Analytical Results for Inflation Correlators at Tree and Loop Levels

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Based on 2205.01692, 2208.13790, 2301.07047, w/ Zhong-Zhi Xianyu

Outline

- Cosmological Collider Physics
  - Signals versus background
  - Helical chemical potential
- Partial Mellin-Barnes representation
  - Full results for tree-level processes
  - Nonlocal signals for loop-level processes
  - Cutting rule and factorization
- Improved cosmological bootstrap
  - From bulk to boundary
  - Folded limit
- Summary & Outlooks

Cosmological Collider Physics

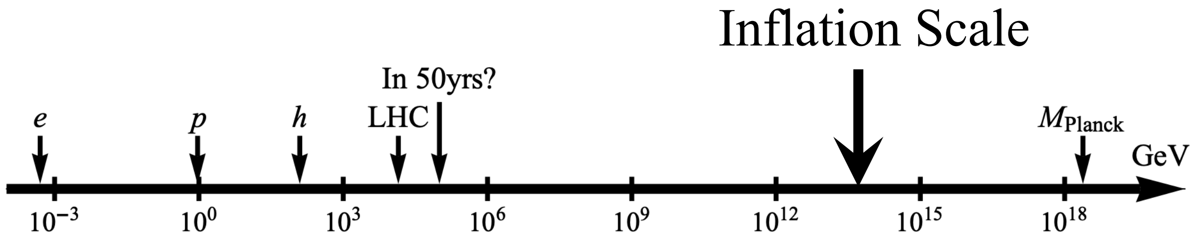
Inflation seems to be the highest energy observable process, with the energy scale characterized by the Hubble parameter, $H \sim 10^{14}\text{GeV}$.

During the inflation, particles with mass $m \gtrsim H$ could be spontaneously produced due to the quantum fluctuations, and then leave imprints in the inflation correlators.

The inflation correlators have some characteristic oscillatory patterns with respect to the logarithm of momentum ratios, which we would call a Cosmological Collider (CC) signal.

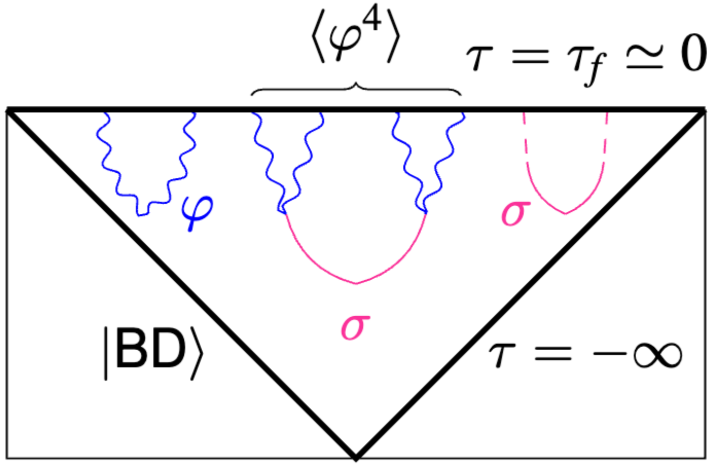
By measuring the frequency of this oscillating signal, we can recover the mass of the massive particle.

Arkani-Hamed, Maldacena: 1503.08043

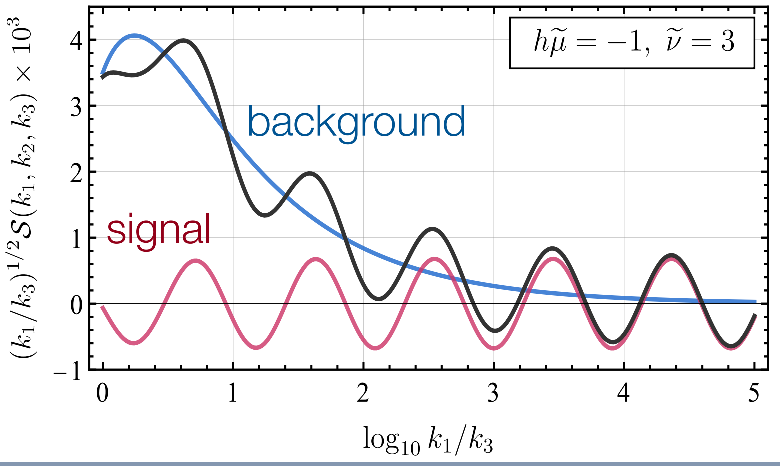


Spacetime metric:

$$ds^2 = a^2(\tau)[-d\tau^2 + d\boldsymbol{x}^2], \qquad a(\tau) = -\frac{1}{H\tau}.$$



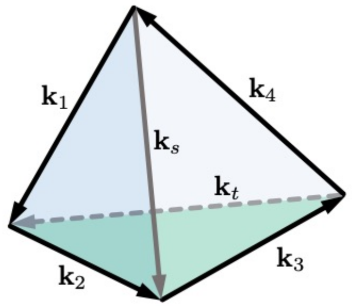
Particle production in dS. Here σ denotes the heavy particle and φ denotes the inflaton.



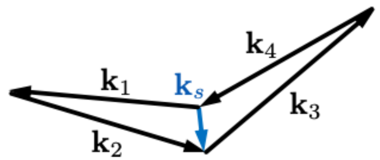
Example of a 3pt correlator (bispectrum).

Cosmological Collider Physics

- 2pt correlator is trivial (with no CC signal).
- 3pt correlator is easier for measurement.
- 4pt correlator is easier to calculate.



