Equivalent $SU(3)_f$ approaches for two-body Anti-triplet charmed baryon decays

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Outline:
1. Introduction
2. Formalism
3. Results
4. Summary

Introduction (IRA and TDA)

• IRA: irreducible $SU(3)_f$ approach (amplitude)

$\mathbf{B}_c$, $\mathbf{B}$, M, and $\mathcal{H}_{eff}$ in the irreducible forms of $SU(3)_f$,

connected as the invariant $SU(3)_f$ amplitudes (a_i)

[Savage, Springer, PRD42, 1527 (1990)]

$\mathcal{H}_{\rm IRA} \sim c_- H(6) + c_+ H(\overline{15})$, $c_- > c_+$

a_1, a_2, a_3, QCD-favored

a_4, a_5, a_6, a_7, QCD-disfavored

C.D. Lu, W. Wang and F.S. Yu [PRD93, 056008 (2016)],

"Test flavor SU(3) symmetry in exclusive Λ_c decays,"

Geng, Hsiao, Y.H. Lin and L.L. Liu [PLB776, 265 (2017)]

able to explain $\mathcal{B}(\mathbf{B}_c \to \mathbf{B}M)$

Introduction (IRA and TDA)

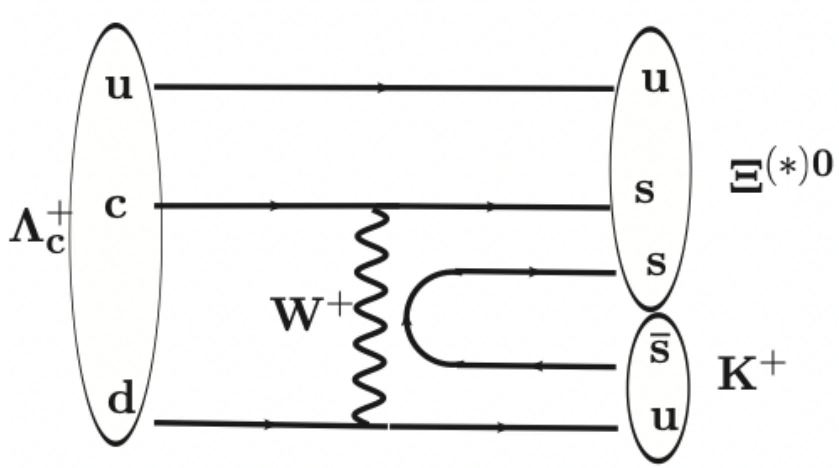
- One prefers topological diagrams for the decay:

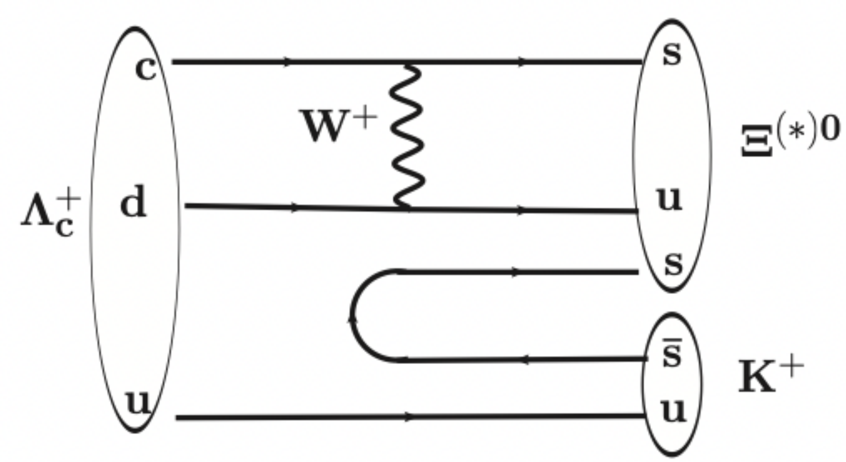
factorizable, non-factorizable, W-boson exchange

BESIII [PLB783, 200 (2018)]

Feynman diagrams of $\Lambda_c^+ \to \Xi^{(*)0} K^+$

IRA: $\mathcal{M}(\Lambda_c^+ \to \Xi^0 K^+) = -2(a_2 - \frac{a_4+a_7}{2})$





Theoretical approaches

Pole model + current algebra:

J. Zou, F. Xu, G. Meng and H.Y. Cheng, PRD101, 014011 (2020),

"Two-body hadronic weak decays of antitriplet charmed baryons."

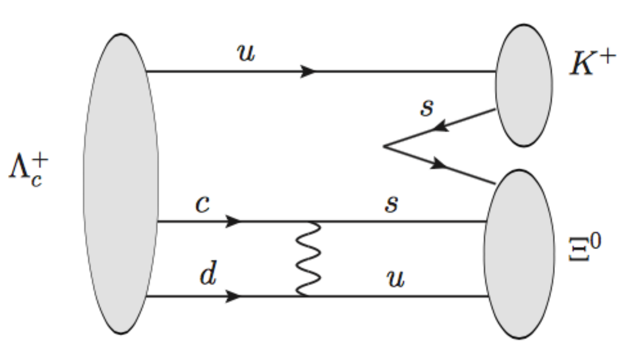
Long-distance triangle rescattering:

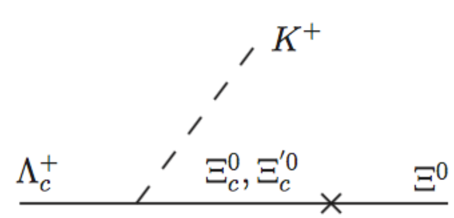
H.W. Ke and X.Q. Li [PRD102, 113013 (2020)]

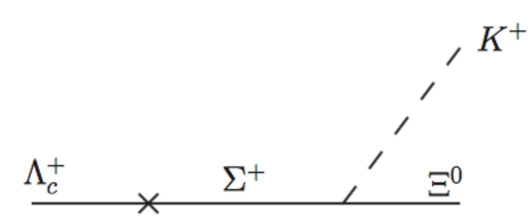
"A natural interpretation on the data of $\Lambda_c \to \Sigma\pi$."

Yu, Hsiao [PLB820, 136586 (2021)]

"Cabibbo-favored $\Lambda_c^+ \to \Lambda a_0(980)^+$ decay in the final state interaction."







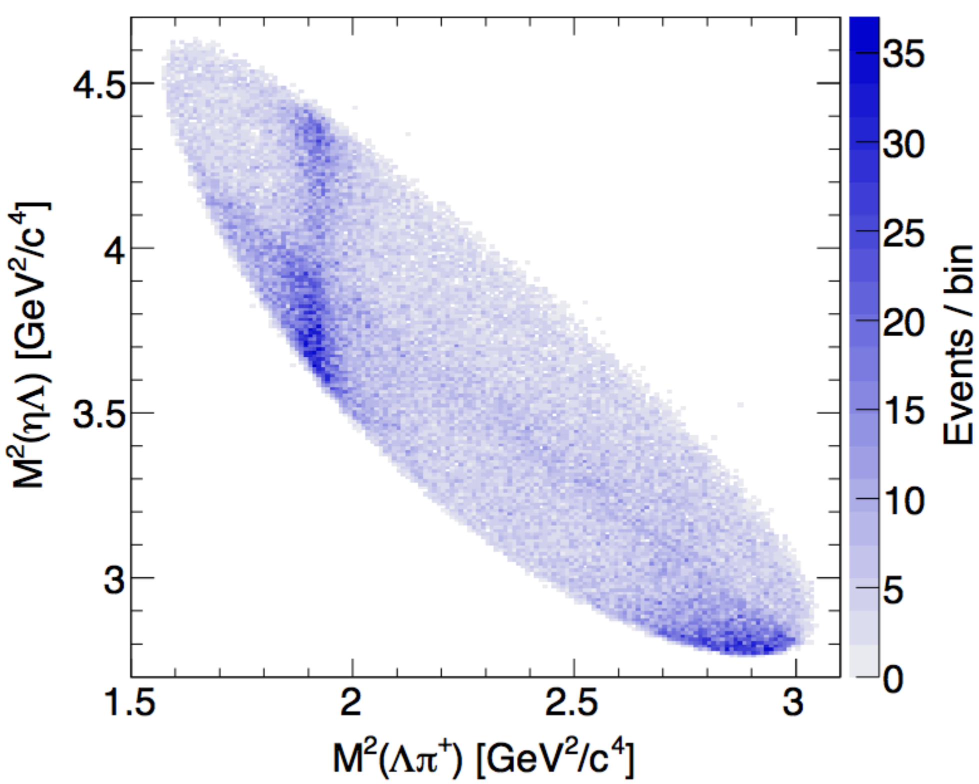
The total branching fraction [Belle, PRD103, 052005 (2021)]

$$\mathcal{B}(\Lambda_c^+ \to \Lambda\eta\pi^+) = (18.4 \pm 0.2 \pm 0.9 \pm 0.9) \times 10^{-3},$$

$$\mathcal{B}(\Lambda_c^+ \to \Lambda^*\pi^+, \Lambda^* \to \Lambda\eta) = (3.5 \pm 0.5) \times 10^{-3},$$

$$\mathcal{B}(\Lambda_c^+ \to \Sigma^{*+}\eta, \Sigma^{*+} \to \Lambda\pi^+) = (10.5 \pm 1.2) \times 10^{-3},$$

with $\Lambda^* \equiv \Lambda(1670)$ and $\Sigma^* \equiv \Sigma(1385)$.

