Giant correlators

at Quantum level

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Based on the work

YJ, Yu Wu, Yang Zhang**,** *to appear (hopefully soon...)*

Motivations

QFT in *real* physics

- The ***process XXX*** is very important observable in a certain experiment.....
- .....We want to compute XXX ***at N...NLO***....
- ....important to ***compute loop integrals***......
- ...... Use ***AMFlow*** at some point....
- Match with ***experimental data***

Some cool figures
from Experiments

with error bars

QFT in ***mathematical*** physics

$\mathcal{N} = 4$ Supersymmetric Yang–Mills Theory

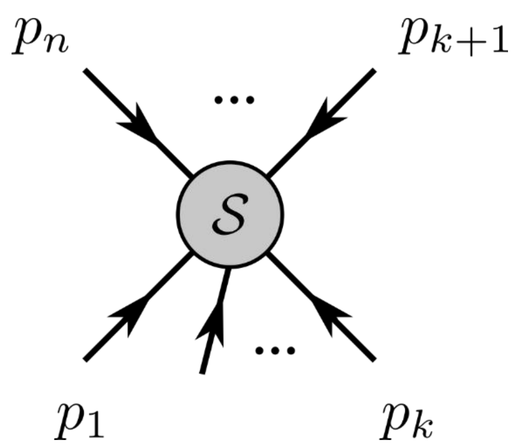
reduce SUSY → **QCD**

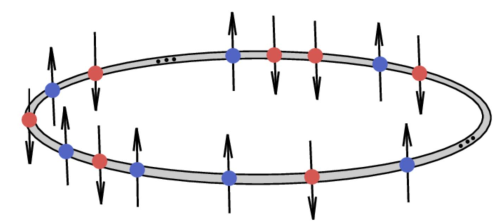
AdS/CFT → **Quantum gravity**

→ **Interacting CFT**

Correlators in $N = 4$ SYM

- Correlators are crucial to understand gravity and field theory
- Fruitful laboratary for developing new technologies





Understand gravity

Develop QFT techniques

Non-perturbative approaches

Restuls for single trace operators

$$\langle \mathcal{O}_{k_1}(x_1)\mathcal{O}_{k_2}(x_2)\mathcal{O}_{k_3}(x_3)\mathcal{O}_{k_4}(x_4)\rangle$$

Half-BPS, single-trace operators

$$\mathcal{O}_{k_i}(x_i) = \mathrm{Tr}(Y_i^I \Phi_I)^{k_i}(x_i)$$

- Operators dual to graviton and KK modes
- 2-pt and 3-pt functions are protected
- 4-pt functions are no longer protected
- Have been studied since early days of AdS/CFT

State-of-the-art

Weak coupling

Perturbation theory

Lagrangian insertion + symmetry + loop integrals

Integrand

3 and 4-loops [Eden, Heslop, Korchemsky, Sokatchev 2012]

10-loops [Boujaily, Heslop, Tran 2016]

Full result

Full results are known up to 3-loops

4-loop & 5-loop integrals are hard in general, possible in OPE limit

OPE data up to 5-loops [Chicherin, Georgoudis, Goncalves, Pereira, 2018]

State-of-the-art

Strong coupling

Bootstrap approach

Superconformal symmetry + bootstrap

Supergravity limit

4pt-function fully determined [Rastelli, Zhou 2016]

5pt are also available [Goncalves, Meneghelli, Pereira, Zhou 2023]

Loop level (in 1/N expansion)

One-loop [Alday-Bissi-Perlmutter 2017, Aprile-Drummond-Heslop-Paul 2019]

Two-loops [Huang-Yuan 2021, Drummond-Paul 2022]

State-of-the-art
Finite coupling

Integrability, localization

Special limits and integrated correlators

Integrability

Operators with large R-charges [Coronado 2018]

mirror corrections surpressed

Localization

Integrated correlation functions [Binder, Chester, Pufu, Wang 2019]

Not exactly the 4pt-function, but contain useful results

Works for finite N [Dorigoni, Chester, Green, Pufu, Wang, Wen,... 2019-2023]

What about other 4-pt functions ?

