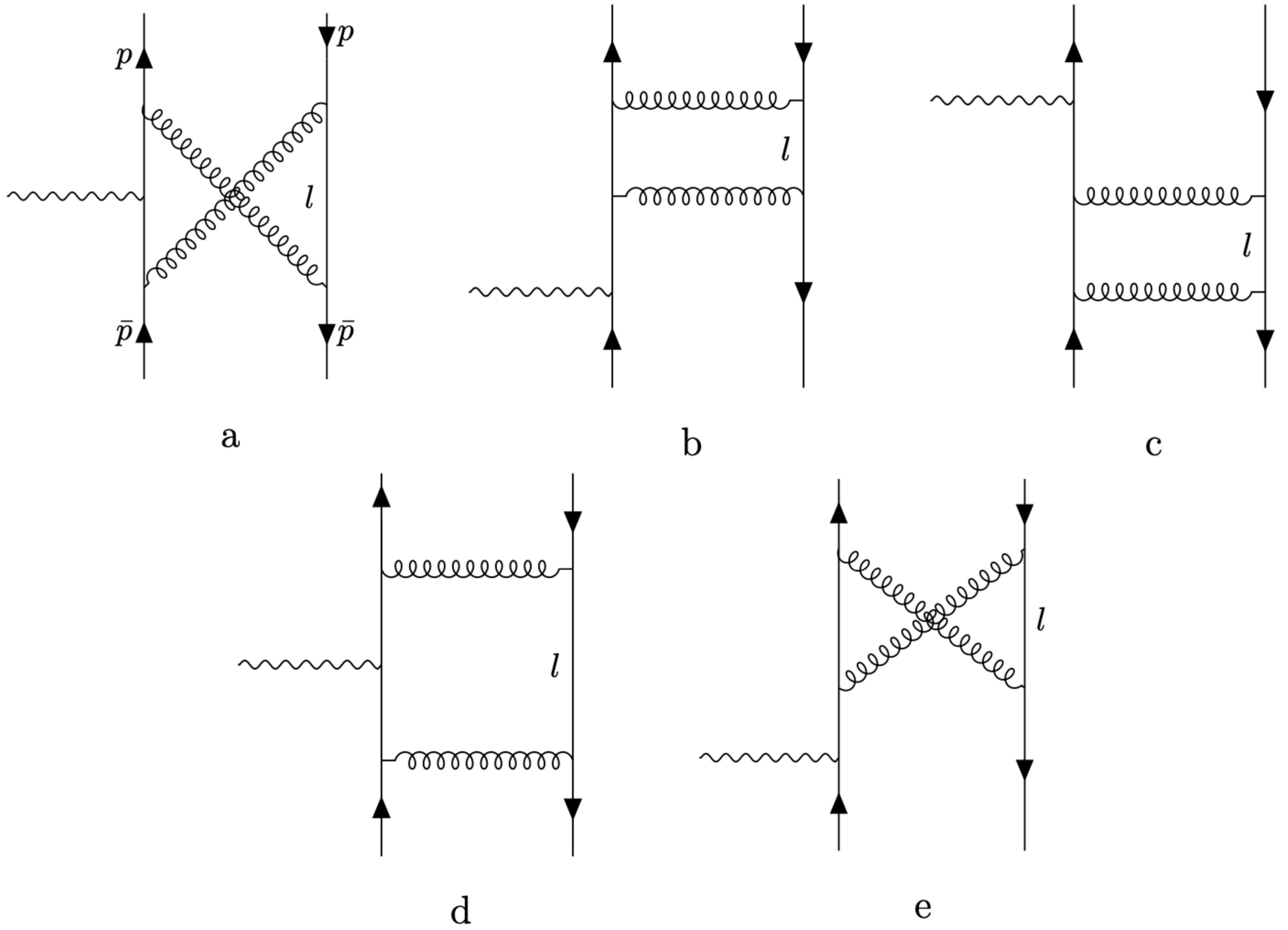
Bodwin, Petrelli, 2002

Short distance amplitude at LO

$$i\mathcal{M}^{\rho}_{LO} = \frac{-256\pi e_c \alpha_s C_F}{m_c Q^4}\epsilon^{\rho\lambda\mu\nu} p_\mu \bar{p}_\nu \epsilon^*_\lambda.$$

One loop

Just consider diagrams contributing to endpoint double logarithm.



FeynCalc, FIRE, PackageX

$m_c \to 0$

diagram	$\log^2 \frac{m^2}{Q^2} \times i\mathcal{M}_{LO}$
a:	$\frac{\alpha_s}{96\pi}$
b:	$\frac{\alpha_s}{12\pi}$
c:	$\frac{\alpha_s}{6\pi}$
d:	$\frac{\alpha_s}{24\pi}$
e:	0

Only endpoint double logarithm.
a,e: contain endpoint and Sudakov double logarithms

