



北京计算科学研究中心  
第三届量子场论及其应用研讨会



高能对撞机上双粲偶素产生的NNLO-QCD修正

**NNLO-QCD Correction of Exclusive Double Quarkonium  
Production at High Energy Colliders**

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# Outline

01

• 研究背景

02

•  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + \eta_c$  NNLO振幅研究

03

•  $e^+e^- \rightarrow Z \rightarrow J/\psi + J/\psi$  NNLO振幅研究

04

• 总结



# 高精度物理时代

- 正在运营、建设、筹划中的超级加速器：
  - 高亮度: **LHC / SuperKEKB**
  - 高能量: **ILC/ FCC-ee/FCC/EIC/ CEPC/EicC**

- 前沿问题瓶颈: 积分约化  
求解主积分

圈数, 外腿数  $I+E$

Process class	Examples	Current status	Possible pheno motivated goal
$2 \rightarrow 1$	$H, W, Z,$	$N^3LO$	$N^4LO$
$2 \rightarrow 2$	Jet inclusive, diboson, top-pair, photon-jet, ...	NNLO	$N^3LO$
$2 \rightarrow 3$	ttH, diphoton+jet, WW/ZZ/ZW + jet, top pair + jet ...	NLO <small>First NNLO result: triphoton production [Chawdhry, Czakon, Mitov, Poncelet, '19]</small>	NNLO

顶夸克衰变

精确检验  
标准模型



发展新物理

[Gudrun Heinrich. "Collider physics at the precision frontier". Physics Reports, 2021-08,922: 1-69.](#)



# QCD & Quarkonium

- 重夸克偶素是研究 QCD 完美场所
- 偶素轻子衰变的精确测量对于
  1. 确定部分子分布函数
  2. 探测 Quark-Gluon 等离子体
  3. 探测新物理信号有重要意义
- QCD 及相关的有效理论如 HQET/ NRQCD 等 是研究夸克偶素相关物理基础
- 比如，在本世纪初，NRQCD 相关物理过程的 NLO 研究得到实验的检验
- 近年来，夸克偶素相关过程的 NNLO 研究越来越受关注

[arxiv:hep-ph/0305102](https://arxiv.org/abs/hep-ph/0305102)



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# B工厂中 $J/\psi + \eta_c$ 产生

2002年, B工厂双粲偶素的产生的疑难。

$B[\geq 2]$ : 两个以上带电径迹的分支比;

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4) \text{ fb},$$

and BaBar [6]

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1}) \text{ fb},$$

领头阶的NRQCD预言 [Braaten]: 2.3-5.5fb

研究者从不同角度研究了其可能性, 如:

$Y \rightarrow J/\psi + \eta_c$ 的截面;

光锥因子化方法等 [Ma, Si];

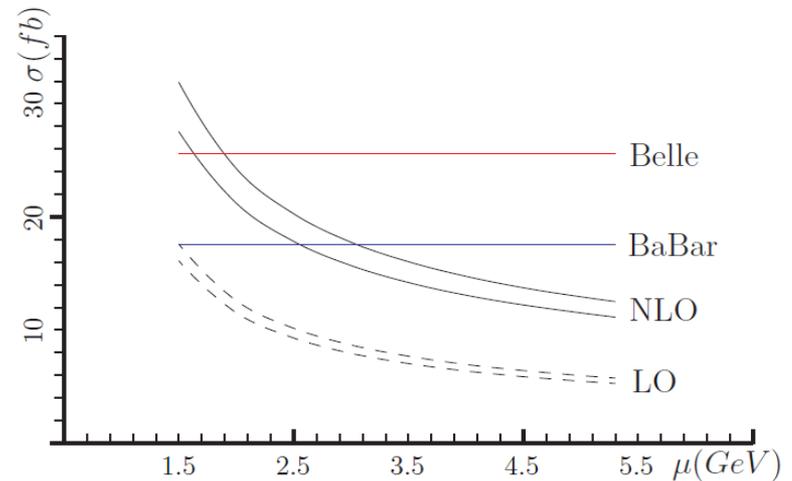
QWG: “与标准模型差异最大的问题之一”

接下来的一段时期, 这个疑难吸引了许多学者的研究兴趣:

2006, NLO计算 [Chao];

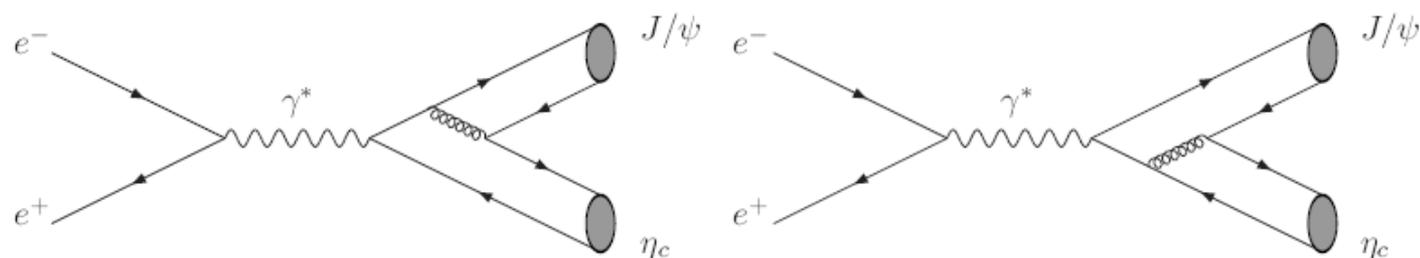
2007解析 [Wang];

QCD+相对论修正 [Jia];





# NRQCD因子化



+2 flipped diagrams

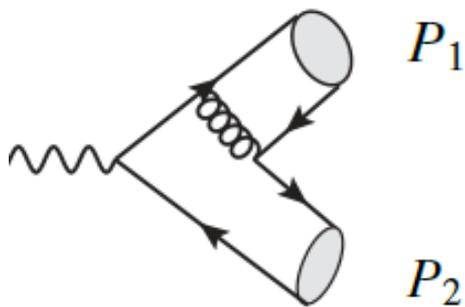
在 NRQCD 因子化框架下， $e^+e^- \rightarrow J/\psi + \eta_c$  的微分截面表示为，

$$d\sigma = d\hat{\sigma}_{e^+e^- \rightarrow c\bar{c}[n_1]+c\bar{c}[n_2]} \langle O^{J/\psi}(n_1) \rangle \langle O^{\eta_c}(n_2) \rangle$$

