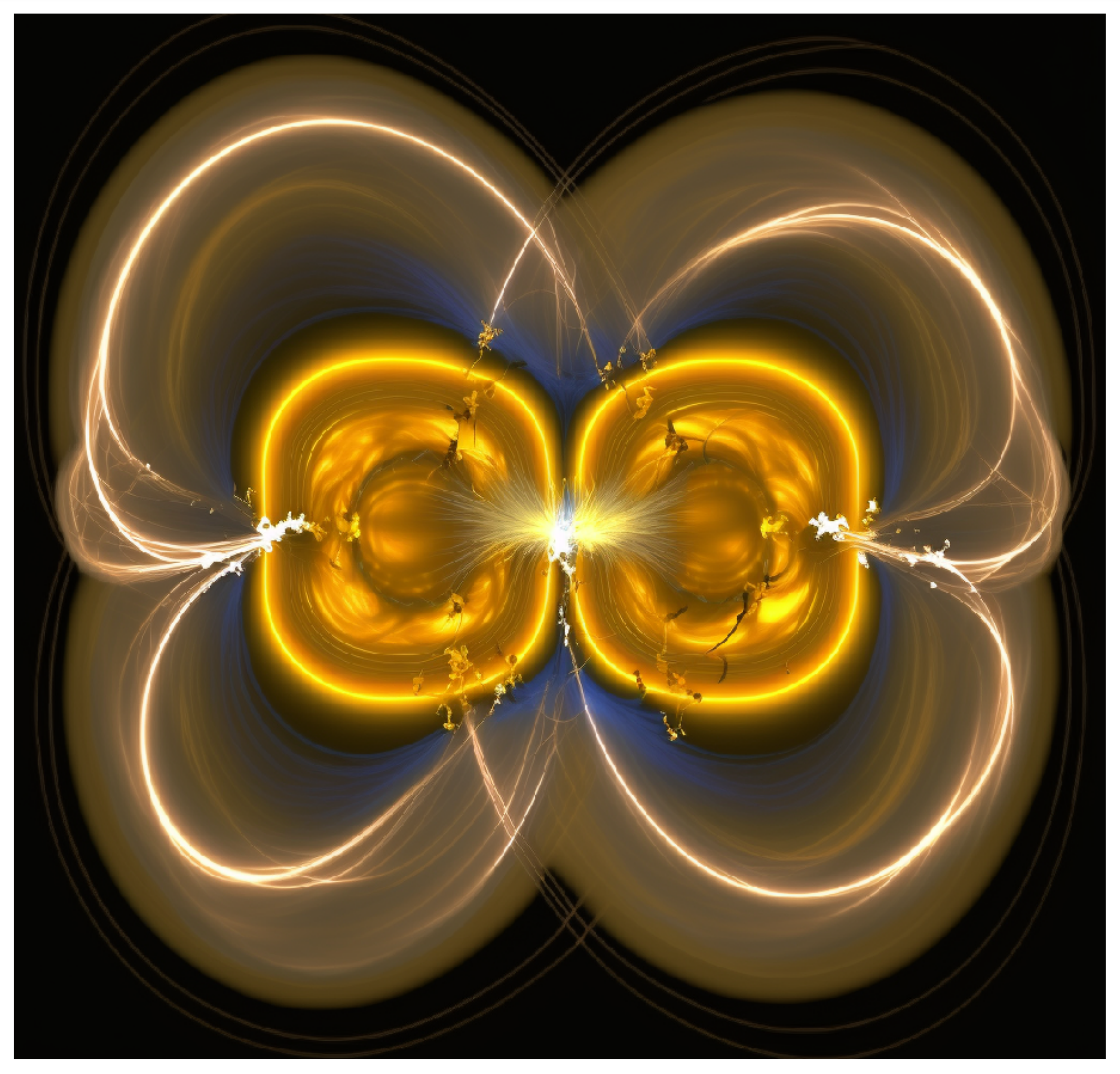
Holographic CME Current

华南师范大学 | 殷雷

Holographic QCD Seminar XIV



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Outline

1. 寻找和计算 CME 电流的困难
2. 全息理论计算 CME 电流的一种思路
3. 三点函数的一些结果和讨论

Yin, L., Hou, D., & Ren, H. C. (2021). Chiral magnetic effect and three-point function from AdS/CFT correspondence. *Journal of High Energy Physics*, *2021*(9), 1-36.