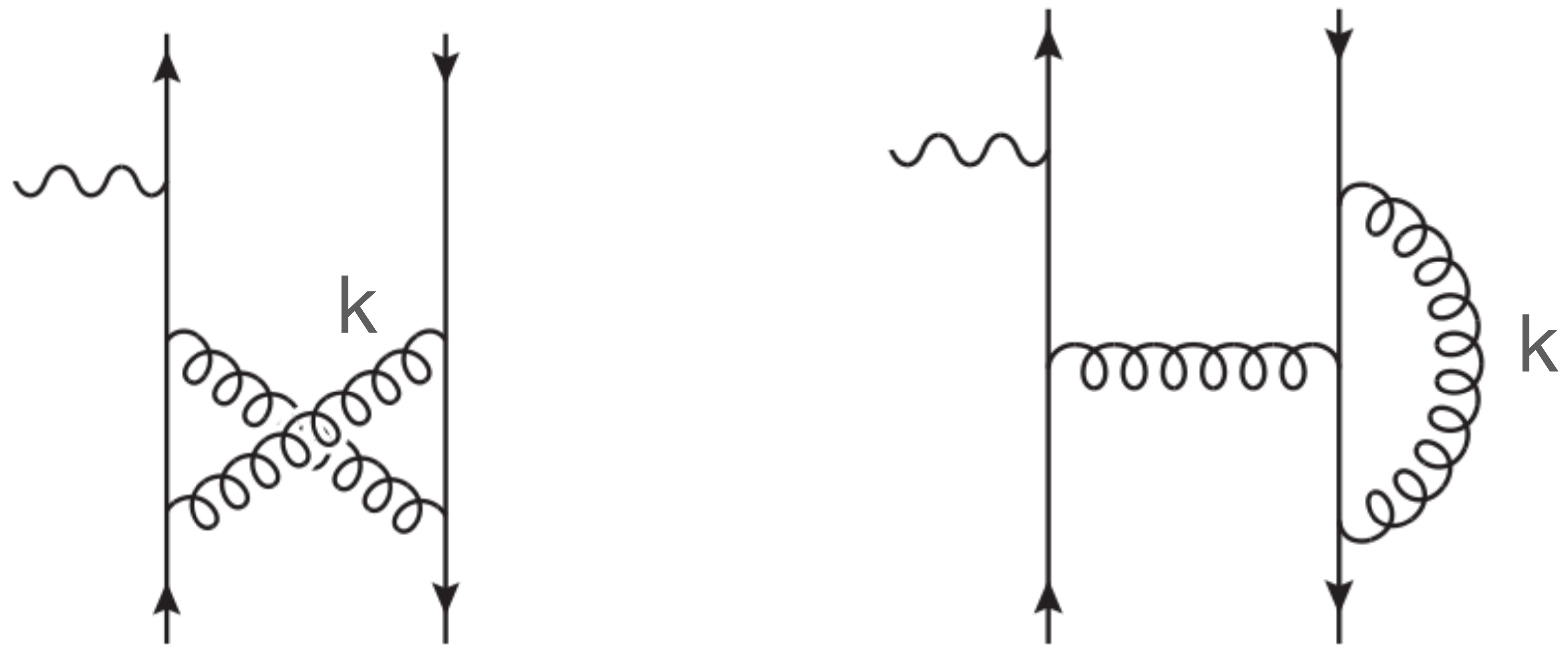
The analytical calculation of the two loop diagram is very complicated.

One loop

Method of regions Beneke, Smirnov, 1998

diagram	$\log^2 \frac{m^2}{Q^2} \times i\mathcal{M}_{LO}$
a:	$\frac{-\alpha_s}{96\pi}$
b:	$\frac{\alpha_s}{12\pi}$
c:	$\frac{\alpha_s}{6\pi}$
d:	$\frac{\alpha_s}{24\pi}$
e:	$\frac{-\alpha_s}{96\pi}$

Endpoint double logarithm



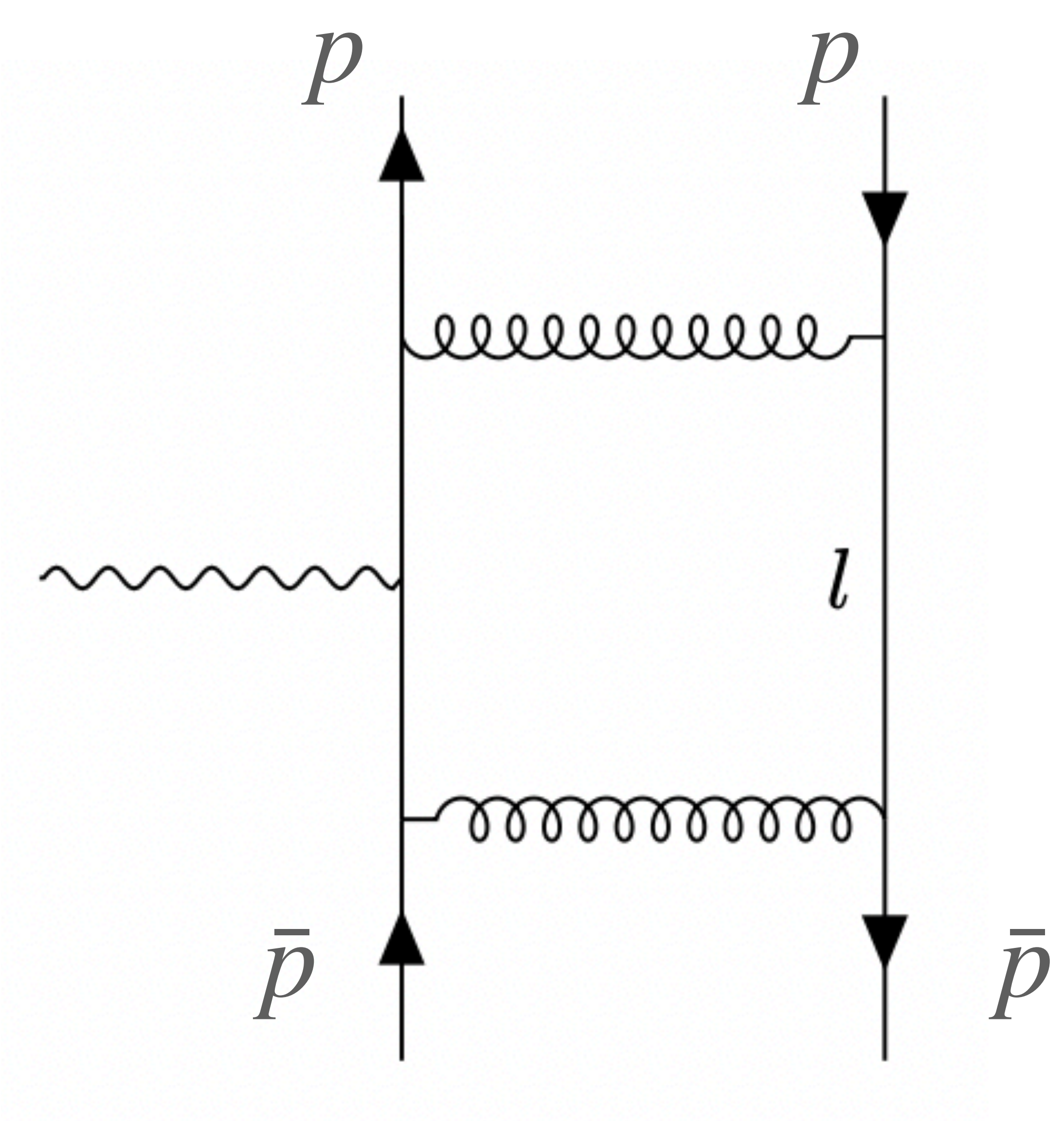
The Sudakov double logarithms canceled after summing all diagrams.

