Bootstrapping One-Loop Inflation Correlators with the Spectral Decomposition

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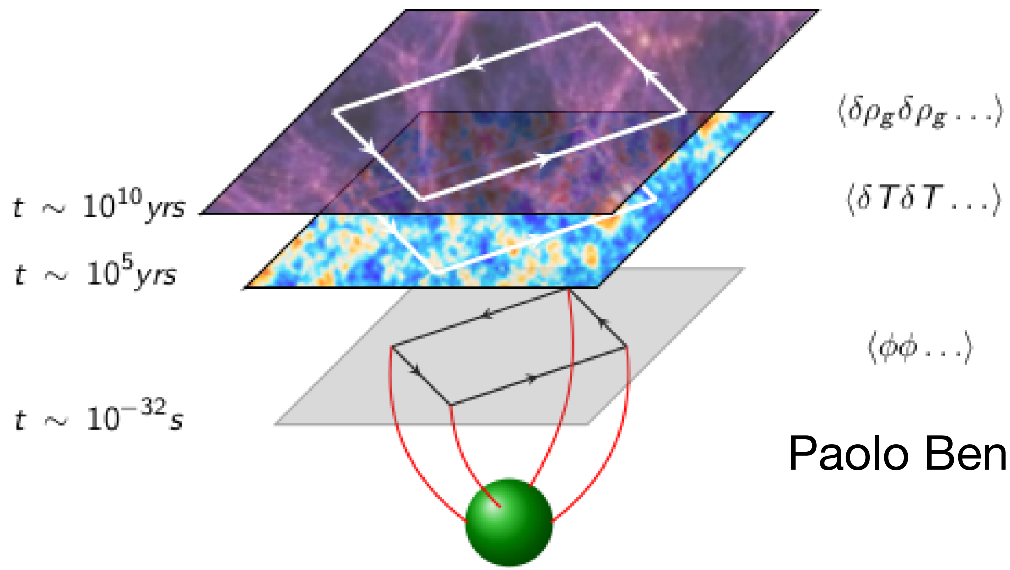
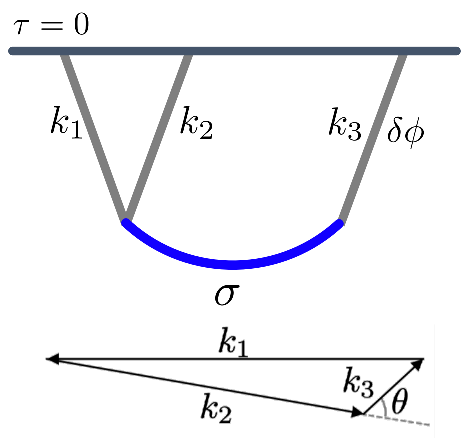
Based on JHEP 04 (2023) 103 [arXiv: 2211.03810] with Zhong-Zhi Xianyu

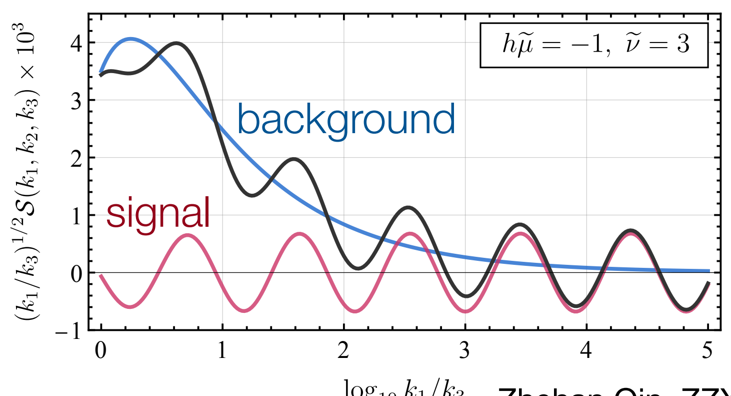
Cosmological collider physics: Make use of the high energies of the early universe to study fundamental particle physics at high scale



Paolo Benincasa, 2203.15220

The late-time oscillations of massive modes show up in the squeezed inflaton correlators





Zhehan Qin, ZZX, 2208.13790

Why One-Loop?

Leading cosmological collider signal appears at the 1-loop level rather than the tree level in many particle models of cosmological collider physics

The signal-generating states have to be produced and annihilated in pairs (Charged particles, spinning particles)

Most of the Standard Model states in the symmetric phase

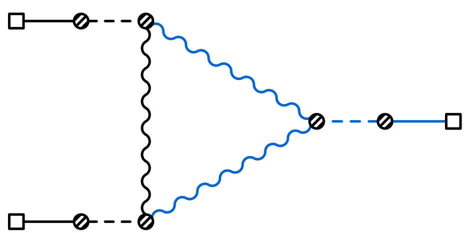
New physics states such as heavy neutrinos, Kaluza-Klein states

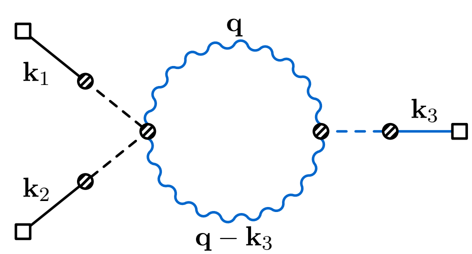
Chen, Wang, ZX: 1610.06597, 1612.08122, 1805.02656; Lu, Wang, ZX, 1907.07390; Hook, Huang, Racco, 1907.10624, 1908.00019; Kumar and Sundrum, 1811.11200

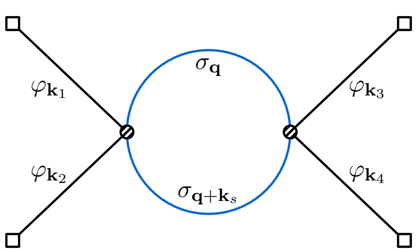
1-loop processes are more enhanced relative to the tree-level process

