Precision Study of Gluon Saturation: Experimental Analysis versus Theoretical Approach

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G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, Phys. Rev. Lett. **108**, 122301 (2012).
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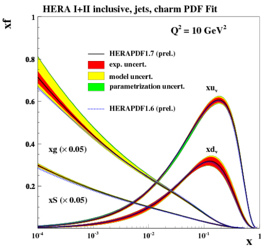
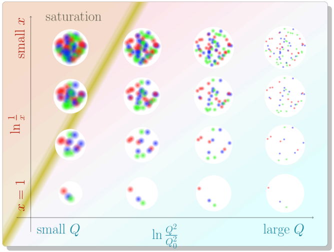
Saturation Physics (Color Glass Condensate)

Describe Emergent Phenomenon of the ultra-dense QCD cold matter.

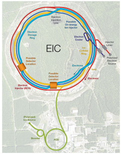


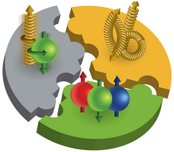
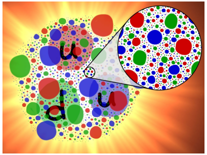


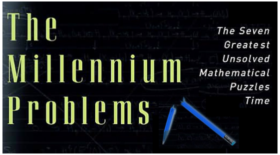
- Gluon density grows rapidly as x gets small. BFKL evolution!
- Resummation of the $\alpha_s \ln \frac{1}{x}$ $\Rightarrow$ BFKL equation. Hard at NLO! (20 years)
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK/JIMWLK) $\Rightarrow$ ultra-dense gluonic matter
- Saturation = Multiple Scattering (MV model) + Small-x (high energy) evolution

Ultimate Questions and Challenges in QCD

To understand our physical world, we have to understand QCD!



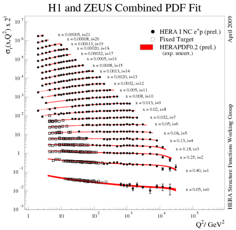


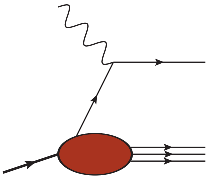
Three pillars of EIC Physics:

- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

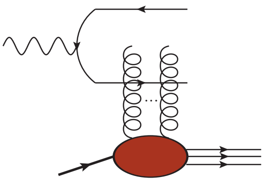
EICs: keys to unlocking these mysteries! Many opportunities will be in front of us!

Dual Descriptions of Deep Inelastic Scattering





Bjorken frame



Dipole frame

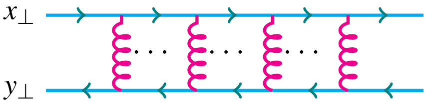
- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

$$F_2(x,Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\rm em}} S_\perp \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$

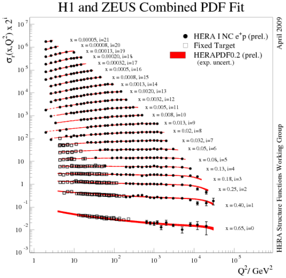


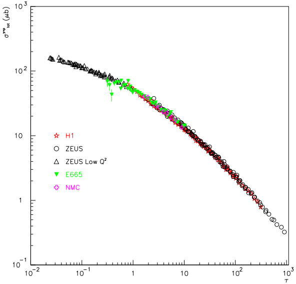
- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c} \left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle.$$

Geometrical Scaling in DIS

[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]





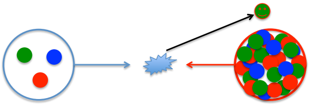
- All data ($x \leq 0.01$, $Q^2 \leq 450 GeV^2$) is function of a single variable $\tau = Q^2/Q_s^2$.
- Define $Q_s^2(x) = (x_0/x)^\lambda \mathrm{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$.

Forward hadron production in pA collisions

Use dilute objects to probe dense targets

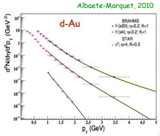
[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

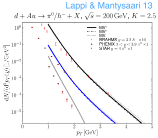
$$\frac{d\sigma_{\text{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}}e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}}e^{-y} \ll 1$ gluon

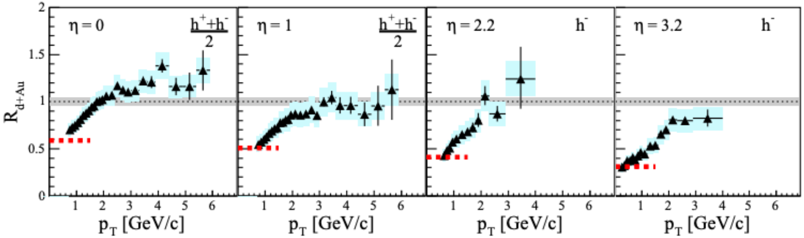




- Proton: Collinear PDFs and FFs (Strongly depends on μ^2).; Nucleus: Small-x gluon!
- Early attempts: [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]
- Full NLO: [Chirilli, BX and Yuan, 12]

d+Au collisions at RHIC

$$\frac{d + Au \to h + X}{p + p \to h + X} \quad \Rightarrow \quad R_{d+Au} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{d^2N_{d+Au}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