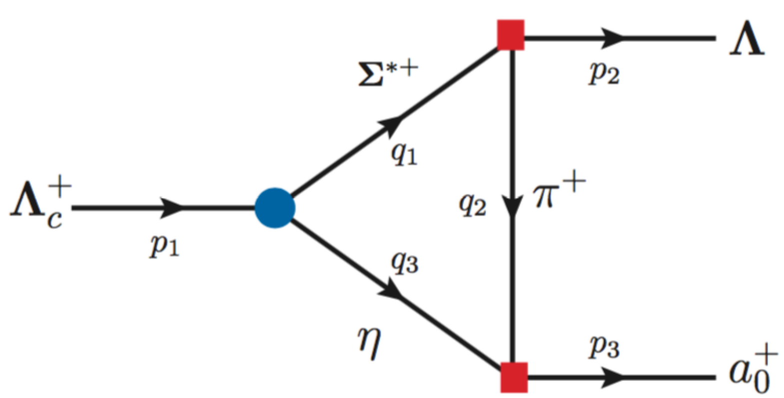


A possible $\Lambda_c^+ \to \Lambda a_0^+, a_0^+ \to \eta\pi^+$ process.

Cabibbo-favored $\Lambda_c^+ \to \Lambda a_0(980)^+$ decay in the final state interaction

Yao Yu[1,*] and Yu-Kuo Hsiao[2,†]

[PLB820, 136586 (2021)]



$$\mathcal{B}(\Lambda_c^+ \to \Lambda a_0^+) = (1.7^{+2.8}_{-1.0} \pm 0.3) \times 10^{-3}$$

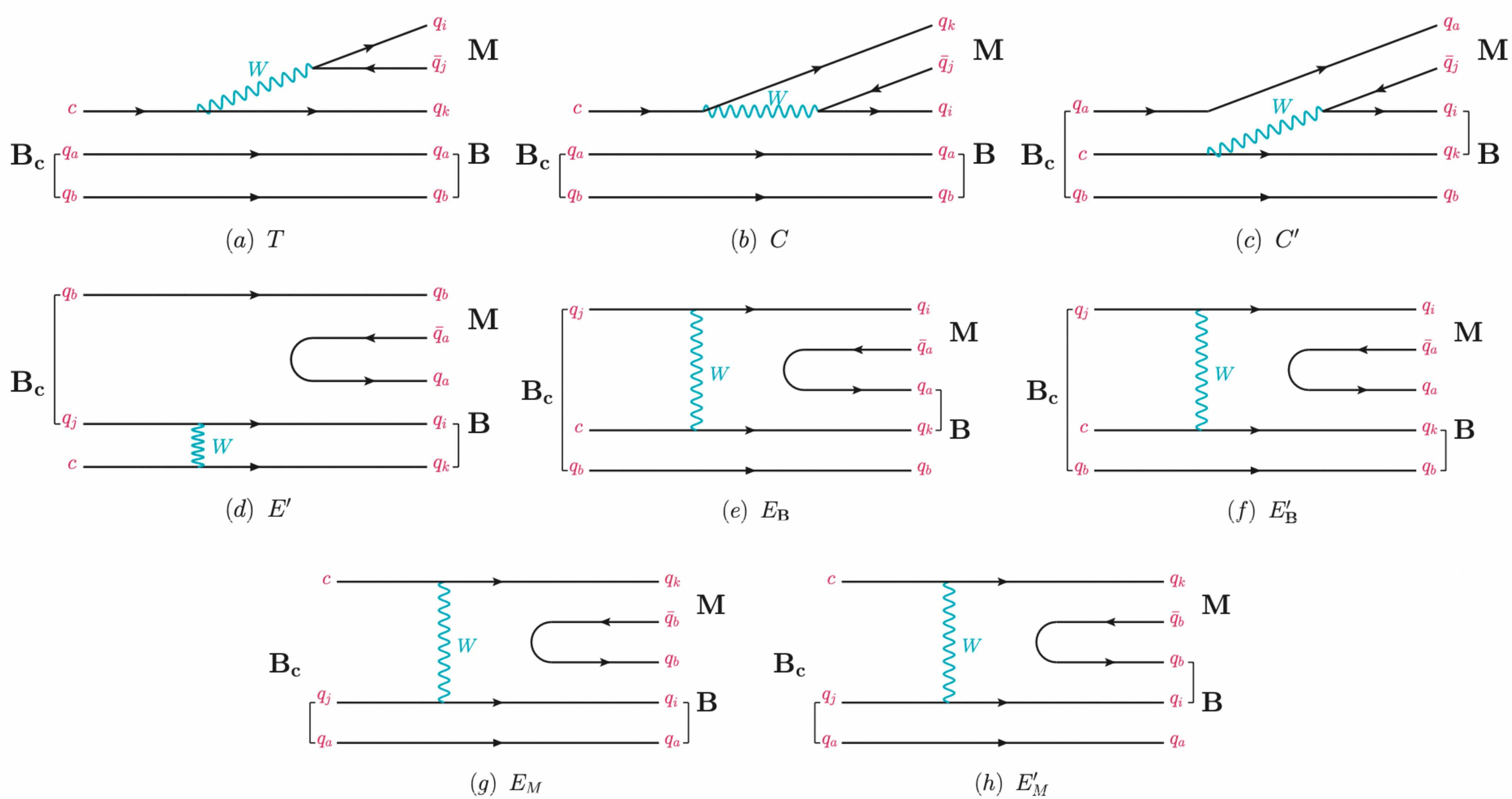
$$\mathcal{B}(\Lambda_c^+ \to p f_0) = (3.5 \pm 2.3) \times 10^{-3}$$

measured in 1990

• TDA: topological $SU(3)_f$ approach (amplitude)

$SU(3)_f$ symmetry+topological diagrams

more approachable, more information



Ξ_c^0, Ξ_c^+ and Λ_c^+: $(ds - sd)c$, $(su - us)c$ and $(ud - du)c$

$\mathbf{B} \sim (q_a - q_k)q_b\,[(q_i - q_a)q_b]$ for $E_{\mathbf{B}(M)}$, $\mathbf{B} \sim (q_k - q_b)q_a\,[(q_b - q_i)q_a]$ for $E'_{\mathbf{B}(M)}$

• Studies with TDA

Y. Kohara [PRD44, 2799 (1991)]

"Quark diagram analysis of charmed baryon decays,"

L.L. Chau, H.Y. Cheng and B. Tseng [PRD54, 2132 (1996)]

"Analysis of two-body decays of charmed baryons using the quark diagram scheme"

H.J. Zhao, Y.L. Wang, Hsiao and Y. Yu [JHEP02, 165 (2020)]

"A diagrammatic analysis of two-body charmed baryon decays with flavor symmetry."

Hsiao, Q. Yi, S.T. Cai and H.J. Zhao [EPJC80, 1067 (2020)]

"Two-body charmed baryon decays involving decuplet baryon in the quark-diagram scheme."

Equivalent $SU(3)_f$ approaches for $\mathbf{B}_c \to \mathbf{B}M$

- Both based on $SU(3)_f$ symmetry,

TDA and IRA can be equivalent

X.G. He and W. Wang [CPC42, 103108 (2018)]

"Flavor SU(3) Topological Diagram and

Irreducible Representation Amplitudes

for Heavy Meson Charmless Hadronic Decays: Mismatch and Equivalence,"

X.G. He, Y.J. Shi and W. Wang [EPJC80, 359 (2020)]

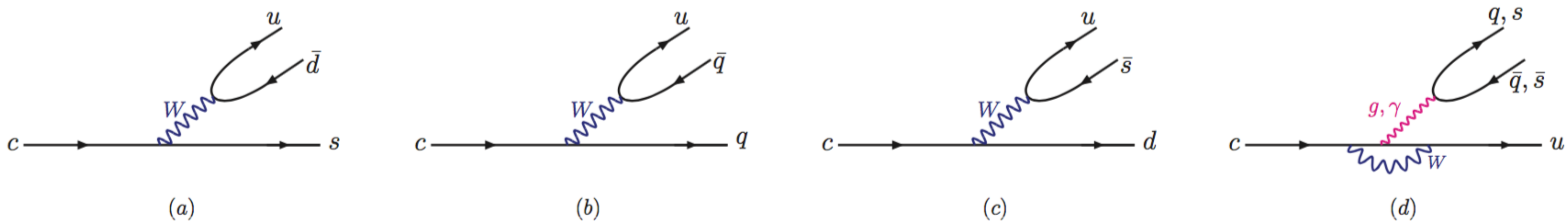
"Unification of Flavor SU(3) Analyses of Heavy Hadron Weak Decays,"

Hsiao, Q. Yi, S.T. Cai and H.J. Zhao [EPJC80, 1067 (2020)]

"Two-body charmed baryon decays involving decuplet baryon

in the quark-diagram scheme."