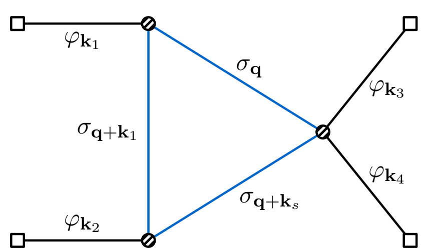
Chemical-potential-enhanced signals in the 3-point functions

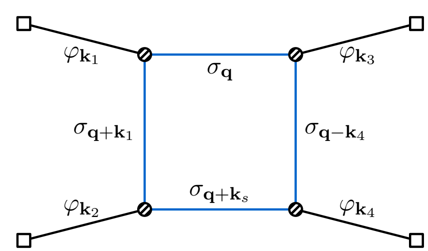
Wang, ZX, 1910.12876, 2004.02887; Tong, ZX, 2203.06349



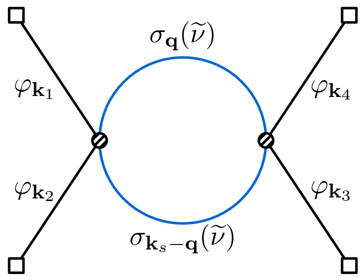








Schwinger-Keldysh Integral



External line: massless scalar (inflaton)
Internal line: scalar of mass $m > dH/2$

Loop seed integral:

$$\mathcal{J}_{\widetilde{\nu}}^{p_1 p_2}(r_1, r_2) \equiv -\frac{1}{2}\sum_{\mathsf{a},\mathsf{b}=\pm} \mathsf{ab}\, k_s^{d+2+p_{12}} \int_{-\infty}^{0} \mathrm{d}\tau_1 \mathrm{d}\tau_2 (-\tau_1)^{p_1}(-\tau_2)^{p_2} e^{\mathrm{i}\mathsf{a}k_{12}\tau_1 + \mathrm{i}\mathsf{b}k_{34}\tau_2}\, \mathcal{Q}_{\widetilde{\nu},\mathsf{ab}}\big(k_s;\tau_1,\tau_2\big)$$

$$\mathcal{Q}_{\widetilde{\nu},\mathsf{ab}}\big(k_s;\tau_1,\tau_2\big) \equiv \int \frac{\mathrm{d}^d\mathbf{q}}{(2\pi)^d} D_{\widetilde{\nu},\mathsf{ab}}\Big(q;\tau_1,\tau_2\Big) D_{\widetilde{\nu},\mathsf{ab}}\Big(|\mathbf{k}_s-\mathbf{q}|;\tau_1,\tau_2\Big)$$

$$r_1 = \frac{k_s}{k_1+k_2}$$

$$r_2 = \frac{k_s}{k_3+k_4}$$

Massive scalar propagator:

$$D_{\pm\pm}(k;\tau_1,\tau_2) = D_{\gtrless}(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2) + D_{\lessgtr}(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1),$$

$$D_{\pm\mp}(k;\tau_1,\tau_2) = D_{\lessgtr}(k;\tau_1,\tau_2)$$

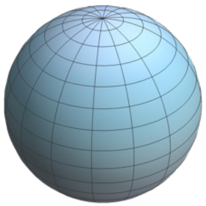
$$D_{<}(k;\tau_1,\tau_2) \equiv D_{>}^{*}(k;\tau_1,\tau_2)$$

$$D_{>}(k;\tau_1,\tau_2) = \frac{\pi}{4} e^{-\pi\widetilde{\nu}} H^2 (\tau_1\tau_2)^{3/2} \mathrm{H}_{\mathrm{i}\widetilde{\nu}}^{(1)}(-k\tau_1)\mathrm{H}_{-\mathrm{i}\widetilde{\nu}}^{(2)}(-k\tau_2)$$

$$\widetilde{\nu} \equiv \sqrt{m^2/H^2 - d^2/4}$$

Lack of symmetries
Build-in time ordering
Complicated mode functions

Spectral decomposition



$$S^{d+1}$$

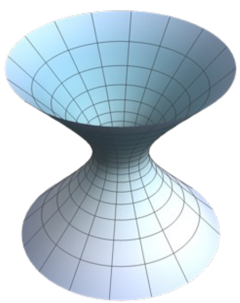
$$D_{\widetilde{\nu}}(x,y) = \sum_{\vec{L}} \frac{1}{(L+d/2)^2+\widetilde{\nu}^2} Y_{\vec{L}}(x) Y^*_{\vec{L}}(y)$$

Bubble function

$$D^2_{\widetilde{\nu}}(x,y) = \sum_{\vec{L}} B_{\widetilde{\nu}}(L) Y_{\vec{L}}(x) Y^*_{\vec{L}}(y)$$

Wick Rotation

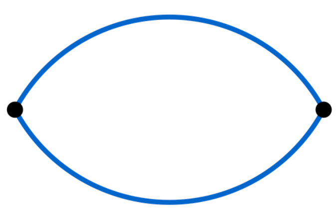
Analytic continuation



$$\mathrm{dS}_{d+1}$$

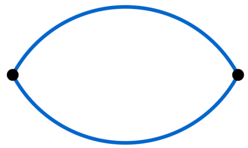
$$D^2_{\widetilde{\nu}}(x,y) = \int_{-\infty}^{\infty} \mathrm{d}\widetilde{\nu}' \frac{\widetilde{\nu}'}{\pi \mathrm{i}} \rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') D_{\widetilde{\nu}}(x,y)$$