J/ψ and η_c are color neutral

Bodwin, Chung, 2014

Framework

Example:



$$iM^{(d,NLO)\rho} = \mu^{2\epsilon} \int \frac{d^D l}{(2\pi)^D}$$

$$\times \frac{(-i)^2 Tr[(ig_s\gamma^\mu)\Pi_3(ig_s\gamma_\mu)i(\not{l} + 2\not{p} + m)(ie_q\gamma^\rho)i(\not{l} - 2\not{\bar{p}} + m)(ig_s\gamma^\nu)\Pi_1(ig_s\gamma_\nu)i(\not{l} + m)]}{[l^2 - m^2](l+p)^2(l-\bar{p})^2[(l+2p)^2 - m^2][(l-2\bar{p})^2 - m^2]}$$

kinematic: $p = (n \cdot p, \bar{n} \cdot p, 0) \sim Q^2(r^2, 1, 0)$ $\bar{p} = (n \cdot p, \bar{n} \cdot \bar{p}, 0) \sim Q^2(1, r^2, 0)$

Expansion parameter : $r = \dfrac{m}{Q} \ll 1$ $4(p + \bar{p})^2 = Q^2$

Framework

Method of regions:

$$\begin{aligned}&\text{hard region}: l \sim (1,1,1)Q,\\&\text{collinear region}: l \sim (r^2,1,r)Q,\\&\text{anti-collinear region}: l \sim (1,r^2,r)Q,\\&\text{soft region}: l \sim (r,r,r)Q.\end{aligned}$$

$$i\mathcal{M}^{\rho}_{NLO} = \mathcal{A} \times i\mathcal{M}^{\rho}_{LO}$$

$$\begin{aligned}\mathcal{A} =&\mathcal{A}_H + (\mathcal{A}_C - \mathcal{A}_S) + (\mathcal{A}_{\bar{C}} - \mathcal{A}_S) + \mathcal{A}_S\\=&\mathcal{A}_H + \mathcal{A}_C + \mathcal{A}_{\bar{C}} - \mathcal{A}_S\end{aligned}$$

Zero bin subtraction

Idilbi, Mehen, 2007

Framework

Introduce regulators Δ_1 and Δ_2 to concretize endpoint divergence.

$$\frac{1}{(l+p)^2} \to \frac{1}{(l+p)^2+\Delta_1}$$
$$\frac{1}{(l-\bar{p})^2} \to \frac{1}{(l-\bar{p})^2+\Delta_2}$$

$$\Delta_1, \Delta_2 \to 0$$

soft region:

$l \sim (r, r, r)Q$

$$\mathcal{A}_S = -\frac{1}{4} i\pi\alpha_s C_F Q^2 \mu^{2\epsilon} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2-m^2)\left[(n\cdot l)Q/2+\Delta_1\right]\left[-(\bar{n}\cdot l)Q/2+\Delta_2\right]}$$

We adopt cut-off UV regularization.

$$\int_0^\infty dl_T^2 (l_T)^{-2\epsilon} \int d(n\cdot l) \int d(\bar{n}\cdot l) \to \int_0^{Q^2} dl_T^2 \int d(n\cdot l) \int d(\bar{n}\cdot l)$$ **finite**

When $l_T^2 > Q^2$, the poles in collinear, anti-collinear, and soft regions will cancel with the hard region.

Framework

$$\mathcal{A}_S = -\frac{1}{256\pi^3} i\alpha_s C_F Q^2 \Omega_{d-2} \int_0^{Q^2} dl_T^2 \int d(n\cdot l) \int d(\bar{n}\cdot l)$$
$$\times \frac{1}{(-l_T^2 + (n\cdot l)(\bar{n}\cdot l) - m^2)\,[(n\cdot l)Q/2 + \Delta_1][-(\bar{n}\cdot l)Q/2 + \Delta_2]}$$

We first integrate over $n \cdot l$ by choosing the residue in $l^2 - m^2 = 0$.

$$\mathcal{A}_S = -\frac{1}{32\pi^2} \alpha_s C_F Q^4 \Omega_{d-2} \int_0^{Q^2} dl_T^2 \int_{-\infty}^{0} d(\bar{n}\cdot l) \frac{1}{\left(l_T^2 + m^2 + \frac{2\Delta_1(\bar{n}\cdot l)}{Q}\right)[(\bar{n}\cdot l) - \frac{2\Delta_2}{Q}]}$$

We divide the integration of $\bar{n} \cdot l$ into two parts.

