Black Hole Superradiance and its Termination from Companions

Yi Wang (王一) The Hong Kong University of Science and Technology

Reference: 2205.10527 with Xi Tong (童曦) and Hui-Yu Zhu (朱慧宇)

See also: 2009.11106 with Qianhang Ding (丁乾航), Xi Tong (童曦)

2106.13484 with Xi Tong (童曦) and Hui-Yu Zhu (朱慧宇)

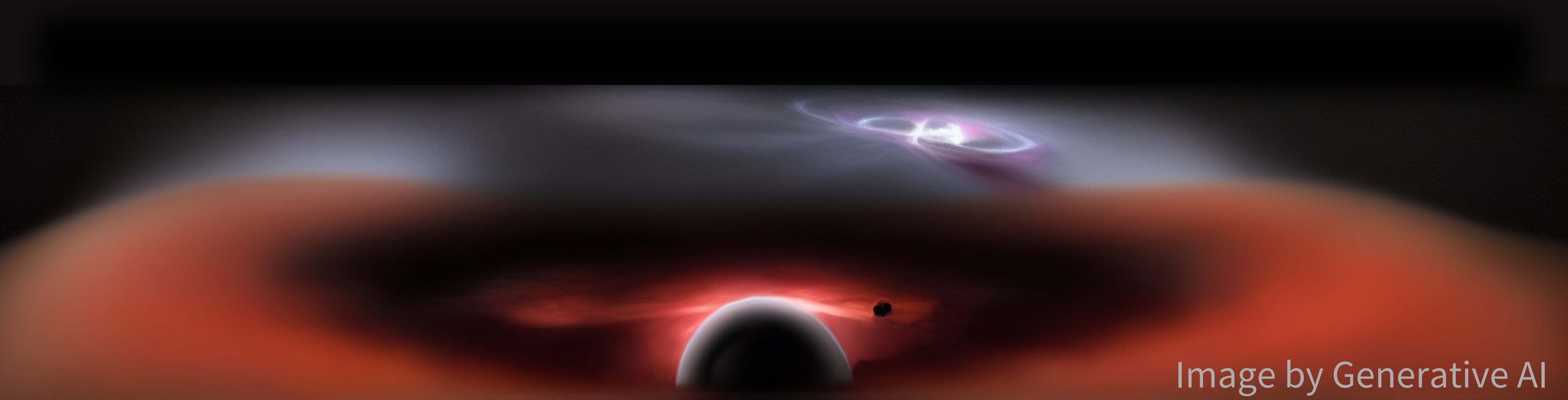


Image by Generative AI

Black hole physics: on-going breakthroughs

- Gravitational wave
- Imaging
- Origin: 100 $M_\odot$? Supermassive? PBH?
- Black hole entropy & information
- Tests of GR, dark matter, …

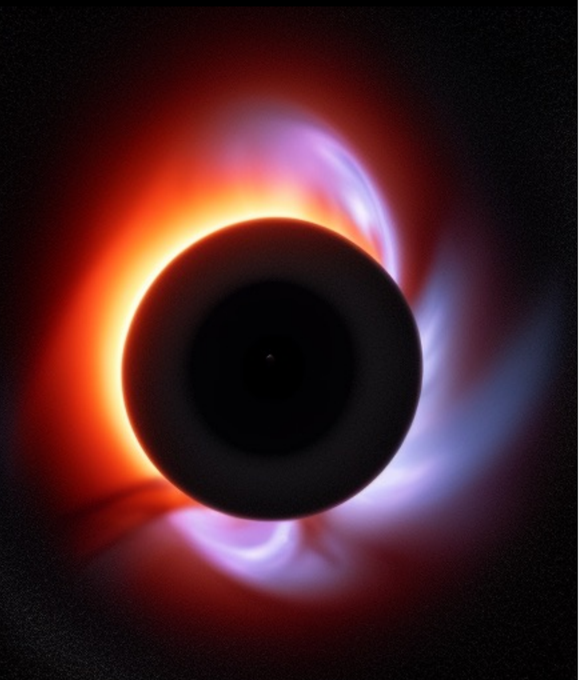
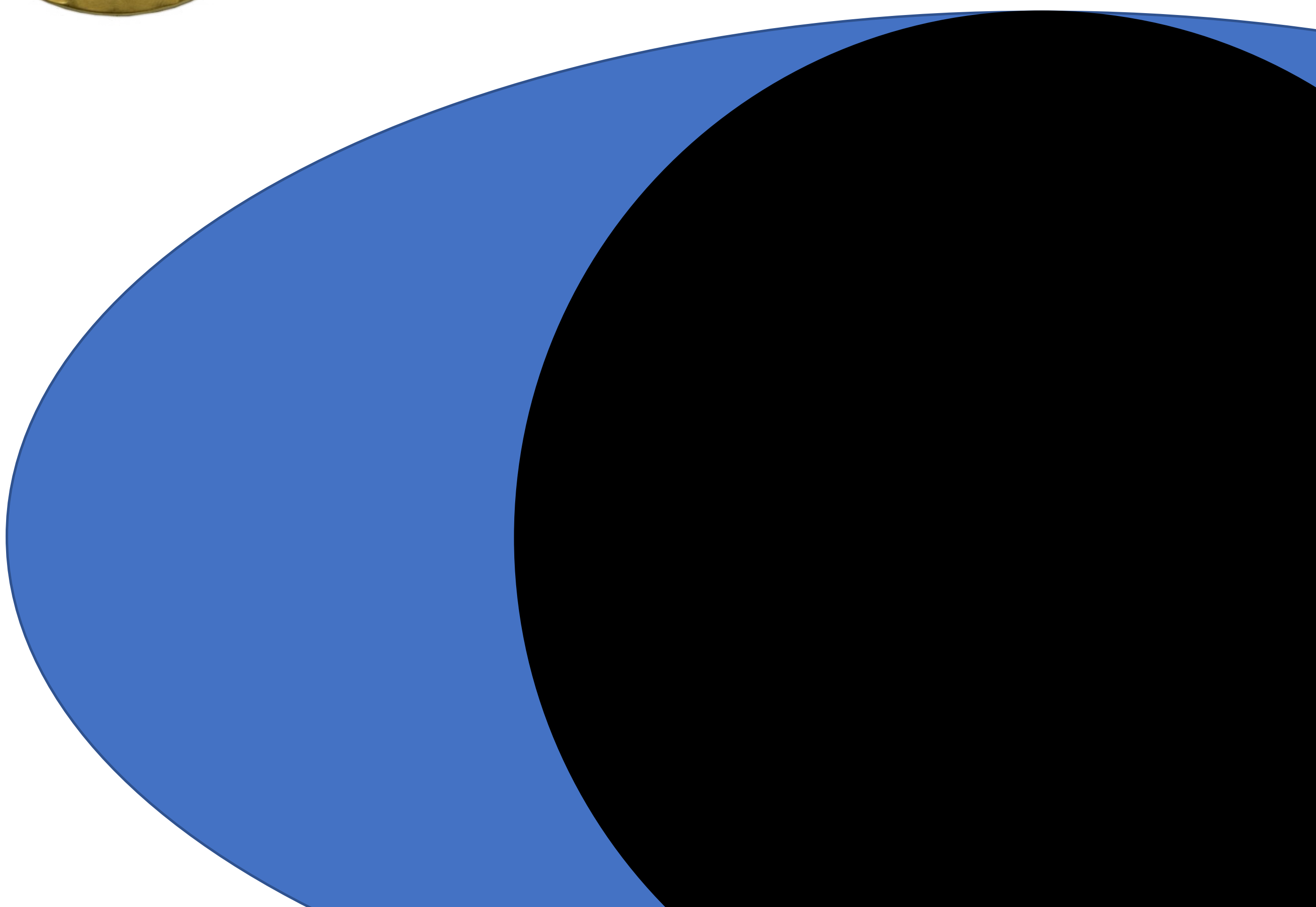


Image by Generative AI

Introduction

Penrose Process and BH Superradiance

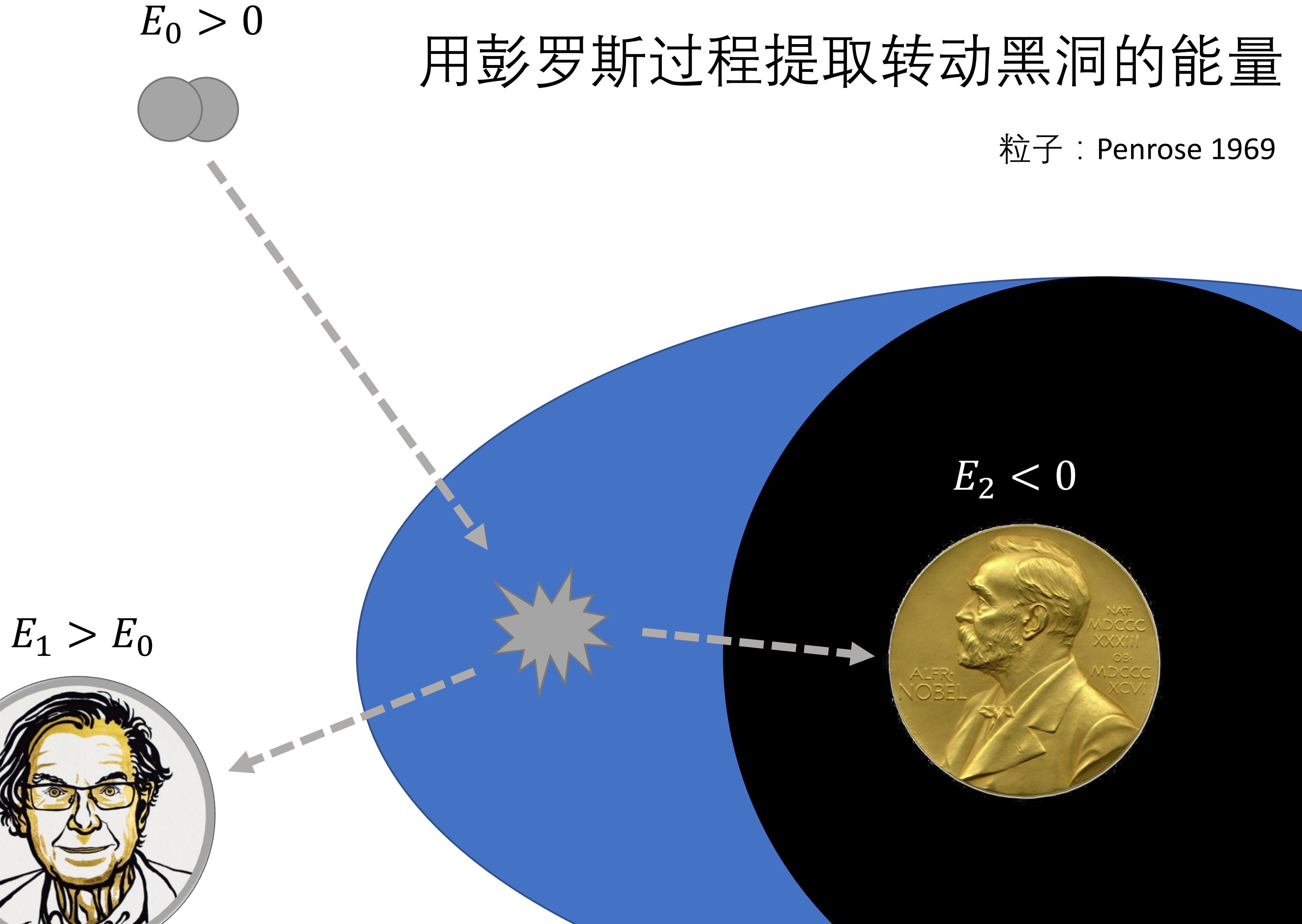

NAT·
MDCCC
XXXIII
OB·
MDCCC
XCVI




NAT·
MDCCC
XXXIII
OB·
MDCCC
XCVI

用彭罗斯过程提取转动黑洞的能量

粒子：Penrose 1969

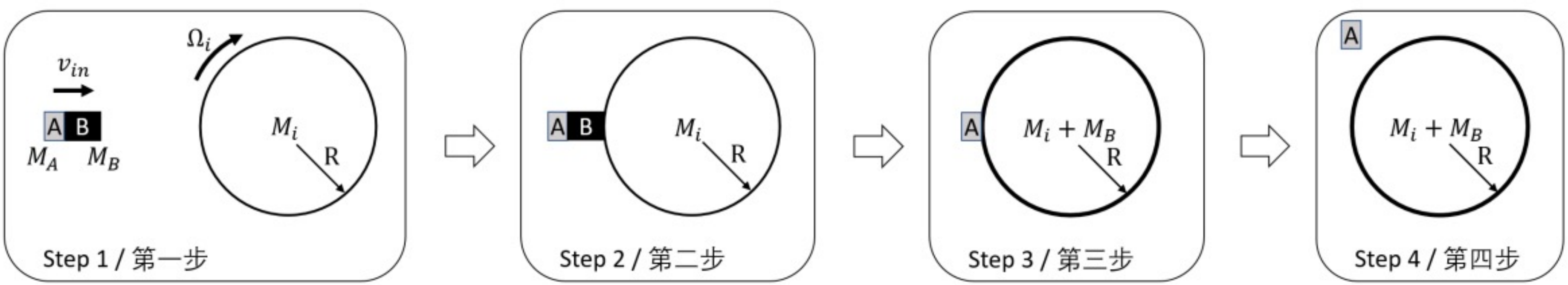


How does the Penrose Process work? A Newtonian analog

Pan Pearl River Delta Physics Olympiad 2021
2021 年泛珠三角及中华名校物理奥林匹克邀请赛
Sponsored by Institute for Advanced Study, HKUST
香港科技大学高等研究院赞助

Simplified Chinese Part-2 (Total 2 Problems, 60 Points)
简体版卷-2（共2题，60分）

(1:30 pm – 5:00 pm, 15 May 2021)



How does the Penrose Process work?