CME Current, Chiral Matter and Response to Magnetic Field

Ohm's law

Normal current

Response to a Electric Field $\mathbf{J} \propto C\,\mathbf{E}$

→ Ordinary Medium

Chiral Magnetic Effect

CME current

Response to a Magnetic Field $\mathbf{J} \propto C_A\,\mathbf{B}$ Quantum anomaly-induced process

Strong Magnetic Field

Chiral Anomaly

→ Chiral Medium

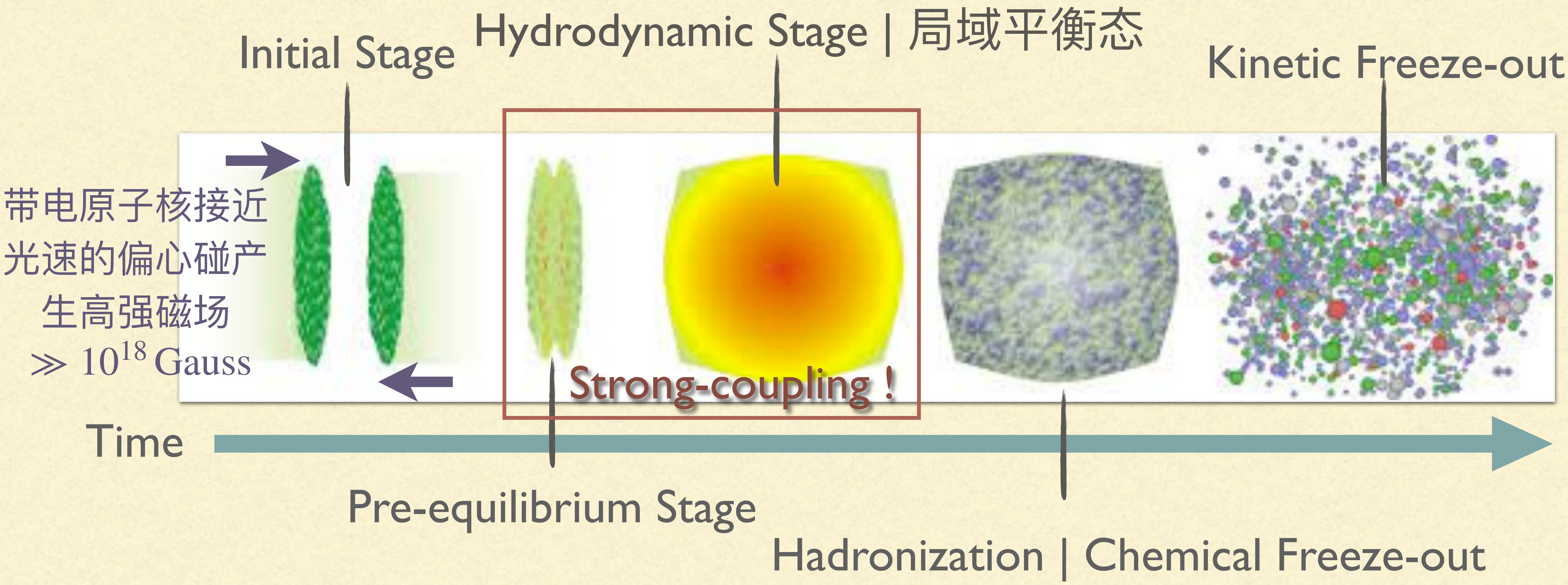
A Physical System

Chiral Imbalance

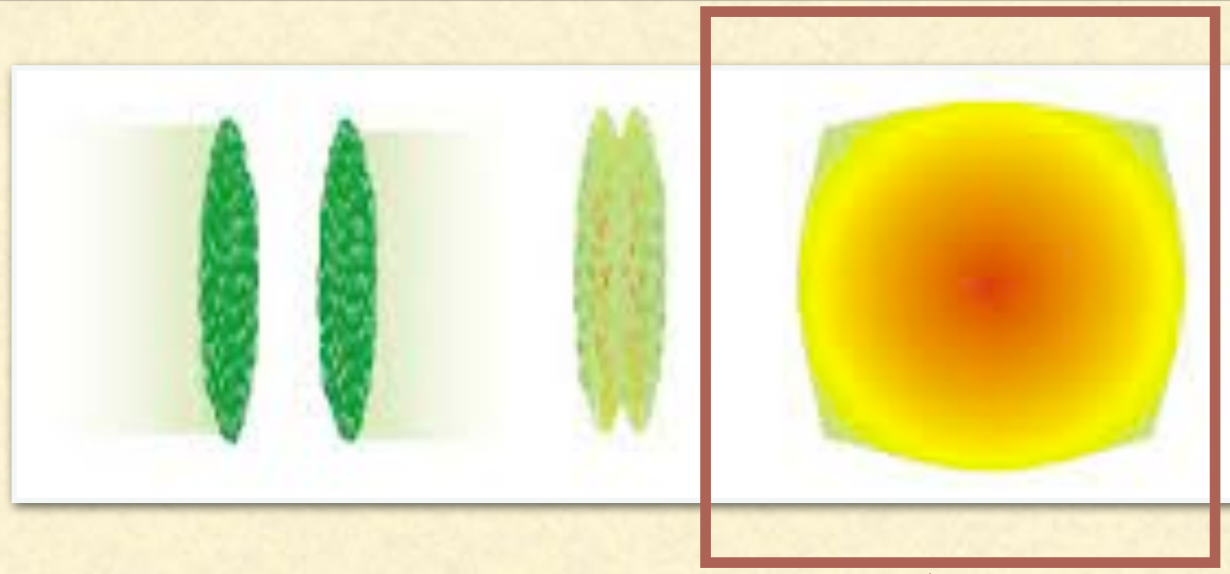
Axial Current non-conservation: $\partial_\mu J_A \neq 0$