For example

$$\langle \mathcal{D}_1(x_1)\mathcal{D}_2(x_2)\mathcal{O}_{k_1}(x_3)\mathcal{O}_{k_2}(x_4)\rangle$$

$$\mathcal{D}_i(x_i) = \det(Y_i^I \Phi_I)(x_i)$$

- *Determinant-like operators* known as *giant gravitons*
- Dual to D-branes in the bulk
- Physically they are equally important

What about other 4-pt functions ?

Or

$$\langle \mathcal{D}_1(x_1)\mathcal{D}_2(x_2)\mathcal{D}_3(x_3)\mathcal{D}_4(x_4)\rangle$$

$$\mathcal{D}_i(x_i) = \det(Y_i^I \Phi_I)(x_i)$$

- *Determinant-like operators* known as *giant gravitons*
- Dual to D-branes in the bulk
- Physically they are equally important

State-of-the-art

Much less is known !

Weak coupling

Wick contractions are hard [Corley, Jevicki, Ramgoolam 2001]

One-loop for 4 giants [Vescovi 2021]

Two-loop for 2 giants + 2 single trace [YJ, Komatsu, Vescovi 2019]

Strong coupling

Almost no results

Finite coupling

Absolutely no results

Goal

Compute [to 3-loop order]

$$\langle \mathcal{D}_1(x_1)\mathcal{D}_2(x_2)\mathcal{O}_{k_1}(x_3)\mathcal{O}_{k_2}(x_4)\rangle$$

Motivation 1

To initiate systematic study of these type of correlators

Motivation 2

To provide data for integrability predictions

Techniques

Main Procedure

1. Ansatz for *integrand*

Lagrangian insertion + symmetry

2. Conformal *integrals*

Write the result in a set of integrals

3. Fix coefficients

Perform OPE analysis order by order

Lagrangian insertion

n-point correlation function

$$G_n = \int \mathcal{D}\Phi \, e^{-iS_{\mathcal{N}=4}} \mathcal{O}(x_1) \ldots \mathcal{O}(x_n)$$

Perturbative expansion

$$G_n = G_n^{(0)} + g^2 \, G_n^{(1)} + g^4 \, G_n^{(2)} + \cdots$$

Observation

$$g^2 \frac{\partial G_n}{\partial g^2} = -i \int \mathrm{d}^4 y \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \mathcal{L}_{\mathcal{N}=4}(y) \rangle_{(0)} + O(g^4)$$

Trade loop computations to tree-level computations

Lagrangian insertion

The DDOO correlator

$$G_{\{2,2\}} = \langle \mathcal{D}_1(x_1)\mathcal{D}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle$$

The operators are given by

$$\mathcal{D}_1(x_1) = \det(Y_1^I\phi_I)(x_1), \qquad \mathcal{D}_2(x_2) = \det(Y_2^I\phi_I)(x_2)$$

$$\mathcal{O}_2(x_3) = \mathrm{Tr}(Y_3^I\phi_I)^2(x_3), \qquad \mathcal{O}_2(x_4) = \mathrm{Tr}(Y_4^I\phi_I)^2(x_4)$$

By Lagrangian insertion

$$G^{(\ell)}_{\{2,2\}} = \int \mathrm{d}^4x_5 \ldots \mathrm{d}^4x_{4+\ell} \langle \mathcal{D}_1(x_1)\ldots\mathcal{O}_2(x_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{4+\ell})\rangle_0$$

Integrand

How to compute

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = \langle \mathcal{D}_1(x_1) \ldots \mathcal{O}_2(x_4) \mathcal{L}(x_5) \ldots \mathcal{L}(x_{4+\ell}) \rangle_0$$

- Feynman diagram : Possible, but complicated
- Strategy: Make full use of symmetry

Superconformal symmetry

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234} \times d_{12}^{N-2} \times F(x_i)$$