$$\mathcal{A}_S = -\frac{1}{32\pi^2} i\alpha_s C_F Q^4 \Omega_{d-2} \int_0^{Q^2} dl_T^2 \int_{-Q}^{0} d(\bar{n}\cdot l) \frac{1}{(l_T^2 + m^2)\,[(\bar{n}\cdot l) - \frac{2\Delta_2}{Q}]} + \int_{-\infty}^{-Q} d(\bar{n}\cdot l) \frac{1}{\left(l_T^2 + m^2 + \frac{2\Delta_1(\bar{n}\cdot l)}{Q}\right)(\bar{n}\cdot l)}$$

$$= \frac{1}{32\pi^2} \alpha_s C_F \Omega_{d-2} \int_0^{Q^2} dl_T^2 \frac{\ln\frac{Q^2}{\Delta_2}}{(l_T^2 + m^2)} + \frac{\ln\frac{l_T^2 + m^2}{\Delta_1}}{\left(l_T^2 + m^2 + \frac{2\Delta_1(\bar{n}\cdot l)}{Q}\right)}$$

Framework

Keeping only double logarithm,

soft region:

$l \sim (r, r, r)Q$

$$\mathcal{A}_S = -\frac{\alpha_s \log^2 r}{24\pi} + \frac{\alpha_s \log\frac{\Delta_2}{Q^2}\log r}{12\pi} + \frac{\alpha_s \log\frac{\Delta_1}{Q^2}\log r}{12\pi}$$

collinear region:

$l \sim (\lambda^2, 1, \lambda)Q$

$$\mathcal{A}_C = \frac{\alpha_s \log\frac{\Delta_2}{Q^2}\log r}{12\pi}$$

The final result is independent of Δ_1, Δ_2

anti-collinear region:

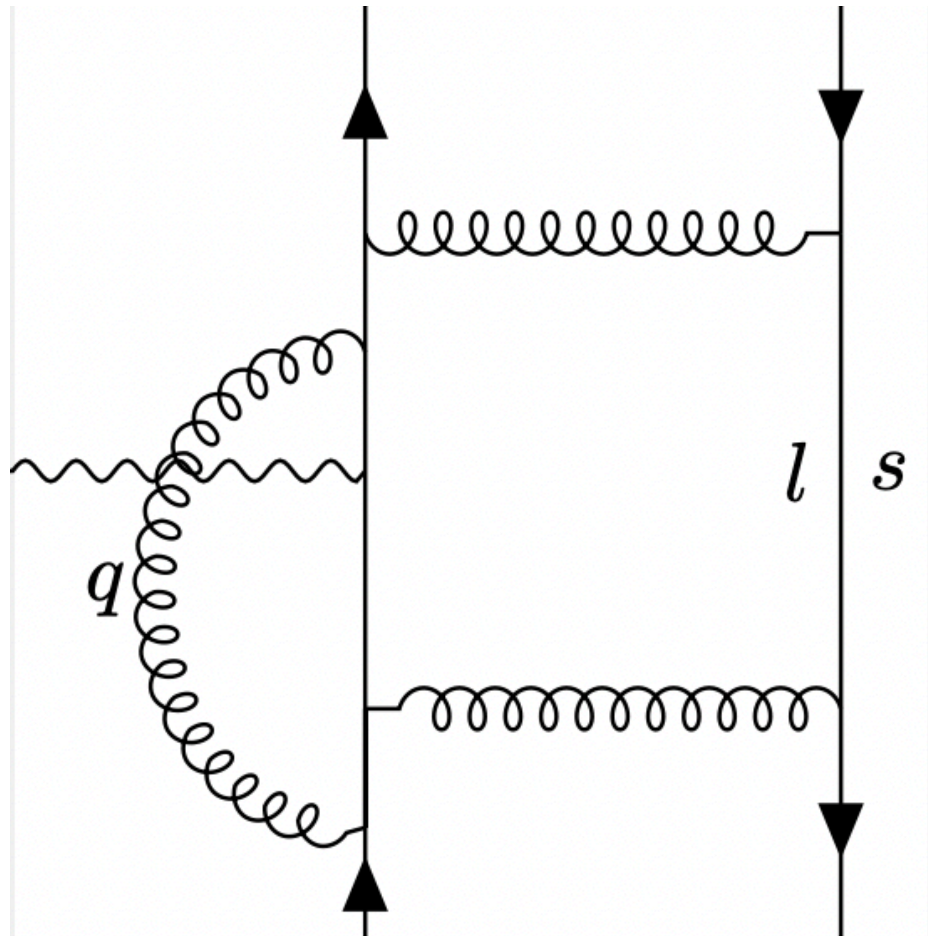
$$\mathcal{A}_{\bar{C}} = \frac{\alpha_s \log\frac{\Delta_1}{Q^2}\log r}{12\pi}$$

$l \sim (1, \lambda^2, \lambda)Q$

Setting $\Delta_1, \Delta_2 = Q^2$, the collinear and anti-collinear contributions vanish.

We need to analyze only the soft region to get the large double logarithms.

