



International Collaboration on Advanced
Neutron Sources (ICANS XXIV)

Unveiling local magnetic correlations: the development of magnetic pair distribution function at CSNS

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Tongji University



About me



- 2011-2015 Fudan University B.S.
- 2015-2016 Columbia University M.S.
- 2017-2021 Columbia University Ph.D.
- 2017-2020 Oak Ridge National Lab Joint Ph.D.
- 2021-2022 UCLA Postdoc
- 2022-Now Tongji University Assistant Professor





Acknowledgements



Simon Billinge
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ORNL



Benjamin Frandsen
Brigham Young Univ.



Wen Yin
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CSNS, Dongguan, China



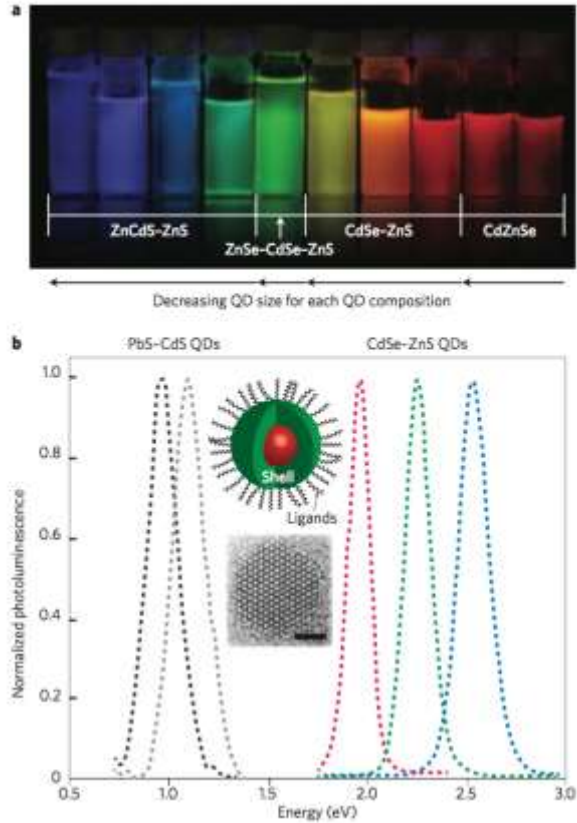
SNS, ORNL, USA

Collaborators:

- Juping Xu (CSNS)
- Yanzhong Pei (Tongji Univ.)
- He Lin (SSRF)
- Emil Bozin (BNL)
- Yuanpeng Zhang (SNS)
- Matthew Krogstad (ANL)
- ...

Structure-Property

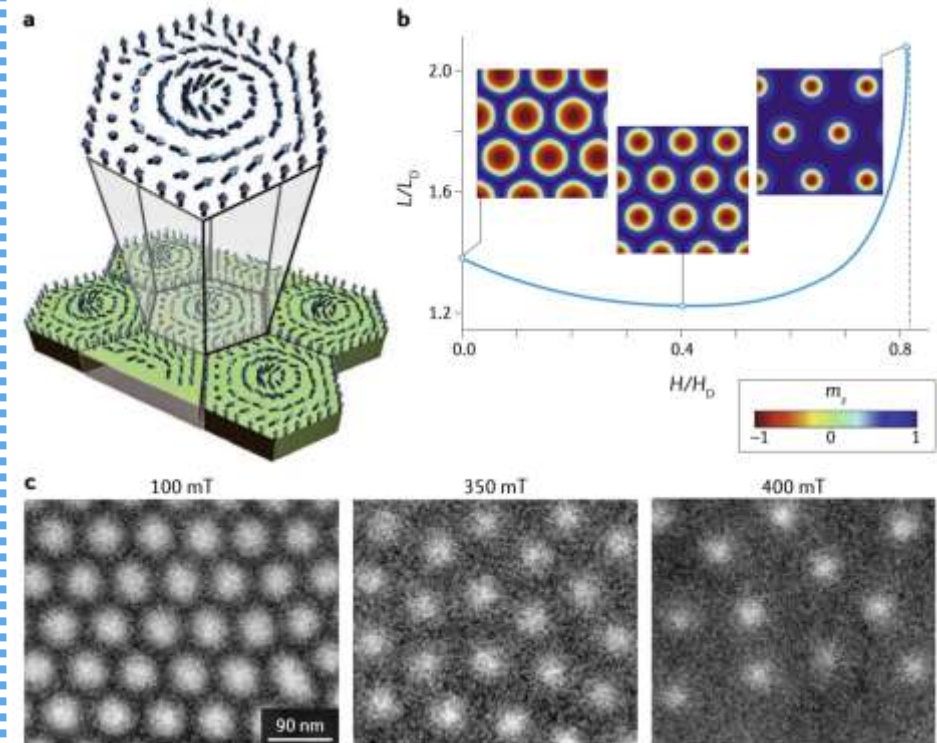
Atomic Structure



**Tunable optical properties
in quantum dots**

Y. Shirasaki, et al. *Nat. Photonics* 2013, 7, 1.

Magnetic Structure



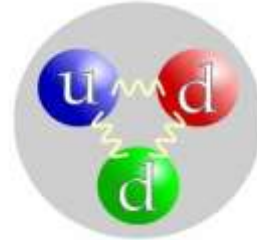
**Magnetic skyrmions for
spintronic applications**

A. Bogdanov and C. Panagopoulos. *Nat. Rev. Phys.* 2020, 2, 9.

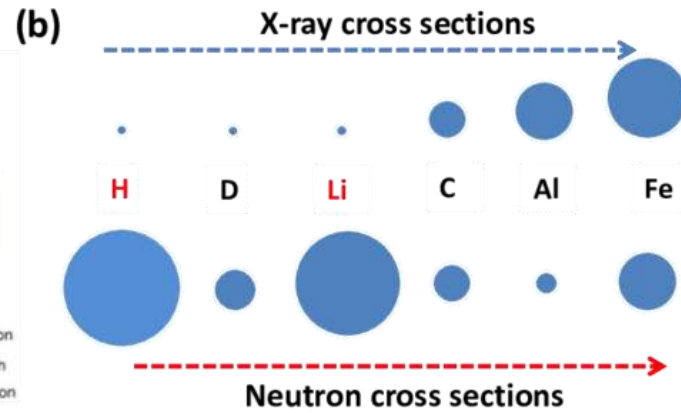
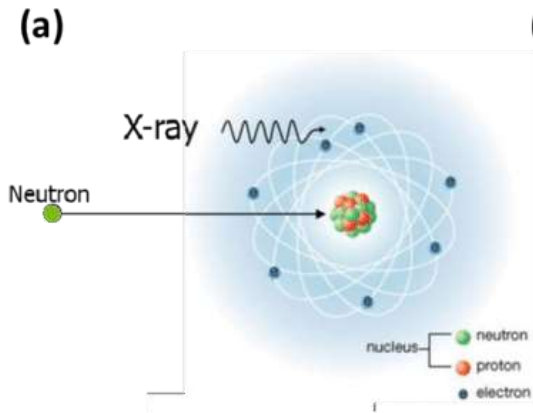


Neutron diffraction

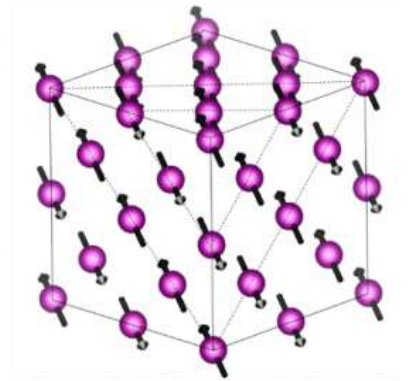
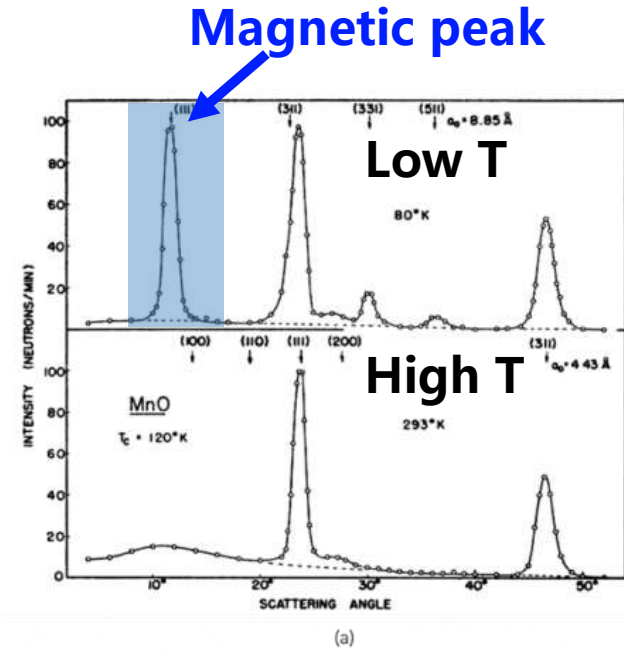
Neutrons



Complementary to x-rays



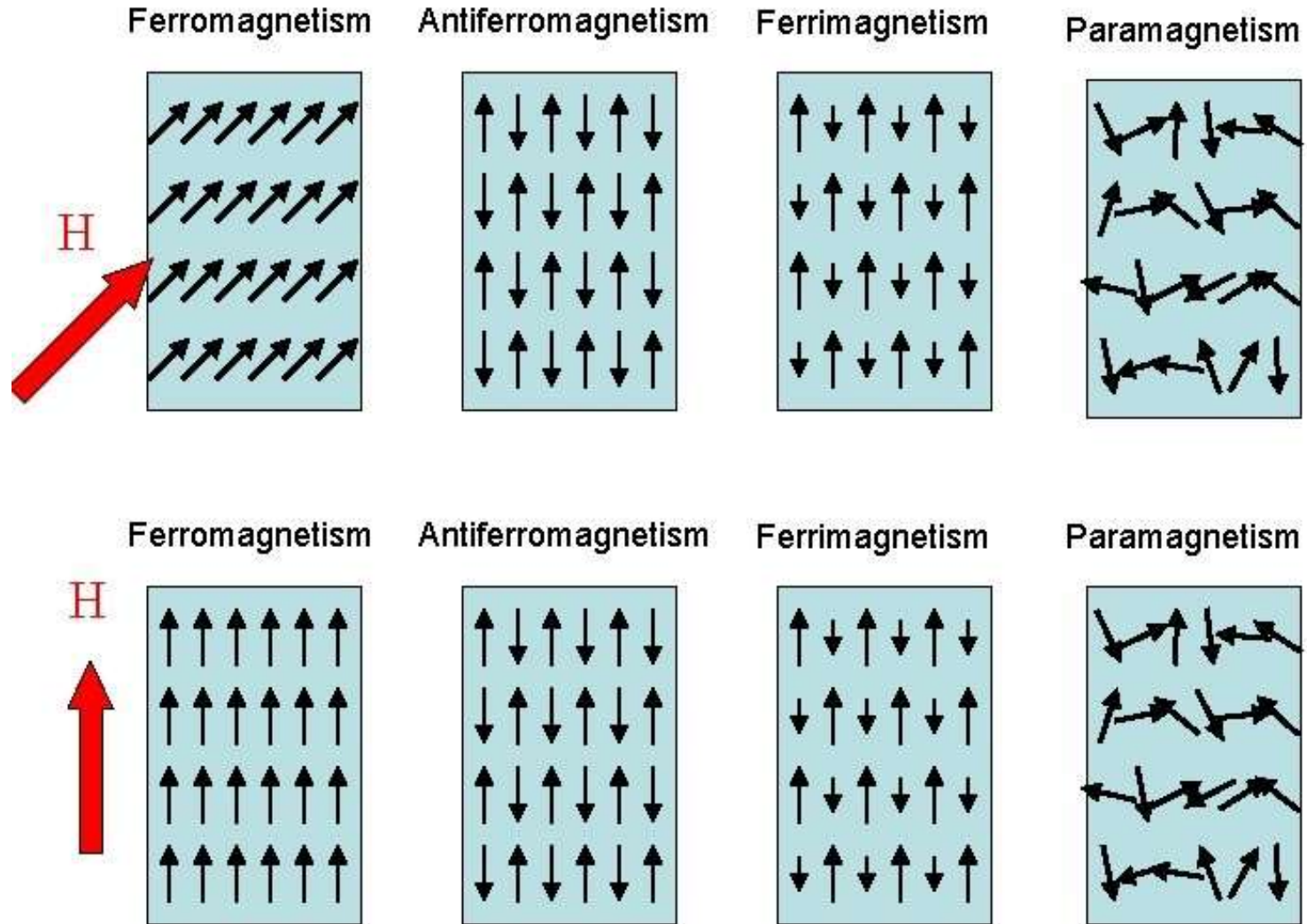
Revealing magnetic structures



(b)



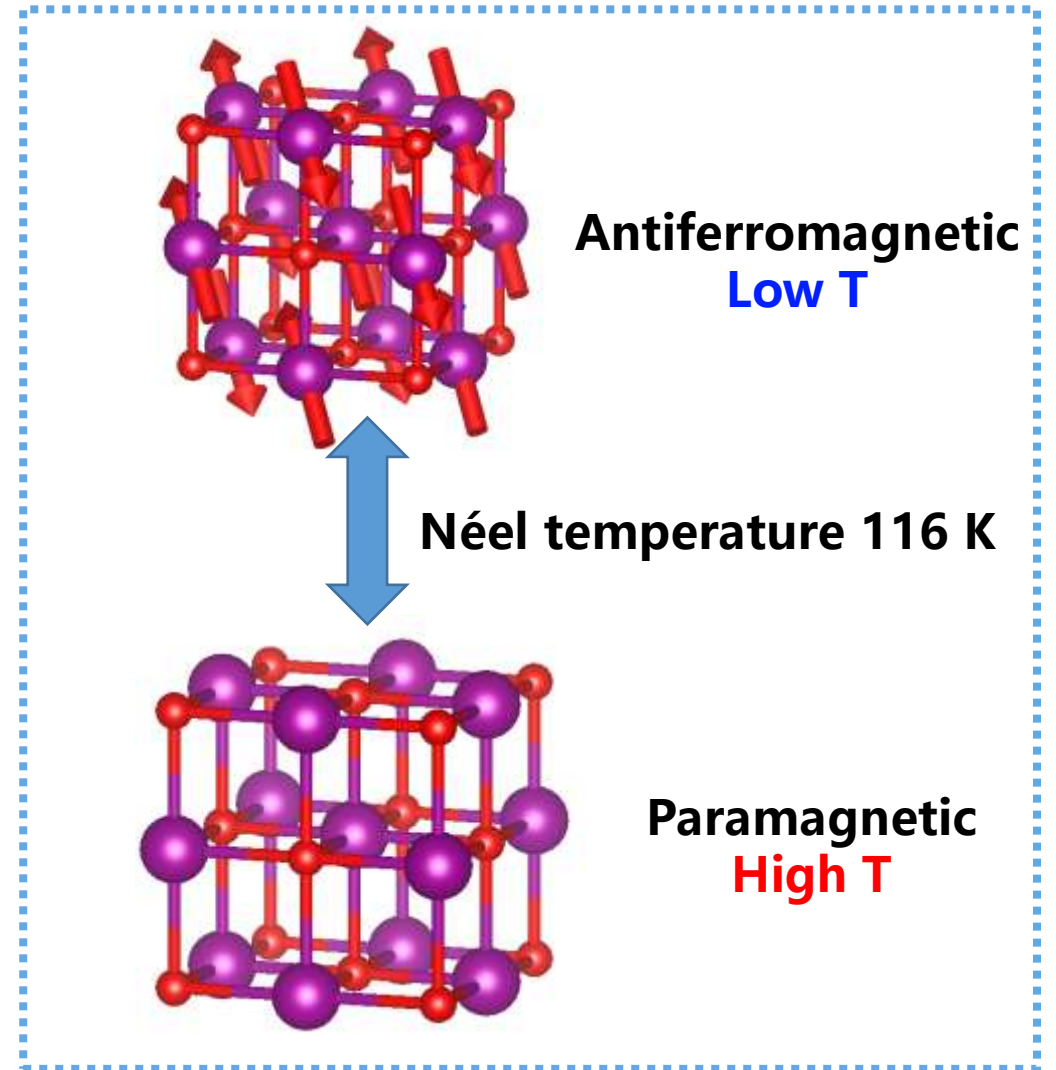
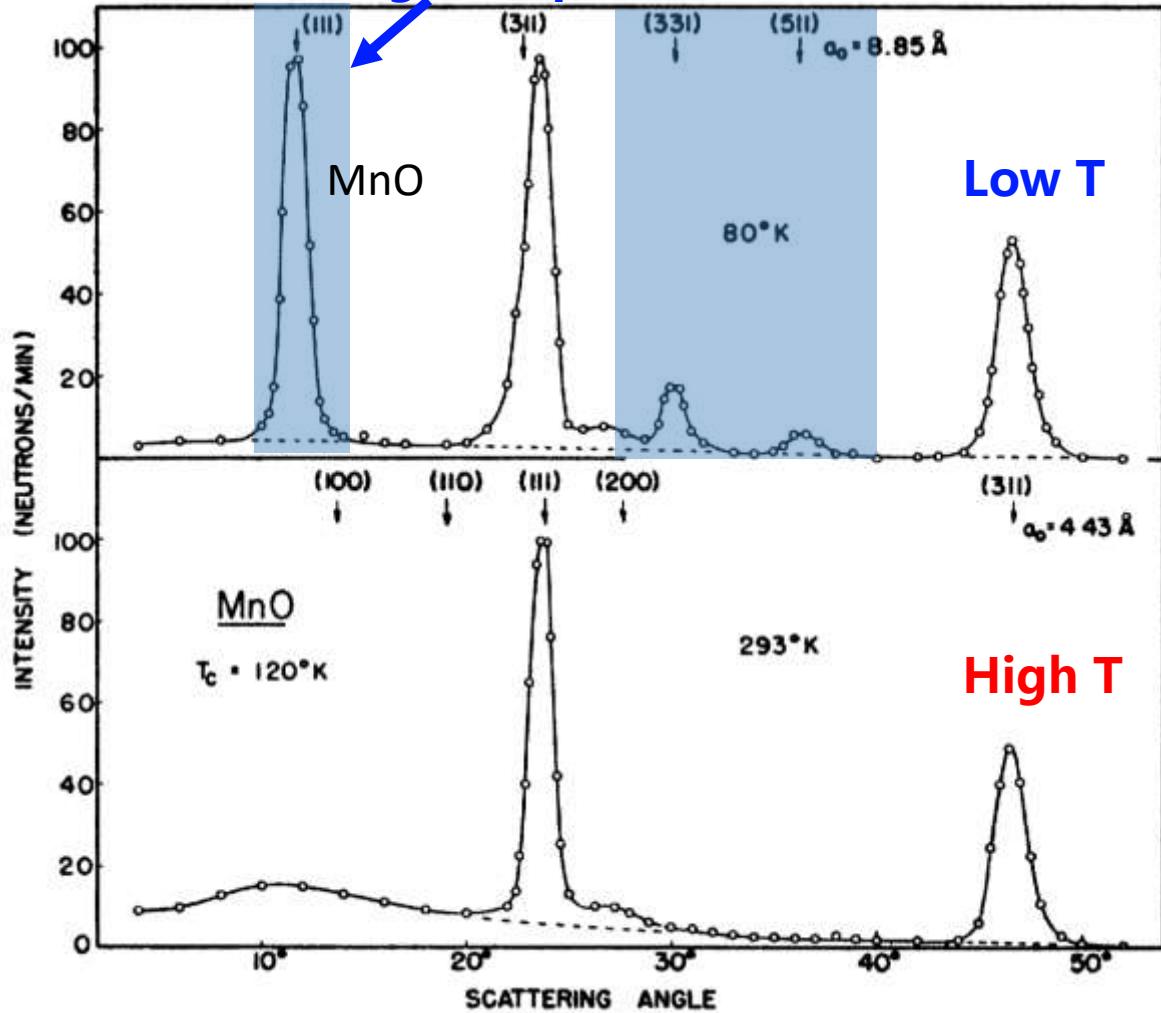
Magnetic structures





MnO

(111) Magnetic peak

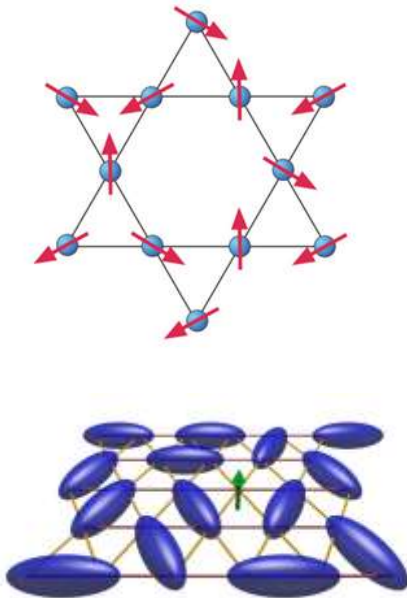




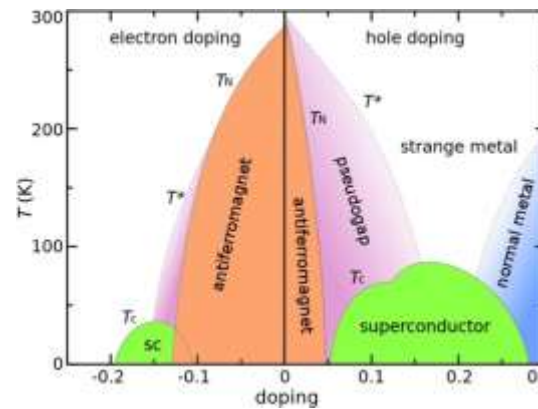
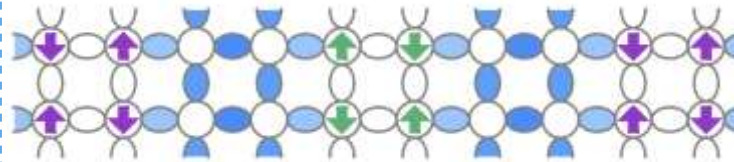
Local magnetic structure

- The short-range local magnetic correlations are important for understanding exotic properties in advanced condensed matters.