General momentum configuration.



Squeezed limit configuration.

In general the 4pt function has the form:

$$\langle \varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\varphi_{\boldsymbol{k}_4}\rangle' \sim K(\theta_i)\times J(k_1,k_2,k_3,k_4,k_s,k_t).$$

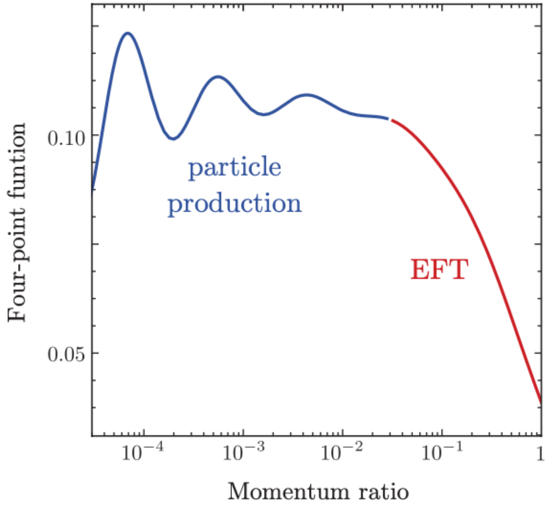
Here the factor $K(\theta_i)$ is purely kinematic. In the squeezed limit $k_s\to 0$, the dynamic piece can be divided into three part:

$$\lim_{k_s\to 0} J = J_{\mathrm{EFT}} + J_{\mathrm{L}} + J_{\mathrm{NL}}.$$

- J_{EFT}: EFT term, or the background piece. Fully analytic in both r_1 and r_2.
- J_{L}: Local signal, proportional to $(r_1/r_2)^{\pm i\omega}$. Analytic in k_s.
- J_{NL}: Nonlocal signal, proportional to $(r_1 r_2)^{\pm i\omega}$. Nonanalytic in k_s.

Oscillatory pattern:

$$A\,(r_1 r_2)^{i\omega} + \mathrm{c.c.} = 2|A|\cos[\omega\log(r_1 r_2) + \vartheta].$$



Baumann, et al: 1811.00024.

Example of a 4pt correlator.

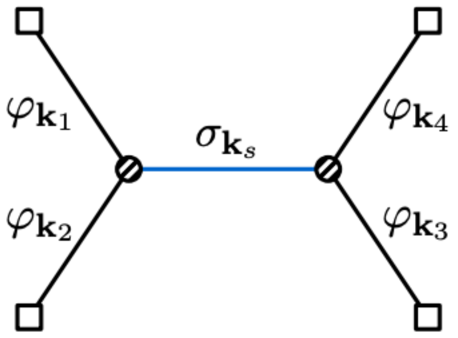
Convention:

$$k_{ij\cdots} \equiv k_i + k_j + \cdots.$$

Momentum ratios:

$$r_1 = \frac{k_s}{k_{12}}, \qquad r_2 = \frac{k_s}{k_{34}}.$$

Inflation Correlators are Hard to Calculate!



$$\sim -\frac{\lambda^2}{E_s^2-k_s^2-m^2} \text{ in Minkowski.}$$

$$= -\frac{\lambda_T^2}{16k_1\cdots k_4}\sum_{\mathsf{a,b}}\mathsf{ab}\int_{-\infty}^0 \mathrm{d}\tau_1\mathrm{d}\tau_2\, e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2). \quad \text{in de Sitter.}$$

(See Chen, Wang, Xianyu: 1703.10166 for SK formalism.)

$$D_{-+}(k_s;\tau_1,\tau_2) = \frac{\pi}{4}e^{-\pi\tilde{\nu}}(\tau_1\tau_2)^{3/2}\mathrm{H}^{(1)}_{\mathrm{i}\tilde{\nu}}(-k_s\tau_1)\mathrm{H}^{(2)}_{-\mathrm{i}\tilde{\nu}}(-k_s\tau_2)$$

$$D_{+-}(k_s;\tau_1,\tau_2) = D^*_{-+}(k_s;\tau_1,\tau_2),$$

$$D_{++}(k_s;\tau_1,\tau_2) = D_{-+}(k_s;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{+-}(k_s;\tau_1,\tau_2)\theta(\tau_2-\tau_1),$$

$$D_{--}(k_s;\tau_1,\tau_2) = D_{+-}(k_s;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{-+}(k_s;\tau_1,\tau_2)\theta(\tau_2-\tau_1).$$

Why?

- Lack of time translation symmetry.
- Special functions in the mode function.
- Time ordering is encountered.
- Inflation patch is dS-boost-breaking.

Helical Chemical Potential

Usually the CC signals suffer the Boltzmann expression $\sim e^{-2\pi m}$ for large mass, so one must break some symmetry for a large signal. Consider the following action for spin-1 gauge boson with an extra axion-type interaction:

$$S = \int \mathrm{d}^4 x \left[\sqrt{-g} \left(-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right) + \frac{\phi}{4\Lambda} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Wang, Xianyu, 2004.02887.

This action is both P-violating & dS-boost breaking.