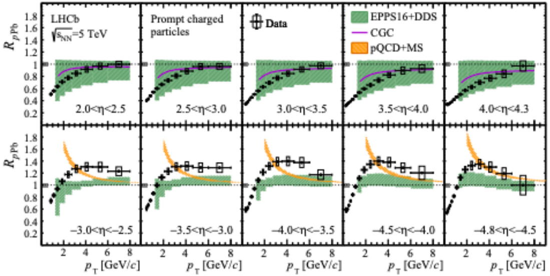
BRAHMS

- Cronin effect at middle rapidity
- Rapidity evolution of the nuclear modification factors R_{d+Au}
- Promising evidence for gluon saturation effects

New LHCb Results

[R. Aaet al. (LHCb Collaboration), Phys. Rev. Lett. 128 (2022) 142004]

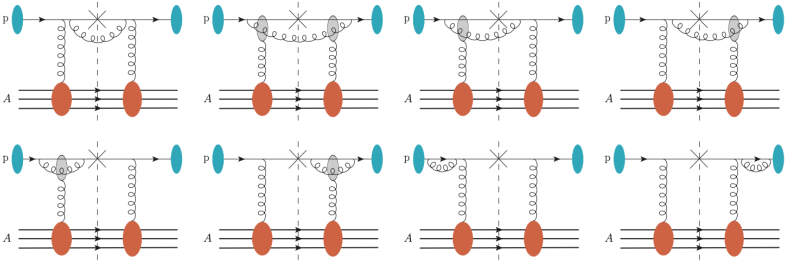
$$R_{pPb} = \frac{1}{\langle N_{\rm coll} \rangle} \frac{d^2N_{p+Pb}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



- Rapidity evolution of the nuclear modification factors R_{pPb} similar to RHIC

NLO diagrams in the $q \to q$ channel

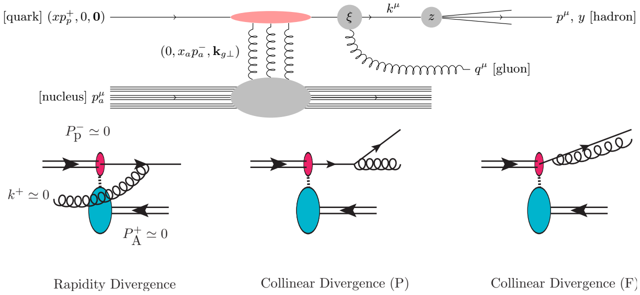
G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, ▸ Phys. Rev. Lett. **108**, 122301 (2012).



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space ⇒Divergences!.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process [Chirilli, BX and Yuan, 12]



- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

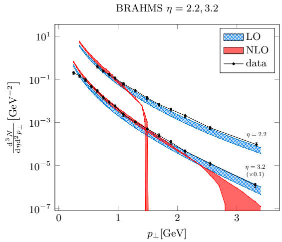
$$d\sigma = \int x f_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int x f_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

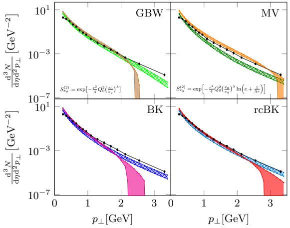
Consistent implementation should include all the NLO α_s corrections.

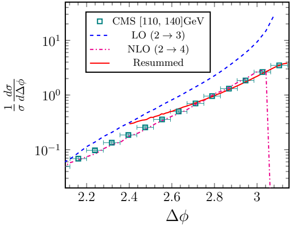
- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. [Chirilli, BX and Yuan, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]







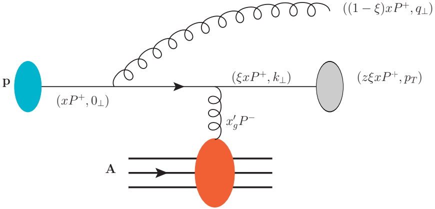
- Reduced factorization scale dependence!
- Catastrophe: Negative NLO cross-sections at high p_T.
- Fixed order calculation in field theories is not guaranteed to be positive.
- Similar example, dijet angular correlation NLO becomes negative $\Delta\phi \sim \pi$.
- Rapidity sub with kinematic constraints. [Watanabe, Xiao, Yuan, Zaslavsky, 15]

