Studies on 3D spiral injection scheme for compact storage rings

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Presentation based on these Reference:

1. M. A. Rehman, “A Validation Study on the Novel Three Dimensional Spiral Injection Scheme with the Electron Beam for Muon $g-2$/EDM Experiment”, PhD. thesis, SOKENDAI, 2020, http://id.nii.ac.jp/1013/00006023/
2. M. A. Rehman et al., “The First Trial of XY-Coupled Beam Phase Space Matching for Three-Dimensional Spiral Injection”, doi:10.18429/ JACoW-IPAC2021-MOPAB16
3. M. A. Rehman et al ., 2019 J. Phys.: Conf. Ser. 1350 012151

I performed these studies were performed during 2015 to 2020 at KEK/Tsukuba.

Content

- Introduction
  - Three-Dimensional Spiral Injection Scheme
- Spiral Injection Test Experiment (SITE) at KEK
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- Phase Space Matching
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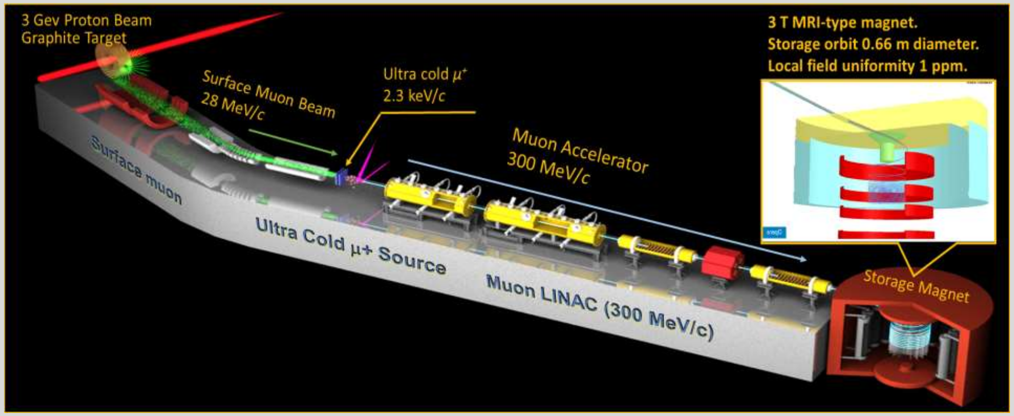
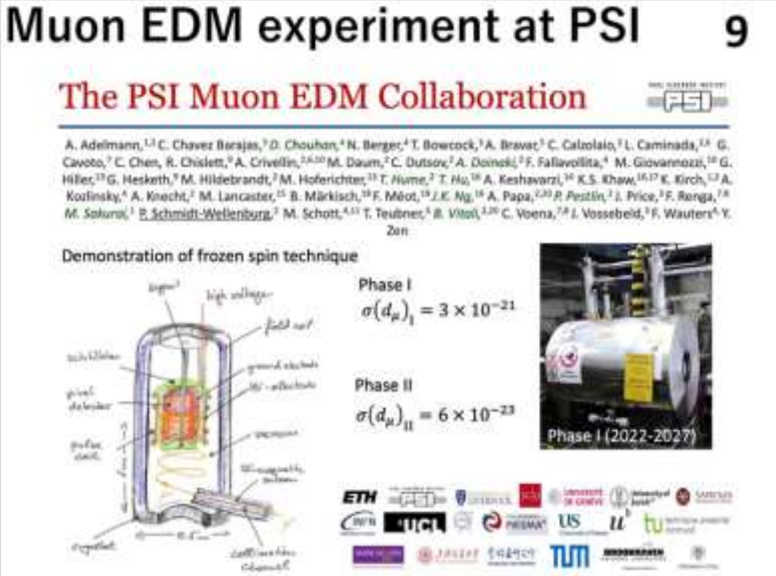
- Summary/Conclusion

Why inject beam spirally?

- **There is a growing interest of compact muon storage scheme for ultra precise measurement of muon g-2 and EDM.**
- **In order to overcome the challenge of low injection efficiency with conventional scheme a new injection scheme was proposed for J-PARC muon g-2/EDM experiment**

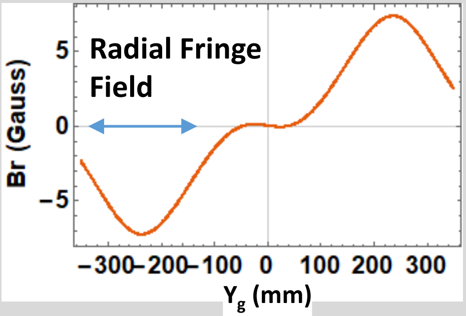


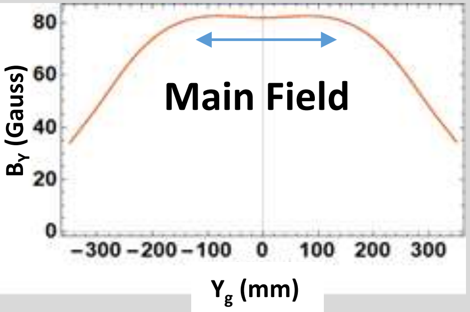
https://doi.org/10.1093/ptep/ptz030

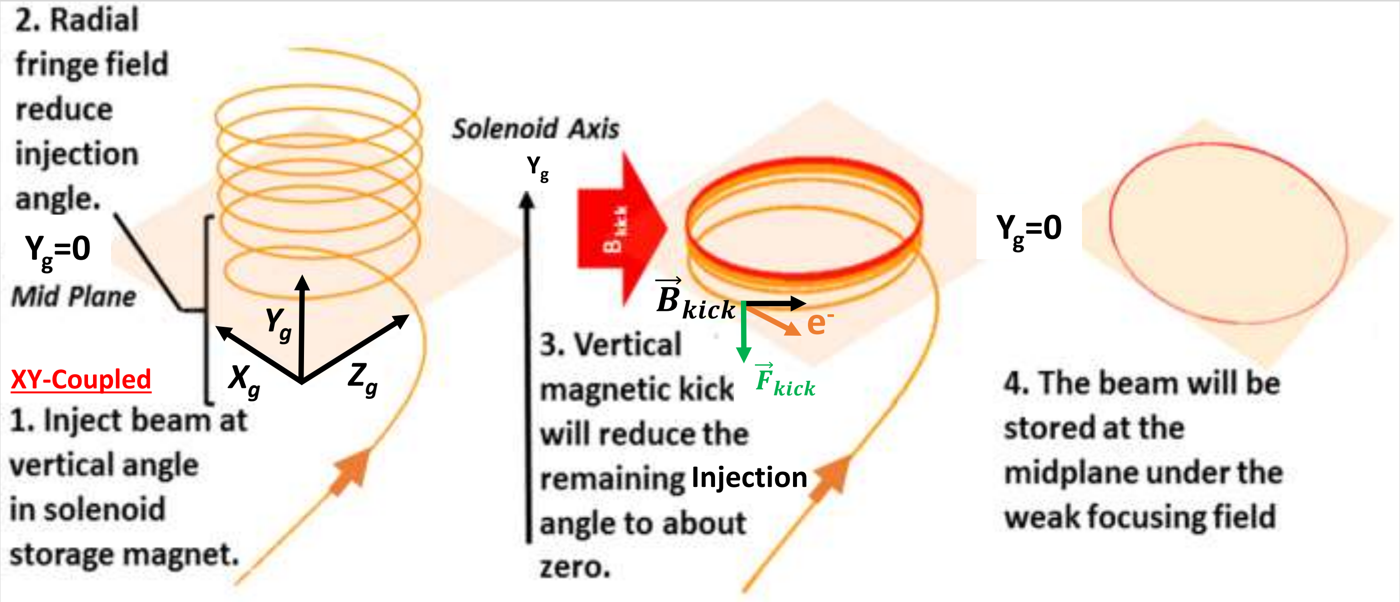


How to inject beam spirally?

To resolve technical challenges a new 3D Spiral Injection scheme has been invented







The Elegance and Advantages

- Smooth connection between injection and storage sections: No need of Inflector
- All in one storage magnet, reduce source of error fields
- No need to kick within a single turn: Relax Kicker Requirements