量子态  $n_1 = {}^3S_1^{[1]}$ ,  $n_2 = {}^1S_0^{[1]}$



# $\gamma^* \rightarrow J/\psi + \eta_c$ 产生的两圈振幅



$$\frac{1024ieN_c C_F m_c^3 g_s^2}{3s^2} \epsilon^{\alpha_2 \mu_1 P_1 P_2},$$

树图: 4

单圈图: 86

两圈图: 2664

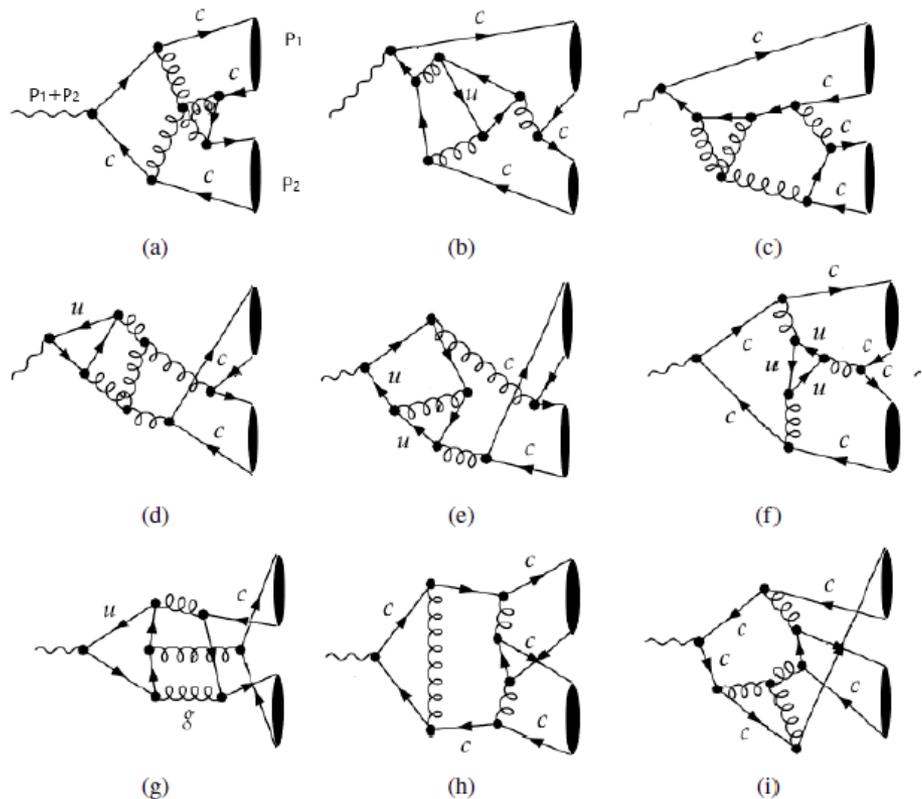
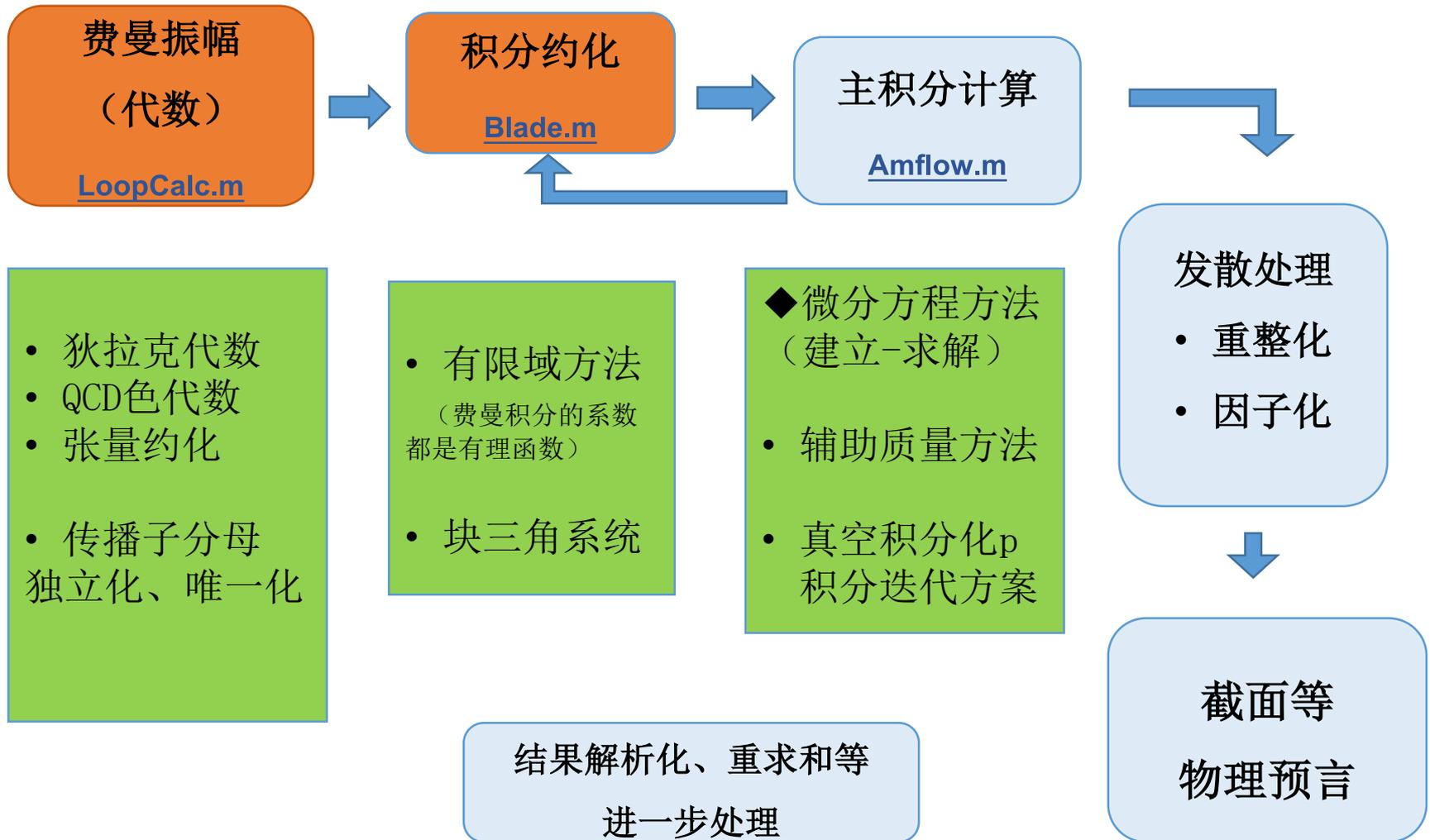


图 3.5 部分两圈费曼图



# 微扰计算方法

模块化-自动化





# 振幅代数计算

- 圈数，外腿数  $L+E$
- 费曼图数；
- 洛伦兹结构数， $\gamma$  矩阵链的长度；
- 色因子数，色矩阵链长度；

$$\frac{1}{MC^2} i EL g_s^6 (\epsilon^{\mu_1 P_1 I_1 I_2} (16 D P_1^{a^2} MC^2 - 64 P_1^{a^2} MC^2 + 16 D P_2^{a^2} MC^2 - 64 P_2^{a^2} MC^2 + 8 Ss^2 P_2^{a^2} - 2 Ss^2 D P_2^{a^2}) +$$

$$\epsilon^{P_1 P_2 I_1 I_2} (16 D g^{a^2 \mu_1} MC^2 - 64 g^{a^2 \mu_1} MC^2 + 2 D^2 P_1^{a^2} P_1^{\mu_1} - 20 D P_1^{a^2} P_1^{\mu_1} + 48 P_1^{a^2} P_1^{\mu_1} + 2 D^2 P_2^{a^2} P_1^{\mu_1} - 20 D P_2^{a^2} P_1^{\mu_1} + 48 P_2^{a^2} P_1^{\mu_1}) +$$

$$\epsilon^{a^2 P_1 I_1 I_2} (16 D P_2^{\mu_1} MC^2 - 64 P_2^{\mu_1} MC^2 + 24 Ss^2 P_1^{\mu_1} + Ss^2 D^2 P_1^{\mu_1} - 10 Ss^2 D P_1^{\mu_1}) - \epsilon^{a^2 \mu_1 P_1 I_2} (38 Ss^2 MC^2 + 4 Ss^2 D^2 MC^2 -$$

$$24 Ss^2 D MC^2 + 16 D P_1 / (1) MC^2 - 64 P_1 / (1) MC^2 + 16 D P_2 / (1) MC^2 - 64 P_2 / (1) MC^2 + 38 Ss^2 P_2 / (1) + 2 Ss^2 D^2 P_2 / (1) - 17 Ss^2 D P_2 / (1)) +$$

$$\epsilon^{\mu_1 P_1 P_2 I_2} (8 D^2 P_1^{a^2} MC^2 - 48 D P_1^{a^2} MC^2 + 76 P_1^{a^2} MC^2 + 8 D^2 P_2^{a^2} MC^2 - 48 D P_2^{a^2} MC^2 + 76 P_2^{a^2} MC^2 - 16 D / (1)^{a^2} MC^2 + 64 / (1)^{a^2} MC^2 +$$

$$4 D P_2^{a^2} P_1 / (1) - 16 P_2^{a^2} P_1 / (1) + 4 D^2 P_1^{a^2} P_2 / (1) - 34 D P_1^{a^2} P_2 / (1) + 76 P_1^{a^2} P_2 / (1) + 4 D^2 P_2^{a^2} P_2 / (1) - 26 D P_2^{a^2} P_2 / (1) + 44 P_2^{a^2} P_2 / (1)) +$$

$$\epsilon^{a^2 P_1 P_2 I_2} (-4 D^2 P_1^{\mu_1} MC^2 + 20 D P_1^{\mu_1} MC^2 - 28 P_1^{\mu_1} MC^2 - 16 D / (1)^{\mu_1} MC^2 + 64 / (1)^{\mu_1} MC^2 - 2 D^2 P_1^{\mu_1} P_1 / (1) + 20 D P_1^{\mu_1} P_1 / (1) -$$

$$48 P_1^{\mu_1} P_1 / (1) - 4 D^2 P_1^{\mu_1} P_2 / (1) + 34 D P_1^{\mu_1} P_2 / (1) - 76 P_1^{\mu_1} P_2 / (1)) - \epsilon^{a^2 \mu_1 P_1 I_1} (42 Ss^2 MC^2 + 4 Ss^2 D^2 MC^2 - 28 Ss^2 D MC^2 -$$

$$16 D P_1 / (2) MC^2 + 64 P_1 / (2) MC^2 - 16 D P_2 / (2) MC^2 + 64 P_2 / (2) MC^2 - 42 Ss^2 P_2 / (2) - 2 Ss^2 D^2 P_2 / (2) + 19 Ss^2 D P_2 / (2)) +$$

$$\epsilon^{\mu_1 P_1 P_2 I_1} (8 D^2 P_1^{a^2} MC^2 - 56 D P_1^{a^2} MC^2 + 84 P_1^{a^2} MC^2 + 8 D^2 P_2^{a^2} MC^2 - 56 D P_2^{a^2} MC^2 + 84 P_2^{a^2} MC^2 + 16 D / (2)^{a^2} MC^2 - 64 / (2)^{a^2} MC^2 -$$

$$4 D P_2^{a^2} P_1 / (2) + 16 P_2^{a^2} P_1 / (2) - 4 D^2 P_1^{a^2} P_2 / (2) + 38 D P_1^{a^2} P_2 / (2) - 84 P_1^{a^2} P_2 / (2) - 4 D^2 P_2^{a^2} P_2 / (2) + 30 D P_2^{a^2} P_2 / (2) - 52 P_2^{a^2} P_2 / (2)) +$$