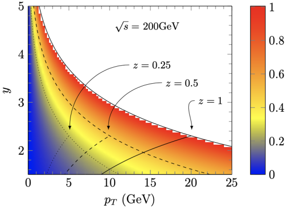
Extending the applicability of CGC calculation

- Goal: find a solution within our current factorization (exactly resum $\alpha_s \ln 1/x_g$) to extend the applicability of CGC. Other scheme choices certainly is possible.
- A lot of logs arise in pQCD loop-calculations: DGLAP, small-x, threshold, Sudakov.
- Breakdown of α_s expansion occurs due to the appearance of logs in certain PS.
- Demonstrate onset of saturation and visualize smooth transition to dilute regime.
- Add'l consideration: numerically challenging due to limited computing resources.
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20;]

Gluon Radiation at the Threshold

Near threshold: radiated gluon has to be soft! $\tau = \frac{p_\perp e^y}{\sqrt{s}}$ density ($\tau = x_p \xi z \leq 1$)





- Gluon momentum: $q^+ = (1-\xi)p_q^+ \to 0$; Clearly there are soft-collinear logs
- Introduce an additional semi-hard scale Λ^2.
- Competition between Q_s and soft gluon emissions.

Threshold Logarithms

Y. Shi, L. Wang, S.Y. Wei, Bo-Wen Xiao, ▸ Phys. Rev. Lett. **128**, 202302 (2022).

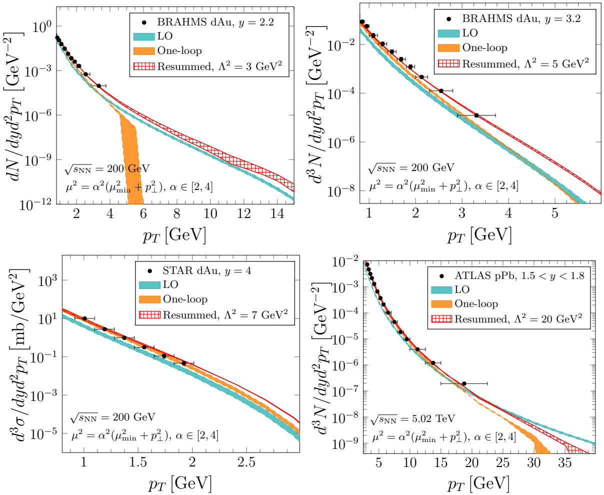
- Numerical integration (8-d in total) is notoriously hard in $r_\perp$ space. Go to $k_\perp$ space.
- In the coordinate space, we can identify two types of logarithms

$$\text{single log: } \ln\frac{k_\perp^2}{\mu_r^2} \to \ln\frac{k_\perp^2}{\Lambda^2}, \quad \ln\frac{\mu^2}{\mu_r^2} \to \ln\frac{\mu^2}{\Lambda^2}; \quad \text{double log: } \ln^2\frac{k_\perp^2}{\mu_r^2} \to \ln^2\frac{k_\perp^2}{\Lambda^2},$$

with $\mu_r \equiv c_0/r_\perp$ with $c_0 = 2e^{-\gamma_E}$.

- Introduce a semi-hard auxiliary scale $\Lambda^2 \sim \mu_r^2 \gg \Lambda_{QCD}^2$. Identify dominant $r_\perp$!
- Dependences on μ^2, Λ^2 cancel order by order. Choose “natural" values at fixed order.
- For running coupling, $\Lambda^2 = \Lambda_{QCD}^2\left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}$. Akin to CSS & Catani *et al*.

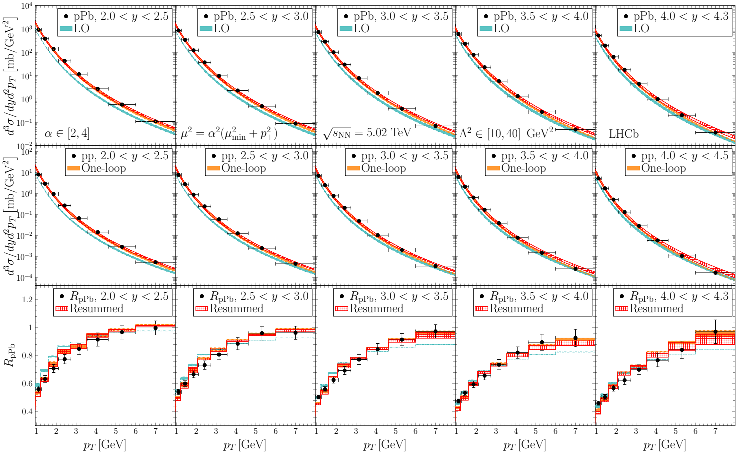
Numerical Results for pA spectra



- $\mu^2 = \alpha^2(\mu_{\min}^2 + p_T^2)$ & $\alpha \in [2, 4]$;
- RHIC: $\Lambda^2 \sim Q_s^2$; LHC, larger Λ^2.
- $\mu \sim Q \geq 2k_\perp$ ($\alpha > 2$) at high p_T. $2 \to 2$ hard scattering.
- Estimate higher order correction by varying μ and Λ.
- Threshold enhancement for σ.
- Nice agreement with data across many orders of magnitudes for different energies and p_T ranges!