Formalism



$$\mathcal{H}_{eff} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i \left(V_{cs} V_{ud} O_i + V_{cq} V_{uq} O_i^q + V_{cd} V_{us} O_i' \right)$$

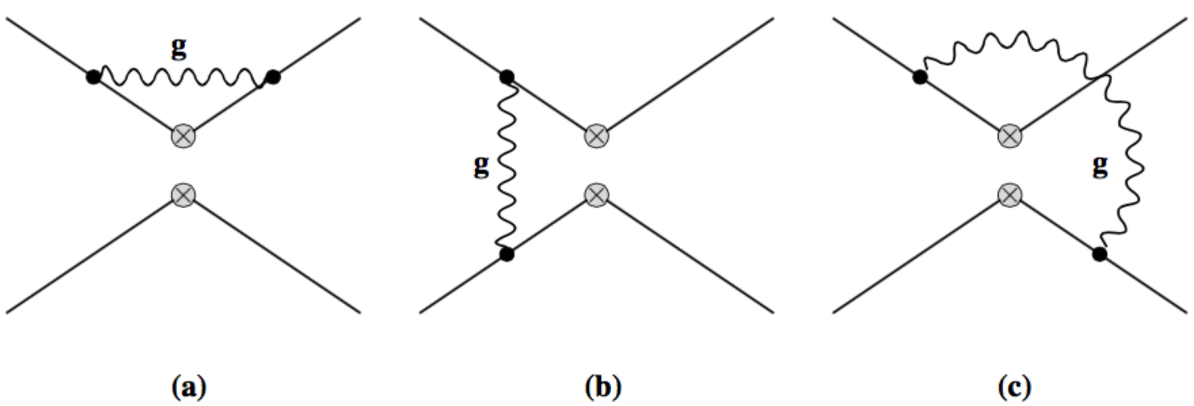
$$(V_{cs}V_{ud}, V_{cd}V_{ud}, V_{cs}V_{us}, V_{cd}V_{us}) = (1, -s_c, s_c, -s_c^2)$$

$$O_\pm = \frac{1}{2}\left[(\bar{u}d)(\bar{s}c) \pm (\bar{s}d)(\bar{u}c)\right]$$

$$O_\pm^q = \frac{1}{2}\left[(\bar{u}q)(\bar{q}c) \pm (\bar{q}q)(\bar{u}c)\right]$$

$$O_\pm' = \frac{1}{2}\left[(\bar{u}s)(\bar{d}c) \pm (\bar{d}s)(\bar{u}c)\right]$$

$$(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$$



$$(c_+, c_-) = (0.76, 1.78)$$

SU(3) flavor symmetry

Under the $SU(3)_f$ symmetry

$(\bar{q}_1 q_2)(\bar{q}_3 c) \sim (\bar{q}^i q_k \bar{q}^j)c$

$q_i = (u, d, s)$ represent the triplet of 3

To be decomposed as irreducible forms:

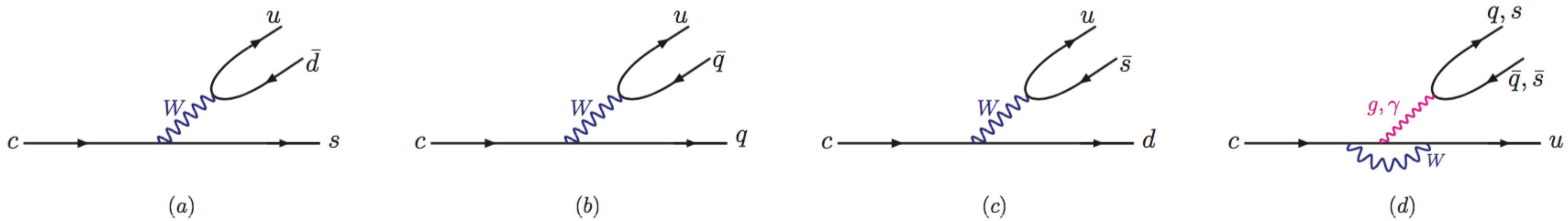
$(\bar{3} \times 3 \times \bar{3})c = (\bar{3} + \bar{3}' + 6 + \overline{15})c$

$$O_{-(+)} \sim \mathcal{O}_{6(\overline{15})} = \frac{1}{2}(\bar{u}d\bar{s} \mp \bar{s}d\bar{u})c\,,$$

$$O^q_{-(+)} \sim \mathcal{O}^q_{6(\overline{15})} = \frac{1}{2}(\bar{u}q\bar{q} \mp \bar{q}q\bar{u})c\,,$$

$$O'_{-(+)} \sim \mathcal{O}'_{6(\overline{15})} = \frac{1}{2}(\bar{u}s\bar{d} \mp \bar{d}s\bar{u})c\,,$$

Formalism



- IRA

$$\mathcal{H}_{\rm IRA} = c_- \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_+ H(\overline{15})^{ij}_k$$

$$\mathbf{B}_c(\mathbf{B}_{c\,i}) = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$$

$$\mathbf{B}^i_j : (n,\ p,\ , \Sigma^{\pm,0}, \Xi^{-,0},\ \Lambda)$$

$$M^i_j : (\pi^{\pm,0},\ K^{\pm},\ K^0,\ \bar{K}^0,\ \eta)$$

- TDA

$$\mathcal{H}_{\rm TDA} = H^{ki}_j\ [H^{31}_2 = 1]$$

$$\mathbf{B}_c(\mathbf{B}_c{}^{ij}) = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}$$

$$\mathbf{B}_{ijk} = \epsilon_{ijl}\mathbf{B}^l_k$$

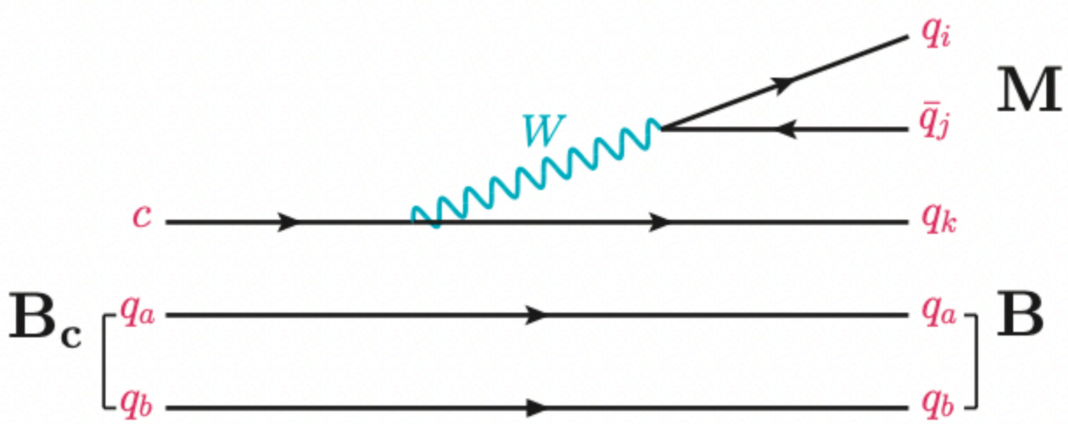
$$M^i_j : (\pi^{\pm,0},\ K^{\pm},\ K^0,\ \bar{K}^0,\ \eta)$$

$$\mathcal{M}_{\rm IRA} = \mathcal{M}_6 + \mathcal{M}_{\overline{15}}\,,$$

$$\mathcal{M}_6 = a_1 H_{ij}(6) T^{ik} \mathbf{B}^l_k M^j_l + a_2 H_{ij}(6) T^{ik} M^l_k \mathbf{B}^j_l + a_3 H_{ij}(6) \mathbf{B}^i_k M^j_l T^{kl}\,,$$

$$\mathcal{M}_{\overline{15}} = a_4 H^{li}_k(\overline{15}) \mathbf{B}_{cj} M^j_i \mathbf{B}^l_k + a_5 \mathbf{B}^i_j M^l_i H(\overline{15})^{jk}_l \mathbf{B}_{ck}$$
$$+ a_6 \mathbf{B}^k_l M^i_j H(\overline{15})^{jl}_i \mathbf{B}_{ck} + a_7 \mathbf{B}^l_i M^i_j H(\overline{15})^{jk}_l \mathbf{B}_{ck}\,,$$

$$\mathcal{M}_{\rm TDA} = T \mathbf{B}_c{}^{ab} H^{ki}_j \mathbf{B}_{abk} M^i_j + C \mathbf{B}_c{}^{ab} H^{ki}_j \mathbf{B}_{abi} M^k_j + C' \mathbf{B}_c{}^{ab} H^{ki}_j \mathbf{B}_{ikb} M^a_j$$
$$+ E_{\mathbf{B}} \mathbf{B}_c{}^{jb} H^{ki}_j \mathbf{B}_{kab} M^i_a + E'_{\mathbf{B}} \mathbf{B}_c{}^{jb} H^{ki}_j \mathbf{B}_{kba} M^i_a$$
$$+ E_M \mathbf{B}_c{}^{jb} H^{ki}_j \mathbf{B}_{iba} M^k_a + E'_M \mathbf{B}_c{}^{jb} H^{ki}_j \mathbf{B}_{iab} M^k_a + E' \mathbf{B}_c{}^{jb} H^{ki}_j \mathbf{B}_{ika} M^b_a\,,$$