Integrand

How to compute

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = \langle \mathcal{D}_1(x_1)\ldots\mathcal{O}_2(x_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{4+\ell})\rangle_0$$

- Feynman diagram : Possible, but complicated
- Strategy: Make full use of symmetry

Superconformal symmetry

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234} \times d_{12}^{N-2} \times F(x_i)$$

$$R_{1234} = \frac{(z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{z\bar{z}(1-z)(1-\bar{z})} d_{13}^2 d_{24}^2 x_{13}^2 x_{24}^2$$

A universal factor from Wald identity

[Nirschl, Osborn 2004]

Integrand

How to compute

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = \langle \mathcal{D}_1(x_1)\ldots\mathcal{O}_2(x_4)\mathcal{L}(x_5)\ldots\mathcal{L}(x_{4+\ell})\rangle_0$$

- Feynman diagram : Possible, but complicated
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Superconformal symmetry

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234} \times d_{12}^{N-2} \times F(x_i)$$

Conservation of harmonic variables

$$\text{Total number of } \{Y_1, Y_2, Y_3, Y_4\} = \{N, N, 2, 2\}$$

$$R_{1234} \text{ number of } \{Y_1, Y_2, Y_3, Y_4\} = \{2, 2, 2, 2\}$$

$$d_{ij} = \frac{Y_i \cdot Y_j}{x_{ij}^2}$$

Integrand

How to compute

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = \langle \mathcal{D}_1(x_1) \dots \mathcal{O}_2(x_4) \mathcal{L}(x_5) \dots \mathcal{L}(x_{4+\ell}) \rangle_0$$

- Feynman diagram : Possible, but complicated
- Strategy: Make full use of symmetry

Superconformal symmetry

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234} \times d_{12}^{N-2} \times \boxed{F(x_i)}$$

A rational function independent of *Y*’s

Integrand

How to compute

$$\mathcal{G}^{(\ell)}_{\{2,2\}} = R_{1234} \times d_{12}^{N-2} \times F(x_i)$$

Analysis of divergences

General structure

$$F(x_i) = \frac{P^{(\ell)}(x_i)}{\prod_{\substack{1\le p\le 4\\ 5\le q\le 4+\ell}} x_{pq}^2 \prod_{5\le p<q\le 4+\ell} x_{pq}^2}$$

$P^{(\ell)}(x_i)$: Polynomial of x_{ij}^2,

Degree $2(\ell-1)$ at each point

Integrand

One-loop

$$P^{(1)}(x_i) = \text{const}$$

Two-loop

Linear combinations

$$P^{(2)}(x_i) = c_1\, P_1^{(2)}(x_i) + c_2\, P_2^{(2)}(x_i)$$

$$P_1^{(2)}(x_i) = x_{12}^2 x_{34}^2 x_{56}^2 + S_2 \times S_4 \text{ permutations}$$
$$P_2^{(2)}(x_i) = x_{13}^2 x_{24}^2 x_{56}^2 + S_2 \times S_4 \text{ permutations}$$