$$\epsilon^{a^2 P_1 P_2 I_1} (-4 D^2 P_1^{\mu_1} MC^2 + 24 D P_1^{\mu_1} MC^2 - 32 P_1^{\mu_1} MC^2 + 16 D / (2)^{\mu_1} MC^2 - 64 / (2)^{\mu_1} MC^2 + 2 D^2 P_1^{\mu_1} P_1 / (2) -$$

$$20 D P_1^{\mu_1} P_1 / (2) + 48 P_1^{\mu_1} P_1 / (2) + 4 D^2 P_1^{\mu_1} P_2 / (2) - 38 D P_1^{\mu_1} P_2 / (2) + 84 P_1^{\mu_1} P_2 / (2)) +$$

$$\epsilon^{a^2 \mu_1 P_1 P_2} (16 D MC^4 - 16 MC^4 + 8 D^2 P_1 / (1) MC^2 - 56 D P_1 / (1) MC^2 + 84 P_1 / (1) MC^2 + 8 D^2 P_1 / (2) MC^2 - 48 D P_1 / (2) MC^2 + 76 P_1 / (2) MC^2 +$$

$$8 D^2 P_2 / (1) MC^2 - 52 D P_2 / (1) MC^2 + 80 P_2 / (1) MC^2 + 8 D^2 P_2 / (2) MC^2 - 52 D P_2 / (2) MC^2 + 80 P_2 / (2) MC^2 + 16 D / (1) / (2) MC^2 -$$

$$32 / (1) / (2) MC^2 + 4 D^2 P_1 / (2) P_2 / (1) - 34 D P_1 / (2) P_2 / (1) + 76 P_1 / (2) P_2 / (1) - 4 D^2 P_1 / (1) P_2 / (2) + 38 D P_1 / (1) P_2 / (2) - 84 P_1 / (1) P_2 / (2))$$



# 缩小积分规模

- 张量积分约化为标量积分
- 圈动量的唯一化表示
- 部分分式化
- 积分Family分类
- 积分约化为主积分

	费曼图数	Families	积分数	主积分数
$J/\psi + \eta_c$	2664	167	16586	< 80



# 积分约化：IBP系统建立、求解

➤ 目标：高圈振幅表示为——主积分的线性组合。

➤ 方法：有限域方法 (Finite-Field)  
块三角系统 (Block-Triangular)

关鑫

[Xin Guan, Xiao Liu and Yan-Qing Ma. \*Chin. Phys. C\*, 2020, 44\(9\): 093106.](#)

[T. Peraro, \*JHEP\* 12, 030 \(2016\), arXiv:1608.01902 \[hep-ph\].](#)

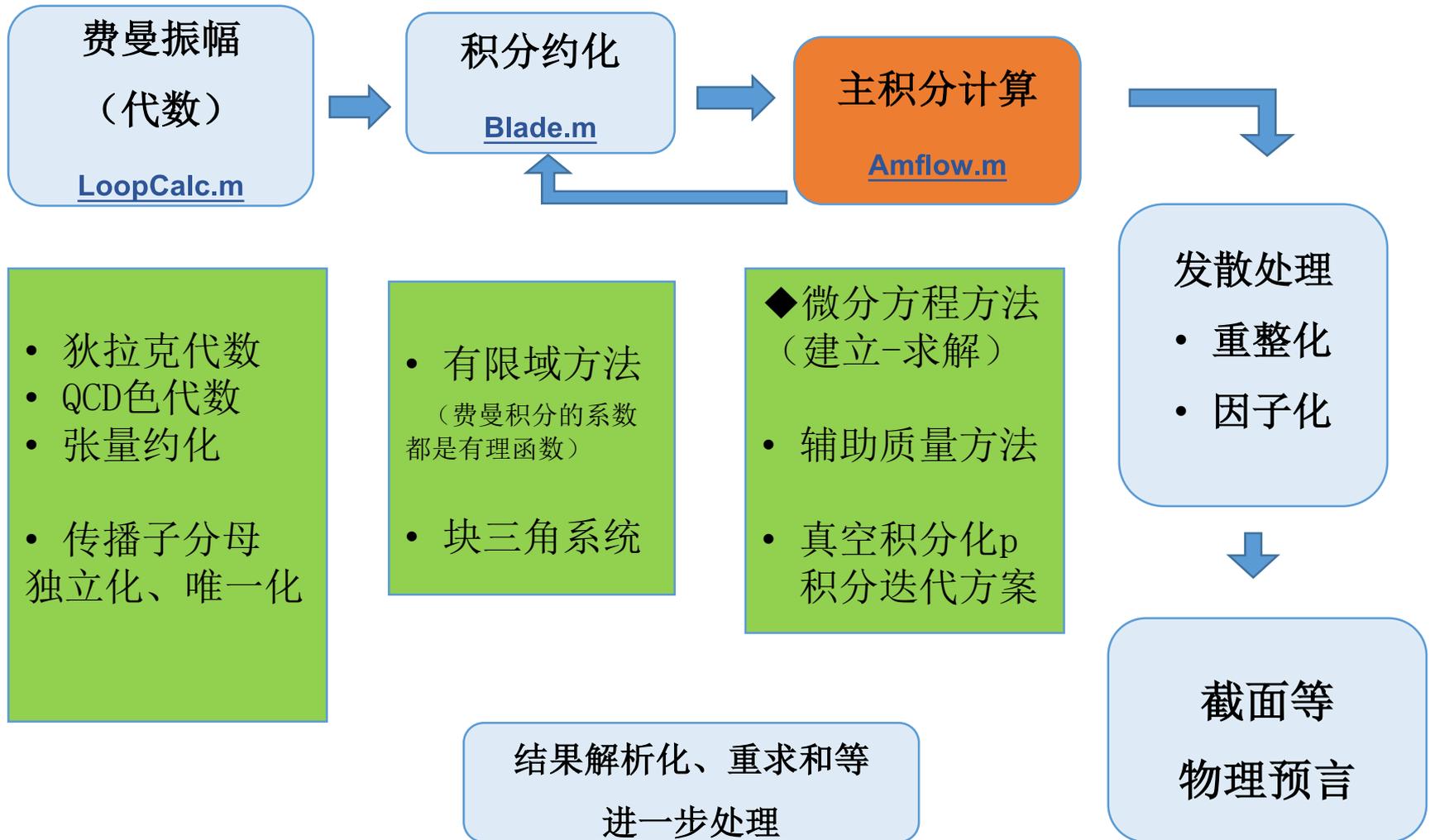
➤ 结果：

Family 序号	1	2	3	4	5	6	7	8	9	10
主积分数	49	66	78	72	72	72	54	49	71	42
积分数	1419	1372	543	196	582	409	108	192	225	299



# 微扰计算方法

模块化-自动化





# 主积分计算

- 辅助质量流方法 (Auxiliary Mass Flow)

$$I_{\vec{\nu}}^{\text{mod}}(\epsilon, \eta) = \int \prod_{i=1}^L \frac{d^D \ell_i}{i\pi^{D/2}} \frac{\mathcal{D}_{K+1}^{-\nu_{K+1}} \cdots \mathcal{D}_N^{-\nu_N}}{(\mathcal{D}_1 + \lambda_1 \times i\eta)^{\nu_1} \cdots (\mathcal{D}_K + \lambda_K \times i\eta)^{\nu_K}},$$
$$\frac{1}{[(\ell + p)^2 - m^2 + i\eta]^\nu}$$
$$= \frac{1}{(\ell^2 + i\eta)^\nu} \sum_{j=0}^{\infty} \frac{(\nu)_j}{j!} \left( -\frac{2\ell \cdot p + p^2 - m^2}{\ell^2 + i\eta} \right)^j,$$

$$\vec{I}^{\text{mod}}(\epsilon, \eta) = \sum_{\mu \in S} \sum_{k=0}^{k_\mu} \eta^\mu \log^k(\eta) \sum_{n=0}^{\infty} \vec{C}_{\mu, k, n}(\epsilon) \eta^n,$$

- 辅助质量演化，化为等质量真空积分
- 化为低一圈传播子积分，问题迭代发展成线性代数问题
- 高效并行
- 本工作精度：600位有效数字



# 微分方程方法

建立微分方程：(IBP)

$$d\vec{f} = \left( \sum_{s \in S} A_s ds \right) \vec{f},$$

m	$\text{Li}_m(-2)$	$\text{Li}_m(-1)$	$\text{Li}_m(2)$
1	$-\ln(3)$	$-\ln(2)$	$-i\pi$
2	$\text{Li}_2(-2)$	$-\frac{\pi^2}{12}$	$\text{Li}_2(2)$
3	$\text{Li}_3(-2)$	$-\frac{3\zeta(3)}{4}$	$\text{Li}_3(2)$
4	$\text{Li}_4(-2)$	$-\frac{7\pi^4}{720}$	$\text{Li}_4(2)$