Spectral function



$$= \int \mathrm{d}\widetilde{\nu}' \, \frac{\widetilde{\nu}'}{\pi \mathrm{i}} \rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') \times$$

Spectral decomposition

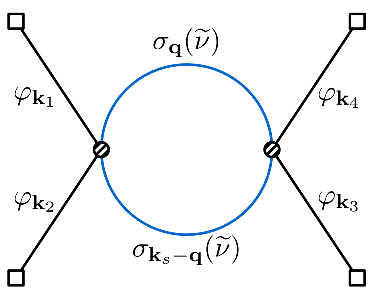


$$= \int \mathrm{d}\widetilde{\nu}' \, \frac{\widetilde{\nu}'}{\pi \mathrm{i}} \rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') \times$$

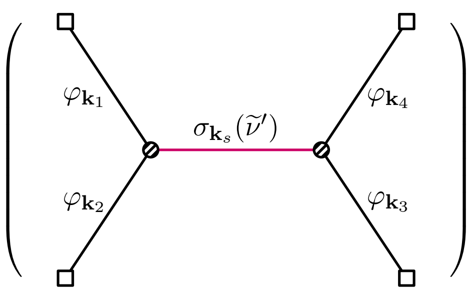
$$\mathcal{Q}_{\widetilde{\nu},\mathsf{ab}}\big(k_s;\tau_1,\tau_2\big) = \int_{-\infty}^{+\infty} \mathrm{d}\widetilde{\nu}' \, \frac{\widetilde{\nu}'}{\pi \mathrm{i}} \rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') D_{\widetilde{\nu}',\mathsf{ab}}\big(k_s;\tau_1,\tau_2\big)$$

Loop seed integral → Integral of tree seed integral $\mathcal{I}^{p_1p_2}_{\widetilde{\nu}}(r_1,r_2)$

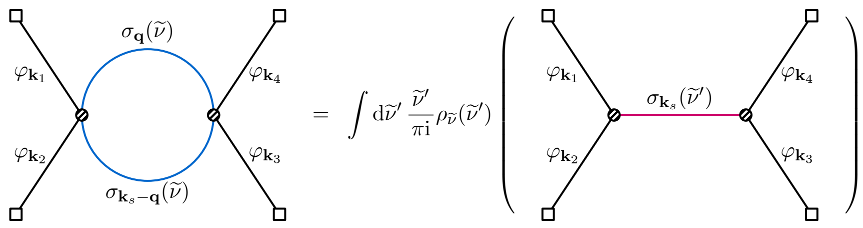
$$\mathcal{J}^{p_1p_2}_{\widetilde{\nu}}(r_1,r_2) = \int_{-\infty}^{+\infty} \mathrm{d}\widetilde{\nu}' \, \frac{\widetilde{\nu}'}{2\pi \mathrm{i}} \rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') \mathcal{I}^{p_1p_2}_{\widetilde{\nu}'}(r_1,r_2)$$



$$= \int \mathrm{d}\widetilde{\nu}' \, \frac{\widetilde{\nu}'}{\pi \mathrm{i}} \rho_{\widetilde{\nu}}(\widetilde{\nu}')$$



Spectral decomposition



$$\mathcal{J}^{p_1p_2}_{\widetilde\nu}(r_1,r_2)=\int_{-\infty}^{+\infty}\mathrm{d}\widetilde\nu'\,\frac{\widetilde\nu'}{2\pi\mathrm{i}}\rho^{\mathrm{dS}}_{\widetilde\nu}(\widetilde\nu')\mathcal{I}^{p_1p_2}_{\widetilde\nu'}(r_1,r_2)$$

$$r_1=\frac{k_s}{k_1+k_2}\qquad r_2=\frac{k_s}{k_3+k_4}$$

Tree seed integral: Zhehan Qin, ZX, 2205.01692, 2208.13790

$$\mathcal{I}^{p_1p_2}_{\widetilde\nu}(r_1,r_2)=\mathcal{I}^{p_1p_2}_{\mathrm{NL},\widetilde\nu}(r_1,r_2)+\mathcal{I}^{p_1p_2}_{\mathrm{L},\widetilde\nu}(r_1,r_2)+\mathcal{I}^{p_1p_2}_{\mathrm{BG},\widetilde\nu}(r_1,r_2)$$

$$\mathcal{I}^{p_1p_2}_{\mathrm{NL},\widetilde\nu}(r_1,r_2)=\mathcal{C}^{p_1p_2}_{\mathrm{i}\widetilde\nu,d}\mathbf{F}^{p_1}_{\mathrm{i}\widetilde\nu,d}(r_1)\mathbf{F}^{p_2}_{\mathrm{i}\widetilde\nu,d}(r_2)(r_1r_2)^{+\mathrm{i}\widetilde\nu}+\mathrm{c.c.}\qquad \mathcal{I}^{p_1p_2}_{\mathrm{L},\widetilde\nu}(r_1,r_2)=-\mathcal{C}^{p_1p_2}_{\mathrm{i}\widetilde\nu,d}\mathbf{F}^{p_1}_{\mathrm{i}\widetilde\nu,d}(r_1)\mathbf{F}^{p_2}_{-\mathrm{i}\widetilde\nu,d}(r_2)\left(\frac{r_1}{r_2}\right)^{+\mathrm{i}\widetilde\nu}+\mathrm{c.c.}$$

$$\mathcal{I}^{p_1p_2}_{\mathrm{BG},\widetilde\nu}(r_1,r_2)=\sum_{\ell,m=0}^{\infty}\frac{(-1)^{\ell+1}\sin[\frac{\pi}{2}(p_{12}+d)](\ell+1)_{2m+d+p_{12}+1}}{2^{2m+1}\left(\frac{\ell-\mathrm{i}\widetilde\nu+p_2+1}{2}+\frac{d}{4}\right)_{m+1}\left(\frac{\ell+\mathrm{i}\widetilde\nu+p_2+1}{2}+\frac{d}{4}\right)_{m+1}}r_1^{2m+d+p_{12}+2}\left(\frac{r_1}{r_2}\right)^{\ell}$$