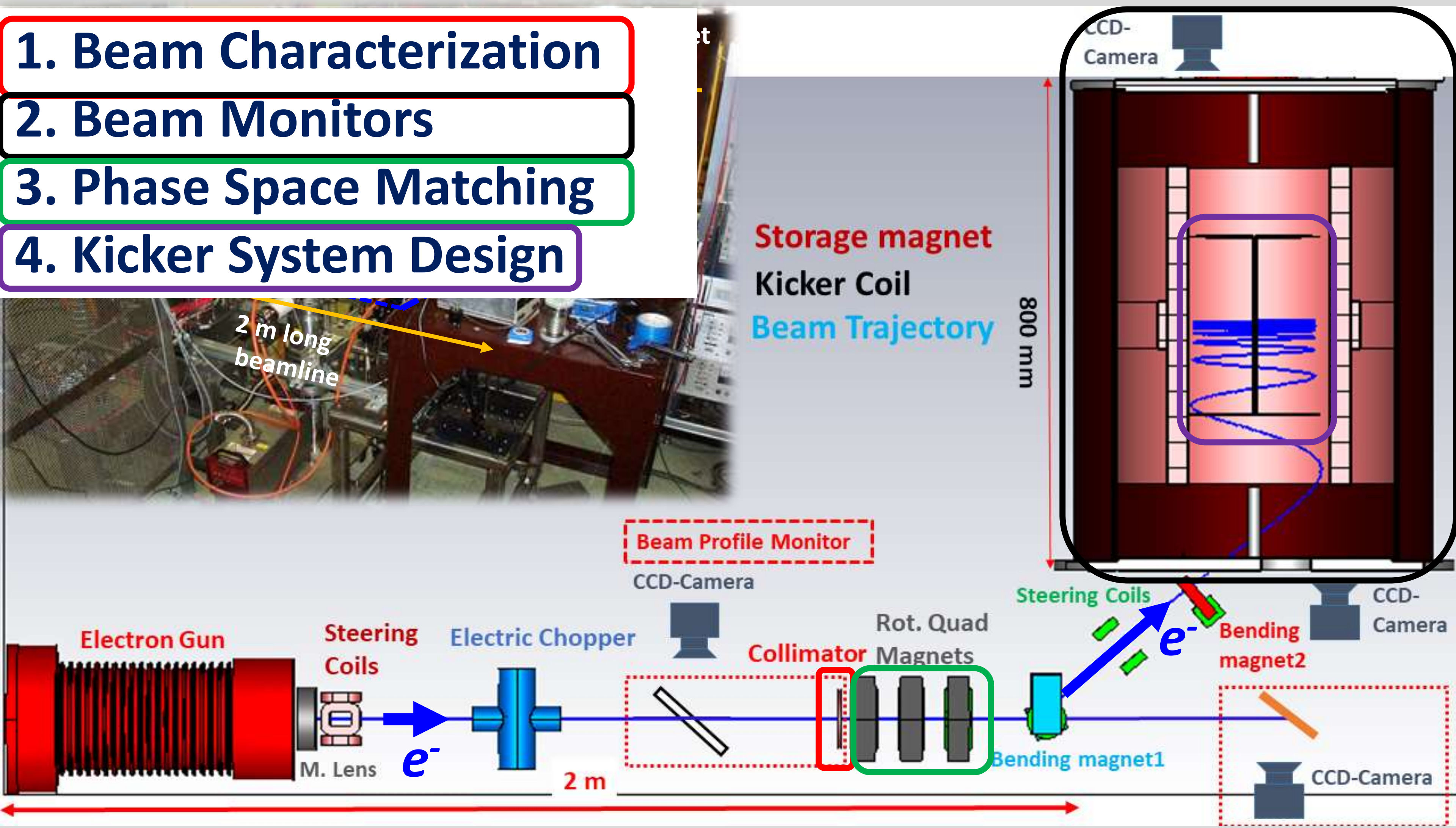
However, Unprecedented

Therefore, it was indispensable to prove the feasibility of this new scheme.

Spiral Injection Test Experiment (SITE)

Spiral Injection Test Experiment Setup at KEK Tsukuba Campus

1. Beam Characterization
2. Beam Monitors
3. Phase Space Matching
4. Kicker System Design

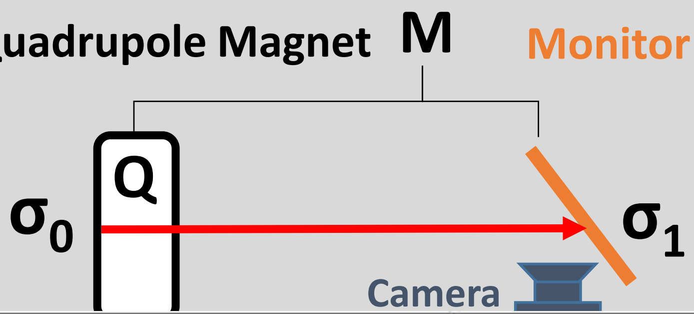


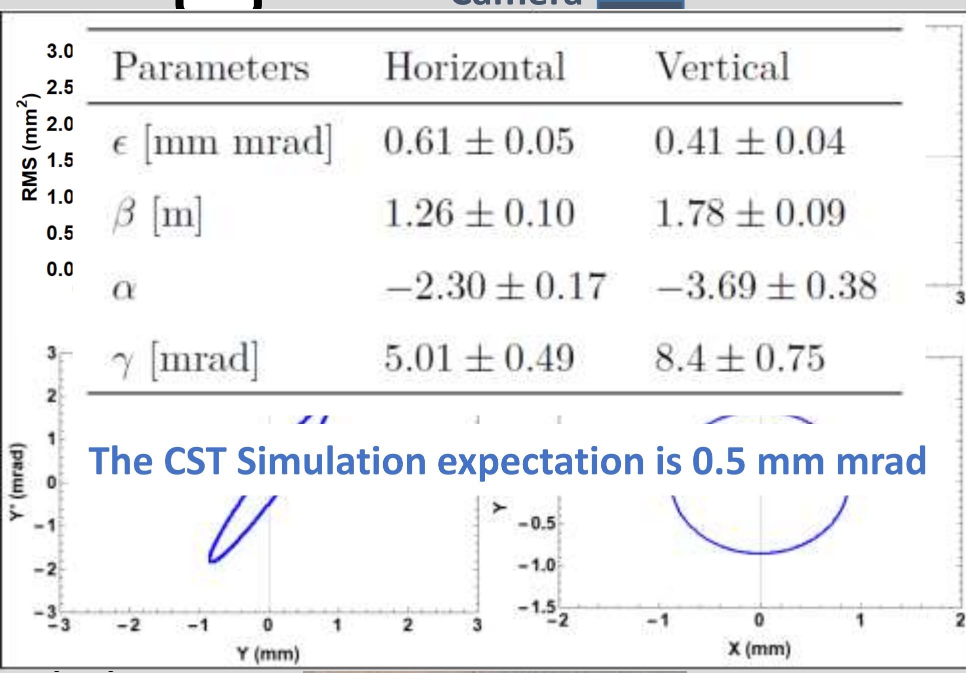
1. Beam Characterization

To design a beamline → Knowledge of beam phase space is necessary

Quadrupole scan

To estimate beam emittance



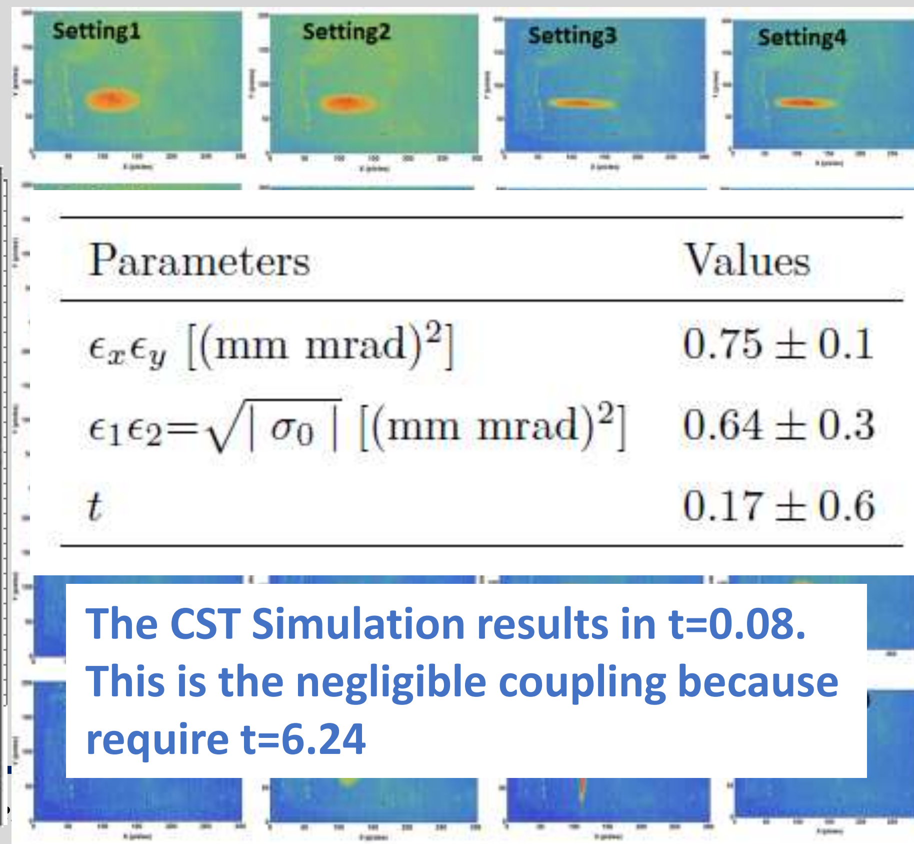


Parameters	Horizontal	Vertical
ϵ [mm mrad]	0.61 ± 0.05	0.41 ± 0.04
β [m]	1.26 ± 0.10	1.78 ± 0.09
α	-2.30 ± 0.17	-3.69 ± 0.38
γ [mrad]	5.01 ± 0.49	8.4 ± 0.75

The CST Simulation expectation is 0.5 mm mrad

Coupling Measurement

- Twenty measurements were taken for this evaluation
- A method of least square was used to solve system of overdetermined equation



Parameters	Values
$\epsilon_x \epsilon_y$ [(mm mrad)^2]	0.75 ± 0.1
$\epsilon_1 \epsilon_2 = \sqrt{\mid \sigma_0 \mid}$ [(mm mrad)^2]	0.64 ± 0.3
t	0.17 ± 0.6

The CST Simulation results in t=0.08. This is the negligible coupling because require t=6.24

2. Beam Monitors for Storage Magnet

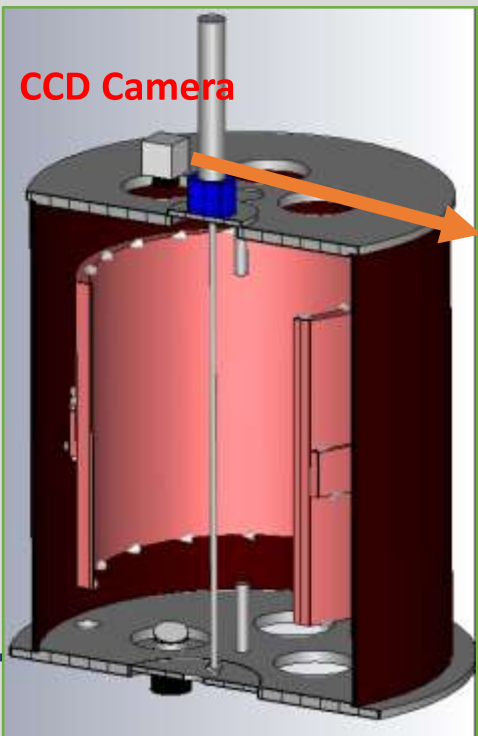
Gas Monito

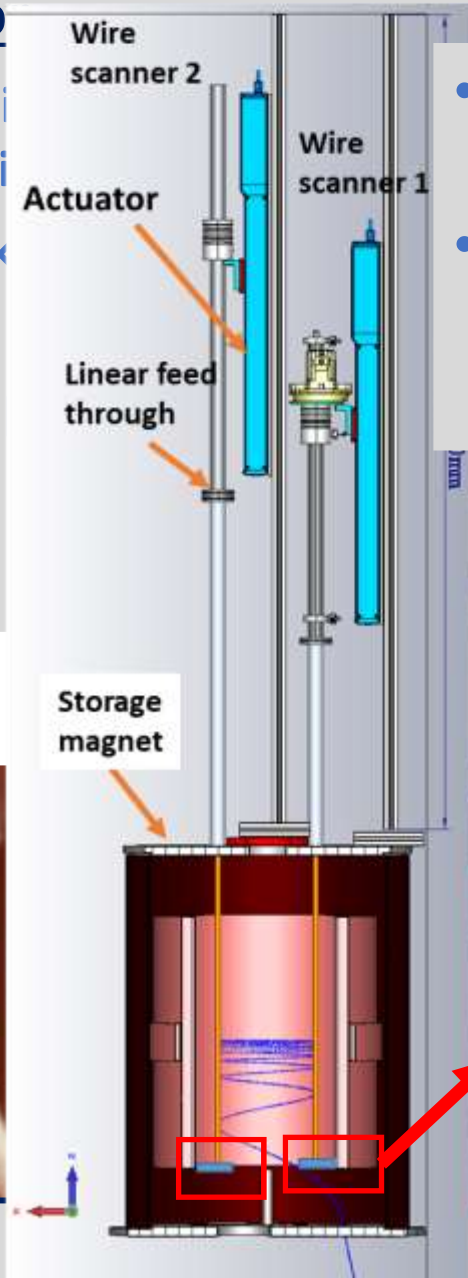
Observe fluorescent li
the beam and N_2 gas i