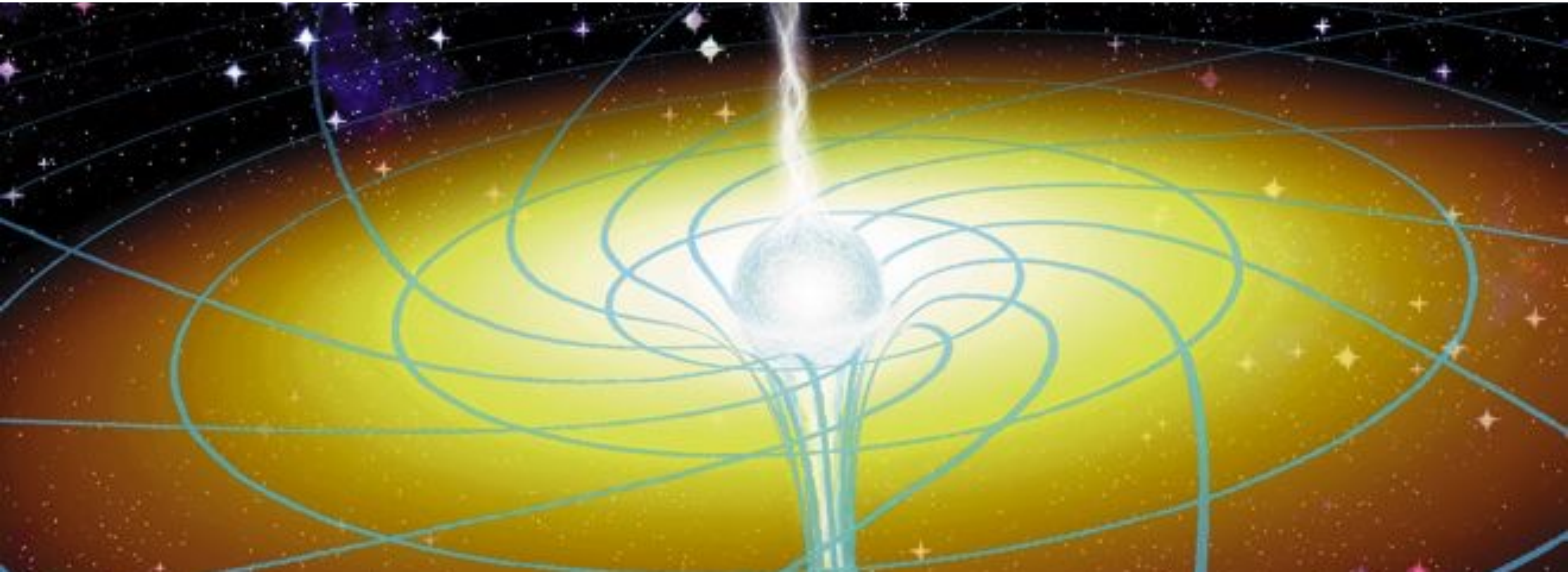
A bit more math of a rotating (Kerr) BH

The Kerr background

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left(a\,dt - (r^2+a^2)\,d\phi\right)^2$$

$$\Delta \equiv r^2 - 2Mr + a^2 \qquad \Sigma \equiv r^2 + a^2\cos^2\theta$$

$$r_\pm = M \pm \sqrt{M^2 - a^2} \qquad \Omega_H \equiv a/2Mr_+$$



Kerr: has Killing vector $\hat{t} = \partial_t$

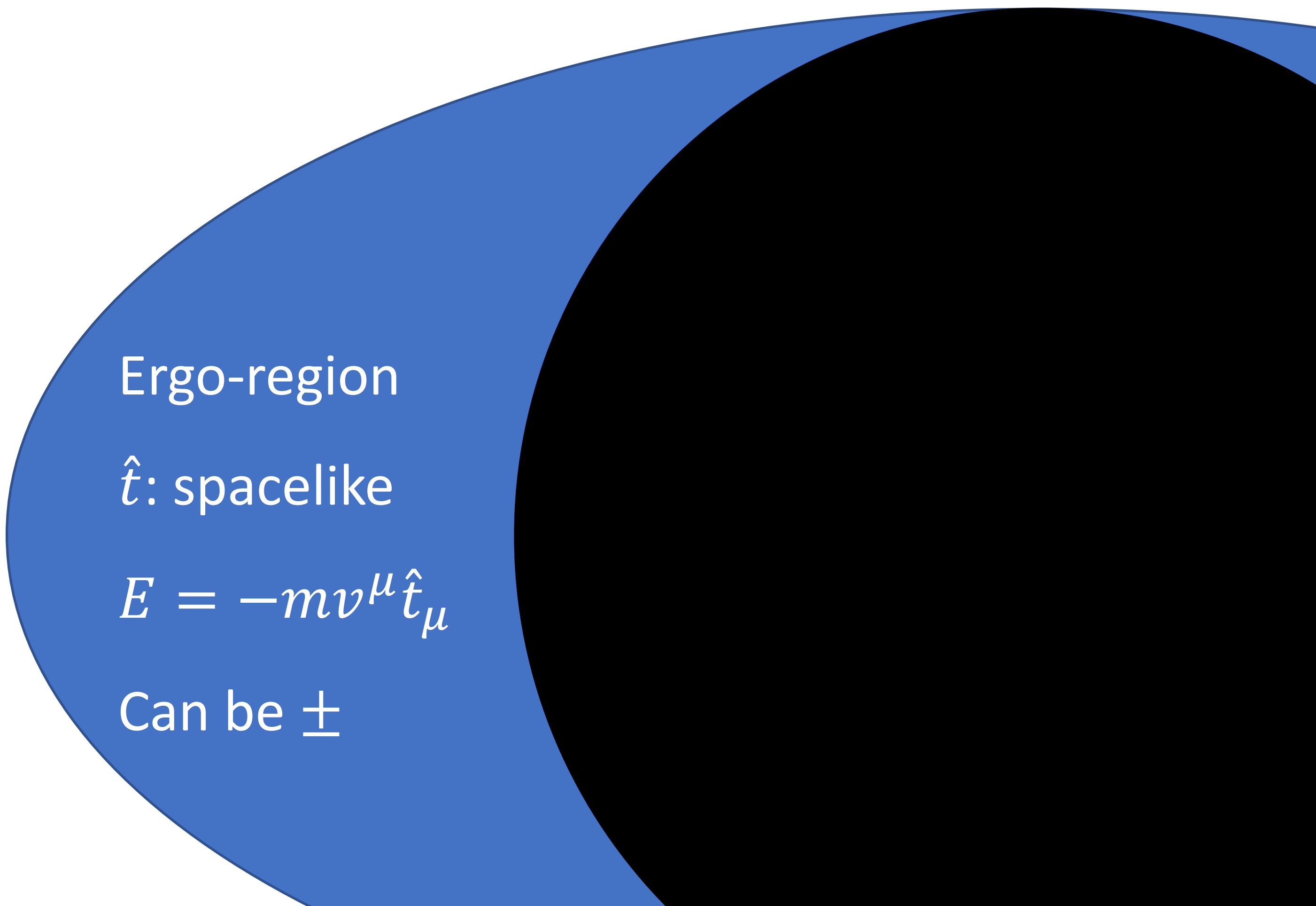
Killing energy $E = -p^\mu \hat{t}_\mu = -m \frac{dx^\mu}{d\tau} \hat{t}_\mu = -m v^\mu \hat{t}_\mu$

is conserved along geodesic

Exterior region

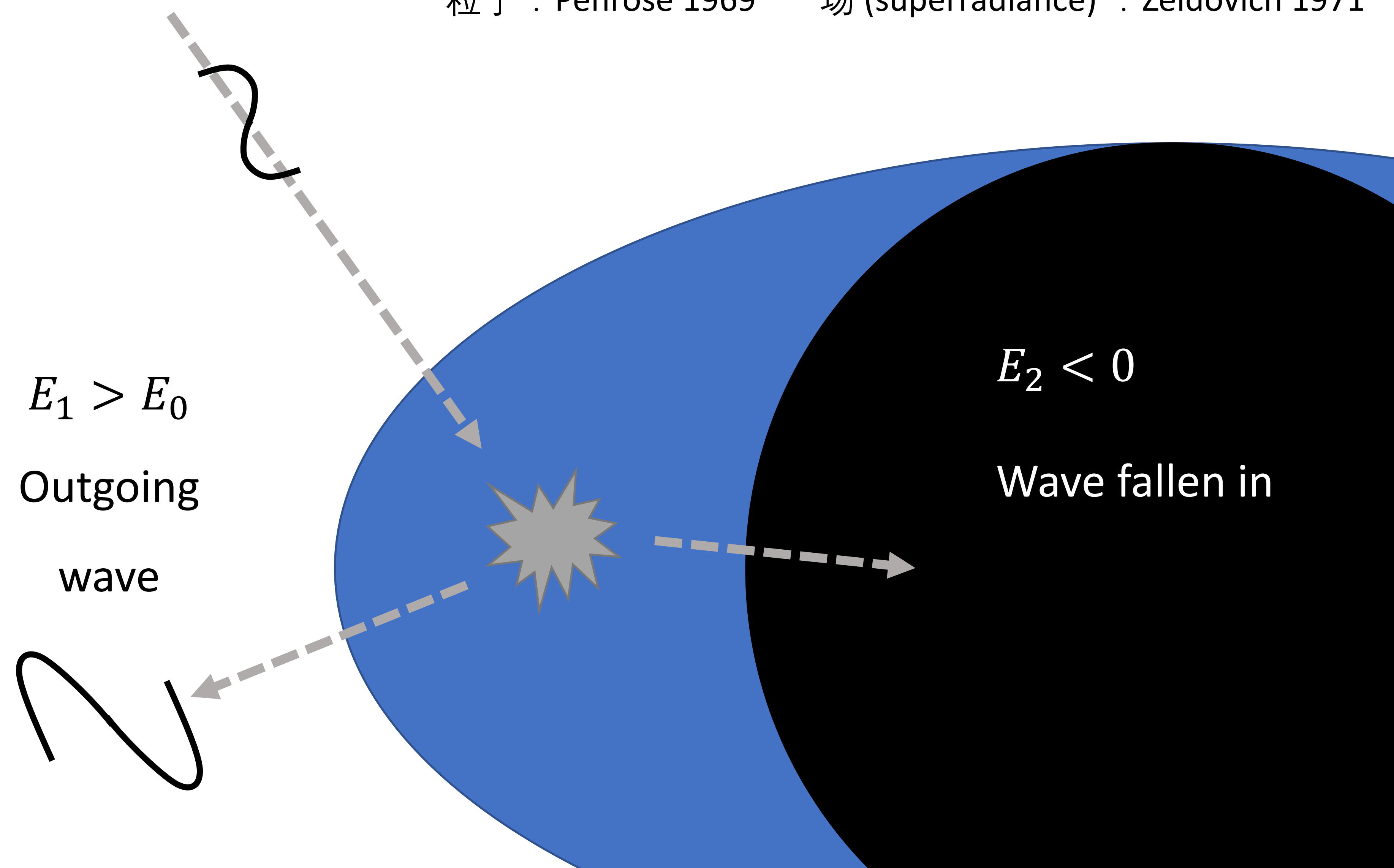
$\hat{t}$: timelike

$E = -m v^\mu \hat{t}_\mu > 0$



用彭罗斯过程提取转动黑洞的能量

粒子：Penrose 1969　　场 (superradiance) ：Zeldovich 1971



From the Penrose Process

to superradiance:

How to re-use the

out-going wave energy?