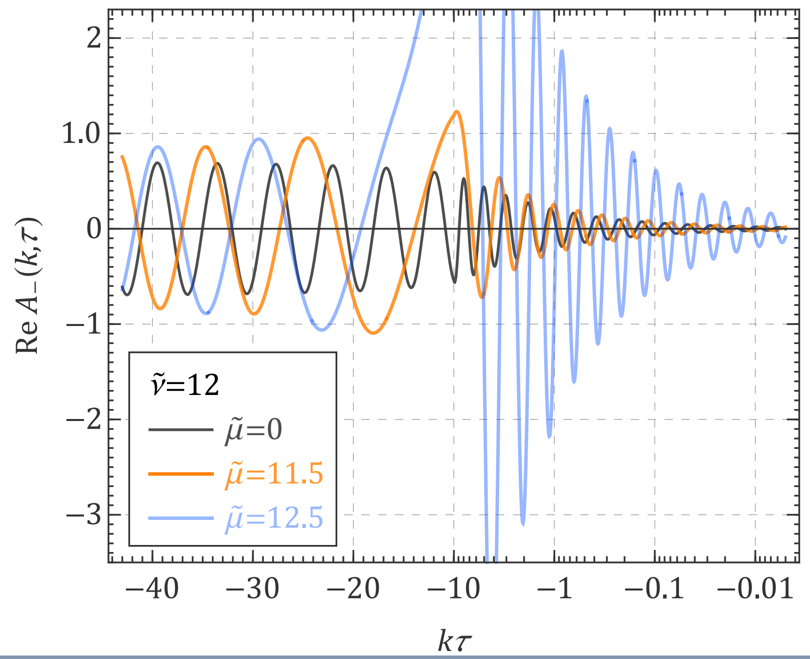
With a rolling background: $\phi = \dot{\phi}_0 t + \text{const.}$, and defining the chemical potential: $\mu = \dot{\phi}_0/\Lambda$, the EoM reads:

$$A_0'' - \partial_j^2 A_0 + (2aHA_0)' + a^2 m^2 A_0 = 0,$$
$$A_i'' - \partial_j^2 A_i + 2aH\partial_i A_0 + a^2 m^2 A_i + 2a\mu\epsilon_{ijk}\partial^j A^k = 0.$$

We can then solve the mode functions:

$$B^{(\pm)}(k,\tau) = \frac{e^{\mp\pi\tilde{\mu}/2}}{\sqrt{2k}} \mathrm{W}_{\pm \mathrm{i}\tilde{\mu}, \mathrm{i}\tilde{\nu}}(2\mathrm{i}k\tau),$$
$$B^{(T)}(k,\tau) = \frac{\sqrt{\pi}k}{2m} e^{-\pi\tilde{\nu}/2} H(-\tau)^{3/2} \mathrm{H}^{(1)}_{\mathrm{i}\tilde{\nu}}(-k\tau)$$

We find that the mode function (and thus the CC signals) for one helicity state is enhanced exponentially by the chemical potential!



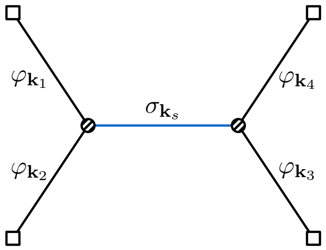
Seed Integrals

We define dimensionless seed integrals for convenience. Inflation correlators can be expressed as a combination of the seed integrals. Our task is then to calculate these seed integrals.

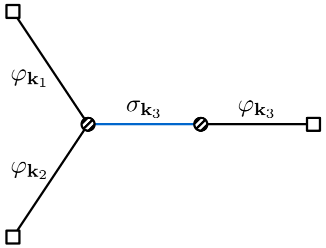
- Scalar seed: $$\mathcal{I}^{p_1p_2}_{\mathsf{ab}}(r_1,r_2)\equiv -\mathsf{ab}\,k_s^{5+p_1+p_2}\int_{-\infty}^{0}\mathrm{d}\tau_1\mathrm{d}\tau_2\,(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2)$$

$$\Delta\mathscr{L}=\frac{1}{2}\lambda a^2\varphi'^2\sigma \qquad \mathcal{T}_\varphi=\frac{\lambda^2}{16k_1k_2k_3k_4k_s^5}\sum_{\mathsf{a,b}=\pm}\mathcal{I}^{00}_{\mathsf{ab}}(r_1,r_2)$$

$$\Delta\mathscr{L}=\frac{1}{2}\lambda_c a^4\phi_c^2\sigma \qquad \mathcal{T}_{\phi_c}=\frac{\lambda_c^2\tau_f^4}{16k_1k_2k_3k_4k_s}\sum_{\mathsf{a,b}=\pm}\mathcal{I}^{-2,-2}_{\mathsf{ab}}(r_1,r_2)$$



$$\mathcal{B}_\phi=\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle_3'=\frac{\lambda\lambda_2}{8k_1k_2k_3^4}\lim_{r_2\to1^-}\sum_{\mathsf{a,b}=\pm}\mathcal{I}^{0,-2}_{\mathsf{ab}}(r_1,r_2)$$



- Helical vector seed: $$\mathcal{I}^{(\lambda)p_1p_2}_{\mathsf{ab}}\equiv -\,\mathsf{ab}\,k_s^{3+p_1+p_2}\int_{-\infty}^{0}\mathrm{d}\tau_1\mathrm{d}\tau_2\,(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D^{(\lambda)}_{\mathsf{ab}}(k_s;\tau_1,\tau_2)$$

$$D^{(\pm)}_{>}(k;\tau_1,\tau_2)=\frac{e^{\mp\pi\widetilde{\mu}}}{2k}\mathrm{W}_{\pm\mathrm{i}\widetilde{\mu},\mathrm{i}\widetilde{\nu}}(2\mathrm{i}k\tau_1)\mathrm{W}_{\mp\mathrm{i}\widetilde{\mu},\mathrm{i}\widetilde{\nu}}(-2\mathrm{i}k\tau_2)$$