Gas Pressure: 1.5×

Beam Current: 50

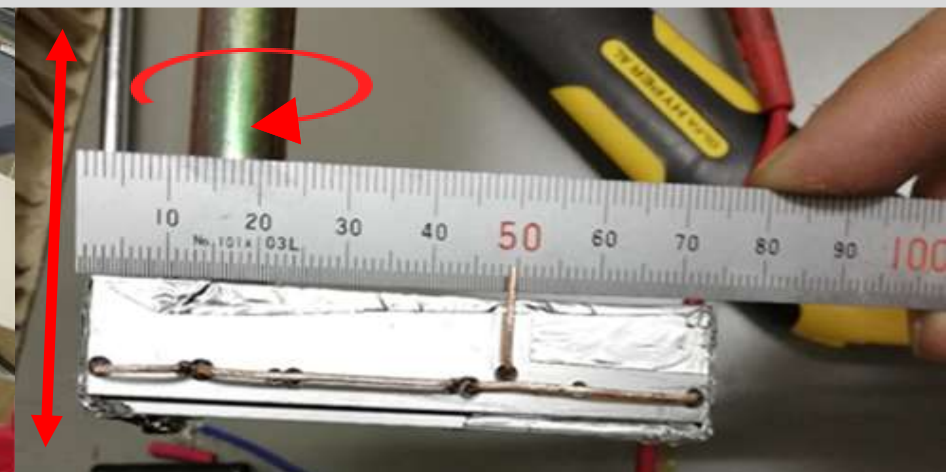
Black Sheet: To su
reflection

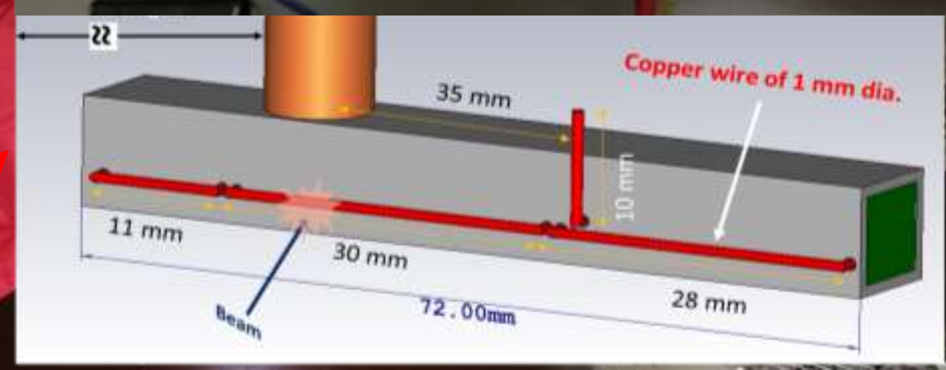




Driving and support system of wire scanner

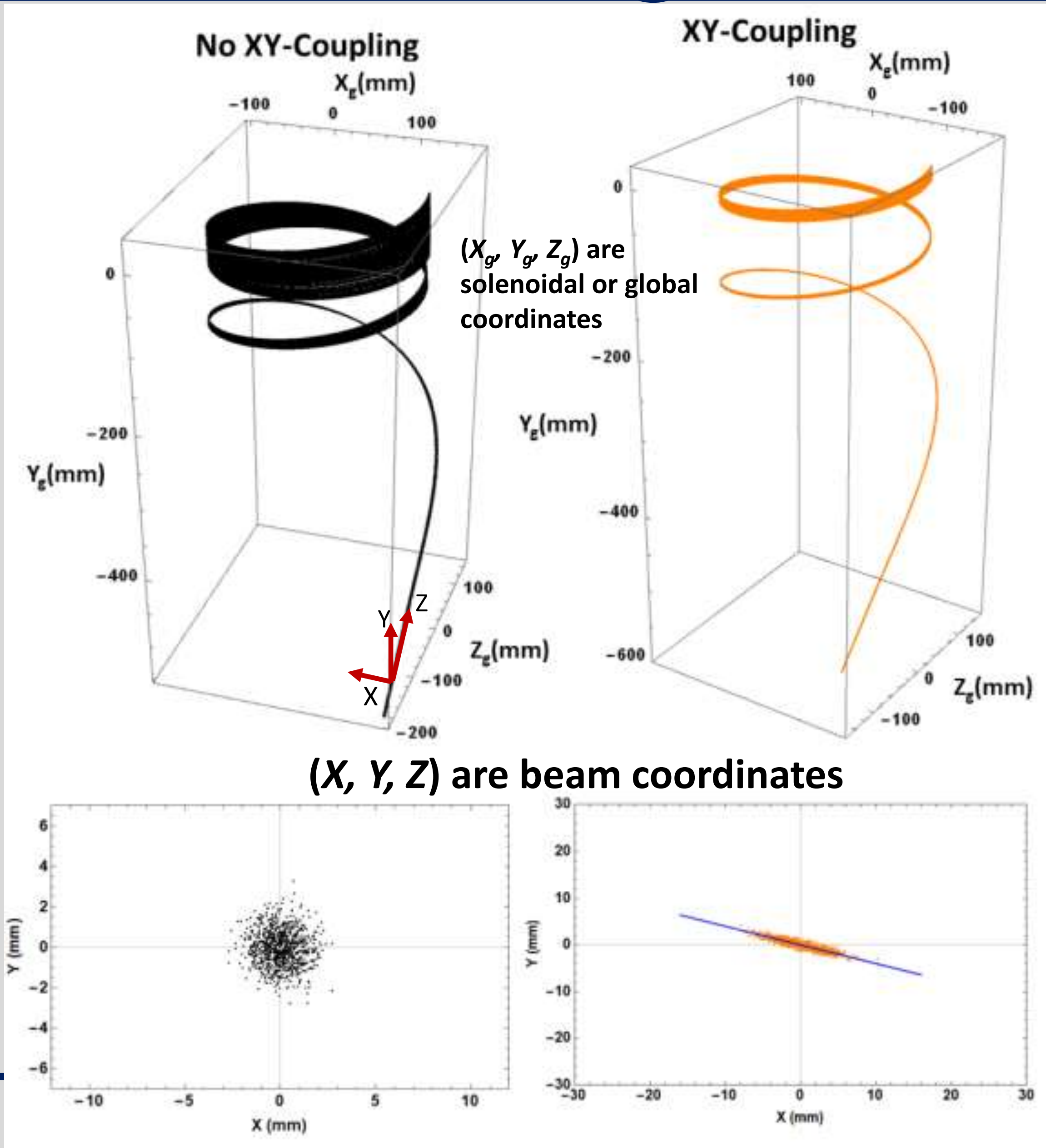
- The beam hits the wire and deposits a corresponding current.
- The voltage across a resistor is proportional to instantaneous current absorbed in the wire





3. Phase Space Matching for SITE

- Due to the axial symmetric field of the solenoid magnet an appropriate XY coupled beam is required
- Moreover, particles at different vertical positions face different radial field and eventually vertical blow-up
- Phase space matching is essential to avoid vertical blow-up



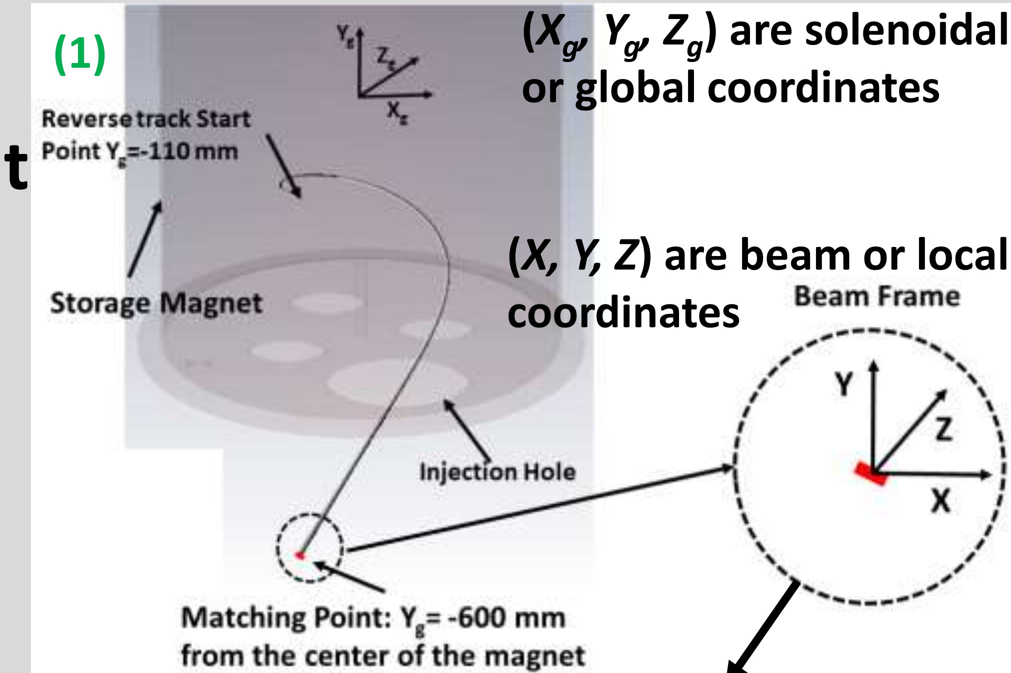
A method to calculate XY-Coupling requirements