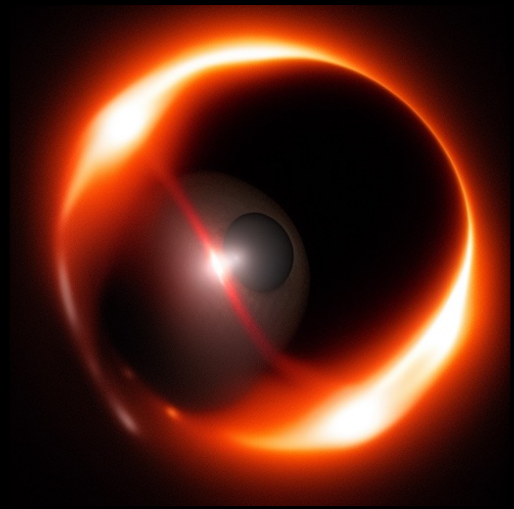


Image by Generative AI

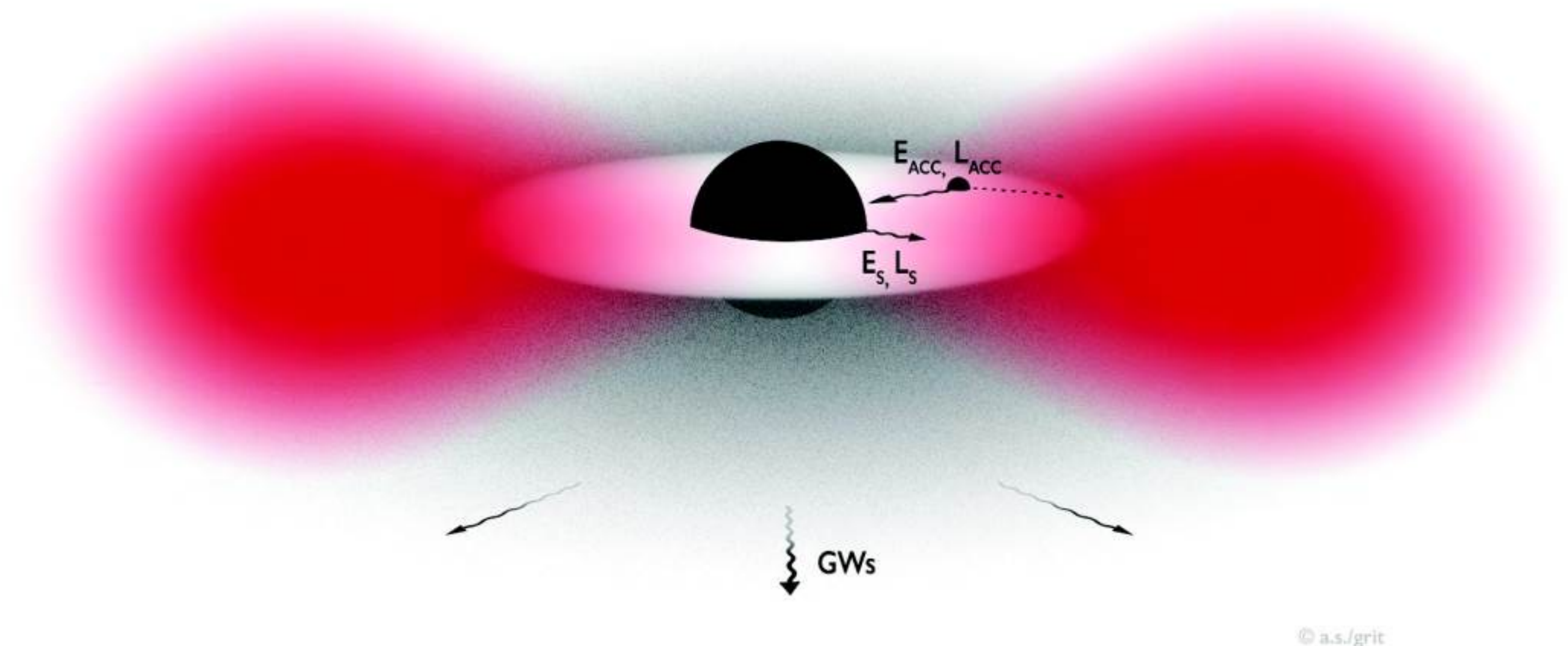
黑洞炸弹？

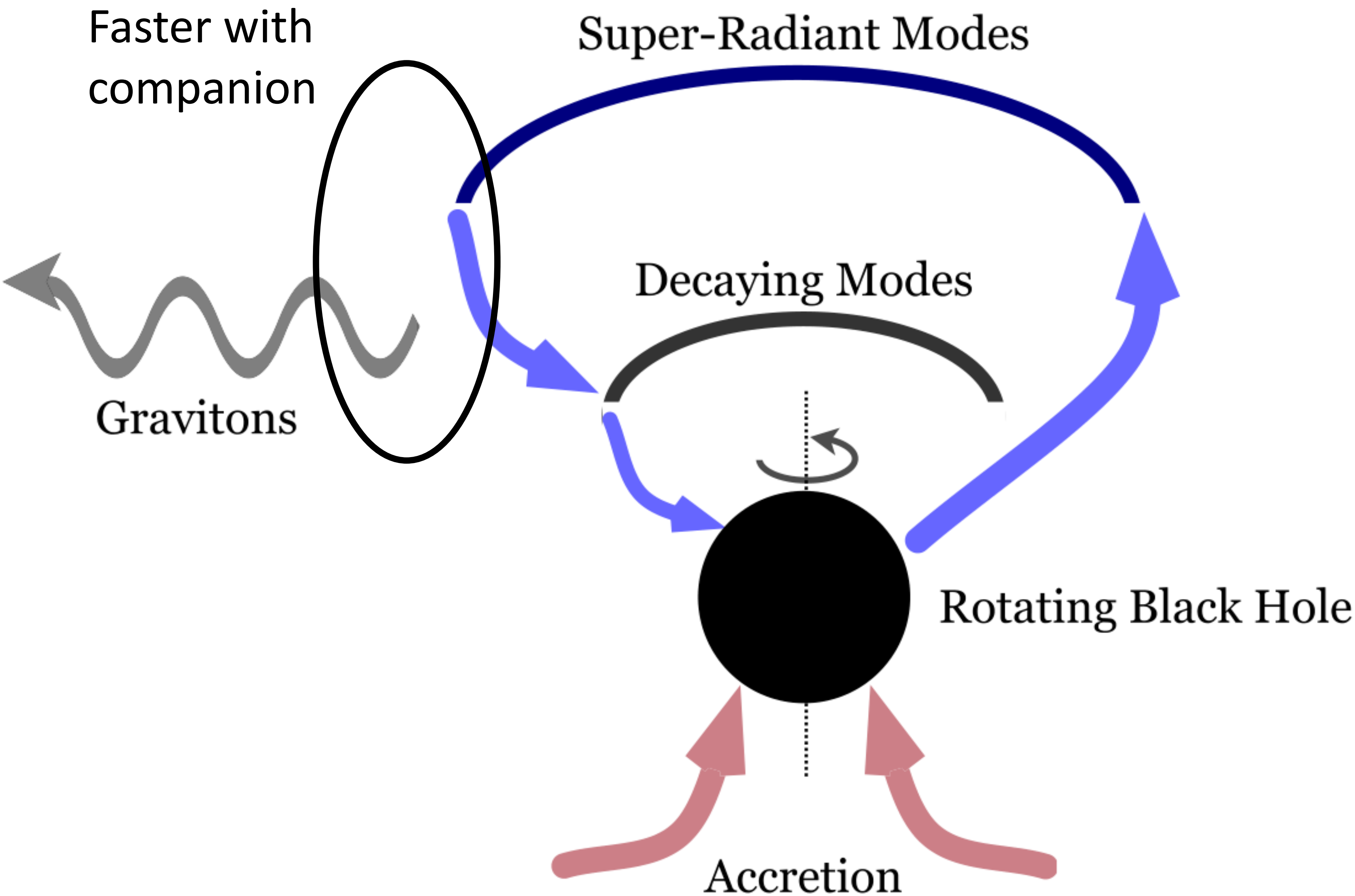
Press, Teukolsky 1972


以照镜子为生的"黑洞文明"？

超辐射 Superradiance

黑洞生轴子的气？ ⇒ 引力原子





“String Axiverse”, Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell, 2010
See also “Superradiance”, Brito, Cardoso, Pani, 2020

What happens

for a superradiant BH

with a companion star?

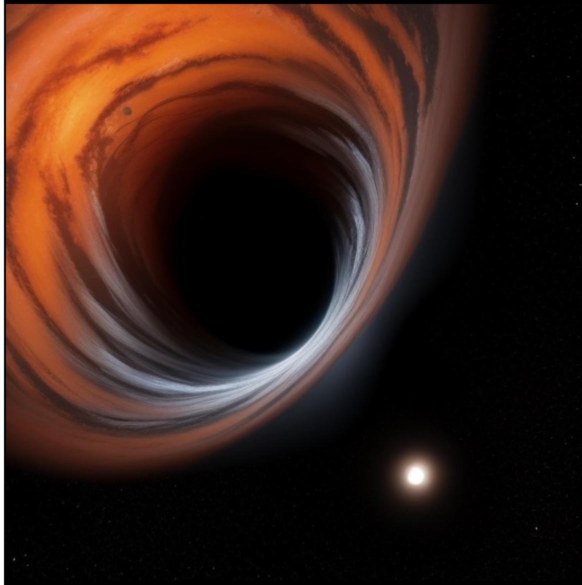
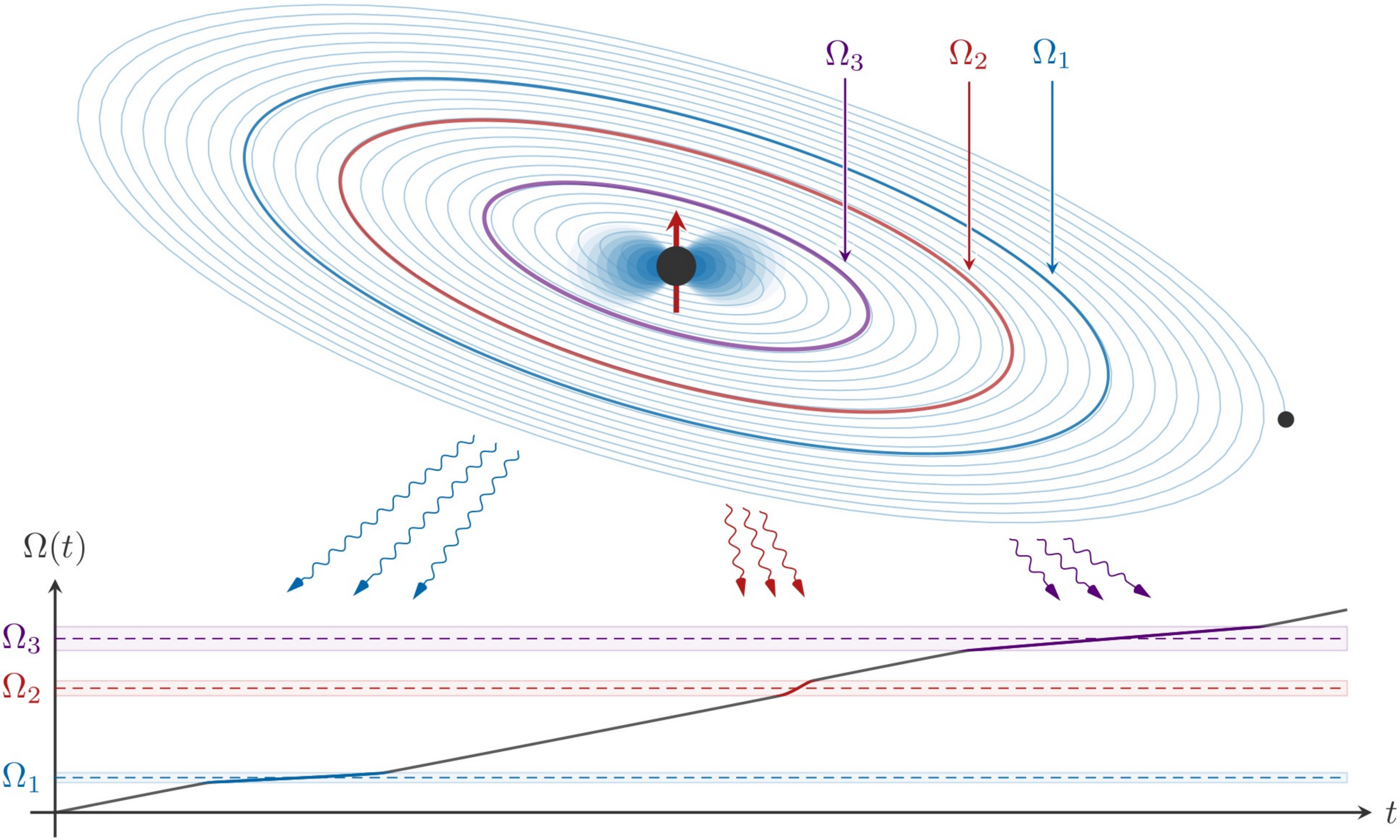
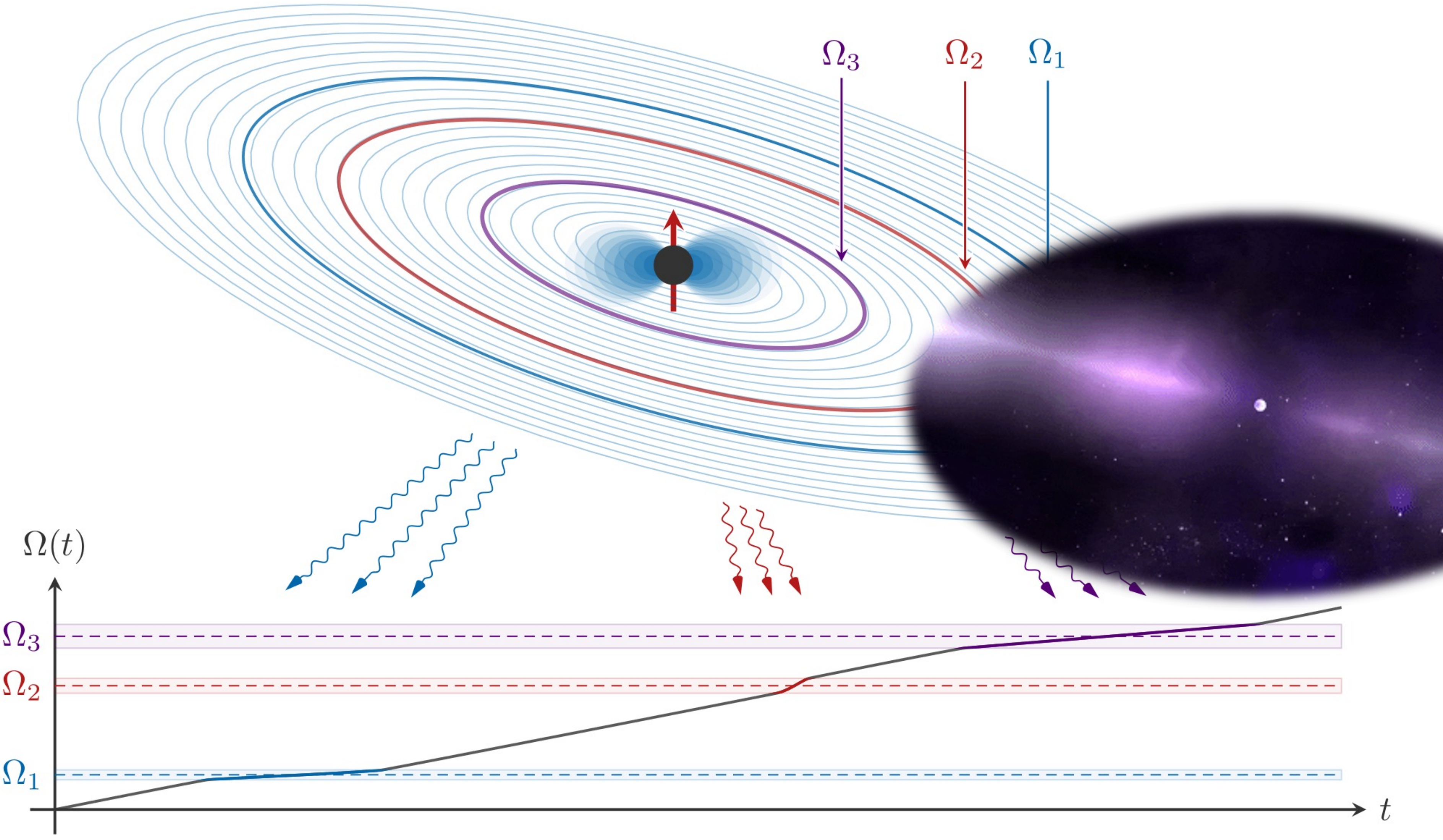