$$\mathcal{C}^{p_1p_2}_{\mathrm{i}\widetilde\nu,d}\equiv\frac{1}{8}\csc^2(\pi\mathrm{i}\widetilde\nu)\left\{\cos\frac{\pi\bar p_{12}}{2}+\cos\left[\pi\left(\mathrm{i}\widetilde\nu+\frac{p_{12}+d}{2}\right)\right]\right\}\qquad \mathbf{F}^{p}_{\mathrm{i}\widetilde\nu,d}(r)\equiv(2r)^{p+d/2+1}\times{}_2\mathcal{F}_1\left[\begin{matrix}\frac{d}{4}+\frac{1}{2}+\frac{p}{2}+\frac{\mathrm{i}\widetilde\nu}{2},\frac{d}{4}+1+\frac{p}{2}+\frac{\mathrm{i}\widetilde\nu}{2}\\1+\mathrm{i}\widetilde\nu\end{matrix}\middle|r^2\right]$$

Spectral function: Marolf, Morrison, 1006.0035

UV divergence; dimensional regularization

$$\rho^{\mathrm{dS}}_{\widetilde\nu}(\widetilde\nu')=\frac{1}{(4\pi)^{(d+1)/2}}\frac{\cos[\pi(\frac{d}{2}-\mathrm{i}\widetilde\nu)]}{\sin(-\pi\mathrm{i}\widetilde\nu)}\Gamma\left[\begin{matrix}\frac{3-d}{2},\frac{d}{2}-\mathrm{i}\widetilde\nu\\\frac{2-d}{2}-\mathrm{i}\widetilde\nu\end{matrix}\right]$$
$$\times{}_7\mathcal{F}_6\left[\begin{matrix}\frac{2-d}{2}+\mathrm{i}\widetilde\nu'-\mathrm{i}\widetilde\nu,\frac{3-d/2+\mathrm{i}\widetilde\nu'-\mathrm{i}\widetilde\nu}{2},\frac{2-d}{2},\frac{2-d}{2}-\mathrm{i}\widetilde\nu,\frac{2-d}{2}+\mathrm{i}\widetilde\nu',\frac{\mathrm{i}\widetilde\nu'-2\mathrm{i}\widetilde\nu+d/2}{2},\frac{\mathrm{i}\widetilde\nu'+d/2}{2}\\\frac{1-d/2+\mathrm{i}\widetilde\nu'-\mathrm{i}\widetilde\nu}{2},1+\mathrm{i}\widetilde\nu'-\mathrm{i}\widetilde\nu,1+\mathrm{i}\widetilde\nu',1-\mathrm{i}\widetilde\nu,\frac{4+\mathrm{i}\widetilde\nu'-3d/2}{2},\frac{4+\mathrm{i}\widetilde\nu'-2\mathrm{i}\widetilde\nu-3d/2}{2}\end{matrix}\middle|1\right]$$
$$+(\widetilde\nu\to-\widetilde\nu)$$

Spectral Integral by Residue Theorem

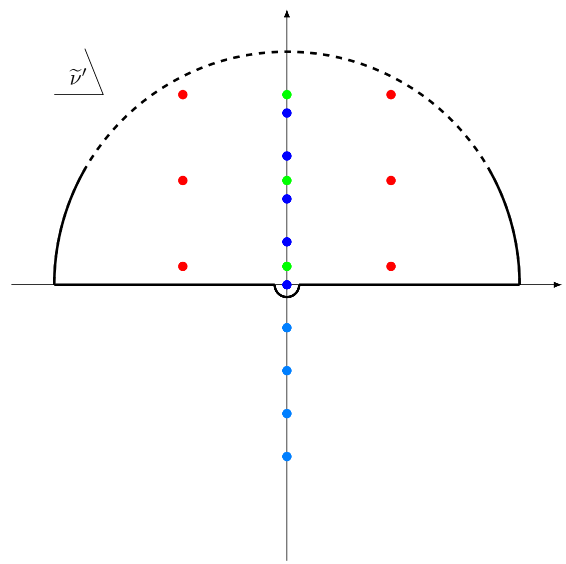
Contributions from the nonlocal tree integral

$$\mathcal{I}^{p_1p_2}_{\mathrm{NL},\widetilde{\nu}}(r_1,r_2)=\mathcal{C}^{p_1p_2}_{\mathrm{i}\widetilde{\nu},d}\,\mathbf{F}^{p_1}_{\mathrm{i}\widetilde{\nu},d}(r_1)\,\mathbf{F}^{p_2}_{\mathrm{i}\widetilde{\nu},d}(r_2)\,(r_1r_2)^{+\mathrm{i}\widetilde{\nu}}+\text{c.c.}$$

$$\mathcal{C}^{p_1p_2}_{\mathrm{i}\widetilde{\nu},d}\equiv\frac{1}{8}\csc^2(\pi\mathrm{i}\widetilde{\nu})\Big\{\cos\frac{\pi\bar{p}_{12}}{2}+\cos\Big[\pi\Big(\mathrm{i}\widetilde{\nu}+\frac{p_{12}+d}{2}\Big)\Big]\Big\}\qquad \mathbf{F}^{p}_{\mathrm{i}\widetilde{\nu},d}(r)\equiv(2r)^{p+d/2+1}\times{}_2\mathcal{F}_1\left[\begin{matrix}\frac{d}{4}+\frac{1}{2}+\frac{p}{2}+\frac{\mathrm{i}\widetilde{\nu}}{2},\ \frac{d}{4}+1+\frac{p}{2}+\frac{\mathrm{i}\widetilde{\nu}}{2}\\ 1+\mathrm{i}\widetilde{\nu}\end{matrix}\middle|\, r^2\right]$$