Partial Mellin-Barnes representation

- For QFT in flat spacetime, Fourier transform is useful, since its the kernel is the eigenmode of translation.
- In dS, there is no more time translation, but we have dilation symmetry. The corresponding integral transform is the so-called Mellin transform:

$$F(s) = \int_0^\infty dz\, z^{s-1} f(z), \qquad f(z) = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i}\, z^{-s} F(s).$$

- Partial MB Rep: Only perform the inverse Mellin transform to the internal modes.
- Scalar propagators: **ZQ**, Xianyu: 2205.01692, 2208.13790.

$$D_{\pm\mp}(k;\tau_1,\tau_2) = \frac{1}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\, e^{\mp\mathrm{i}\pi(s_1-s_2)}\left(\frac{k}{2}\right)^{-2s_{12}}(-\tau_1)^{-2s_1+3/2}(-\tau_2)^{-2s_2+3/2}$$
$$\times\,\Gamma\left[s_1-\frac{\mathrm{i}\widetilde{\nu}}{2}, s_1+\frac{\mathrm{i}\widetilde{\nu}}{2}, s_2-\frac{\mathrm{i}\widetilde{\nu}}{2}, s_2+\frac{\mathrm{i}\widetilde{\nu}}{2}\right],$$
$$D_{\pm\pm}(k;\tau_1,\tau_2) = D_{\mp\pm}(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2) + D_{\pm\mp}(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1).$$

- The time integrals and the loop integrals are then simplified. The price is the integral of the Mellin variables.

Partial Mellin-Barnes representation

- The opposite-sign seed integral becomes:

$$\mathcal{I}^{p_1p_2}_{\pm\mp}(r_1,r_2)=\frac{1}{4\pi}e^{\mp\mathrm{i}\pi(p_1-p_2)/2}r_1^{5/2+p_1}r_2^{5/2+p_2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\left(\frac{r_1}{2}\right)^{-2s_1}\left(\frac{r_2}{2}\right)^{-2s_2}$$
$$\times\,\Gamma\left[p_1+\tfrac{5}{2}-2s_1,p_2+\tfrac{5}{2}-2s_2,s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right].$$

- Under the rearrangement: $D_{\pm\pm}(k;\tau_1,\tau_2)=D_{\gtrless}(k;\tau_1,\tau_2)+\Big[D_{\lessgtr}(k;\tau_1,\tau_2)-D_{\gtrless}(k;\tau_1,\tau_2)\Big]\theta(\tau_2-\tau_1).$

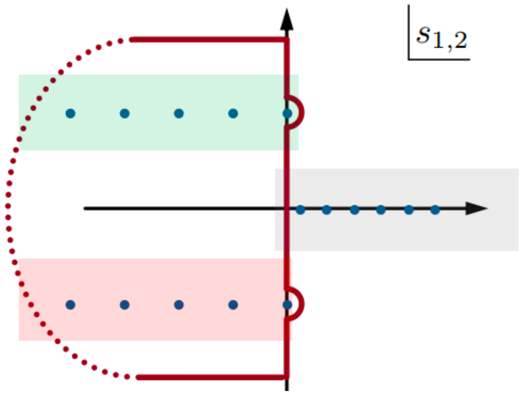
The same-sign seed integral becomes: $\mathcal{I}^{p_1p_2}_{\pm\pm}(r_1,r_2)=\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{F},>}(r_1,r_2)+\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{TO},>}(r_1,r_2).\quad(r_1<r_2)$

$$\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{F},>}=\frac{1}{4\pi}e^{\mp\mathrm{i}\pi(p_1+p_2)/2}r_1^{5/2+p_1}r_2^{5/2+p_2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\left(\pm\mathrm{i}e^{\pm2\mathrm{i}\pi s_1}\right)\left(\frac{r_1}{2}\right)^{-2s_1}\left(\frac{r_2}{2}\right)^{-2s_2}$$
$$\times\,\Gamma\left[p_1+\tfrac{5}{2}-2s_1,p_2+\tfrac{5}{2}-2s_2,s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right],$$

$$\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{TO},>}=\frac{1}{4\pi}e^{\mp\mathrm{i}\pi(p_1+p_2)/2}r_1^{5+p_1+p_2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\left(\mp\mathrm{i}e^{\pm2\mathrm{i}\pi s_1}\pm\mathrm{i}e^{\pm2\mathrm{i}\pi s_2}\right)\left(\frac{r_1}{2}\right)^{-2s_{12}}$$
$$\times\,\Gamma\left[p_2+\tfrac{5}{2}-2s_2,p_1+p_2+5-2s_{12},s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right]$$
$$\times\,{}_2\widetilde{\mathrm{F}}_1\left[\begin{matrix}p_2+\frac{5}{2}-2s_2,p_1+p_2+5-2s_{12}\\ p_2+\frac{7}{2}-2s_2\end{matrix}\middle|-\frac{r_1}{r_2}\right].$$

- Then we only need to finish the Mellin integral using the residue theorem.

