A unique γ_5 prescription for the Standard Model in Dimensional Regularization

陈龙

山东大学物理学院

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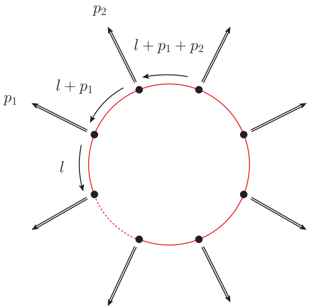
Dimensional Regularization

Dimensional Regularization (DR) for Feynman integrals [Hooft, Veltman 72; Bollini, Giambiagi 72]

$$g^{\mu}_{\ \mu} = D \equiv 4 - 2\epsilon$$

$$\frac{\mathrm{d}^4 l}{(2\pi)^4} \to \mu_{\mathrm{DR}}^{2\epsilon} \frac{\mathrm{d}^D l}{(2\pi)^D}$$



- Invariant under arbitrary loop momentum shifts (→ **IBP**)
- Complex analytic/meromorphic functions of kinematics and D
- Lorentz and gauge symmetries manifestly preserved (in non-chiral theory)
- UV and IR divergences regularized with one (mass-dimensionless) $\epsilon = \frac{4-D}{2}$
  ("*One ring to rule them all*")

The γ_5-issue in DR: absence of an AC-γ_5

Assuming $\{\gamma^\mu , \gamma^\nu\} = 2\, g^{\mu\nu}$ and $\{\gamma^\mu , \gamma_5\} = 0$ in $g^\mu_\mu = D$ dimenisons:

- Bring γ_α through the 4 γ_{μ_i} and use $\gamma_\alpha \gamma^\alpha = D\, \hat{1}$:

$$\mathrm{Tr}\Big[\gamma_\alpha \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma^\alpha \gamma_5\Big] = (D-8)\, \mathrm{Tr}\Big[\gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_5\Big]$$

- Cyclicity of the trace and anticommutativity of γ_5:

$$\mathrm{Tr}\Big[\gamma_\alpha \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma^\alpha \gamma_5\Big] = -D\, \mathrm{Tr}\Big[\gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_5\Big]$$

- Resulting

$$(D-4)\, \mathrm{Tr}\Big[\gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} \gamma_{\mu_4} \gamma_5\Big] = 0$$

No fully anticommuting $\gamma_5 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$ in D≠4 dimensions

The γ_5-issue in DR: chirality-violation by a non-AC-γ_5

Yet, γ_5's anticommutativity $\Delta_5^{ac} \equiv \{\gamma^\mu , \gamma_5\} = 0$ is crucial for **Chiral** Lorentz/Gauge symmetries in 4 dimensions.

- $\Delta_5^{ac} \sim \mathcal{O}(\epsilon) \neq 0$ implies **loss** of chirality-conservation by the regulator in use!

$$\bar{\psi}\, \gamma^\mu \gamma_5 \psi \quad \neq \quad \bar{\psi}_L\, \gamma^\mu \gamma_5 \psi_L \,+\, \bar{\psi}_R\, \gamma^\mu \gamma_5 \psi_R$$

  with $\psi_{L/R} \equiv \frac{1}{2}\big(\hat{1} \pm \gamma_5\big)\, \psi$

- Chirality of a spinor is **not** preserved under Lorentz transformation!

$$\begin{aligned} \big[\gamma_5 \,,\, \sigma^{\mu\nu}\big] &\quad \neq \quad 0 \,, \\ \big[\gamma_5 \,,\, \Lambda(\theta)\big] &\quad \neq \quad 0 \end{aligned}$$

  with $\sigma^{\mu\nu} \equiv \frac{1}{2}\big[\gamma^\mu \,,\, \gamma^\nu\big]$ and $\Lambda(\theta) \equiv \exp\big(\theta_{\mu\nu}\, \sigma^{\mu\nu}\big)$.

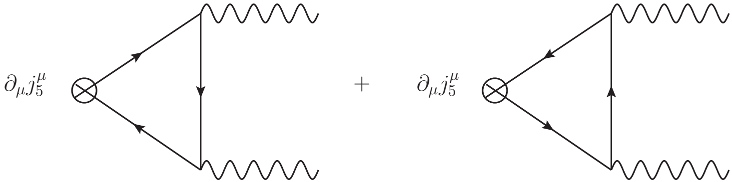
Axial-current conservation and chiral Ward identities are thus expected to be affected at $\mathcal{O}(\epsilon)$

γ_5-issue in DR: the Axial Anomaly

A naive use of a formally anticommuting γ_5 in DR (pretending not vanishing),

$$\mathrm{Tr}\Big[\gamma_5\,\gamma_{\mu_1}\,\cdots\,\gamma_{\mu_N}\Big] \;=\; -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\Big[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\,\gamma_{\mu_1}\,\cdots\,\gamma_{\mu_N}\Big]$$

does not (always) work, most prominently, signified in the failure to reproduce the following famous axial anomaly:



"vanishes" with *translational invariant loop integrals* and an *anticommuting* γ_5.

The perturbative pinch identity

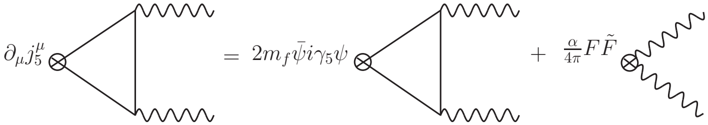
$$(\not{p}_1 + \not{p}_2)\,\gamma_5 = \cancel{(\not{l} + \not{p}_1 - m)\,\gamma_5 + \gamma_5\,(\not{l} - \not{p}_2 - m)} \;+\; 2\,m\,\gamma_5 + \Delta_5^{ac}(l)$$

The Adler-Bell-Jackiw Anomaly

The anomalous axial-vector divergence equation [Adler 69; Bell, Jackiw 69]

$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi = 2m_f\bar{\psi}\, i\gamma_5\,\psi - \frac{\alpha}{4\pi}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\,.$$