表 3.2 本文中无理数基底的基本元素

求解微分方程：

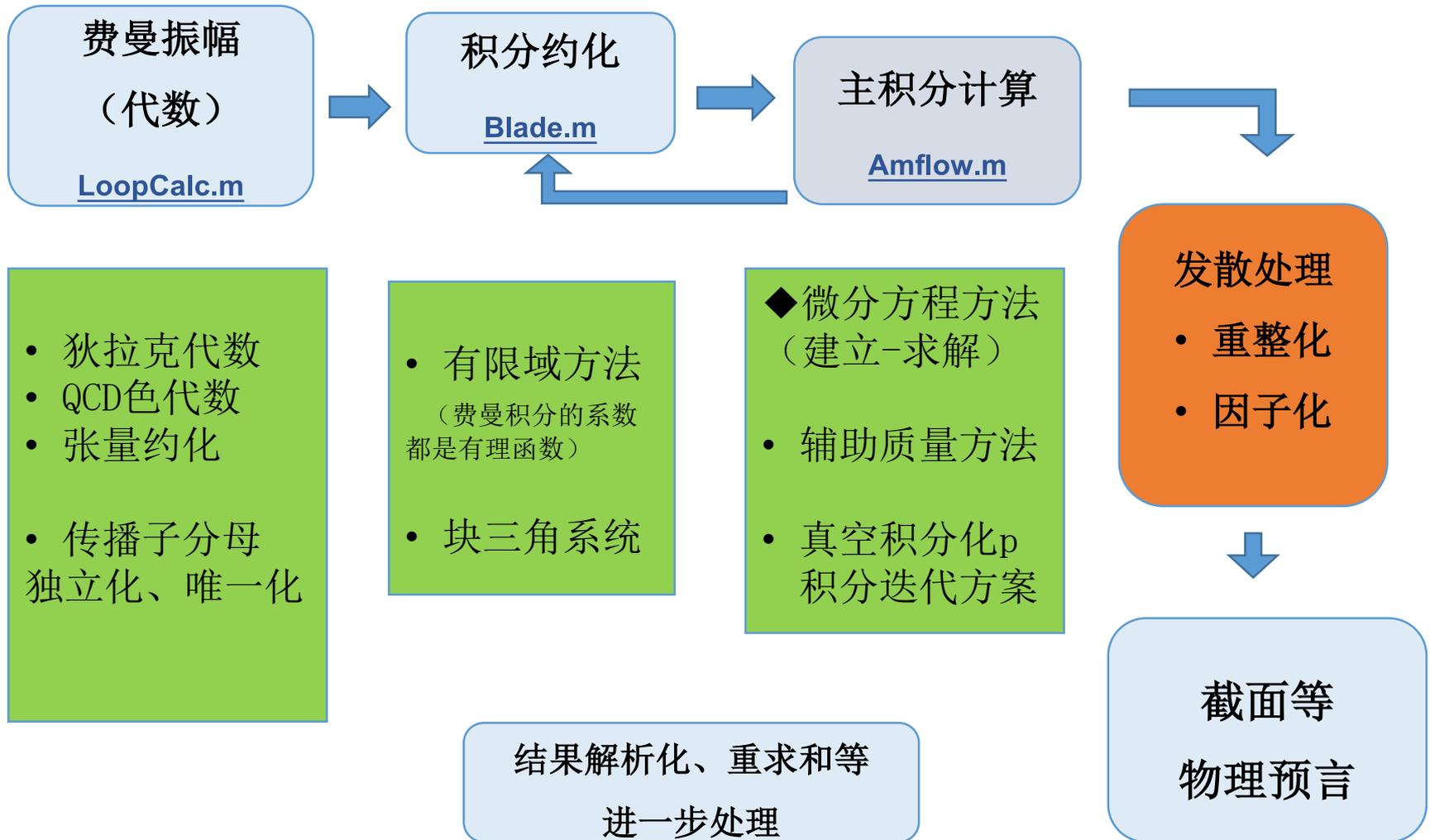
$$\mathcal{A}(\epsilon, s) = \sum C_f f(\epsilon, s) = \sum_{\mu, k, n} c_{\mu, k, n}(\epsilon) s^{\mu(\epsilon)} \ln^k(s) s^n$$

PSLQ整数关系算法



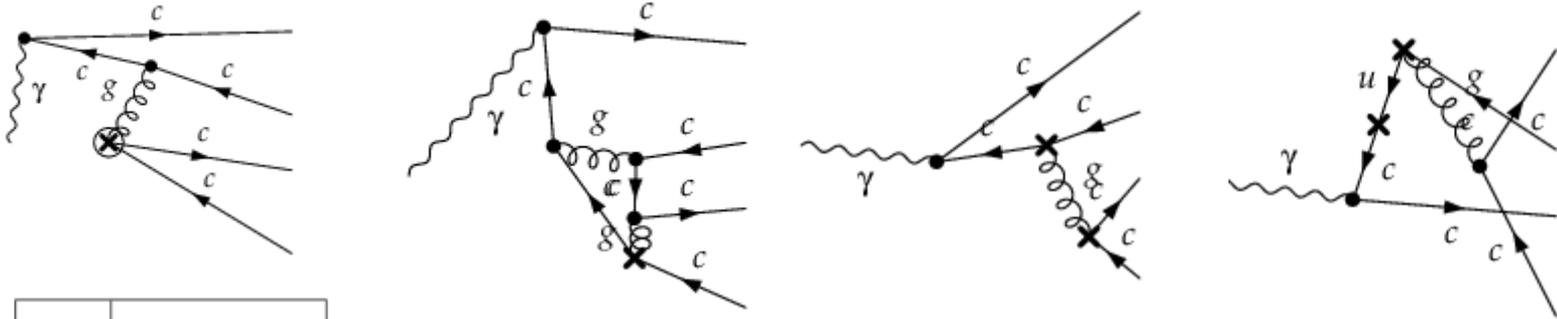
# 微扰计算方法

模块化-自动化





# 重整化



$Z_{2l}$	轻夸克场强
$Z_2$	重夸克场强
$Z_m$	重夸克质量
$Z_3$	胶子场强
$Z_{3h}$	鬼场场强
$Z_g$	强耦合常数

$$\begin{aligned}
 Z_Q = 1 + & \left( \frac{\alpha_s^{(n_f)}}{2\pi} \right) C_F \left\{ -\frac{3}{2\epsilon} - 2 - \frac{3}{2}l_\mu - 4\epsilon - 2\epsilon l_\mu - \frac{3}{4}\epsilon l_\mu^2 - \frac{\pi^2}{8}\epsilon - 8\epsilon^2 - 4\epsilon^2 l_\mu \right. \\
 & - \epsilon^2 l_\mu^2 - \frac{1}{4}\epsilon^2 l_\mu^3 - \frac{\pi^2}{6}\epsilon^2 - \frac{\pi^2}{8}\epsilon^2 l_\mu + \frac{1}{2}\epsilon^2 \zeta_3 \left. \right\} + \left( \frac{\alpha_s^{(n_f)}}{2\pi} \right)^2 C_F \left\{ T_F n_h \left[ \frac{1}{4\epsilon} + \frac{1}{\epsilon} l_\mu \right. \right. \\
 & + \frac{947}{72} + \frac{11}{6} l_\mu + \frac{3}{2} l_\mu^2 - \frac{5\pi^2}{4} \left. \right] + T_F n_l \left[ -\frac{1}{2\epsilon^2} + \frac{11}{12\epsilon} + \frac{113}{24} + \frac{19}{6} l_\mu + \frac{1}{2} l_\mu^2 + \frac{\pi^2}{3} \right] \\
 & + C_F \left[ \frac{9}{8\epsilon^2} + \frac{51}{16\epsilon} + \frac{9}{4\epsilon} l_\mu + \frac{433}{32} + \frac{51}{8} l_\mu + \frac{9}{4} l_\mu^2 - \frac{49\pi^2}{16} + 4 \ln 2\pi^2 - 6\zeta_3 \right] \\
 & \left. + C_A \left[ \frac{11}{8\epsilon^2} - \frac{127}{48\epsilon} - \frac{1705}{96} - \frac{215}{24} l_\mu - \frac{11}{8} l_\mu^2 + \frac{5\pi^2}{4} - 2 \ln 2\pi^2 + 3\zeta_3 \right] \right\} \quad (15)
 \end{aligned}$$

P. Barnreuther, M. Czakon and P. Fiedler. "Virtual amplitudes and threshold behaviour of hadronic top-quark pair-production cross sections". JHEP, 2014, 02: 078.