Two loop double logarithm



region: **Pysecdec, Asy** Heinrich, Jahn, Jones et al. 2022; Jantzen, Smirnov, Smirnov, 2012

soft(s) $q \sim (r, r, r)Q$ soft-anti-collinear(sc) $q \sim (r^3, r, r^2)Q$

anti-collinear($\bar{c}$) $q \sim (r^2, 1, r)Q$ hard-anticollinear($h\bar{c}$) $q \sim (r, 1, \sqrt{r})Q$

$$s: \quad \frac{1}{\epsilon^2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log(l\cdot\bar{n}) + \log(l\cdot n)}{\epsilon (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{(\log(l\cdot\bar{n}) + \log(l\cdot n))^2}{2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})}$$

$$h\bar{c}: \quad \frac{1}{\epsilon^2 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log(l\cdot\bar{n})}{\epsilon (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log^2(l\cdot\bar{n})}{2 ((m^2 - l^2)(l\cdot n)(l\cdot \bar{n}))}$$

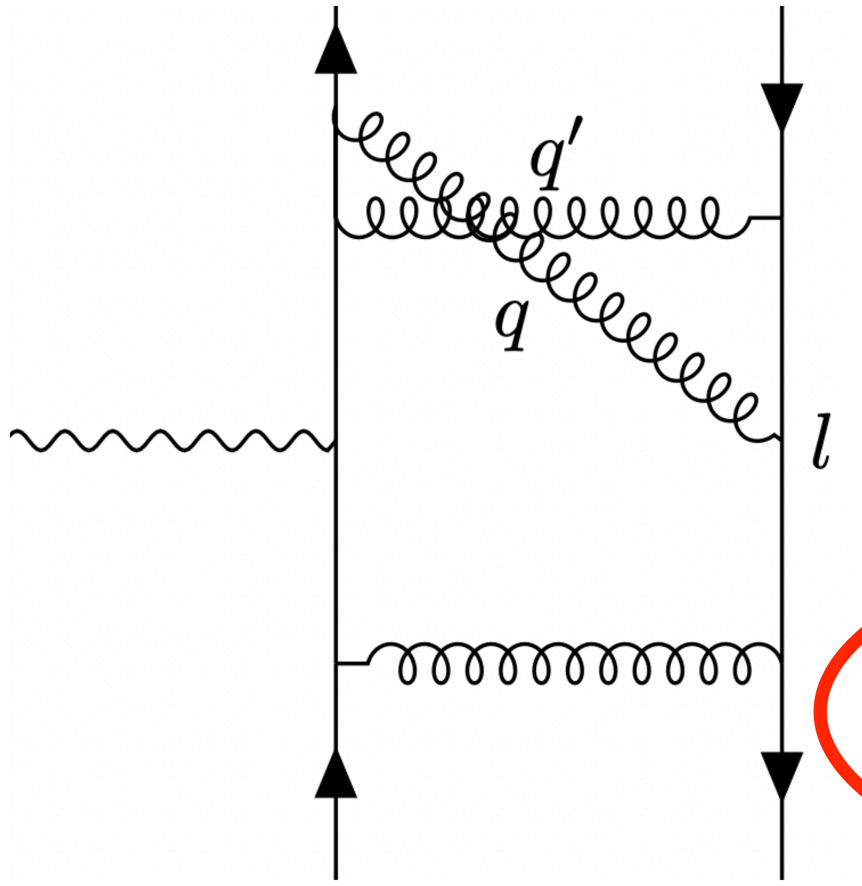
$$\bar{c}: \quad \frac{1}{2\epsilon^2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log(m^2)}{2\epsilon ((m^2 - l^2)(l\cdot n)(l\cdot \bar{n}))} + \frac{\log^2(m^2)}{4 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})}$$

$$s\bar{c}: \quad \frac{1}{2\epsilon^2 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(l\cdot n) + \log(m^2)}{2\epsilon ((l^2 - m^2)(l\cdot n)(l\cdot \bar{n}))} + \frac{(2\log(l\cdot n) + \log(m^2))^2}{4 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})}$$

$$s + h\bar{c} + \bar{c} + s\bar{c}$$

$$-\frac{\log^2(l\cdot n)}{2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log(m^2)\log(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log(l\cdot n)\log(l\cdot \bar{n})}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})}$$

Two loop double logarithm



hard-collinear(hc) $q \sim (r, 1, \sqrt{r})$

collinear(c) $q \sim (r^2, 1, r)$

soft-collinear(sc) $q \sim (r^3, r, r^2)$

collinear(c') $q' \sim (r^2, 1, r)$

soft(s) $q \sim (r, r, r)$

→ Introduce a regulator η $\qquad \dfrac{1}{(q'^2)^{1+\eta}}$

$$s: \frac{\frac{1}{(l^2-m^2)(l\cdot n)(l\cdot \bar{n})\,\epsilon} + \frac{\log(m^2)}{(m^2-l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)}{\eta} + \left(\frac{1}{\epsilon^2 (l^2-m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log(l^2-m^2)+\log(l\cdot n)}{\epsilon(m^2-l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(m^2)\log(l\cdot n) + \log^2(l^2-m^2) + (\log(m^2)-\log(l^2-m^2))^2}{2\left((m^2-l^2)(l\cdot n)(l\cdot \bar{n})\right)} + O(\epsilon^1)\right) + O(\eta^1)$$

$$c': \frac{\frac{1}{(m^2-l^2)(l\cdot n)(l\cdot \bar{n})\,\epsilon} + \frac{\log(m^2)}{(l^2-m^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)}{\eta} + \left(\frac{1}{\epsilon^2 (l^2-m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log^2(m^2)}{2(m^2-l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)\right) + O(\eta^1)$$