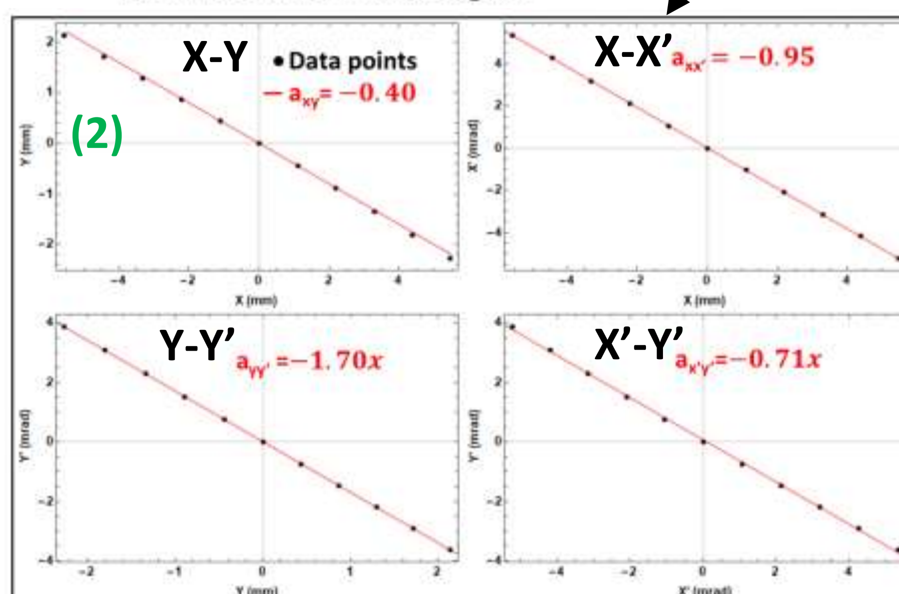
1. Consider a flat (radially) distribution at the kick point and reverse track it to the matching point (outside of the storage magnet)

2. Calculate slopes of beam phase space correlations at the matching point.

(X, Y, Z) are the beam or local coordinates

$$X' = \frac{P_x}{|P|} \quad Y' = \frac{P_y}{|P|}$$





A method to calculate XY-Coupling requirements

3. Apply transfer matrix consists of coupling parameters (R1,R2,R3,R4) and Twiss parameters to input phase space

$$X = MX_0 \qquad M = U_{out}^{-1} D U_{in}^{-1}$$

Twiss Parameters

$$\begin{pmatrix} D_x & 0 \\ 0 & D_y \end{pmatrix}$$

Coupling Parameters

$$U_{out} = \begin{pmatrix} \mu & 0 & -R_4 & R_2 \\ 0 & \mu & R_3 & -R_1 \\ R_1 & R_2 & \mu & 0 \\ R_3 & R_4 & 0 & \mu \end{pmatrix}$$

$U_{in}^{-1} = Identity$ in our case,

From Coupling Measurement. Slide#15

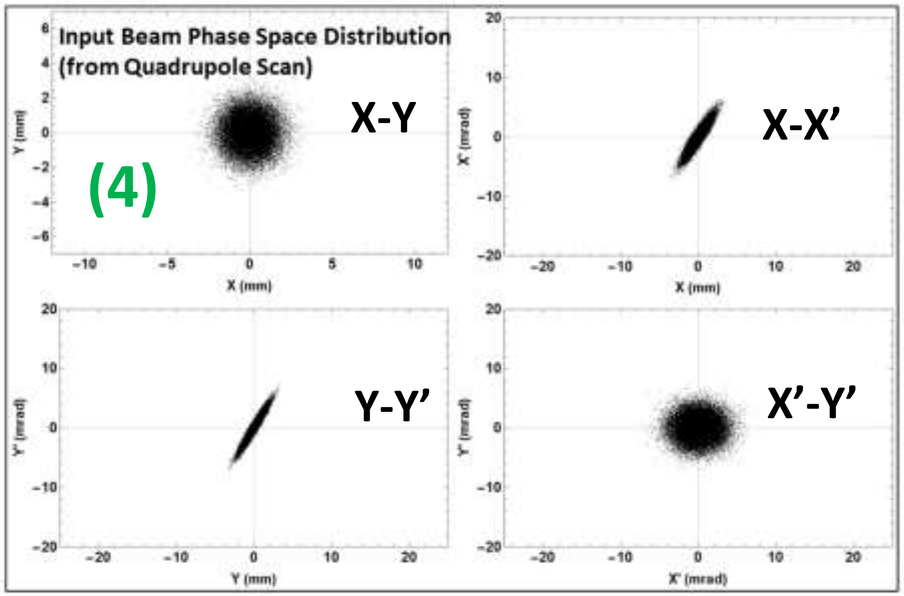
$$M = U_{out}^{-1} D$$

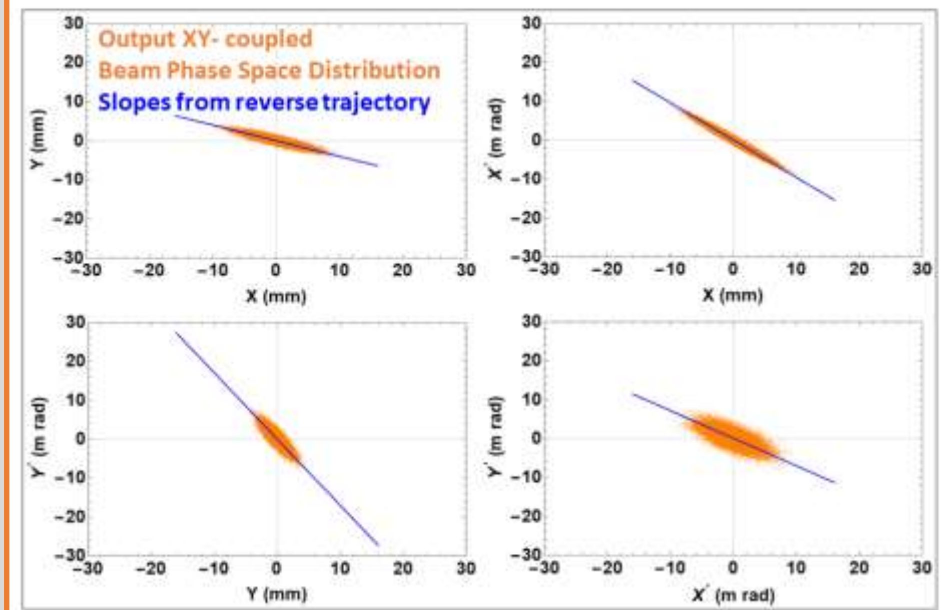
Non-zero values of (*R1, R2, R3, R4*) shows coupling

A method to calculate XY-Coupling requirements

4. Iteration of R1-R4 until slope of input distribution match required slope

$$X = MX_0$$





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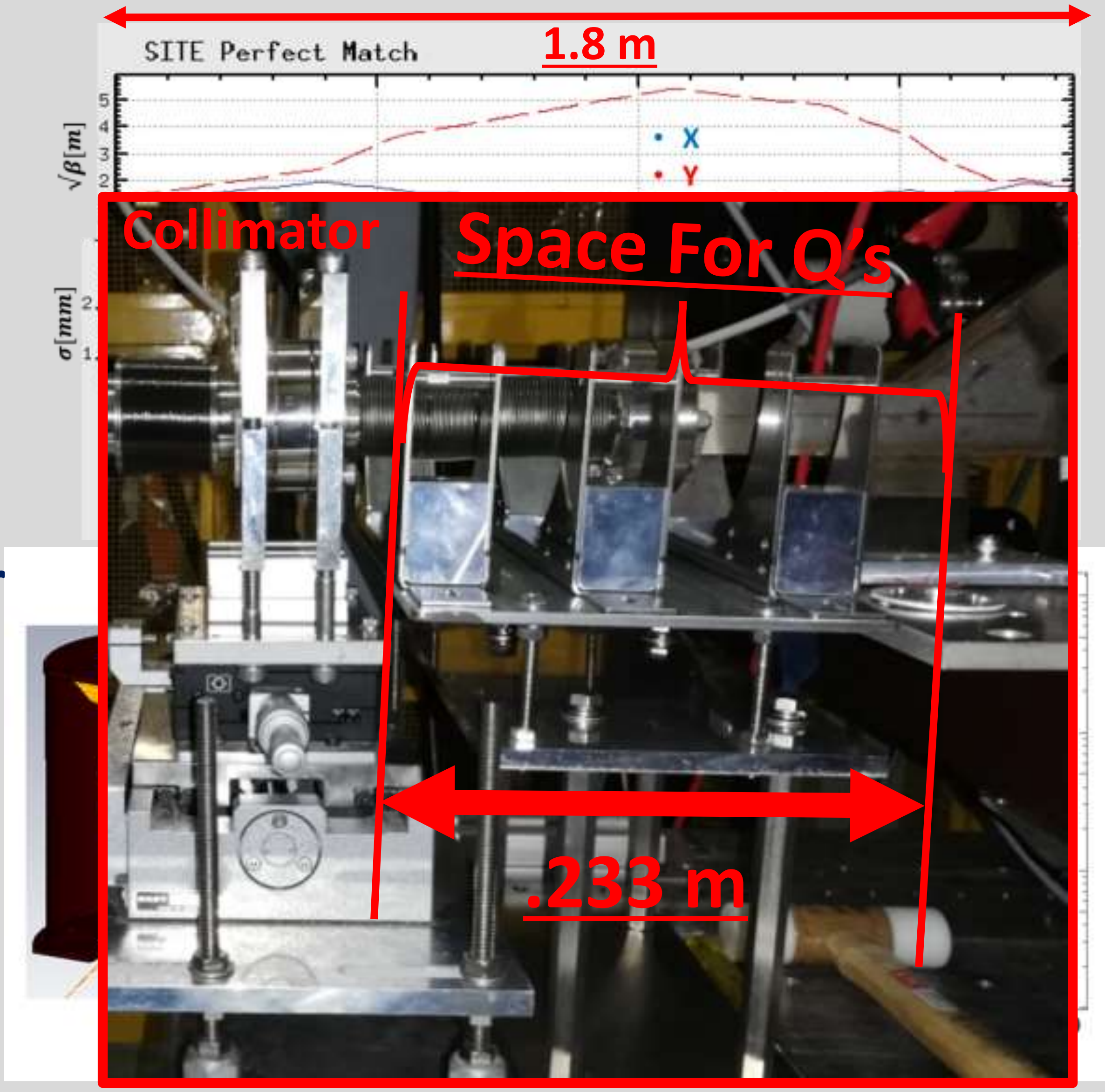
α	-2.30 ± 0.17	-3.69 ± 0.38
γ [mrad]	5.02 ± 0.49	8.4 ± 0.75
R_1, R_2, R_3, R_4	0	
$\sqrt{\lvert \sigma_{input} \rvert}$	0.25	