# 红外发散减除

$$\mathcal{A}^{(2)}(\epsilon, \alpha_s) = \frac{1}{\epsilon} \frac{4i(N_c - 1)^2(N_c + 1)^2(5N_c^2 - 2)}{9\pi^2 N_c^2} t^2 + \mathcal{O}(\epsilon^0 t^2) + \mathcal{O}(t^3)$$

$$\begin{aligned} \gamma_{J/\psi} &= -\frac{\pi^2}{12} C_F (2C_F + 3C_A), \\ \gamma_{\eta_c} &= -\frac{\pi^2}{4} C_F (2C_F + C_A). \end{aligned}$$

$$\frac{\alpha_s^2}{\pi^2} (\gamma_{J/\psi} + \gamma_{\eta_c}) \frac{-512i(N_c^2 - 1)}{3} t^2 = \frac{4i(N_c - 1)^2(N_c + 1)^2(5N_c^2 - 2)}{9\pi^2 N_c^2} t^2$$

$$\tilde{\mathcal{A}}^{(2)} = \mathcal{A}^{(2)} + \mathcal{A}^{(0)} \alpha_s^2(\mu_R) \left( \frac{\mu_\Lambda^2 e^{\gamma_E}}{\mu_R^2 4\pi} \right)^{-2\epsilon} \left( \frac{4C_F^2}{3} + C_F C_A \right) \frac{1}{2\epsilon}.$$



# J/ψ + η<sub>c</sub> 振幅解析结果 I

➤ Born 振幅  $-\frac{1024ieN_cC_Fm_c^3g_s^2}{3s^2}\epsilon^{\alpha_2\mu_1P_1P_2} \rightarrow -\frac{4096it^2}{3}$

➤ NLO 振幅 ( t = 1/s )

$$\frac{128it^2(-150\log(\mu^2) - 39\log^2(t) + (-3(9 + 82\log(2)) - 78i\pi)\log(t) + 2\pi^2 - 27i\pi + 92 - 177\log^2(2) - 246i\pi\log(2) - 639\log(2))}{27\pi^2}$$



# J/ψ + η<sub>c</sub> 振幅解析结果 II

➤ Born 振幅

$$-\frac{4096 i e \epsilon^{\alpha 2 \mu 1 P 1 P 2} m_c^3 g_s^2}{3 s^2} \rightarrow -\frac{512}{3} i (N_c^2 - 1) t^2$$

➤ NNLO 振幅 ( t = 1/s )

$$h^{(2l)}(t) = t^2 \left[ L_{2,4}^{(2l)} \ln^4(t) + L_{2,3}^{(2l)} \ln^3(t) + L_{2,2}^{(2l)} \ln^2(t) + L_{2,1}^{(2l)} \ln(t) + L_{2,0}^{(2l)} \right]$$

领头对数项:

$$L_{2,4}^{(2l)} = -\frac{i(N_c^2 - 1)^2 (7N_c^2 - 19)}{144\pi^4 N_c^2}.$$

次领头对数项:

$$\begin{aligned} L_{2,3}^{(2l)} = & \frac{2i(N_c^2 - 1)(7N_c^2 - 11)n_{\text{lf}}}{27\pi^4 N_c} + \frac{(N_c^2 - 1)^2 (7N_c^2 - 19)}{36\pi^3 N_c^2} \\ & - \frac{i(N_c^2 - 1)}{54\pi^4 N_c^2} [N_c^4(79 + 87 \ln(2)) - 31N_c^3 - 2N_c^2(40 + 141 \ln(2)) \\ & + N_c(59 - 24 \ln(2)) - 99 + 189 \ln(2)] \end{aligned}$$



# J/ψ + η<sub>c</sub> 振幅解析结果 III

次次领头对数项:

$$\begin{aligned}
 L_{2,2}^{(2l)} = & -\frac{8i(N_c^2 - 1)n_{lf}^2}{27\pi^4} \\
 & \frac{i(N_c^2 - 4)(N_c^2 - 1)(n_{lm} - 2n_{lp})(2\ln^2(2) - 1)}{12\pi^4 N_c} \\
 & - \frac{2(N_c^2 - 1)(7N_c^2 - 11)n_{lf}}{9\pi^3 N_c} \\
 & + \frac{i(N_c^2 - 1)n_{lf}(N_c^2(327 + 498\ln(2)) - 100N_c - 61 - 906\ln(2))}{108\pi^4 N_c} \\
 & + \frac{i(N_c^2 - 1)(7N_c^2 - 11)n_{lf}\ln(\mu^2)}{9\pi^4 N_c} \\
 & + \frac{(N_c^2 - 1)}{18\pi^3 N_c^2} [N_c^4(79 + 87\ln(2)) - 31N_c^3 - 2N_c^2(40 + 141\ln(2)) \\
 & + N_c(59 - 24\ln(2)) - 99 + 189\ln(2)] \\
 & + \frac{i(N_c^2 - 1)}{432\pi^4 N_c^2} \left[ N_c^4 \left( -2563 + 162\pi^2 - 3672\ln^2(2) - 7374\ln(2) \right) + \right. \\
 & 12N_c^3 \left( 148 + \pi^2 - 36\ln^2(2) + 226\ln(2) \right) \\
 & - 2N_c^2 \left( 926 + 309\pi^2 - 6408\ln^2(2) - 6186\ln(2) \right) \\
 & \left. + 8N_c \left( -26 + 297\ln^2(2) - 579\ln(2) \right) + 420\pi^2 + 9 \left( 255 - 960\ln^2(2) + 602\ln(2) \right) \right] \\
 & - \frac{i(11N_c - 2)(N_c^2 - 1)(7N_c^2 - 11)\log(\mu^2)}{18\pi^4 N_c}
 \end{aligned} \tag{3.39}$$



# Double Log 重求和

➤ Born 振幅  $-\frac{512}{3} i(N_c^2 - 1)t^2$

➤ NLO 振幅领头对数项(  $t = 1/s$  ):

$$h^{(1l)}(t) = t^2 \left[ L_{2,2}^{(1l)} \ln^2(t) + L_{2,1}^{(1l)} \ln(t) + L_{2,0}^{(1l)} \right]$$

$$L_{2,2}^{(1l)} = -\frac{4i(N_c^2 - 1)(7N_c^2 - 11)}{3\pi^2 N_c},$$

张大江

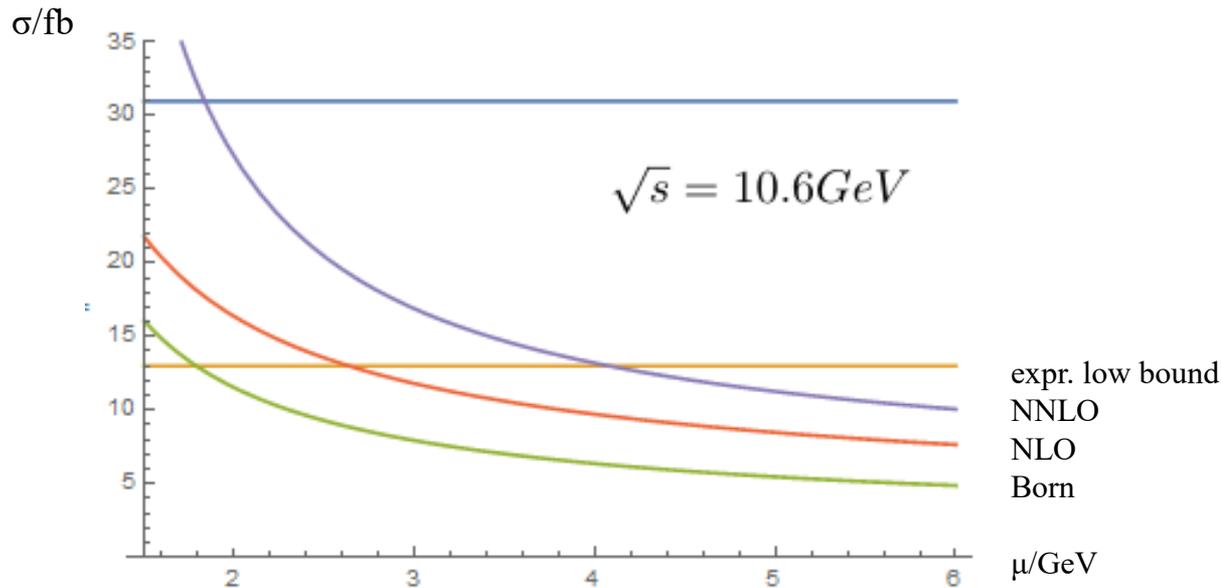
➤ NNLO 振幅领头对数项(  $t = 1/s$  ):

$$h^{(2l)}(t) = t^2 \left[ L_{2,4}^{(2l)} \ln^4(t) + L_{2,3}^{(2l)} \ln^3(t) + L_{2,2}^{(2l)} \ln^2(t) + L_{2,1}^{(2l)} \ln(t) + L_{2,0}^{(2l)} \right]$$

$$L_{2,4}^{(2l)} = -\frac{i(N_c^2 - 1)^2(7N_c^2 - 19)}{144\pi^4 N_c^2}.$$



# $J/\psi + \eta_c$ 截面数值结果 I



$\sqrt{s}/\text{GeV}$	LO/fb	NLO/fb	NNLO/fb
10.6	5.2721	11.3655	17.3453
250	$3.14718 \times 10^{-11}$	$1.58064 \times 10^{-10}$	$4.30898 \times 10^{-10}$

表 3.6 截面数值结果, 重整化标度  $\mu = \sqrt{s}/2$ , 因子化标度  $\mu_\Lambda = m_c = 1.5\text{GeV}$



# J/ψ + η<sub>c</sub>截面数值结果对比 II

$\sqrt{s}/\text{GeV}$	LO/fb	NLO/fb	NNLO/fb
10.6	5.2721	11.3655	17.3453
250	$3.14718 \times 10^{-11}$	$1.58064 \times 10^{-10}$	$4.30898 \times 10^{-10}$