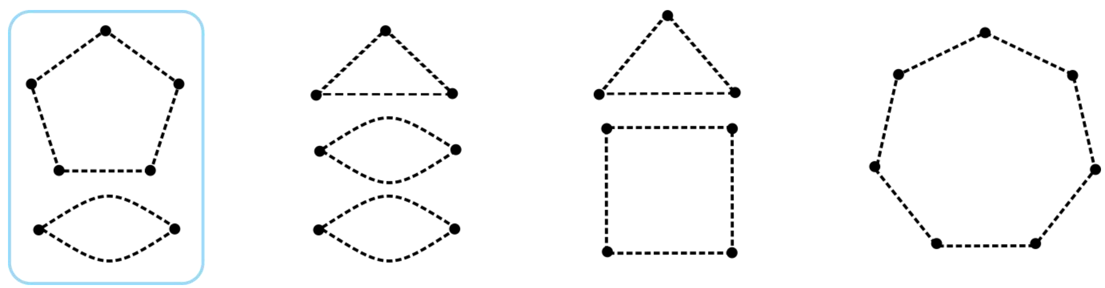
S_2 : permutations of x_1, x_2

S_4 : permutations of x_3, x_4, x_5, x_6

Integrand

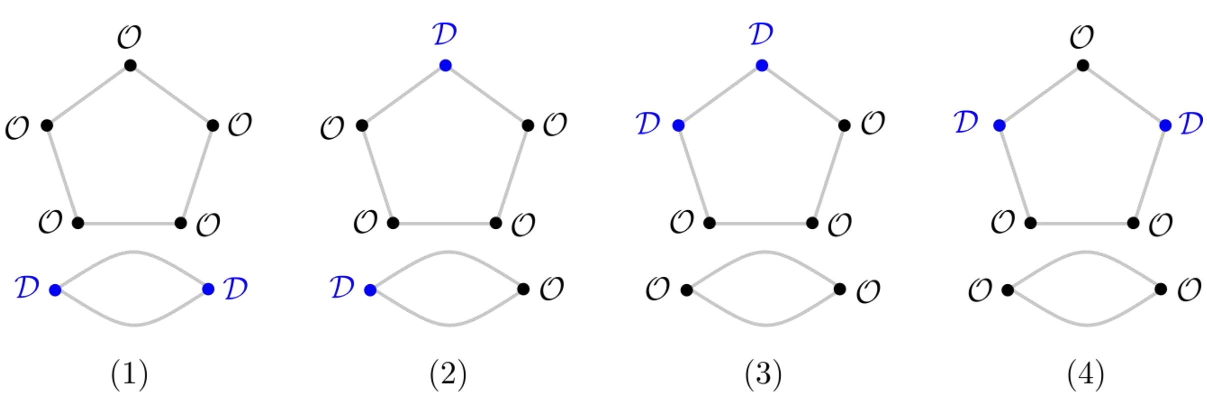
Three-loop

Possible topologies



planarity !

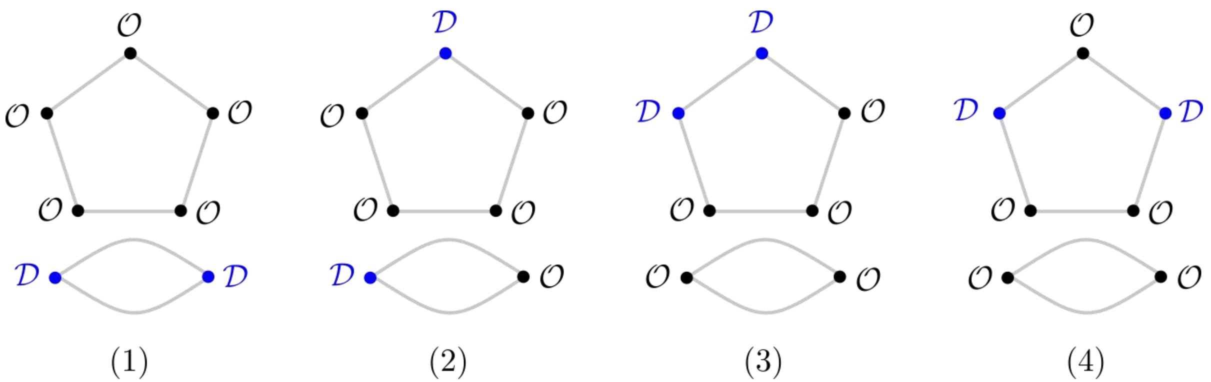
Different polynomials



Integrand

Three-loop

Ansatz



$$P^{(3)} = c_1\, P^{(3)}_1 + c_2\, P^{(3)}_2 + c_3\, P^{(3)}_3 + c_4\, P^{(3)}_4$$