Diagrammatically,



- The one-loop VVA-triangle is regularization independant under vector-current conservation
  - Form factor decomposition [Rosenberg 63]
  - Proper shift of linearly-divergant integrals [Adler, Bell, Jackiw 69, 85]
  - Dispersion relations [Dolgov, Zakharov 71]

- The Adler-Bardeen theorem [Adler, Bardeen 69] : “one-loop” exact

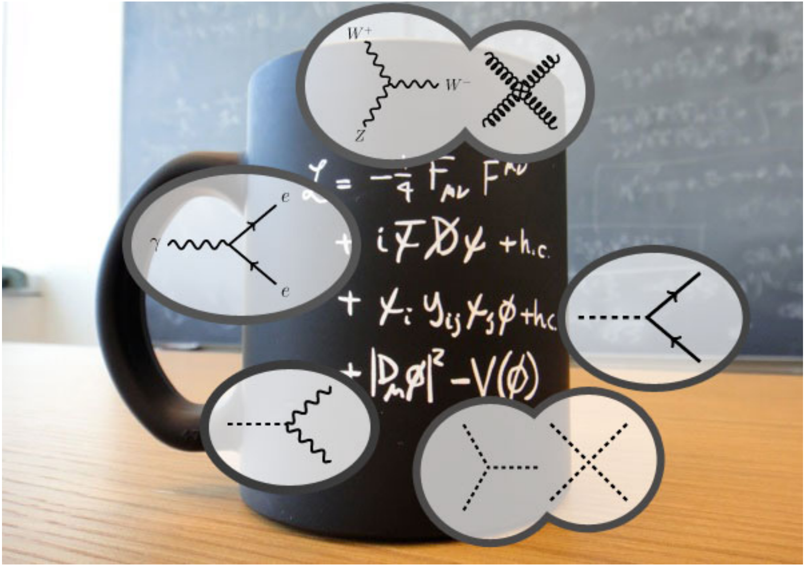
A wonderful recount by Adler [hep-th/0405040]

The intriguing axial anomaly in QFT

- **Gauge/internal anomalies** must cancel !
  - The Standard Model is *anomaly free*
  - Constraints on gauge couplings of New particles
  - Anomaly matching ['t Hooft et al. 80], Spontaneous chiral symmetry breaking ...
- **Global/external anomalies** are allowed and important
  - $\pi \to \gamma\gamma$ decay [Steinberger 49; Sutherland, Veltman 67; Adler 69; Bell, Jackiw 69]
    $\partial_\mu J^\mu_{5,(3)} = f_\pi \, m_\pi^2 \, \pi_{(3)} \; - \; \frac{\alpha}{8\pi} \, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$
  - $U(1)_A/\eta'$ problem [Weinberg 75; 't Hooft 76]
  - Strong CP problem and Axion [Peccei, Quinn 77] ...
- **Practical applications of renormalization of anomalous $\bar{\psi}\gamma^\mu\gamma_5\psi$**
  - Treatment of the singlet axial-current operator in heavy-top EFT [Chetyrkin, Kühn 91 93; **LC**, Czakon, Niggetiedt 21]
  - Structure of the *non-decoupling* heavy-quark-mass logarithms [Collins, Wilczek, Zee 78; Chetyrkin, Kühn 93; **LC**, Czakon 22]
  - Polarized structure and splitting functions [Matiounine, Smith, Neerven; Moch, Vermaseren, Vogt; Blümlein, Marquard, Schneider, Schönwald; Tarasov, Venugopalan...]
  - ......

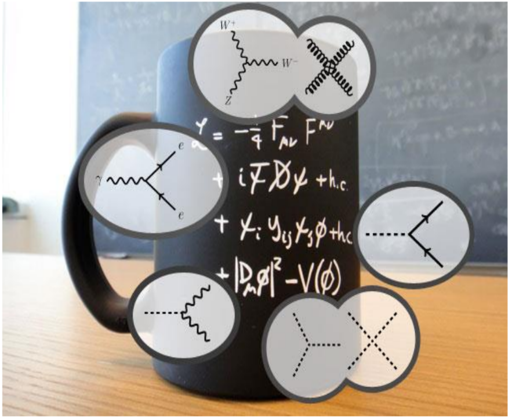
The SM Lagrangian

The Standard Model is a Quantum Field Theory based on the *chiral* gauge group $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ with spontaneous symmetry breaking by Higgs potential.



The SM Lagrangian at a closer look

The SM Lagrangian



$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu +$$
$$\bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2} \partial_\nu Z^0_\mu \partial_\nu Z^0_\mu -$$
$$\frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2} \partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- -$$
$$\frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2} M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h -$$
$$igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) +$$
$$A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu +$$
$$g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 +$$
$$2H\phi^+\phi^-] - \frac{1}{8} g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- +$$
$$2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac{1}{2} g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2} g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] +$$
$$\frac{1}{2} g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- -$$
$$W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) -$$
$$\frac{1}{4} g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4} g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 -$$
$$1)^2\phi^+\phi^-] - \frac{1}{2} g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2} ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$\frac{1}{2} g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2} ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 -$$
$$1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}_j^\lambda(\gamma\partial +$$
$$m_u^\lambda)u_j^\lambda - \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + igs_wA_\mu\left[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)\right] +$$
$$\frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(\tfrac{4}{3} s_w^2 - 1 - \gamma^5)u_j^\lambda) +$$
$$(\bar{d}_j^\lambda\gamma^\mu(1 - \tfrac{8}{3} s_w^2 - \gamma^5)d_j^\lambda)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)] +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) +$$
$$\phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1 -$$
$$\gamma^5)d_j^\kappa) + m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) -$$
$$m_u^\kappa(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1-\gamma^5)u_j^\kappa] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda) -$$
$$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 +$$
$$\bar{Y}\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) +$$
$$igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) +$$
$$igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \frac{1}{2} gM[\bar{X}^+X^+H +$$
$$\bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ -$$
$$\bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + \frac{1}{2} igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]$$