$$\begin{aligned}\rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}')=&\ \frac{1}{(4\pi)^{(d+1)/2}}\frac{\cos[\pi(\frac{d}{2}-\mathrm{i}\widetilde{\nu})]}{\sin(-\pi\mathrm{i}\widetilde{\nu})}\Gamma\left[\begin{matrix}\frac{3-d}{2},\ \frac{d}{2}-\mathrm{i}\widetilde{\nu}\\ \frac{2-d}{2}-\mathrm{i}\widetilde{\nu}\end{matrix}\right]\\ &\times{}_7\mathcal{F}_6\left[\begin{matrix}\frac{2-d}{2}+\mathrm{i}\widetilde{\nu}'-\mathrm{i}\widetilde{\nu},\ \frac{3-d/2+\mathrm{i}\widetilde{\nu}'-\mathrm{i}\widetilde{\nu}}{2},\ \frac{2-d}{2},\ \frac{2-d}{2}-\mathrm{i}\widetilde{\nu},\ \frac{2-d}{2}+\mathrm{i}\widetilde{\nu}',\ \frac{\mathrm{i}\widetilde{\nu}'-2\mathrm{i}\widetilde{\nu}+d/2}{2},\ \frac{\mathrm{i}\widetilde{\nu}'+d/2}{2}\\ \frac{1-d/2+\mathrm{i}\widetilde{\nu}'-\mathrm{i}\widetilde{\nu}}{2},\ 1+\mathrm{i}\widetilde{\nu}'-\mathrm{i}\widetilde{\nu},\ 1+\mathrm{i}\widetilde{\nu}',\ 1-\mathrm{i}\widetilde{\nu},\ \frac{4+\mathrm{i}\widetilde{\nu}'-3d/2}{2},\ \frac{4+\mathrm{i}\widetilde{\nu}'-2\mathrm{i}\widetilde{\nu}-3d/2}{2}\end{matrix}\middle|\,1\right]\\ &+(\widetilde{\nu}\to-\widetilde{\nu})\end{aligned}$$

Poles	$(r_1r_2)^{+\mathrm{i}\widetilde{\nu}'}$ term	$(r_1r_2)^{-\mathrm{i}\widetilde{\nu}'}$ term
Set 1A:	-	$\widetilde{\nu}'=\mathrm{i}d/2\pm2\widetilde{\nu}+2\mathrm{i}n,$
Set 1B:	-	$\widetilde{\nu}'=\mathrm{i}d/2+2\mathrm{i}n,$
Set 1C:	$\widetilde{\nu}'=-\mathrm{i}n,\quad(n\neq0)$	$\widetilde{\nu}'=+\mathrm{i}n.$



Results

$$\mathcal{J}_{\widetilde{\nu}}^{p_1p_2}(r_1,r_2) = \mathcal{J}_{\rm NS}^{p_1p_2}(r_1,r_2) + \mathcal{J}_{\rm LS}^{p_1p_2}(r_1,r_2) + \mathcal{J}_{\rm LT}^{p_1p_2}(r_1,r_2) + \mathcal{J}_{\rm BG}^{p_1p_2}(r_1,r_2)$$

ZX, Hongyu Zhang, 2211.03810

- Nonlocal Signal $\quad \mathcal{J}_{\rm NS}^{p_1p_2}(r_1,r_2) \propto (r_1r_2)^{\pm 2\mathrm{i}\widetilde{\nu}}$

$$\mathcal{J}_{\rm NS}^{p_1p_2} = -\frac{(r_1r_2)^{d/2+2\mathrm{i}\widetilde{\nu}}\sin[\pi(\frac{d}{2}+2\mathrm{i}\widetilde{\nu})]}{8\pi^{d/2}\Gamma(\frac{d}{2})\sin^2(\pi\mathrm{i}\widetilde{\nu})}\sum_{n=0}^{\infty}\frac{(1+n)_{\frac{d}{2}-1}\left[(1+\mathrm{i}\widetilde{\nu}+n)_{\frac{d}{2}-1}\right]^2(1+2\mathrm{i}\widetilde{\nu}+n)_{\frac{d}{2}-1}}{(1+2\mathrm{i}\widetilde{\nu}+2n)_{d-1}}$$
$$\times\left(\frac{d}{2}+2\mathrm{i}\widetilde{\nu}+2n\right)\mathcal{C}^{p_1p_2}_{2\mathrm{i}\widetilde{\nu}+d/2+2n,d}\mathbf{F}^{p_1}_{2\mathrm{i}\widetilde{\nu}+d/2+2n,d}(r_1)\mathbf{F}^{p_2}_{2\mathrm{i}\widetilde{\nu}+d/2+2n,d}(r_2)(r_1r_2)^{2n}+\text{c.c.}$$

- Local Signal: $\quad \mathcal{J}_{\rm LS}^{p_1p_2}(r_1,r_2) \propto \left(\frac{r_1}{r_2}\right)^{\pm 2\mathrm{i}\widetilde{\nu}}$

$$r_1 = \frac{k_s}{k_1+k_2}$$
$$r_2 = \frac{k_s}{k_3+k_4}$$

- Logarithmic tail: $\quad \mathcal{J}_{\rm LT}^{p_1p_2}(r_1,r_2) \propto \log r_2$

(Vanishes when $d=3$)