The appropriate contour and poles are the following:



Recall that the MB rep for the propagator:

$$D_{\pm\mp}(k;\tau_1,\tau_2) = \frac{1}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\,e^{\mp\mathrm{i}\pi(s_1-s_2)}\left(\frac{k}{2}\right)^{-2s_{12}}(-\tau_1)^{-2s_1+3/2}(-\tau_2)^{-2s_2+3/2}$$
$$\times\Gamma\left[s_1-\frac{\mathrm{i}\widetilde{\nu}}{2}, s_1+\frac{\mathrm{i}\widetilde{\nu}}{2}, s_2-\frac{\mathrm{i}\widetilde{\nu}}{2}, s_2+\frac{\mathrm{i}\widetilde{\nu}}{2}\right],$$
$$D_{\pm\pm}(k;\tau_1,\tau_2) = D_{\mp\pm}(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{\pm\mp}(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1).$$

For factorized part:

$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2 = -n_2 \pm \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => local signal

$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2 = -n_2 \mp \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => nonlocal signal

For time-ordered (non-factorized) part:

$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2 = -n_2 \pm \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => background

$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2 = -n_2 \mp \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => 0

The four propagators become identical at the nonlocal poles. So we can replace D_{ab} by Re$[D]$ to calculate the nonlocal signal!

Bootstrap Equations

- Original bootstrap: Symmetry implies differential equations. Baumann, et al: 1811.00024, 1910.14051, 2005.04234.
- Our bootstrap: EoM of propagators (mode functions) implies ODE for correlators. Applicable to dS-boost breaking cases. **ZQ**, Xianyu: 2208.13790, 2301.07047.

$$(\tau_1^2\partial_{\tau_1}^2-2\tau_1\partial_{\tau_1}+k_s^2\tau_1^2+m^2)D_{\pm\mp}(k_s;\tau_1,\tau_2)=0$$

$$(\tau_1^2\partial_{\tau_1}^2-2\tau_1\partial_{\tau_1}+k_s^2\tau_1^2+m^2)D_{\pm\pm}(k_s;\tau_1,\tau_2)=\mp\mathrm{i}\tau_1^2\tau_2^2\delta(\tau_1-\tau_2)$$

Insert the differential operator in the integrand before the propagator, then commute it with the time integral using IBP.

$$\mathcal{I}_{\mathsf{ab}}^{-2,-2}(r_1,r_2)\equiv-\mathsf{ab}\,k_s\int_{-\infty}^{0}\frac{\mathrm{d}\tau_1}{\tau_1^2}\frac{\mathrm{d}\tau_2}{\tau_2^2}\,e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2).$$

$$\left[(r_1^2-r_1^4)\partial_{r_1}^2-2r_1^3\partial_{r_1}+\left(\widetilde{\nu}^2+\frac{1}{4}\right)\right]\mathcal{I}_{\pm\mp}^{-2,-2}(r_1,r_2)=0$$

$$\left[(r_1^2-r_1^4)\partial_{r_1}^2-2r_1^3\partial_{r_1}+\left(\widetilde{\nu}^2+\frac{1}{4}\right)\right]\mathcal{I}_{\pm\pm}^{-2,-2}(r_1,r_2)=\frac{r_1r_2}{r_1+r_2}$$

Solutions

$$\left[(r_1^2 - r_1^4)\partial_{r_1}^2 - 2r_1^3\partial_{r_1} + \left(\widetilde{\nu}^2 + \frac{1}{4}\right)\right]\mathcal{I}_{\pm\mp}^{-2,-2}(r_1, r_2) = 0$$

$$\left[(r_1^2 - r_1^4)\partial_{r_1}^2 - 2r_1^3\partial_{r_1} + \left(\widetilde{\nu}^2 + \frac{1}{4}\right)\right]\mathcal{I}_{\pm\pm}^{-2,-2}(r_1, r_2) = \frac{r_1 r_2}{r_1 + r_2}$$

As an ODE, the solution to the bootstrap equations is the sum of two parts:

- Homogeneous solution:

The homogeneous solution can be expressed in terms of the hypergeometric functions. In particular, the homogeneous solution is factorized and nonanalytic in $r_{1,2}$, so it corresponds to the signals.

- Inhomogeneous solution:

We solve the inhomogeneous solution with an ansatz: $\mathcal{Z}(r_1, r_2) = r_1 \sum_{m,n=0}^{\infty} (-1)^n \mathcal{Z}_{m,n} r_1^m \left(\frac{r_1}{r_2}\right)^n,$

$Z_{0,0}$ can be given by setting appropriate boundary conditions. We find the inhomogeneous solution is analytic in both r_1 and r_2, so it is exactly the background.

Improved Cosmological Bootstrap

- We find a change of variables: $u_{1,2} \equiv 2r_{1,2}/(1+r_{1,2})$ is crucial for analytically solving the bootstrap equation for vector seed integral: **ZQ**, Xianyu: 2208.13790.

$$\left[(u_1^2 - u_1^3)\partial_{u_1}^2 - (1 \pm \mathrm{i}h\widetilde{\mu})u_1^2\partial_{u_1} + \left(\widetilde{\nu}^2 + \frac{1}{4}\right)\right]\mathcal{I}_{\pm\mp}^{(h)-1,-1}(u_1,u_2) = 0$$

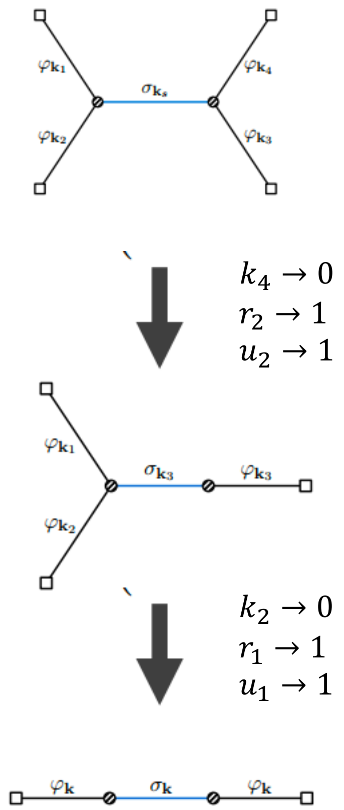
$$\left[(u_1^2 - u_1^3)\partial_{u_1}^2 - (1 \pm \mathrm{i}h\widetilde{\mu})u_1^2\partial_{u_1} + \left(\widetilde{\nu}^2 + \frac{1}{4}\right)\right]\mathcal{I}_{\pm\pm}^{(h)-1,-1}(u_1,u_2) = \frac{1}{2}\frac{u_1u_2}{u_1+u_2-u_1u_2}$$

