Precision Predictions for Top-quark Width

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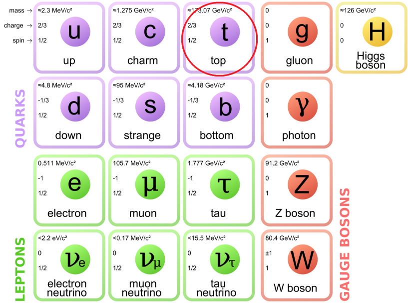
Motivation

Top-quark is the heaviest elementary particle in the Standard Model.

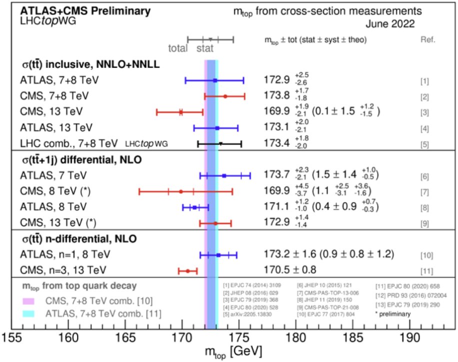
Top-quark provides the strongest coupling to the SM Higgs boson and opens doors to new physics.



Motivation

Top-quark mass is the one of the fundamental parameters in Standard Model.

Summary of the top-mass analyses at the LHC.

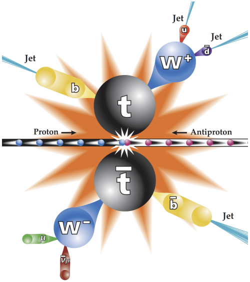


Motivation

Top decay width Γ_t is one of fundamental properties of top-quark.

Due to its large mass, Γ_t is expected to be very large.

The measurement of Γ_t could hint at new-physics.



[Denisov, Vellidis 2015]

Motivation

The top-quark decays almost exclusively to Wb. $\Gamma_t = \Gamma_t(t \to Wb)$.

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

[PDG 2022]

At LHC, the direct measurement is model independent but less precise, $\Gamma_t = 1.9 \pm 0.5$ GeV by ATLAS [ATLAS 2019].

The indirect measurement is model dependent but more precise, $\Gamma_t = 1.36 \pm 0.02\ ({\rm stat.})^{+0.14}_{-0.11}\ ({\rm syst.})$ GeV by CMS [CMS, 2014], which is the most precise measurement for Γ_t by now.

In the future e^+e^- collider, Γ_t can be measured with an uncertainty of 30 MeV [Martinez, Miquel 2019].

Motivation

On the theoretical side,

NLO QCD corrections [Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991]

NLO EW corrections [Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991]

Asymptotic analytic results of NNLO QCD corrections using $m_W \to 0$ and $m_W \to m_t$ [Czarnecki, Melnikov 1999, Chetyrkin, Harlander, Seidensticker, Steinhauser 1999, Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005]

Numerical result of full NNLO QCD corrections [Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013]

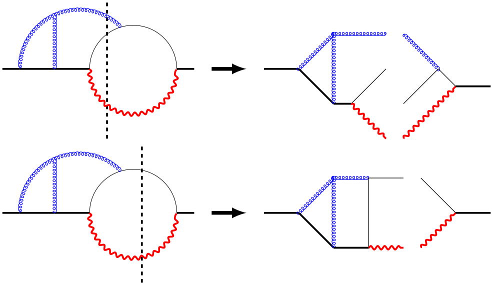
Full analytic results of NNLO QCD corrections [Chen, Li, Wang, Wang 2022]

Optical Theorem

Consider the three-loop self-energy diagrams Σ for $t \to Wb \to t$

$$\Gamma_t = \frac{\text{Im}(\Sigma)}{m_t} \quad (1)$$

The imaginary part comes from cut diagrams. For example,



The complicated phase space integration can be avoided.

Scalar Integrals

For $t \to Wb \to t$, b quark is massless.

After spin summation

$$u(k, m_t)\bar{u}(k, m_t) = \not{k} + m_t \tag{2}$$

the amplitudes can be written as the linear combination of scalar integrals. Then the IBP reduction can be used.

Master Integrals Calculations

Method: canonical differential equations – see L.L.Yang's and L.B.Chen's talks.

Analytic calculations of three loop master integrals are non-trivial.

Two important ingredients: – see L.L.Yang's and L.B.Chen's talks.

1. Construct canonical form (ϵ form, $d\log$ form)

2. Boundary conditions – `AMFlow` [Liu, Ma 2022] and PSLQ method [Ferguson, Beiley, Arno 1992 1999]

Results: harmonic polylogarithms (HPLs), multiple polylogarithms (GPLs)

Analytic Results

Combining analytic results of master integrals and IBP relations, the bare amplitudes are obtained.

After renomarization, QCD corrections of Γ_t up to NNLO.