Parameters	Horizontal	Vertical
β_{out} [m]	3	3
α_{out}	3	5.19
$\sqrt{\lvert \sigma_{output} \rvert}$	0.25	
t	6.24	

Parameters	Values
R_1	-0.50
R_2	-0.60
R_3	-0.99
R_4	-1.09

Beamline for phase space matching

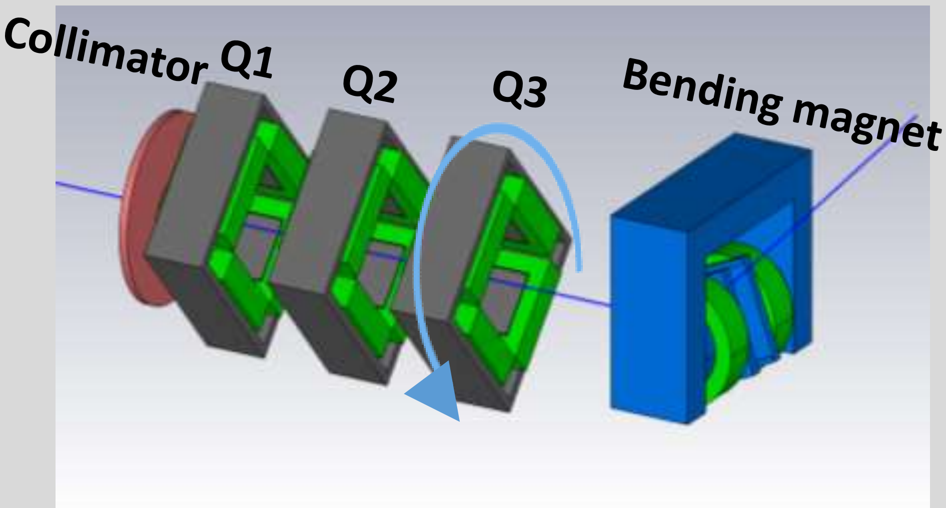
- **SAD program had been utilized to calculate beamline to produce required phase space for the injection**
- **At least Six Quadrupole magnets are required for the perfect matching**
- **Limited space on beamline 6-Q's cannot be placed**



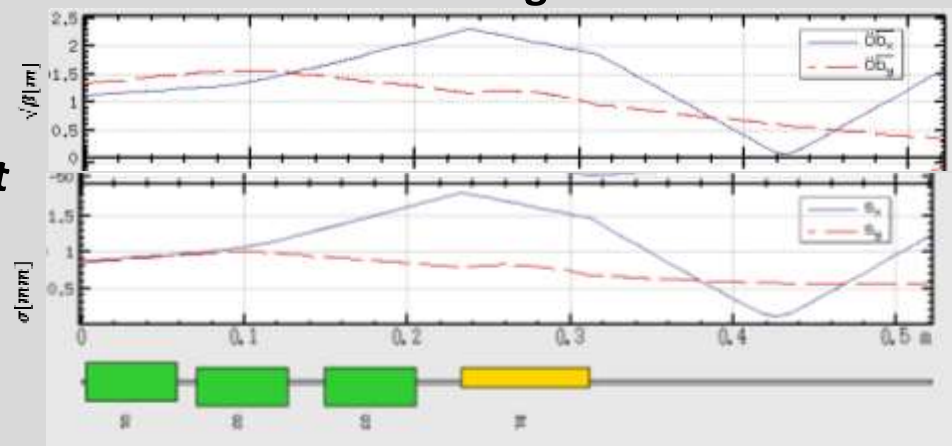
3. Phase Space Matching (Device)

Three rotatable quadrupole magnets can apply appropriate XY-coupling to reduce the blow-up of the beam in vertical direction