Image by Generative AI



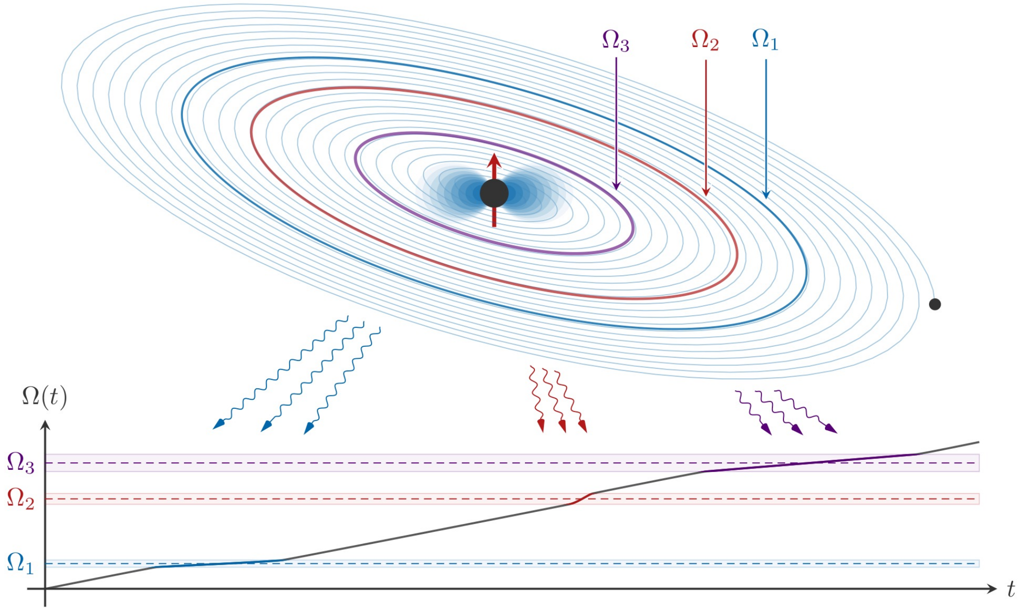
“Gravitational Collider Physics” (GCP), Baumann, Chia, Porto, Stout, 2019

Qianhang Ding, Xi Tong, YW, 2020

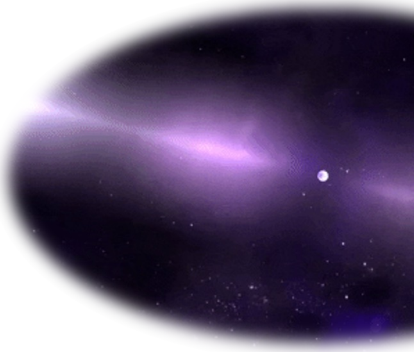


“Gravitational Collider Physics” (GCP), Baumann, Chia, Porto, Stout, 2019

Xi Tong, YW, Hui-Yu Zhu, 2020



Hyper-fine structures



"Gravitational Collider Physics" (GCP), Baumann, Chia, Porto, Stout, 2019

Outline

1. Introduction (✓)
2. Scalar cloud over Kerr (←)
3. GCP Bohr transitions
4. Termination of superradiance
5. Pulsar timing, hyperfine

Ref of 2 and 3:
Brito, Cardoso, Pani, 2020
Baumann, Chia, Porto 2018
Baumann, Chia, Stout, Haar 2019
Baumann, Chia, Porto, Stout, 2019

Ref: Tong, YW, Zhu, 2022

Ref:
Ding, Tong, YW, 2020
Tong, YW, Zhu, 2021

符号约定

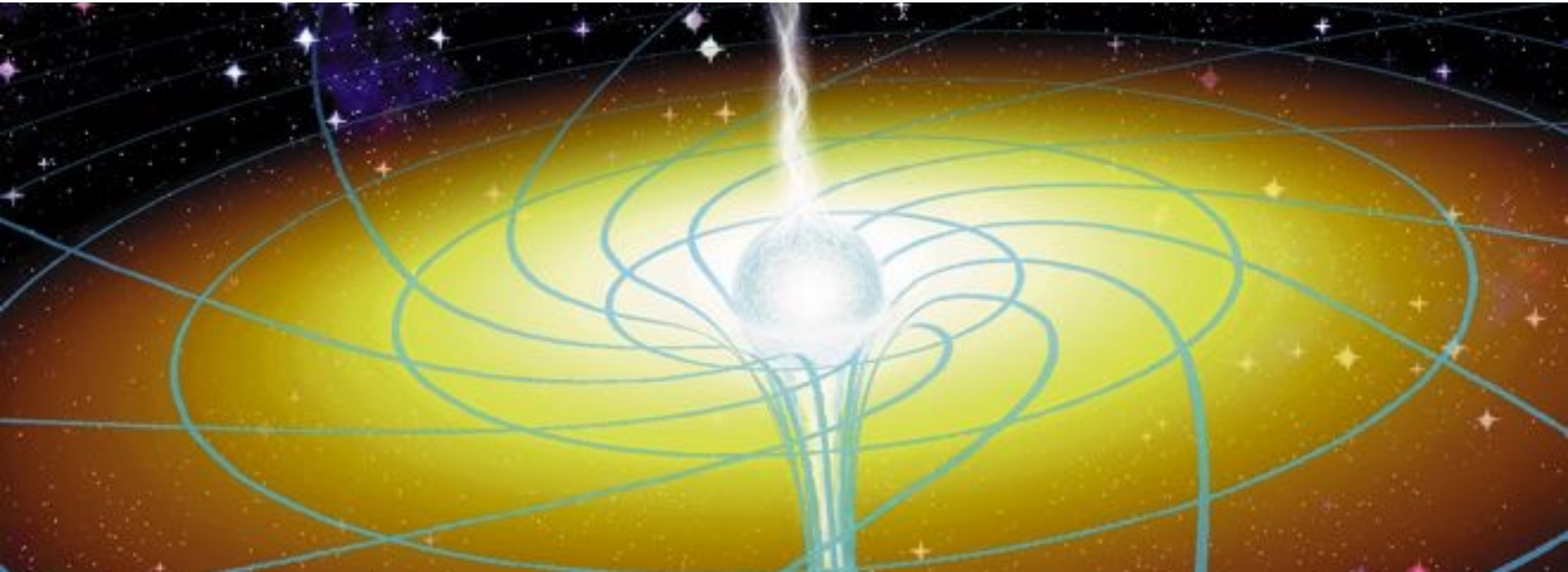
标量场内禀	标量场模式	黑洞
μ: 标量场质量	m: 磁量子数（Y_{lm}）	M: 黑洞质量
	ω: 频率（$e^{-i\omega t}$）	Ω_H: 外视界拖曳速度（零角动量粒子在视界的角速度）

The Kerr background

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left(a\,dt - (r^2+a^2)\,d\phi\right)^2$$

$$\Delta \equiv r^2 - 2Mr + a^2 \qquad \Sigma \equiv r^2 + a^2\cos^2\theta$$

$$r_\pm = M \pm \sqrt{M^2 - a^2} \qquad \Omega_H \equiv a/2Mr_+$$



Scalar EoM in the Kerr background

$$\left(g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \mu^2\right)\Phi(t,\mathbf{r}) = 0$$

Ansatz:

$$\Phi(t,\boldsymbol{r}) = \frac{1}{\sqrt{2\mu}}\left[\psi(t,\mathbf{r})\,e^{-i\mu t} + \psi^*(t,\mathbf{r})e^{+i\mu t}\right]$$

Non-relativistic approximation: Schrodinger-like eq

$$i\frac{\partial}{\partial t}\psi(t,\mathbf{r}) = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + \Delta V\right)\psi(t,\mathbf{r})$$

$$\alpha \equiv \frac{GM\mu}{\hbar c} \simeq 0.02\left(\frac{M}{3M_\odot}\right)\left(\frac{\mu}{10^{-12}\,\mathrm{eV}}\right)$$

ΔV: higher order in α

Separation of variables:

$$\psi_{n\ell m}(t, \mathbf{r}) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)\, e^{-i(\omega_{n\ell m} - \mu)t}$$

$$\omega = E + i\Gamma$$

After solving the Schrodinger-like equation:

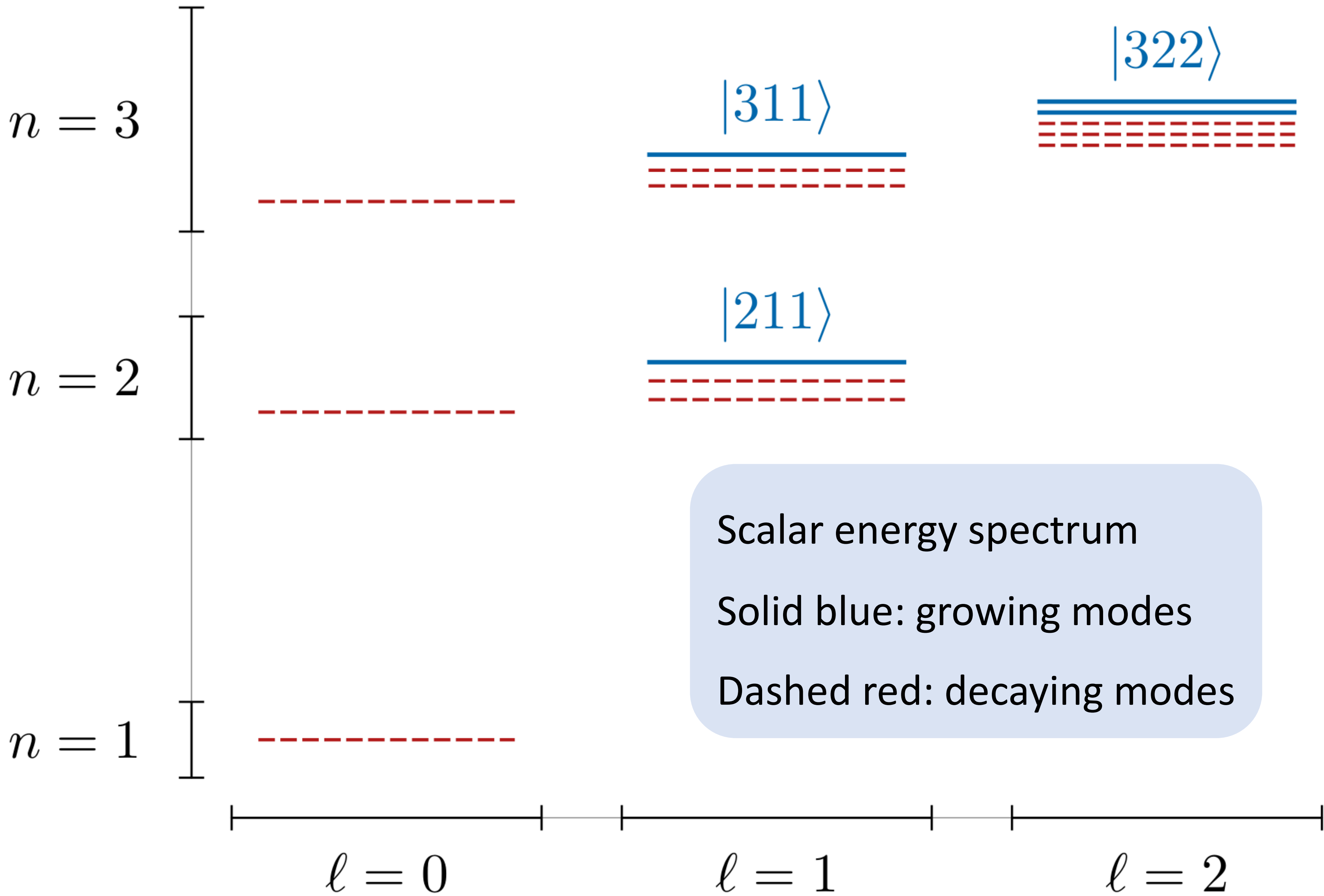
$$E_{n\ell m} = \mu\left(1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} - \frac{(3n - 2\ell - 1)\,\alpha^4}{n^4\,(\ell + 1/2)} + \frac{2\tilde{a}m\,\alpha^5}{n^3\ell(\ell + 1/2)(\ell + 1)} + \mathcal{O}(\alpha^6)\right)$$

$$\Gamma_{n\ell m} = 2\tilde{r}_+ C_{n\ell}\, g_{\ell m}(\tilde{a}, \alpha, \omega)\,(m\Omega_{\rm H} - \omega_{n\ell m})\,\alpha^{4\ell+5}$$