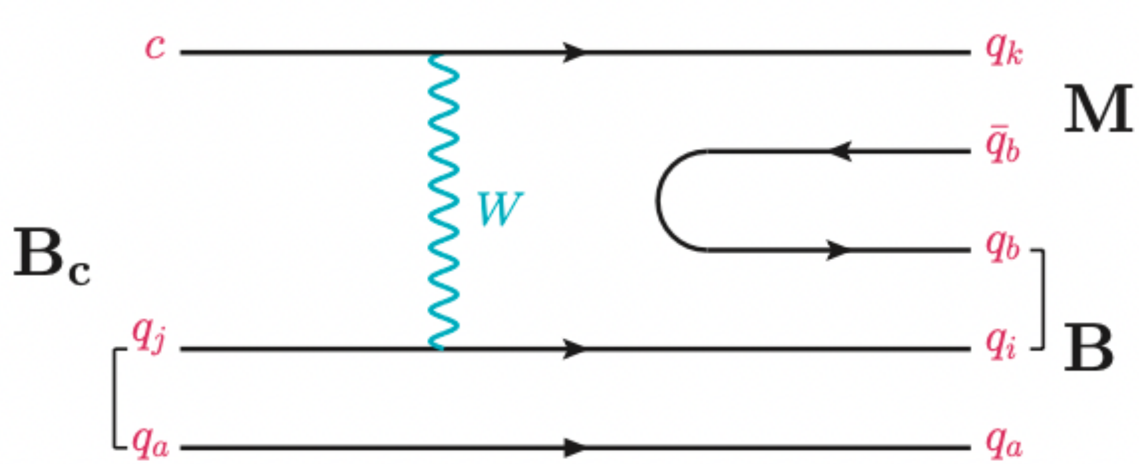


(a) T

$$T^{ij} \equiv \mathbf{B}_{ck}\epsilon^{ijk}$$

Decay mode	$M_{\rm TDA}$	$M_{\rm IRA}$
$\Lambda_c^+ \to \Lambda^0\pi^+$	$-\frac{1}{\sqrt{6}}(4T+C'-E_{\mathbf{B}}-E')$	$-\frac{\sqrt{6}}{3}(a_1+a_2+a_3-\frac{a_5-2a_6+a_7}{2})$
$\Lambda_c^+ \to \Sigma^0\pi^+$	$\frac{1}{\sqrt{2}}(C'+E_{\mathbf{B}}-E')$	$-\sqrt{2}(a_1-a_2-a_3-\frac{a_5-a_7}{2})$
$\Lambda_c^+ \to \Sigma^+\pi^0$	$-\frac{1}{\sqrt{2}}(C'+E_{\mathbf{B}}-E')$	$\sqrt{2}(a_1-a_2-a_3-\frac{a_5-a_7}{2})$
$\Lambda_c^+ \to \Xi^0 K^+$	$E'^{(s)}$	$-2(a_2-\frac{a_4+a_7}{2})$
$\Lambda_c^+ \to p\bar{K}^0$	$2C-E'_M$	$-2(a_1-\frac{a_5+a_6}{2})$
$\Lambda_c^+ \to \Sigma^+\eta$	$[\frac{1}{\sqrt{2}}(C'-E_{\mathbf{B}}+E')c\phi-E_M'^{(s)}s\phi]$	$\sqrt{2}c\phi(-a_1-a_2+a_3+\frac{a_5+a_7}{2})-s\phi a_4$
$\Xi_c^+ \to \Sigma^+\bar{K}^0$	$-2C+C'$	$2(a_3-\frac{a_4+a_6}{2})$
$\Xi_c^+ \to \Xi^0\pi^+$	$-2T-C'$	$2(a_3+\frac{a_4+a_6}{2})$
$\Xi_c^0 \to \Lambda^0\bar{K}^0$	$\frac{1}{\sqrt{6}}(2C+C'-E_M-2E'_M-E')$	$-\frac{\sqrt{6}}{3}(2a_1-a_2-a_3+\frac{2a_5-a_6-a_7}{2})$
$\Xi_c^0 \to \Sigma^0\bar{K}^0$	$\frac{1}{\sqrt{2}}(2C-C'+E_M+E')$	$-\sqrt{2}(a_2+a_3-\frac{a_6-a_7}{2})$
$\Xi_c^0 \to \Sigma^+K^-$	$-E_M-E'$	$2(a_2+\frac{a_4+a_7}{2})$
$\Xi_c^0 \to \Xi^0\pi^0$	$\frac{1}{\sqrt{2}}(E_{\mathbf{B}}+C')$	$-\sqrt{2}(a_1-a_3-\frac{a_4-a_5}{2})$
$\Xi_c^0 \to \Xi^-\pi^+$	$2T-E_{\mathbf{B}}$	$2(a_1+\frac{a_5+a_6}{2})$
$\Xi_c^0 \to \Xi^0\eta$	$[\frac{1}{\sqrt{2}}(E_{\mathbf{B}}-C')c\phi+(E_M^{(s)}+E_M'^{(s)}+E'^{(s)})s\phi]$	$\sqrt{2}c\phi(a_1-a_3+\frac{a_4+a_5}{2})-2s\phi(a_2+\frac{a_7}{2})$

$E_M'^s$ for $g \to s\bar{s}$



(h) E'_M

Using $\mathcal{M}_{\mathbf{IRA}} = \mathcal{M}_{\mathbf{TDA}}$

$$(T, C, C') = (a_1 + \tfrac{a_4+a_6}{2}, -a_1 + \tfrac{a_4+a_6}{2}, -2a_1 + 2a_3)$$

$$(E_{\mathbf{B}}, E_M, E') = (a_4, -2a_4 - 2a_7, -2a_2 + a_4 + a_7)$$

$$E'_M = E_{\mathbf{B}}$$

$$(a_1, a_2, a_3) = (\tfrac{T-C}{2}, -\tfrac{E_M+2E'}{4}, \tfrac{T-C+C'}{2})$$

$$(a_4, a_5, a_6, a_7) = (E_{\mathbf{B}}, 0, T + C - E_{\mathbf{B}}, -E_{\mathbf{B}} - \tfrac{E_M}{2})$$

such that we "topologize" the $SU(3)_f$ invariant amplitudes.

Global fit

$$\chi^2 = \Sigma_i [(\mathcal{B}^i_{th} - \mathcal{B}^i_{ex})/\sigma^i_{ex}]^2$$

$$|T|, |C|e^{i\delta_C}, |C'|e^{i\delta_{C'}}, |E_{\mathbf{B}}|e^{i\delta_{E_{\mathbf{B}}}}, |E_M|e^{i\delta_{E_M}}, |E'|e^{i\delta_{E'}} \,,$$

$$E^s_{\mathbf{B}} = E'^{\,s}_M = |E^s_{\mathbf{B}}|e^{i\delta_{E^s_{\mathbf{B}}}}$$

Branching fraction	Data
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0\pi^+)$	1.31 ± 0.09
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+)$	1.22 ± 0.11
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0)$	1.25 ± 0.10
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$	0.55 ± 0.07
$10^2\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0)$	3.18 ± 0.16
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta)$	0.44 ± 0.20
$10^3\mathcal{B}(\Xi_c^+ \to \Sigma^+\bar{K}^0)$	
$10^3\mathcal{B}(\Xi_c^+ \to \Xi^0\pi^+)$	16.0 ± 8.0
$10^3\mathcal{B}(\Xi_c^0 \to \Lambda^0\bar{K}^0)$	8.24 ± 2.44
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^0\bar{K}^0)$	1.38 ± 0.48
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^+ K^-)$	2.21 ± 0.68
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^0\pi^0)$	
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$	18.0 ± 5.2
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^0\eta)$	

	TDA (S1)	TDA (S2)
χ^2	4.5	5.7
$n.d.f$	5	4
$\|T\|$	0.23 ± 0.02	0.24 ± 0.02
$\|C\|$	0.26 ± 0.01	0.23 ± 0.02
$\|C'\|$	0.34 ± 0.02	0.32 ± 0.03
$\|E_{\mathbf{B}}\|$	0.22 ± 0.03	0.22 ± 0.05
$\|E^s_{\mathbf{B}}\|$		0.37 ± 0.06
$\|E_M\|$	0.40 ± 0.03	0.38 ± 0.03
$\|E'\|$	0.24 ± 0.02	0.23 ± 0.02
δ_C	$(183.2 \pm 9.6)^\circ$	$(179.5 \pm 12.9)^\circ$
$\delta_{C'}$	$(163.7 \pm 5.0)^\circ$	$(149.7 \pm 6.7)^\circ$
$\delta_{E_{\mathbf{B}}}$	$(-100.3 \pm 7.1)^\circ$	$(-93.6 \pm 8.2)^\circ$
$\delta_{E^s_{\mathbf{B}}}$		$(43.3 \pm 8.0)^\circ$
δ_{E_M}	$(100.3 \pm 8.0)^\circ$	$(113.2 \pm 10.7)^\circ$
$\delta_{E'}$	$(-71.1 \pm 6.7)^\circ$	$(-50.1 \pm 12.3)^\circ$

TABLE I. Cabibbo-allowed (CA) branching fractions.