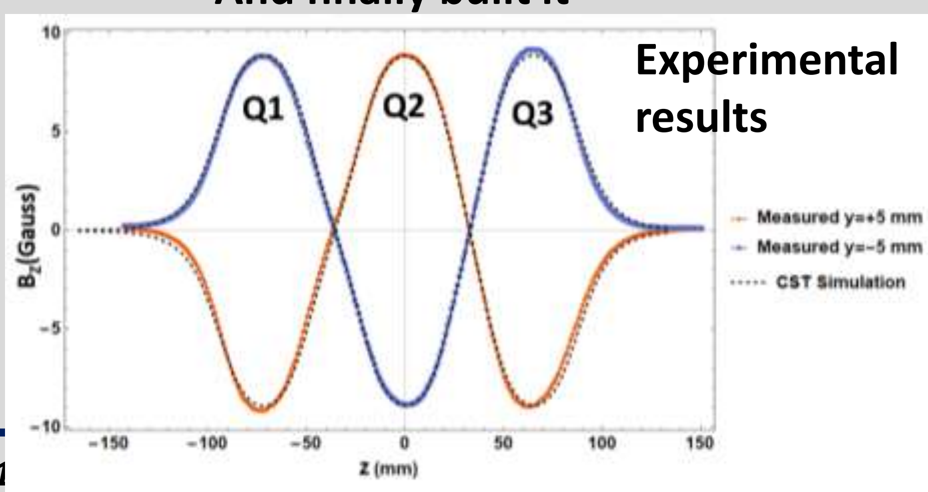
Design of Quadrupole Magnet in CST-EM

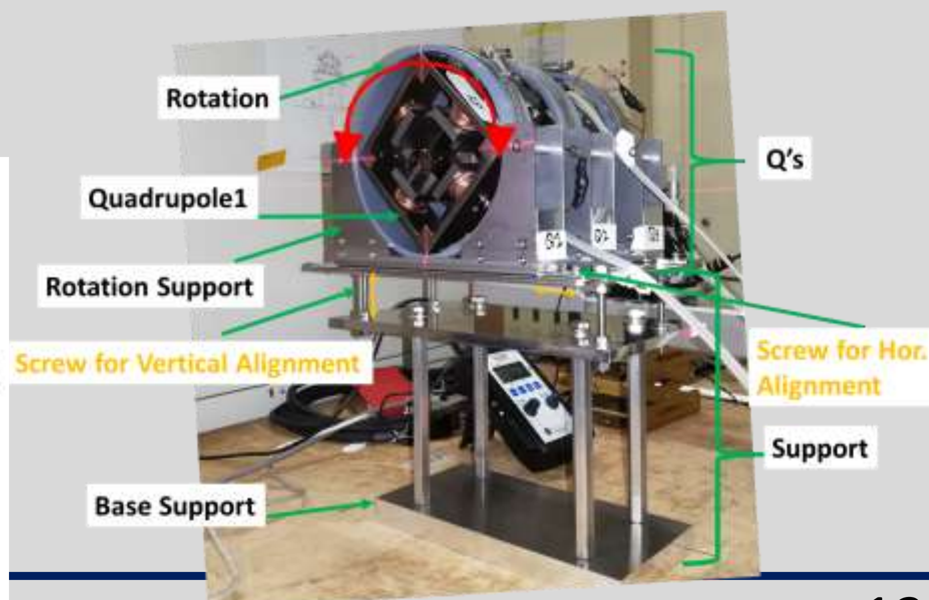


Tracking in SAD

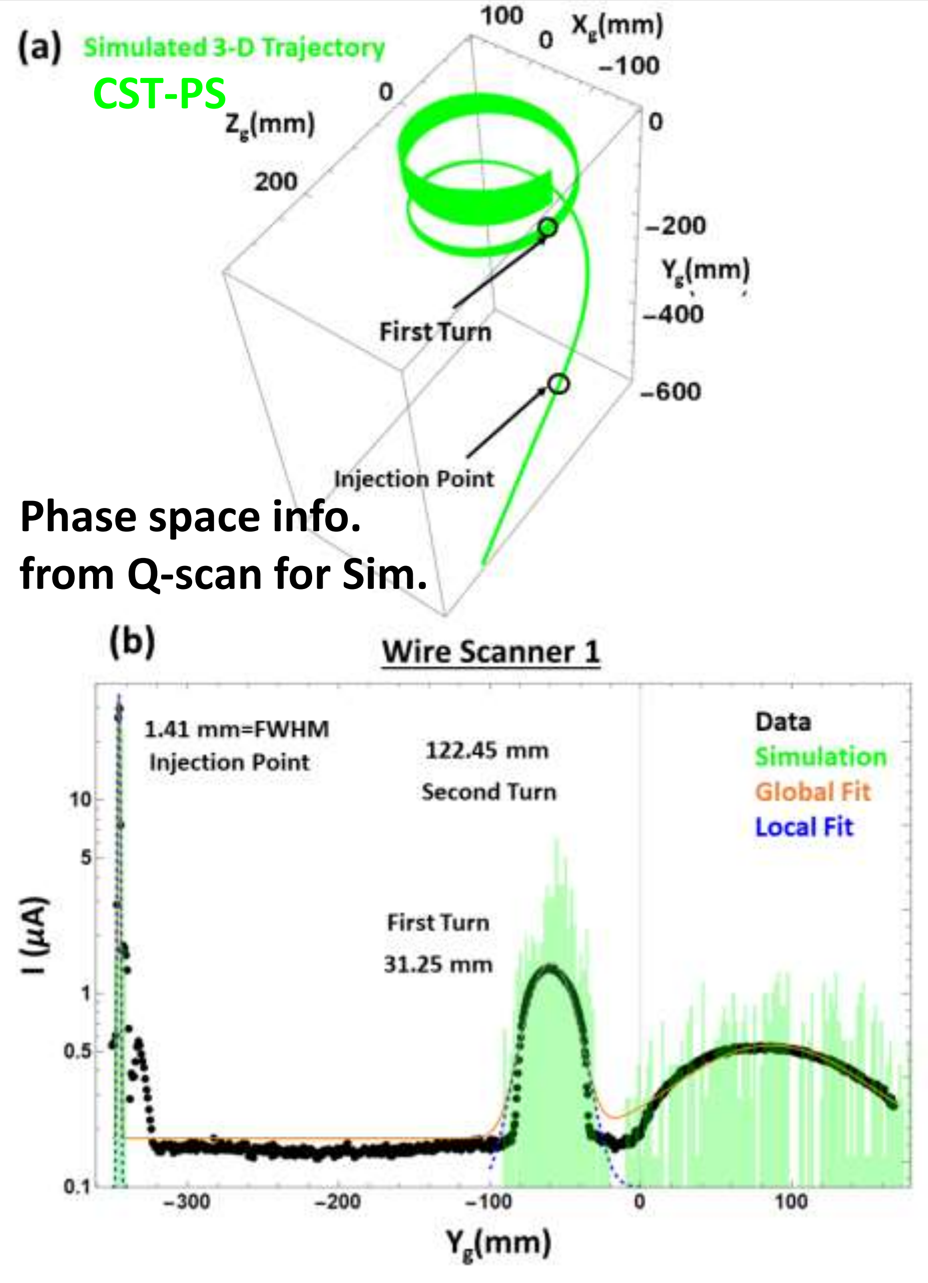


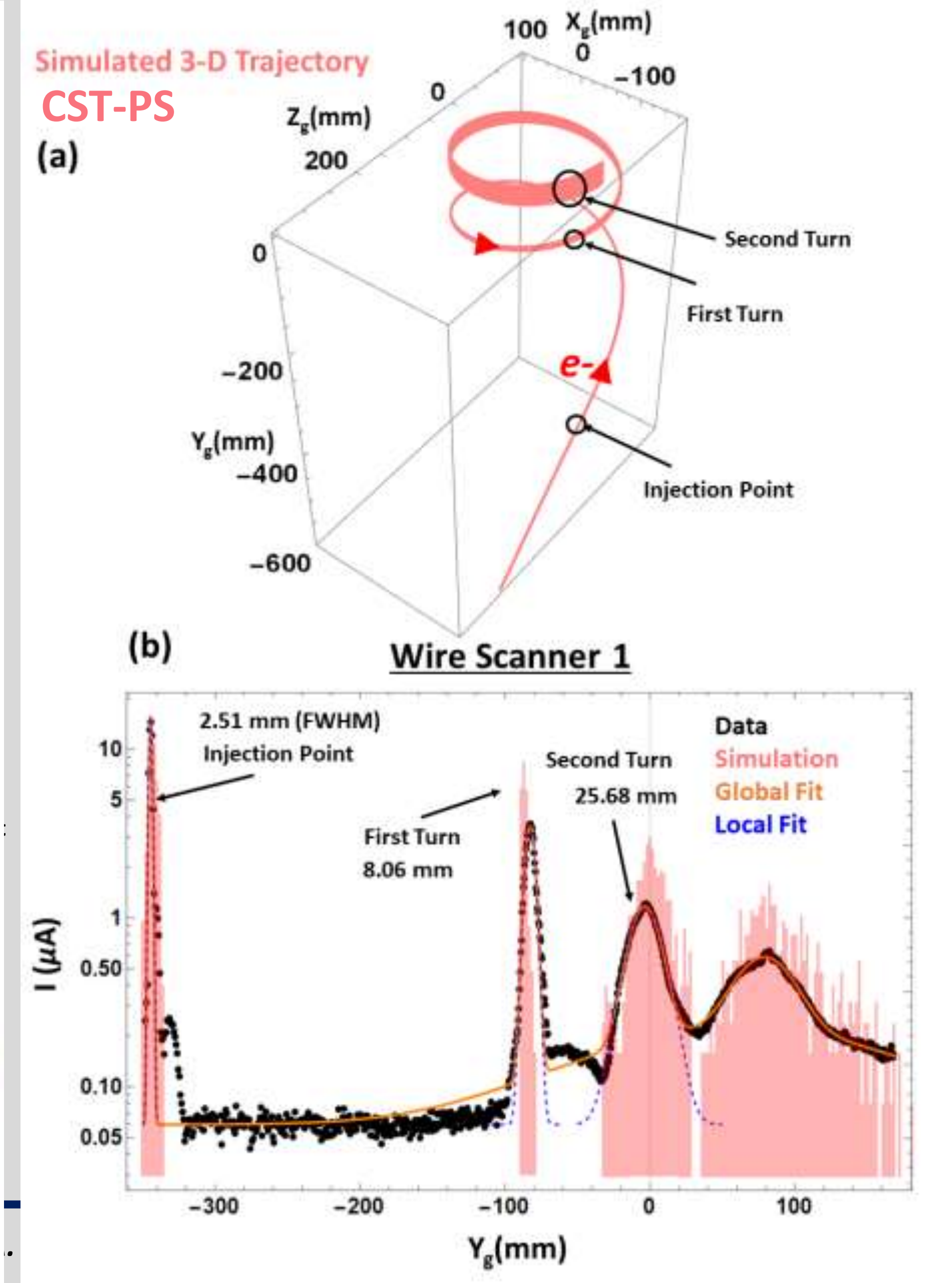
And finally built it





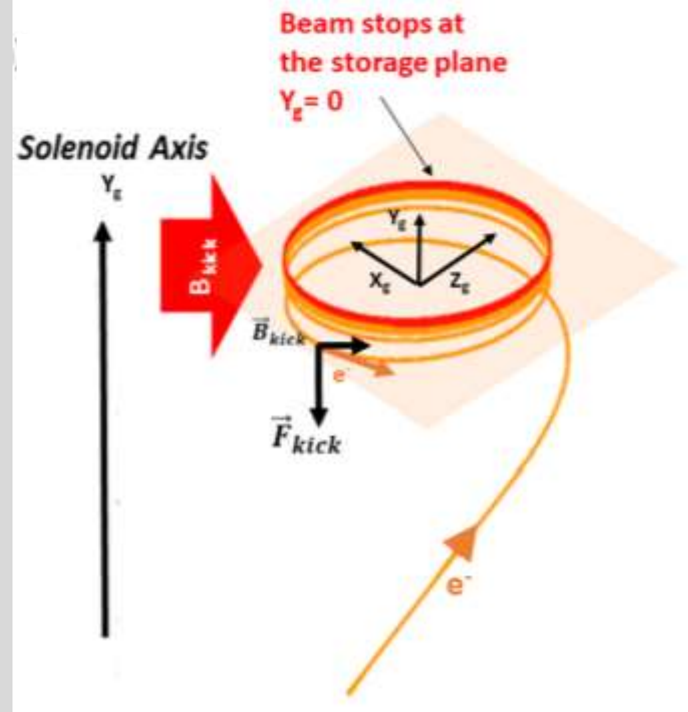
3. Phase Space Matching (Results)





Concept of Kicker

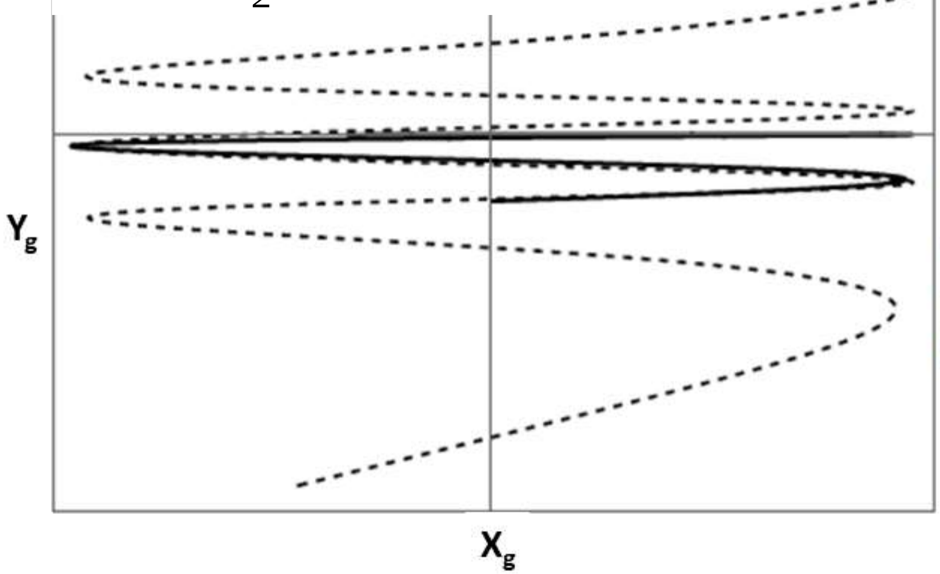
- The radial fringe field decrease the pitch angle of the beam. But do not decrease it to zero as it approach to the mid-plane of magnet
- A vertical kicker reduce the pitch angle to nearly zero as it reach to mid-plane of the