Note the sign of Γ

- Negative: decay
- Positive ($\frac{\omega}{m} < \Omega_H$) : superradiance

Angular velocity of wave from $e^{-i\omega t + im\phi}$


$n = 3$
$n = 2$
$n = 1$
$|311\rangle$
$|322\rangle$
$|211\rangle$
Scalar energy spectrum
Solid blue: growing modes
Dashed red: decaying modes
$\ell = 0$
$\ell = 1$
$\ell = 2$

For initially extremal Kerr,

the mass and angular momentum for leading states

	$m=1$	$m=2$	$m=3$	$\cdots$
Scalar: $\vert n\ell m\rangle$	$\vert 211\rangle$	$\vert 322\rangle$	$\vert 433\rangle$	$\cdots$
Vector: $\vert n\ell jm\rangle$	$\vert 1011\rangle$	$\vert 2122\rangle$	$\vert 3233\rangle$	$\cdots$
$M_{c,0}/M$	$\alpha-4\alpha^2$	α^2	$2\alpha^2/9$	$\cdots$
$S_{c,0}/M^2$	$1-4\alpha$	α	$2\alpha/9$	$\cdots$

$$T_{211}^{(\mathrm{growth})} \sim \frac{10^6 \ \mathrm{yrs}}{\tilde{a}} \left(\frac{M_B}{M_\odot}\right) \left(\frac{0.012}{\alpha}\right)^9$$

$$T_{211}^{(\mathrm{deplete})} \sim 10^8 \ \mathrm{yrs} \left(\frac{M_B}{M_\odot}\right) \left(\frac{0.053}{\alpha}\right)^{15}$$

$$T_{322}^{(\mathrm{growth})} \sim \frac{10^6 \ \mathrm{yrs}}{\tilde{a}} \left(\frac{M_B}{M_\odot}\right) \left(\frac{0.08}{\alpha}\right)^{13}$$

$$T_{322}^{(\mathrm{deplete})} \sim 10^8 \ \mathrm{yrs} \left(\frac{M_B}{M_\odot}\right) \left(\frac{0.18}{\alpha}\right)^{20}$$

Outline

1. Introduction (✓)
2. Scalar cloud over Kerr (✓)
3. GCP Bohr transitions (←)
4. Termination of superradiance
5. Pulsar timing, hyperfine

Selection rules

$$(\mathrm{S1}) \quad -m' + m_* + m = 0\,,$$

$$(\mathrm{S2}) \quad \ell + \ell_* + \ell' = 2p\,, \text{ for } p \in \mathbb{Z}\,,$$

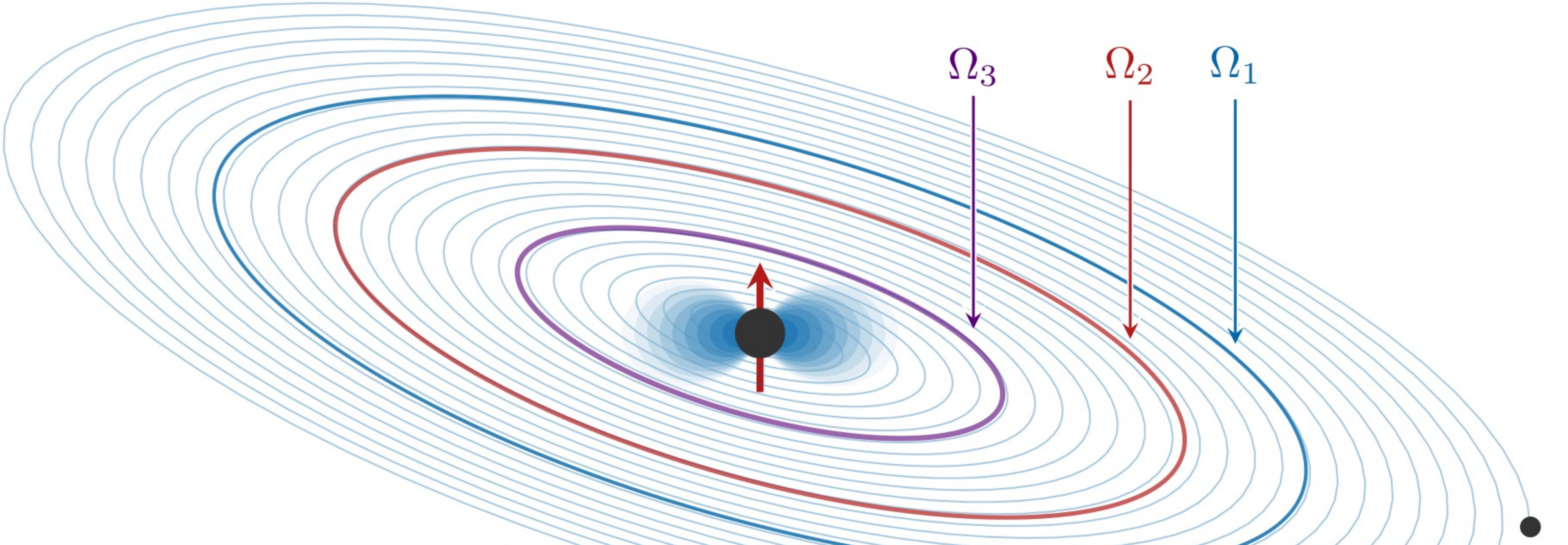
$$(\mathrm{S3}) \quad |\ell - \ell'| \leq \ell_* \leq \ell + \ell'\,.$$

ℓ, m: old axion state

ℓ', m': new axion state

ℓ_*, m_*: perturbation

(no dipole $\ell_* = 1$)



Two-state transitions

$$\mathcal{H}_{\mathrm{D}}(t) = \mathcal{U}^\dagger \mathcal{H} \mathcal{U} - i\mathcal{U}^\dagger \frac{\mathrm{d}\mathcal{U}}{\mathrm{d}t} = \begin{pmatrix} (\Delta m \Omega(t) - \Delta E)/2 & \eta \\ \eta & -(\Delta m \Omega(t) - \Delta E)/2 \end{pmatrix}$$

small gravitational perturbation

Landau-Zener transition happens when this vanishes

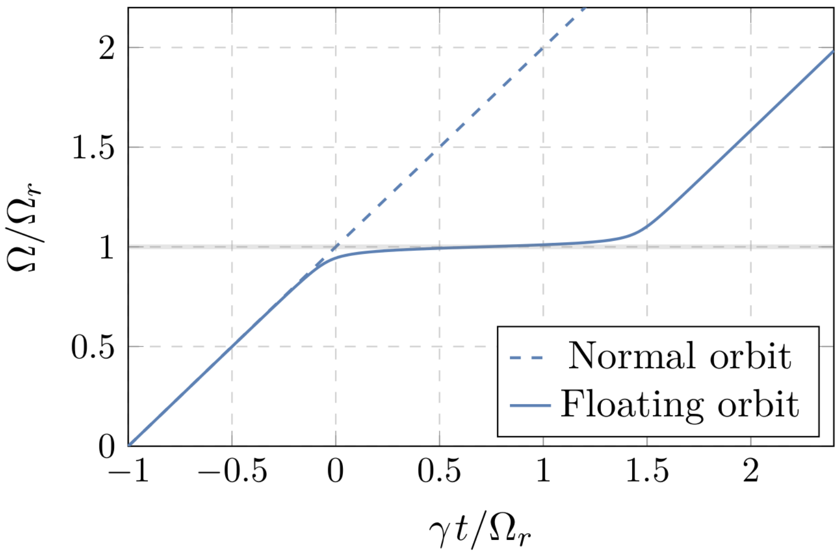
Period of the perturbation for Landau-Zener transition:

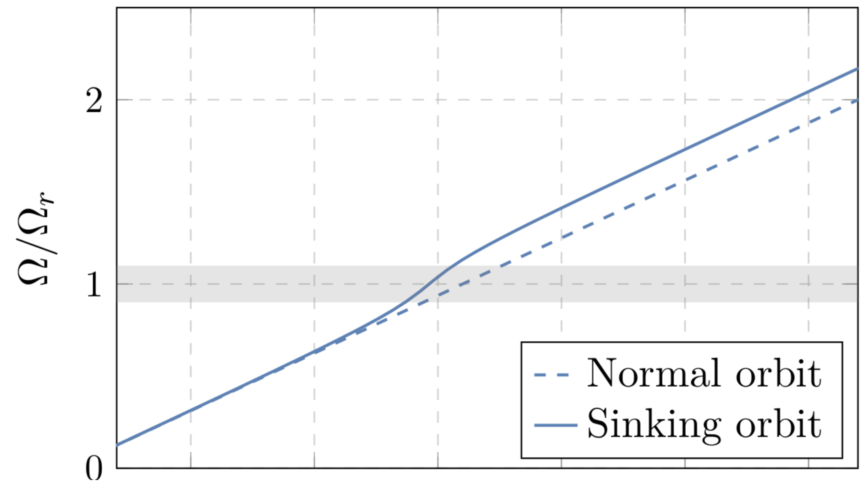
$$P_r = 2\pi \left| \frac{\Delta m_{ab}}{\Delta E_{ab}} \right|, \;\; \Delta E_{ab} = \frac{\mu \alpha^2}{2} \left(\frac{1}{n_a^2} - \frac{1}{n_b^2} \right)$$

$$\frac{\Delta P_r}{P_r} = 2 R_{ab} \frac{q}{1+q}, \;\; R_{ab} \lesssim 0.3$$

$$\dot{P} = -\frac{96}{5} (2\pi)^{8/3} (G M_B)^{5/3} \frac{q}{(1+q)^{1/3}} P^{-5/3} \times \frac{1}{1 \pm \delta}$$

$$\delta = \frac{\Delta t_c}{\Delta t} \simeq \frac{1}{4} \frac{(1+q)^{4/3}}{q^2} \left(\frac{\alpha}{0.07} \right) \frac{S_{c,0}}{M_B^2}$$