$$P^{(3)}_1(x_i) = (x^4_{12})(x^2_{34}x^2_{45}x^2_{56}x^2_{67}x^2_{73}) + S_2 \times S_5 \text{ permutations}\,,$$
$$P^{(3)}_2(x_i) = (x^4_{13})(x^2_{24}x^2_{45}x^2_{56}x^2_{67}x^2_{72}) + S_2 \times S_5 \text{ permutations}$$
$$P^{(3)}_3(x_i) = (x^4_{67})(x^2_{12}x^2_{23}x^2_{34}x^2_{45}x^2_{51}) + S_2 \times S_5 \text{ permutations}\,,$$
$$P^{(3)}_4(x_i) = (x^4_{67})(x^2_{13}x^2_{32}x^2_{24}x^2_{45}x^2_{51}) + S_2 \times S_5 \text{ permutations}\,.$$

Integrals

Loop integrals

Important for both correlators and amplitudes !

One-loop conformal integral

$$F^{(1)}(z,\bar{z}) = \frac{x_{13}^2 x_{24}^2}{\pi^2} \int \frac{\mathrm{d}^4 x_5}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2}$$

Can be computed analytically

$$F^{(1)}(z,\bar{z}) = \frac{1}{z-\bar{z}} \left(2\mathrm{Li}_2(z) - 2\mathrm{Li}_2(\bar{z}) + \ln(z\bar{z}) \ln \frac{1-z}{1-\bar{z}} \right)$$

Integrals

Two-loop conformal integral

$$F^{(2)}(z,\bar{z}) = \frac{x_{13}^2 x_{24}^2 x_{14}^2}{\pi^4} \int \frac{\mathrm{d}^4 x_5 \mathrm{d}^4 x_6}{x_{15}^2 x_{25}^2 x_{45}^2 x_{56}^2 x_{16}^2 x_{36}^2 x_{46}^2}$$

Can also computed analytically.

Two-loop result

$$G^{(2)}_{\{2,2\}} \propto \left(c_1\, z\bar{z} + \frac{c_2}{4}(1-z)(1-\bar{z}) + \frac{c_2}{4}\right)\left(F^{(1)}(z,\bar{z})\right)^2 + c_2\, F^{(2)}_{z} + 2(c_1 + c_2/4) F^{(2)}_{1-z} + c_2\, F^{(2)}_{z/z-1}$$

with unknown coefficients to be fixed.

Integrals

Three-loop conformal integrals

$$T(1,2;3,4) = \frac{x_{34}^2}{(-4\pi^2)^3}\int \frac{\mathrm{d}^4x_5\mathrm{d}^4x_6\mathrm{d}^4x_7\, x_{17}^2}{(x_{15}^2x_{35}^2)(x_{16}^2x_{46}^2)(x_{37}^2x_{27}^2x_{47}^2)x_{56}^2x_{57}^2x_{67}^2},$$

$$E(1,2;3,4) = \frac{x_{23}^2x_{24}^2}{(-4\pi^2)^3}\int \frac{\mathrm{d}^4x_5\mathrm{d}^4x_6\mathrm{d}^4x_7\, x_{16}^2}{(x_{15}^2x_{25}^2x_{35}^2)x_{56}^2(x_{26}^2x_{36}^2x_{46}^2)x_{67}^2(x_{17}^2x_{27}^2x_{47}^2)},$$

$$L(1,2;3,4) = \frac{x_{34}^4}{(-4\pi^2)^3}\int \frac{\mathrm{d}^4x_5\mathrm{d}^4x_6\mathrm{d}^4x_7}{(x_{15}^2x_{35}^2x_{45}^2)x_{56}^2(x_{36}^2x_{46}^2)x_{67}^2(x_{27}^2x_{37}^2x_{47}^2)},$$

$$gh(1,2;3,4) = \frac{x_{12}^2x_{34}^4}{(-4\pi^2)^3}\int \frac{\mathrm{d}^4x_5\mathrm{d}^4x_6\mathrm{d}^4x_7}{(x_{15}^2x_{25}^2x_{35}^2x_{45}^2)(x_{16}^2x_{36}^2x_{46}^2)(x_{27}^2x_{37}^2x_{47}^2)x_{67}^2},$$

$$H(1,2;3,4) = \frac{x_{14}^2x_{23}^2x_{34}^2}{(-4\pi^2)^3}\int \frac{\mathrm{d}^4x_5\mathrm{d}^4x_6\mathrm{d}^4x_7\, x_{57}^2}{(x_{15}^2x_{25}^2x_{35}^2x_{45}^2)x_{56}^2(x_{36}^2x_{46}^2)x_{67}^2(x_{17}^2x_{27}^2x_{37}^2x_{47}^2)}.$$