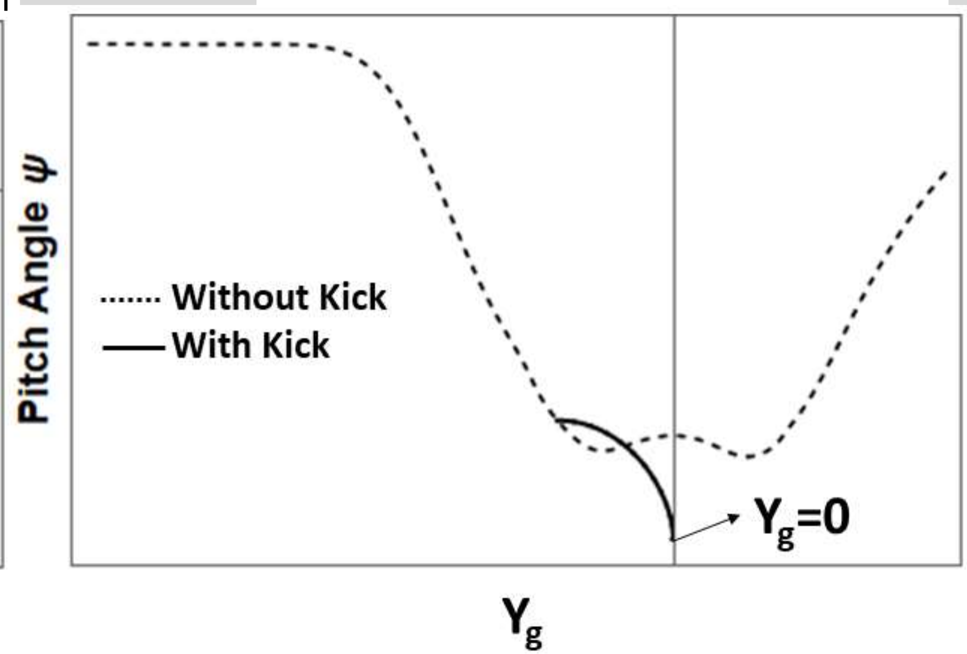


$$\vec{B}_{kick} = \vec{B}_{coil}.Sin\left(\frac{2\pi t}{T_{kick}}\right)$$

$$while, t < \frac{T_{kick}}{2}$$

$$pitch\ angle = \psi = \frac{P_y}{|P|}$$





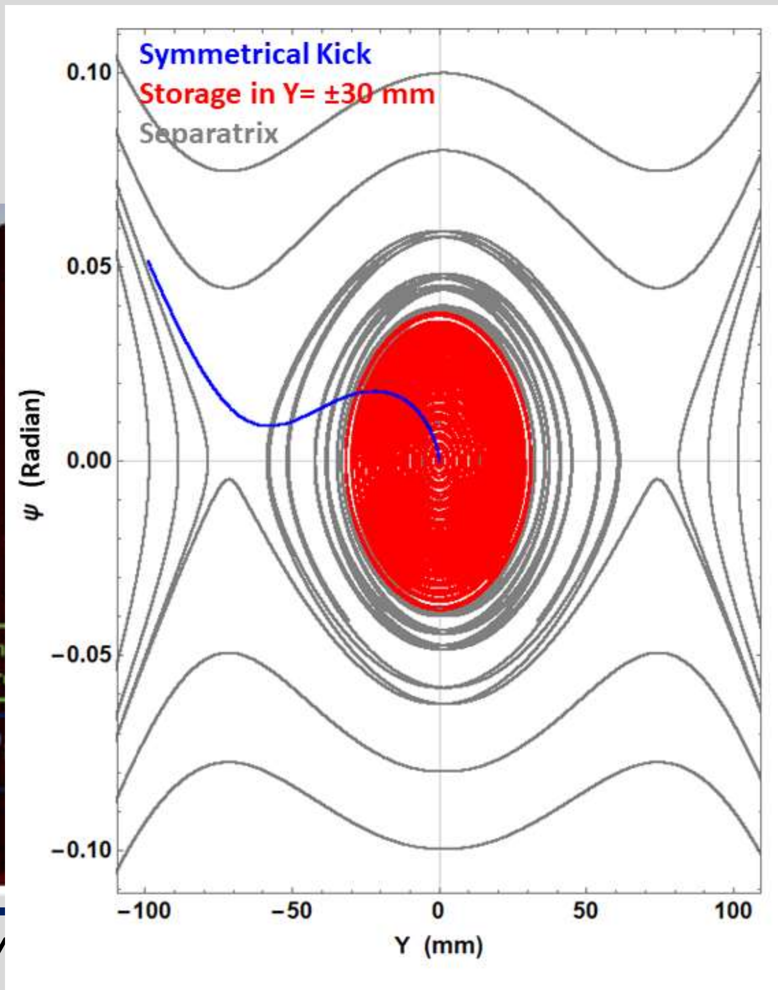
Kicker design and parameters

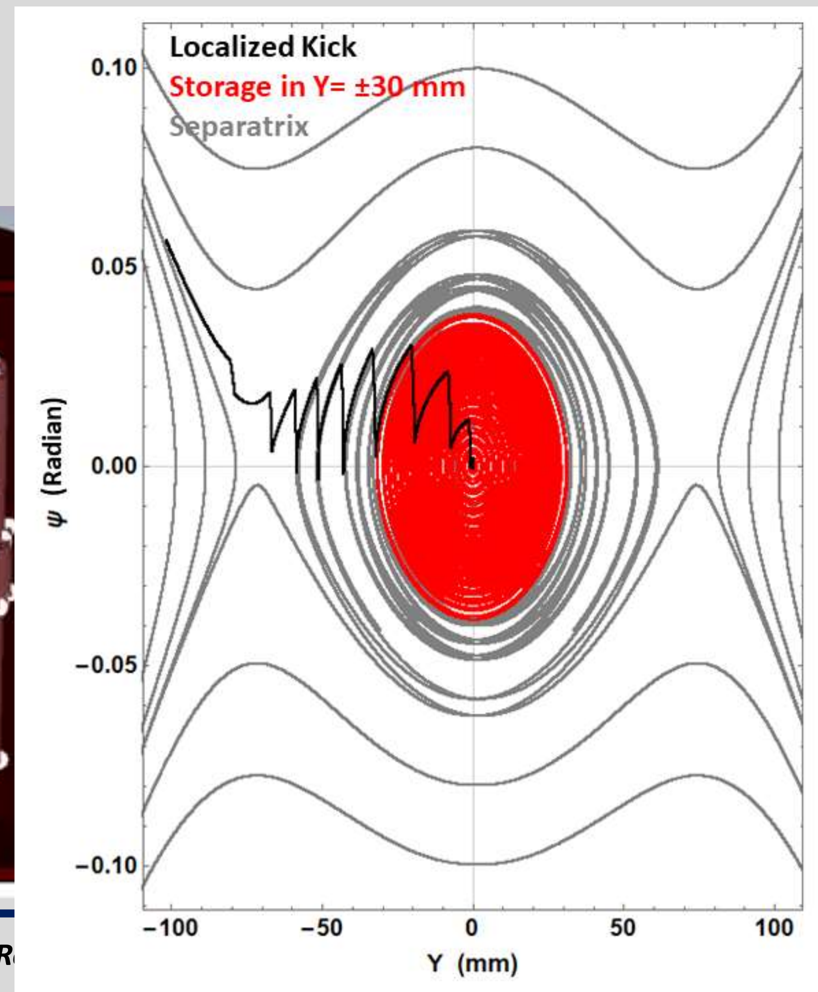
- **Two kinds of kicker has been**

Kicker parameters

Tracking in Kicker field

-



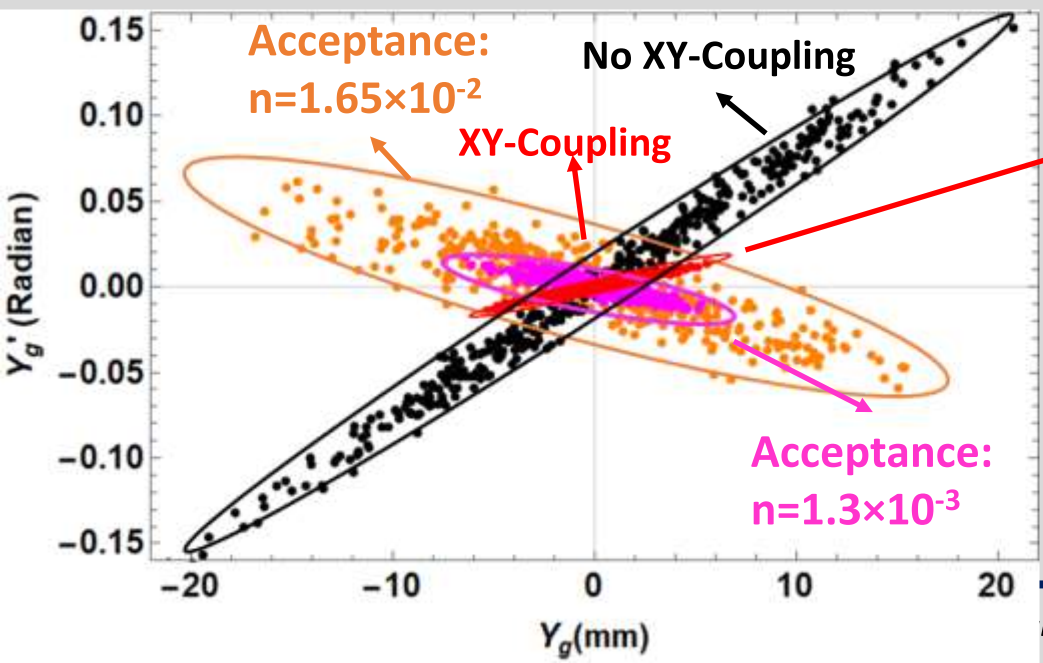


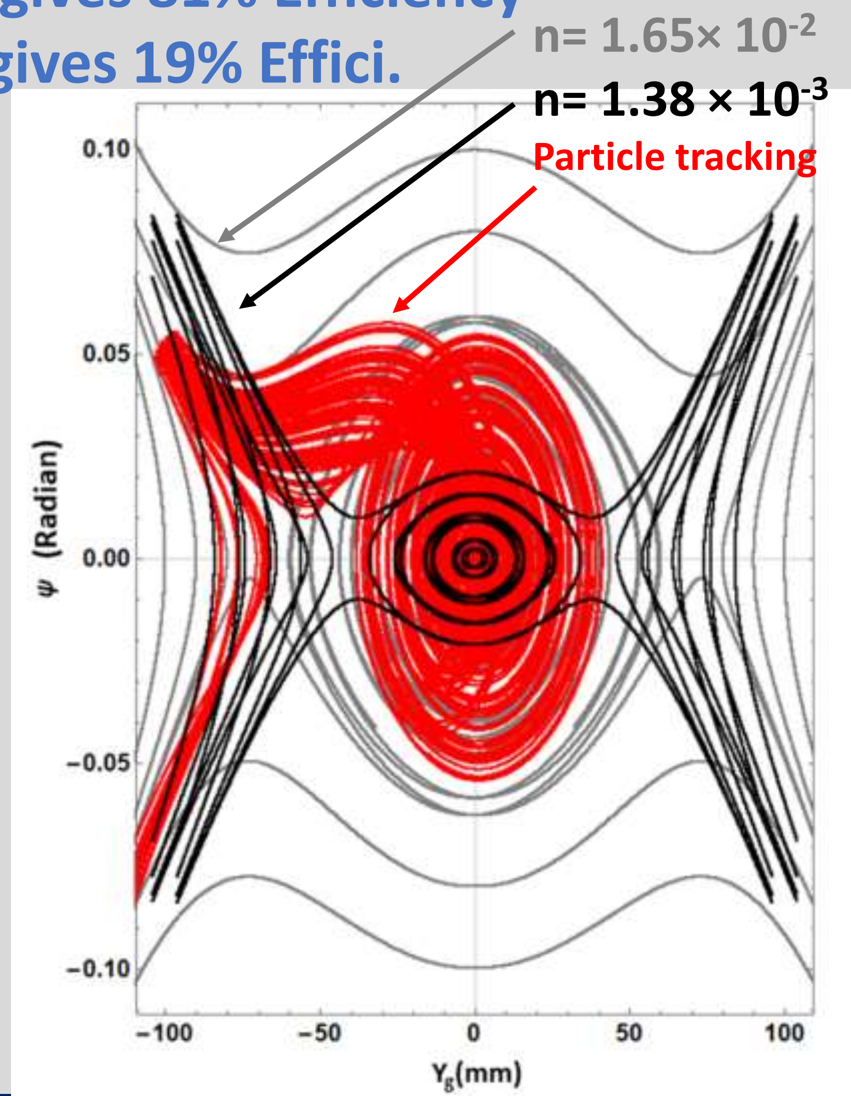
Injection Efficiency

- Multi-particle tracking to determine storage efficiency
- XY-Coupled and field index 1.65×10^{-2} gives 81% Efficiency
- XY-Coupled and field index 1.38×10^{-3} gives 19% Effici.