The anomaly system: QGP

特征：在非常小的范围里；非常短的时间内发生了剧烈和强劲的物理变化



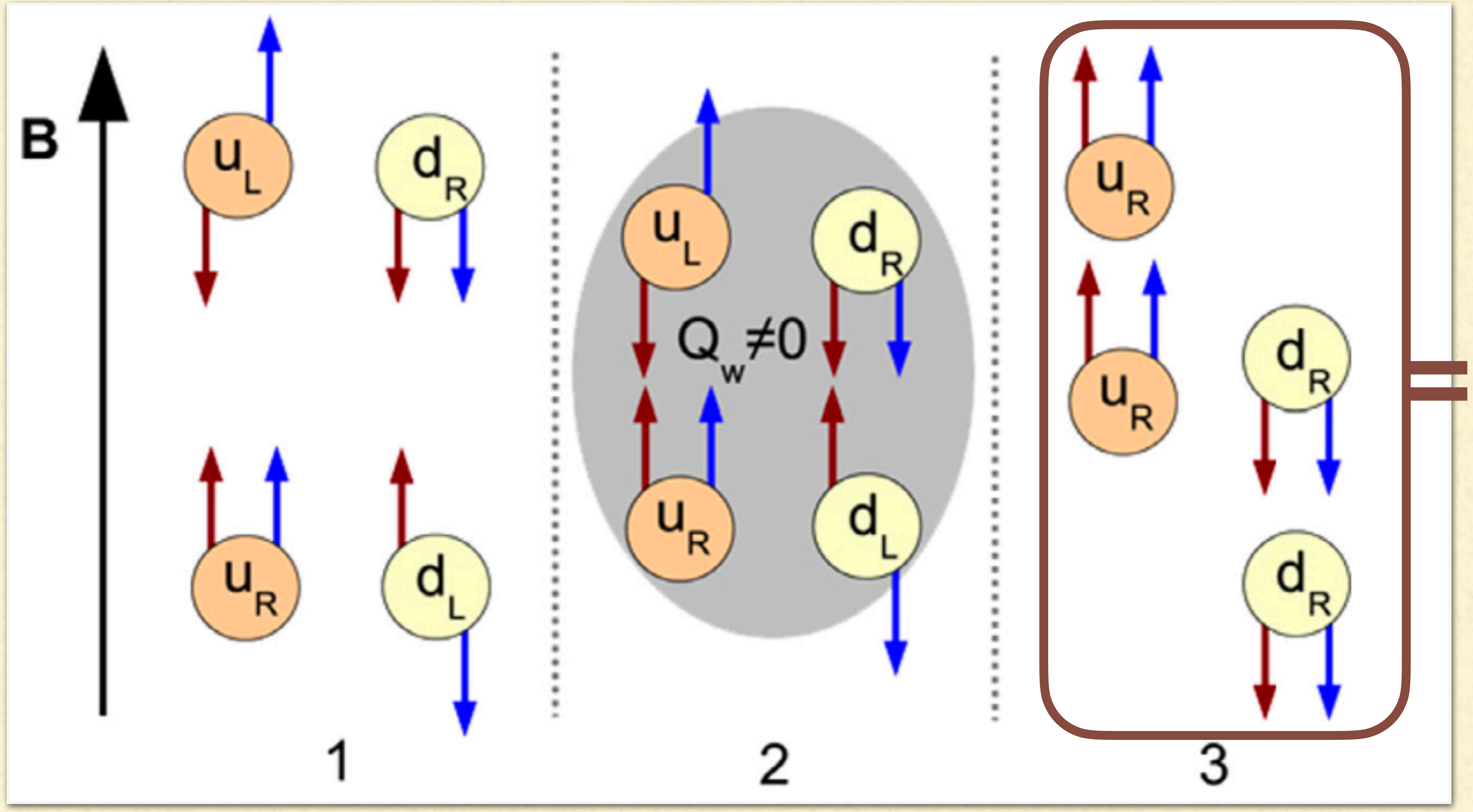
Time Scale: ~ 10 femtoseconds (10^{-15} s)   Length Scale: ~ 10 femtometers (10^{-15} m)