+: floating orbit, $\Delta E < 0$, cloud releases energy

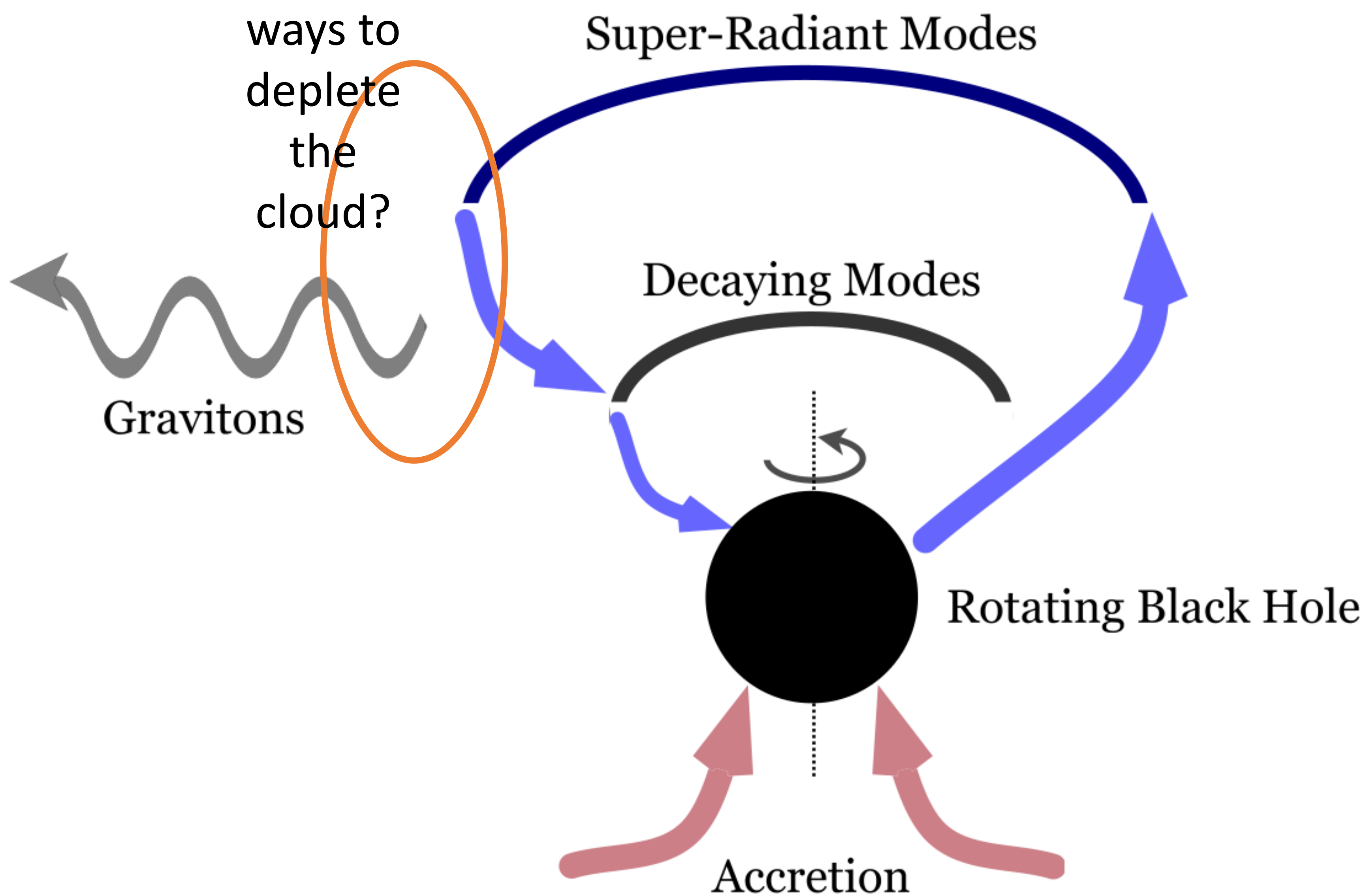
- : sinking orbit , $\Delta E > 0$, cloud gets energy

$$\dot{P} = -\frac{96}{5}(2\pi)^{8/3}(GM_B)^{5/3}\frac{q}{(1+q)^{1/3}}P^{-5/3} \times \frac{1}{1 \pm \delta}$$

$$\delta = \frac{\Delta t_c}{\Delta t} \simeq \frac{1}{4}\frac{(1+q)^{4/3}}{q^2}\left(\frac{\alpha}{0.07}\right)\frac{S_{c,0}}{M_B^2}$$

Outline

1. Introduction (✓)
2. Scalar cloud over Kerr (✓)
3. GCP Bohr transitions (✓)
4. Termination of superradiance (←)
5. Pulsar timing, fine & hyperfine

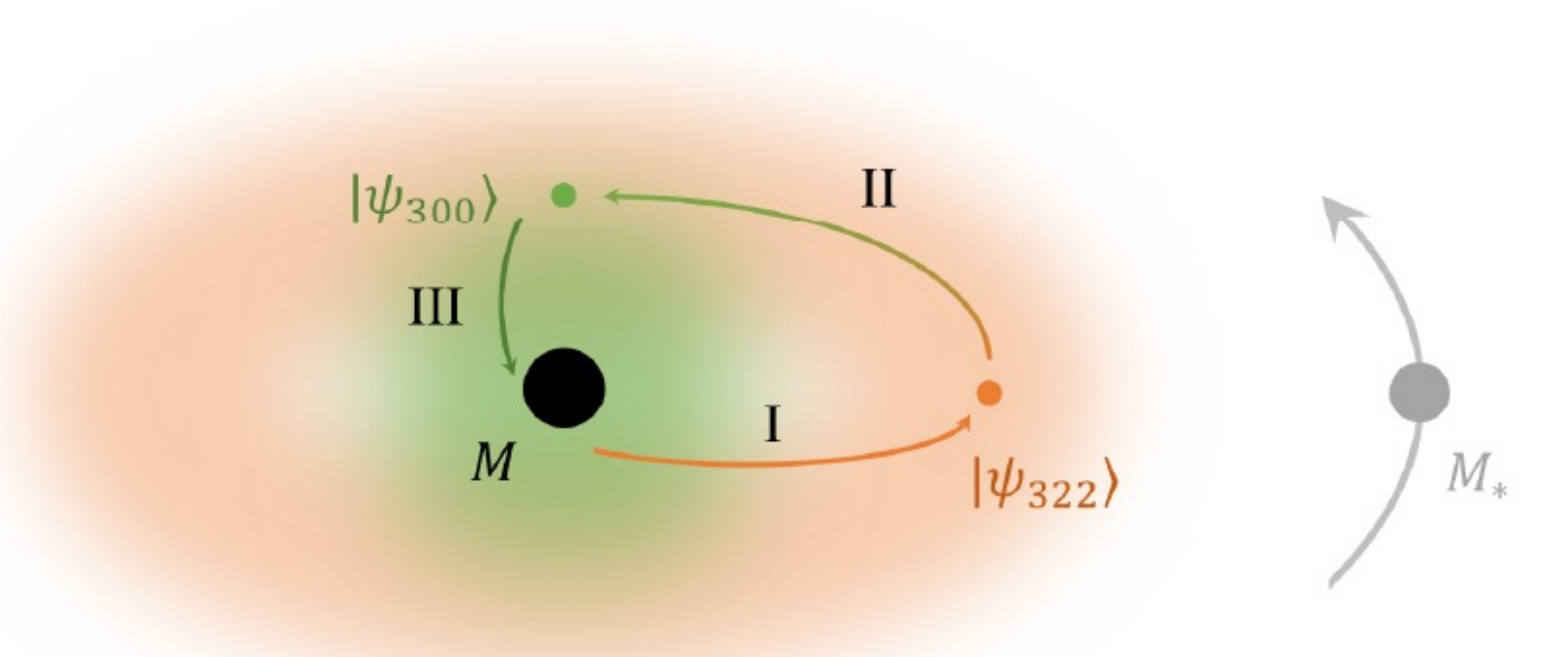

ways to
deplete
the
cloud?
Super-Radiant Modes
Decaying Modes
Gravitons
Rotating Black Hole
Accretion

GCP requires a companion:

Tidal perturbation not only

(1) Trigger GCP resonance, but also

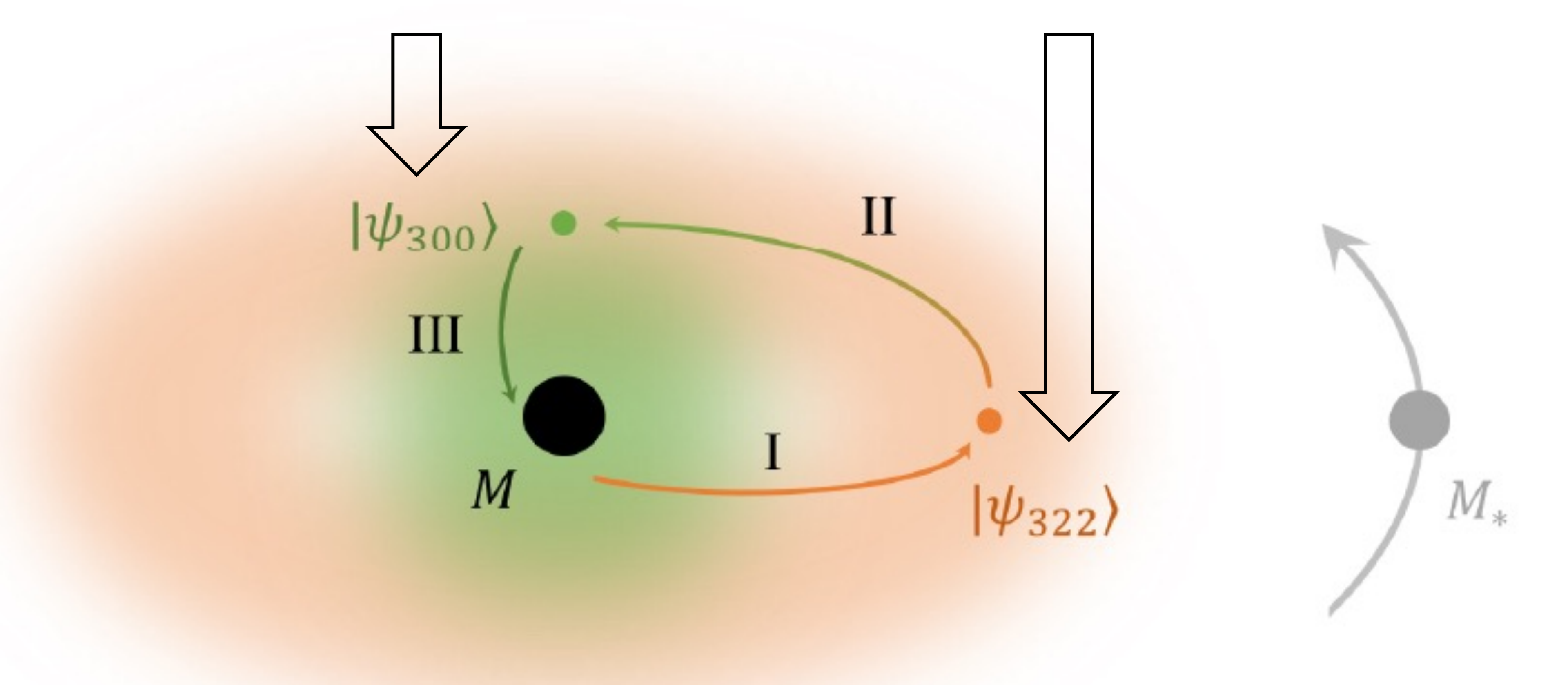
(2) Trigger off-resonance decay



Mixed by M_*. How to solve the mixing?

Unstable

More stable



Mixed by M_*. How to solve the mixing?

Simple estimate: adiabatic approximation

$$H = \begin{pmatrix} \omega_1 + V_{11} & V_{12} \\ V_{21} & \omega_2 + V_{22} \end{pmatrix} \equiv \begin{pmatrix} \bar{E}_1 + i\Gamma_1 & \eta^* \\ \eta & \bar{E}_2 + i\Gamma_2 \end{pmatrix}$$

WKB solution of $c_i(t) \equiv \langle \psi_i | \psi(t) \rangle$

$$c_i(t) = C_{i+} e^{-i \int \lambda_+ dt} + C_{i-} e^{-i \int \lambda_- dt} \ , \ i = 1, 2$$

$\lambda_\pm$: instantaneous eigenvalues of H

$$\lambda_\pm \equiv \frac{\bar{\omega}_1 + \bar{\omega}_2}{2} \pm \sqrt{|\eta|^2 + \left(\frac{\bar{\omega}_1 - \bar{\omega}_2}{2} \right)^2}$$

$$\simeq \begin{cases} \bar{E}_1 + \frac{|\eta|^2}{\bar{E}_1 - \bar{E}_2} + i \left[\Gamma_1 - \frac{\Gamma_1 - \Gamma_2}{(\bar{E}_1 - \bar{E}_2)^2} |\eta|^2 \right], & + \\ \bar{E}_2 + \frac{|\eta|^2}{\bar{E}_2 - \bar{E}_1} + i \left[\Gamma_2 - \frac{\Gamma_2 - \Gamma_1}{(\bar{E}_1 - \bar{E}_2)^2} |\eta|^2 \right], & - \end{cases}$$

$$\tilde{\Gamma}_1 \equiv \Gamma_1 + \Delta\Gamma_1, \ \Delta\Gamma_1 \simeq -\frac{\Gamma_1 - \Gamma_2}{(\bar{E}_1 - \bar{E}_2)^2} |\eta(R_*)|^2 < 0$$

Example:

$$E_{322} - E_{300} \simeq 0.04 \ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^5 \left(\frac{M}{10M_\odot}\right)^{-1}$$

$$\Gamma_{322} \simeq 3 \times 10^{-13} \ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{13} \left(\frac{M}{10M_\odot}\right)^{-1}$$

$$\Gamma_{300} \simeq -3 \times 10^{-3} \ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^5 \left(\frac{M}{10M_\odot}\right)^{-1}$$

$$\Delta\Gamma_{322} \simeq -7 \times 10^3 \frac{q^2}{\alpha^{10}} \frac{M^5}{R_*^6}$$

$$\simeq -0.6 \times 10^{-13} \ \mathrm{s}^{-1} \left(\frac{\alpha}{0.1}\right)^{-10} \left(\frac{q}{0.2}\right)^2 \left(\frac{M}{10M_\odot}\right)^{-1}$$