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right], \tag{3}$$

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}. \tag{4}$$

The LO and NLO corrections are

$$X_0 = (2w+1)(w-1)^2,$$

$$X_1 = C_F \Bigg(X_0 \Big(-2H_{0,1}(w) + H_0(w)H_1(w) - \frac{\pi^2}{3} \Big) + \frac{1}{2}(4w+5)(w-1)^2 H_1(w)$$
$$+ w(2w^2+w-1)H_0(w) + \frac{1}{4}(6w^3-15w^2+4w+5) \Bigg) \tag{5}$$

Analytic Results

According to color structure,

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2\right], \tag{6}$$

$$X_2 = C_F(T_R n_l X_l + T_R n_h X_h + C_F X_F + C_A X_A) \tag{7}$$

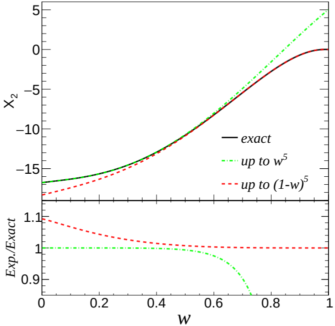
$$\begin{aligned}
X_l &= -\frac{X_0}{3}\left[H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3)\right] + g_l(w), \\
X_F &= \frac{1}{12} X_0 [-6\left(2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3)H_1(w)\right) - \pi^2 H_{1,0}(w)] \\
&+ (X_0 + 4w)\left(-\frac{1}{6}\pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w)\right) \\
&+ \frac{1}{12}\left(18w^3 - 3w^2 + 76w + 15\right)\pi^2 H_{0,1}(w) - \frac{1}{2}\left(4w^3 - 2w^2 + 4w + 3\right) H_{0,0,0,1}(w) \\
&+ \frac{1}{2}\left(4w^3 - 2w^2 + 16w + 3\right) H_{0,0,1,0}(w) + w\left(2w^2 - 7w - 16\right) H_{0,0,1,1}(w) \\
&- \frac{1}{2}\left(2w^3 - 11w^2 - 28w - 1\right) H_{0,1,1,0}(w) + \frac{1}{720}\pi^4\left(42w^3 - 191w^2 - 328w - 11\right) + g_F(w).
\end{aligned}$$

Cross Check

Master integrals are confirmed by numerical check with `AMFlow`.

Two different gauges of W boson have been used to cross check.

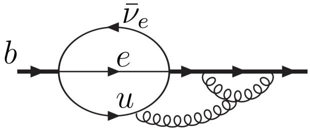
The result expanded in $w=0$ and $w=1$ $(w=m_W^2/m_t^2)$ coincides with [Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005].



Relations With Other Process

Our results can be taken as the invariant mass spectrum in semileptonic $b \to uW^*$

Integrating over w $(w = m_W^2/m_t^2)$ from 0 to 1, reproduce NNLO QCD corrections in semileptonic decay $\Gamma(b \to X_u e \bar{\nu}_e)$ [Ritbergen 1999].



Off-Shell W Boson

Including the W boson width $\Gamma_W = 2.085$ GeV, the Γ_t becomes [Jezabek, Kuhn 1989]

$$\tilde{\Gamma}_t \equiv \Gamma(t \to W^* b) = \frac{1}{\pi} \int_0^{m_t^2} dq^2 \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \Gamma_t(q^2/m_t^2), \quad (8)$$

In the narrow width limit, $\Gamma_W \to 0$, $\tilde{\Gamma}_t \to \Gamma_t$.

$$\tilde{\Gamma}_t = \Gamma_0 \left[\tilde{X}_0 + \frac{\alpha_s}{\pi} \tilde{X}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \tilde{X}_2 \right], \quad r = \frac{\Gamma_W}{m_W}, \quad w = \frac{m_W^2}{m_t^2}$$

$$\tilde{X}_0 = \frac{1}{2\pi} \big(-(2(r-i)w - i((r-i)w+i)^2 G(w+irw,1)) - ((r+i)w-i)^2 2(r+i)w + iG(w-irw,1) - 4r(1-2w)w \big), \quad (9)$$

$$\tilde{X}_1 = \frac{1}{18\pi} \big(((r+i)w-i)(2(4\pi^2-9)(r+i)^2 w^2 + (4\pi^2-27)(1-ir)w + 4\pi^2 - 15) G(w-iw,1) + (r-i)w-i)(2(4\pi^2-9)(r-i)^2 w^2 + (4\pi^2-27)(1+ir)w + 4\pi^2 - 15) G(w+iw,1) + \cdots \big) \quad (10)$$

Numerical Results

Input parameters from [P.D.G 2022]

$$
\begin{aligned}
m_t &= 172.69 \text{ GeV}, \quad m_b = 4.78 \text{ GeV}, \\
m_W &= 80.377 \text{ GeV}, \quad \Gamma_W = 2.085 \text{ GeV}, \\
m_Z &= 91.1876 \text{ GeV}, \quad G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}, \\
|V_{tb}| &= 1, \quad \alpha_s(m_Z) = 0.1179.
\end{aligned} \tag{11}
$$

$\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

$$
\begin{aligned}
\Gamma_t = \Gamma_t^{(0)}[&(1 + \delta_b^{(0)} + \delta_W^{(0)}) \\
&+ (\delta_b^{(1)} + \delta_W^{(1)} + \delta_{\text{EW}}^{(1)} + \delta_{\text{QCD}}^{(1)}) \\
&+ (\delta_b^{(2)} + \delta_W^{(2)} + \delta_{\text{EW}}^{(2)} + \delta_{\text{QCD}}^{(2)} + \delta_{\text{EW}\times\text{QCD}}^{(2)})]
\end{aligned} \tag{12}
$$

Numerical Results

Corrections in percentage (%) normalized by the LO width $\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\rm EW}^{(i)}$	$\delta_{\rm QCD}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	1.361
NNLO	$*$	0.030	$*$	-2.070	1.331

QCD corrections are dominant.