CME 电流具有 Chiral non-equilibrium 的特征

净拓扑荷不为零：夸克（反夸克）手性改变

Chiral Medium



CME 电流产生

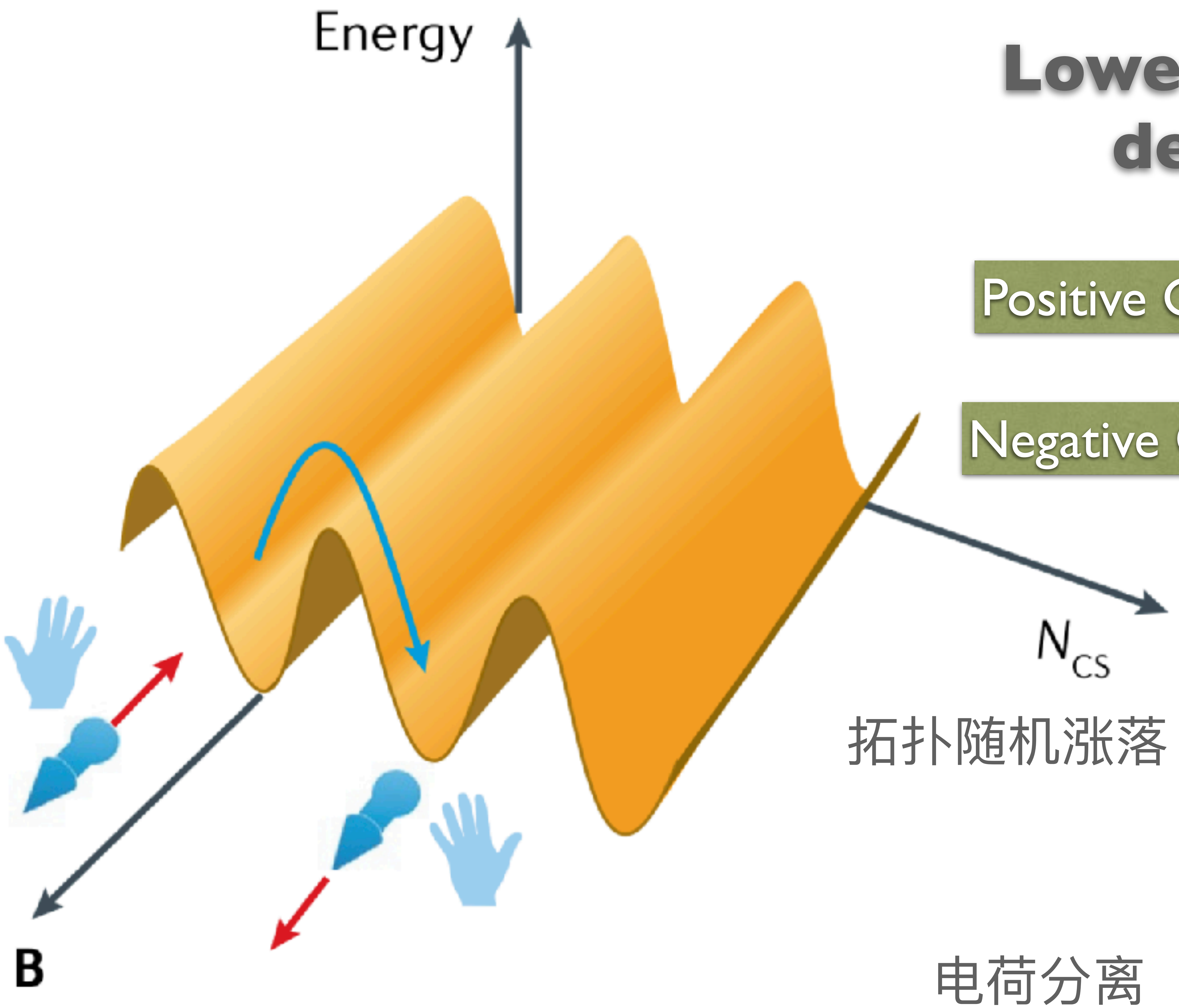
Strong magnetic field

+

Chiral Imbalance

Topological excitation of gluon field

For Chiral Matter: massless quarks or anti-quarks



Lowest Landau Level: Electric charges determine the direction of spin

Positive Charge ⟷ Spin $\vec{S}$ aligned along Magnetic Field $\vec{B}$

Negative Charge ⟷ Spin $\vec{S}$ opposite to Magnetic Field $\vec{B}$

拓扑随机涨落 → 动量方向变更 → 手征改变 → 手征不均衡 → CME 电流 → 电荷分离

1.Kharzeev, D. E., & Liao, J. (2021). *Nature Reviews Physics*, *3*(1), 55-63.

2.Huang, A., Shi, S., Lin, S., Guo, X., & Liao, J. (2023). *Physical Review D*, *107*(3), 034012.

Difficulties in Finding the CME Current Signal

Background effects	CME signal	v.s	Elliptic Flow ….
Event-by-event fluctuations	CME signal	v.s	Various Fluctuations in the Data
Finite size effects	CME signal	v.s	Extracting from Non-equilibrium
Magnetic field uncertainties	Theoretical Prediction	v.s	Real Situation in QGP
……			

1. Shi, S., Zhang, H., Hou, D. and Liao, J., (2020). *Physical Review Letters*, *125*(24), p.242301.
2. Kharzeev, D. E., McLerran, L. D., & Warringa, H. J. (2008). Nuclear Physics A, 803(3-4), 227-253.
3. Skokov, V., Illarionov, A., & Toneev, V. (2009). International Journal of Modern Physics A, 24(30), 5925-5932.
4. Bzdak, A., Koch, V., Liao, J., & Voloshin, S. (2012). Physics Letters B, 710(1), 171-175.
5. Wang, G., Huang, X. G., & Yang, Y. (2019). Progress in Particle and Nuclear Physics, 107, 237-302.
6. …..