- It is also useful for deriving closed-form formula for 3pt and 2pt correlators:

To derive a 3pt correlator, we should set $r_2 = 1 = u_2$ in the 4pt correlator. We find the source term in the RHS becomes simply $\sim u_1$. This allows us to solve the bootstrap equation in a closed-form.

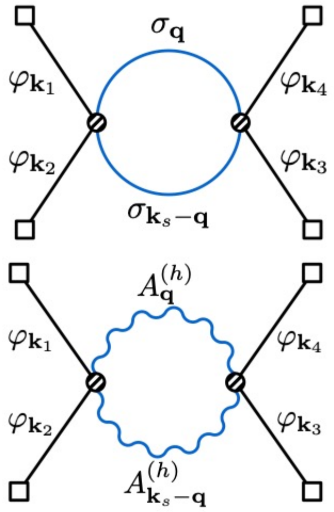
Similarly, to derive a 2pt correlator we should then set $r_1 = 1 = u_1$ in the 3pt correlator. This can be done if carefully dealing with the spurious divergence.

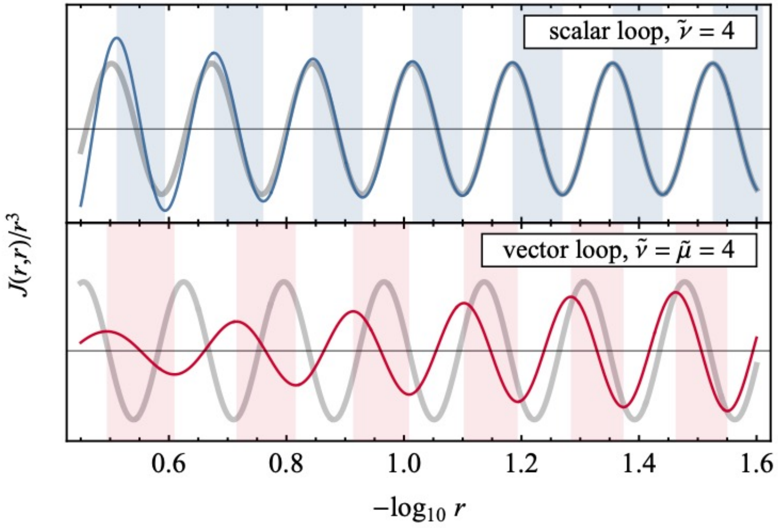
ZQ, Xianyu: 2301.07047.



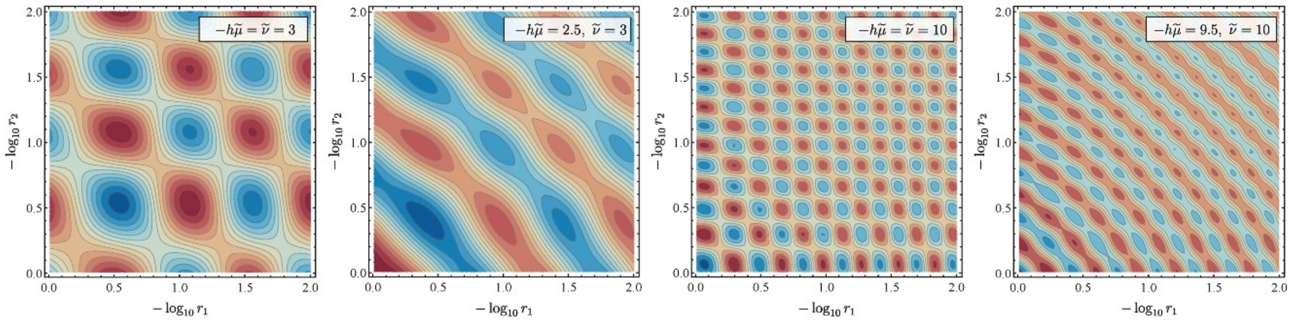
Deriving 3pt and 2pt correlator by taking single and double folded limits, respectively.