Branching fraction	pole model	IRA (with a_6)	IRA (with all a_i)	TDA [This work] (S1, S2)	Data
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0\pi^+)$	1.30	1.27 ± 0.07	1.307 ± 0.069	$(1.27 \pm 0.22, 1.24 \pm 0.30)$	1.31 ± 0.09
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+)$	2.24	1.26 ± 0.06	1.272 ± 0.056	$(1.22 \pm 0.23, 1.24 \pm 0.30)$	1.22 ± 0.11
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\pi^0)$	2.24	1.26 ± 0.06	1.283 ± 0.057	$(1.22 \pm 0.23, 1.24 \pm 0.30)$	1.25 ± 0.10
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$	0.73	0.57 ± 0.09	0.548 ± 0.068	$(0.54 \pm 0.07, 0.51 \pm 0.07)$	0.55 ± 0.07
$10^2\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0)$	2.11	3.14 ± 0.15	3.174 ± 0.154	$(3.18 \pm 0.64, 3.10 \pm 0.80)$	3.18 ± 0.16
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+\eta)$	0.74	0.29 ± 0.12	0.45 ± 0.19	$(0.42 \pm 0.18, 0.57 \pm 0.26)$	0.44 ± 0.20
$10^3\mathcal{B}(\Xi_c^+ \to \Sigma^+\bar{K}^0)$	2.0	$7.8^{+10.2}_{-\ 7.8}$	10.6 ± 14.0	$(12.7 \pm 7.0, 14.7^{+10.7}_{-\ 8.4})$	
$10^3\mathcal{B}(\Xi_c^+ \to \Xi^0\pi^+)$	17.2	4.2 ± 1.7	5.4 ± 1.8	$(7.0^{+3.3}_{-2.6}, 15.7 \pm 6.2)$	16.0 ± 8.0
$10^3\mathcal{B}(\Xi_c^0 \to \Lambda^0\bar{K}^0)$	13.3	14.2 ± 0.09	6.68 ± 1.30	$(9.85 \pm 2.26, 10.0 \pm 2.9)$	8.24 ± 2.44
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^0\bar{K}^0)$	0.4	$0.9^{+1.1}_{-0.9}$	1.38 ± 0.48	$(1.48^{+1.27}_{-0.92}, 1.46^{+1.57}_{-1.09})$	1.38 ± 0.48
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^+ K^-)$	4.6	7.6 ± 1.4	2.21 ± 0.68	$(2.21^{+0.37}_{-0.16}, 2.25^{+1.8}_{-1.1})$	2.21 ± 0.68
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^0\pi^0)$	18.2	10.0 ± 1.4	2.56 ± 0.93	$(6.0 \pm 1.2, 3.6 \pm 1.2)$	
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$	64.7	29.5 ± 1.4	12.1 ± 2.1	$(24.5 \pm 3.7, 23.3 \pm 4.5)$	18.0 ± 5.2
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^0\eta)$	26.7	13.0 ± 2.3		$(4.2^{+1.6}_{-1.3}, 7.3 \pm 3.2)$	

Explain the data

- $\Xi_c^0 \to \Xi^- K^+$ and $\Xi_c^0 \to \Xi^- \pi^+$

data: $\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$=0.02

theory: $\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) \simeq (-\sin\theta_c)^2 \simeq 0.05$

$\Rightarrow SU(3)_f$ symmetry breaking

IRA: at least 3 new parameters for breaking.

Savage [PLB257, 414 (1991)]

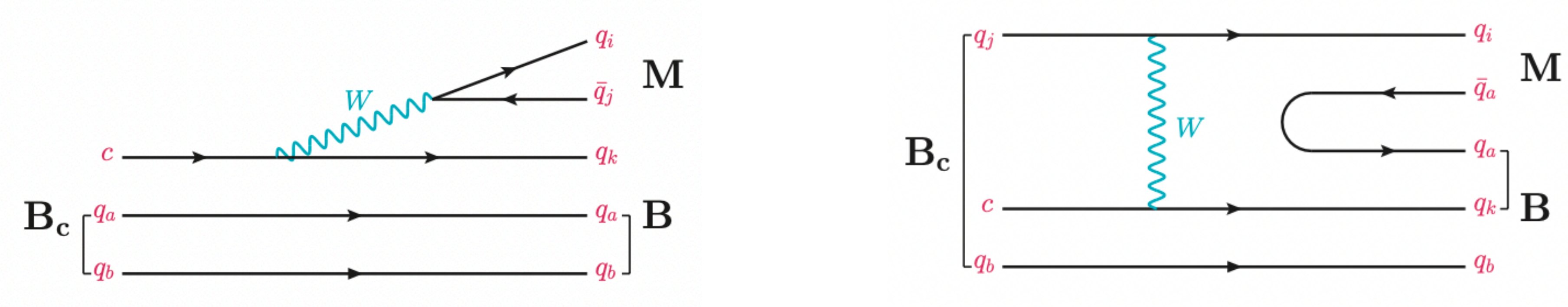
"SU(3) violations in the nonleptonic decay of charmed hadrons,"

Geng, Hsiao, C.W. Liu and T.H. Tsai [EPJC78, 593 (2018)]

"SU(3) symmetry breaking in charmed baryon decays."

TDA: $\mathcal{R}(\Xi_c^0) \equiv \frac{\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)}{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)} = s_c^2 \frac{(2T - E_{\mathbf{B}}^{(s)})^2}{(2T - E_{\mathbf{B}})^2}$

$E_{\mathbf{B}}^s$ for $g \to s\bar{s}$



(a) T (e) $E_{\mathbf{B}}$

	TDA (S1)	TDA (S2)
χ^2	4.5	5.7
$n.d.f$	5	4
$\|T\|$	0.23 ± 0.02	0.24 ± 0.02
$\|C\|$	0.26 ± 0.01	0.23 ± 0.02
$\|C'\|$	0.34 ± 0.02	0.32 ± 0.03
$\|E_{\mathbf{B}}\|$	0.22 ± 0.03	0.22 ± 0.05
$\|E^s_{\mathbf{B}}\|$		0.37 ± 0.06
$\|E_M\|$	0.40 ± 0.03	0.38 ± 0.03
$\|E'\|$	0.24 ± 0.02	0.23 ± 0.02
δ_C	$(183.2 \pm 9.6)^\circ$	$(179.5 \pm 12.9)^\circ$
$\delta_{C'}$	$(163.7 \pm 5.0)^\circ$	$(149.7 \pm 6.7)^\circ$
$\delta_{E_{\mathbf{B}}}$	$(-100.3 \pm 7.1)^\circ$	$(-93.6 \pm 8.2)^\circ$
$\delta_{E^s_{\mathbf{B}}}$		$(43.3 \pm 8.0)^\circ$
δ_{E_M}	$(100.3 \pm 8.0)^\circ$	$(113.2 \pm 10.7)^\circ$
$\delta_{E'}$	$(-71.1 \pm 6.7)^\circ$	$(-50.1 \pm 12.3)^\circ$

$$\mathcal{B}_{\rm S1}(\Xi_c^0 \to \Xi^- K^+) = (11.8 \pm 1.8) \times 10^{-4}$$

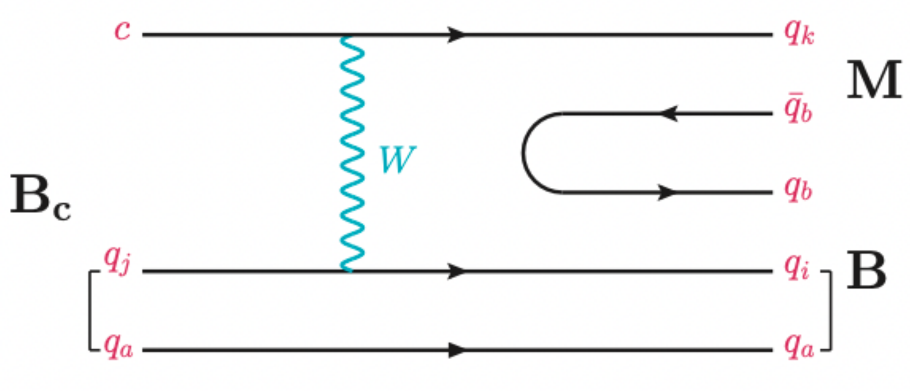
$$\mathcal{B}_{\rm S2}(\Xi_c^0 \to \Xi^- K^+) = (4.1 \pm 2.8) \times 10^{-4}$$

$$\mathcal{B}_{ex}(\Xi_c^0 \to \Xi^- K^+) = (3.9 \pm 1.2) \times 10^{-4}$$

$$E^s_{\mathbf{B}} = |n_q| e^{i\delta_{n_q}} E_{\mathbf{B}}$$

$$|n_q| = 1.2 \pm 0.4,\ \delta_{n_q} = (50.3 \pm 11.5)^\circ$$

Explain the data



(g) E_M

- CA $\Xi_c^0 \to \mathbf{B}M$

$\mathcal{B}_{ex}(\Xi_c^0 \to \Lambda^0\bar{K}^0, \Sigma^0\bar{K}^0, \Sigma^+K^-)$

$= (8.24 \pm 2.44, 1.38 \pm 0.48, 2.21 \pm 0.68) \times 10^{-3}$

Belle [PRD105, L011102 (2022)]

TDA: $\mathcal{M}(\Xi_c^0 \to \Lambda^0\bar{K}^0, \Sigma^0\bar{K}^0, \Sigma^+K^-) = \frac{1}{\sqrt{6}}(2C + C' - E_M - 2E'_M - E'),$

$\frac{1}{\sqrt{2}}(2C - C' + E_M + E'),\ -E_M - E'$

E_M only appearing in $\Xi_c^0 \to \mathbf{B}M$, instead of $\Xi_c^+, \Lambda_c^+ \to \mathbf{B}M$,

cannot be neglected ($E_M = -2a_4 - 2a_7$).