Two loop double logarithm

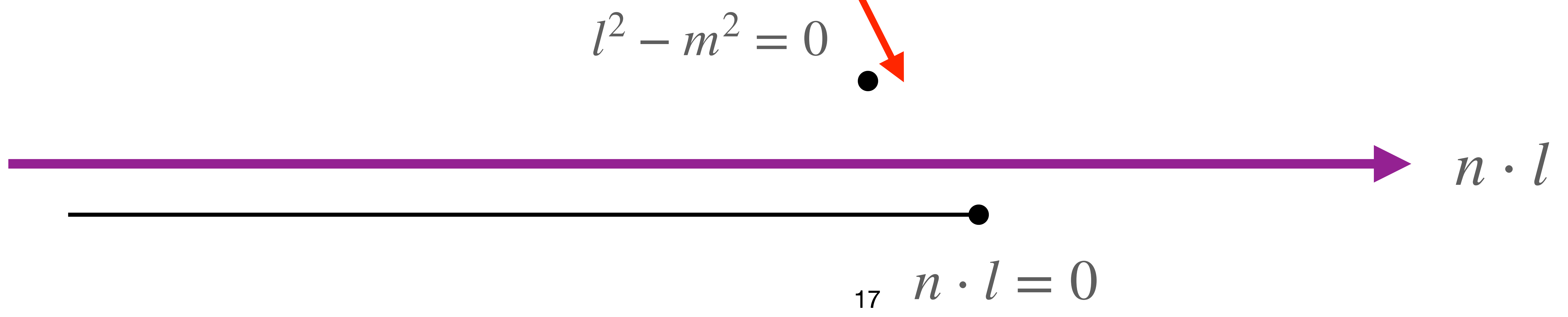
$$hc: \quad \frac{2}{\epsilon^2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{2\log(l\cdot n)}{\epsilon (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log^2(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

$$c: \quad \frac{1}{2\epsilon^2 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log(m^2)}{2\epsilon((l^2 - m^2)(l\cdot n)(l\cdot \bar{n}))} + \frac{\log^2(m^2)}{4(l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

$$sc: \quad \frac{1}{\epsilon^2 (2m^2 (l\cdot n)(l\cdot \bar{n}) - 2l^2 (l\cdot n)(l\cdot \bar{n}))} + \frac{\log(l^2 - m^2) - \log(l\cdot n) + \frac{\log(m^2)}{2}}{\epsilon (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{(2\log(l^2 - m^2) - 2\log(l\cdot n) + \log(m^2))^2}{4(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

$$s + hc + c + sc + c': \quad \frac{2\log^2(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(m^2)\log(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(l^2 - m^2)\log(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{2\log(m^2)\log(l^2 - m^2)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})}$$

Pole structure :



Two loop double logarithm

$$hc: \quad \frac{2}{\epsilon^2 (m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{2\log(l\cdot n)}{\epsilon (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log^2(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

$$c: \quad \frac{1}{2\epsilon^2 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} - \frac{\log(m^2)}{2\epsilon ((l^2 - m^2)(l\cdot n)(l\cdot \bar{n}))} + \frac{\log^2(m^2)}{4(l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

$$sc: \quad \frac{1}{\epsilon^2 (2m^2 (l\cdot n)(l\cdot \bar{n}) - 2l^2 (l\cdot n)(l\cdot \bar{n}))} + \frac{\log(l^2 - m^2) - \log(l\cdot n) + \frac{\log(m^2)}{2}}{\epsilon (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{(2\log(l^2 - m^2) - 2\log(l\cdot n) + \log(m^2))^2}{4(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)$$

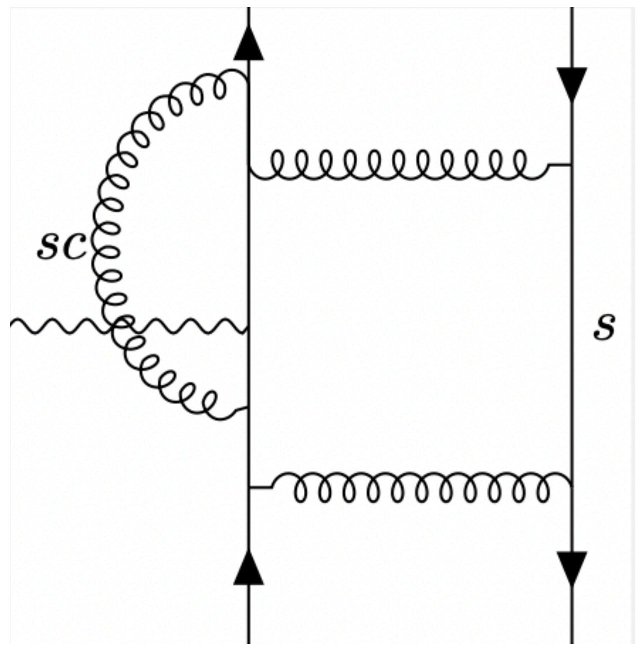
$$c': \quad \frac{\frac{1}{(m^2-l^2)(l\cdot n)(l\cdot \bar{n})\epsilon} + \frac{\log(m^2)}{(l^2-m^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)}{\eta} + \left(\frac{1}{\epsilon^2 (l^2 - m^2)(l\cdot n)(l\cdot \bar{n})} + \frac{\log^2(m^2)}{2(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + O(\epsilon^1)\right) + O(\eta^1)$$

$$s + hc + c + sc + c': \quad \frac{2\log^2(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(m^2)\log(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} - \frac{2\log(l^2 - m^2)\log(l\cdot n)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})} + \frac{2\log(m^2)\log(l^2 - m^2)}{(m^2 - l^2)(l\cdot n)(l\cdot \bar{n})}$$

Pole structure :

Two loop double logarithm

The contribution from the soft-collinear region and the soft-anti-collinear region vanishes in the sum of all diagrams.