Various Equilibriums

which kinds of equilibrium in high energy experiments ?

Thermodynamic Equilibrium	Macroscopic Properties: Temperature, Pressure, …., Chemical Potential Local Equilibrium = Local Thermodynamic Equilibrium
Electromagnetic Equilibrium	Spacetime-independent Electric and Magnetic Field Charges and Currents are distributed in such a way that the fields are stable.
Chiral Equilibrium	Homogeneous and Stable Chiral Imbalance The difference in the number of life-handed and right-handed quarks(anti-quarks) doesn't vary in spacetime
。。。。。。。	

One Local Equilibriums & Two non-equilibrium

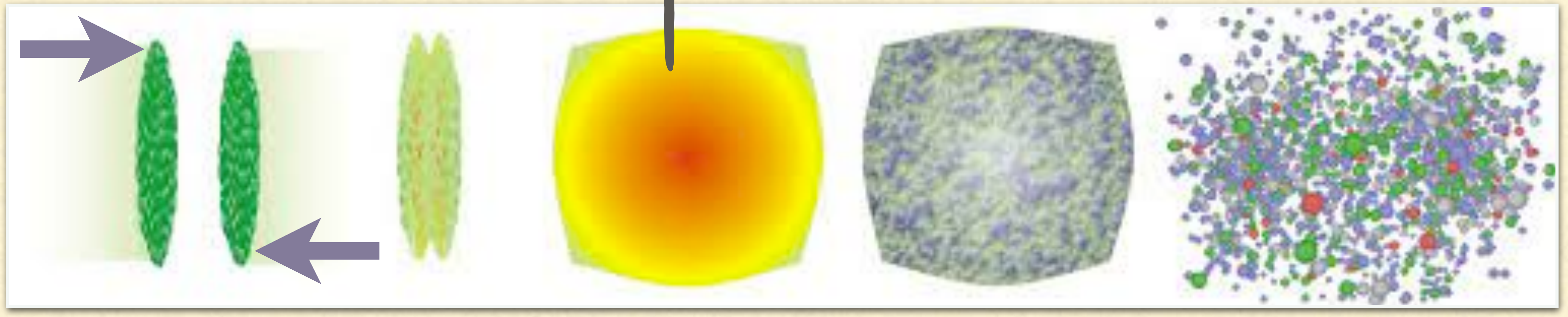
Electromagnetic non-Equilibrium

Hydrodynamic Stage

Local Thermodynamic Equilibrium

EM-field varies rapidly !

Long-wavelength Description works !