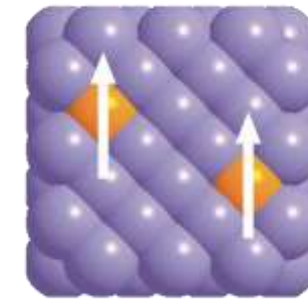
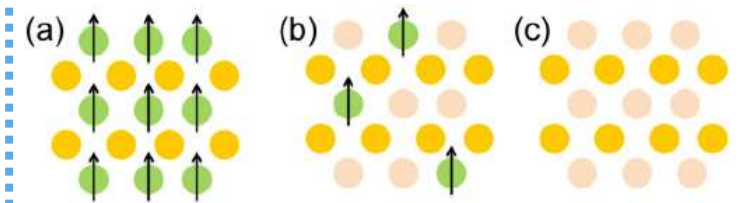
Spin fluctuations in frustrated magnetism



Spinstripe correlations in cuprate superconductors



Spin order in diluted magnetic semiconductors

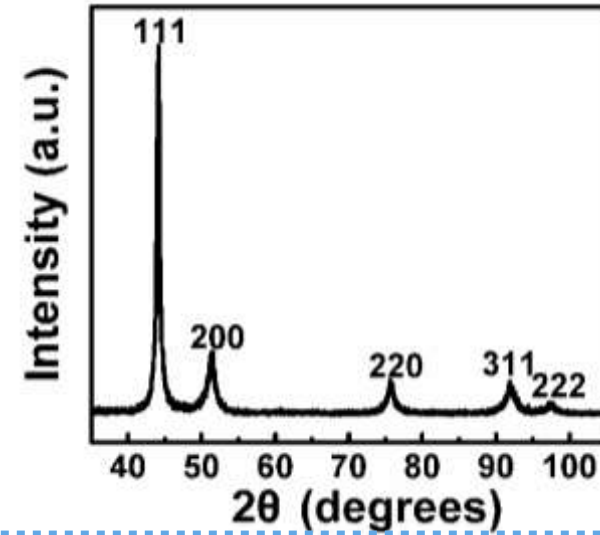
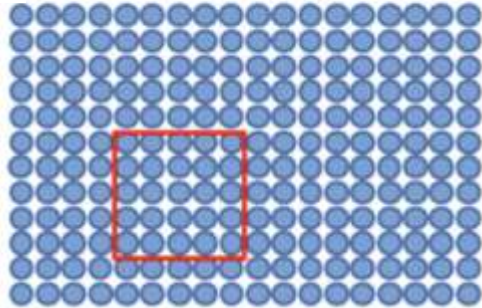




Local structure probe

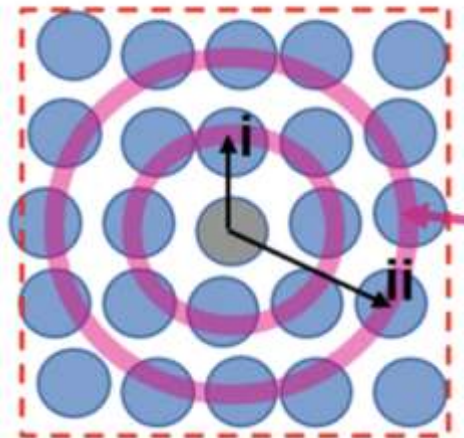
Conventional diffraction

Global structure

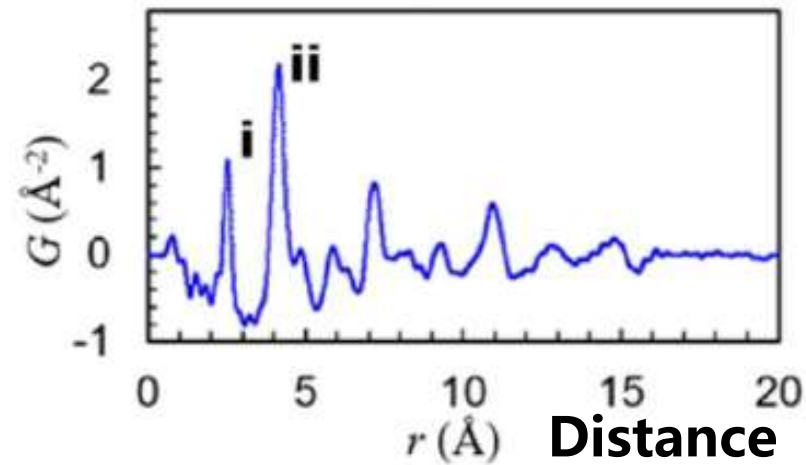


Pair distribution function

Local structure



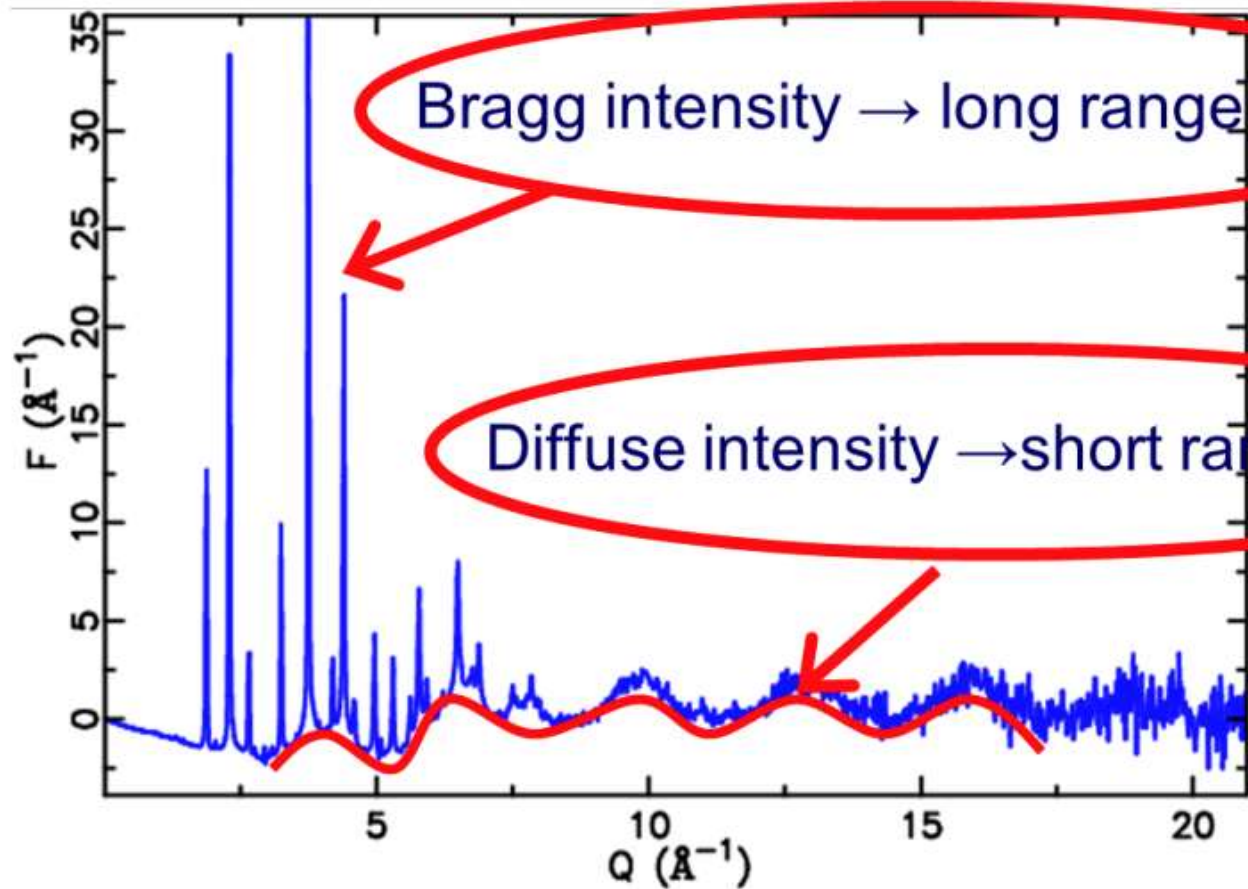
Probability



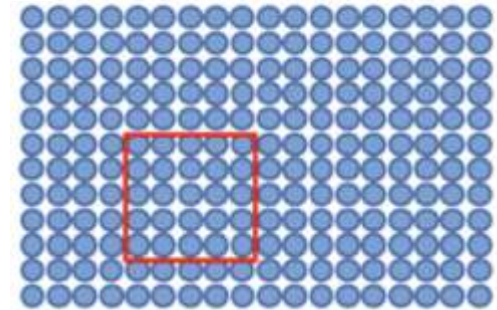


Local structure probe

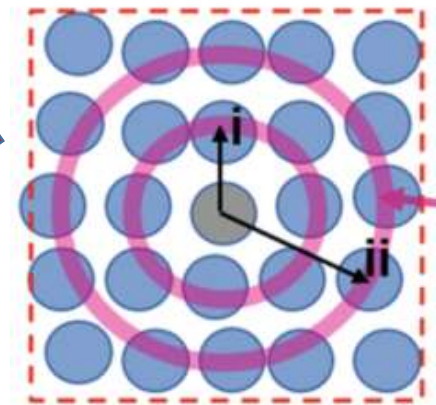
- Total scattering technique



Global structure



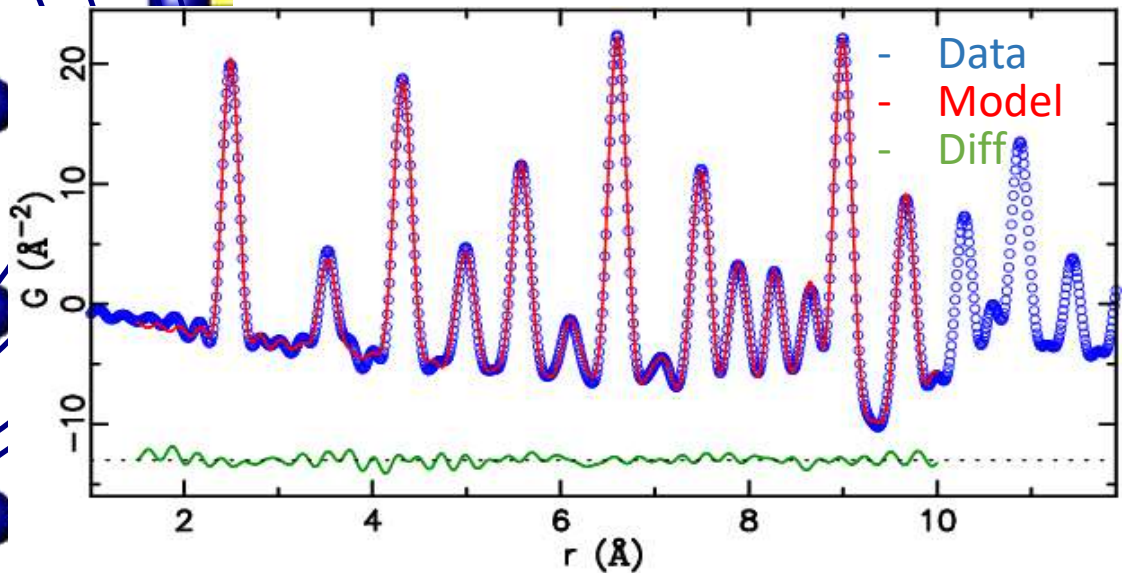
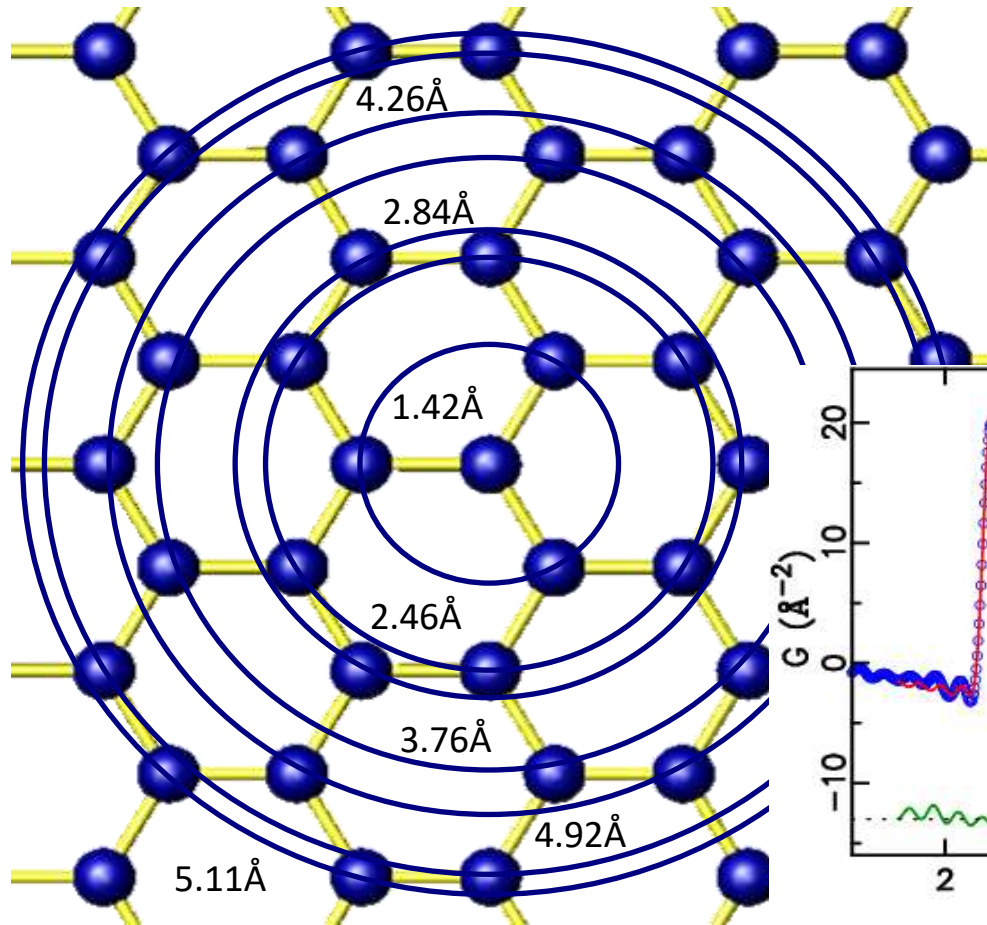
Local structure





Pair Distribution Function (PDF)

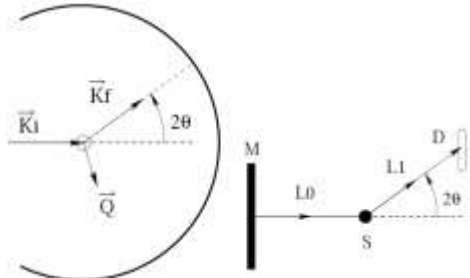
- Local structure probe.
- PDF gives the probability of finding an atom at a distance "r" from a given atom.
- Neutron, x-ray, electron....



Sub-nanometer resolution

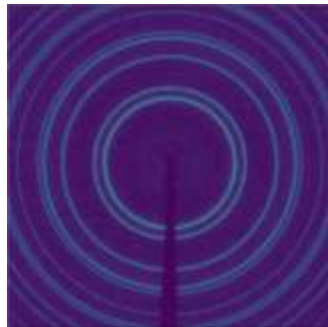


1D-PDF data processing



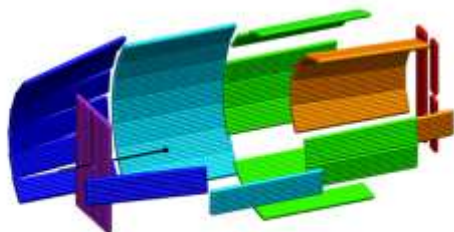
Scattering vector:
 $Q = 4\pi \sin(\theta) / \lambda$

X-ray

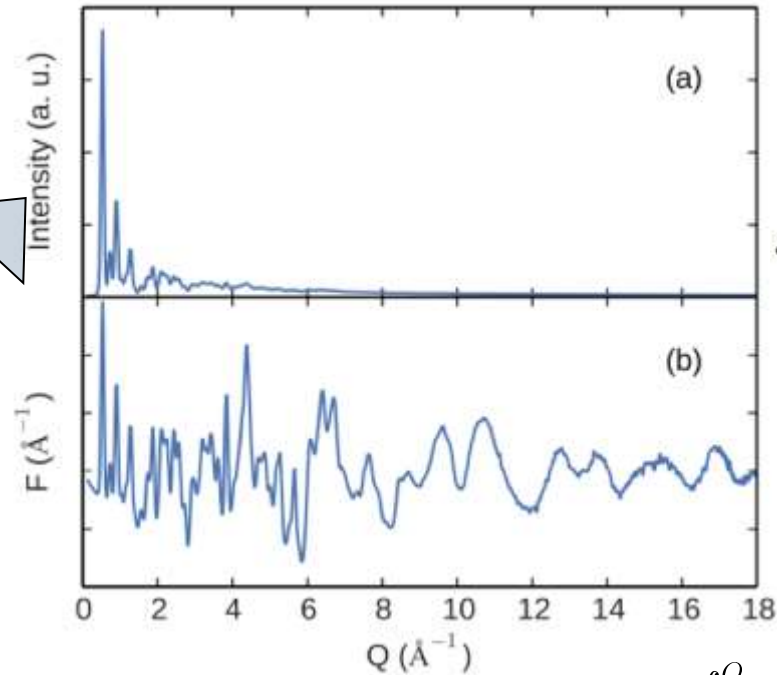


2D Detector

Neutron



TOF Bank Detectors

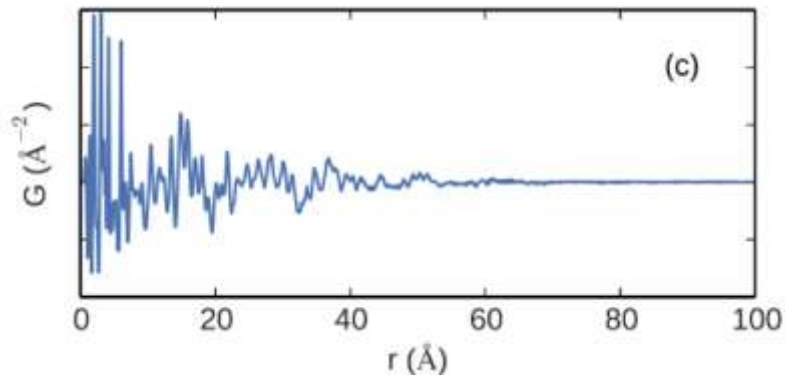


Integrated 1D data

$$S(Q) - 1 = \frac{I(Q)}{N \langle f \rangle^2} - \frac{\langle f^2 \rangle}{\langle f \rangle^2}$$
$$F(Q) = Q[S(Q) - 1]$$

Structure function

$$G(r) = (2/\pi) \int_{Q_{min}}^{Q_{max}} F(Q) \sin(Qr) dQ$$



PDF

PDFgetN, PDFgetX



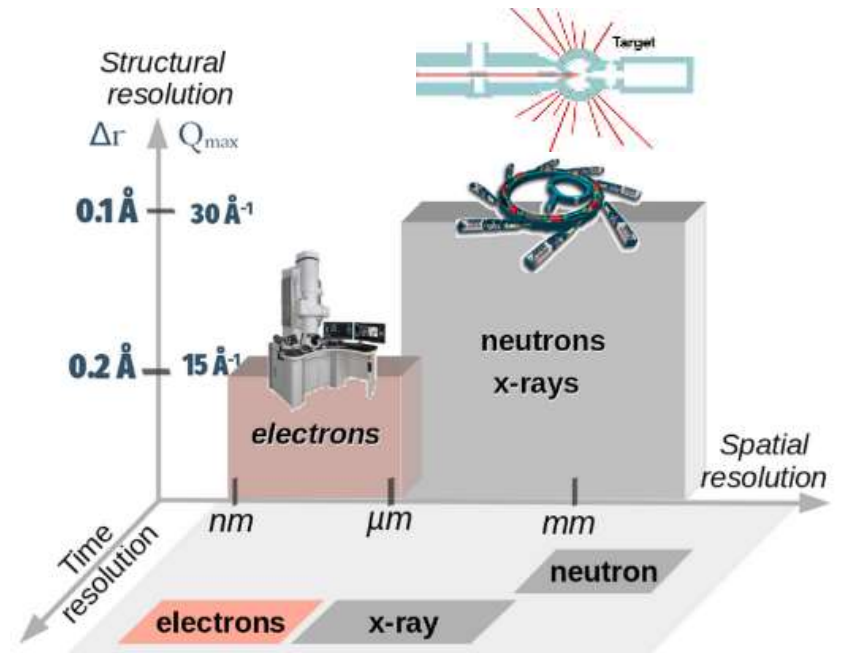
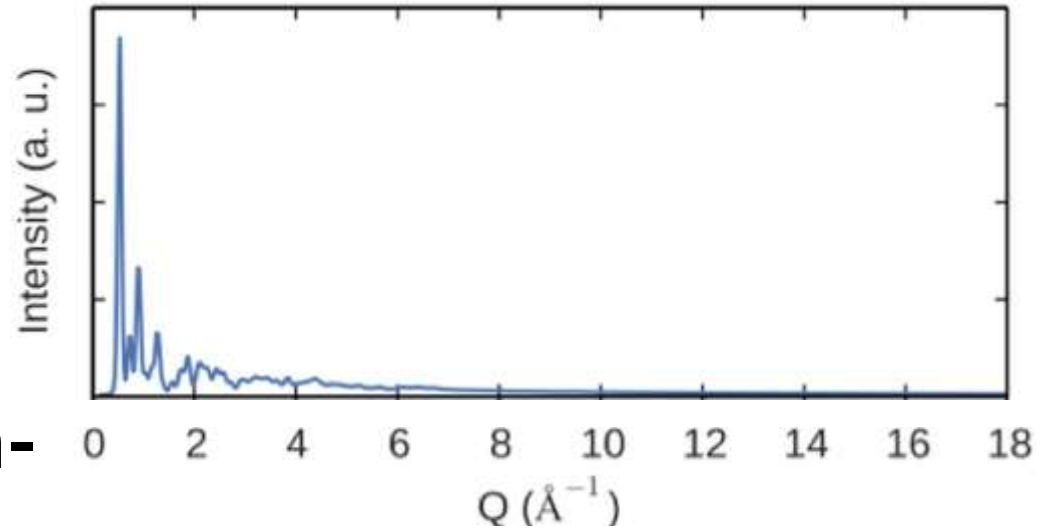
Why do we need spallation neutron source?

- **Low-Q: mostly Bragg peaks.**
- **High-Q: weak diffuse scattering.**

$$Q = \frac{4\pi \sin \theta}{\lambda}$$

- **We need short-wavelength (high-energy), to get large Q_{\max} .**
- **To improve real space resolution Δr .**

$$\Delta r \sim \frac{\pi}{Q_{\max}}$$

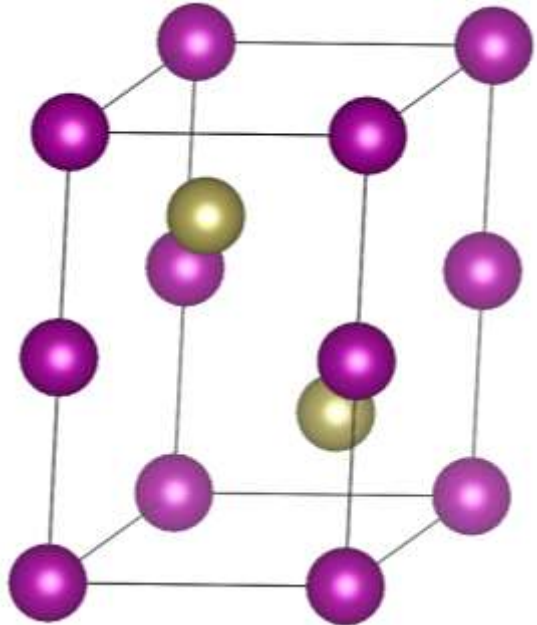




Local structure probe

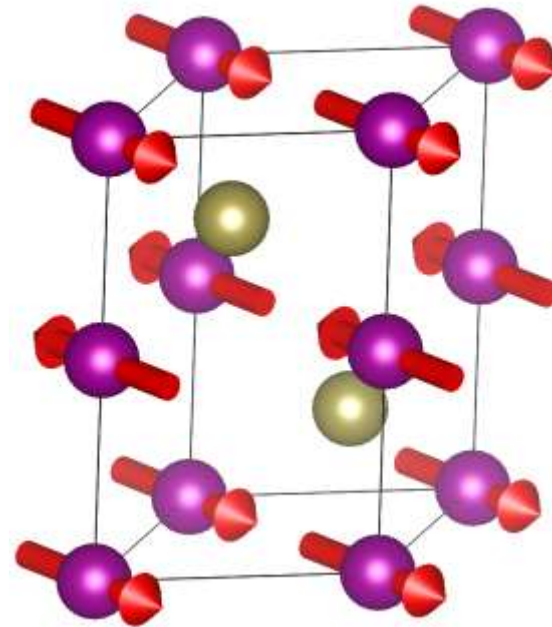
- In a neutron total scattering experiment, we can obtain both!