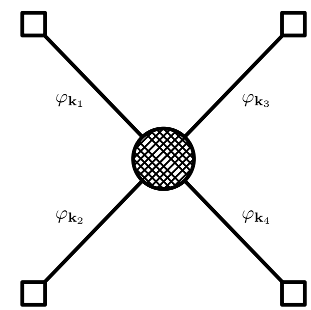
- Background: $\quad \mathcal{J}_{\rm BG}^{p_1p_2}(r_1,r_2)$ analytic in the squeezed limit

Consistency Check

UV Divergence

The UV divergence is manifestly local and is identical to flat space result

$$\mathcal{L}_{\phi_c,\widetilde{\nu}}\Big|_{s,\text{div.}} = \frac{1}{(4\pi)^2}\frac{2}{3-d}\frac{\tau_f^4}{16k_1k_2k_3k_4k_{1234}} + \mathcal{O}\Big((3-d)^0\Big)$$



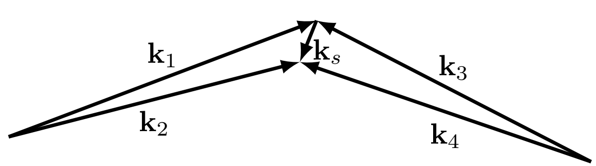
Large Mass Limit

Identical to flat-space result

$$\lim_{\widetilde{\nu}\to\infty}\frac{\tau_f^4}{16k_1k_2k_3k_4k_s}\mathcal{J}_{(3)}^{-2,-2} \sim -\frac{1}{16k_1k_2k_3k_4k_{1234}}\frac{1}{(4\pi)^2}\left[\log\frac{m^2}{\mu_R^2} + \mathcal{O}(\widetilde{\nu}^{-2})\right]$$

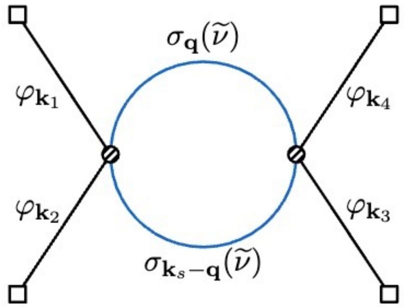
Squeezed Limit

The full expression matches a direct integration in the sqeezed limit



One-Loop Trispectrum

$$\mathcal{L}_{\varphi,\widetilde{\nu}} = \frac{1}{16k_1k_2k_3k_4k_s^5}\left[\mathcal{J}_{\widetilde{\nu}}^{00}(r_1,r_2)\right]_{\overline{\mathrm{MS}}}$$



$$\mathcal{L}_{\varphi,\widetilde{\nu}} = \frac{1}{16k_1k_2k_3k_4(k_{12}k_{34})^{5/2}}\left[\widehat{\mathcal{J}}_{\mathrm{NS}}(r_1,r_2)+\widehat{\mathcal{J}}_{\mathrm{LS}}(r_1,r_2)+\widehat{\mathcal{J}}_{\mathrm{BG}}(r_1,r_2)\right]$$

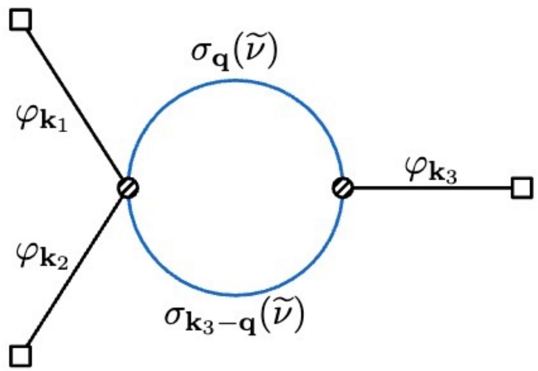
$$\widehat{\mathcal{J}}_{\mathrm{NS}} = \frac{2(r_1r_2)^{3/2+2\mathrm{i}\widetilde{\nu}}}{\pi^2\cos(2\pi\mathrm{i}\widetilde{\nu})}\sum_{n=0}^{\infty}\frac{(1+n)_{\frac{1}{2}}\left[(1+\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}\right]^2(1+2\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}}{(1+2\mathrm{i}\widetilde{\nu}+2n)_2}\left(\tfrac{3}{2}+2\mathrm{i}\widetilde{\nu}+2n\right)$$
$$\times\,{}_2\mathcal{F}_1\left[\begin{matrix}2+\mathrm{i}\widetilde{\nu}+n,\ \frac{5}{2}+\mathrm{i}\widetilde{\nu}+n\\ \frac{5}{2}+2\mathrm{i}\widetilde{\nu}+2n\end{matrix}\middle|\,r_1^2\right]{}_2\mathcal{F}_1\left[\begin{matrix}2+\mathrm{i}\widetilde{\nu}+n,\ \frac{5}{2}+\mathrm{i}\widetilde{\nu}+n\\ \frac{5}{2}+2\mathrm{i}\widetilde{\nu}+2n\end{matrix}\middle|\,r_2^2\right](r_1r_2)^{2n}+\mathrm{c.c.}$$

$$\widehat{\mathcal{J}}_{\mathrm{LS}} = -\frac{2(r_1/r_2)^{3/2+2\mathrm{i}\widetilde{\nu}}}{\pi^2\cos(2\pi\mathrm{i}\widetilde{\nu})}\sum_{n=0}^{\infty}\frac{(1+n)_{\frac{1}{2}}\left[(1+\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}\right]^2(1+2\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}}{(1+2\mathrm{i}\widetilde{\nu}+2n)_2}\left(\tfrac{3}{2}+2\mathrm{i}\widetilde{\nu}+2n\right)$$
$$\times\,{}_2\mathcal{F}_1\left[\begin{matrix}2+\mathrm{i}\widetilde{\nu}+n,\ \frac{5}{2}+\mathrm{i}\widetilde{\nu}+n\\ \frac{5}{2}+2\mathrm{i}\widetilde{\nu}+2n\end{matrix}\middle|\,r_1^2\right]{}_2\mathcal{F}_1\left[\begin{matrix}\frac{1}{2}-\mathrm{i}\widetilde{\nu}-n,\ 1-\mathrm{i}\widetilde{\nu}-n\\ -\frac{1}{2}-2\mathrm{i}\widetilde{\nu}-2n\end{matrix}\middle|\,r_2^2\right]\left(\frac{r_1}{r_2}\right)^{2n}+\mathrm{c.c.}$$