Chiral non-Equilibrium

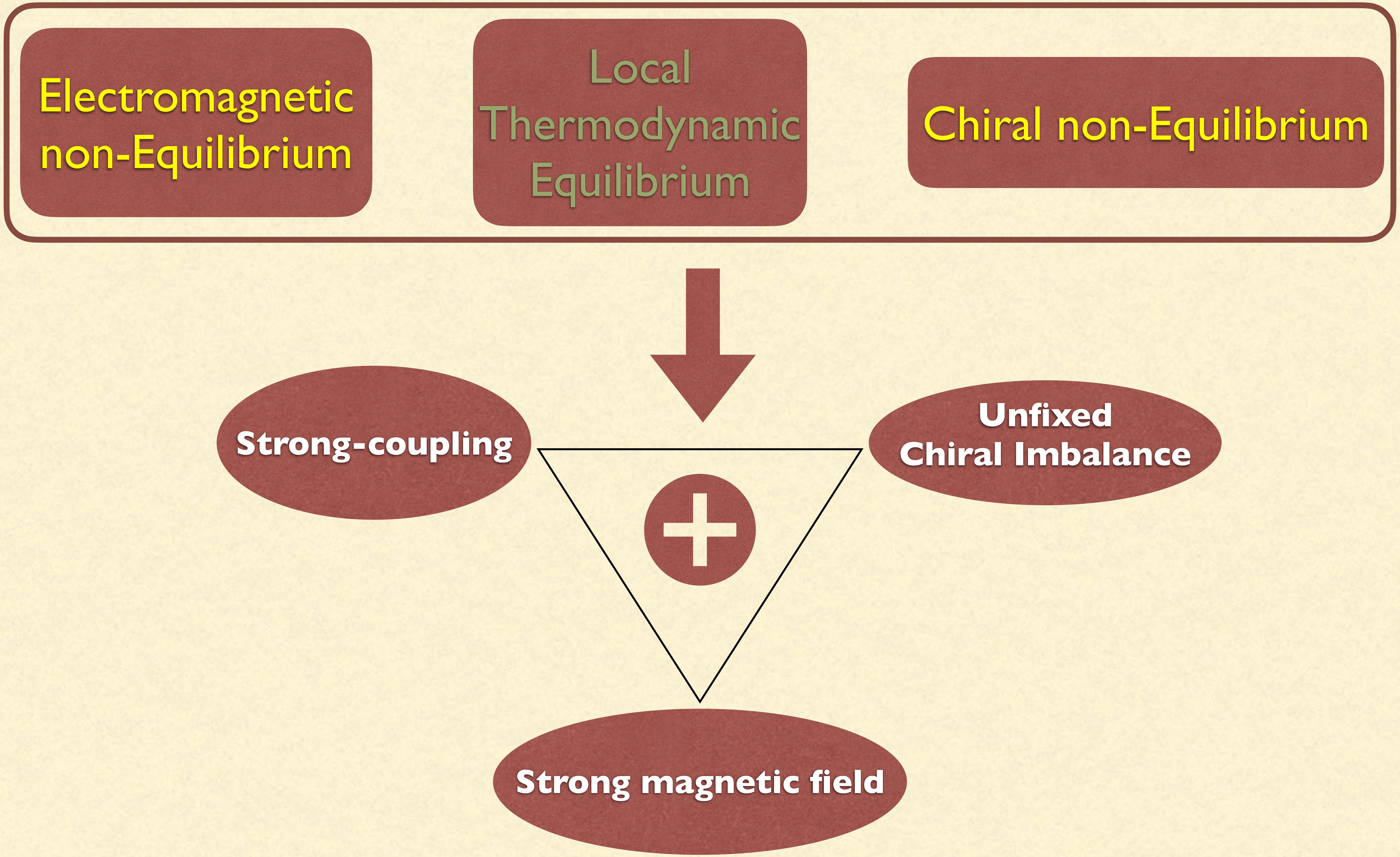
胶子场真空的非平庸涨落激发导致随机的「手征不均衡」

QGP火球的剧烈迅猛演化

Uniform Chiral Imbalance

$$\mu_A = \frac{\mu_R - \mu_L}{2}$$

Theoretical Challenges in CME



AdS/CFT解决方案

$D=3+1;\ \mathcal{N}=4$ SYM = IIB Strings on $AdS_5 \times S^5$

Large 't Hooft limit: $\lambda \gg 1$

Large N_c limit: $N_c \to \infty$

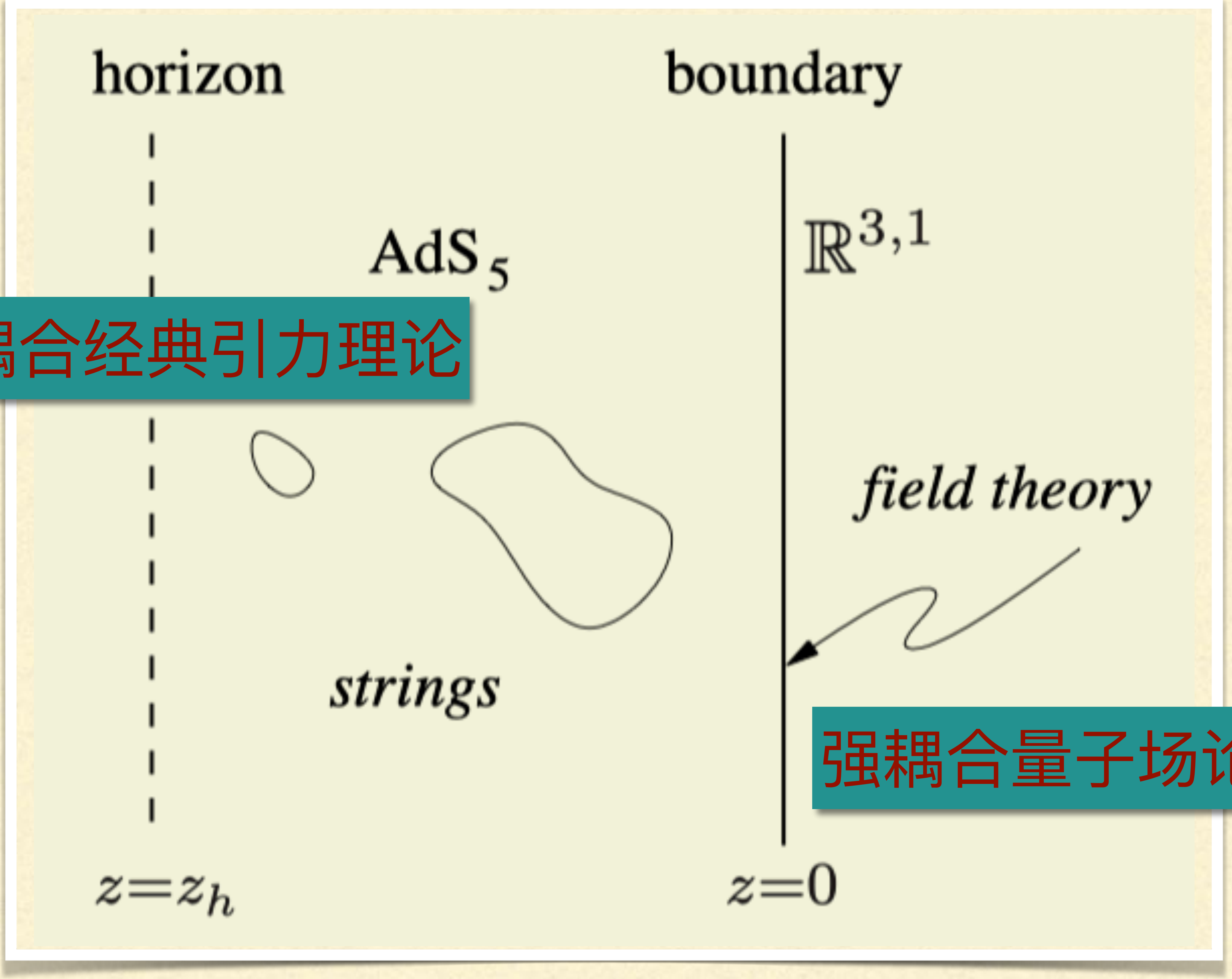
强耦合量子场论的自由度

↕

经典引力理论的动力学

1. 相变序参量...
2. 自由能，热力学参数（温度，熵，化学势，轴化学势）...
3. 各类关联函数，电极化率，磁化率...
4. 粘滞系数...
5. 手征不均衡的量度

引力场方程组，regular conditions，稳定性条件。。。。。



CME Current $J^{\mu}(q) = \Pi^{\mu\nu}(q)V_{\nu}(q) + J^{\mu}_{CME}(q)$

Local Thermodynamic Equilibrium

$$J^{\mu}_{CME}(q) = J^{\mu}(q) + J_{AVV}(q)$$

Electromagnetic non-Equilibrium

Uniform Chiral Imbalance 可以定义 μ_A → 磁场具有时空依赖 $\vec{J} = 8C\,\mu_A\,\vec{B}$

Local Chiral Equilibrium

1. Yee, H.-U. (2011). Journal of High Energy Physics, 2011(11), 1-18.