Atomic PDF



Local atomic structure

Magnetic PDF



Local magnetic structure



mPDF theory

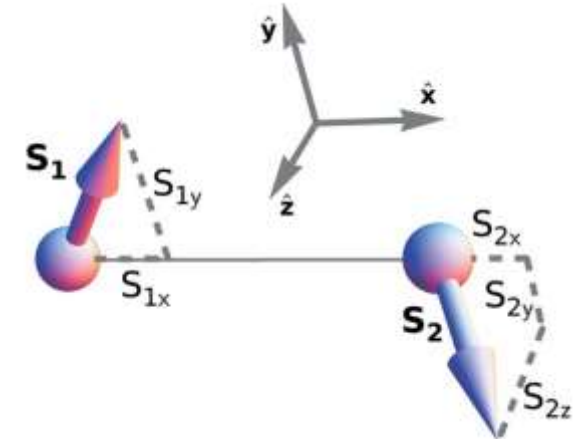
The orientationally averaged magnetic scattering (Blech and Averbach. Physics, 1964):

$$\frac{d\sigma}{d\Omega} = \frac{2}{3} NS(S+1)(\gamma r_0)^2 f^2 + (\gamma r_0)^2 f^2 \times \sum_{i \neq j} \left\{ A_{ij} \frac{\sin \kappa r_{ij}}{\kappa r_{ij}} + B_{ij} \left[\frac{\sin \kappa r_{ij}}{(\kappa r_{ij})^3} - \frac{\cos \kappa r_{ij}}{(\kappa r_{ij})^2} \right] \right\}$$

Self-scattering (i=j)

$$A_{ij} = \langle S_i^y S_j^y \rangle \quad B_{ij} = 2 \langle S_i^x S_j^x \rangle - \langle S_i^y S_j^y \rangle$$

$$S(\kappa) = \frac{d\sigma/d\Omega}{\frac{2}{3} NS(S+1)(\gamma r_0)^2 f^2} = 1 + \frac{1}{N} \frac{3}{2S(S+1)} \times \sum_{i \neq j} \left\{ A_{ij} \frac{\sin \kappa r_{ij}}{\kappa r_{ij}} + B_{ij} \left[\frac{\sin \kappa r_{ij}}{(\kappa r_{ij})^3} - \frac{\cos \kappa r_{ij}}{(\kappa r_{ij})^2} \right] \right\}$$



$$\hat{\mathbf{x}} = \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|} \quad \text{and} \quad \hat{\mathbf{y}} = \frac{\mathbf{S}_i - \hat{\mathbf{x}}(\mathbf{S}_i \cdot \hat{\mathbf{x}})}{|\mathbf{S}_i - \hat{\mathbf{x}}(\mathbf{S}_i \cdot \hat{\mathbf{x}})|}$$



mPDF theory

- **Reduced structure function** $F(\kappa) = \kappa[S(\kappa) - 1]$

$$F(\kappa) = \frac{1}{N} \frac{3}{2S(S+1)} \times \sum_{i \neq j} \left[A_{ij} \frac{\sin \kappa r_{ij}}{r_{ij}} + B_{ij} \left(\frac{\sin \kappa r_{ij}}{\kappa^2 r_{ij}^3} - \frac{\cos \kappa r_{ij}}{\kappa r_{ij}^2} \right) \right]$$

- **mPDF via Fourier transform**

$$f(r) = \frac{2}{\pi} \int_0^{\infty} d\kappa F(\kappa) \sin \kappa r$$

mPDF contains both **spatial** and **orientational** magnetic correlations.

$$= \frac{1}{N} \frac{3}{2S(S+1)} \sum_{i \neq j} \left\{ \frac{A_{ij}}{r} \delta(r - r_{ij}) + B_{ij} \frac{r}{r_{ij}^3} [1 - \Theta(r - r_{ij})] \right\}$$

- **Atomic PDF**

$$f_{\text{atPDF}}(r) = (1/rN) \sum_{i \neq j} \delta(r - r_{ij})$$



mPDF theory

$$f(r) = \frac{2}{\pi} \int_0^{\infty} Q \left[\frac{I_m}{\frac{2}{3} N_s S(S+1) (\gamma r_0)^2 f_m^2(Q)} - 1 \right] \sin Qr \, dQ \quad (1) \quad \leftarrow \text{Experimental mPDF}$$

$$= \frac{1}{N_s} \frac{3}{2S(S+1)} \sum_{i \neq j} \left[\frac{A_{ij}}{r} \delta(r - r_{ij}) + B_{ij} \frac{r}{r_{ij}^3} \Theta(r_{ij} - r) \right], \quad (2) \quad \leftarrow \text{Simulated mPDF}$$

- $f_m(Q)$ is the magnetic form factor

X-ray $F_{hkl} = \sum f_j \exp(2\pi i(hx + ky + lz)) e^{-2W}$

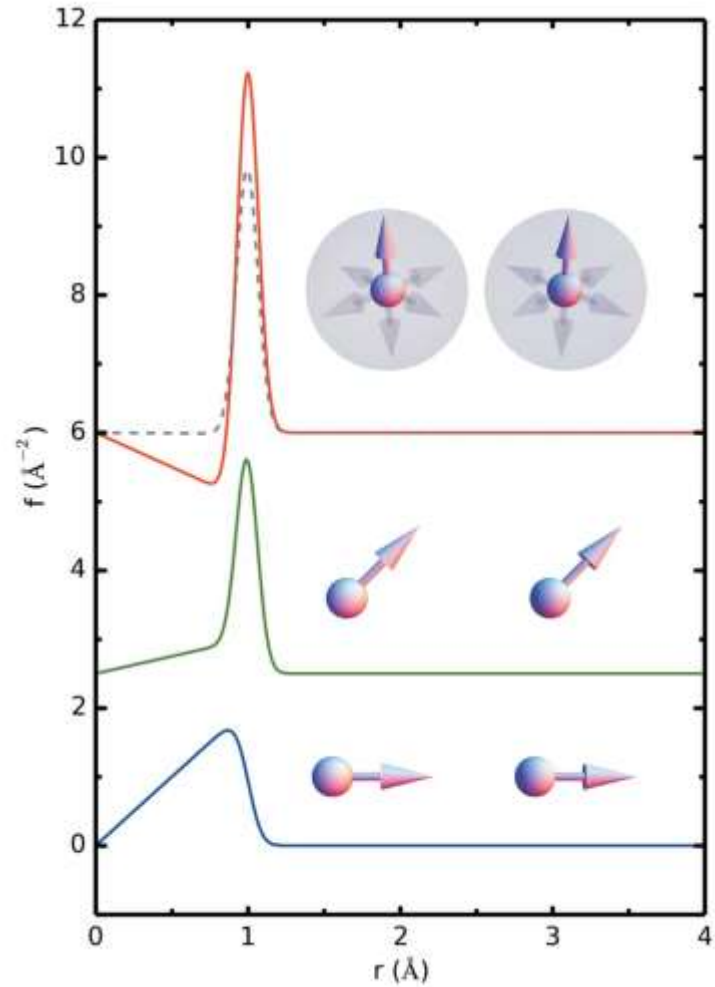
Neutron $F_{hkl} = \sum b_j \exp(2\pi i(hx + ky + lz)) e^{-2W}$

Magnetic $F_{hkl} = \sum q_j f_{Mj} \exp(2\pi i(hx + ky + lz)) e^{-2W}$



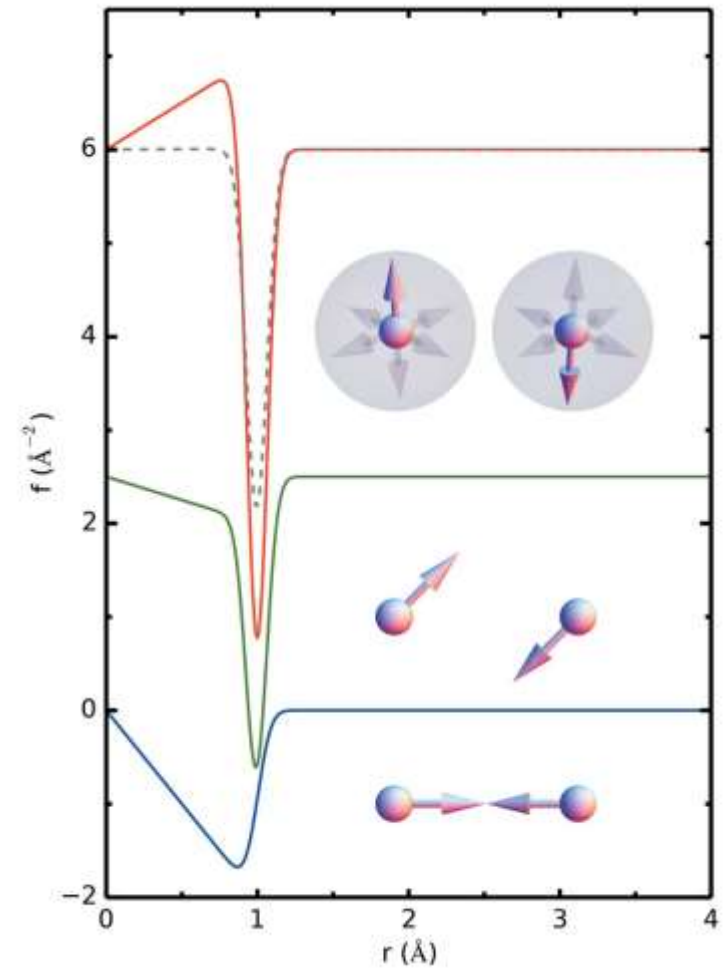
mPDF

Ferromagnetic



Positive peak

Antiferromagnetic

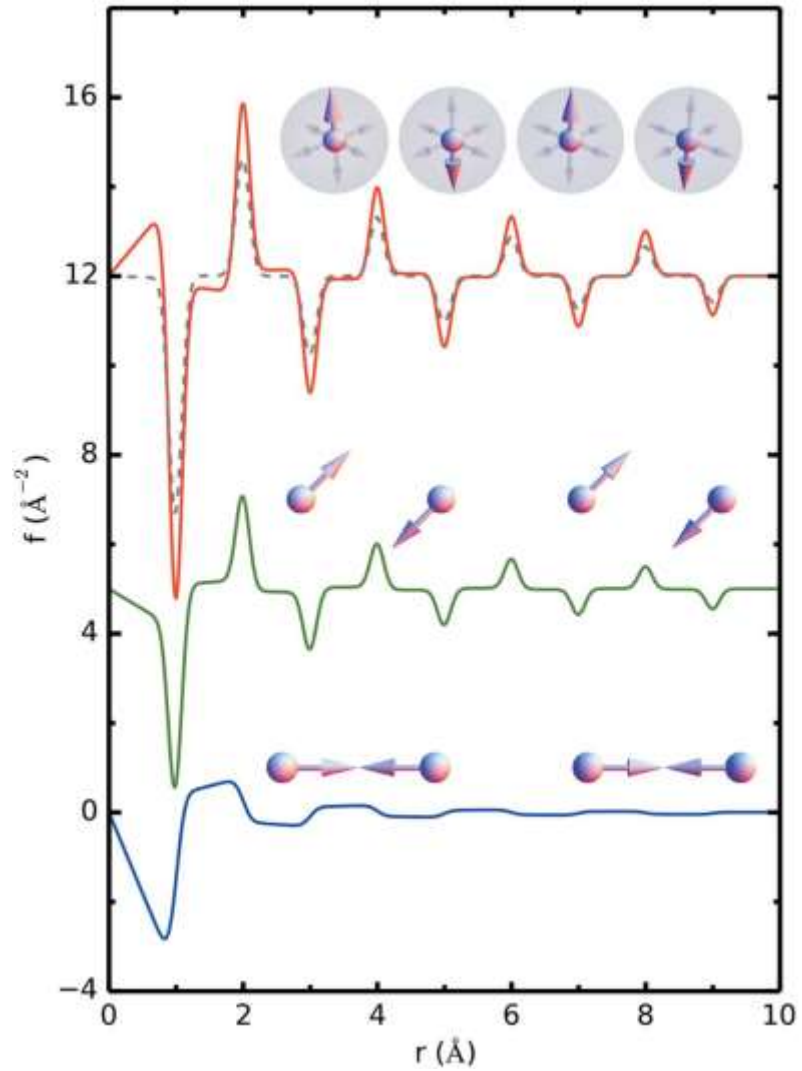


Negative peak

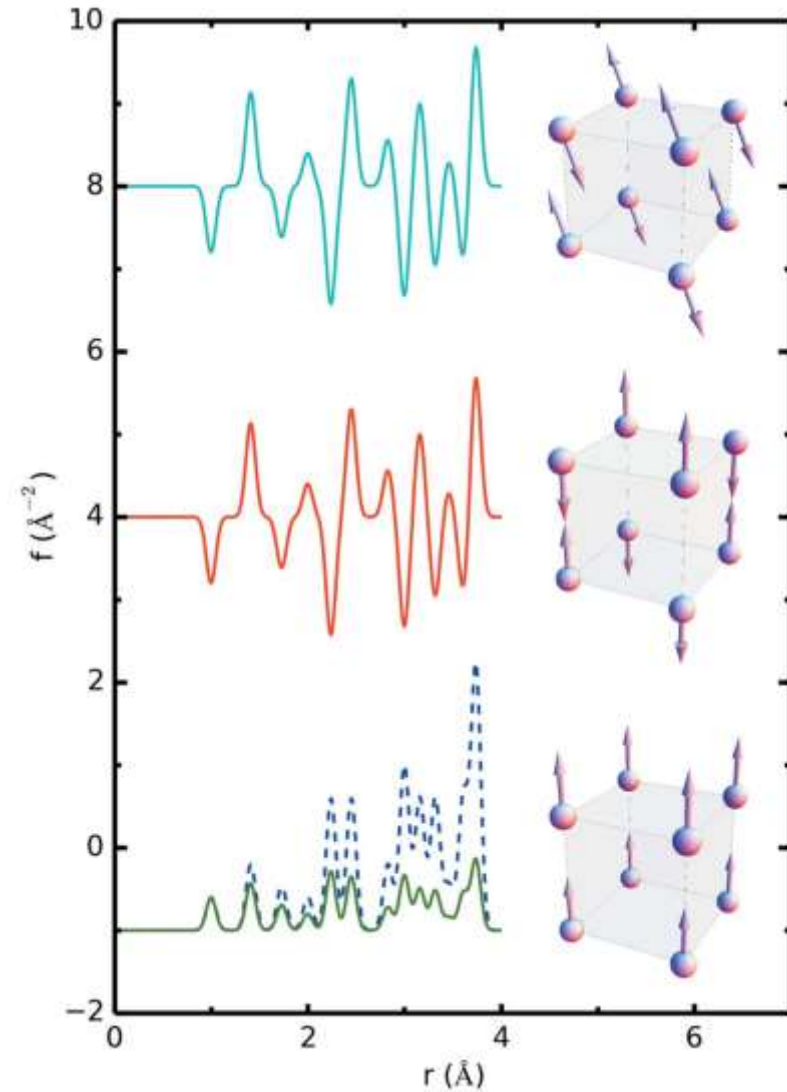


mPDF

1D structure



3D structure

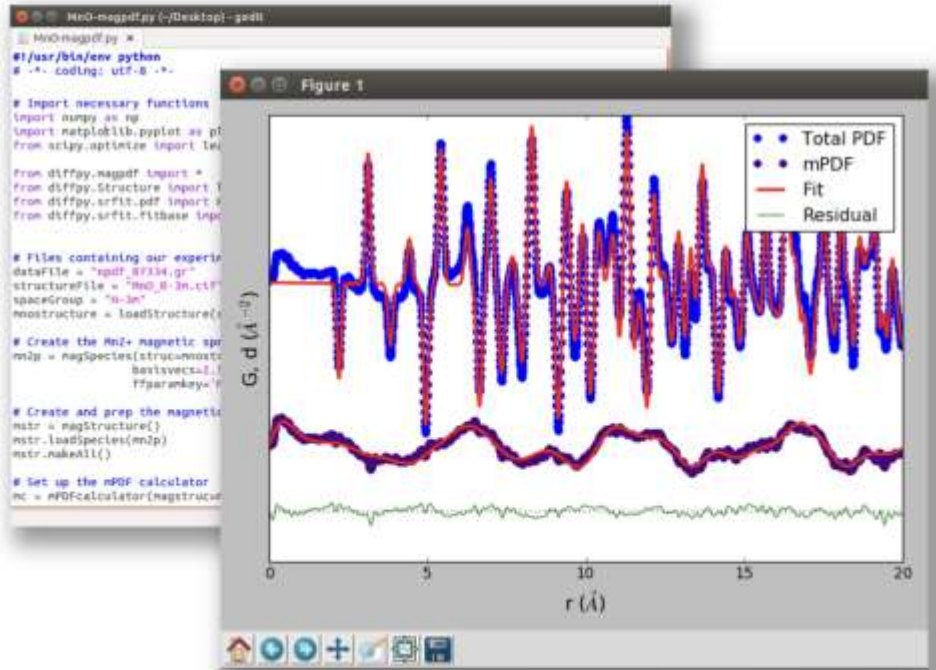




DiffPy.mPDF

mpdf

The diffpy.mpdf package provides a convenient method for computing the magnetic PDF (mPDF) from magnetic structures and performing fits to neutron total scattering data. The mPDF is calculated by an MPDFcalculator object, which extracts the spin positions and spin vectors from a MagStructure object that the MPDFcalculator takes as input. The MagStructure object in turn can contain multiple MagSpecies objects, which generate magnetic configurations based on a diffpy.structure object and a set of propagation vectors and basis vectors provided by the user. Alternatively, the user can manually define a magnetic unit cell that will be used to generate the magnetic structure, or the magnetic structure can be defined simply as lists of spin positions and spin vectors provided by the user.



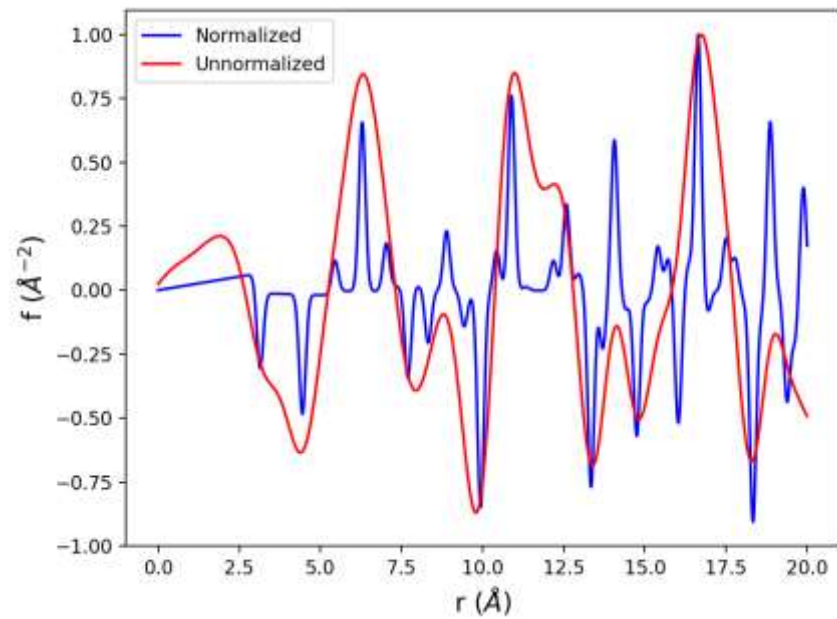
```
1 #!/usr/bin/env python
2
3 import numpy as np
4 import matplotlib.pyplot as plt
5 from diffpy.mpdf import *
6 from diffpy.structure import loadStructure
7 from diffpy.srfit.fitbase import Profile, FitContribution
8 from diffpy.srfit.fitbase import FitRecipe, FitResults
9
10
11 # read in the mcif
12 mcif = '1.31_MnO.mcif'
13 mstr = create_from_mcif(mcif, ftparamkey='Mn2')
14
15 # adjust the unit cell parameters to agree with the results of the
16 a_fit = 4.44864 # fill in the value from the fit
17 alpha_fit = 90.2175 # fill in the value from the fit
18 mstr.struc.lattice.a = 2*a_fit # multiply by two because the magnet
19 mstr.struc.lattice.b = 2*a_fit
20 mstr.struc.lattice.c = 2*a_fit
21 mstr.struc.lattice.alpha = alpha_fit
22 mstr.struc.lattice.beta = alpha_fit
23 mstr.struc.lattice.gamma = alpha_fit
24
25 mstr.makeAll()
26
27
28 fit_file = 'LY4T20_RUN19625_Gr.fgr'
29 r, gcalc, __, __, gdifff = np.loadtxt(fit_file, skiprows=12).T
30 gobs = gcalc + gdifff
31
32 qmag = 1.0*adiff # we set the experimental mPDF to the fit residual
```