$\mathcal{B}_{\mathrm{S1}}(\Xi_c^0 \to \Lambda^0\bar{K}^0, \Sigma^0\bar{K}^0, \Sigma^+K^-)$

$= (9.85 \pm 2.26, 1.48^{+1.27}_{-0.92}, 2.21^{+0.37}_{-0.16}) \times 10^{-3}$

$\mathcal{B}_{\mathrm{S2}}(\Xi_c^0 \to \Lambda^0\bar{K}^0, \Sigma^0\bar{K}^0, \Sigma^+K^-)$

$= (10.0 \pm 2.9, 1.46^{+1.57}_{-1.09}, 2.25^{+1.8}_{-1.1}) \times 10^{-3}$

Explain the data

- $\Lambda_c^+ \to p\pi^0$ and $\Lambda_c^+ \to n\pi^+$

$\mathcal{B}_{ex}(\Lambda_c^+ \to p\pi^0) > 3 \times 10^{-4}$, BESIII [PRD95, 111102 (2017)]

$\mathcal{B}_{ex}(\Lambda_c^+ \to p\pi^0) > 0.8 \times 10^{-4}$, Belle [PRD103, 072004 (2021)]

$\mathcal{B}_{ex}(\Lambda_c^+ \to n\pi^+) = (6.6 \pm 1.3) \times 10^{-4}$, BESIII [PRL128, 142001 (2022)]

- $\mathcal{B}_{ex}(\Lambda_c^+ \to n\pi^+) \gg \mathcal{B}_{ex}(\Lambda_c^+ \to p\pi^0)$

IRA: $\mathcal{M}(\Lambda_c^+ \to n\pi^+, p\pi^0) \sim A + B, (A - B)/\sqrt{2}$

$(A, B) = (a_2 + a_3 - a_7/2, a_6/2)$

Causing the constructive and destructive interferences.

- Theoretical results

$\mathcal{B}_{\text{S1,S2}}(\Lambda_c^+ \to p\pi^0) = (0.3^{+1.0}_{-0.3}, 0.4^{+1.7}_{-0.4}) \times 10^{-4}$

$\mathcal{B}_{\text{S1,S2}}(\Lambda_c^+ \to n\pi^+) = (7.6 \pm 1.7, 8.3 \pm 2.6) \times 10^{-4}$

Consistent with the analyses with IRA

Partly restoring the QCD-disfavored parameters

Geng, C.W. Liu and T.H. Tsai [PLB794, 19 (2019)]

With all parameters

C.P. Jia, D. Wang and F.S. Yu [NPB956, 115048 (2020)]

F. Huang, Z.P. Xing and X.G. He [JHEP **03**, 143 (2022)]

With all parameters+$SU(3)_f$ breaking

H. Zhong, F. Xu, Q. Wen and Y. Gu [JHEP02, 235 (2023)]

$\mathbf{B_c} \to \mathbf{B}^* M$ with TDA

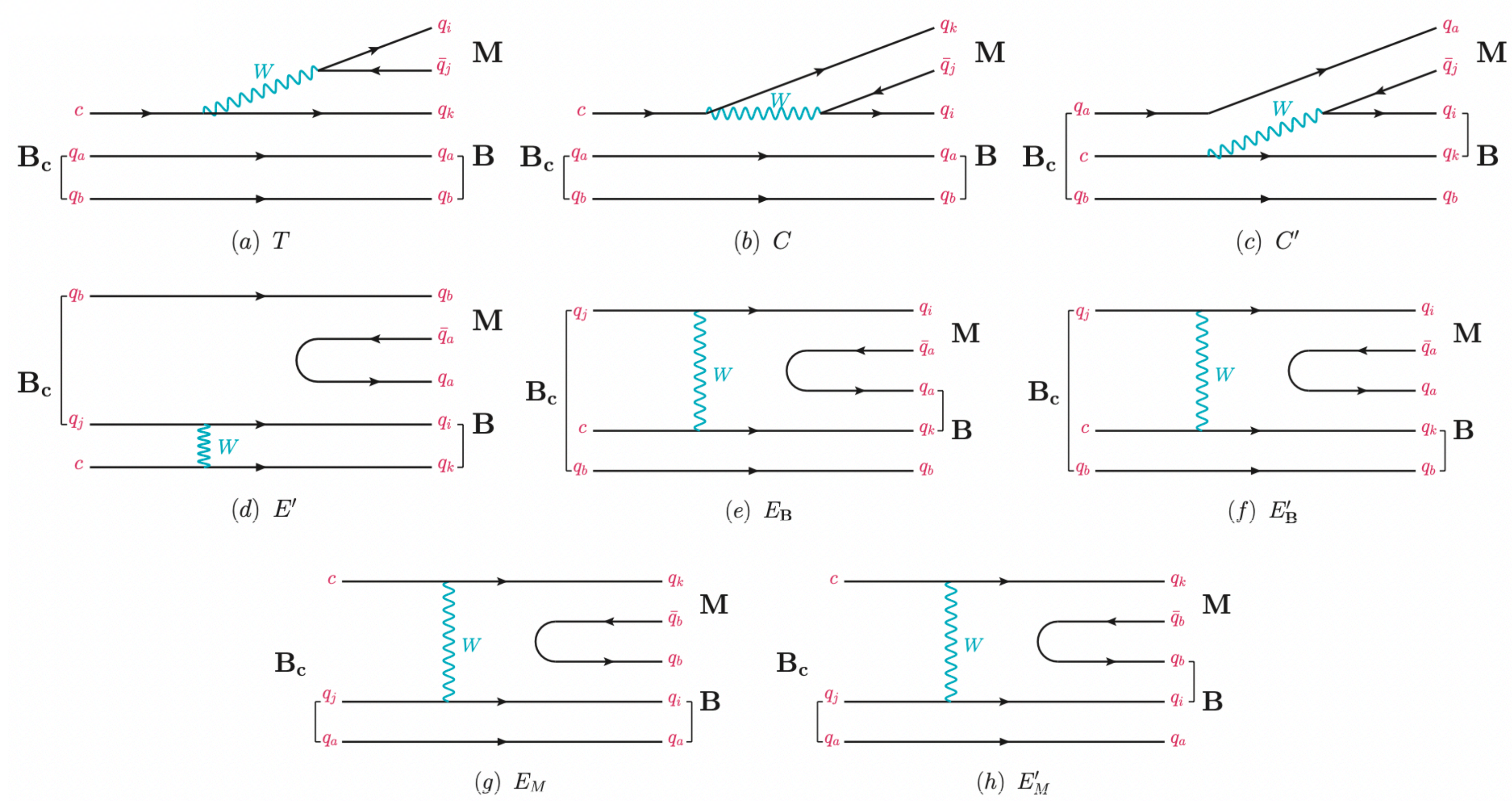
$$\mathbf{B}(\mathbf{B}^i_j) = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}.$$

$$\mathbf{B}_{ijk} = \epsilon_{ijl}\mathbf{B}^l_k$$

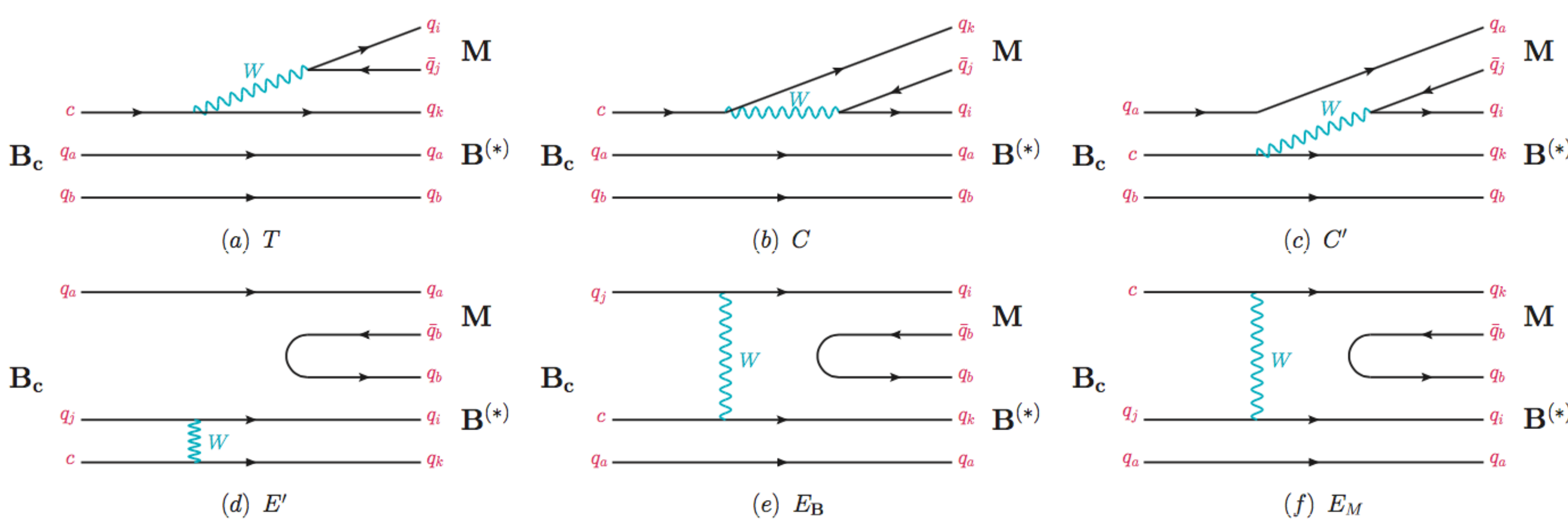
$$\mathbf{B}^*_{ijk} = \frac{1}{\sqrt{3}} \times$$

$$\left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma^{*+} \\ \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma^{*-} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \\ \Xi^{*0} & \Xi^{*-} & \sqrt{3}\Omega^- \end{pmatrix} \right).$$