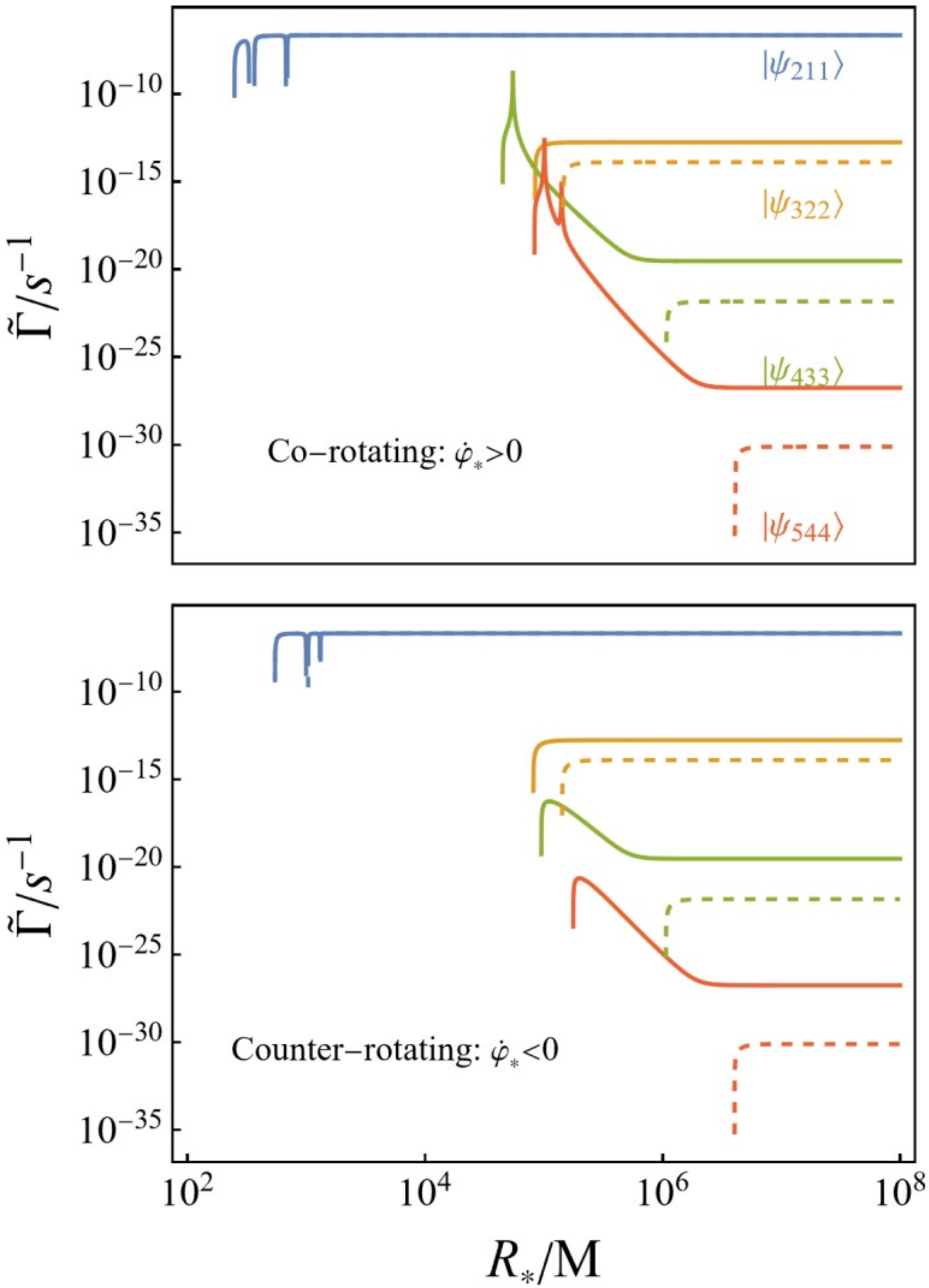
More careful calculation: the same thing in co-rotating frame

$$H_D = \begin{pmatrix} \bar{E}_1 + i\Gamma_1 - m_1\dot{\varphi}_* & |\eta| \\ |\eta| & \bar{E}_2 + i\Gamma_2 - m_2\dot{\varphi}_* \end{pmatrix}$$

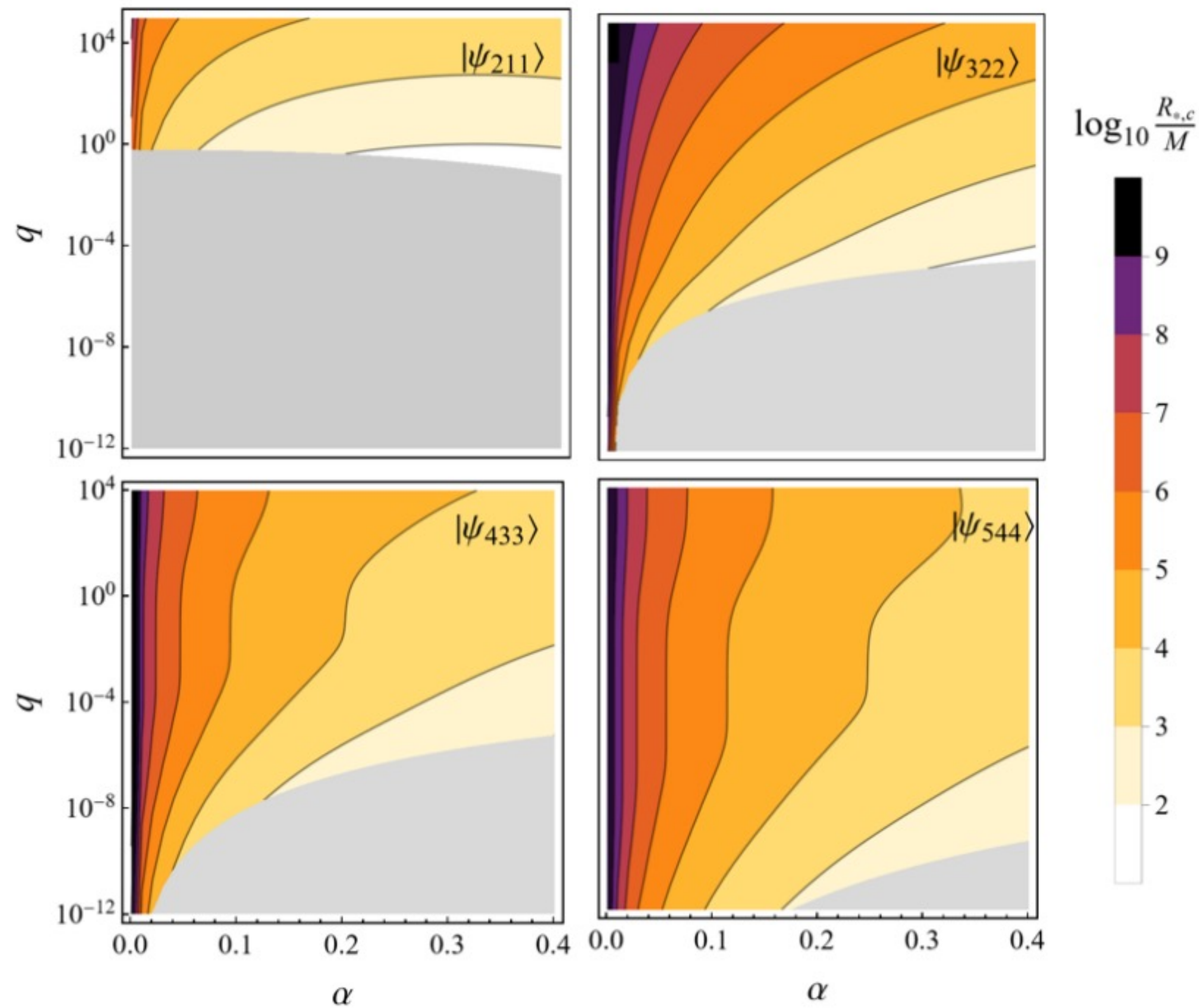
$$\Delta\Gamma_1 \simeq -\frac{\Gamma_1 - \Gamma_2}{\left[\bar{E}_1 - \bar{E}_2 - (m_1 - m_2)\dot{\varphi}_*(R_*)\right]^2}|\eta(R_*)|^2$$

Summing over all decay channels:

$$\Delta\Gamma_1 \simeq -\sum_{i=n'l'm'} \frac{\Gamma_1 - \Gamma_i}{\left[\bar{E}_1 - \bar{E}_i - (m_1 - m_i)\dot{\varphi}_*(R_*)\right]^2}|\eta_{1i}(R_*)|^2$$



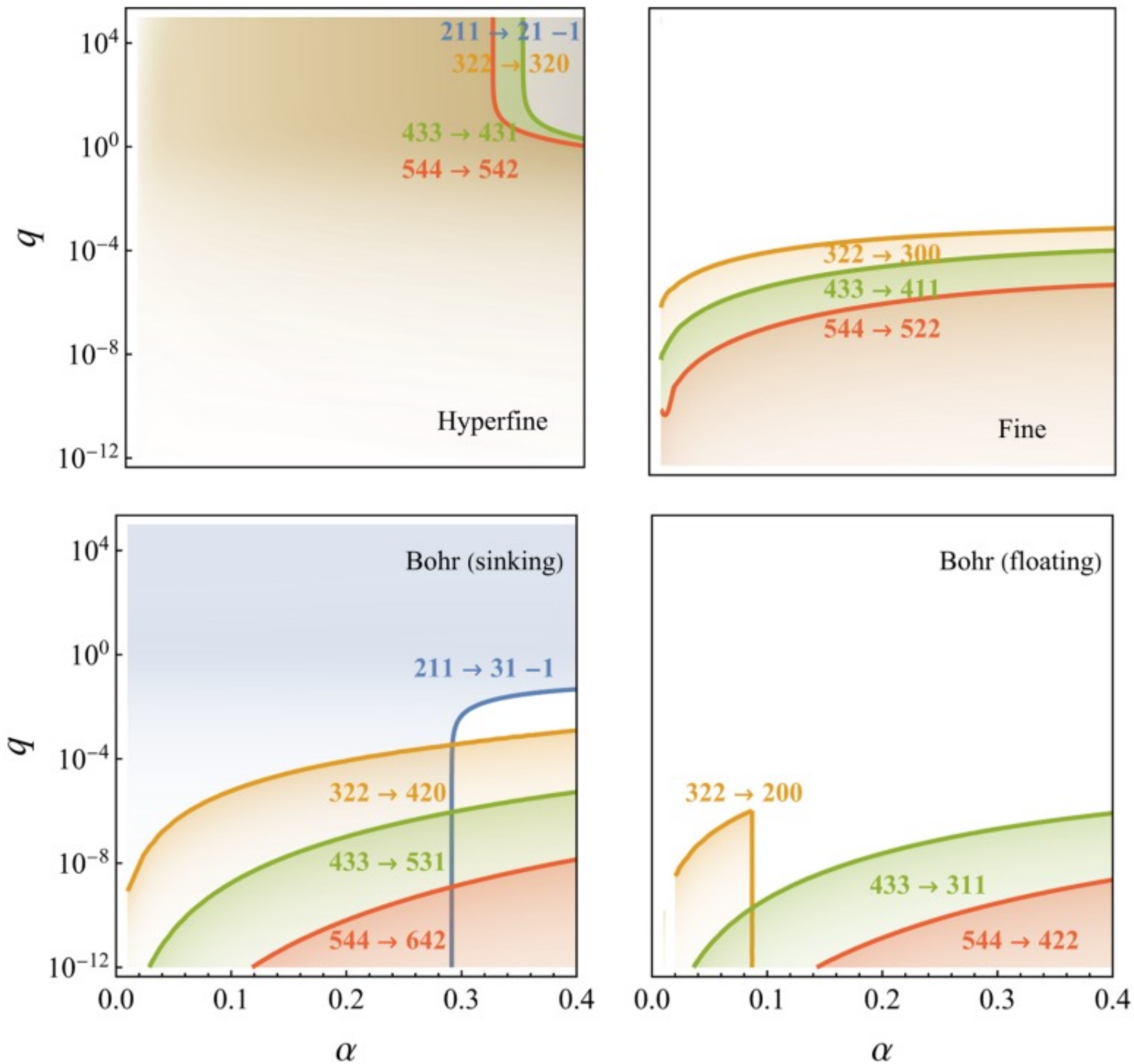
For $\alpha = 0.1, q = 0.2$



Critical distance of R_*: $R_{*,c}$

Gray: $R_{*,c}$ < (cloud size), our calculation breaks down

Implication to GCP: Cloud can decay before resonance



Safe transitions (shaded == allowed; white == cannot resonant)

Off-resonance GCP signals:

$$(\dot{P})_{\rm GR} = -\frac{96}{5}(2\pi)^{8/3}\frac{q}{(1+q)^{1/3}}M^{5/3}P^{-5/3}$$

$$(\dot{P})_{\rm C} = -3(2\pi)^{1/3}(1+q)^{-2/3}\frac{S_{c,0}m_1}{M^2}\frac{d|c_1(t)|^2}{dt}M^{1/3}P^{2/3}$$