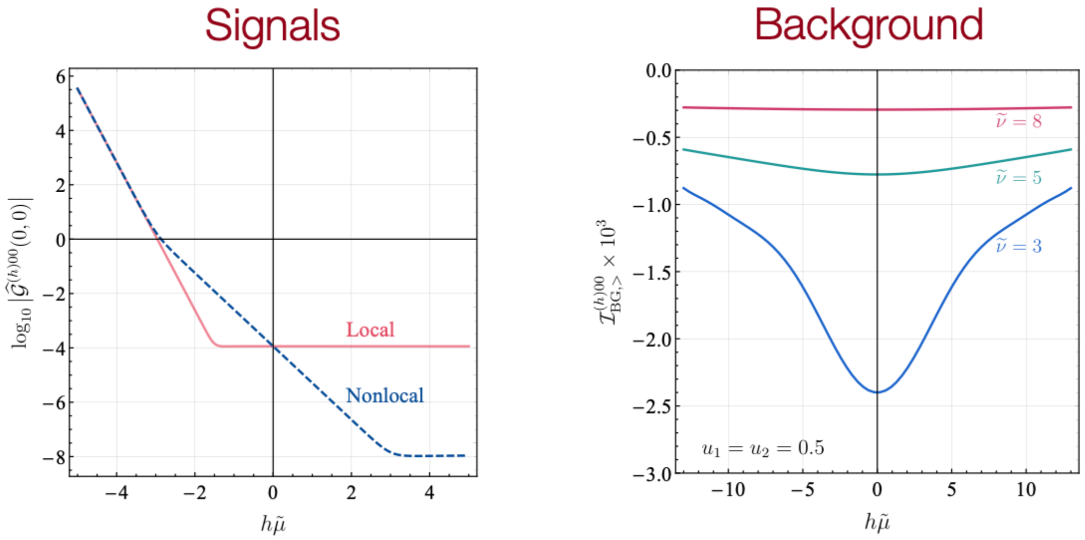
Other Results





Different particle species and different types of interaction give rise to different phases.
ZQ, Xianyu: 2205.01692.





Local and nonlocal signals with chemical potential.
ZQ, Xianyu: 2208.13790.

Signals exponentially sensitive to chemical potential. Background insensitive.
ZQ, Xianyu: 2208.13790.

Summary & Outlooks

- Calculation of inflation correlators are important (but difficult).
  - Useful for particle model buildings, parameter scanning, template design, …
  - Amplitudes in dS are least understood among the three maximally symmetric spacetimes.
  - Inflation provides a very high energy scale at $H \sim 10^{14}\mathrm{GeV}$.
- Two very useful methods are developed, both applicable to helical correlators, and tree level calculations are basically solved.
  - Partial MB representation.
  - Bootstrap equations
- Cutting rule as a byproduct.
- Outlooks:
  - Full results for general tree graph.
  - Nonlocal signal for general loop graph. **ZQ**, Xianyu: 2304.13295, 2308.xxxxx.
  - Local signal for general loop graph graph (on going).
  - Deeper understanding of analytical structure of inflation correlators.

Thanks

- $I_{++}^{-2,-2}$ as an example:

$$\mathcal{I}_{\pm\pm}^{p_1p_2}(r_1,r_2)=\mathcal{I}_{\pm\pm,\mathrm{F}}^{p_1p_2}(r_1,r_2)+\mathcal{I}_{\pm\pm,\mathrm{TO}}^{p_1p_2}(r_1,r_2)$$

$$\mathcal{I}_{++,\mathrm{F},>}^{-2,-2}(r_1,r_2)\equiv -\,k_s\int_{-\infty}^{0}\frac{\mathrm{d}\tau_1}{\tau_1^2}\frac{\mathrm{d}\tau_2}{\tau_2^2}e^{\pm\mathrm{i}(k_{12}\tau_1+k_{34}\tau_2)}D_>(k_s;\tau_1,\tau_2)$$

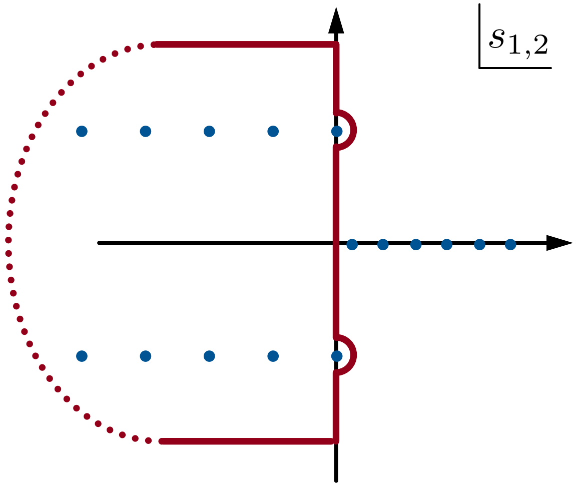
$$=\frac{(r_1r_2)^{1/2}}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}(\mathrm{i}e^{2\mathrm{i}\pi s_1})\Big(\frac{r_1}{2}\Big)^{-2s_1}\Big(\frac{r_2}{2}\Big)^{-2s_2}$$

$$\times\,\Gamma\Big[\tfrac{1}{2}-2s_1,\tfrac{1}{2}-2s_2,s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\Big]$$

$$\mathcal{I}_{++,\mathrm{TO},>}^{-2,-2}=\frac{(r_1r_2)^{1/2}}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}(-\mathrm{i}e^{2\mathrm{i}\pi s_1}+\mathrm{i}e^{2\mathrm{i}\pi s_2})\Big(\frac{r_1}{2}\Big)^{-2s_{12}}$$

$$\times\,\Gamma\Big[\tfrac{1}{2}-2s_2,1-2s_{12},s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\Big]$$

$$\times\,{}_2\widetilde{\mathrm{F}}_1\left[\begin{matrix}\tfrac{1}{2}-2s_2,1-2s_{12}\\ \tfrac{3}{2}-2s_2\end{matrix}\,\middle|\,-\frac{r_1}{r_2}\right]$$



$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\pm\frac{\mathrm{i}\widetilde{\nu}}{2}$$