表 3.6 截面数值结果，重整化标度  $\mu = \sqrt{s}/2$ ，因子化标度  $\mu_\Lambda = m_c = 1.5\text{GeV}$

$m(\text{GeV})$	$\mu_R$	LO	NLO	NNLO
1.5	$\sqrt{s}/2$	$5.05^{+0.92+2.31}_{-0.99-1.49}$	$10.54^{+2.86+3.92}_{-2.60-2.66}$	$15.00^{+5.03+4.29}_{-4.14-3.14}$

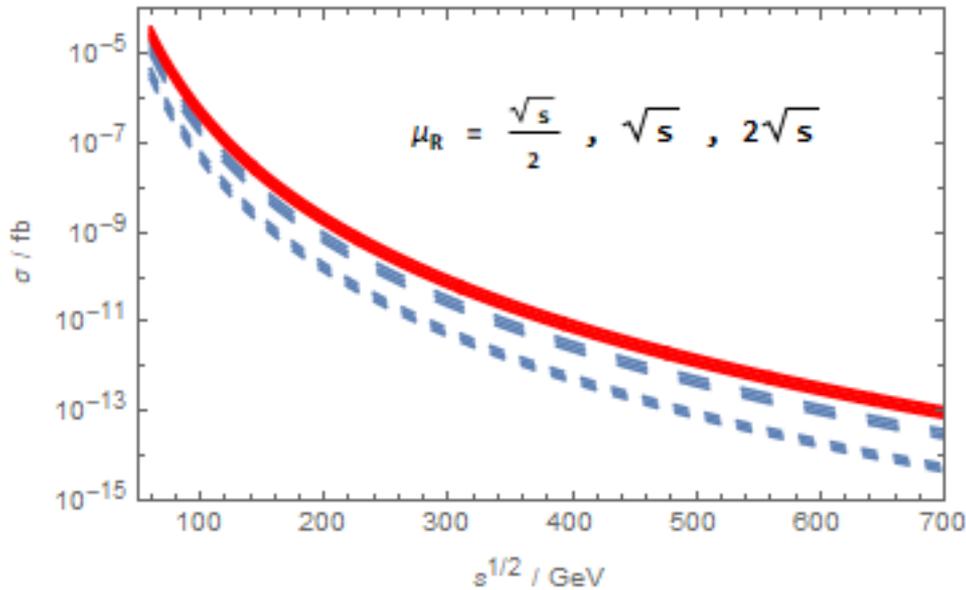
[Feng Feng, Yu Jia, Zhewen Mo et al. "Next-to-next-to-leading-order QCD corrections to  \$e^+e^- \rightarrow J/\psi + \eta\_c\$  at B factories". 2019-01.](#)

		$\alpha_s^2$ -terms	$\alpha_s^3$ -terms	$\alpha_s^4$ -terms	Total
$\mu_\Lambda = m_c$	$\mu_R = 2m_c$	7.40	$7.04 + 0.13$	$2.17 + 0.28$	$16.61 + 0.41$
	$\mu_R = \sqrt{s}/2$	5.06	$5.57 - 0.05$	$3.43 - 0.08$	$14.06 - 0.13$
$\mu_\Lambda = 1\text{GeV}$	$\mu_R = 2m_c$	7.40	$7.04 + 0.13$	$4.62 + 0.27$	$19.06 + 0.40$
	$\mu_R = \sqrt{s}/2$	5.06	$5.57 - 0.05$	$4.58 - 0.09$	$15.21 - 0.14$

[Xu-Dong Huang, Bin Gong and Jian-Xiong Wang. "Next-to-next-to-leading-order QCD corrections  \$\rightarrow J/\psi + \eta\_c\$  production at the B factories". JHEP, 2023, 02: 049.](#)



# $J/\psi + \eta_c$ 截面数值结果 II



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ILC	250GeV-1TeV
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FCC-ee	91.2, 160, 240, 365 (GeV)
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FCC	100TeV
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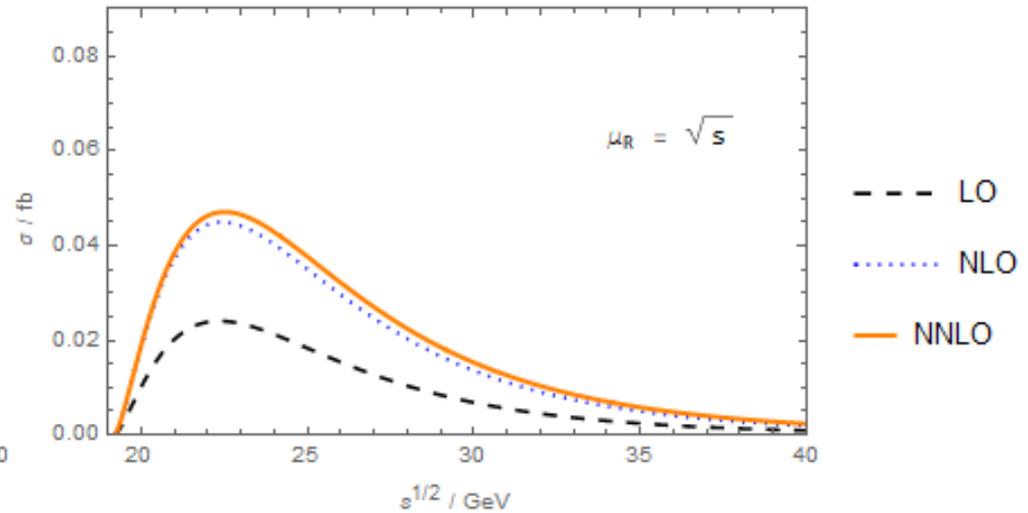
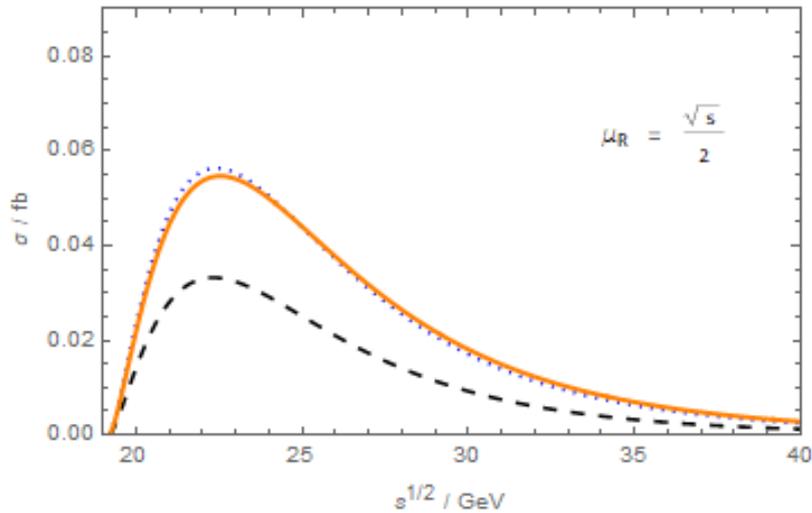
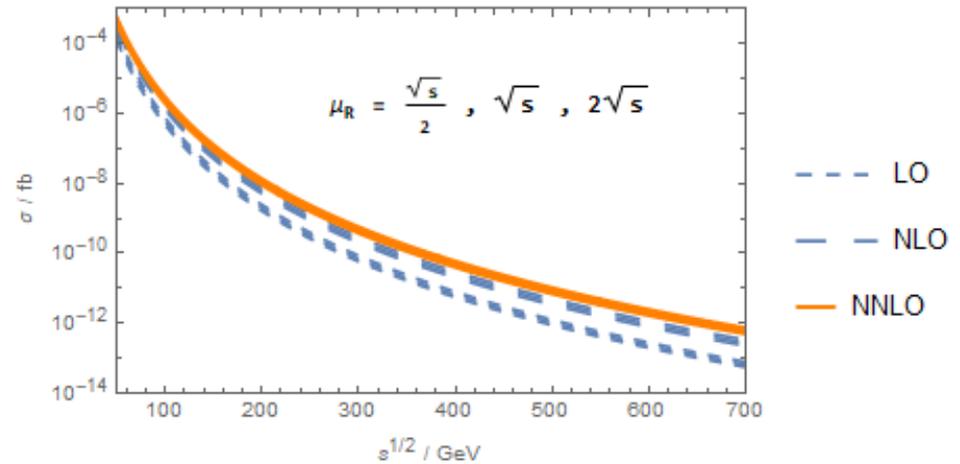
表 8: 未来对撞机设计能量



# $\Upsilon + \eta_b$ 截面数值结果

	$m_Q$	$n_{lf}$	$n_{lp}$	$n_{lm}$	$ R_H(0) ^2$
$J/\psi + \eta_c$	1.5GeV	3	1	2	0.978GeV <sup>3</sup>
$\Upsilon + \eta_b$	4.8GeV	4	2	2	3.46GeV <sup>3</sup> [100]