NLO EW correction is 1.683%.

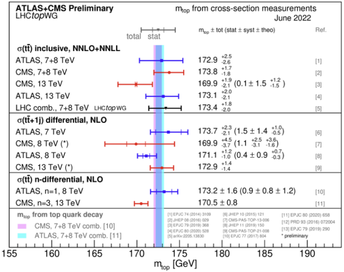
The off-shell W boson effect at NNLO is further suppressed.

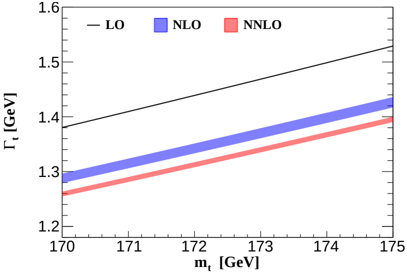
The b quark mass correction at NLO is not severely suppressed compared to the LO due to the large logarithms.

Top-quark Mass

Top-quark mass varies from 170 GeV to 175 GeV.

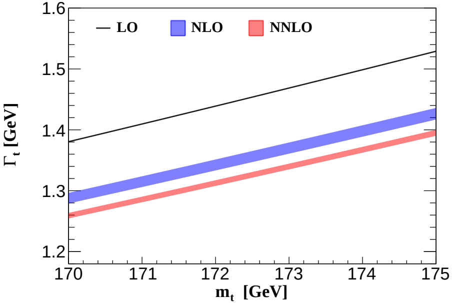
The width changes from 1.258 GeV to 1.394 GeV.





Theoretical Uncertainties

QCD renormalization scale $\mu \in [m_t/2, 2m_t]$, the variation is about $\pm 0.8\%$ and $\pm 0.4\%$ at NLO and NNLO.



$\overline{\text{MS}}$ scheme differs from on-shell scheme -3.79% and 0.09% at NLO and NNLO.

Missing NNNLO QCD contribution would be of the order of 0.4%.

Theoretical Uncertainties

The uncertainties at NNLO from $\alpha_s(m_Z) = 0.1179 \pm 0.0009$ and $m_W = 80.377 \pm 0.012$ GeV [P.D.G 2022] are 0.1% and 0.01%.

The deviation between the α and G_F scheme in the EW correction is 0.1% at NLO.

The missing NNLO EW as well as the mixed $\mathrm{EW} \times \mathrm{QCD}$ corrections.

Considering all the possible uncertainties, the uncertainty at NNLO is less than 1%.

Mathematica program TopWidth

Mathematica program TopWidth can be downloaded from

https://github.com/haitaoli1/TopWidth. The package HPL is required [Maitre 2006].

```
<< TopWidth`

        (****** TopWidth-1.0 ******)
    Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang
    TopWidth[QCDorder, mbCorr, WwidthCorr, EWcorr, mu] is provided for top width calculations
    Please cite the paper for reference: arXiv:2212.06341

*-*-*-*-*-* HPL 2.0 *-*-*-*-*-*

Author: Daniel Maitre, University of Zurich
Rules for minimal set loaded for weights: 2, 3, 4, 5, 6.
Rules for minimal set for + - weights loaded for weights: 2, 3, 4, 5, 6.
Table of MZVs loaded up to weight 6
Table of values at I loaded up to weight 6
$HPLFunctions gives a list of the functions of the package.
$HPLOptions gives a list of the options of the package.
More info in hep-ph/0507152, hep-ph/0703052 and at
 http://krone.physik.unizh.ch/~maitreda/HPL/

(* SetParameters[mt, mb, mw, Wwidth, mz, GF] *)
(* If the parameters are not set by the users the code will use the default ones *)
SetParameters[17269/100, 478/100, 80377/1000, 2085/1000, 911876/10000, 11663788 × 10^-12]

(* NNLO decay width *)
TopWidth[2, 1 (* with  mb effects *), 1 (* with  Γw effects*), 1 (* with NLO EW effects *), 17269/100]

1.33051
```

Summary and Outlook

We provide the first full analytic result of top-quark width at NNLO in QCD, which can be used to perform both fast and accurate evaluations.

The off-shell W boson contribution is calculated analytically up to NNLO in QCD.

The most precise top-quark width is predicted to be 1.331 GeV for mt = 172.69 GeV with the total theoretical uncertainty less than 1%.

The next target is NNNLO QCD corrections for top-quark width.