<https://www.diffpy.org/products/mPDF.html>

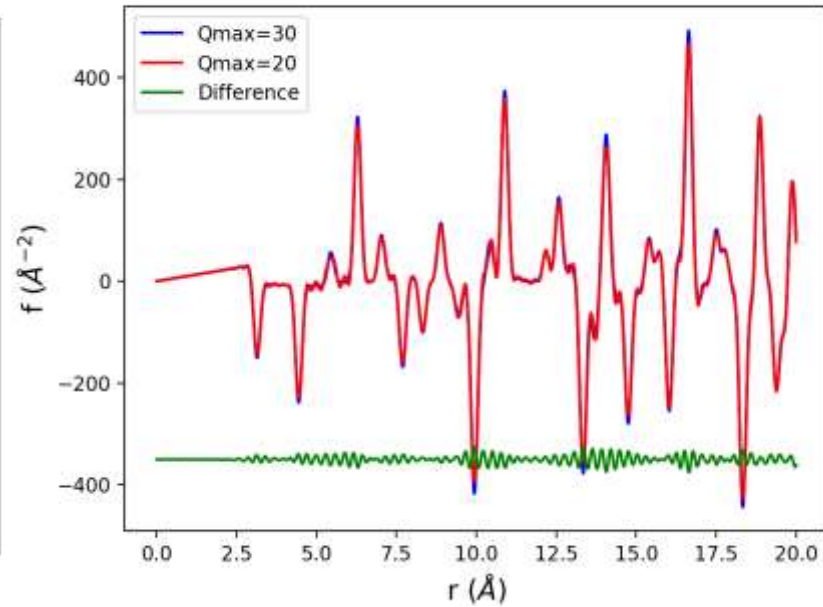


DiffPy.mPDF

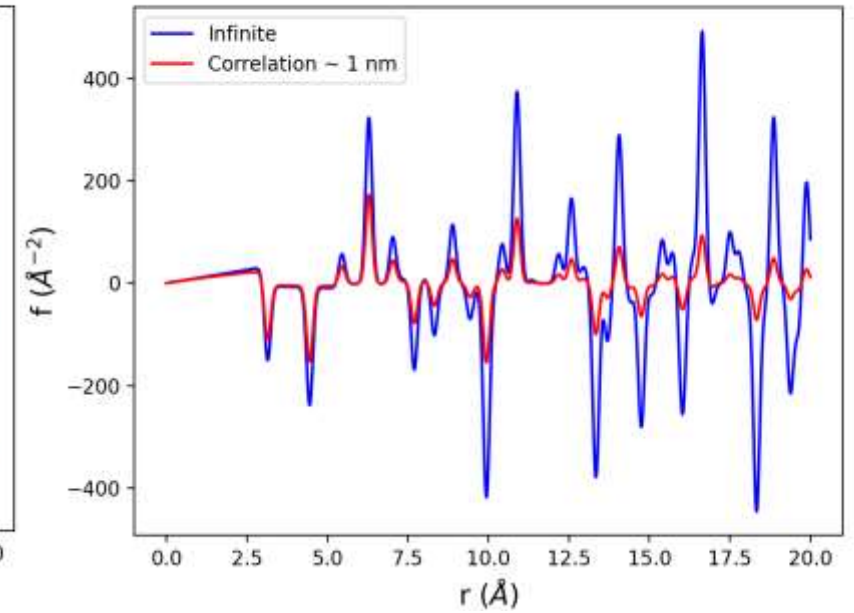
Normalized v.s. Unnormalized



Q_{\max} effect



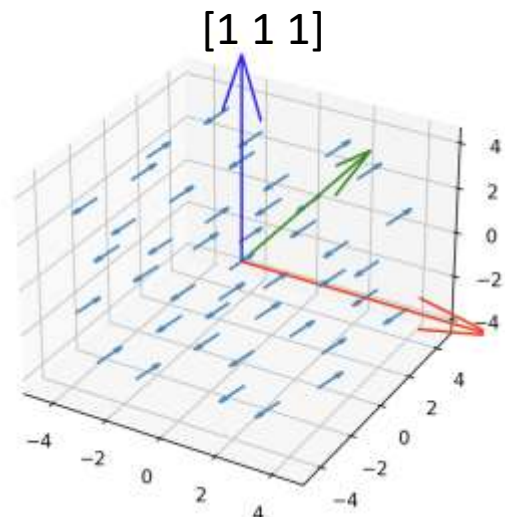
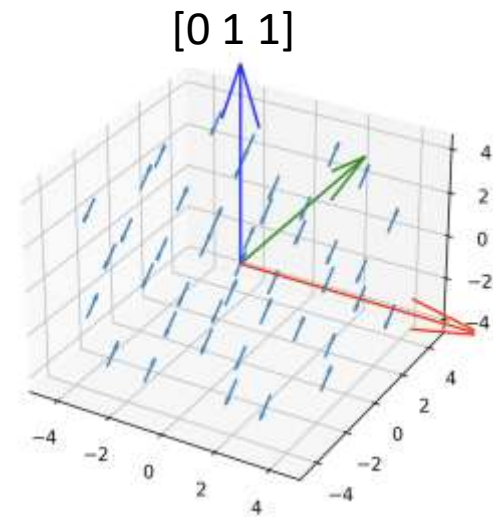
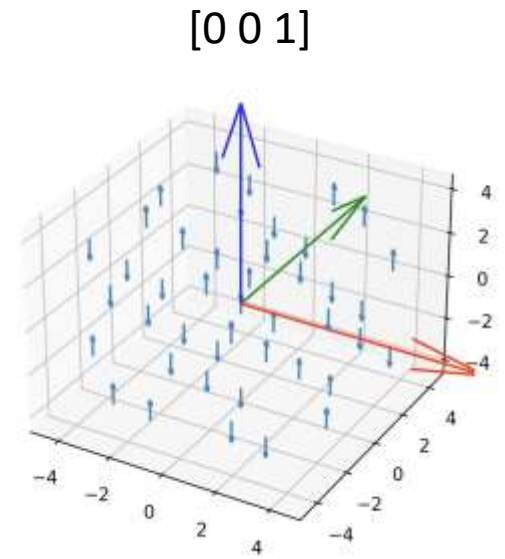
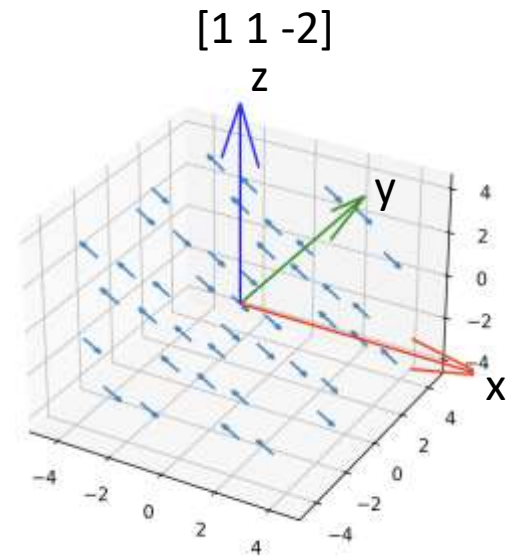
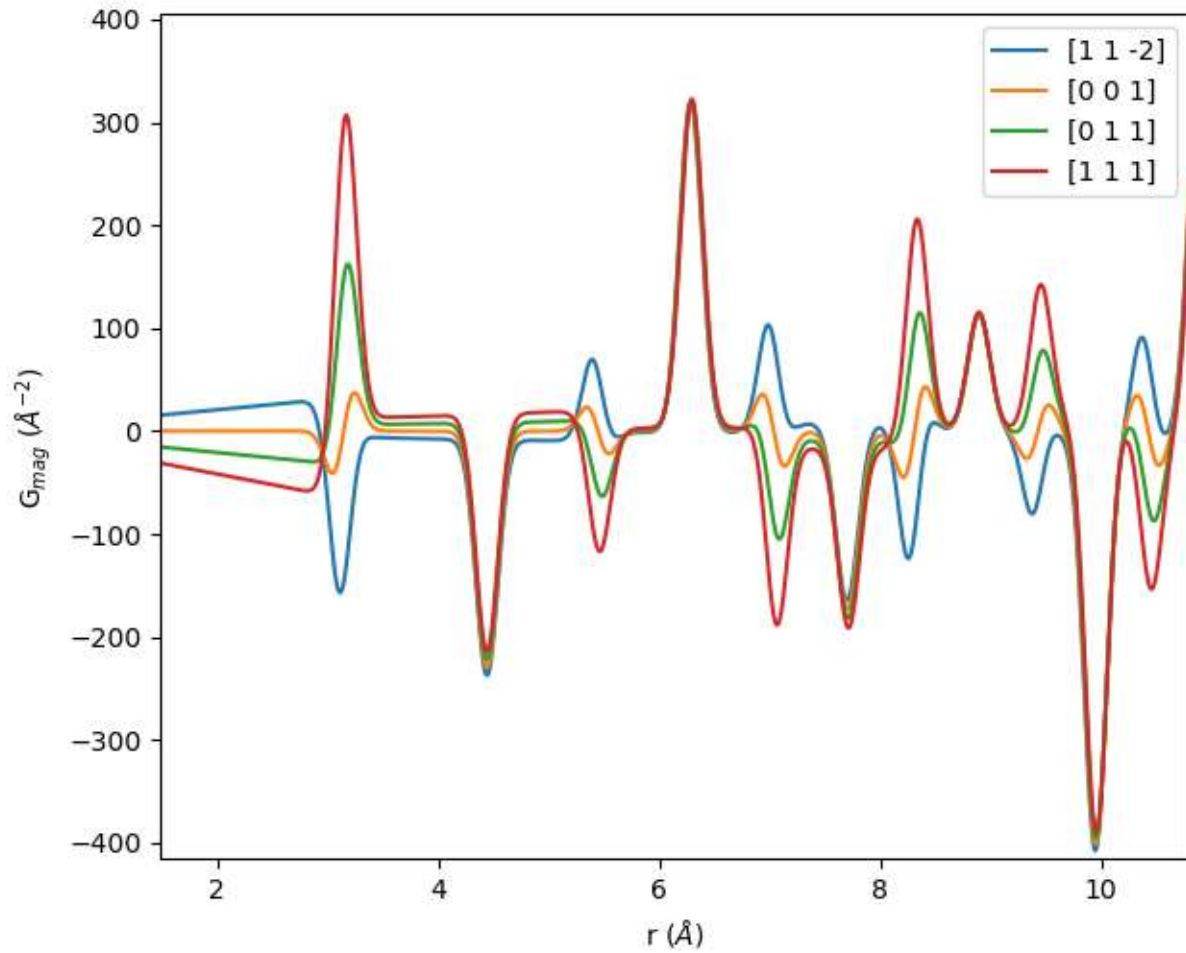
Finite correlation length





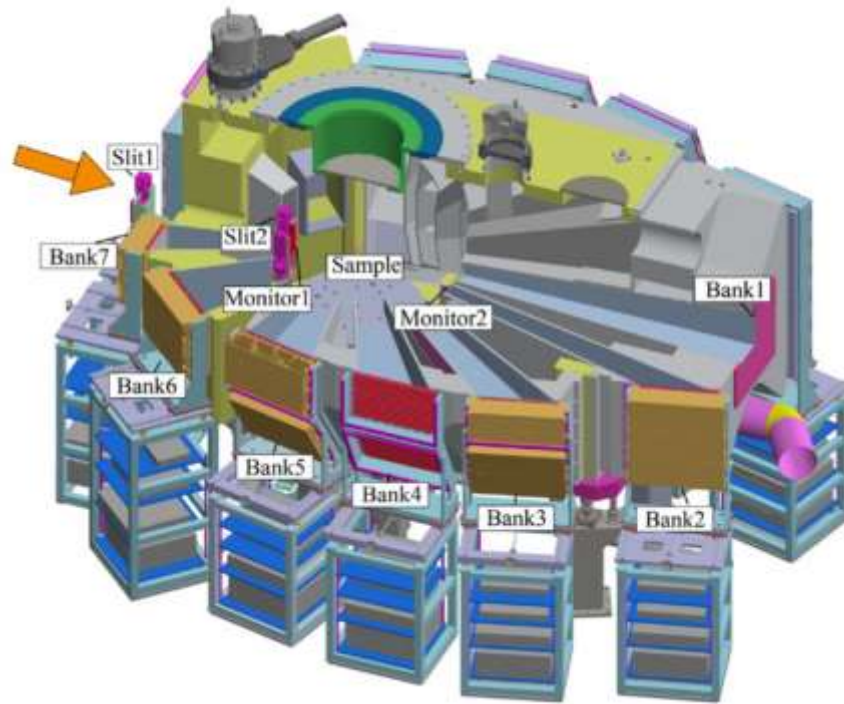
DiffPy.mPDF

Different spin directions



Multi-physics instrument, CSNS

- The first total scattering neutron diffractometer in China.



Nuclear Inst. and Methods in Physics Research, A 1013 (2021) 165642



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journal homepage: www.elsevier.com/locate/nima



Multi-physics instrument: Total scattering neutron time-of-flight diffractometer at China Spallation Neutron Source

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^b Spallation Neutron Source Science Center (SNSSC), Dongguan 523803, China
^c School of Mechanical Engineering, Dongguan University of Technology, Dongguan 523803, China
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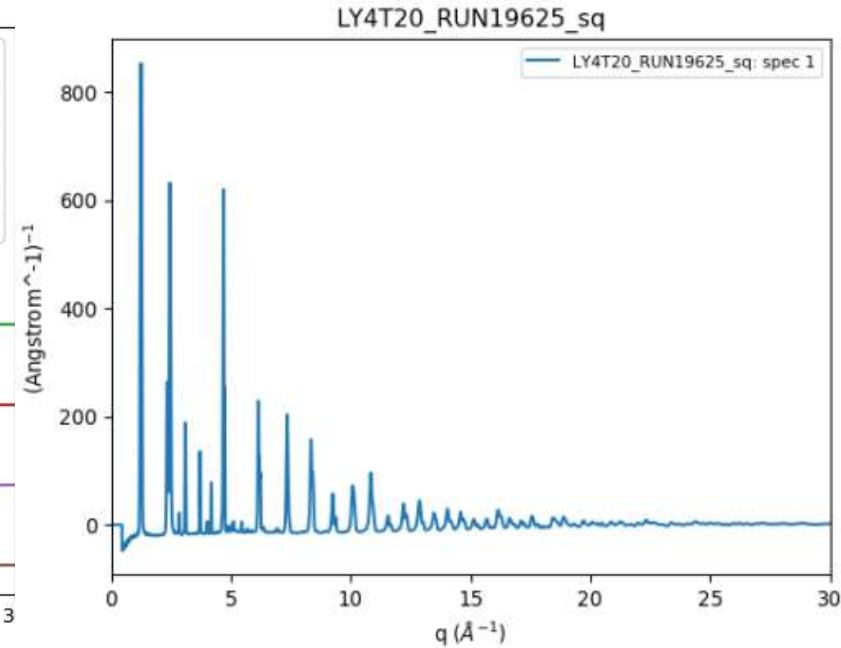
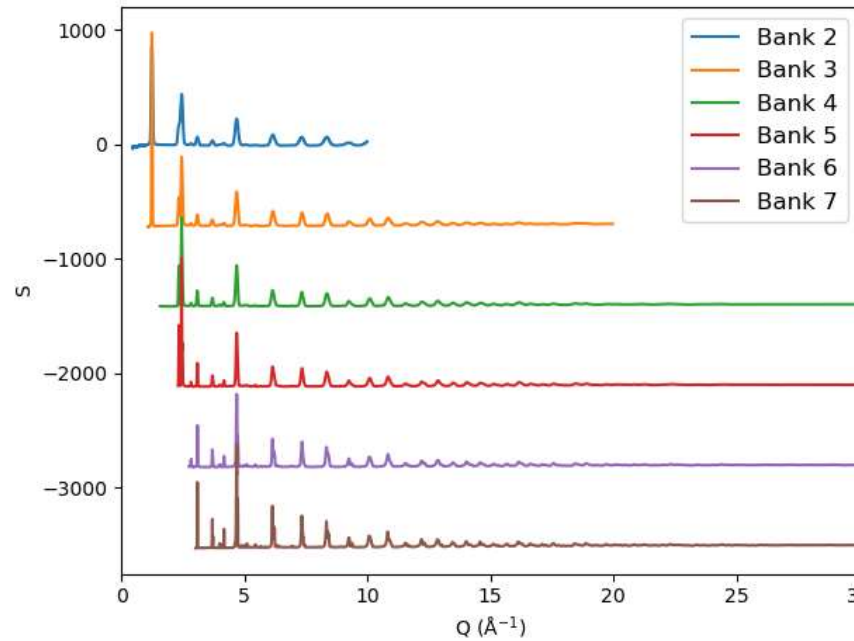
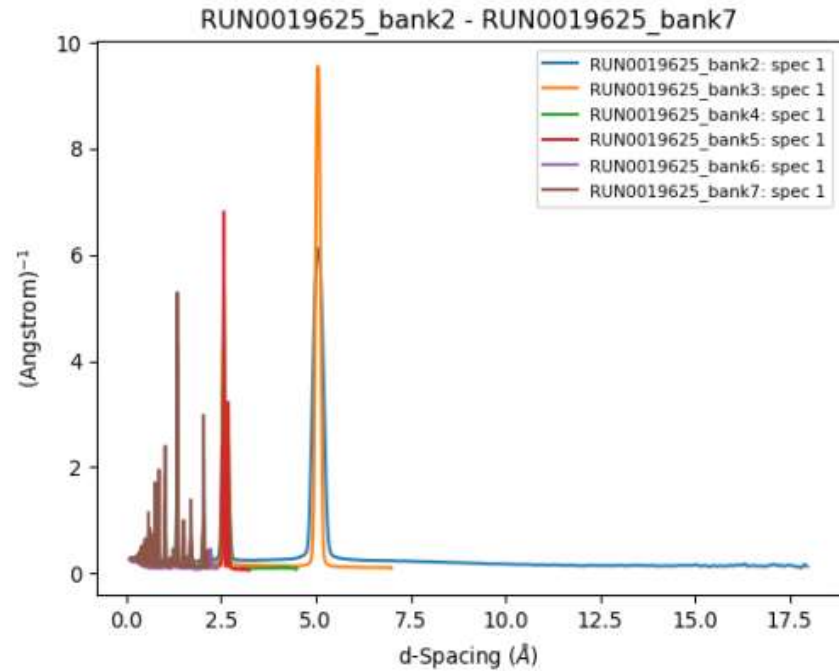
Table 1
Parameters of the MPI detector array.

| Detector bank No. | Secondary flight path L2 (m) | Scattering angle 2θ (Degree) | ³ He tube style | Number of ³ He tubes/modules #1 | Number of ³ He tubes/modules #2 | Calculated dQ/Q @-1 Å | Estimated Q range (Å ⁻¹) |
|-------------------|------------------------------|------------------------------|------------------------------|--|--|-----------------------|--------------------------------------|
| Bank01 | 2.82 | 2.9-5.04 | 0.5 m×1/2 inch | - | 20/1 | - | 0.1-2.7 |
| Bank02 | 2.319-2.711 | 12.54-17.62 | Type-1: 20 atm, 0.5 m×1 inch | - | 96/12 | 6% | 0.45-9 |
| Bank03 | 1.772-2.065 | 31-42.25 | Type-2: 20 atm, 0.3 m×1 inch | 32/4 | 96/12 | 2.1% | 1.1-22 |
| Bank04 | 1.351-1.599 | 51.25-67.32 | Type-2: 20 atm, 0.3 m×1 inch | 48/6 | 112/14 | 1.3% | 1.8-34 |
| Bank05 | 1.131-1.220 | 81.55-105.18 | Type-2: 20 atm, 0.3 m×1 inch | 72/9 | 112/14 | 0.8% | 2.7-49 |
| Bank06 | 1.166-1.191 | 121.80-145.51 | Type-2: 20 atm, 0.3 m×1 inch | 32/4 | 64/8 | 0.55% | 3.6-59 |
| Bank07 | 1.232-1.314 | 157.47-170.00 | Type-2: 20 atm, 0.3 m×1 inch | 64/8 | 64/8 | 0.39% | 4.1-62 |



MnO data

20 K



Experiment setup

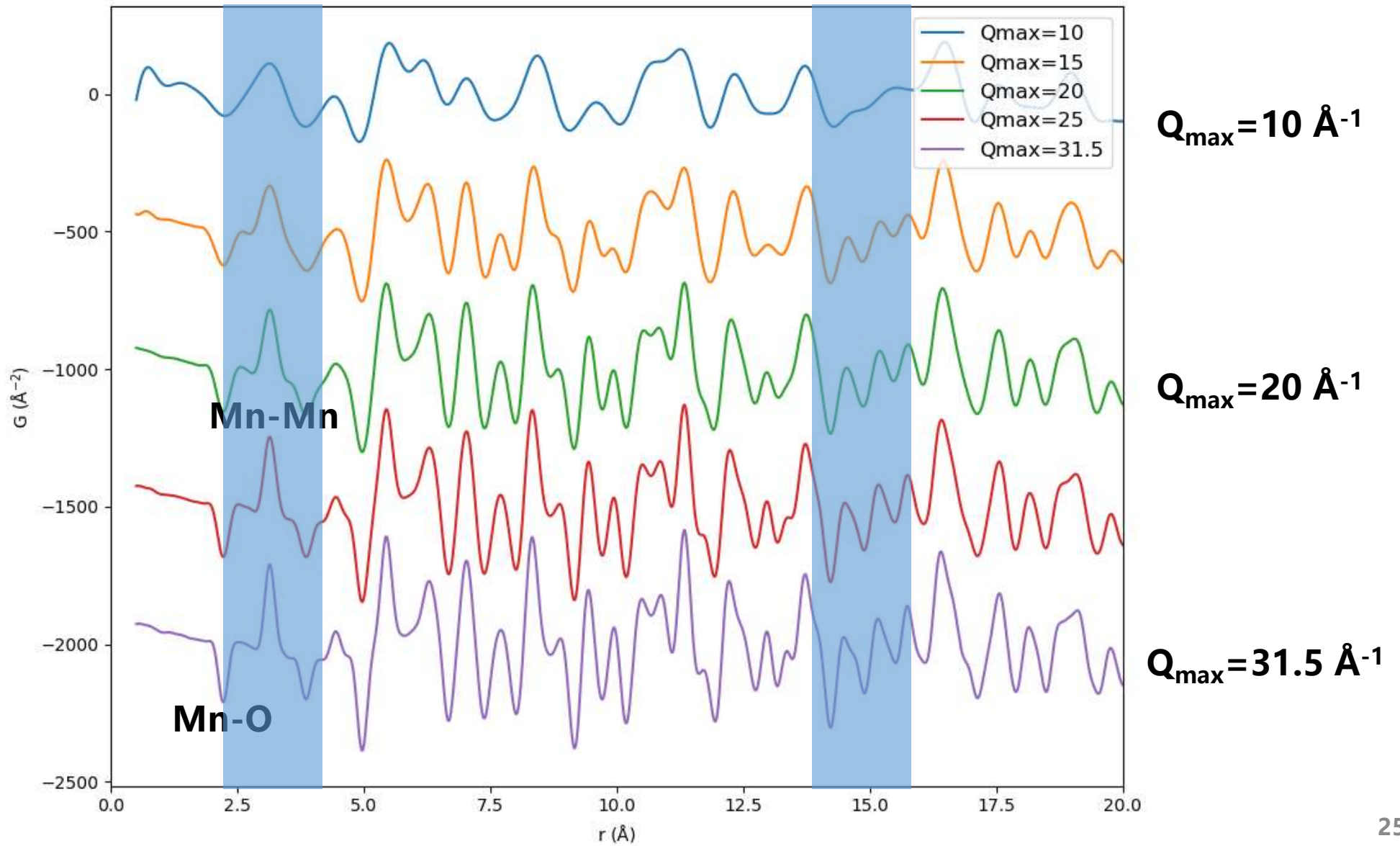
- Sample Environment: CCR06
- Sample holder: 9mm vanadium can
- Sample install: He glovebox
- Sealed ring: In
- Neutron wavelength: 0.1-4.5Å

Data collected at March 2023 @ MPI

Courtesy of Juping Xu (CSNS)



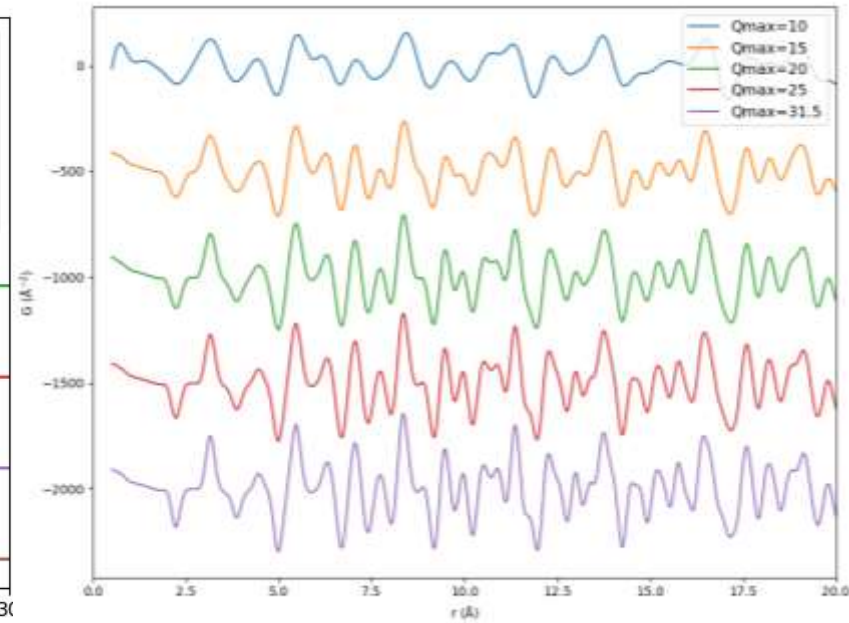
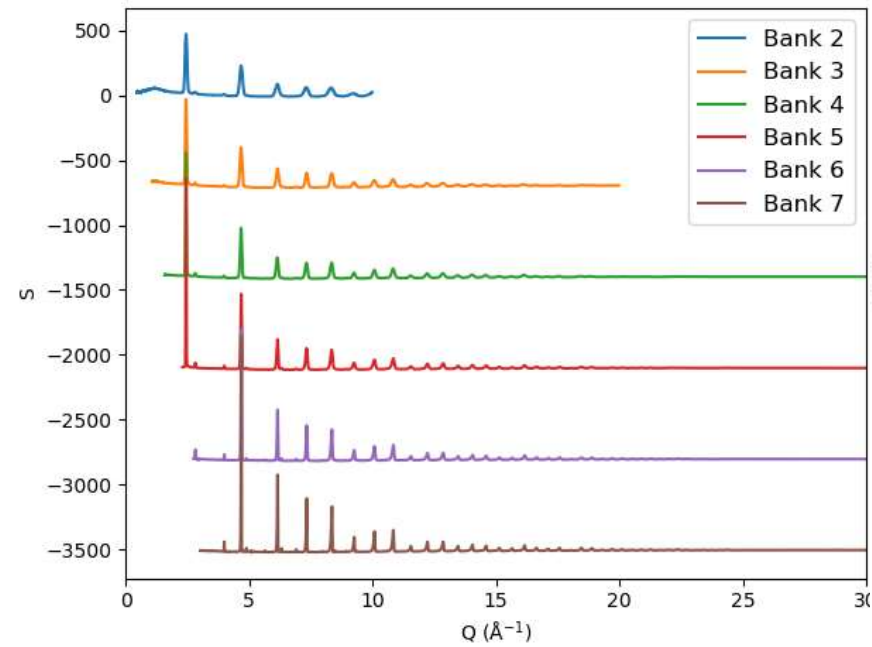
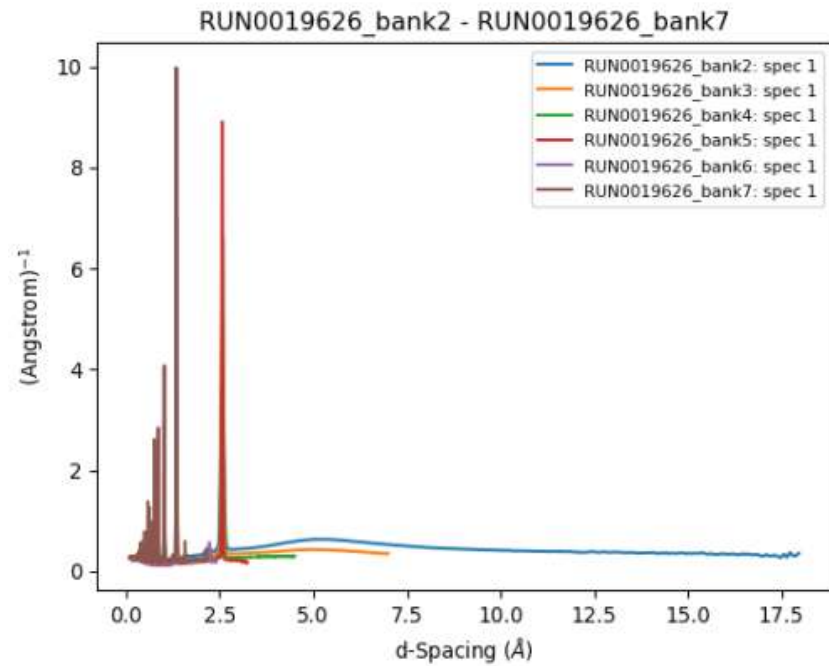
PDF