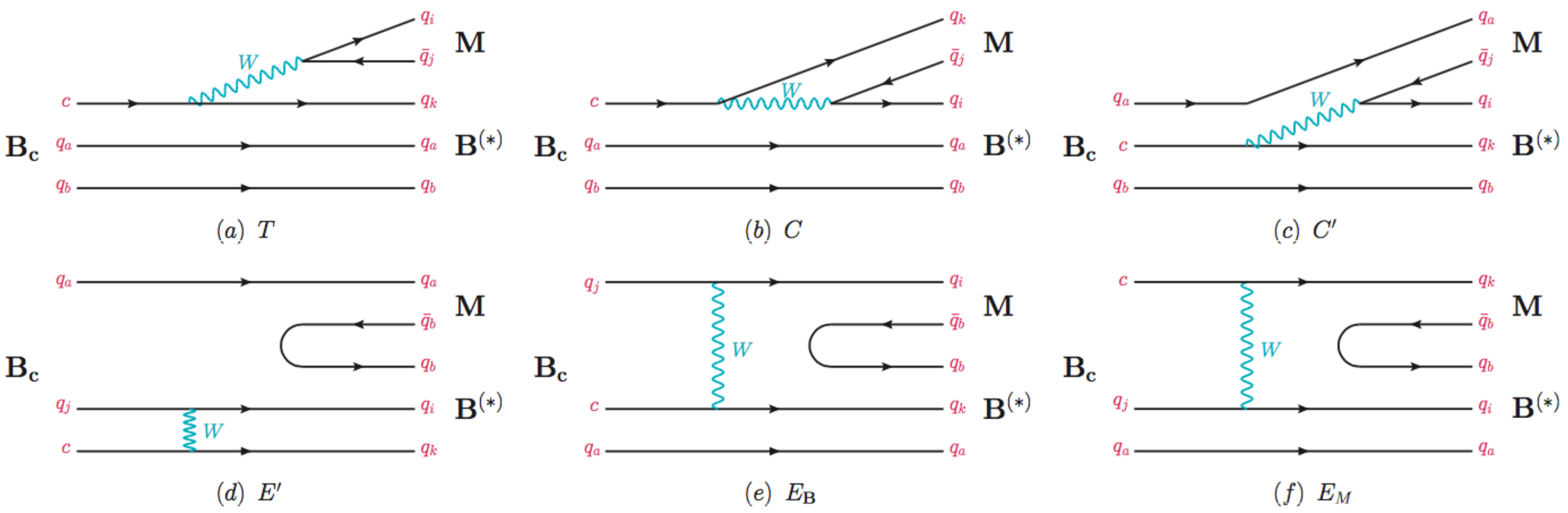
$\mathbf{B_c \to B}M$ with TDA



$\mathbf{B_c \to B^*}M$ with TDA



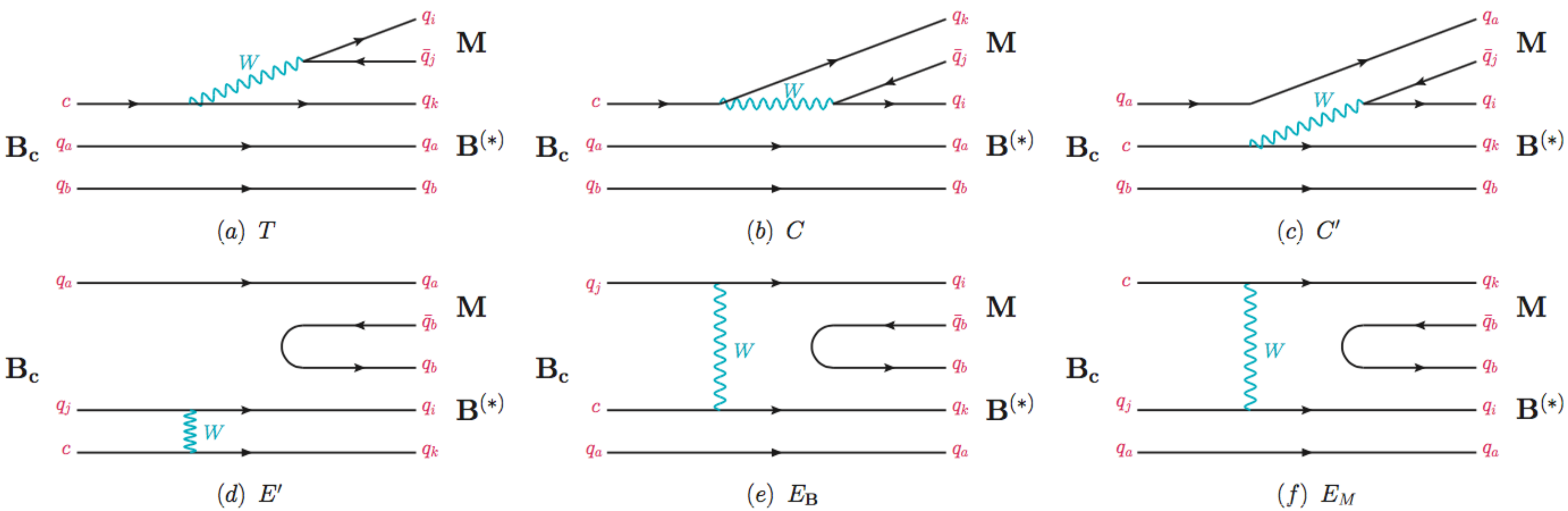
$\mathbf{B_c} \to \mathbf{B}^* M$ with TDA



$$T(\mathbf{B}_c \to \mathbf{B}^* M) = E_{\mathbf{B}} (\mathbf{B}_c)^{ja} H^{ki}_j (\mathbf{B}^*)_{kab} (M)^b_i + E_M (\mathbf{B}_c)^{ja} H^{ki}_j (\mathbf{B}^*)_{iab} (M)^b_k$$
$$+ C' (\mathbf{B}_c)^{ab} H^{ki}_j (\mathbf{B}^*)_{ikb} (M)^j_a + E' (\mathbf{B}_c)^{ka} H^{ki}_j (\mathbf{B}^*)_{ikb} (M)^b_a \,.$$

$$T(\mathbf{B}_c \to \mathbf{B}' M) =$$
$$a_8 (\mathbf{B}')_{ijk} (\mathbf{B}_c)_l H_{nm}(6) (M)^i_o \epsilon^{jln} \epsilon^{kmo} + a_9 (\mathbf{B}')_{ijk} (M)^i_l H(\overline{15})^{jn}_m (\mathbf{B}_c)_n \epsilon^{klm}$$
$$+ a_{10} (\mathbf{B}')_{ijk} (M)^i_l H(\overline{15})^{jk}_m (\mathbf{B}_c)_n \epsilon^{lmn} + a_{11} (\mathbf{B}')_{ijk} (M)^l_m H(\overline{15})^{ij}_l (\mathbf{B}_c)_n \epsilon^{kmn} \,,$$

$\mathbf{B_c \to B^* }M$ with TDA



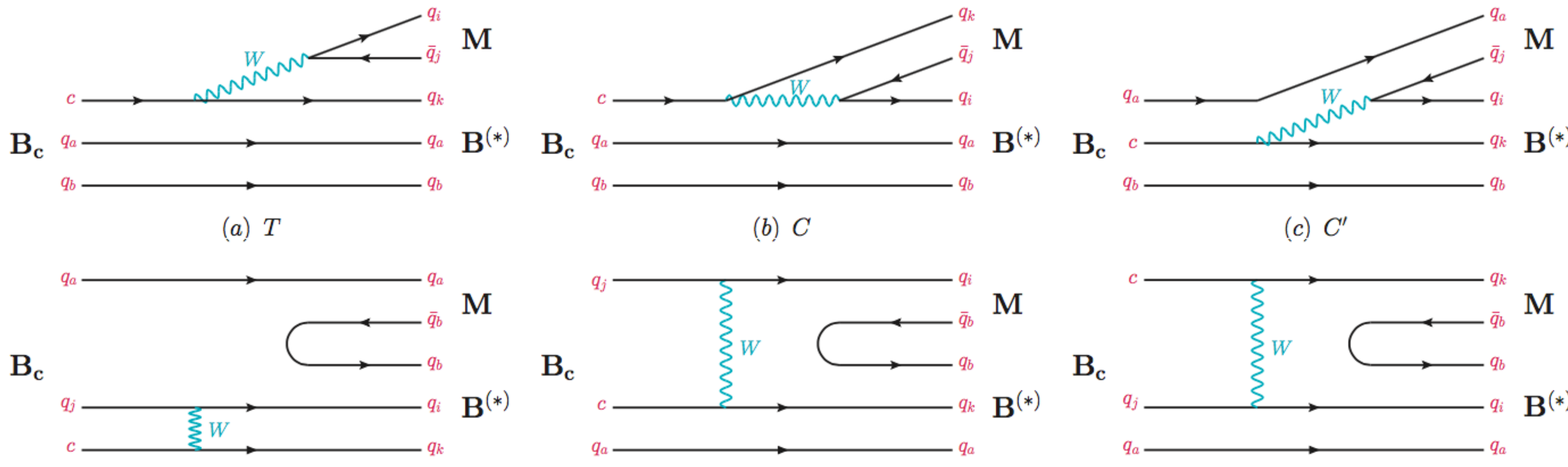
$$T(\mathbf{B}_c \to \mathbf{B}^* M) = E_{\mathbf{B}}(\mathbf{B}_c)^{ja} H^{ki}_j (\mathbf{B}^*)_{kab}(M)^b_i + E_M(\mathbf{B}_c)^{ja} H^{ki}_j (\mathbf{B}^*)_{iab}(M)^b_k$$

$$+C'(\mathbf{B}_c)^{ab} H^{ki}_j (\mathbf{B}^*)_{ikb}(M)^j_a + E'(\mathbf{B}_c)^{ka} H^{ki}_j (\mathbf{B}^*)_{ikb}(M)^b_a \,.$$

$$T(\mathbf{B}_c \to \mathbf{B}' M) =$$

$$a_8(\mathbf{B}')_{ijk}(\mathbf{B}_c)_l H_{nm}(6)(M)^i_o \epsilon^{jln}\epsilon^{kmo} + a_9(\mathbf{B}')_{ijk}(M)^i_l H(\overline{15})^{jn}_m (\mathbf{B}_c)_n \epsilon^{klm}$$

$$+a_{10}(\mathbf{B}')_{ijk}(M)^i_l H(\overline{15})^{jk}_m (\mathbf{B}_c)_n \epsilon^{lmn} + a_{11}(\mathbf{B}')_{ijk}(M)^l_m H(\overline{15})^{ij}_l (\mathbf{B}_c)_n \epsilon^{kmn} \,,$$



(a) T (b) C (c) C'
(d) E' (e) E_B (f) E_M

- $\mathbf{B}_c$ with $(q_a q_b - q_b q_a)c$ cannot be turned into $\mathbf{B}^*(q_a q_b q_{k(i)})$:

(T, C) give no contributions to $\mathbf{B}_c \to \mathbf{B}^* M$.