Comparison with the new LHCb data

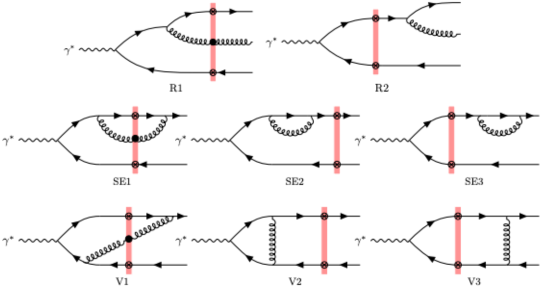
[Shi, Wang, Wei, Xiao, 21] ‣ 2112.06975 [hep-ph] [LHCb: 2108.13115]



- Sudakov/Threshold resummation help stabilize NLO.
- Precision test needs reliable NLO calculation from CGC.
- Proof of concept for NLO predictive power.
- Agreement with forward LHCb data.
- Rapidity evolution.

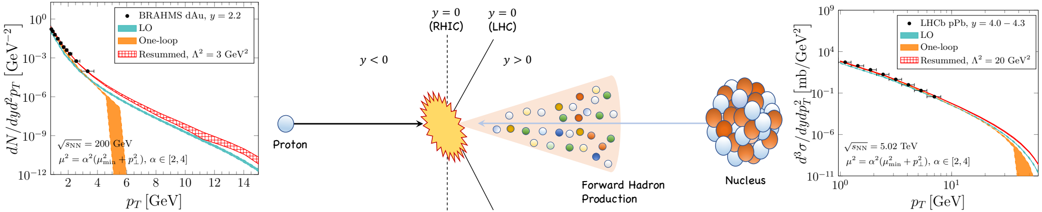
NLO DIS dijets

[Caucal, Salazar, Schenke, Stebel and Venugopalan, 2304.03304]
see also [Taels, Altinoluk, Beuf and Marquet, 2204.11650]



- Proved factorization at one-loop.
- Resummation of small-x and Sudakov logarithms.
- Provide more reliable predictions for measurements at future EIC.

Summary



- Ten-Year Odyssey in NLO hadron productions in pA collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC.
- Next Goal:Global analysis for CGC combining data from pA and DIS.
- Exciting time of NLO CGC phenomenology with the upcoming EIC.

Threshold resummation in the CGC formalism

Threshold logarithms: Sudakov soft gluon part and Collinear (plus-distribution) part.

- Soft single and double logs ($\ln k_\perp^2/\Lambda^2$, $\ln^2 k_\perp^2/\Lambda^2$) are resummed via Sudakov factor. Performing Fouier transformations

$$\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \ln \frac{\mu^2}{\mu_r^2} e^{-ik_\perp \cdot r_\perp} = -\int \frac{d^2 l_\perp}{\pi l_\perp^2} \left[F(k_\perp + l_\perp) - J_0(\frac{c_0}{\mu} l_\perp) F(k_\perp) \right]$$

$$= \quad -\frac{1}{\pi} \int \frac{d^2 l_\perp}{(l_\perp - k_\perp)^2} \left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2 + (l_\perp - k_\perp)^2} F(k_\perp) \right] + F(k_\perp) \ln \frac{\mu^2}{\Lambda^2}.$$

- Two equivalent methods to resum the collinear part ($P_{ab}(\xi) \ln \Lambda^2/\mu^2$): 1. Reverse DGLAP evolution; 2. RGE method (threshold limit $\xi \to 1$).
- Introduce forward threshold quark jet function $\Delta^q(\Lambda^2, \mu^2, \omega)$, which satisfies

$$\frac{d\Delta^q(\omega)}{d \ln \mu^2} = -\frac{d\Delta^q(\omega)}{d \ln \Lambda^2} = -\frac{\alpha_s C_F}{\pi} \left[\ln \omega + \frac{3}{4} \right] \Delta^q(\omega) + \frac{\alpha_s C_F}{\pi} \int_0^\omega d\omega' \frac{\Delta^q(\omega) - \Delta^q(\omega')}{\omega - \omega'}$$

- Consistent with the threshold resummation in SCET[Becher, Neubert, 06]!