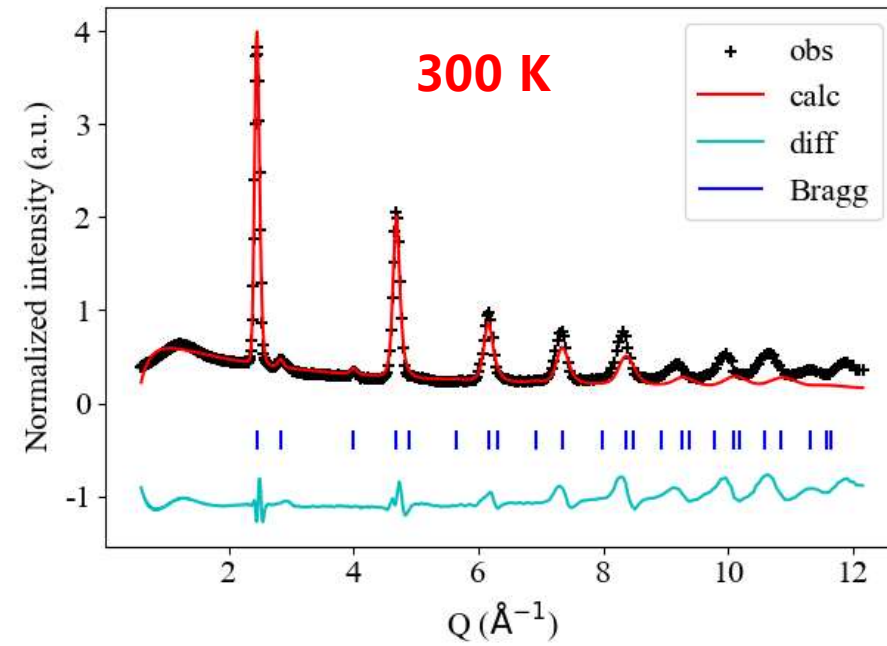
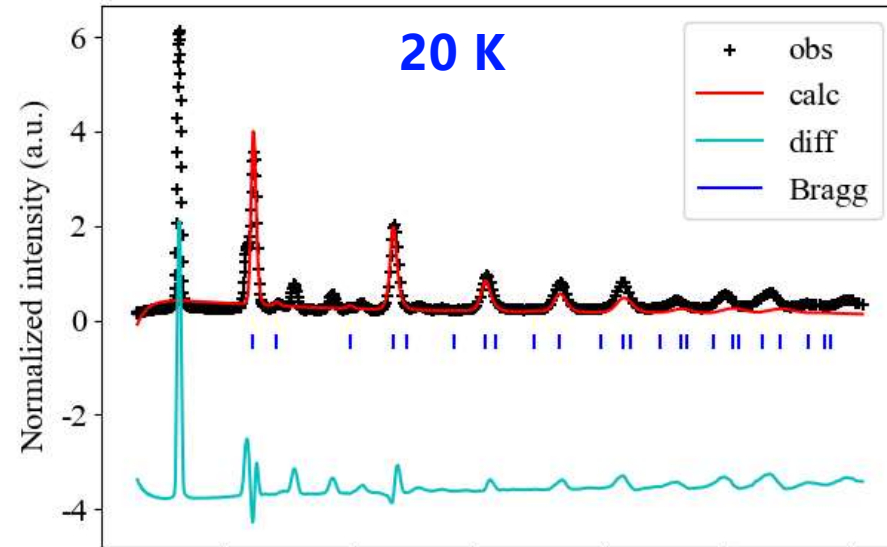
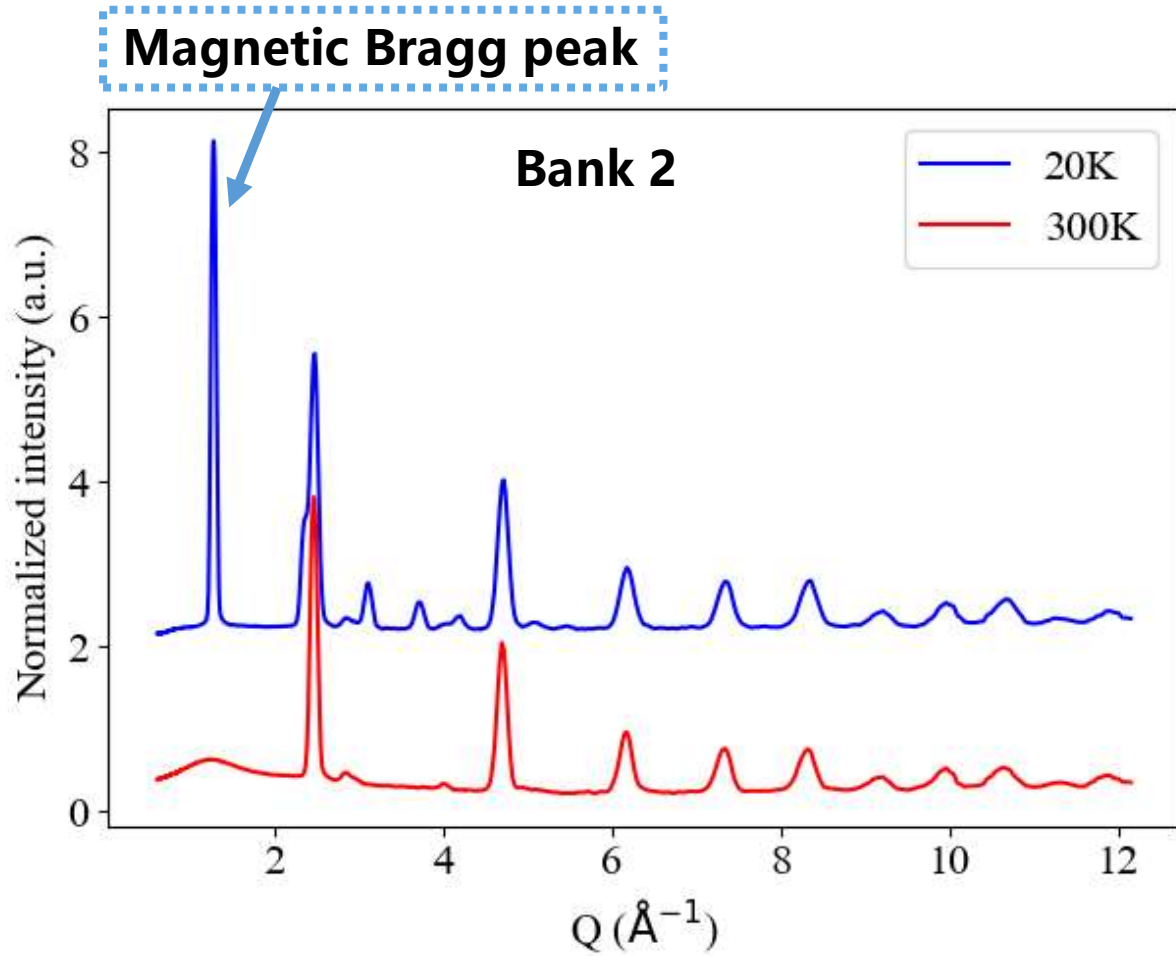
MnO data

300 K



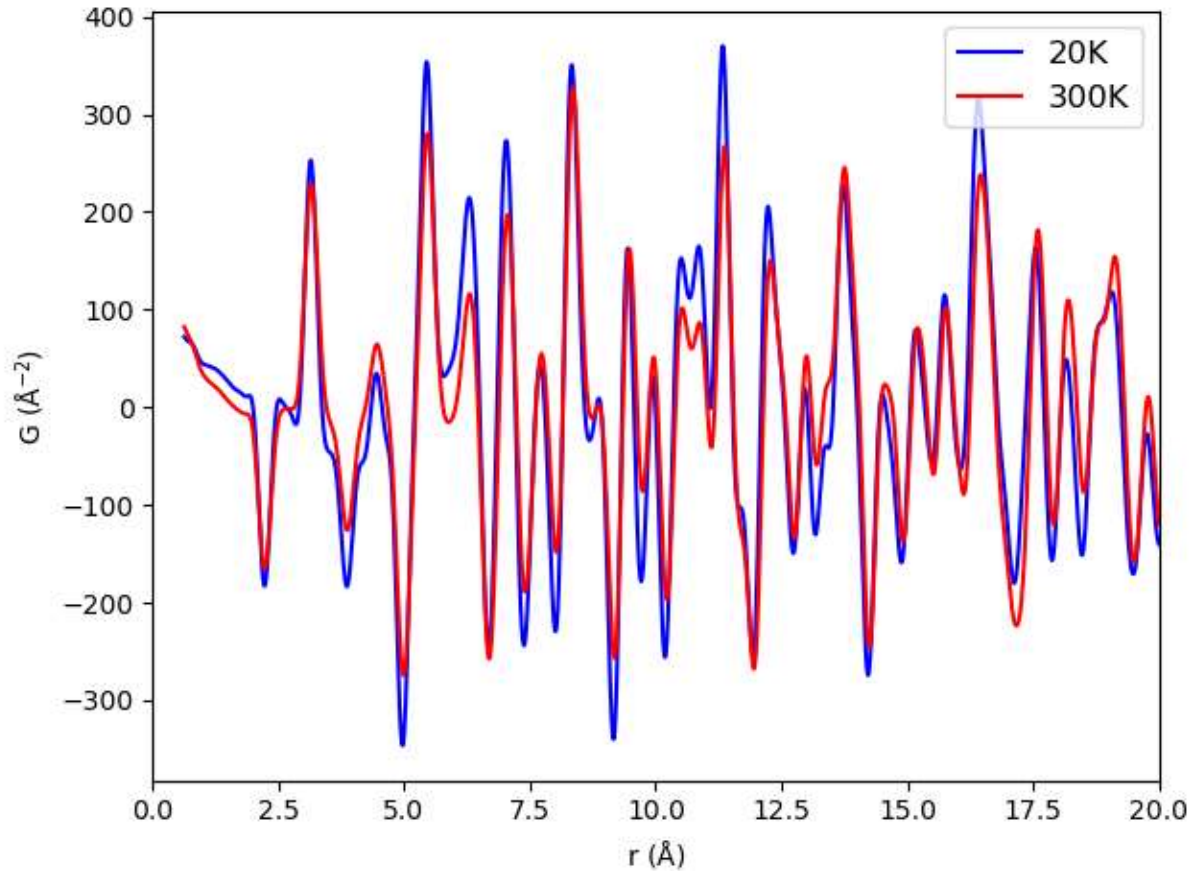


Reciprocal space data analysis

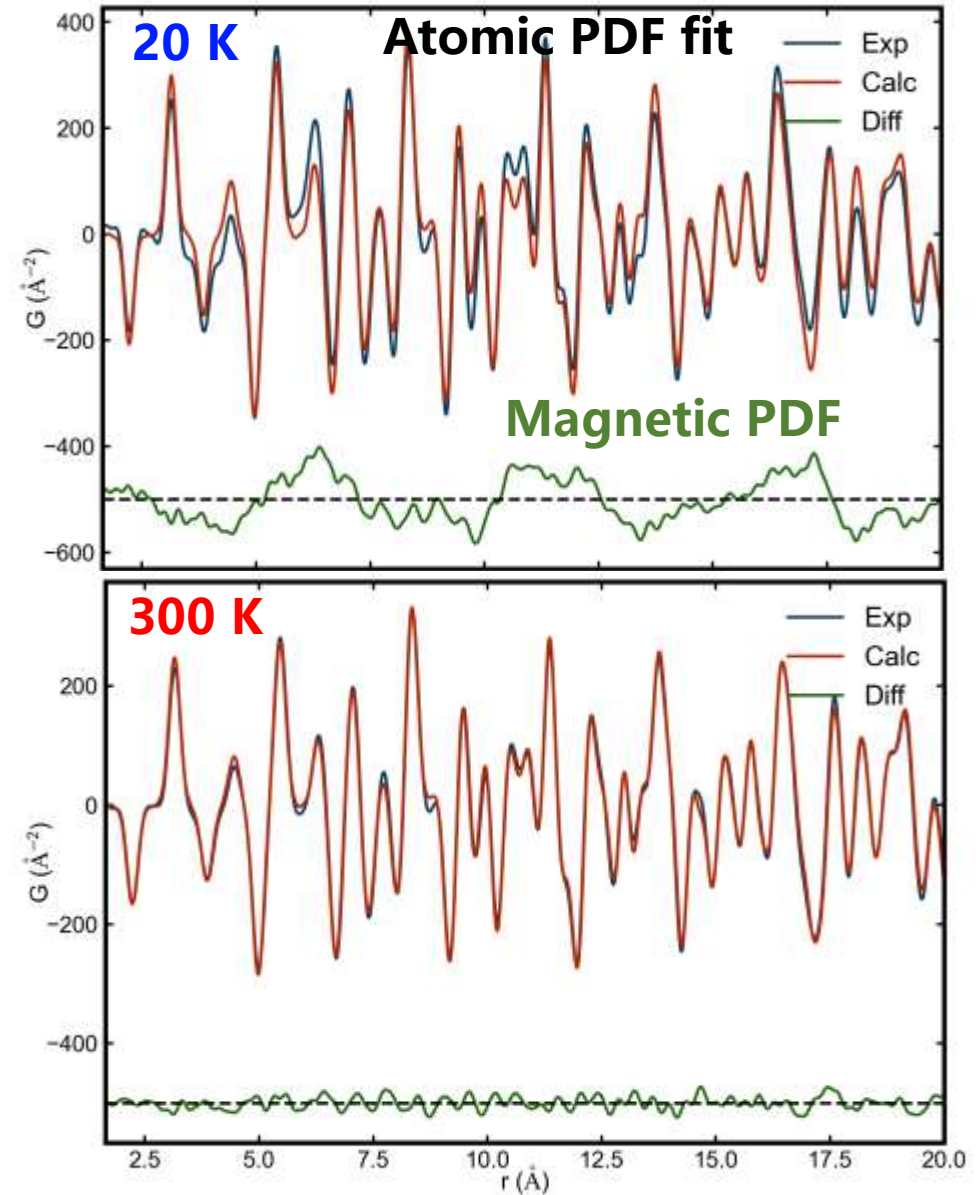




Real space atomic PDF data analysis

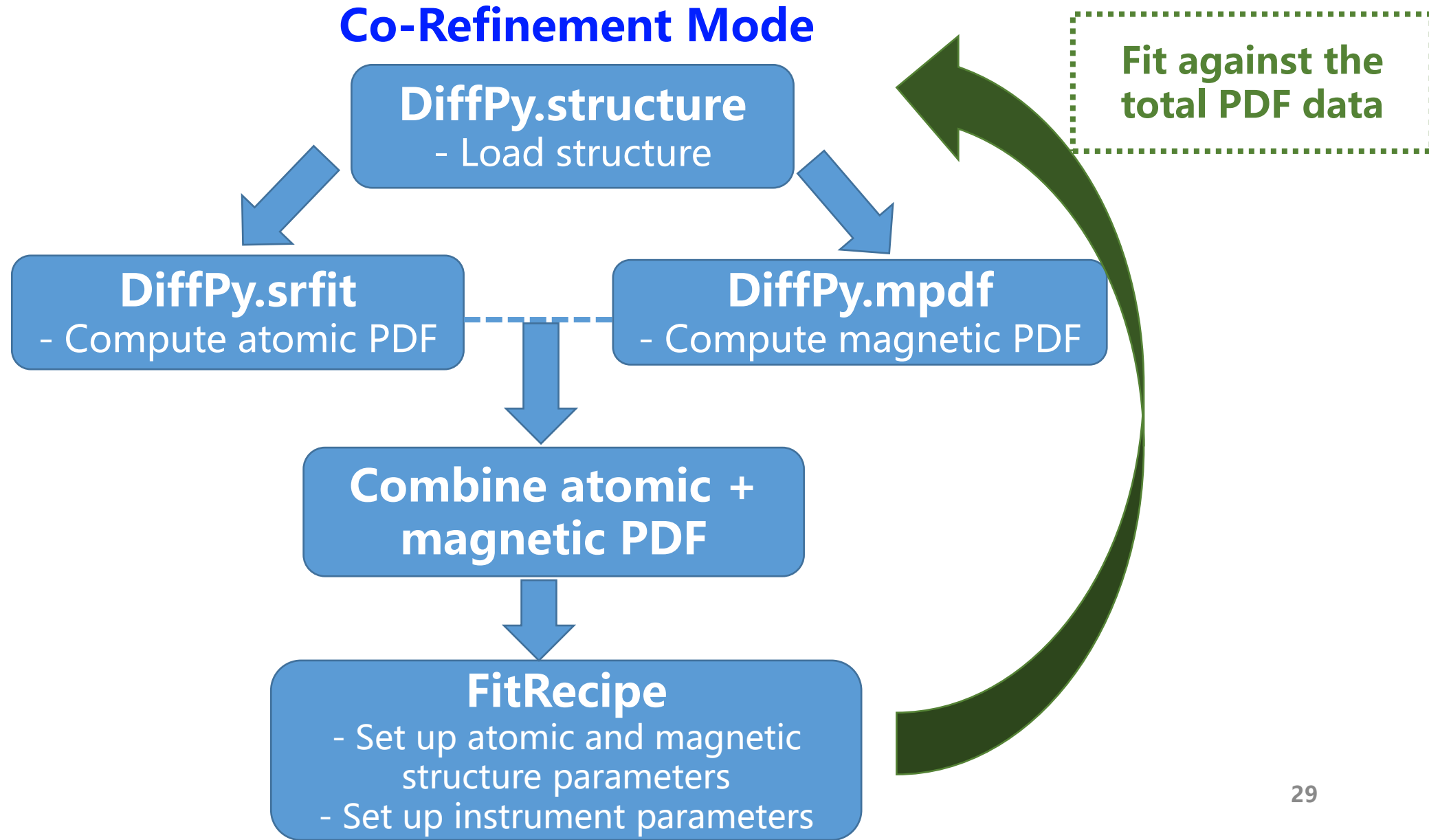


- After fitting atomic structure, the magnetic signals left are prominent.





Software framework





Software framework

Toggle Mode

DiffPy.structure
- Load structure

Atomic

DiffPy.srfit
- Compute atomic PDF



FitRecipe
- Set up atomic structure parameters
- Set up instrument parameters

Magnetic

DiffPy.mpdf
- Compute magnetic PDF

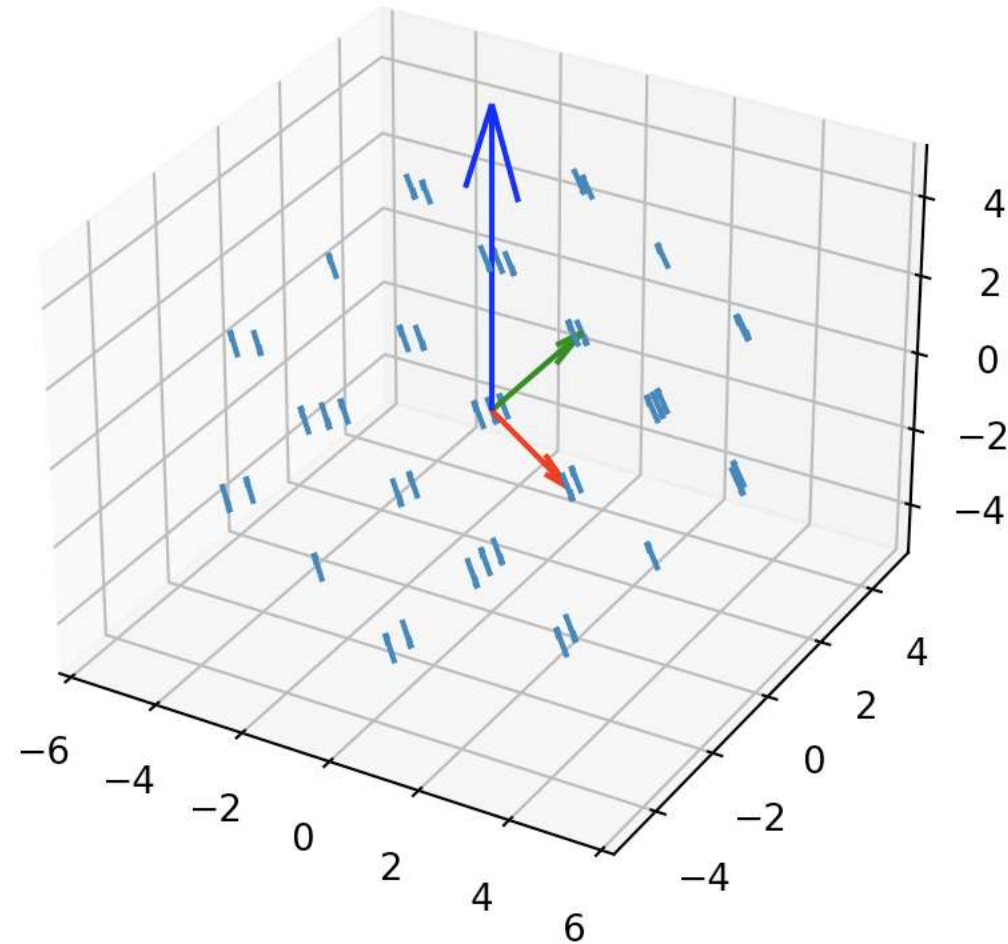


FitRecipe
- Set up magnetic structure parameters
- Set up instrument parameters

Iterative fit
against atomic
and magnetic
PDF data



Magnetic structure



Set up initial magnetic structure

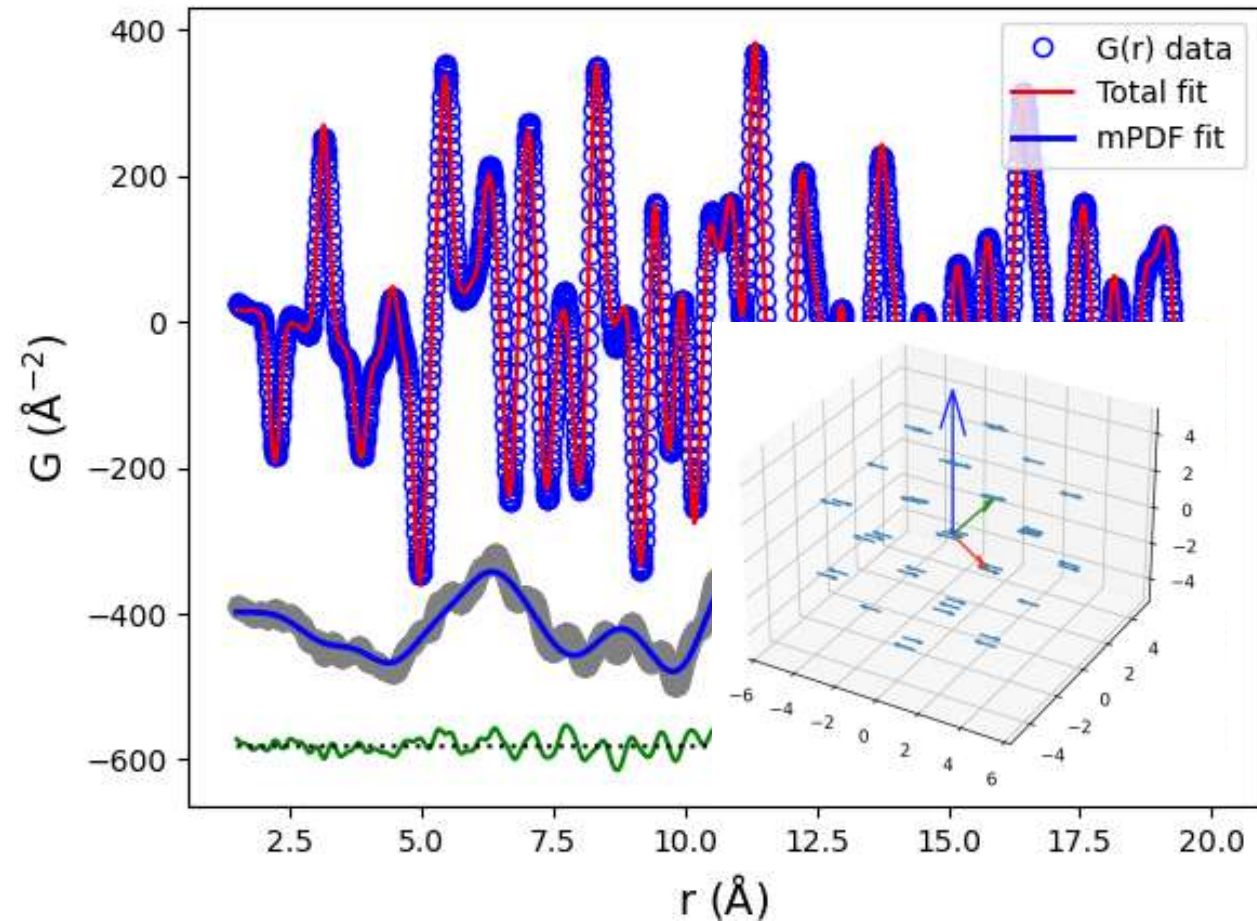
- Magnetic basis vector = $[1, -1, 0]$
- Magnetic propagation vector = $[0, 0, 1.5]$