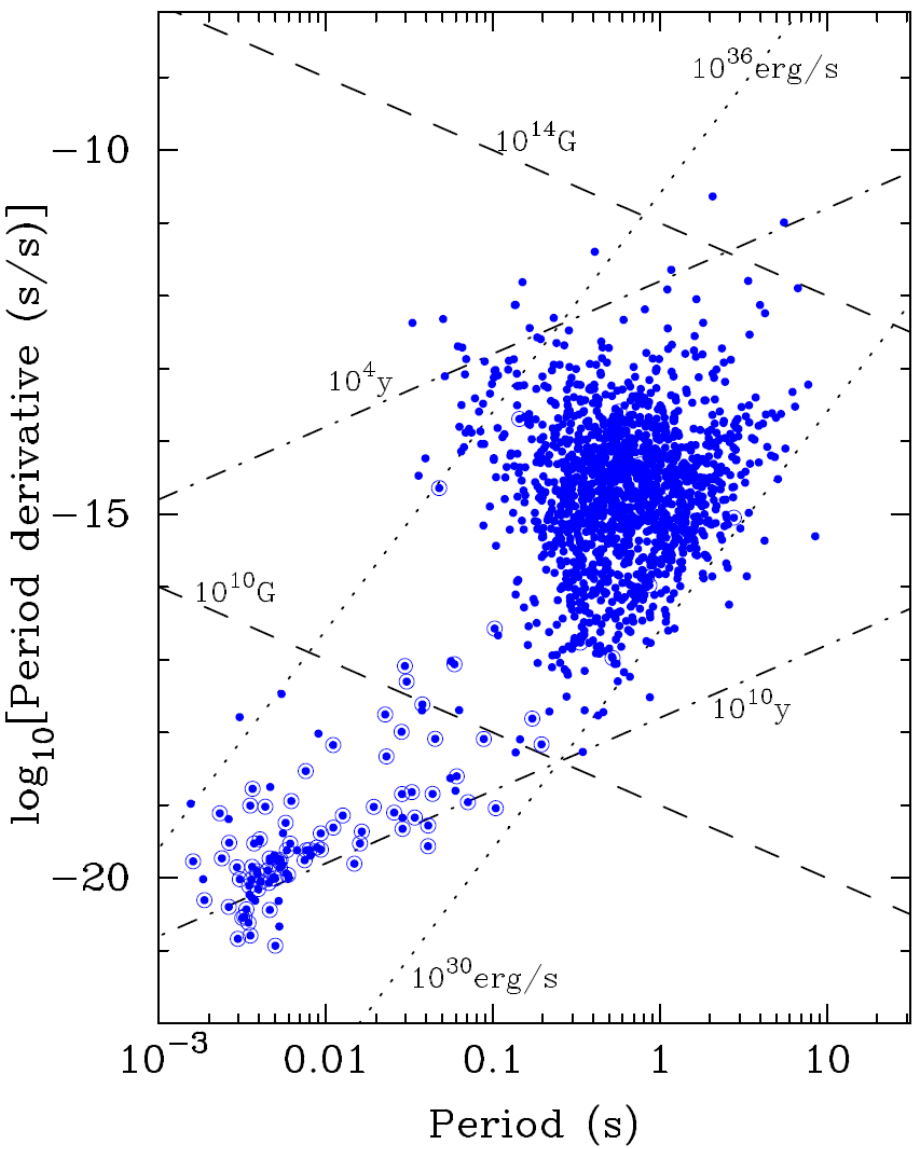
For example: the 322 state:

$$\frac{(\dot{P})_{\rm C}}{(\dot{P})_{\rm GR}}$$

$$\simeq -15|c_{322}|^2\left(\frac{\alpha}{0.1}\right)^{-9}\frac{q}{(1+q)^{7/3}}\left(\frac{M}{10M_\odot}\right)^{5/3}\left(\frac{P}{1\ \rm hr}\right)^{-5/3}$$

Outline

1. Introduction (✓)
2. Scalar cloud over Kerr (✓)
3. GCP Bohr transitions (✓)
4. Termination of superradiance (✓)
5. Pulsar timing, hyperfine (←)



Binary pulsars

"Binary and Millisecond Pulsars", Lorimer, 2008

Pulsar–black hole binaries in the Galactic Centre

Claude-André Faucher-Giguère ✉, Abraham Loeb

Monthly Notices of the Royal Astronomical Society, Volume 415, Issue 4, August 2011, Pages 3951–3961, https://doi.org/10.1111/j.1365-2966.2011.19019.x

... ...

therefore important. We show that if the central parsec around Sgr A* harbours a cluster of ~25 000 stellar BHs (as predicted by mass-segregation arguments) and if it is also rich in recycled pulsar binaries (by analogy with globular clusters), then three-body exchange interactions should produce PSR–BHs in the Galactic Centre. Simple estimates of the formation rate and survival time of

... ...

While theoretical considerations suggest that a certain fraction of pulsars should have black hole (BH) companions (e.g. Narayan, Piran & Shemi 1991; Phinney 1991; Portegies Zwart & Yungelson 1998; Bethe & Brown 1999; Belczynski, Kalogera & Bulik 2002; Sipior & Sigurdsson 2002; Pfahl, Podsiadlowski & Rappaport 2005) and pulsar–black hole (PSR–BH) binaries are a holy grail of pulsar astronomy, none has been found to date. The discovery of such a system would represent a major step forward both from the astrophysical point of view and for the unique gravity tests that it might allow. It is therefore important to develop an understanding of where

Black hole/pulsar binaries in the Galaxy

Yong Shao, Xiang-Dong Li

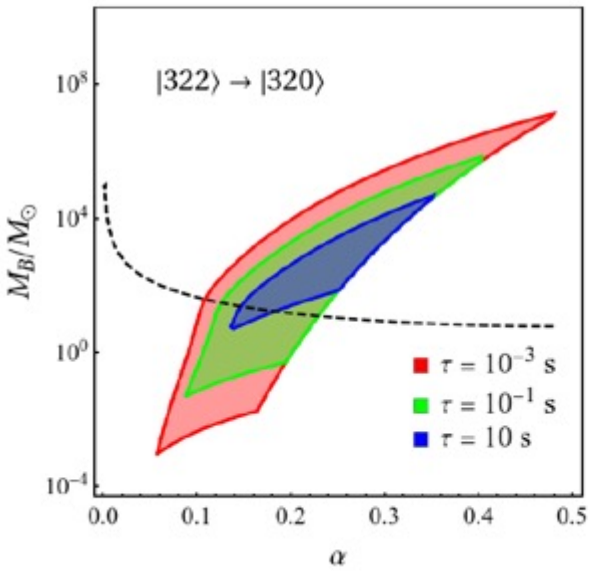
Monthly Notices of the Royal Astronomical Society: Letters, Volume 477, Issue 1, June 2018, Pages L128–L132, https://doi.org/10.1093/mnrasl/sly063

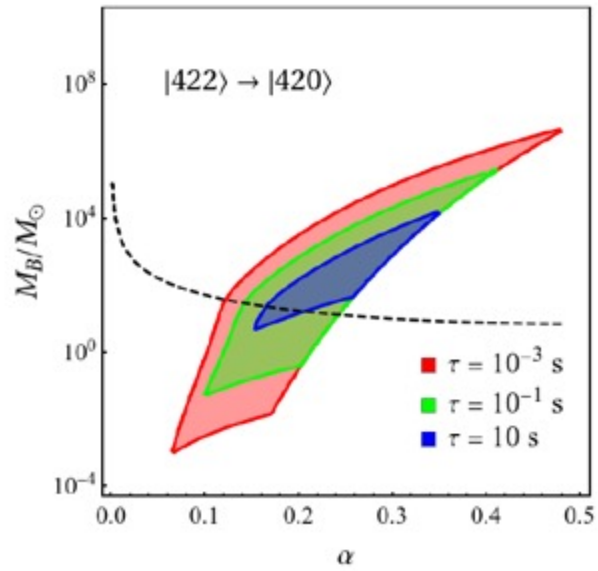
Published: 18 April 2018 **Article history** ▾

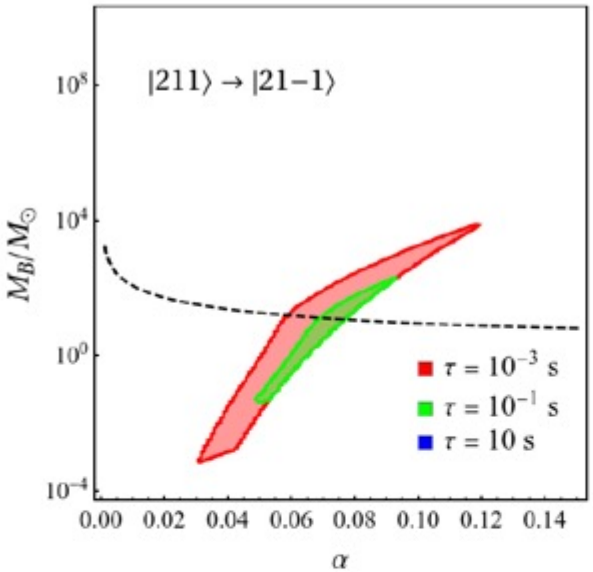
Cite Permissions Share ▾

ABSTRACT

We have performed population synthesis calculation on the formation of binaries containing a black hole (BH) and a neutron star (NS) in the Galactic disc. Some of important input parameters, especially for the treatment of common envelope evolution, are updated in the calculation. We have discussed the uncertainties from the star formation rate of the Galaxy and the velocity distribution of NS kicks on the birthrate ($\sim$0.6--13 M yr^{-1}) of BH/NS binaries. From incident BH/NS binaries, by modelling the orbital evolution due to gravitational wave radiation and the NS evolution as radio pulsars, we obtain the distributions of the observable parameters such as the orbital period, eccentricity, and pulse period of the BH/pulsar binaries. We estimate that there may be ~3–80 BH/pulsar binaries in the Galactic disc and around 10 per cent of them could be detected by the Five-hundred-metre Aperture Spherical radio Telescope.





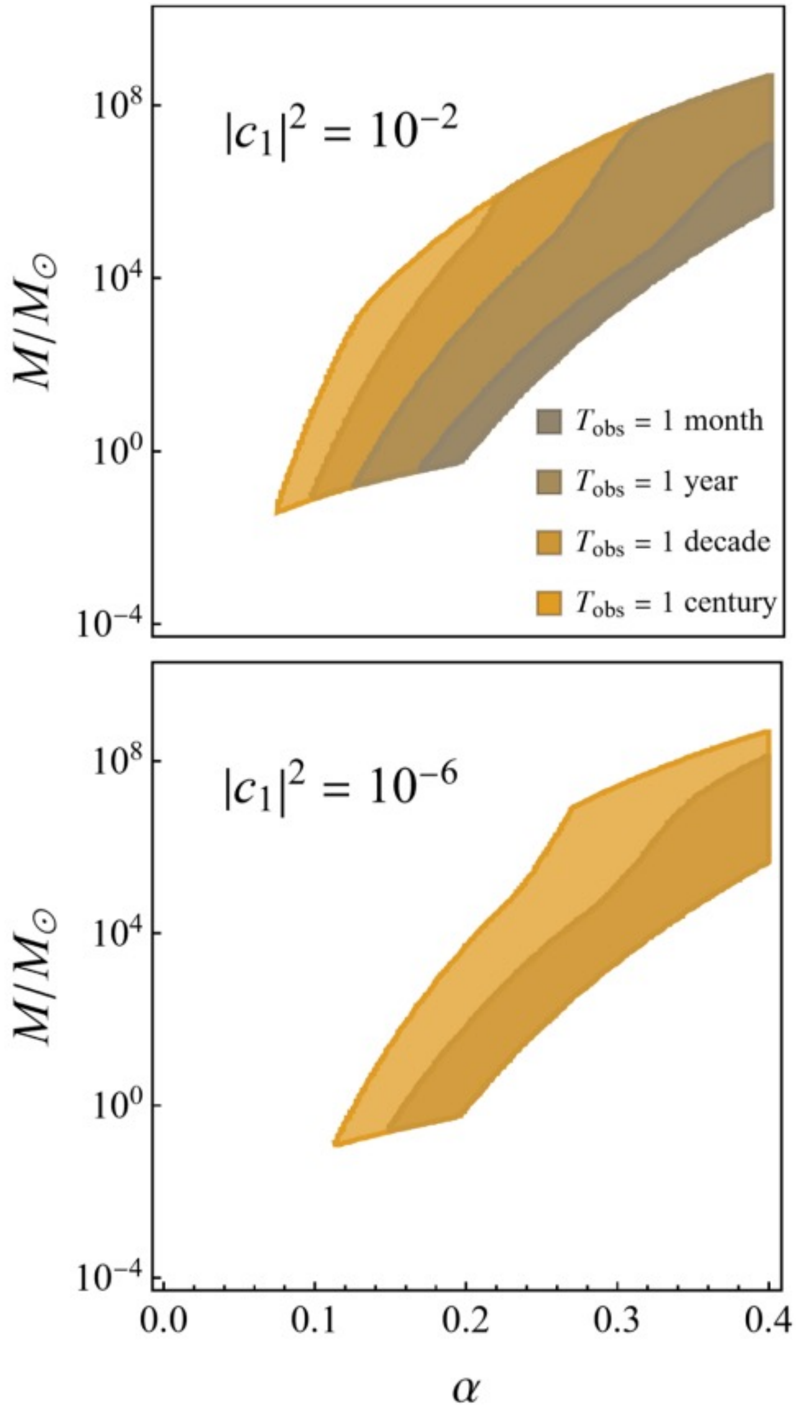


Using pulsars to

search for

resonance GCP

Hyperfine structure



Using pulsars to

search for

off-resonance GCP

Conclusion & outlook:

- GCP Bohr resonance: only 211 remains

  $R_* \sim$ (cloud size), focus on 211 & more careful

- GCP fine structure resonance: not good

- GCP hyperfine structure resonance : good

  But cloud back reaction is big

- Off resonance GCP: good

  Off resonance signal should be

- Axion constraints from BH

  Need reconsideration for BH w

Thank you!

Acknowledgment:

the work was supported in part by

GRF 16306422 & CRF C6017-20GF

by the RGC of Hong Kong, SAR