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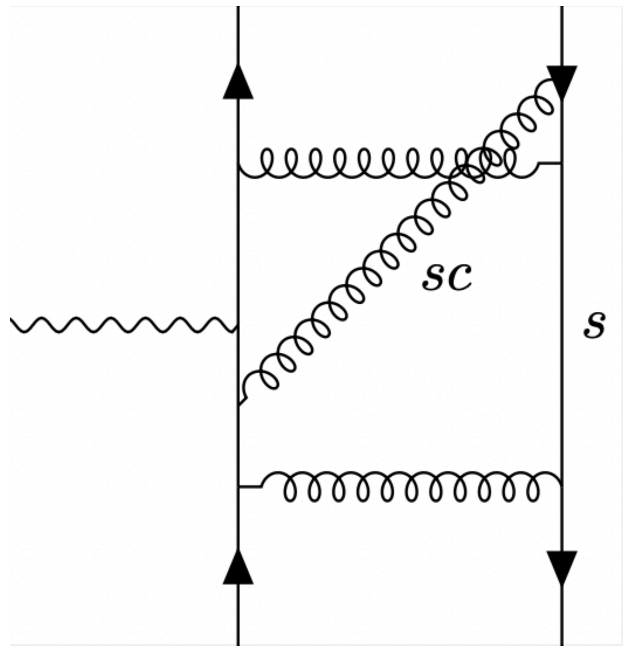

$= 0$

J/ψ **and** η_c **are color neutral**

$$\mathrm{d}_1^{sc} + \mathrm{d}_3^{sc} + \mathrm{d}_6^{sc} + \mathrm{d}_8^{sc} + \mathrm{d}_{10}^{sc} + \mathrm{d}_{12}^{sc} + \mathrm{d}_{14}^{sc} + \mathrm{d}_{16}^{sc} + \mathrm{d}_{18}^{sc} + \mathrm{d}_{20}^{sc} = 0$$

$$\mathrm{d}_5^{s\bar{c}} + \mathrm{d}_6^{s\bar{c}} + \mathrm{d}_7^{s\bar{c}} + \mathrm{d}_8^{s\bar{c}} + \mathrm{d}_{13}^{s\bar{c}} + \mathrm{d}_{14}^{s\bar{c}} + \mathrm{d}_{15}^{s\bar{c}} + \mathrm{d}_{16}^{s\bar{c}} + \mathrm{d}_{22}^{s\bar{c}} + \mathrm{d}_{24}^{s\bar{c}} = 0$$

Two loop double logarithm

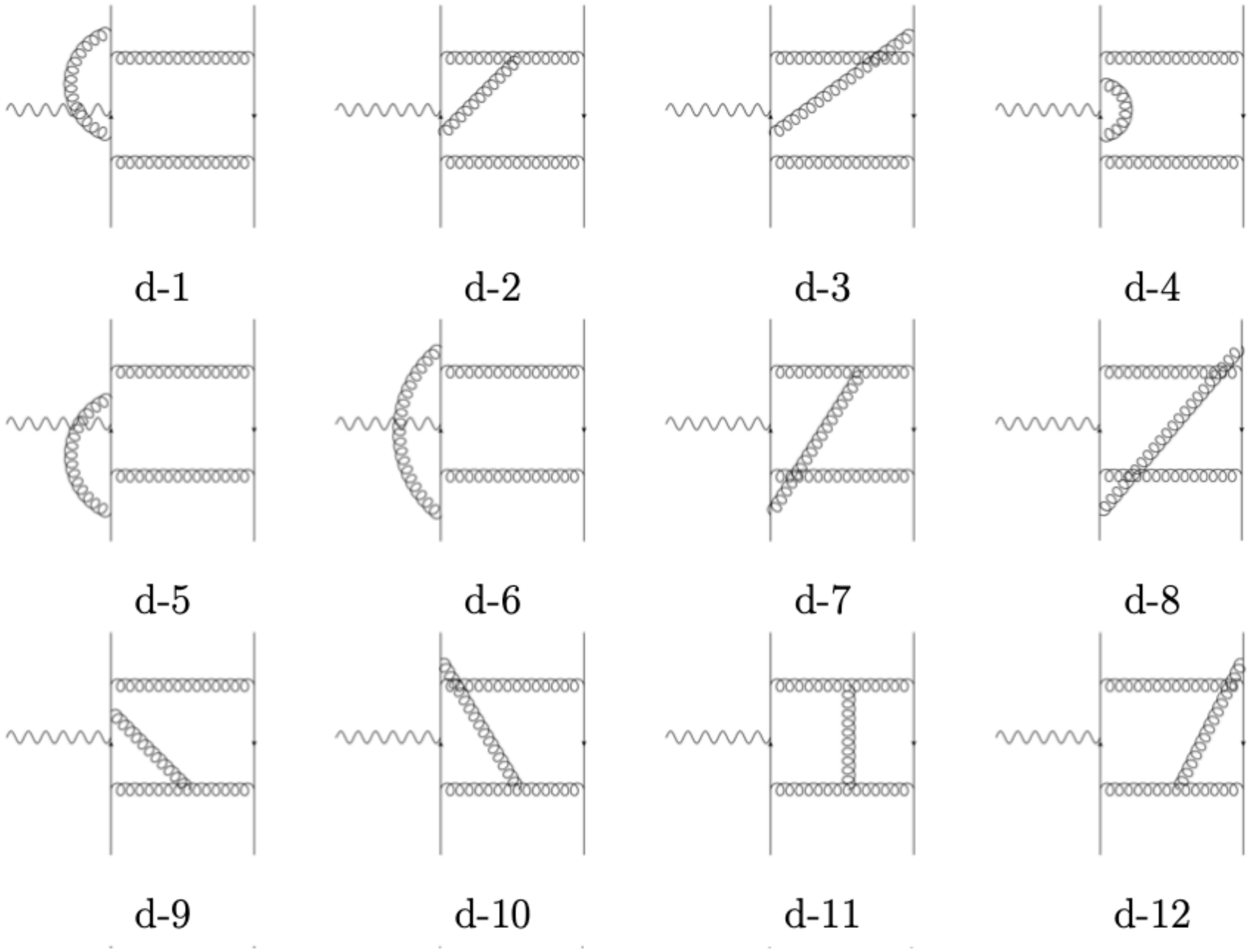

diagram	$\log^4 \frac{m^2}{Q^2} \times i\mathcal{M}_{LO}$
d-1:	$\frac{5\alpha_s^2}{3456\pi^2}$
d-2:	$\frac{-\alpha_s^2}{384\pi^2}$
d-3:	$\frac{-11\alpha_s^2}{1728\pi^2}$
d-4:	$\frac{-\alpha_s^2}{432\pi^2}$
d-5:	$\frac{\alpha_s^2}{864\pi^2}$
d-6 + d-8:	$\frac{\alpha_s^2}{27648\pi^2}$
d-7:	$\frac{7\alpha_s^2}{3072\pi^2}$
d-9:	$\frac{-\alpha_s^2}{384\pi^2}$
d-10:	$\frac{\alpha_s^2}{768\pi^2}$
d-11:	0
d-12:	$\frac{-\alpha_s^2}{1536\pi^2}$