The HV-variants based on non-AC γ_5

The **HV/BM** [72, 79] prescription of γ_5 in dimensional regularization:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

$$\gamma_\mu\gamma_5 \to \frac{1}{2}\Big(\gamma_\mu\gamma_5 - \gamma_5\gamma_\mu\Big) = -\frac{i}{6}\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma\,,$$

$$\Delta_5^{ac} \equiv \{\gamma^\mu\,,\,\gamma_5\} \sim \mathcal{O}(\epsilon) \neq 0\,.$$

"Symmetrization" needed to have a **hermitian** axial-current [Akyeampong, Delbourgo 77; Fujii, Ohta, Taniguchi 81]

Advantages:

- Unambiguous expressions for any Feynman diagrams
- Easy to implement reliably on computer
- In principle applicable to all-order (SM)
- Particularly simple for QCD corrections (Additional renormalisation needed currently known to $\mathcal{O}(a_s^5)$ in $\overline{\mathrm{MS}}$ [**LC**, Czakon 23])

Disadvantages:

- Loss of γ_5 anticommutativity $\to$ Spurious Ward-Takahashi identities violation

  [Bardeen et al 72, Chanowitz et al, Trueman, Kodaira 79]
- Additional γ_5-vertex renormalizations with a-priori unknown coefficients
  $\left[J_5^\mu\right]_R = Z_J\,\bar{\psi}_B\,[\gamma^\mu, \gamma_5]/2\,\psi_B$
  (systematically obtainable to any pertubative order;

  but more structrues needed for SM)
- Traces with high γ powers in case of multiple γ_5

Operator Renormalization in Larin's prescription

The all-order axial-anomaly equation [Adler 69; Adler, Bardeen 69]

$$\left[\partial_\mu J_5^\mu\right]_R = a_s\, n_f\, \mathrm{T}_F\, \left[F\tilde{F}\right]_R$$

with $F\tilde{F} \equiv -\epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$ in QCD with n_f massless quarks.

- The renormalization of the operators involved: [Adler 69; Espriu,Tarrach 82; Breitenlohner,Maison,Stelle 84; Bos 92; Larin 93 ...]

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln\mu^2} \begin{pmatrix} \left[\partial_\mu J_5^\mu\right]_R \\ \left[F\tilde{F}\right]_R \end{pmatrix} = \frac{\mathrm{d}}{\mathrm{d}\ \ln\mu^2} \begin{pmatrix} Z_J & 0 \\ Z_{FJ} & Z_{F\tilde{F}} \end{pmatrix} \cdot \begin{pmatrix} \left[\partial_\mu J_5^\mu\right]_B \\ \left[F\tilde{F}\right]_B \end{pmatrix} = \begin{pmatrix} \gamma_s & 0 \\ \gamma_{FJ} & \gamma_{F\tilde{F}} \end{pmatrix} \cdot \begin{pmatrix} \left[\partial_\mu J_5^\mu\right]_R \\ \left[F\tilde{F}\right]_R \end{pmatrix}$$

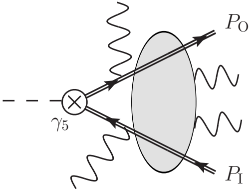
- The so-called Larin's prescription [Larin, Vermaseren 91; Larin 93]:
  - $\epsilon^{\mu\nu\rho\sigma}$ treated outside R-operation formally in D dimensions [Larin, Vermaseren 91; Zijlstra, Neerven 92]
  - Take $Z_{F\tilde{F}}$ and Z_{FJ} in $\overline{\mathrm{MS}}$ then determine $Z_J \equiv Z_5^f\, Z_5^{ms}$ by ABJ eq. with ϵ-independent Z_5^f

- $Z_{F\tilde{F}} = Z_{a_s}$ verified to 4-loop in QCD [Ahmed, **LC**, Czakon 21] later proved exactly [Lüscher, Weisz 21]

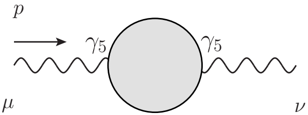
- $Z_J = Z_5^f\, Z_5^{ms}$ with Z_5^{ms} at $\mathcal{O}(a_s^5)$ and Z_5^f at $\mathcal{O}(a_s^4)$ currently known [**LC**, Czakon 21, 22]

No γ_5-odd Dirac-trace, No problem

- If γ_5 is on **open** fermion lines, shift γ_5 anticommutatively into the **external** spinor or projector. [Bardeen 72]



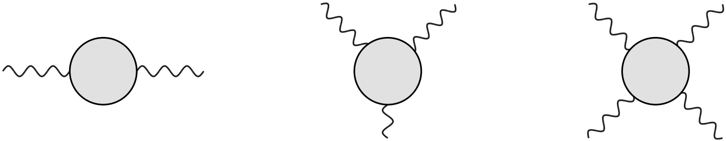
- If γ_5 appear on **closed** fermion chain in **even** numbers, anticommute and $\gamma_5^2 = \hat{1}$ [Chanowitz et.al 79; Gottlieb et.al 79].



γ_5-odd Dirac-trace has no overall UV divergence

What to do with a **closed** fermion chain with **odd** number of γ_5?

- γ_5-odd Dirac-trace has no **overall** UV divergence in SM!



(**No** gauge-boson self-interaction vertices with $\epsilon^{\mu\nu\rho\sigma}$ in Feynman Rules for SM)

- The guiding principles [Kreimer 94]:
  - anticommute γ_5 *outside* of the vertex loop correction with sub-UV-divergences.
  - Sub-UV-divergences must be computed **unambiguously** for all diagrams with (Charge-conjugation and Bose) symmetry ensured.

Why look at the AC-γ_5 prescription again

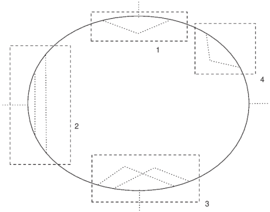
- If a thorough usage of γ_5 avoids the additional renormalizations, does $\left[\partial_\mu J^\mu_{A_5,s}\right]_B = a_s^B\, n_f\, \mathrm{T}_F\, \left[F\tilde{F}\right]_B$ hold **automatically**?