表 3.8 对底粲过程不同取值





skip to summary if time is up

01

• 研究背景

02

•  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + \eta_c$  振幅研究

03

•  $e^+e^- \rightarrow Z \rightarrow J/\psi + J/\psi$  振幅研究

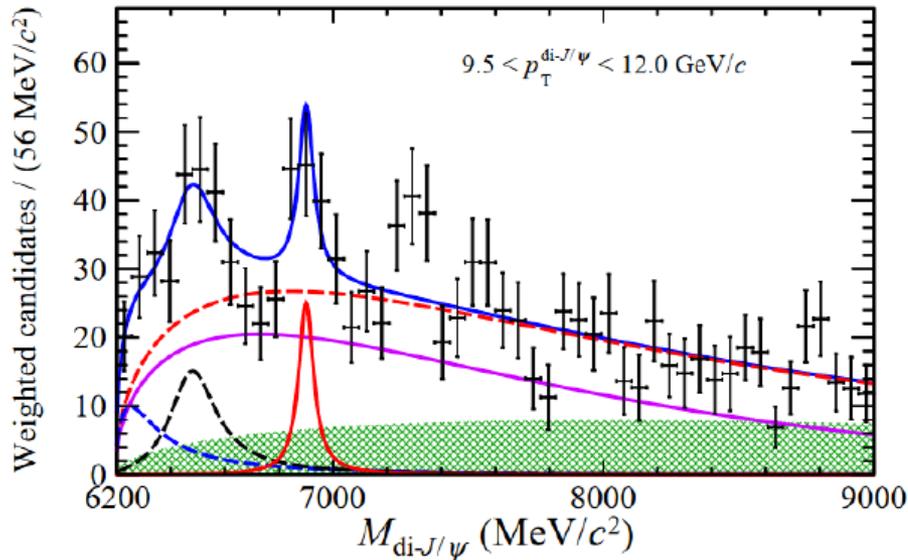
04

• 总结

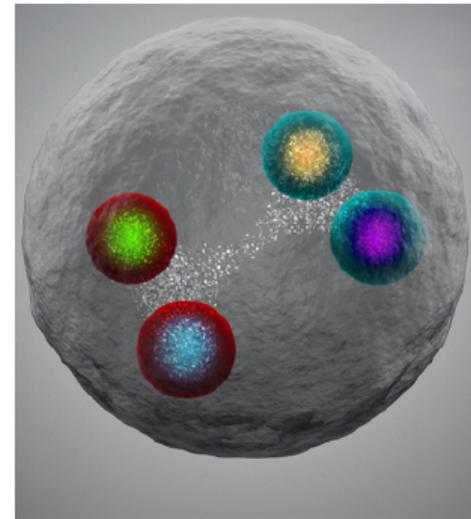


# LHC上 $Z \rightarrow J/\psi + J/\psi$

- B工厂上没有找到足量双 $J/\psi$ 产生的实例 (ILC/CEPC/FCC-ee)
- 2019年, CMS组宣布寻找Z的四轻子衰变。



(a) 双  $J/\psi$  不变质量谱, 在 6.9GeV 处发现共振峰



(b) 四夸克态

图 4.4 2020 年 LHCb 重要发现 [104]



# J/ψ + J/ψ 产生

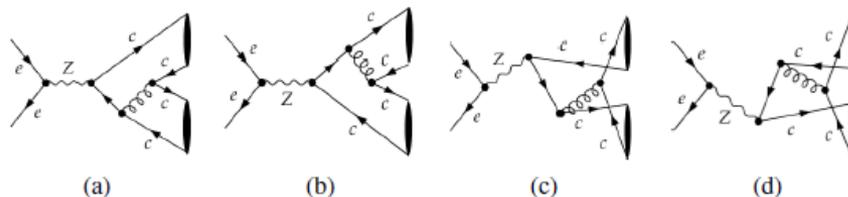


图 4.1  $e^+ + e^- \rightarrow Z \rightarrow J/\psi + J/\psi$  树图

在表格 4.1 中给出领头阶振幅的解析表达式，其中  $\epsilon$  为四阶全反对称张量， $\mu_1$  是  $Z - q - \bar{q}$  顶点的洛伦兹指标， $\alpha_1, \alpha_2$  是矢量粒子  $J/\psi$  的洛伦兹指标。 $\theta_W$  是 Weinberg 弱混合角。维数正规化下， $D = 4 - 2\epsilon$ 。由此可以计算领头阶截面的表达式如下

$\epsilon^{\alpha_1 \alpha_2 \mu_1 P_1}$	$\frac{8iC_F e g_s^2 m_c^2 N_c (16m_c^2 + 4s - sD)}{s^2 \cos\theta_W \sin\theta_W}$
$\epsilon^{\alpha_1 \alpha_2 \mu_1 P_2}$	$-\frac{8iC_F e g_s^2 m_c^2 N_c (16m_c^2 + 4s - sD)}{s^2 \cos\theta_W \sin\theta_W}$
$\epsilon^{\alpha_2 \mu_1 P_1 P_2} P_1^{\alpha_1}$	$-\frac{8iC_F e g_s^2 m_c^2 N_c (D - 6)}{s^2 \cos\theta_W \sin\theta_W}$
$\epsilon^{\alpha_2 \mu_1 P_1 P_2} P_2^{\alpha_1}$	$-\frac{8iC_F e g_s^2 m_c^2 N_c (D - 4)}{s^2 \cos\theta_W \sin\theta_W}$
$\epsilon^{\alpha_1 \mu_1 P_1 P_2} P_2^{\alpha_2}$	$\frac{8iC_F e g_s^2 m_c^2 N_c (D - 4)}{s^2 \cos\theta_W \sin\theta_W}$
$\epsilon^{\alpha_1 \mu_1 P_1 P_2} P_1^{\alpha_2}$	$\frac{8iC_F e g_s^2 m_c^2 N_c (D - 6)}{s^2 \cos\theta_W \sin\theta_W}$

表 4.1 born 振幅表达式



# Z $\rightarrow$ J/ $\psi$ + J/ $\psi$ 振幅的重整化

轴矢流 &  $\gamma_5$  的介入

$\epsilon^{\alpha_1 \alpha_2 \mu_1 P_1}$	$\frac{8iC_F e g_s^2 m_c^2 N_c (16m_c^2 + 4s - sD)}{s^2 \cos\theta_w \sin\theta_w}$
--	---

$\epsilon^i$	$\epsilon^{-4}$	$\epsilon^{-3}$	$\epsilon^{-2}$	$\epsilon^{-1}$	$\epsilon^0$
(loop2+cttm2 Amps )	0	0	0	$-\frac{1120it^2}{27\pi^2} - \frac{400it^2}{9\pi^4}$	-
$\delta Z_A^{(2)}$ ( born Amps )	0	0	0	$\frac{400it^2}{9\pi^4}$	-
$\frac{\alpha_s^2}{\pi^2}$ ( $2\gamma_{J/\psi}$ ) ( born Amps )	0	0	0	0	$-\frac{1120it^2}{27\pi^2}$