- The Körner-Pati-Woo theorem:

K. Miura and T. Minamikawa, Prog. Theor. Phys. **38**, 954 (1967);

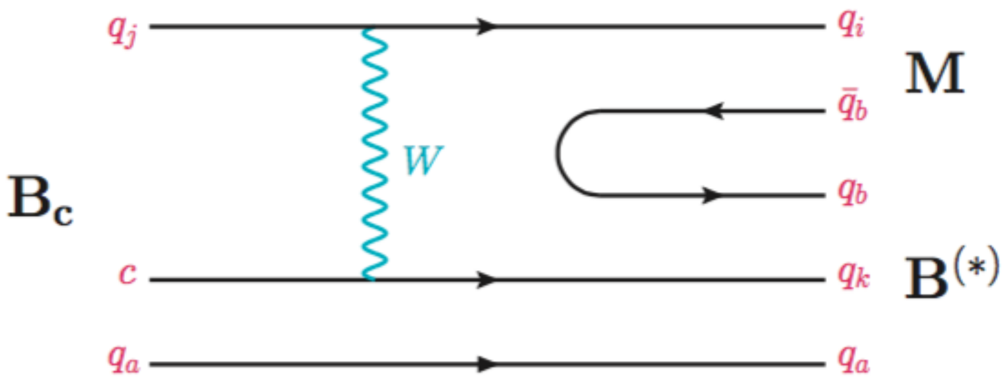
J.G. Korner, NPB25, 282 (1971);

J.C. Pati and C.H. Woo, PRD3, 2920 (1971).

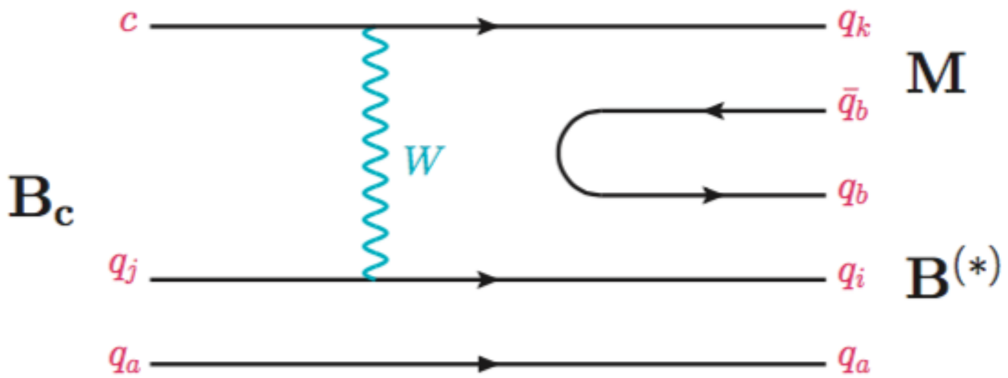
$(\bar{q}_i q_j)_{V-A}(\bar{q}_k c)_{V-A}$: q_i and q_k are color anti-symmetric,

$q_{i,k}$ are flavor anti-symmetric in $\mathbf{B}^{(*)}$,

C' and E' are suppressed in $\mathbf{B}_c \to \mathbf{B}^* M$.



$(e)\ E_{\mathbf{B}}$



$(f)\ E_M$

Equivalence:

$$(E_{\mathbf{B}}, E_M) = (-2a_8 + a_9, 2a_8 + a_9)$$

$$(E', C') = (-2a_9 - 2a_{10}, -2a_{11})$$

$$a_9 = -a_{10} \text{ and } a_{11} = 0$$

Decay modes	T-amp
$\Lambda_c^+ \to \Delta^{++} K^-$	$-\lambda_a E_M$
$\Lambda_c^+ \to \Delta^+ \bar{K}^0$	$-\lambda_a \frac{1}{\sqrt{3}} E_M$
$\Lambda_c^+ \to \Sigma^{*0} \pi^+$	$-\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$
$\Lambda_c^+ \to \Sigma^{*+} \pi^0$	$-\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$
$\Lambda_c^+ \to \Sigma^{*+} \eta$	$-\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} c\phi - \sqrt{2} E_M^{(s)} s\phi)$
$\Lambda_c^+ \to \Sigma^{*+} \eta'$	$-\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} s\phi + \sqrt{2} E_M^{(s)} c\phi)$
$\Lambda_c^+ \to \Xi^{*0} K^+$	$-\lambda_a \frac{1}{\sqrt{3}} E_{\mathbf{B}}$

Decay modes	T-amp
$\Xi_c^0 \to \Sigma^{*+} K^-$	$\lambda_a \frac{1}{\sqrt{3}} E_M$
$\Xi_c^0 \to \Sigma^{*0} \bar{K}^0$	$\lambda_a \frac{1}{\sqrt{6}} E_M$
$\Xi_c^0 \to \Xi^{*-} \pi^+$	$\lambda_a \frac{1}{\sqrt{3}} E_{\mathbf{B}}$
$\Xi_c^0 \to \Xi^{*0} \pi^0$	$\lambda_a \frac{1}{\sqrt{6}} E_{\mathbf{B}}$
$\Xi_c^0 \to \Xi^{*0} \eta$	$\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} c\phi - \sqrt{2} E_M^{(s)} s\phi)$
$\Xi_c^0 \to \Xi^{*0} \eta'$	$\lambda_a \frac{1}{\sqrt{6}} (E_{\mathbf{B}} s\phi + \sqrt{2} E_M^{(s)} c\phi)$
$\Xi_c^0 \to \Omega^- K^+$	$\lambda_a E_{\mathbf{B}}^{(s)}$
$\Xi_c^+ \to \Sigma^{*+} \bar{K}^0$	0
$\Xi_c^+ \to \Xi^{*0} \pi^+$	0

- Three triangle sum rules for $\mathbf{B}_c \to \Delta\pi$

$$T(\Lambda_c^+ \to \Delta^0 \pi^+) - T(\Lambda_c^+ \to \Delta^{++} \pi^-) - \sqrt{6} T(\Lambda_c^+ \to \Delta^+ \pi^0) = 0\,,$$

$$T(\Xi_c^+ \to \Delta^0 \pi^+) - T(\Xi_c^+ \to \Delta^{++} \pi^-) - \sqrt{6} T(\Xi_c^+ \to \Delta^+ \pi^0) = 0\,,$$

$$T(\Xi_c^0 \to \Delta^+ \pi^-) - T(\Xi_c^0 \to \Delta^- \pi^+) - \sqrt{6} T(\Xi_c^0 \to \Delta^0 \pi^0) = 0\,.$$

- $C' = 0$ (Körner-Pati-Woo theorem)

resulting in

$T(\Lambda_c^+ \to \Delta^+ K^0, \Delta^0 K^+) = 0,$

$T(\Xi_c^+ \to \Sigma^{*+} \bar{K}^0, \Xi^{*0}\pi^+) = 0.$

Existing models also give $\mathcal{B} = 0$.

In comparison with the data:

$\mathcal{B}_{ex}(\Xi_c^+ \to \Xi^{*0}\pi^+) < 4.0 \times 10^{-3},$

$\mathcal{B}_{ex}(\Xi_c^+ \to \Sigma^{*+}\bar{K}^0) = (2.9 \pm 1.7) \times 10^{-2}.$

- $T(\Lambda_c^+ \to \Sigma^{*0}\pi^+) = T(\Lambda_c^+ \to \Sigma^{*+}\pi^0)$

respects the isospin symmetry.

Summary

• We demonstrated that IRA and TDA for $\mathbf{B}_c \to \mathbf{B}^{(*)}M$

Equivalent $SU(3)_f$ approaches.

• We explained $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+, \Xi^0 \pi^+)$ with

$SU(3)_f$ symmetry breaking.

• $\mathcal{B}(\Lambda_c^+ \to p\pi^0)$ and $\mathcal{B}(\Lambda_c^+ \to n\pi^+)$ receive the contributions from

the destructive and constructive interfering effects, respectively.

• The exchange topology E_M plays a key role in

the $\Xi_c^0 \to \Lambda^0 \bar{K}^0, \Sigma^0 \bar{K}^0, \Sigma^+ K^-$ decays.

• $\mathcal{B}_{ex}(\Xi_c^+ \to \Sigma^{*+} \bar{K}^0) = (2.9 \pm 1.7) \times 10^{-2}$

needs a more careful experimental investigation.

Thank You