> **4 Verification of the Adler Bardeen Theorem in this Scheme** [Kreimer 94]
>
> Here we will give a short proof of the Adler Bardeen Theorem [9]. We can restrict our

- Is it necessary to use the fancy ***non-cyclic* trace** [Kreimer 90, 94; Körner, Kreimer, Schilcher 92]?

- Although having improper points in [Körner, Kreimer, Schilcher 92] corrected, [Kreimer 94] seems to cover only the configuration:

How to **define** the fermion chain **unambiguously** in an **algorithmic** way in general Feynman diagrams in SM?

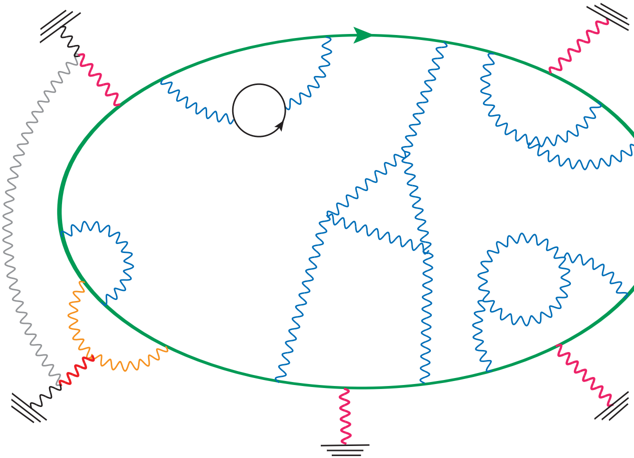


- Does one have to treat $\epsilon^{\mu\nu\rho\sigma}$ in 4 dimensions?

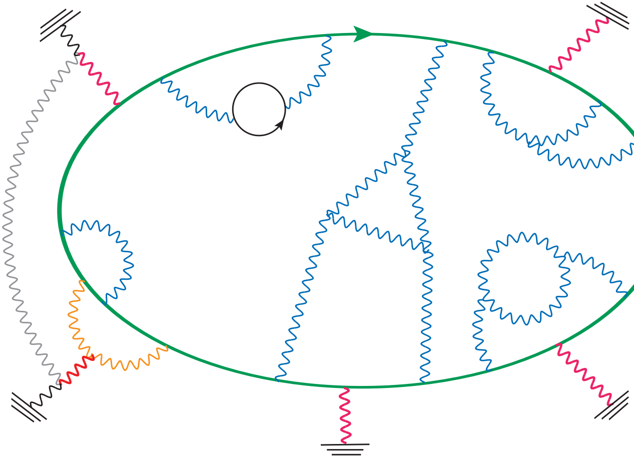
- Does the IR-divergence in loop amplitudes matter?

The Question:

Given a closed fermion chain possibly embedded in **a big Feynman diagram**, what are **its external legs** relevant for defining the γ_5-trace, and how to identify them



(I) Isolate the target fermion loop

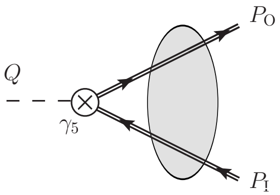


My Answer:

The *minimal* cut to isolate the target fermion loop into a minimal 1PI diagram (containing this fermion loop) with each of its external momenta equal to the difference between certain pair of fermion propagators, plus a **minimal** number of complementary diagrams

(II) maximal 1PI open $G_{5\mathrm{ff}}$

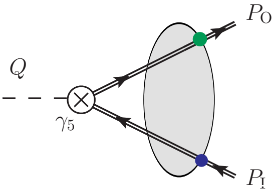
Given a diagram G with one ***external*** (axial-field) A_5 with momentum insertion Q on a fermion chain F_c, identify maximal 1PI open $G_{5\mathrm{ff}}$ in the following **algorithmic** way [LC 23].



- Find all pairs of propagators of F_c satisfying $P_{\mathrm{O}} - P_{\mathrm{I}} = Q$, each qualifies as a *two-fermion cut* (TFC) of F_c;
- Examine each TFC, exclude those leading to prop. with the same momenta (as the cut) in the cut-subchain with A_5, effectively ensuring **1PI** condition;
- Pick up the TFC resulting the **largest 1PI** cut-subchain $G_{5\mathrm{ff}}$ with A_5, identified, respectively, as the **I/O-leg** according to *fermion-flow* direction.

F_c is written out in direction *against* fermion charge flow, but is otherwise allowed to start from ***any*** (!) vertex or propagator cyclically permuted.

(III-1) Symmetrization



- The final expression $\bar{F}_c$ for the fermion chain F_c is *defined* as:

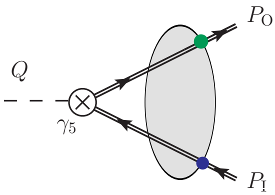
$$\bar{F}_c \equiv \frac{1}{2}\left(F_c^{A_5\to\mathrm{Ih}} + F_c^{A_5\to\mathrm{Ot}}\right)$$

$F_c^{A_5\to\mathrm{Ih}}$ is obtained from F_c by a.c. shifting γ_5 from A_5 to the *head* of the I-leg propagator $S_F^{\mathrm{I}}(P_{\mathrm{I}})$, subsequently replaced by

$$-\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma \equiv \hat{\gamma}_5\,.$$

Similarly for $F_c^{A_5\to\mathrm{Ot}}$, albeit with *head* replaced by *tail*.

- The above symmetrization is **necessary** to ensure Furry's theorem, just like $\gamma_\mu\gamma_5 \to \frac{1}{2}\left(\gamma_\mu\gamma_5 - \gamma_5\gamma_\mu\right)$ in HV.