表 9: Z  $\rightarrow$  J $\psi$  + J $\psi$  振幅重整化

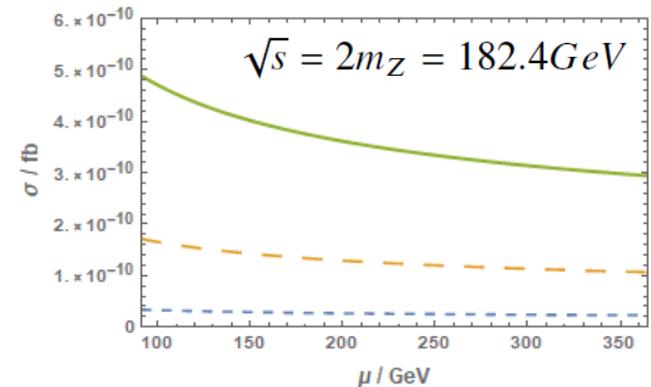
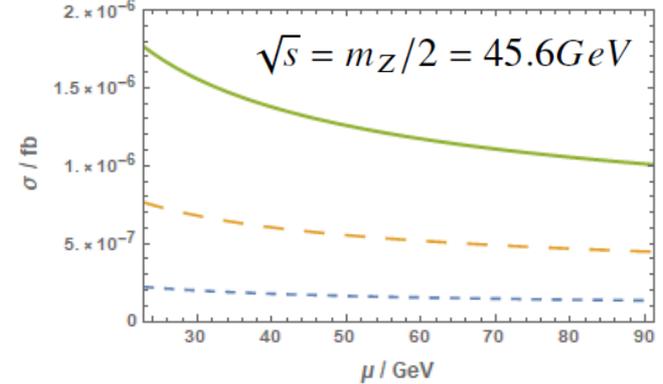
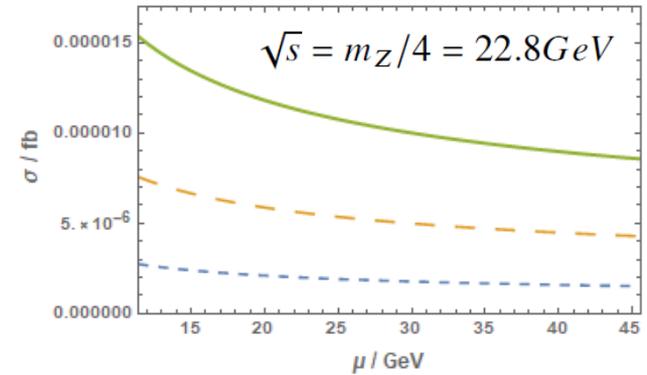
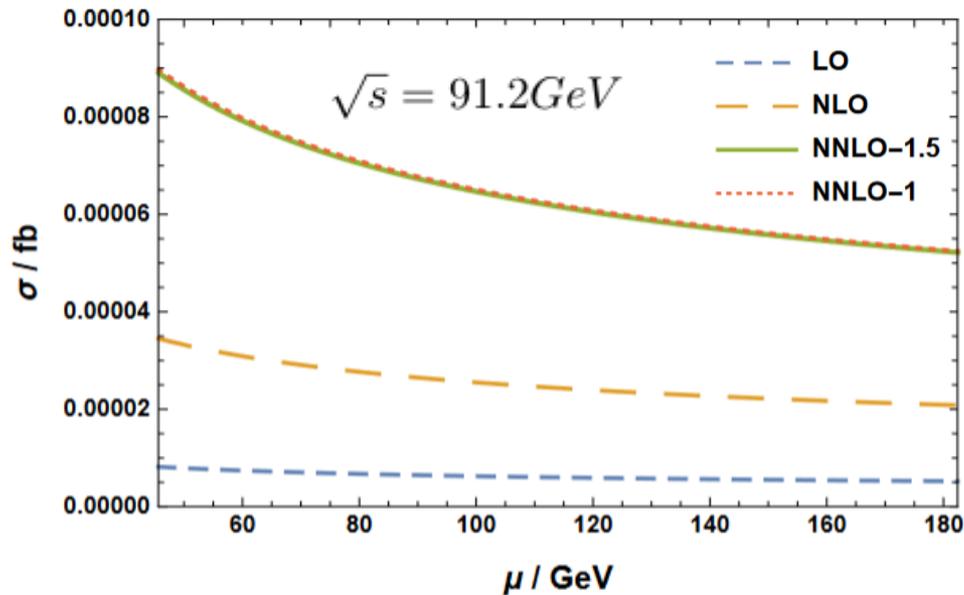
$$Z_{a,NS}^{\overline{MS}} = 1 + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{1}{\epsilon} \left( \frac{11}{24} C_A C_F - \frac{1}{6} C_F T_F n_f \right)$$



# J/ψ + J/ψ产生数值结果 I

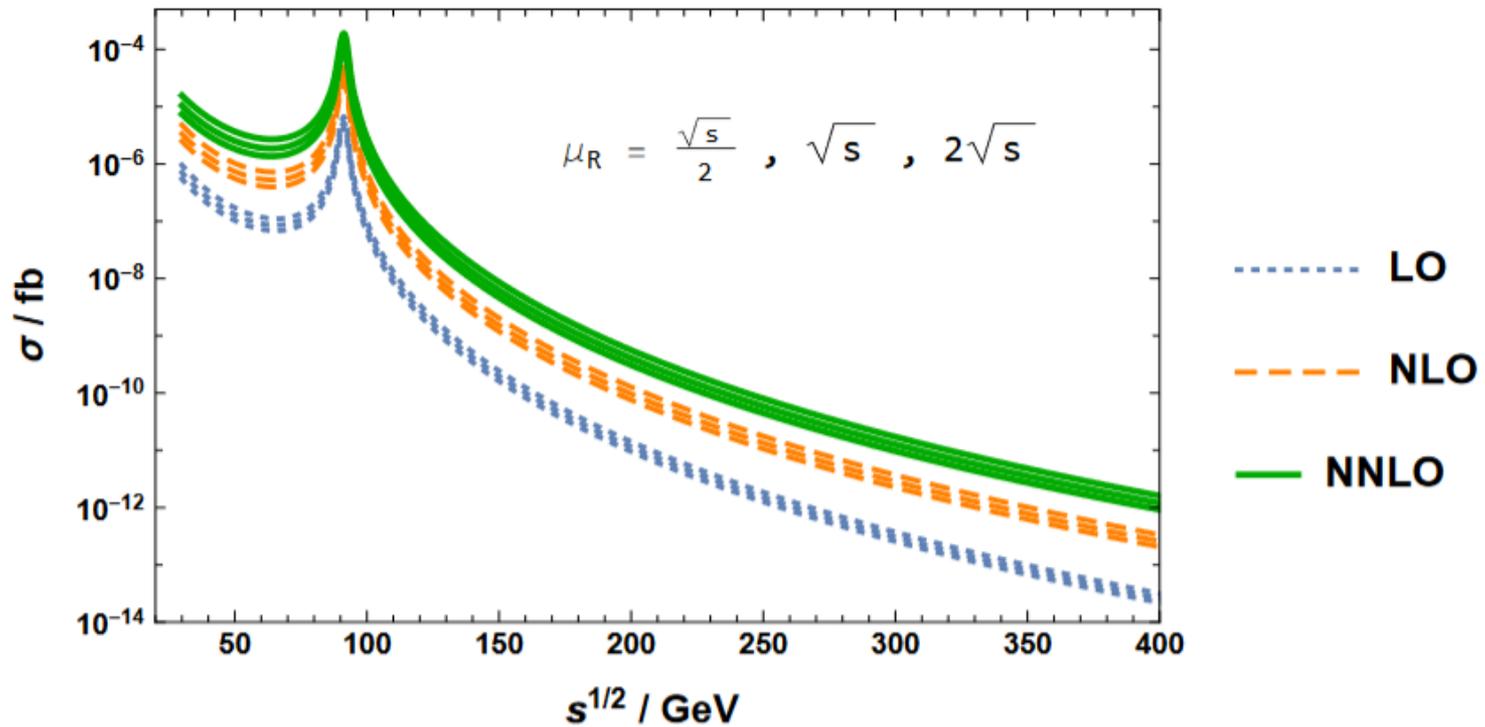
$$R_{J/\psi}^2 = 1.1\text{GeV}^3 \quad m_c = 1.5\text{GeV} \quad m_Z = 91.2\text{GeV}$$

$$\Gamma_Z = 2.5\text{GeV} \quad \sin\theta_w = 38/79 \quad \cos\theta_w = 64/73$$





# J/ψ + J/ψ产生数值结果 II





# 总结

- 完成了 $J/\psi + \eta_c$ 两圈的计算，并获得振幅解析结果，大大改善了同实验的一致性
- 完成了 $Z \rightarrow J/\psi + J/\psi$ 两圈振幅的计算，首次得到 $J/\psi + J/\psi$ 产生的NNLO截面的理论预言。
- 我们发展了一套自动化并且模块化的高圈计算方案及程序

感谢各位的倾听！～～

感谢合作者们：关鑫、陈相、刘霄、马滢青、张鹏



# Backup V: $J/\psi + \eta_c$ 微扰高阶贡献量级

	LO/fb	NLO/fb	NNLO1/fb	NNLO2/fb	部分 NNNLO/fb	部分 NNNNLO/fb
	$O(\alpha_s^2)$	$O(\alpha_s^3)$	$O(\alpha_s^4)$	$O(\alpha_s^4)$	$O(\alpha_s^5)$	$O(\alpha_s^6)$
$\sqrt{s}/GeV$	$ \mathcal{A}^{0l} ^2$	$2\text{Re}[\mathcal{A}^{1l}\mathcal{A}^{0l*}]$	$2\text{Re}[\mathcal{A}^{2l}\mathcal{A}^{0l*}]$	$ \mathcal{A}^{1l} ^2$	$2\text{Re}[\mathcal{A}^{2l}\mathcal{A}^{1l*}]$	$ \mathcal{A}^{2l} ^2$
10.6	3.55886	4.51486	2.56631	1.6709	1.89253	0.535931
91.2	$1.10169 \times 10^{-7}$	$3.25263 \times 10^{-7}$	$2.85491 \times 10^{-7}$	$3.05978 \times 10^{-7}$	$5.37132 \times 10^{-7}$	$2.35729 \times 10^{-7}$
182.4	$3.50059 \times 10^{-10}$	$1.2438 \times 10^{-9}$	$1.16053 \times 10^{-9}$	$1.36713 \times 10^{-9}$	$2.55057 \times 10^{-9}$	$1.18961 \times 10^{-9}$
250	$2.57369 \times 10^{-11}$	$9.87343 \times 10^{-11}$	$9.44552 \times 10^{-11}$	$1.15625 \times 10^{-10}$	$2.21227 \times 10^{-10}$	$1.05819 \times 10^{-10}$
500	$8.38651 \times 10^{-14}$	$3.75406 \times 10^{-13}$	$3.77679 \times 10^{-13}$	$4.99287 \times 10^{-13}$	$1.00557 \times 10^{-12}$	$5.06308 \times 10^{-13}$

表 3.9 截面更  $\alpha_s$  高阶的贡献; 其中重整化能标  $\mu = \sqrt{s}$