$$Q_{\max} = 25 \text{ \AA}^{-1}$$

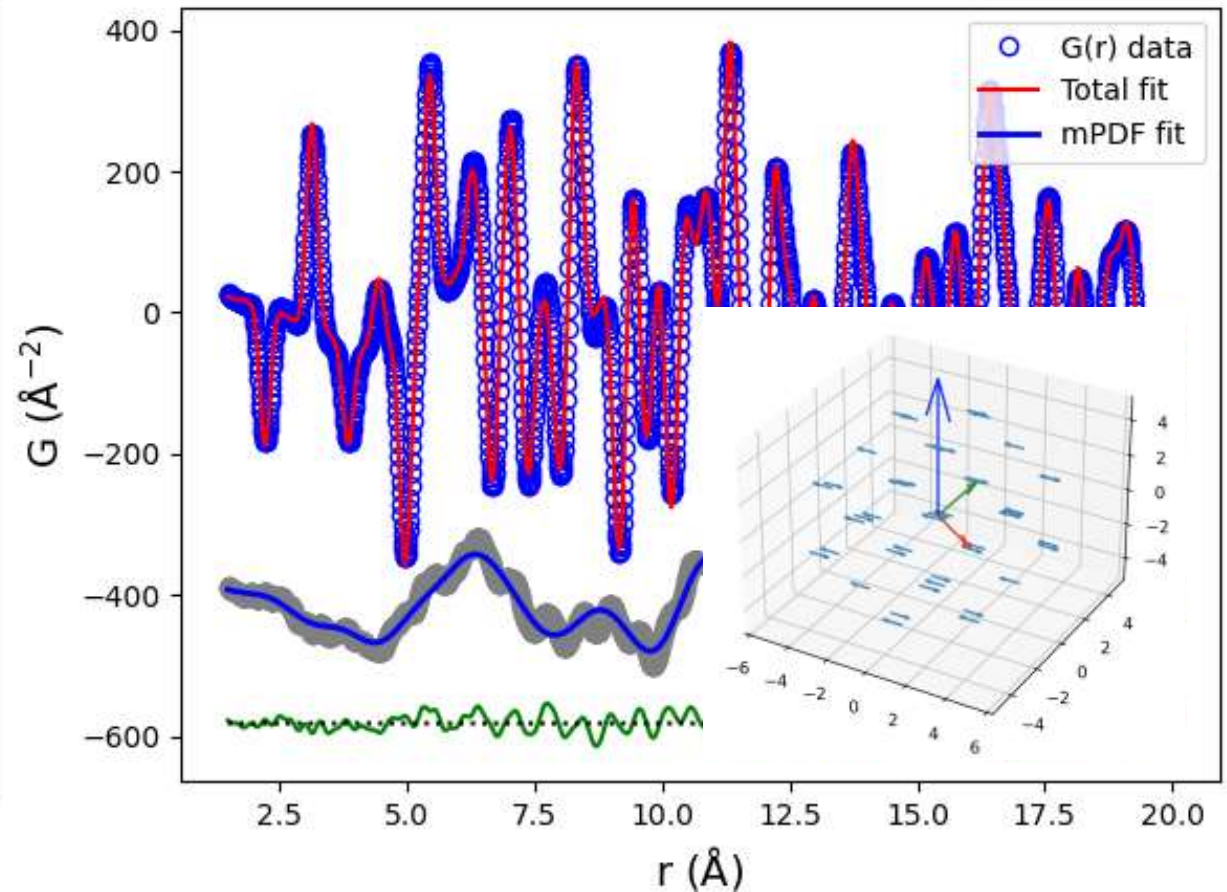
Co-refinement

$$R_w = 0.086$$



Toggle

$$R_w = 0.085$$

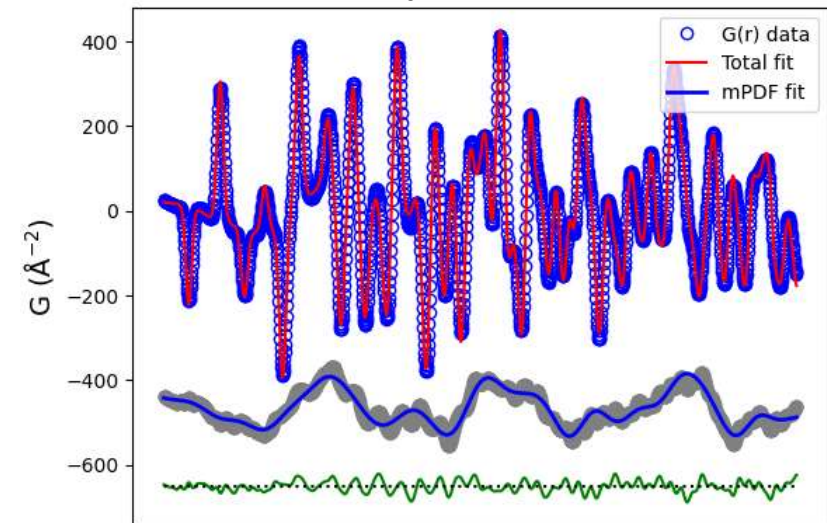


- Co-refinement and toggle algorithms perform similarly well.

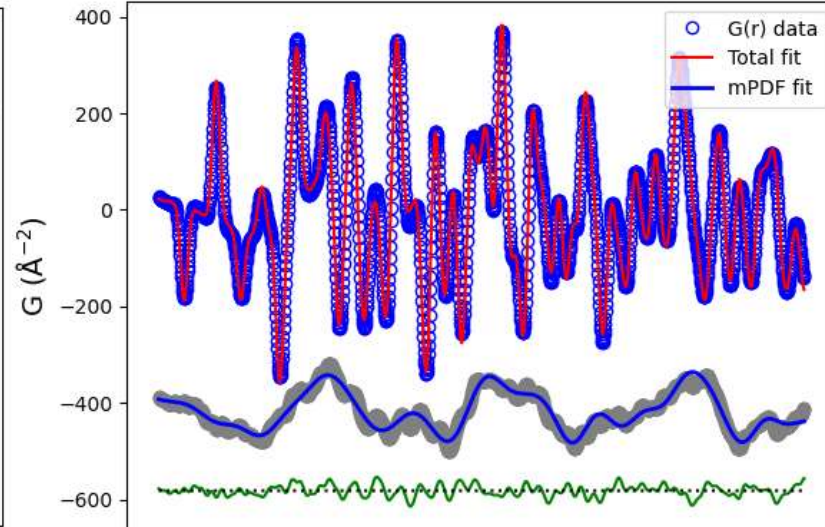


Different Q_{\max}

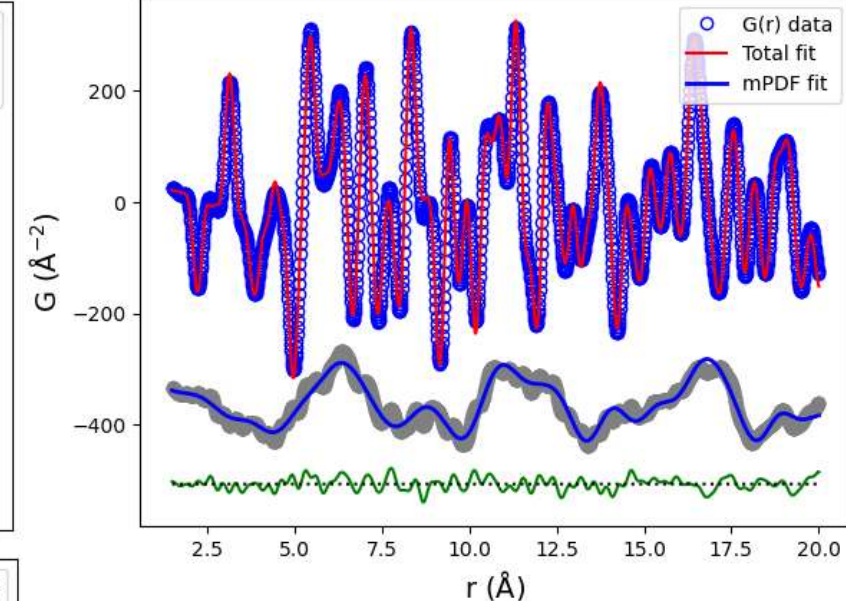
$Q_{\max} = 31.5 \text{ \AA}^{-1}$



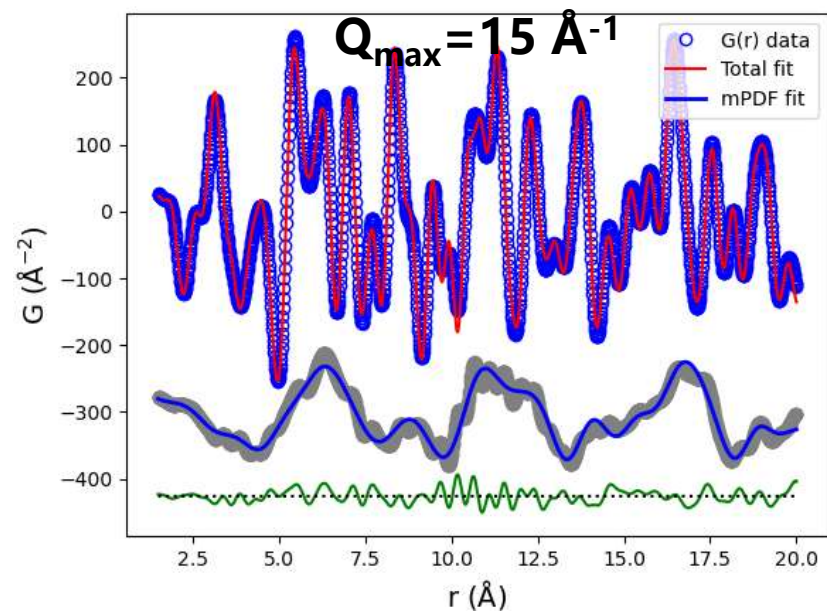
$Q_{\max} = 25 \text{ \AA}^{-1}$



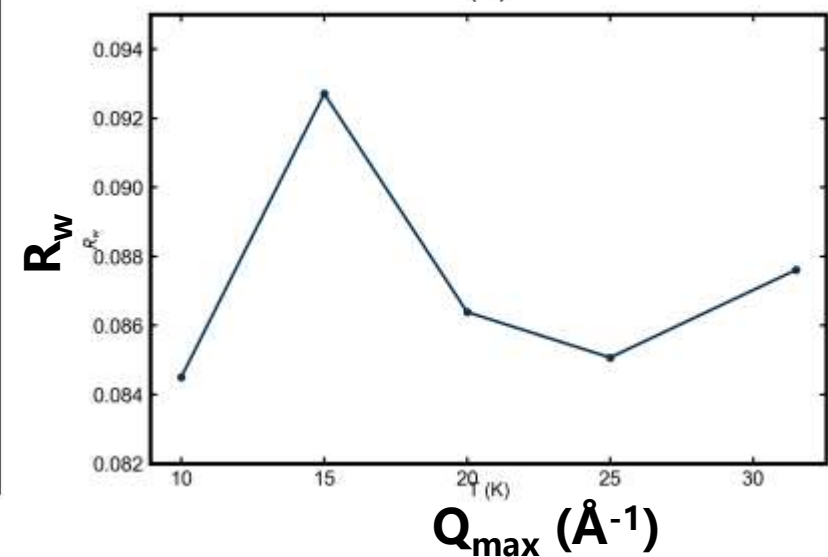
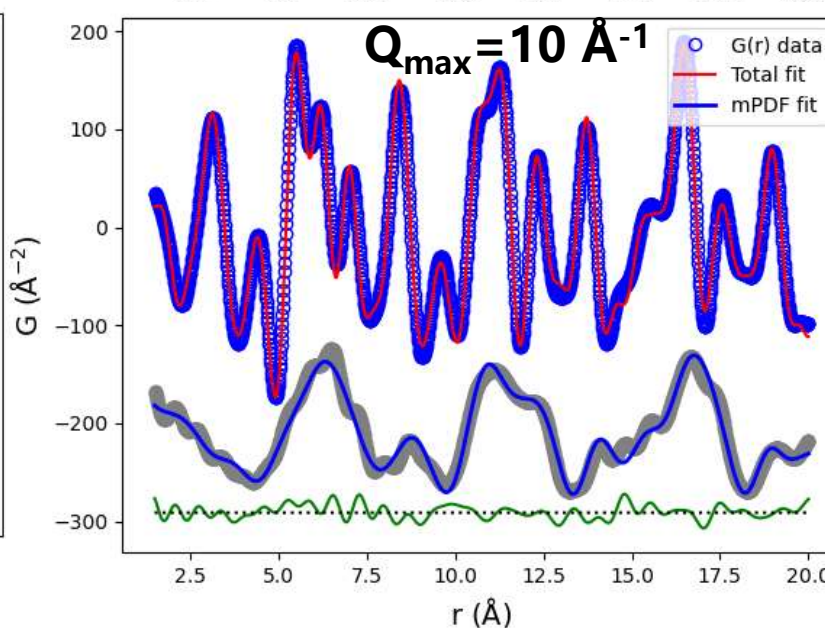
$Q_{\max} = 20 \text{ \AA}^{-1}$



$Q_{\max} = 15 \text{ \AA}^{-1}$



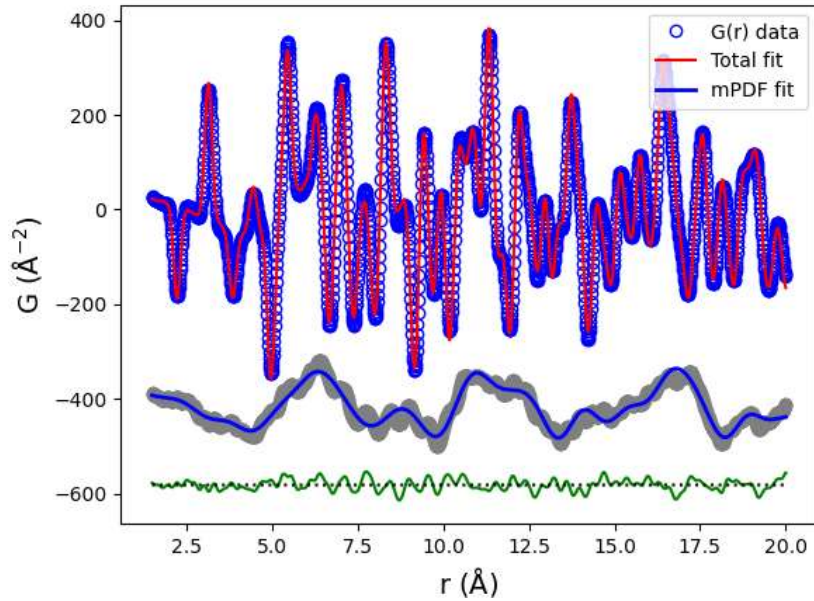
$Q_{\max} = 10 \text{ \AA}^{-1}$





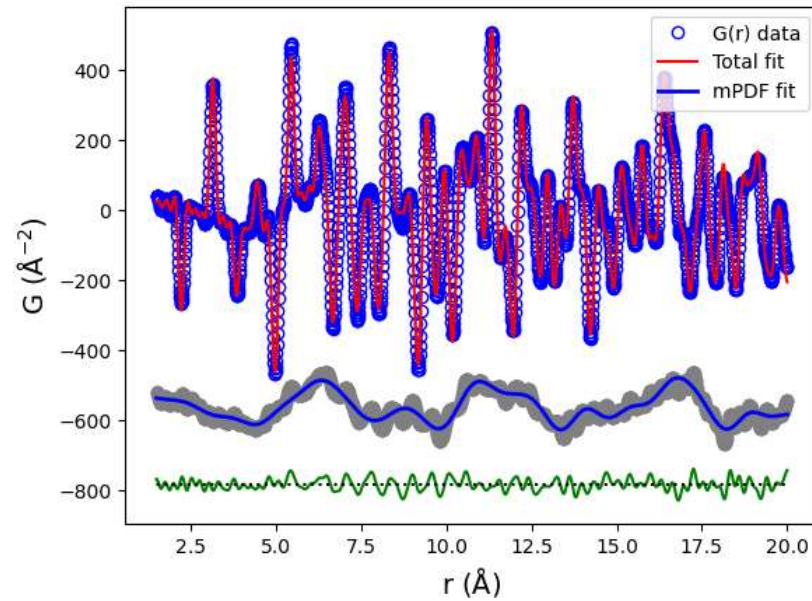
Lorch function

No lorch

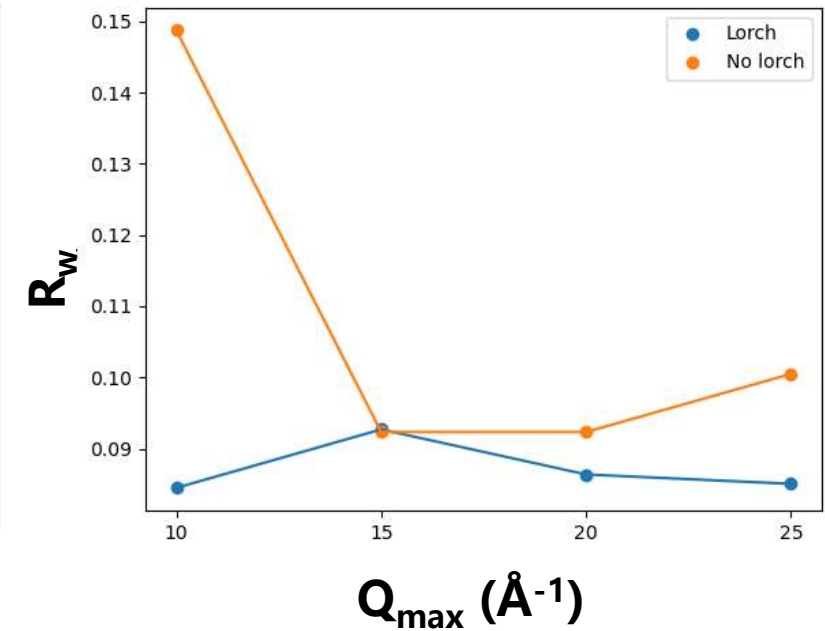


$$Q_{\max} = 25 \text{ \AA}^{-1}$$
$$R_w = 0.100$$

Lorch



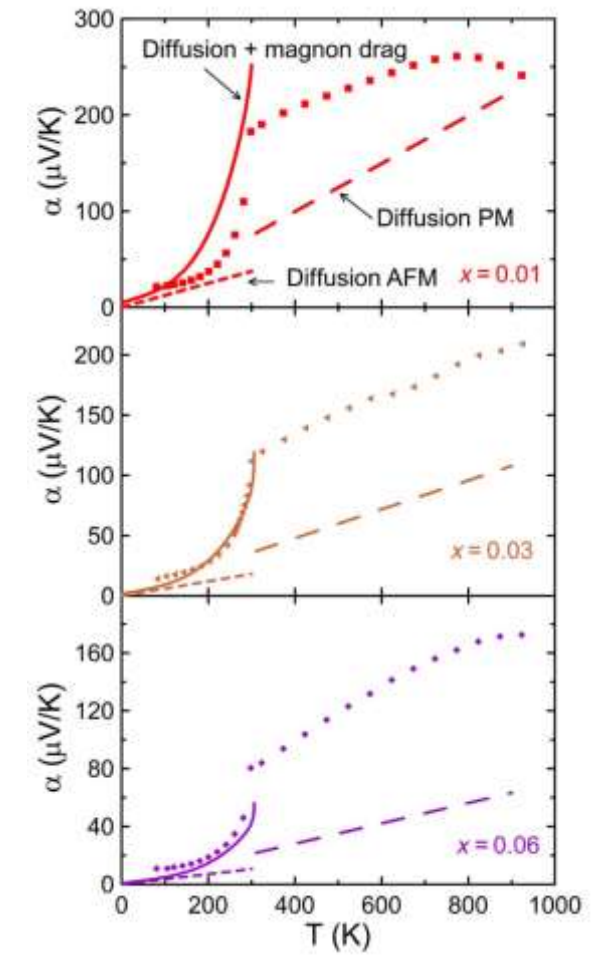
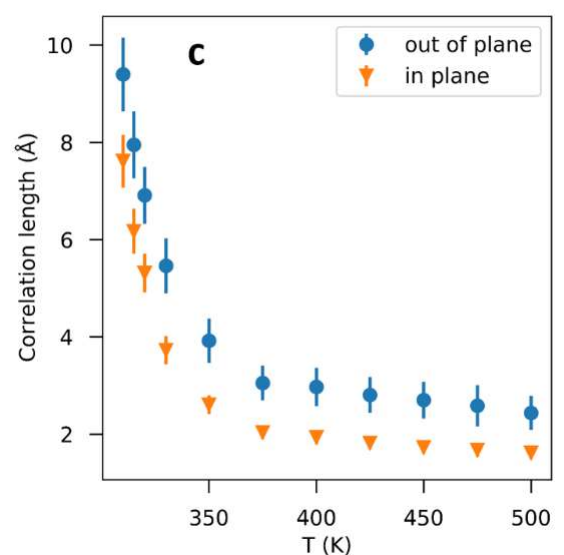
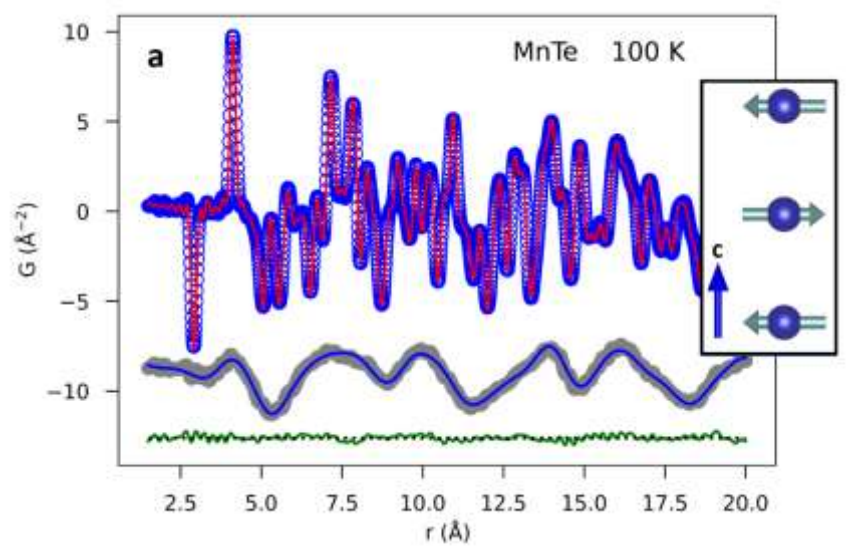
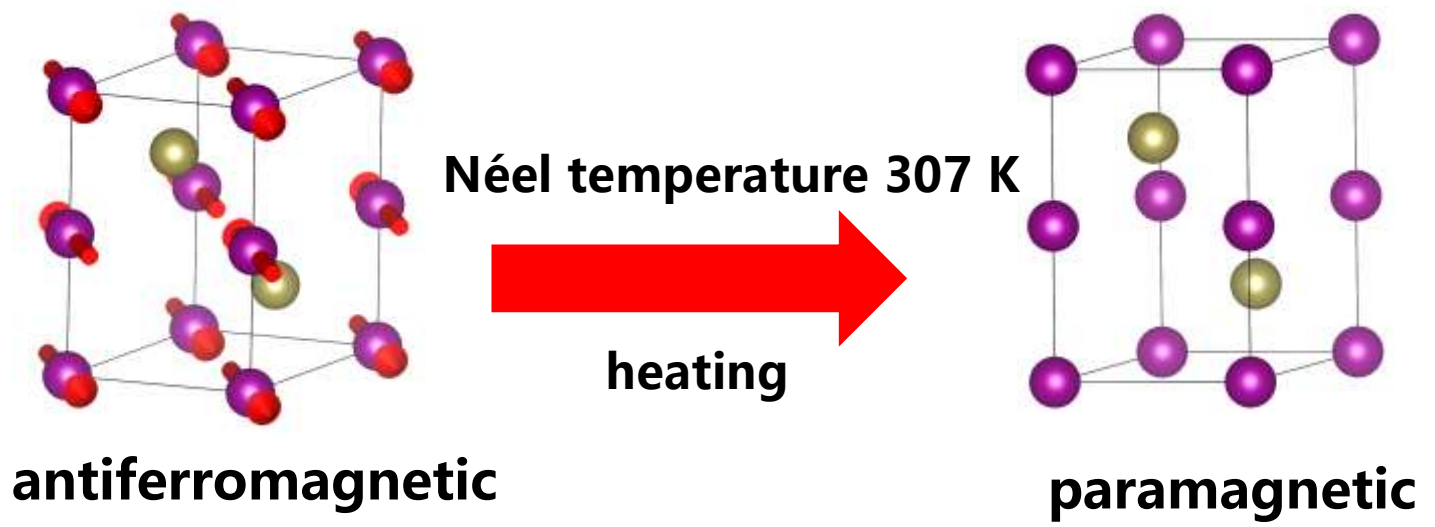
$$Q_{\max} = 25 \text{ \AA}^{-1}$$
$$R_w = 0.085$$





MnTe

- Short-range antiferromagnetic correlations enhanced thermoelectric material.