- If an even number of A_5 on the same F_c in G, $\gamma_5^2 = \hat{1}$ results in a unique trace expression free of γ_5 for F_c.
- If an odd number $N(\geq 3)$ of A_5 on F_c, **another** level of symmetrization is needed to reach an ***unambiguous*** trace for F_c:

$$\bar{\bar{F}}_c \equiv \frac{1}{N} \sum_{i=1}^{N} \bar{F}_c^{[i]} \, .$$

Unlike the original Kreimer prescription, we demand the average for each F_c, *irrespective* of whether A_5 coupled to *identical* gauge or scalar bosons.

(IV) Levi-Civita tensors

Multiple $\epsilon^{\mu\nu\rho\sigma}$ appear if several A_5 on **different (disconnected)** F_c or from projectors.

- $\epsilon^{\mu\nu\rho\sigma}$ **defined only in 4 dimensions**, just like γ_5, in compatible with $\gamma_5^2 = \hat{1}$.

- Lack of the 4-dimensional Schouten identity $\rightarrow$ contraction ordering matters [Breitenlohner, Maison 77; Siegel 80].

$$\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'} = \mathrm{Det}\left[g^{\alpha\alpha'}\right], \quad \text{with } \alpha \in \{\mu,\nu,\rho,\sigma\} \text{ and } \alpha' \in \{\mu',\nu',\rho',\sigma'\},$$

- Still possible to manipulate $\epsilon^{\mu\nu\rho\sigma}$ with D($\neq 4$)-dim. indices for specific problems [Larin, Vermaseren 91; Zijlstra, Neerven 92; Moch, Vermaseren, Vogt 15]
  - No need to implement dimensional splitting
  - Contraction can be done before completing tensor loop integrals in D

In our calculation of vacuum-gg elements: no need $\gamma_5^2 = \hat{1}$ and only one pair of $\epsilon^{\mu\nu\rho\sigma}$.

(IV) Levi-Civita tensors

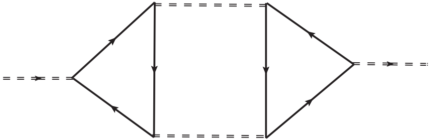
- In our γ_5 prescription, pairs of $\epsilon^{\mu\nu\rho\sigma}$ to be contracted must come from **two independent internal fermion chains** or external "bosonic" projectors, hence no compatibility issue with $\gamma_5^2 = \hat{1}$ (applied always on the same F_c).

- To eliminate the contraction-order ambiguity in $\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\,\cdots$, partition them into two subsets:
  - For external $\epsilon^{\mu\nu\rho\sigma}$, fix an arbitrary contraction-order adopted consistently in all bare amplitudes (which can also be left undone to the very end!)
  - For internal $\epsilon^{\mu\nu\rho\sigma}$, take the symmetric average over all possible pairings (as suggested by Prof. **Y.Q. Ma**), e.g.

$$\left[\epsilon_1\,\epsilon_2\,\epsilon_3\,\epsilon_4\right] \equiv \frac{1}{3}\Big(\left[\epsilon_1\,\epsilon_2\right]\left[\epsilon_3\,\epsilon_4\right] + \left[\epsilon_1\,\epsilon_3\right]\left[\epsilon_2\,\epsilon_4\right] + \left[\epsilon_1\,\epsilon_4\right]\left[\epsilon_2\,\epsilon_3\right]\Big)$$

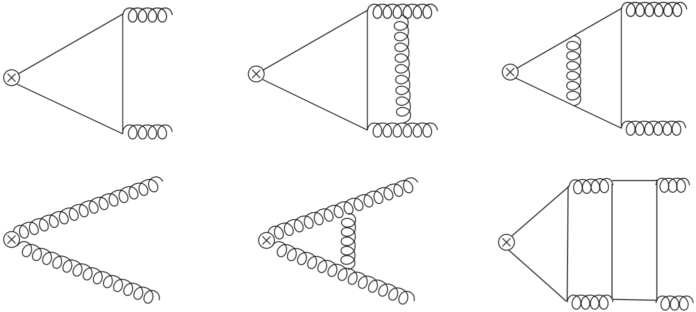
- The resulting spacetime-metric tensors $g_{\mu\nu}$ are set D**-dimensional**.

No issue is expected in application to SM at least to 3-loop:
a *superficially UV-div. 1PI amplitude* with two internal $\epsilon^{\mu\nu\rho\sigma}$ starts from 3-loop which is UV-finite.



VVA diagrams calculated in Kreimer-variant

$$\Gamma^{\mu\mu_1\mu_2}_{lhs}(p_1,p_2) \equiv \int d^4x d^4y\, e^{-ip_1\cdot x - iq\cdot y}\, \langle 0|\hat{\mathrm{T}}\,[J^{\mu}_5(y)\, A^{\mu_1}_a(x)\, A^{\mu_2}_a(0)]\,|0\rangle|_{\mathrm{amp}}$$

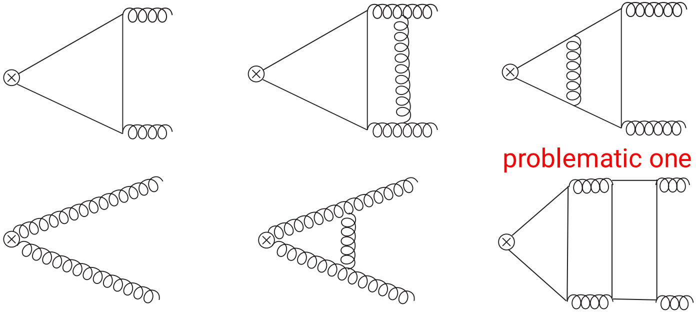


According to Kreimer, seemingly among the common lore, one expects

$$\mathrm{M}_{lhs} - n_f\, \mathrm{T}_F\, \mathrm{M}_{rhs}\Big|_{\epsilon=0} = 0$$

An observation on VVA diagrams in Kreimer-variant

$$\Gamma_{lhs}^{\mu\mu_1\mu_2}(p_1, p_2) \equiv \int d^4x d^4y\, e^{-ip_1\cdot x - iq\cdot y}\, \langle 0|\hat{\mathrm{T}}\,[J_5^{\mu}(y)\, A_a^{\mu_1}(x)\, A_a^{\mu_2}(0)]\,|0\rangle|_{\mathrm{amp}}$$