Two loop double logarithm

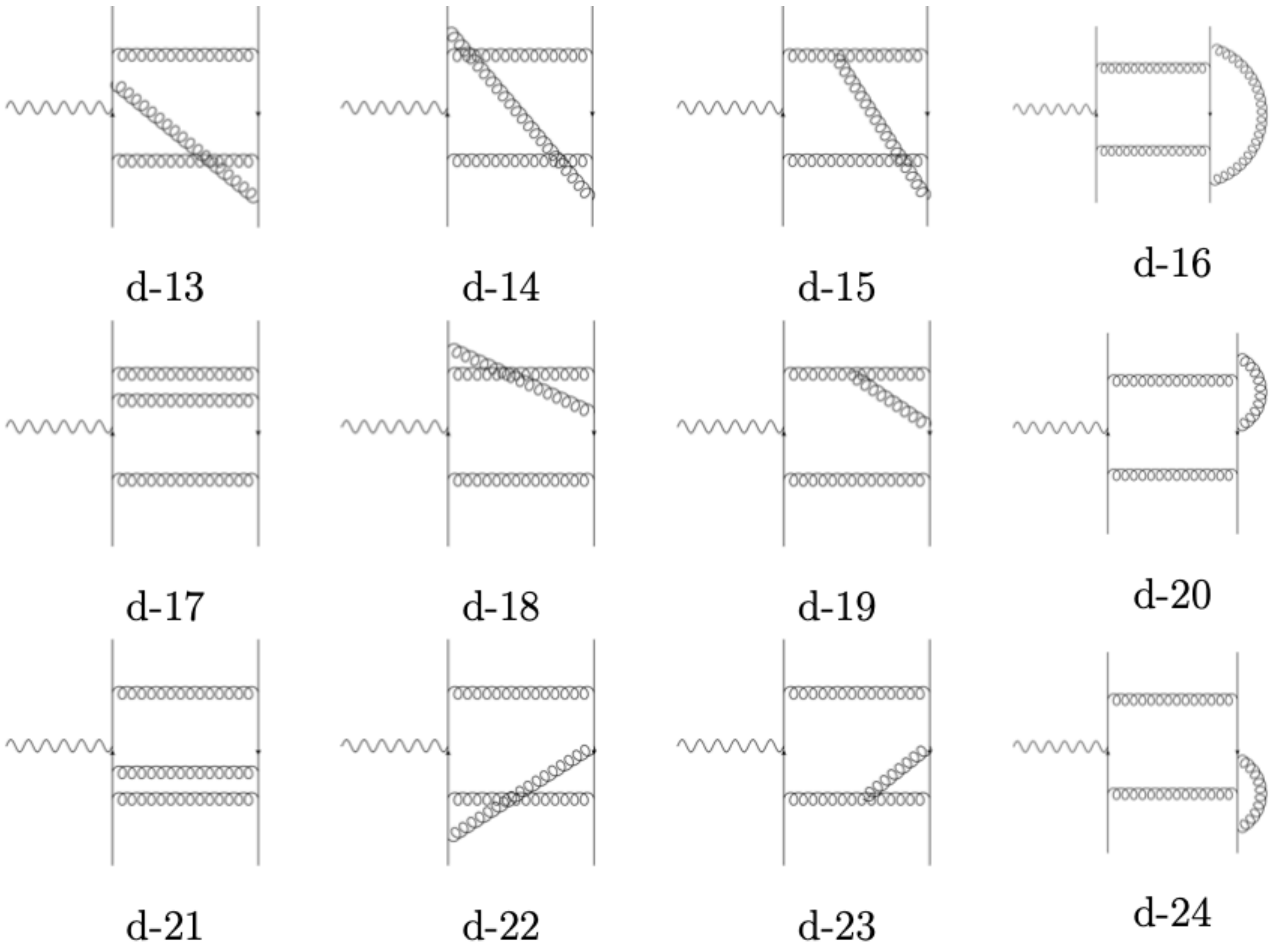

diagram	$\log^4 \frac{m^2}{Q^2} \times i\mathcal{M}_{LO}$
d-13:	$\frac{-\alpha_s^2}{288\pi^2}$
d-14 + d-16:	0
d-15:	$\frac{\alpha_s^2}{1536\pi^2}$
d-17:	$\frac{\alpha_s^2}{216\pi^2}$
d-18 :	$\frac{-\alpha_s^2}{576\pi^2}$
d-19:	0
d-20:	$\frac{\alpha_s^2}{864\pi^2}$
d-21:	0
d-22:	$\frac{-\alpha_s^2}{864\pi^2}$
d-23:	0
d-24:	$\frac{\alpha_s^2}{864\pi^2}$

Conclusion

- Reproduce the endpoint double logarithms at NLO.
- In part of the two-loop diagram, we analyze the regions that contribute to the endpoint double logarithms at leading power.
- The double logarithms coincide with the result extracted numerically using AMFlow diagram by diagram.(See ChuanQi's talk)

Thanks!