Can be computed in terms of single valued harmonic polylogarithms

Three-loop result can be written in terms these integrals

Fixing coefficients

How to fix the unknown coefficients ?

For OOOO-type, use correlator/amplitude duality

Fixing coefficients

How to fix the unknown coefficients ?

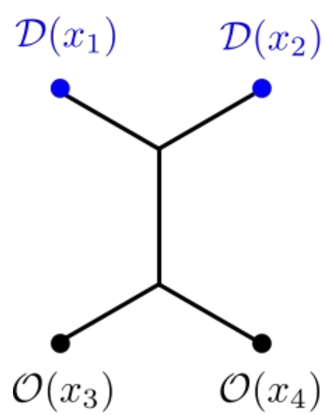
For DDOO-type, not clear whether this duality holds or not

Fixing coefficients

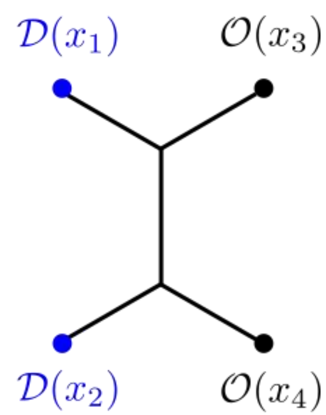
How to fix the unknown coefficients ?

For DDOO-type, not clear whether this duality holds or not

OPE limit analysis

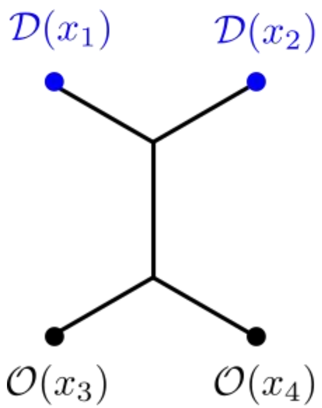


s-channel



t-channel

Fixing coefficients



s-channel

Direct computation

$$\lim_{\substack{x_1\to x_2 \\ x_3 \to x_4}} \frac{G^{(\ell)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = \sum_k c_k \lim_{\substack{u\to 0 \\ v\to 1}} I^{(\ell)}_k$$

OPE analysis

$$\lim_{\substack{x_1\to x_2 \\ x_3 \to x_4}} \frac{G^{(\ell)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = \frac{1}{4^\ell} \mathsf{d}_{\mathcal{K}}(g) \mathsf{c}_{\mathcal{K}}(g) u^{\delta\Delta/2}\Big|_{O(g^{2\ell})}$$

Input

- Leading contribution controlled by Konishi operator
- Anomalous dimension of Konishi (known up to 11-loops)
- Result from previous loop order

Fixing coefficients

Example: two-loop

Direct calculation

$$\lim_{\substack{x_1\to x_2\\ x_3\to x_4}} \frac{G^{(2)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = \frac{3}{2}c_2\,(\ln u)^2 - 8c_2\,\ln u + 12(c_1 + c_2/4)\zeta_3 + 14c_2$$

Perturbative expansion of OPE

$$\mathsf{d}_{\mathcal{K}}(g)\mathsf{c}_{\mathcal{K}}u^{\delta\Delta/2}\Big|_{O(g^4)} = \frac{1}{8}C_0\gamma_1^2(\ln u)^2 + \frac{1}{2}(C_1\gamma_1 + C_0\gamma_2)\ln u + C_2$$