To our surprise (!), we find

$$\mathrm{M}_{lhs} - n_f\, \mathrm{T}_F\, \mathrm{M}_{rhs}\Big|_{\epsilon=0} = a_s^2\, 4\, n_f\, C_F + \mathcal{O}(a_s^3)$$

Interpretation and checks of the observation in Kreimer-variant

The discrepancy

$$\mathrm{M}_{lhs} - n_f \, \mathrm{T}_F \, \mathrm{M}_{rhs}\Big|_{\epsilon=0} = a_s^2 \, 4 \, n_f \, C_F \, + \mathcal{O}(a_s^3)$$

can be manually compensated by

$$\begin{aligned} \left[J^{\mu}_{A_5,s}\right]_R &= Z_s^{A_5} \, \left[J^{\mu}_{A_5,s}\right]_B \, + Z_{JK} \, K^{\mu} \\ Z_{JK} &= -\frac{1}{2} n_f \, C_F \, a_s^2 \, + \mathcal{O}(a_s^3) \, . \end{aligned}$$

Kreimer scheme itself **does not** offer a *constructive* proof for the Adler-Bardeen theorem, the ABJ equation does not hold automatically in bare form in this scheme.

- The same issue appears with massive quarks at on-shell kinematics.
- No such kind of issue observed if the axial-current vertex is replaced by pseudo-scalar vertex.
- The extra pieces cancel in *non-anomalous* combination of contributions from isospin doublet (e.g. top and bottom quarks)

AC v.s. non-AC γ_5

The HV-scheme variants

Advantages:

- Unambiguous expressions for any Feynman diagrams
- Easy to implement reliably on computer
- In principle applicable/doable to any order

Disadvantages:

- Loss of γ_5 anticommutativity $\to$ Spurious WTI violation
- Additional γ_5-vertex renormalizations with a-priori unknown coefficients
- Traces with high γ powers in case of multiple γ_5

Our **revision** of Kreimer scheme

Competitive features:

- Unambiguous expressions for any (non-anomalous) Feynman amplitude in SM
- Easy to implement reliably on computer
- Applicability to **anomaly-free** SM (at least to 3-loop) and to all-order (still a conjecture!)

Advantages:

- **No** spurious violation of non-anomalous WTIs
- **No** need for additional γ_5-vertex renormalizations for *non-anomalous* amplitudes (anomalous diagrams with VVA-type subgraphs needs manual corrections [LC 23])
- **No** traces with exploding γ powers generated by multiple γ_5 insertion

Summary and Outlook

☑ Despite the known issue of γ_5, similarly $\epsilon^{\mu\nu\rho\sigma}$, in D$\neq$4 dimensions, practical prescriptions have been formulated that work successfully.

☑ For the *anomaly-free* Standard Model, our **revision** of Kreimer scheme (with few modification and extension) shall work (without ref. to the fancy *non-cyclic* trace), at least to 3-loop, albeit still a conjecture.

☑ For quantities with external axial anomalies, and in applications to EFTs involving axial currents, γ_5-vertices may need additional renormalization even if treated using the revised AC-γ_5 scheme.

☑ If only QCD corrections are studied (e.g. in EFTs), then NAC-γ_5 seems to be more convenient (the 4-loop R.C.s known).

☒ It is desirable to have the above conjecture scrutinized more stringently.

Summary and Outlook

- ☑ Despite the known issue of γ_5, similarly $\epsilon^{\mu\nu\rho\sigma}$, in D$\neq$4 dimensions, practical prescriptions have been formulated that work successfully.

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谢谢！

Backup Slides

Renormalization of $\bar{\psi}_q \gamma^\mu \gamma_5 \psi_q$

What is $[J^\mu_{5,\,q}]_R$ as in $[J^\mu_5]_R = \sum_{i=1}^{n_l} [J^\mu_{5,\,i}]_R$?

$$[J^\mu_5]_R = Z_J J^\mu_5 = (Z_{ns} + n_l Z_s) \sum_{i=1}^{n_l} \bar{\psi}^B_i \gamma^\mu \gamma_5 \psi^B_i$$

$$\Downarrow$$

$$[J^\mu_{5,\,q}]_R \neq \cancel{(Z_{ns} + n_l Z_s) \bar{\psi}^B_q \gamma^\mu \gamma_5 \psi^B_q}$$

$$= \Big(Z_{ns} \bar{\psi}^B_q \gamma^\mu \gamma_5 \psi^B_q + Z_s \sum_{i=1}^{n_l} \bar{\psi}^B_i \gamma^\mu \gamma_5 \psi^B_i \Big)$$

The renormalized singlet contribution featuring EW a_b:

$$\mathrm{F}^A_{s,b}(a_s, m_t, \mu) = \langle 0 | [J^\mu_{5,b}]_R | b\bar{b} \rangle |_{singlet}$$

$$= Z_{ns} Z_2 \mathrm{F}^A_{s,b}(\hat{a}_s, \hat{m}_t) + Z_s Z_2 \Big(\mathrm{F}^A_{ns}(\hat{a}_s, \hat{m}_t) + \sum_{i=1}^{n_l} \mathrm{F}^A_{s,i}(\hat{a}_s, \hat{m}_t) \Big)$$

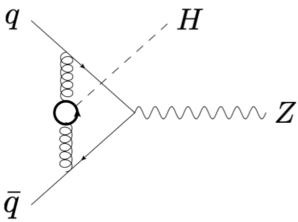
Note: $\mu^2 \frac{\mathrm{d}Z_{ns}}{\mathrm{d}\mu^2} = 0$ while $\mu^2 \frac{\mathrm{d}Z_s}{\mathrm{d}\mu^2} = \bar{\gamma}_s (Z_{ns} + n_l Z_s)$.