$$\widehat{\mathcal{J}}_{\mathrm{BG}} = \sum_{\ell,m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^{\ell+n+1}(\ell+1)_{2m+4}\left(\frac{5}{2}+\ell+2n\right)}{2^{2m}n!(m-n)!\left(\frac{5}{2}+\ell+n\right)_{m+1}}\widehat{\rho}_{\widetilde{\nu}}^{\mathrm{dS}}\left(-\tfrac{\mathrm{i}5}{2}-\mathrm{i}\ell-2\mathrm{i}n\right)r_1^{2m}\left(\frac{r_1}{r_2}\right)^{5/2+\ell}$$

One-Loop Bispectrum

$$\mathcal{B}_{\varphi,\widetilde{\nu}} = \frac{1}{8k_1k_2k_3^4}\Big[\mathcal{J}_{\widetilde{\nu}}^{0,-2}\Big(\frac{k_3}{k_{12}},1^-\Big)\Big]_{\overline{\mathrm{MS}}}$$

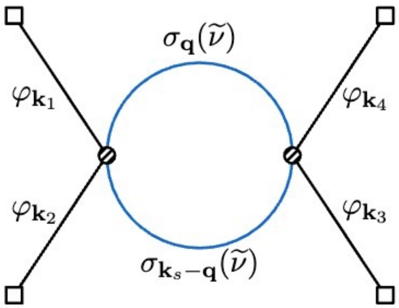


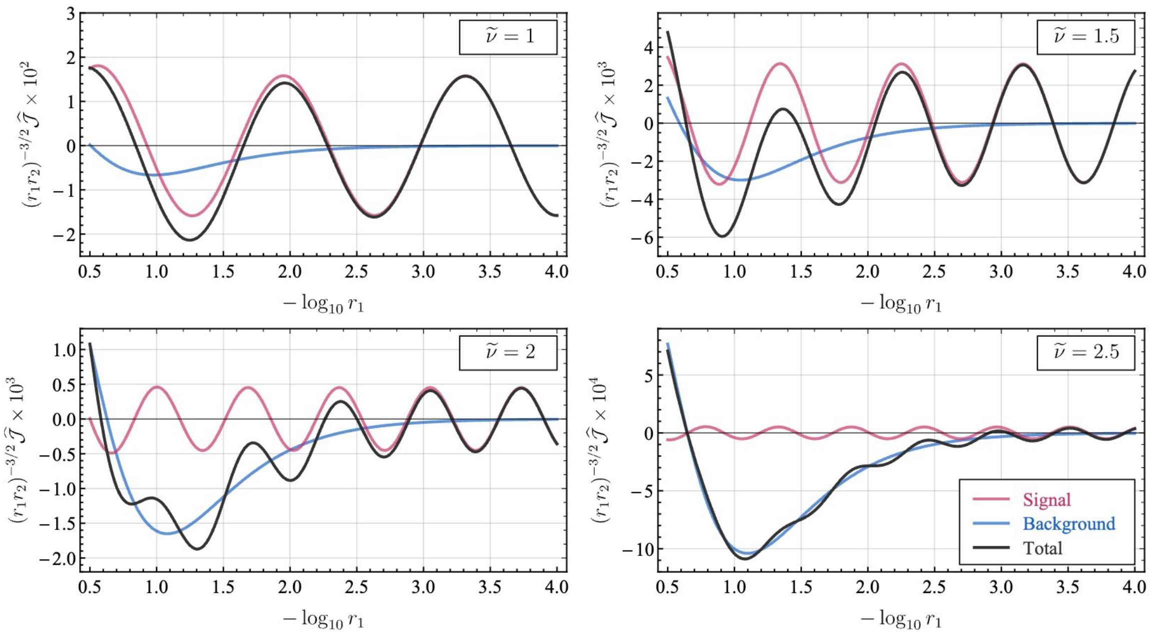
$$\mathcal{B}_{\varphi,\widetilde{\nu}} = \frac{1}{8k_1k_2k_3^4}\Big[\mathcal{S}_{\mathrm{S}}(r)+\mathcal{S}_{\mathrm{BG}}(r)\Big]$$

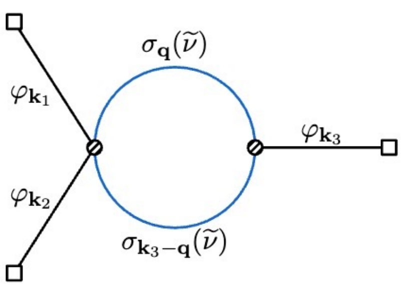
$$\mathcal{S}_{\mathrm{S}}(r) = \frac{r^{4+2\mathrm{i}\widetilde{\nu}}}{8\pi\sin(-2\pi\mathrm{i}\widetilde{\nu})}\sum_{n=0}^{\infty}(3+4\mathrm{i}\widetilde{\nu}+4n)\frac{(1+n)_{\frac{1}{2}}(1+2\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}}{(\frac{1}{2}+\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}(\frac{3}{2}+\mathrm{i}\widetilde{\nu}+n)_{\frac{1}{2}}}$$
$$\times\,{}_2\mathcal{F}_1\left[\begin{matrix}2+\mathrm{i}\widetilde{\nu}+n,\frac{5}{2}+\mathrm{i}\widetilde{\nu}+n\\ \frac{5}{2}+2\mathrm{i}\widetilde{\nu}+2n\end{matrix}\middle|\, r^2\right]r^{2n}+\mathrm{c.c.}$$