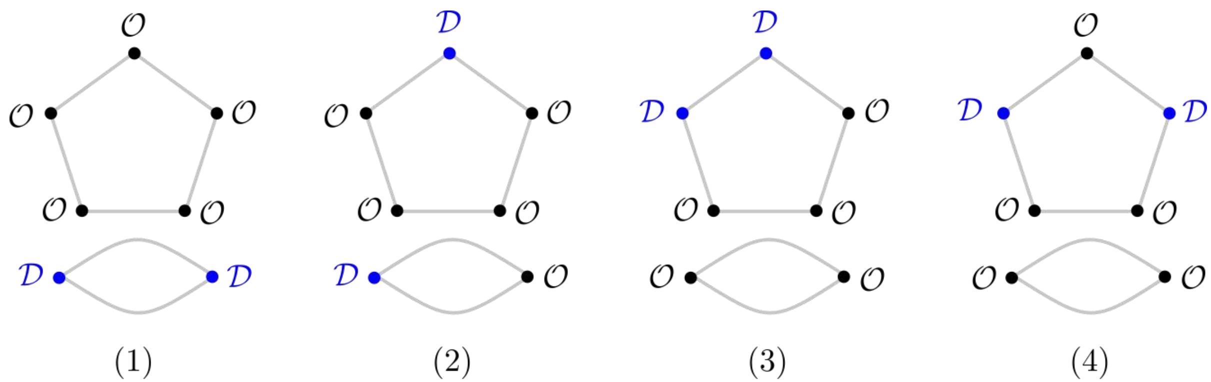
lower loop (tree, one-loop) data

Use the data of the *previous orders* to determine the coefficients

Results

Result

Three-loop ansatz



$$P^{(3)} = c_1\, P_1^{(3)} + c_2\, P_2^{(3)} + c_3\, P_3^{(3)} + c_4\, P_4^{(3)}$$

Using the OPE data recursively, we can fix

$$c_1 = \frac{1}{3}, \quad c_2 = -\frac{1}{3}, \quad c_3 = -\frac{1}{3}, \quad c_4 = \frac{1}{3}$$

Result

In terms of conformal integrals

$$\frac{G^{(3)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = 4\left[gh(1,3;2,4) + gh(1,4;2,3) - gh(1,2;3,4)\right] + 12\left[L(1,3;2,4) + L(1,4;2,3)\right] + 8E(1,2;3,4) + 2\left(1 - \frac{u}{v}\right) H(1,3;2,4) + 2(1-u)H(1,4;2,3)\,.$$

Extract OPE data

OPE coefficient for twist-2 spin-2 operator

$$\mathsf{d}_{\mathcal{K}}(g)\mathsf{c}_{\mathcal{K}}(g) = \frac{1}{3} - 4g^2 + 56g^4 - (768 - 112\zeta_3 + 160\zeta_5)g^6$$

Result

In terms of conformal integrals

$$\frac{G^{(3)}_{\{2,2\}}}{\tilde{R}_{1234}(d_{12})^{N-2}} = 4\left[gh(1,3;2,4) + gh(1,4;2,3) - gh(1,2;3,4)\right] + 12\left[L(1,3;2,4) + L(1,4;2,3)\right] + 8E(1,2;3,4) + 2\left(1-\frac{u}{v}\right)H(1,3;2,4) + 2(1-u)H(1,4;2,3)\,.$$

Extract OPE data

- OPE coefficients for higher spins can also be extracted
- Has interesting relationship with the ones of single trace operators

Conclusions

Multi-point **correlation functions** are important to understand AdS/CFT.

Important results have been made for correlation functions of single trace operators at strong, finite, weak couplings

We initiate the study of operators involving **giant operators** at quantum level

We obtain the result for **DDOO type** of operator up to **3-loop**

Outlook

- **4-loop result**

  Important to go to four-loop order, to test wrapping corrections

- **Longer operators**

  Obtain OPE data for high twist operators

- **Four giants**

  Combining large-N EFT and lagrangian insertion method