Alternative renormalization prescriptions

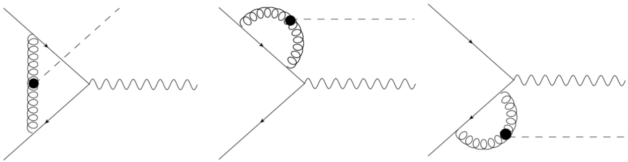
- $\overline{\text{MS}}$ scheme: $Z_5^{ms}\,[J_5^\mu]_B$
  - The ABJ (AWTI) is not respected
  - $\mathrm{F}^A_{s,b}(a_s, m_t) - \mathrm{F}^A_{s,t}(a_s, m_t)$ is still anomalous
- Chetyrkin scheme: $Z^f_{ns}\, Z_5^{ms}\,[J_5^\mu]_B$
  - The ABJ (AWTI) is not respected
  - $\mathrm{F}^A_{s,b}(a_s, m_t) - \mathrm{F}^A_{s,t}(a_s, m_t)$ is non-anomalous (correct)
- Larin scheme: $Z^f_s\, Z_5^{ms}\,\partial_\mu\,[J_5^\mu]_B = a_s\, n_f\, T_F\,(Z_{F\tilde{F}}\,[J_5^\mu]_B + \partial_\mu\,[J_5^\mu]_B)$
  - The ABJ (AWTI) is respected and $\gamma_s \neq 0$
  - $\mathrm{F}^A_{s,b}(a_s, m_t) - \mathrm{F}^A_{s,t}(a_s, m_t)$ is non-anomalous (correct)
- Renormalization-group invariant (RGI) scheme: $Z_{ext}(a_s) \equiv \hat{P}\exp\left(\int_0^{a_s} \frac{-\bar{\gamma}_s(a)}{\beta(a)}\,\frac{\mathrm{d}a}{a}\right)$
  - The ABJ (AWTI) is respected and $\gamma_s = 0$ (no more running!)
  - $Z^{\text{RGI}}_{F\tilde{F}} \neq Z_{\alpha_s}$
  - No more explicit $\ln(\mu^2/m_t^2)$ when expressed w.r.t $\alpha_s(\mu = m_t)$ [LC, Czakon 22].

An Amusing Pitfall in Applying non-AC γ_5 to $q\bar{q} \to ZH$

We **observe** that for the top-loop-induced (y_t-dependant) $q\bar{q} \to ZH$ with a non-AC γ_5: the usual $Z_5^f Z_5^{ms} \bar{\psi}\gamma^\mu\gamma_5\psi$ prescription works for



but not for their counter-parts using effective Higgs-gluon vertex: [Ahmed, Bernreuther, **LC**,Czakon 20]



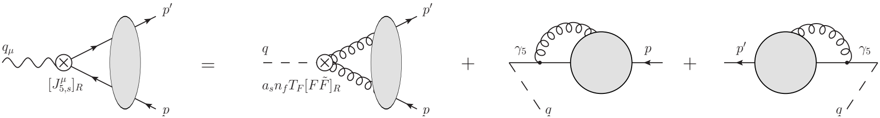
One needs additional counterms on top of the usual renormalized axial-current!

Vacuum-Quark matrix element and AWTI

Much more efficient to extract Z_J by using the off-shell Ward-Takahashi identity for an axial current with a non-anticommuting γ_5 [LC,Czakon 21]

The anomalous Ward-Takahashi identity:

$$q_\mu \, \Gamma^\mu_{5,s}(p', p) = -a_s \, n_f \, \mathrm{T}_F \, \Lambda(p', p) \, + \, \gamma_5 \, \hat{S}^{-1}(p) \, + \, \hat{S}^{-1}(p') \, \gamma_5 \, ,$$



- q can not be 0 to have a non-zero anomaly
- Either p or p' should be 0 to reduce to the propagator-type integrals
- γ_5 on the RHS does not require any renormalization!

Z_5^{ms} up to $\mathcal{O}(a_s^5)$ from 4-loop calculations

- The anomalous dimension of the $[J_5^\mu]_R$:

$$\gamma_s \equiv \frac{\mathrm{d}\ \ln Z_s}{\mathrm{d}\ \ln \mu^2} = \frac{\mathrm{d}\ \ln Z_s^{ms}}{\mathrm{d}\ \ln \mu^2} + \frac{\mathrm{d}\ \ln Z_s^{f}}{\mathrm{d}\ \ln \mu^2}$$
$$= \gamma_s^{ms} + \beta\, \frac{\mathrm{d}\ \ln Z_s^{f}}{\mathrm{d}\ \ln a_s} - \epsilon\, \frac{\mathrm{d}\ \ln Z_s^{f}}{\mathrm{d}\ \ln a_s}\,.$$

- Z_5^{ms} at $\mathcal{O}(a_s^5)$ using ABJ equation with $Z_{F\tilde{F}} = Z_{a_s}$ [LC, Czakon 22]

$$\gamma_s^{ms} = a_s\, n_f\, \mathrm{T}_F\, \gamma_{FJ} - \beta\, \frac{\mathrm{d}\ \ln Z_s^{f}}{\mathrm{d}\ \ln a_s}\,.$$

  - γ_s^{ms} at $\mathcal{O}(a_s^5)$ requires only γ_{FJ} and Z_s^f up to 4-loop (from AWTI) [LC, Czakon 21, 22]
  - Z_5^f at $\mathcal{O}(a_s^5)$ not known yet

The non-Abelian Adler-Bardeen theorem

The equality verified to 4-loop in QCD [Ahmed,**LC**,Czakon 21]:

$$Z_{F\tilde{F}} = Z_{a_s}$$

The ABJ equation in QCD in terms of the *bare* fields:

$$\left(Z_J - n_f \, \mathrm{T}_F \, a_s \, Z_{FJ}\right) \left[\partial_\mu J_5^\mu\right]_B = \hat{a}_s \, n_f \, \mathrm{T}_F \, \left[F\tilde{F}\right]_B$$