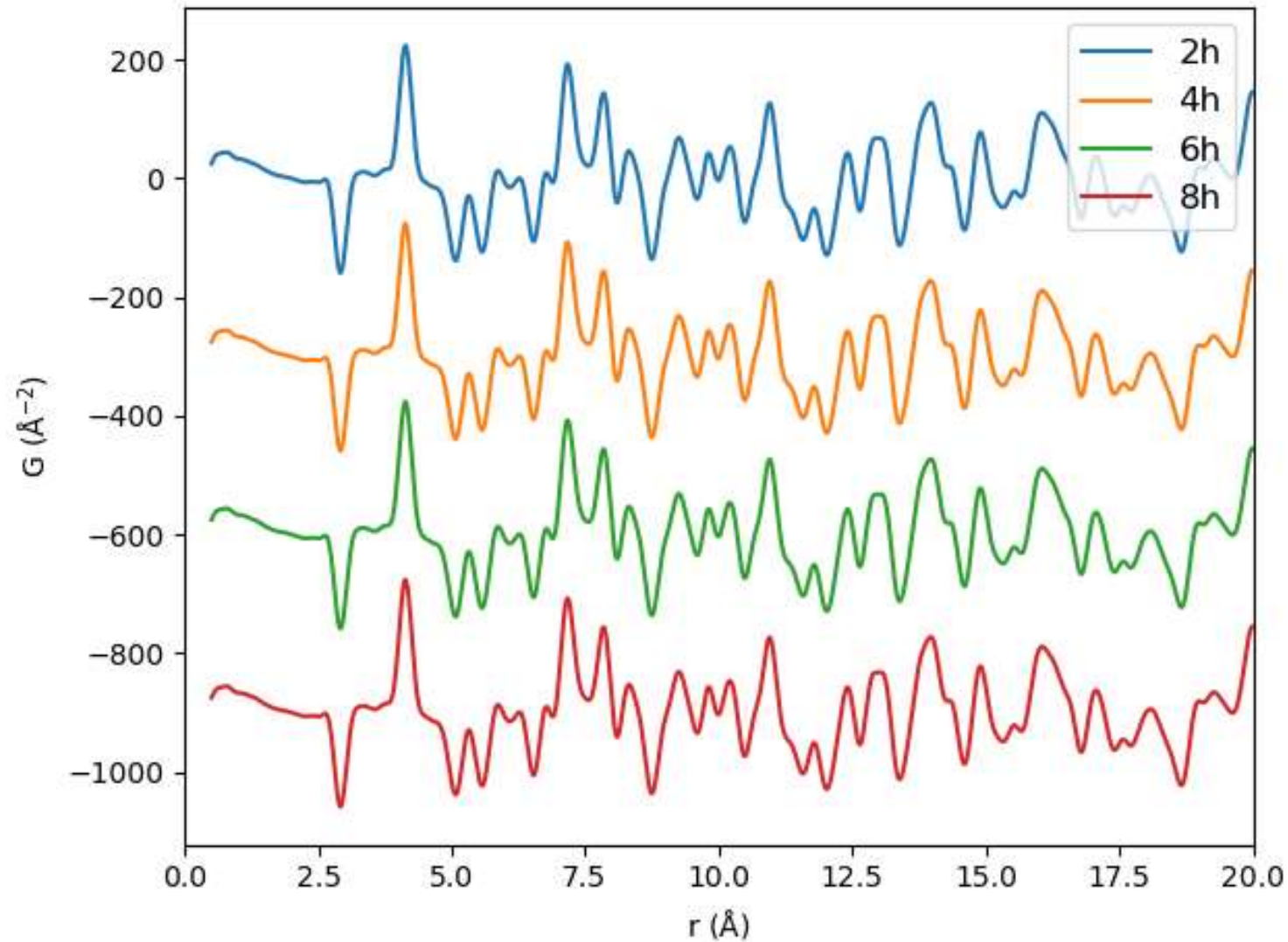


R. Baral, et al., *Matter*, 2022, 5, 6.
Y. Zheng, et al., *Sci. Adv.*, 2019, 5, 9.



Different exposure time

20 K

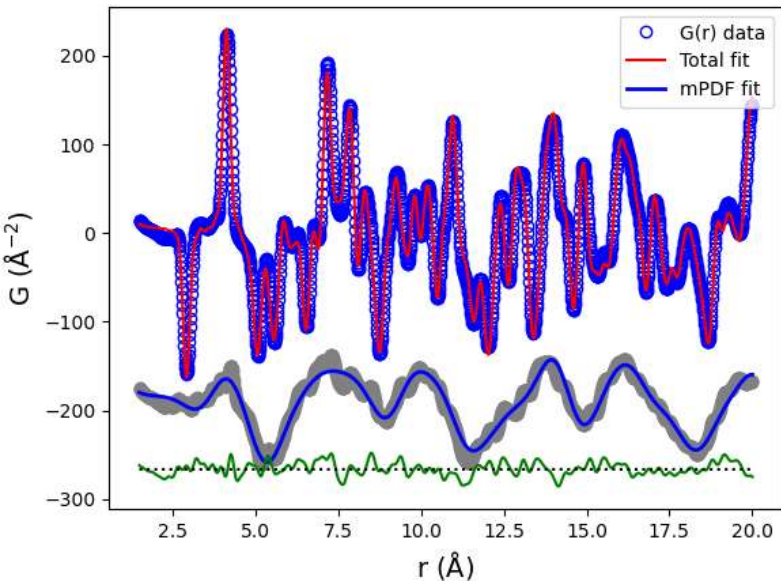


@MPI

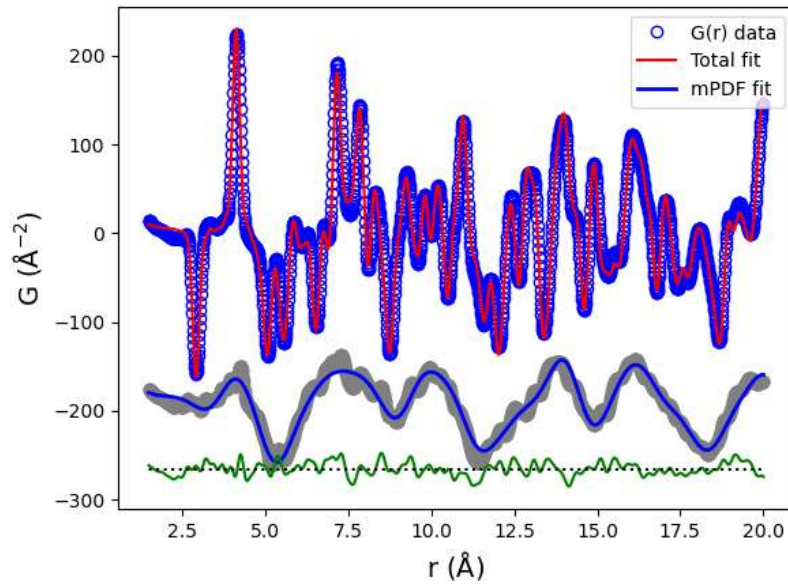


MnTe - different exposure time

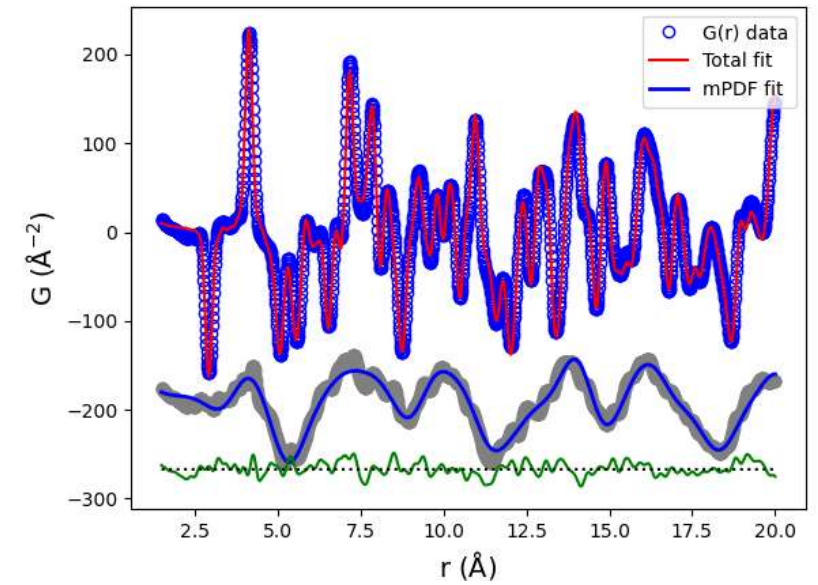
8h
 $R_w = 0.113$



6h
 $R_w = 0.114$



2h
 $R_w = 0.113$





MnTe - different exposure time

| 20 K | 8h | 6h | 2h |
|-------------------------------------|--------|--------|--------|
| a (Å) | 4.138 | 4.138 | 4.139 |
| c (Å) | 6.682 | 6.682 | 6.680 |
| $U_{\text{Mn}11}$ (Å ²) | 0.0067 | 0.0068 | 0.0068 |
| $U_{\text{Mn}33}$ (Å ²) | 0.0132 | 0.0132 | 0.0124 |
| $U_{\text{O}11}$ (Å ²) | 0.0055 | 0.0055 | 0.0055 |
| $U_{\text{O}33}$ (Å ²) | 0.0068 | 0.0067 | 0.0066 |
| Corr. L (Å) | >1000 | >1000 | >1000 |
| R_w | 0.113 | 0.114 | 0.113 |

- High-quality mPDF data can be collected in **2 hours** at MPI.

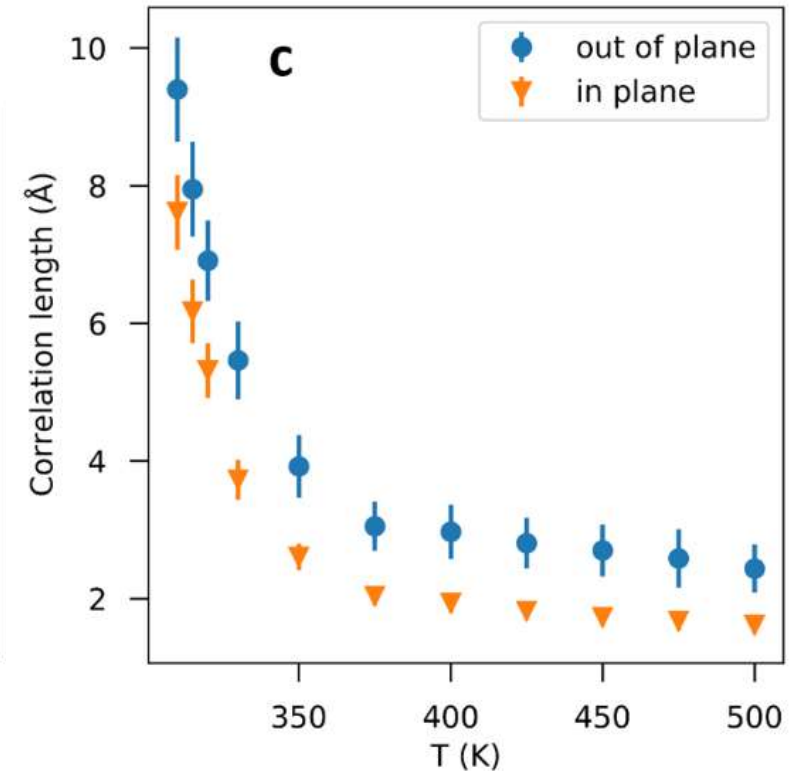
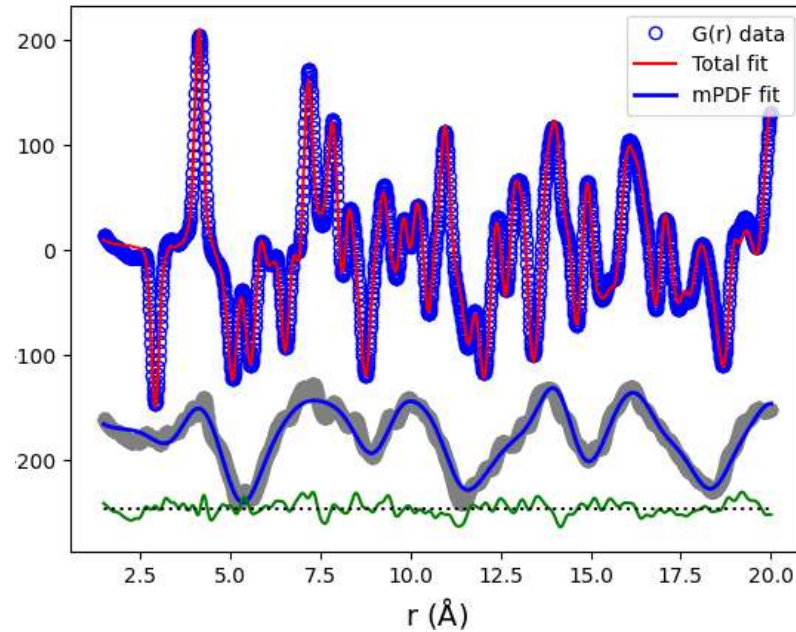
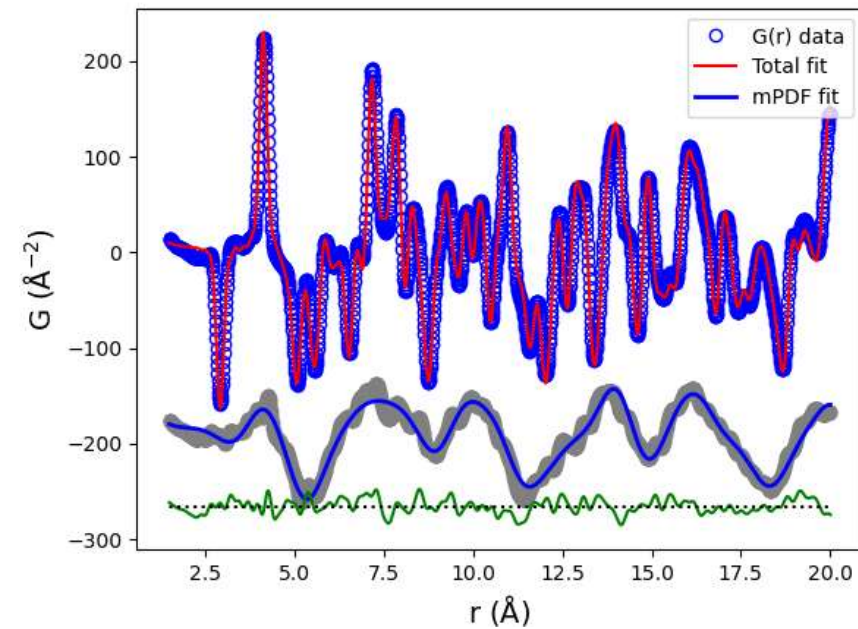


MnTe

Same exposure time

20K

100K



- The short-range antiferromagnetic correlation persists above Néel temperature.



DiffPy



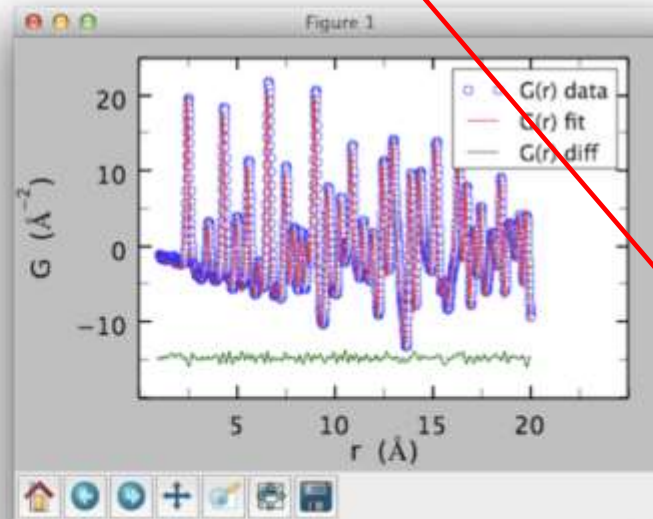
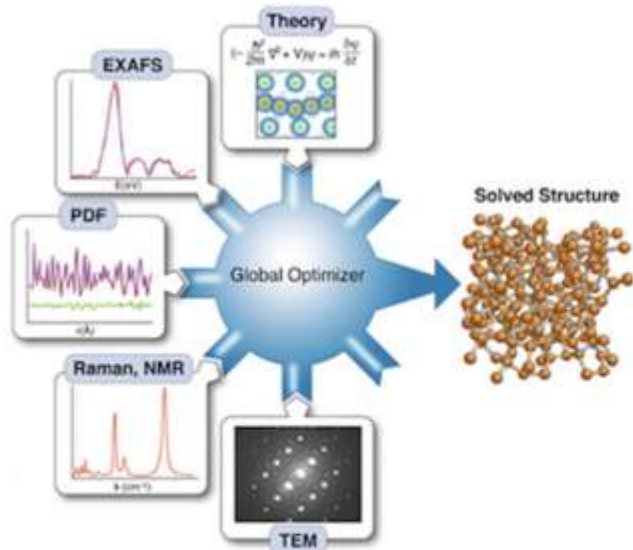
DiffPy-CMI upgrade 3.0 for Python 2 and 3 is now available! (Mar 14, 2019)

Get DiffPy-CMI

Credits

DiffPy - Atomic Structure Analysis in Python

A free and open source software project to provide python software for diffraction analysis and the study of the atomic structure of materials.



The image shows a screenshot of the 'Products' dropdown menu from the DiffPy website. The menu is open, displaying a list of software tools and packages. The items listed are: DiffPy-CMI, xPDFsuite, PDFgetX3, PDFgetN3 and PDFgetS3, PDFgui, SrMise, mPDF, xINTERPDF, and Python Packages.

Release of PDFgui 2.0

<https://diffpy.org>



PDFgui 2.X

- Support mPDF refinement
- UI visualize magnetic structure





PDF in the Cloud

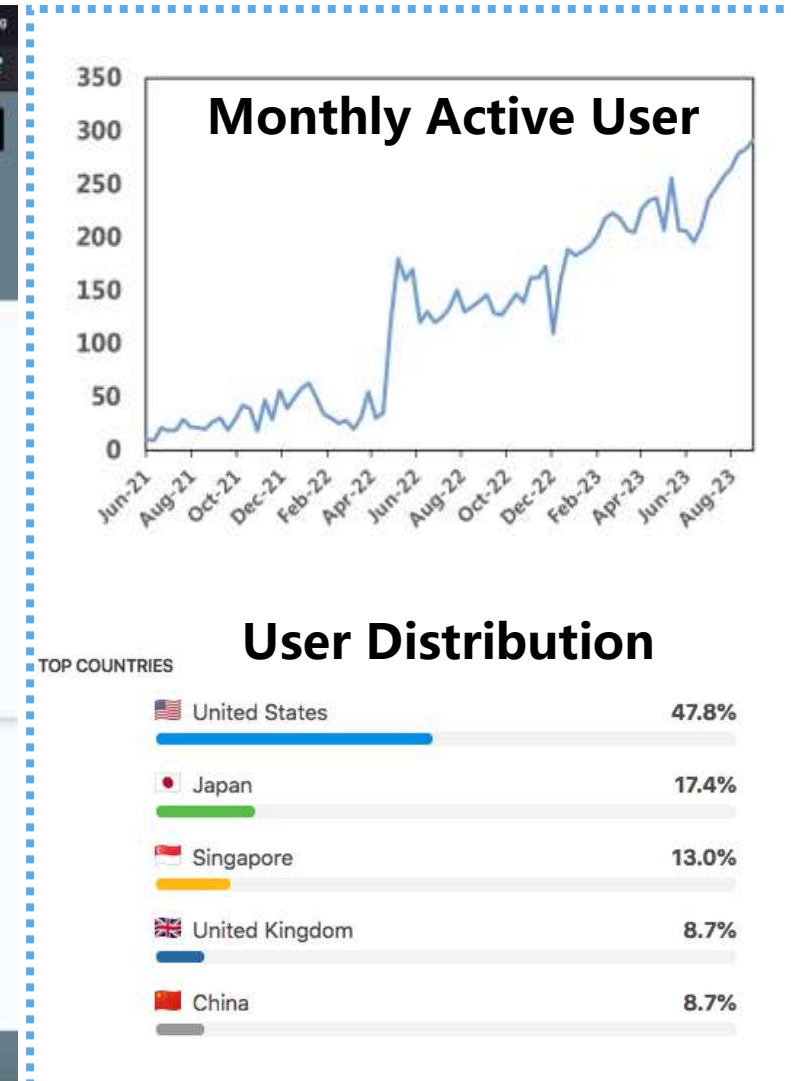
structureMining
Auto search for the best structures from an experimental PDF.
start

spacegroupMining
Auto search for the best space groups from an experimental PDF.
start

similarityMapping
Calculate the correlation between experimental PDFs.
start

nmfMapping
Disentangle structural phase components and their ratios from sets of PDFs or powder diffraction patterns.

About | Contact | Terms of Use | Privacy Policy | Citing



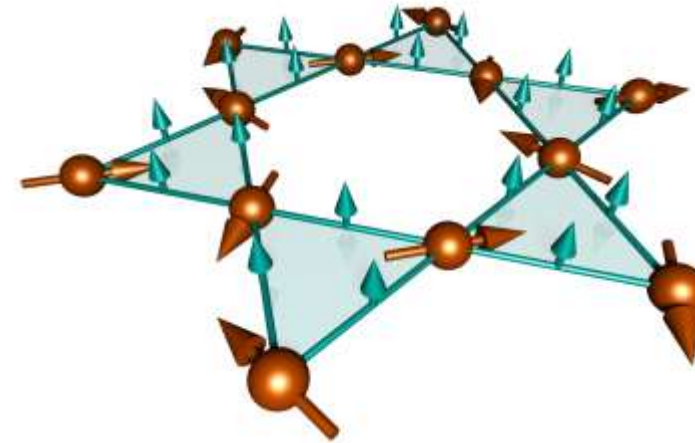
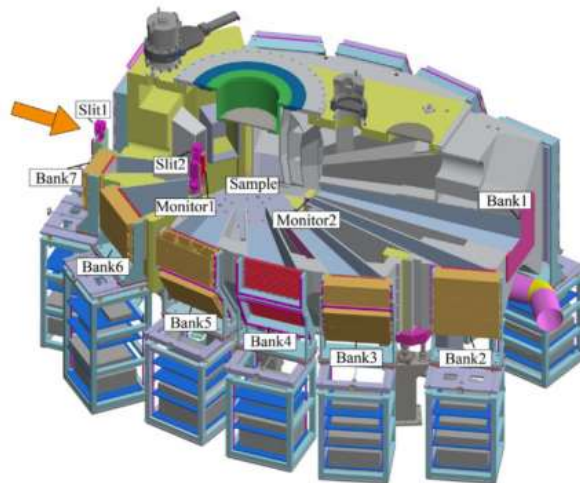
<https://pdfitc.org>

L. Yang, et al. Acta Crystallogr. A (2021) 42



Summary

- The MPI instrument is capable of collecting high quality mPDF data.
- The mPDF may serve as a promising method for exploring local magnetic correlations in complicated condensed matters.



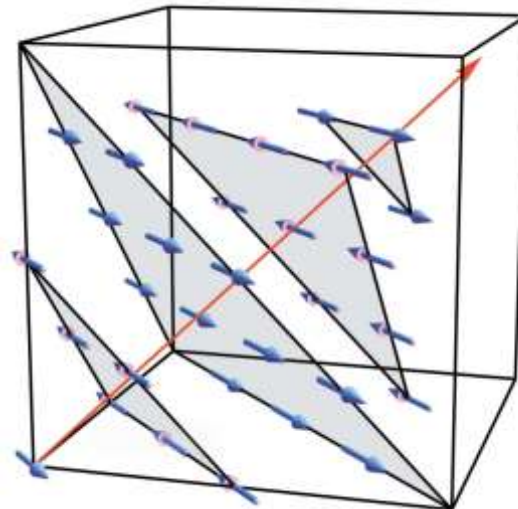
Thank you!





MnO

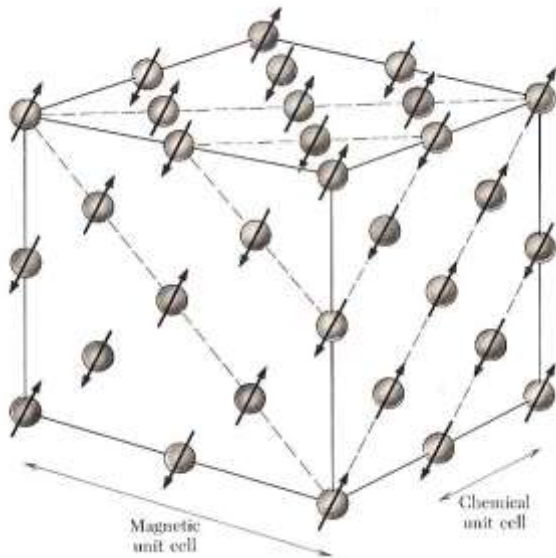
- **Early neutron diffraction studies showed that MnO has the cubic rock-salt structure (s.g.: Fm-3m) at high temperature, and rhombohedral phase (s.g.: R-3m) at low temperature. (Shull et al., 1951; Roth, 1958)**
- **The antiferromagnetic spin arrangement in MnO is compatible only with monoclinic or lower symmetry (Shaked et al., 1988), so the true structural symmetry must be lower than R-3m.**





MnO

- In 1949 Clifford Shull and Ernest Wollan showed the magnetic structure of MnO, which leads to the discovery of **antiferromagnetism**.
- The spin alignment axis lies within the (111) plane, and the spin direction reverses between adjacent sheets along the [111] direction.
- Clifford Shull won the Nobel Prize in Physics in 1994.