2. Landsteiner, K., Megías, E., & Pena-Benitez, F. (2011). Journal of High Energy Physics, 2011(09), 1-18.
3. Landsteiner, K., Megías, E., & Pena-Benitez, F. (2013). Journal of High Energy Physics, 2013(05), 1-20.
4. Bu, Y., Lublinsky, M., & Sharon, A. (2016). Part I. Journal of High Energy Physics, *2016*(11), 1-42.
5. Bu, Y., Lublinsky, M., & Sharon, A. (2017). Part II. *The European Physical Journal C*, *77*, 1-17.
6. ……….

CME Current

$$J^{\mu}_{\text{CME}}(q) = \mathsf{J}^{\mu}(q) + J_{\text{AVV}}(q)$$

Local Thermodynamic Equilibrium

Electromagnetic non-Equilibrium

Chiral non-Equilibrium

$q_1 = (\omega_1, \mathbf{q}_1)$

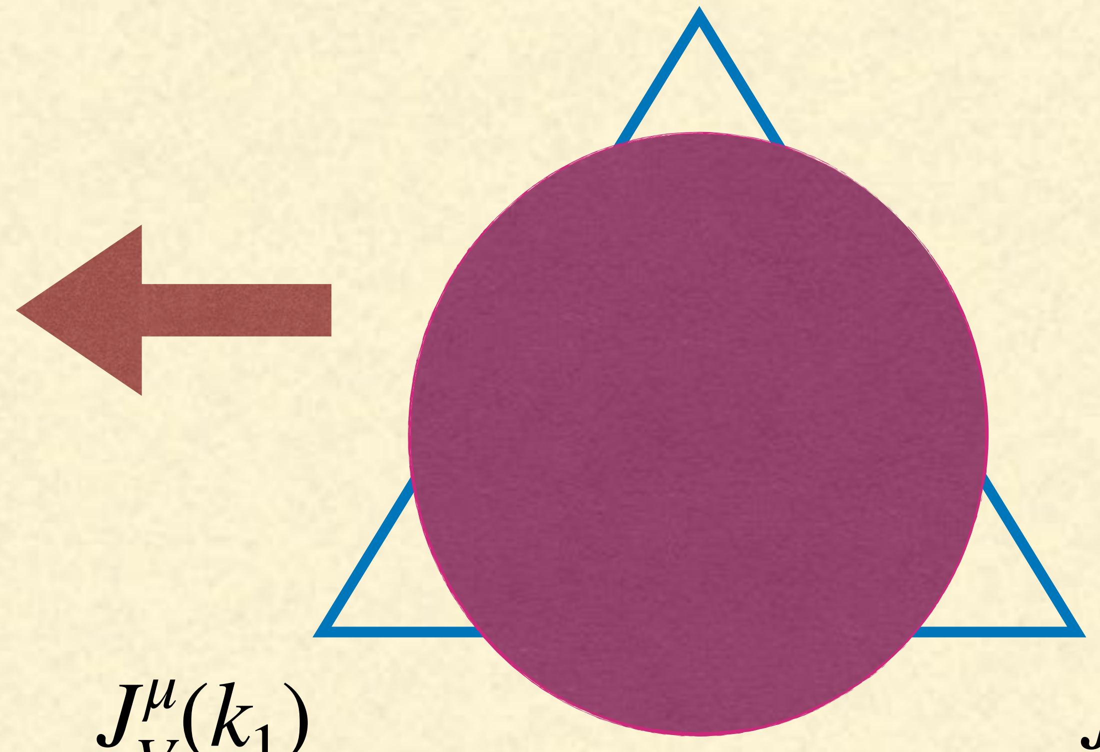
$q_2 = (\omega_2, \mathbf{q}_2)$

$$J^{\mu}_{\text{AVV}}(q) = \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} (2\pi)^4 \delta^4(q_1 + q_2 - q)\ \Lambda^{\mu\nu\rho}(q_1, q_2)\ \mathsf{A}_{\rho}(q_2) \mathsf{V}_{\nu}(q_1)$$