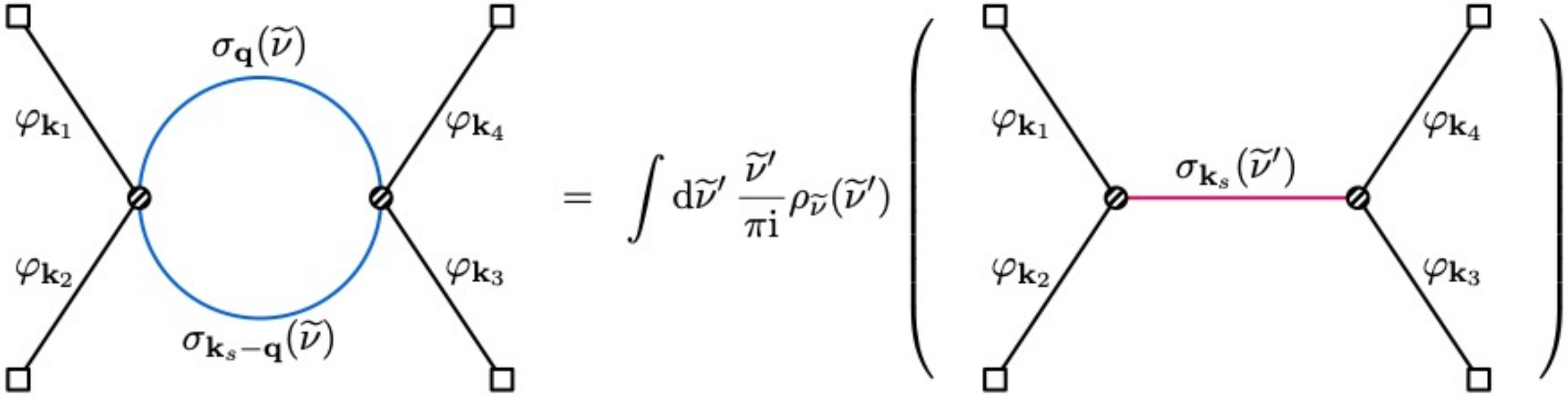
$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\mp\frac{\mathrm{i}\widetilde{\nu}}{2}$$

Contributing to local/nonlocal signals in the Factorized part; and to background/0 in the Time-Ordered part.

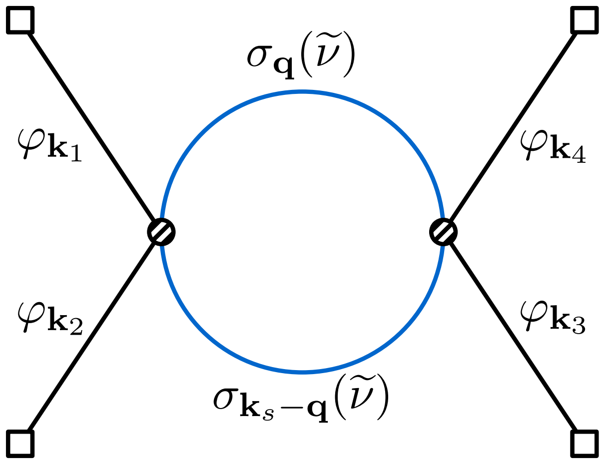
Backup

- Spectral decomposition

$$\int \frac{\mathrm{d}^d\mathbf{q}}{(2\pi)^d} D_{\widetilde{\nu},\mathsf{ab}}\Big(q;\tau_1,\tau_2\Big) D_{\widetilde{\nu},\mathsf{ab}}\Big(|\mathbf{k}_s-\mathbf{q}|;\tau_1,\tau_2\Big) = \int_{-\infty}^{+\infty} \mathrm{d}\widetilde{\nu}'\,\frac{\widetilde{\nu}'}{\pi\mathrm{i}}\rho^{\mathrm{dS}}_{\widetilde{\nu}}(\widetilde{\nu}') D_{\widetilde{\nu}',\mathsf{ab}}\Big(k_s;\tau_1,\tau_2\Big)$$



- PMB for one loop bubble:



$$\begin{aligned}\mathcal{T}_{\mathrm{NL},\varphi'^2\sigma^2} = &\frac{\lambda^2}{256\pi^{7/2}k_1\cdots k_4(k_{12}k_{34})^{5/2}}(1-\cosh 2\pi\widetilde{\nu})\left(\frac{r_1r_2}{4}\right)^{3+2\mathrm{i}\widetilde{\nu}}\\ &\times\sum_{n_1,n_2,n_3,n_4=0}^{\infty}\frac{(-1)^{n_{1234}}}{n_1!n_2!n_3!n_4!}\left(\frac{r_1}{2}\right)^{2n_{13}}\left(\frac{r_2}{2}\right)^{2n_{24}}\\ &\times\Gamma\left[4+2n_{13}+2\mathrm{i}\widetilde{\nu},4+2n_{24}+2\mathrm{i}\widetilde{\nu},-n_1-\mathrm{i}\widetilde{\nu},-n_2-\mathrm{i}\widetilde{\nu},-n_3-\mathrm{i}\widetilde{\nu},-n_4-\mathrm{i}\widetilde{\nu}\right]\\ &\times\Gamma\left[\begin{matrix}\frac{3}{2}+n_{12}+\mathrm{i}\widetilde{\nu},\frac{3}{2}+n_{34}+\mathrm{i}\widetilde{\nu},-n_{1234}-\frac{3}{2}-2\mathrm{i}\widetilde{\nu}\\ -n_{12}-\mathrm{i}\widetilde{\nu},-n_{34}-\mathrm{i}\widetilde{\nu},3+n_{1234}+2\mathrm{i}\widetilde{\nu}\end{matrix}\right]\\ &+\mathrm{c.c.}.\end{aligned}$$