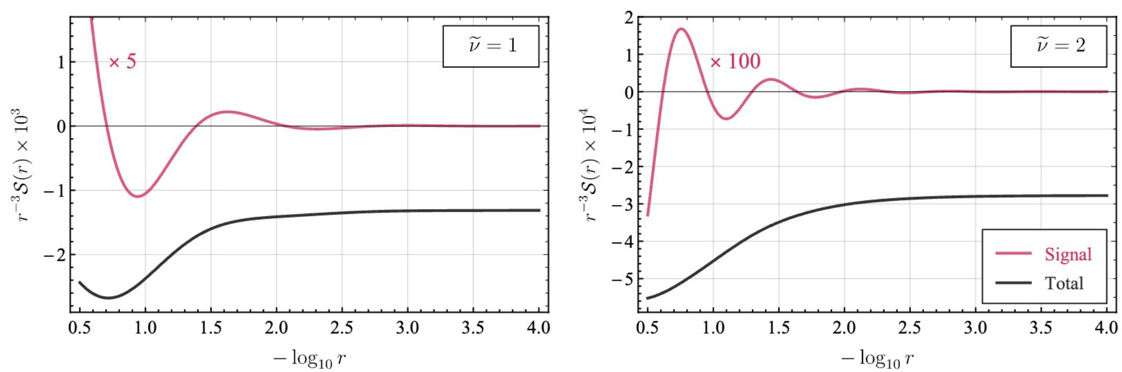
$$\mathcal{S}_{\mathrm{BG}}(r) = -\frac{r^3\widetilde{\nu}\,\mathrm{csch}(2\pi\widetilde{\nu})}{2\pi(1-r^2)^2}$$
$$+\sum_{\ell,m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^{\ell+n}(\ell+1)_{2m+2}(\frac{1}{2}+\ell+2n)}{2^{2m}n!(m-n)!(\frac{1}{2}+\ell+n)_{m+1}}\widehat{\rho}_{\widetilde{\nu}}^{\mathrm{dS}}\big(-\tfrac{\mathrm{i}}{2}-\mathrm{i}\ell-2\mathrm{i}n\big)r^{3+2m+\ell}$$

Trispectrum and Bispectrum

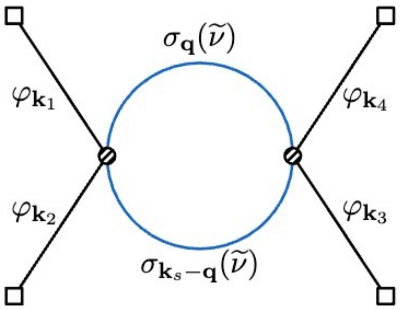








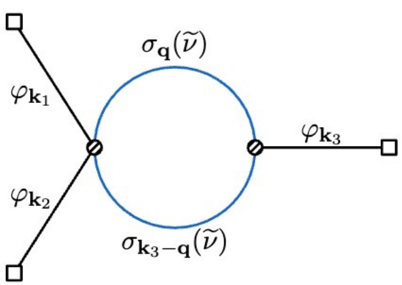
Squeezed and large mass limit



$$\lim_{\widetilde{\nu}\gg 1}\lim_{r_1\ll r_2\ll 1}\widehat{\mathcal{J}}_{\rm NS}=\sqrt{\frac{2}{\pi}}e^{-{\rm i}\pi/4}\widetilde{\nu}^{7/2}e^{-2\pi\widetilde{\nu}}\left(\frac{r_1r_2}{4}\right)^{3/2+2{\rm i}\widetilde{\nu}}+{\rm c.c.}$$

$$\lim_{\widetilde{\nu}\gg 1}\lim_{r_1\ll r_2\ll 1}\widehat{\mathcal{J}}_{\rm LS}=-\sqrt{\frac{2}{\pi}}e^{+{\rm i}\pi/4}\widetilde{\nu}^{7/2}e^{-2\pi\widetilde{\nu}}\left(\frac{r_1}{r_2}\right)^{3/2+2{\rm i}\widetilde{\nu}}+{\rm c.c.}$$

$$\lim_{\widetilde{\nu}\gg 1}\lim_{r_1\ll r_2\ll 1}\widehat{\mathcal{J}}_{\rm BG}=-\frac{3}{2\pi^2}\left(\log\frac{\widetilde{\nu}^2}{\mu_R^2}+\frac{1}{\widetilde{\nu}^2}\right)\left(\frac{r_1}{r_2}\right)^{5/2}$$



$$\lim_{\widetilde{\nu}\gg 1}\lim_{r\ll 1}\mathcal{S}_{\rm S}(r)=-4{\rm i}\widetilde{\nu}^2e^{-2\pi\widetilde{\nu}}\left(\frac{r}{2}\right)^{4+2{\rm i}\widetilde{\nu}}+{\rm c.c.}$$

$$\lim_{\widetilde{\nu}\gg 1}\lim_{r\ll 1}\mathcal{S}_{\rm BG}(r)=\frac{1}{8\pi^2}\left(\log\frac{\widetilde{\nu}^2}{\mu_R^2}+\frac{1}{25\widetilde{\nu}^2}\right)r^3$$

Summary

- First complete analytical results for massive 1-loop correlators by spectral decomposition

- Efficient calculation of loop diagrams; much faster (1 min on a laptop) than direct numerical integration (1 week on a cluster)

- Outlook: derivative couplings; spinning loops; dS breaking loops; other one-loop topologies (triangle, box)

Thank you!