- In an Abelian theory with Pauli-Villar regularization (with an AC γ_5), the coefficient is 1 to all orders [Adler 69; Adler, Bardeen 69]
- The coefficient is not 1 with a NAC γ_5 in DR in QCD, but the LHS current remains ***RG-invariant*** (albeit in D=4 limit):

$$\gamma_{F\tilde{F}} = -\mu^2 \frac{\mathrm{d} \ln a_s}{\mathrm{d}\mu^2} = -\beta \,, \quad \gamma_s|_{\epsilon=0} = n_f \, \mathrm{T}_F \, a_s \, \gamma_{FJ} \,.$$

- An all-order argument of the non-Abelian extension was sketched [Breitenlohner, Maison, Stelle 84]; A proof is completed only recently [Lüscher, Weisz 21]
- However, $Z_J = Z_5^f \, Z_5^{ms}$ needs to be computed order by order ...
  Z_5^f at $\mathcal{O}(a_s^3)$ from 4-loop VVA-amplitude [Ahmed,**LC**,Czakon 21]

A Simplified Recipe Valid for SM @ one-loop

Up to one-loop in SM:

- γ_5 **open** fermion chain: pulled outside to spinors
- Even γ_5 on **closed** fermion chain: anticommute and $\gamma_5^2 = \hat{1}$.
- Odd γ_5 on **closed** fermion chain: apply $\gamma_5^2 = \hat{1}$ first and replace the remaining single γ_5-vertex (pseudo-scalar or axial-current) as in HV/BM/Larin scheme

Absence of divergent γ_5-odd fermion-one-loop in SM $\rightarrow$ averaging or not is irrelevant in 4 dimensions

$$
\begin{aligned}
&-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \\
&\bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \\
&\tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \tfrac{1}{2}m_h^2 H^2 - \partial_\mu\varphi^+\partial_\mu\varphi^- - M^2\varphi^+\varphi^- - \\
&\tfrac{1}{2}\partial_\mu\varphi^0\partial_\mu\varphi^0 - \tfrac{1}{2c_w^2}M\varphi^0\varphi^0 - \beta_h[\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \varphi^0\varphi^0 + 2\varphi^+\varphi^-)] + \tfrac{2M^4}{g^2}\alpha_h - \\
&igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
&W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + \\
&A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
&g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + \\
&g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\varphi^0\varphi^0 + \\
&2H\varphi^+\varphi^-] - \tfrac{1}{8}g^2\alpha_h[H^4 + (\varphi^0)^4 + 4(\varphi^+\varphi^-)^2 + 4(\varphi^0)^2\varphi^+\varphi^- + 4H^2\varphi^+\varphi^- + \\
&2(\varphi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\varphi^0\partial_\mu\varphi^- - \varphi^-\partial_\mu\varphi^0) - \\
&W^-_\mu(\varphi^0\partial_\mu\varphi^+ - \varphi^+\partial_\mu\varphi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\varphi^- - \varphi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\varphi^+ - \varphi^+\partial_\mu H)] + \\
&\tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\varphi^0 - \varphi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\varphi^- - W^-_\mu\varphi^+) + igs_w MA_\mu(W^+_\mu\varphi^- - \\
&W^-_\mu\varphi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\varphi^+\partial_\mu\varphi^- - \varphi^-\partial_\mu\varphi^+) + igs_w A_\mu(\varphi^+\partial_\mu\varphi^- - \varphi^-\partial_\mu\varphi^+) - \\
&\tfrac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\varphi^0)^2 + 2\varphi^+\varphi^-] - \tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\varphi^0)^2 + 2(2s_w^2 - \\
&1)^2\varphi^+\varphi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\varphi^0(W^+_\mu\varphi^- + W^-_\mu\varphi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\varphi^- - W^-_\mu\varphi^+) + \\
&\tfrac{1}{2}g^2 s_w A_\mu\varphi^0(W^+_\mu\varphi^- + W^-_\mu\varphi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\varphi^- - W^-_\mu\varphi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - \\
&1)Z^0_\mu A_\mu\varphi^+\varphi^- - g^1 s_w^2 A_\mu A_\mu\varphi^+\varphi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
&m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \\
&\tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)u^\lambda_j) + \\
&(\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \\
&\tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\varphi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \\
&\varphi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\varphi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\varphi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \\
&\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\varphi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - \\
&m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\varphi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \\
&\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\varphi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \\
&\bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + \\
&igc_w W^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + \\
&igc_w Z^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM[\bar{X}^+X^+H + \\
&\bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H] + \tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\varphi^+ - \bar{X}^-X^0\varphi^-] + \tfrac{1}{2c_w}igM[\bar{X}^0X^-\varphi^+ - \\
&\bar{X}^0X^+\varphi^-] + igMs_w[\bar{X}^0X^-\varphi^+ - \bar{X}^0X^+\varphi^-] + \tfrac{1}{2}igM[\bar{X}^+X^+\varphi^0 - \bar{X}^-X^-\varphi^0]
\end{aligned}
$$

Differences compared to the original Kreimer scheme

1. The scope of this AC-γ_5 prescription shall be limited to just the non-anomalous amplitudes (if one would like to maintain the most celebrated feature of no requiring additional counter-terms); the anomalous axial-current matrix elements still requires counter-terms in this scheme.
2. An explicit (fool-proof) algorithmic procedure involving only the notion of the standard cyclic trace with a constructively defined γ_5, straightforward to be implemented in public computer-algebra tools (No reference to the fancy notion of "non-cyclic trace").
3. We refined the meaning of the (external) "axial vertex" on the closed fermion chain for which the Max1PIopenVFF shall be searched in the general scenarios, as well as the averaging prescription in an algorithmic procedure to reach an unambiguous definition of the trace for an arbitrary Feynman diagram in SM.
4. Our preferred non-4-dimensional treatment of the Levi-Civita tensor shall be applicable (for computing physical observables) in SM up to 3-loop order without any problem.
5. Discussions on how to proceed in the cases of loop diagrams on cuts with intermediate IR divergences present in individual cut diagrams (possibly computed separately and independently), to avoid the introduction of spurious pieces in the final combined results.