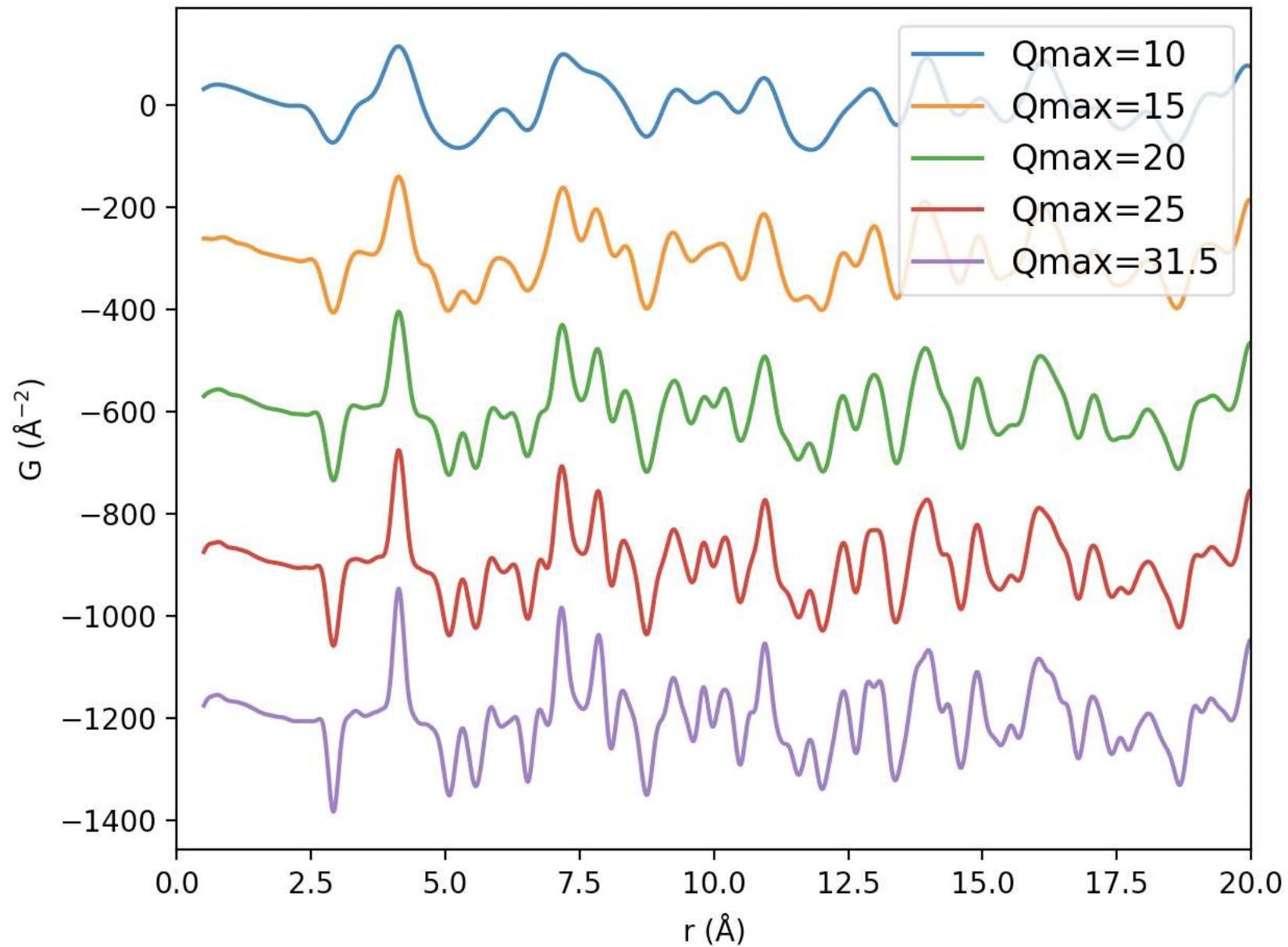


courtesy of ORNL





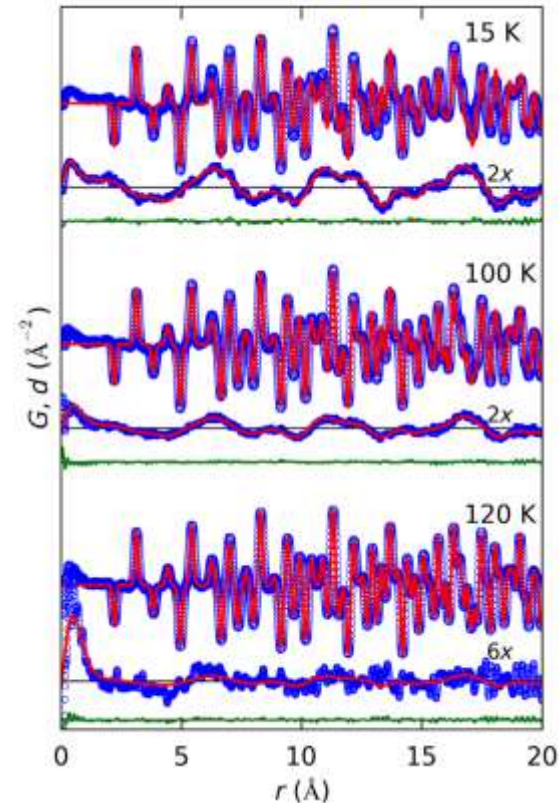
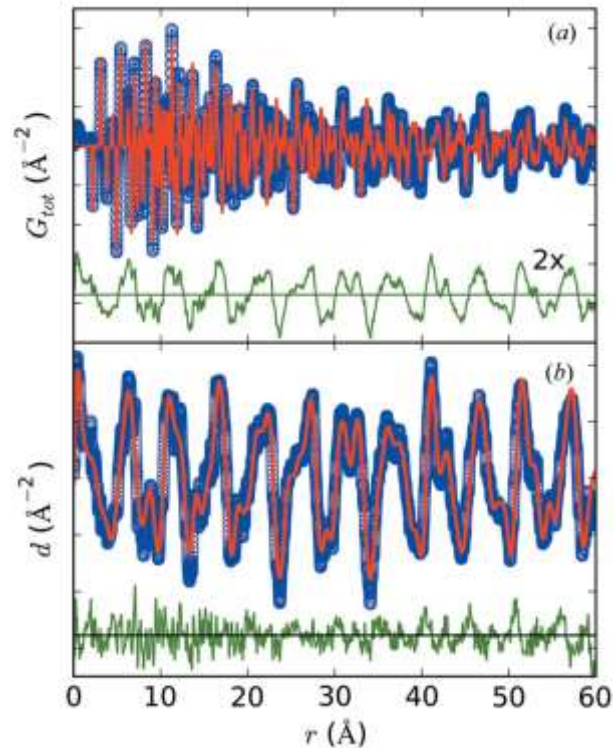
MnTe





Atomic v.s. magnetic PDF

- mPDF is lower in frequency and characteristically broader than the atomic PDF due to the effects of the magnetic form factor.
- mPDF changes more dramatically with temperature.



B. Frandsen, S. J.L. Billinge, *Acta Cryst. A*, 2015, 71, 3.

B. Frandsen, et al., *Phys. Rev. Lett.*, 2016, 116, 19.

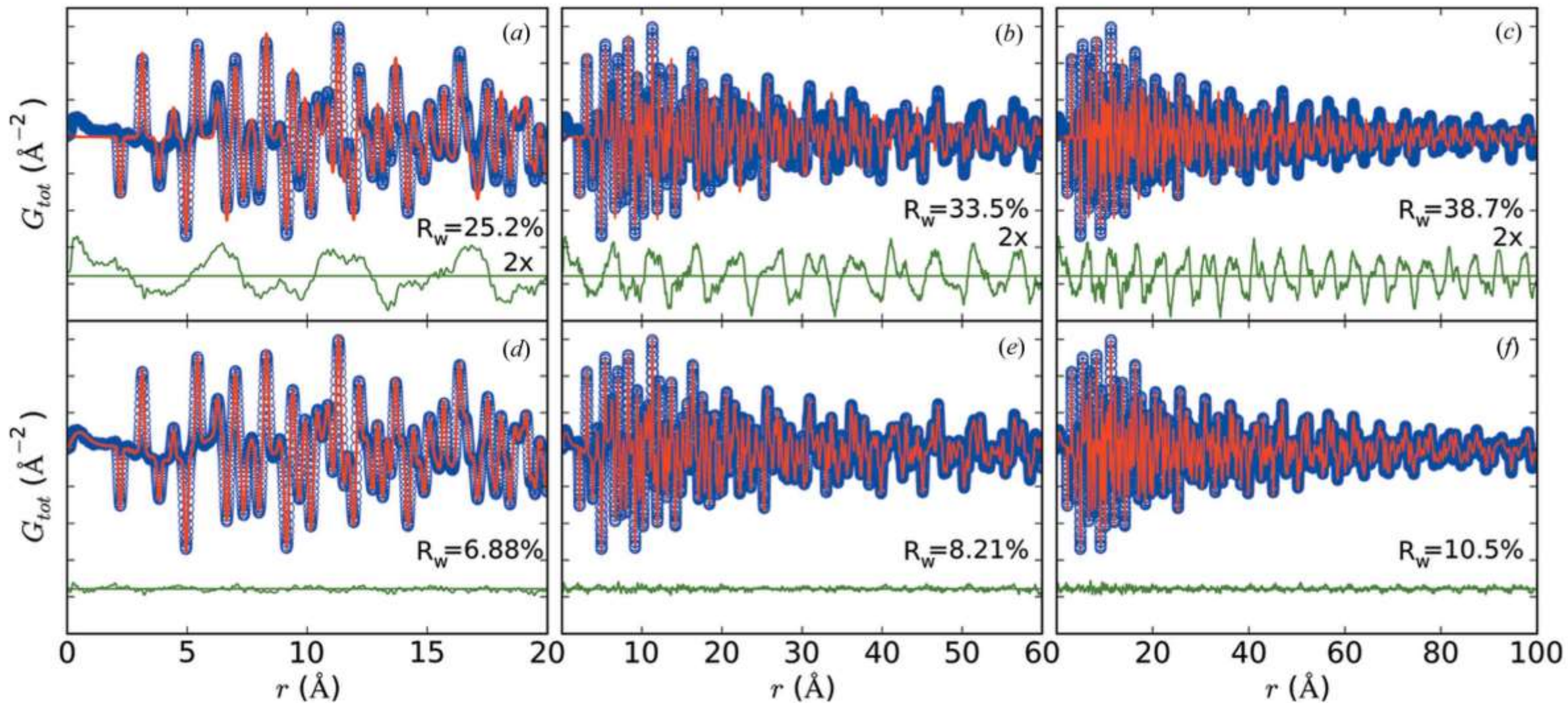
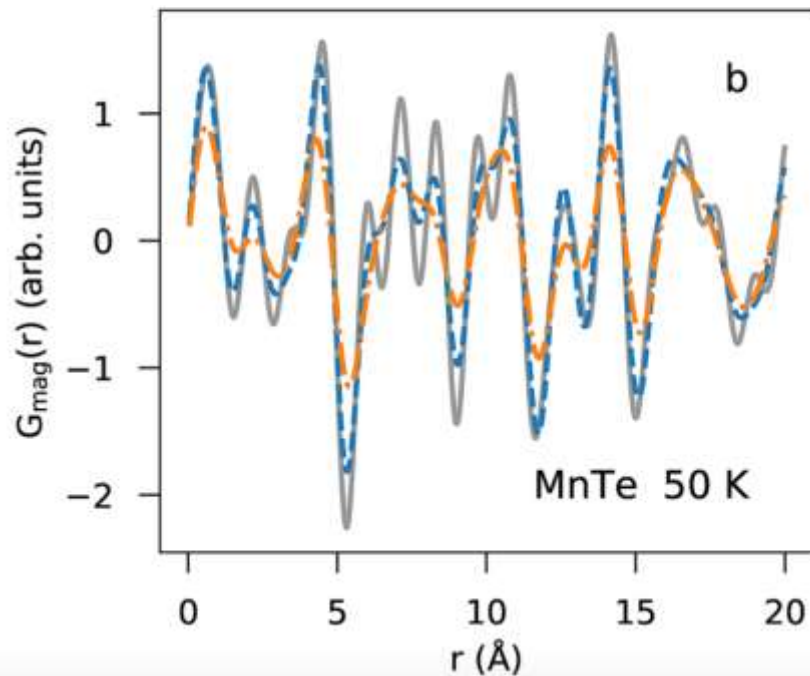
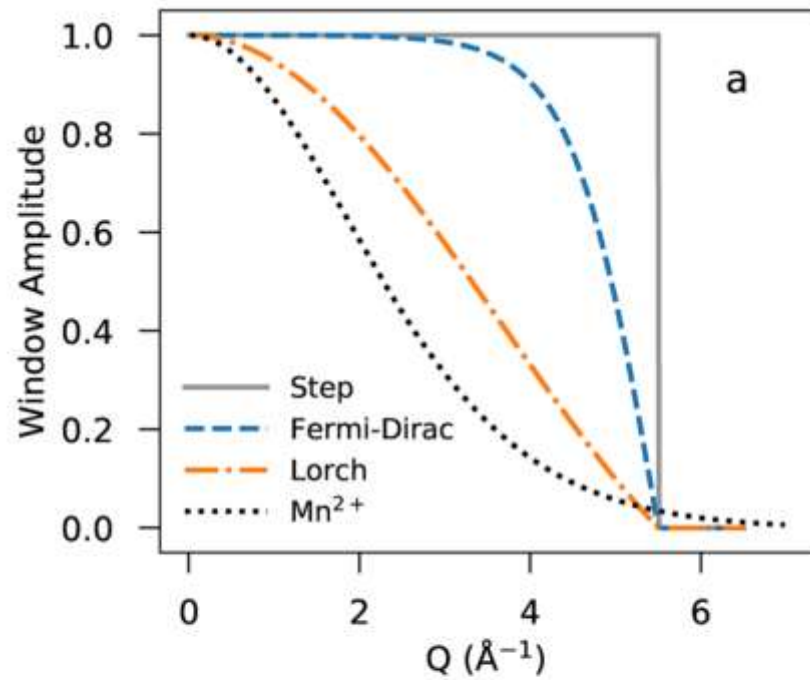


Figure 7 [Save the figure](#)
 Atomic and magnetic PDF refinements of the monoclinic model for three fitting ranges. (a)–(c) Refinement results when only the atomic structure is included in the model. The mPDF $d(r)$ is seen in the difference curves, which are multiplied by two for clarity. (d)–(f) Results of co-refinements of the atomic and magnetic PDFs. In all panels, the blue curve is the experimental data, the red curve is the calculated pattern, and the offset green curve is the difference.



step function

$$w_s(Q) = \begin{cases} 1 & \text{if } Q \leq Q_{\text{max}}, \\ 0 & \text{if } Q > Q_{\text{max}}, \end{cases} \quad (6)$$

a modified Fermi-Dirac function

$$W_{\text{FD}}(Q) = \begin{cases} \frac{2}{e^{(Q-Q_{\text{max}})/\Delta} + 1} - 1 & \text{if } Q \leq Q_{\text{max}}, \\ 0 & \text{if } Q > Q_{\text{max}}, \end{cases} \quad (7)$$

and the conventional Lorch function³³

$$w_L(Q) = \begin{cases} \frac{Q_{\text{max}}}{\pi Q} \sin\left(\frac{\pi Q}{Q_{\text{max}}}\right) & \text{if } Q \leq Q_{\text{max}}, \\ 0 & \text{if } Q > Q_{\text{max}}. \end{cases} \quad (8)$$

B. Frandsen, et al., *J Appl. Phys.*, 2022, 132, 22.



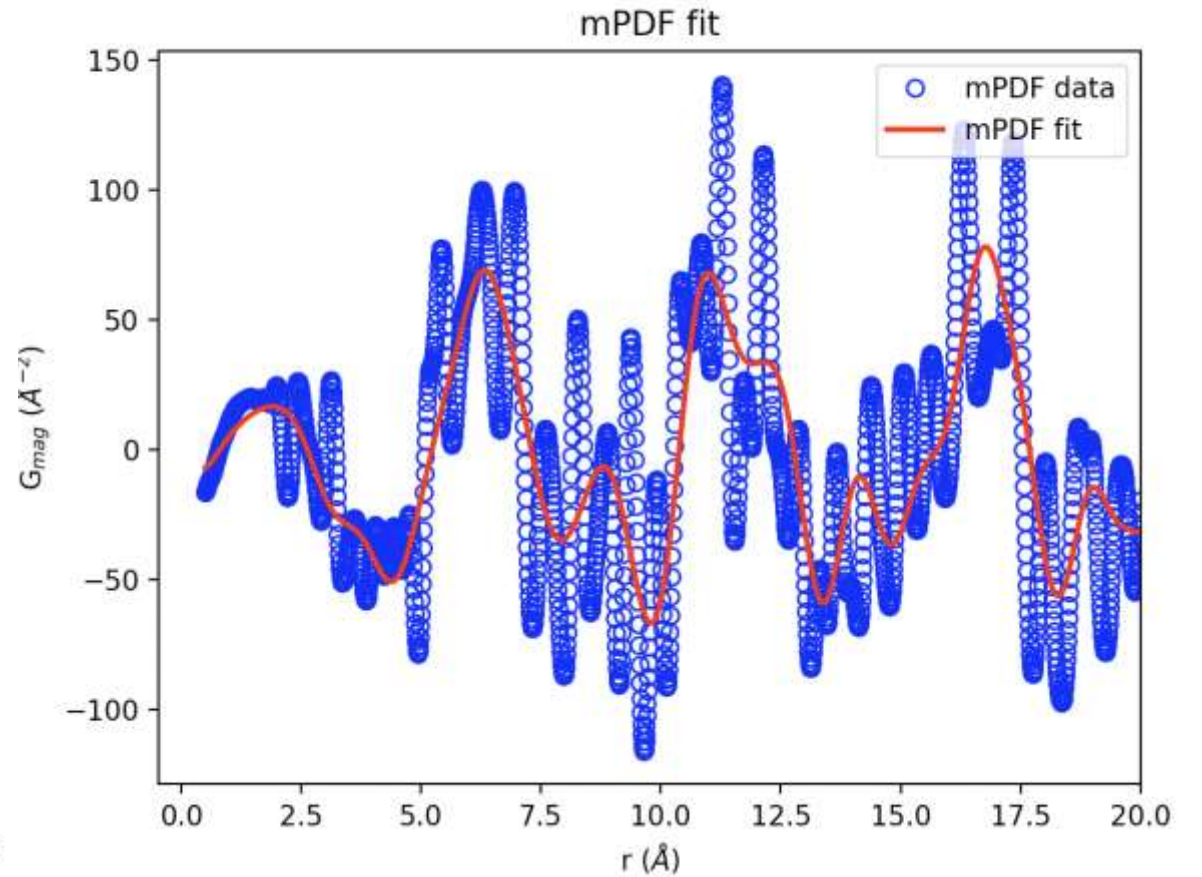
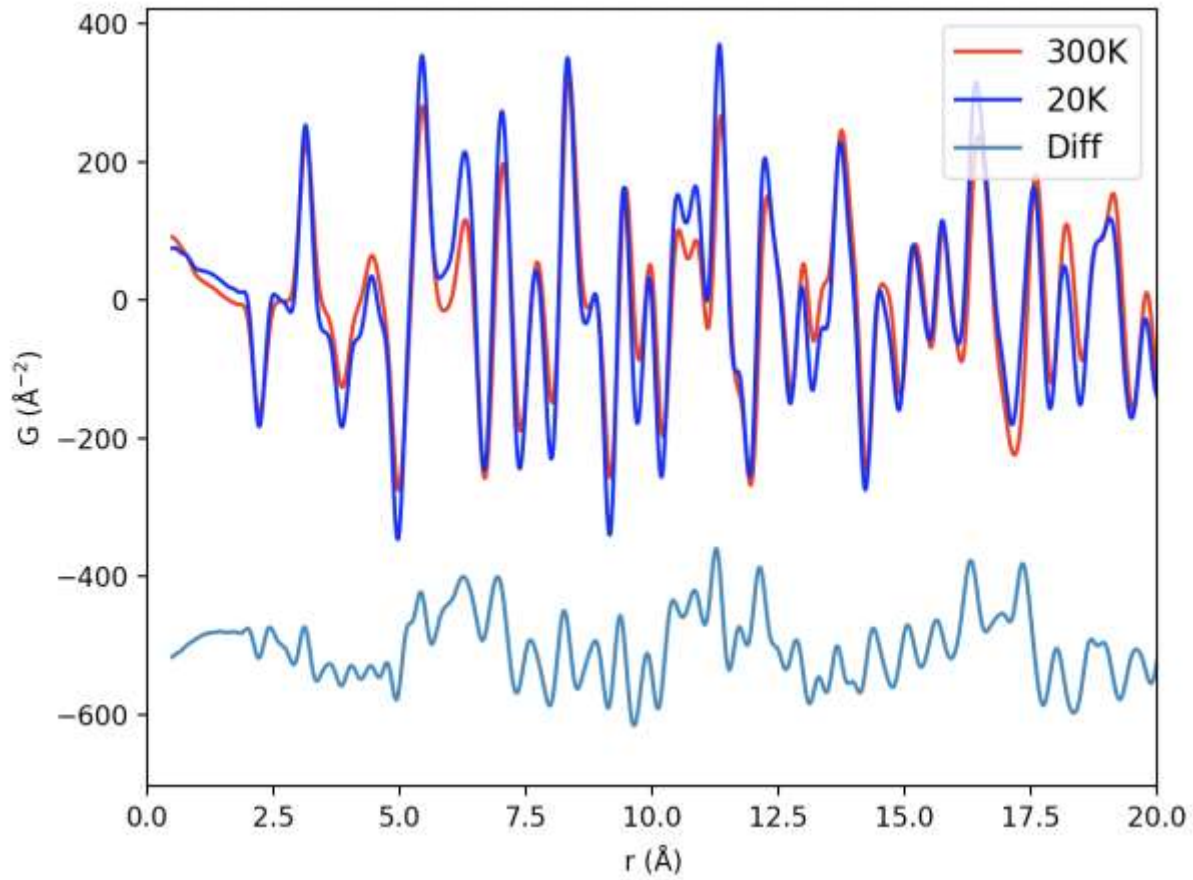
Co-refinement v.s. Toggle

| 20 K | no mPDF fit | Co-refine mPDF fit | Toggle mPDF fit |
|---|---------------|--------------------|-----------------|
| a (Å) | 3.161 | 3.161 | 3.161 |
| c (Å) | 7.628 | 7.619 | 7.618 |
| U_{Mn11} (Å²) | 0.0050 | 0.0056 | 0.0056 |
| U_{Mn33} (Å²) | 0.0044 | 0.0066 | 0.0064 |
| U_{O11} (Å²) | 0.0061 | 0.0060 | 0.0051 |
| U_{O33} (Å²) | 0.0057 | 0.0063 | 0.0066 |
| R_w | 0.30 | 0.086 | 0.085 |

- **Co-refinement and toggle algorithms perform similarly well.**



MnO 20K-300K



$Q_{\text{max}}=25 \text{\AA}^{-1}$, Use Lorch



$$G_{\text{tot}}(r) = \mathcal{F} \left\{ Q \left(\frac{I_n}{N_a \langle b \rangle^2} - \frac{\langle b^2 \rangle}{\langle b \rangle^2} \right) \right\} + \mathcal{F} \left\{ Q \frac{I_m}{N_a \langle b \rangle^2} \right\}$$
$$= G_n(r) + d(r)/N_a \langle b \rangle^2,$$

where $G_n(r)$ is the atomic (nuclear) PDF and $d(r) = \mathcal{F}\{QI_m(Q)\}$ is a quantity that we will call the ‘unnormalized mPDF’, since it does not involve division by the magnetic form factor $f_m(Q)$. A straightforward application of the convolution theorem reveals that

$$d(r) = C_1 \times f(r) * S(r) + C_2 \times \frac{dS}{dr},$$

where C_1 and C_2 are constants related by $C_1/C_2 = -1/(2\pi)^{1/2}$ in the fully ordered state, $*$ represents the convolution operation, and $S(r) = \mathcal{F}\{f_m(Q)\} * \mathcal{F}\{f_m(Q)\}$. The quantity $\mathcal{F}\{f_m(Q)\}$ is closely related to the real-space spin density. Roughly speaking, $d(r)$ is equivalent to the proper mPDF $f(r)$ twice broadened by the spin density with an additional peak at low r produced by the derivative term in equation (6). The two