ID	Condition	n	Effi. (%)
Red	XY-Coupled	1.65×10^{-2}	81
Black	No-Coupling	1.65×10^{-2}	12
	XY-Coupled	1.38×10^{-3}	19

Vertical Phase Space at the kick point





Summary

- ✓ Three-dimensional spiral injection has been established
  - ➢ Goal 1:
    - ✓ Visualized 3-D spiral injection
  - ➢ Goal 2:
    - ✓ Build a beamline to generate required XY-coupling for the storage magnet
    - ✓ Established a way to measure beam XY-coupling
  - ➢ Goal 3
    - ✓ To extract beam quantitative info. a special type of Wire Scanner was designed/commissioned
    - ✓ Beam vertical blow-up was reduced to 8.06±0.21 mm with phase space matching as compared to 31.86 mm without phase space matching
  - ➢ Goal 4
  - ✓ A kicker System has been designed to store the beam
    - ✓ Injection efficiency of 81% is estimated with the current situation of phase space matching

Backup

Muon g-2/EDM Experiment at J-PARC

Spin Precession: BMT equation

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

A new muon g-2/EDM (E34) experiment at J-PARC with very different techniques

BNL, ϵ~1000 mm·mrad E≠0

Magic γ=29.3

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c})\right]$$

J-PARC, ϵ<0.5 mm·mrad E=0, γ=3

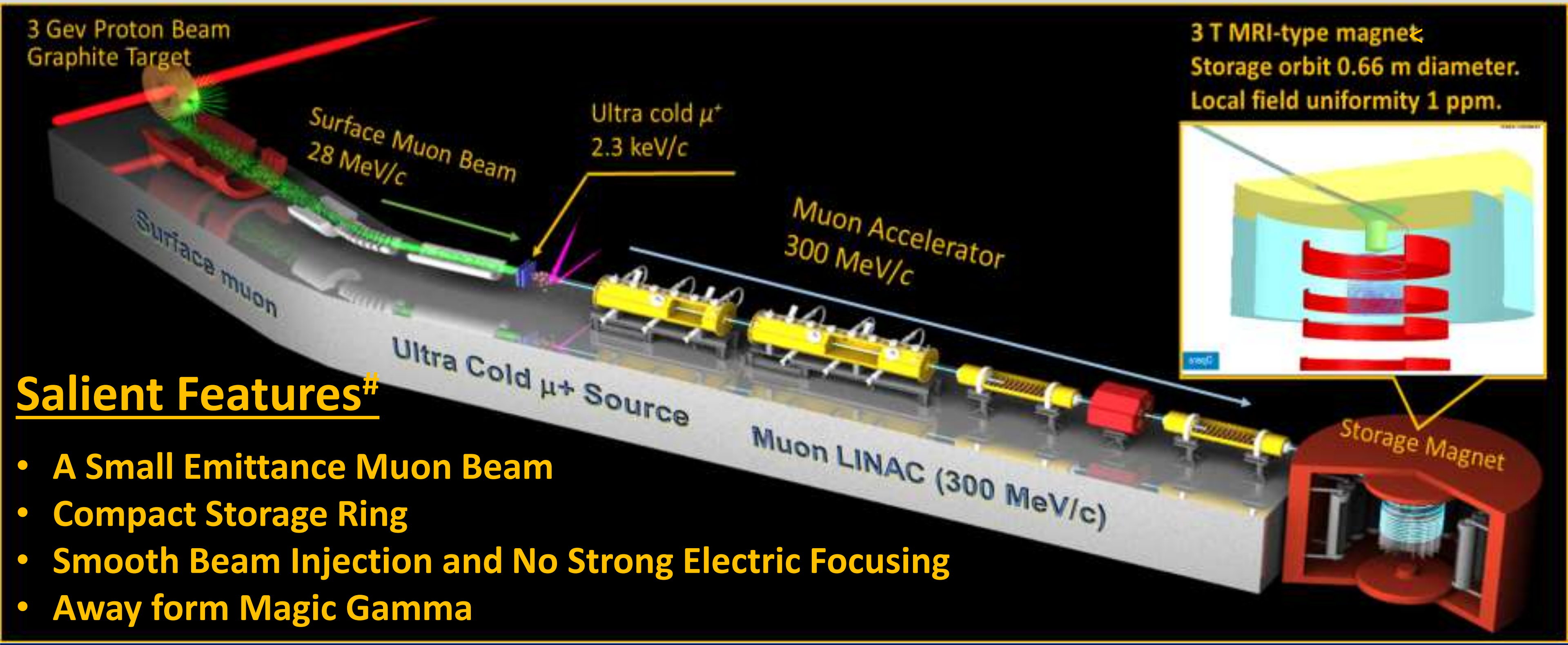
$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

g-2 EDM

Final Goal

$(g-2)_\mu$: 0.1 ppm

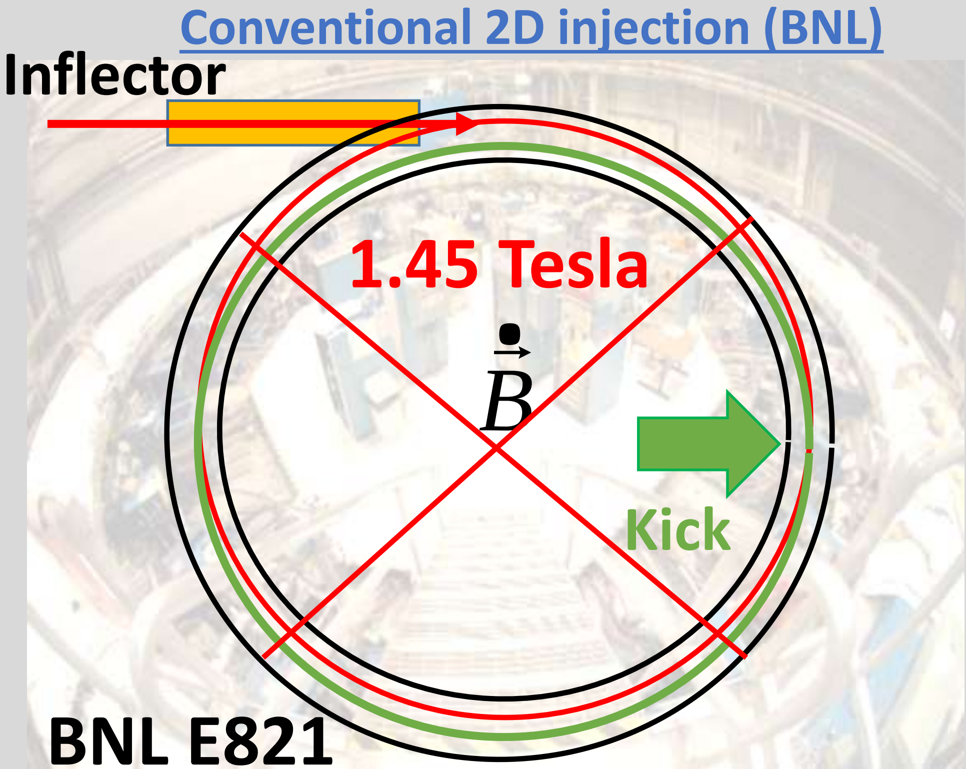
EDM: 1.5×10^{-21} e.cm

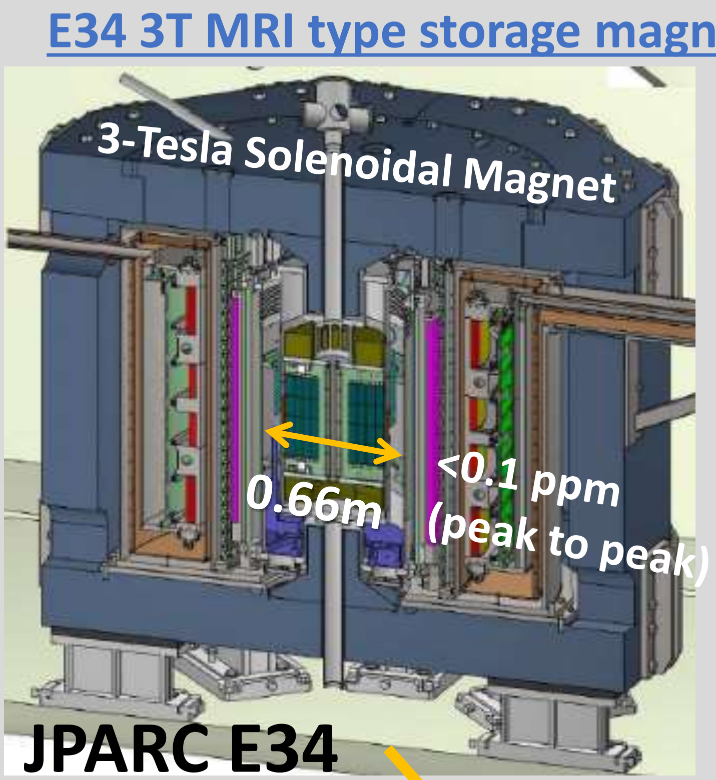


Salient Features#

- A Small Emittance Muon Beam
- Compact Storage Ring
- Smooth Beam Injection and No Strong Electric Focusing
- Away form Magic Gamma

Why inject beam spirally?





14 m orbit, To avoid beam hit at inflector (77 mm), kick angle become 10.8 mrad within 149 ns.

0.66 m Orbit→ Kick angle is 233 mrad within 7.4 ns.
Too stringent inside supercond. magnet
3 T is too high to be canceled by inflector.