Full AVV three-point correlation : $\Lambda^{\mu\nu\rho}(q_1, q_2) = \Delta^{\rho\mu\nu}(-q_1 - q_2, q_1)$

Furry 定理不起作用

$$\langle J^{\rho}_{A}(-k_1 - k_2) J^{\mu}_{V}(k_1) J^{\nu}_{V}(k_2) \rangle = \Delta^{\rho\mu\nu}(k_1, k_2)$$



Task: 求解强耦合三点函数 AVV

Chiral Anomalies in $\mathcal{N}=4$ SYM theory at large N_c and Strong `t Hooft Coupling

$$S = S_{\text{EH}} + S_{\text{MCS}} + S_{\text{c.t.}}$$

$$S_{\text{EH}} = \kappa_{\text{EH}} \int d^5x \sqrt{-g}\,(R - 2\Lambda)$$

Maxwell Sector

$$S_{\text{MCS}} = \kappa_{\text{M}} \int d^5X \sqrt{-g} \left[-\frac{1}{4}F_{\text{V}}^2 - \frac{1}{4}F_{A}^2 + \frac{\kappa_{\text{CS}}}{4\sqrt{2}\kappa_{\text{M}}\sqrt{-g}} \varepsilon^{MNOPQ}\left(3A_M F^{\text{V}}_{NO}F^{\text{V}}_{PQ} + A_M F^{A}_{NO}F^{A}_{PQ}\right)\right]$$

Chern-Simons Sector

$$J^{\mu}_{\text{V}}(x) \equiv \frac{\delta S}{\delta \text{V}_\mu} = \left[-\kappa_{\text{M}} (\text{F}_{\text{V}})^{5\mu}\sqrt{-g} + \frac{3\,\kappa_{\text{CS}}}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma}\,\text{A}_\nu (\text{F}_{\text{V}})_{\rho\sigma} \right]\Bigg|_{\text{AdS-boundary}}$$

$$J^{\mu}_{\text{A}}(x) \equiv \frac{\delta S}{\delta \text{A}_\mu} = \left[-\kappa_{\text{M}} (\text{F}_{\text{A}})^{5\mu}\sqrt{-g} + \frac{\kappa_{\text{CS}}}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma}\,\text{A}_\nu (\text{F}_{\text{A}})_{\rho\sigma} \right]\Bigg|_{\text{AdS-boundary}}$$

$$C = \frac{3\kappa_{\text{CS}}}{4\sqrt{2}}$$

$$\partial_\mu J^{\mu}_{\text{V}}(x) = 0$$

$$\partial_\mu J^{\mu}_{\text{A}}(x) = \frac{C}{3}\epsilon^{\mu\nu\rho\sigma}\left[3\,(F_{\text{V}})_{\mu\nu}(F^{\text{V}})_{\rho\sigma} + (F^{\text{A}})_{\mu\nu}(F^{\text{A}})_{\rho\sigma}\right]\Bigg|_{\text{AdS-boundary}}$$

Troublesome Backreaction

$$\nabla_N\left[(\mathrm{F}^{\mathrm{V}})^{NM}\sqrt{-g}\right] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{\mathrm{CS}}}{\kappa_{\mathrm{M}}}\cdot\epsilon^{MNOPQ}(\mathrm{F}^{\mathrm{A}})_{NO}(\mathrm{F}^{\mathrm{V}})_{PQ}$$

$$\nabla_N\left[(\mathrm{F}^{\mathrm{A}})^{NM}\sqrt{-g}\right] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{\mathrm{CS}}}{\kappa_{\mathrm{M}}}\cdot\frac{1}{2}\epsilon^{MNOPQ}\left[(\mathrm{F}^{\mathrm{V}})_{NO}(\mathrm{F}^{\mathrm{V}})_{PQ}+(\mathrm{F}^{\mathrm{A}})_{NO}(\mathrm{F}^{\mathrm{A}})_{PQ}\right]$$

Key to Probe Limit

$$R_{MN}-\frac{1}{2}Rg_{MN}-\Lambda\, g_{MN} = -\frac{\kappa_{\mathrm{M}}}{\kappa_{\mathrm{EH}}}T_{MN}$$

Bad News: $\{\kappa_{\mathrm{M}}, \kappa_{\mathrm{EH}}\} \to N_c^2$ as the large N_c limit !

$$T_{MN} = \frac{2}{\kappa_{\mathrm{M}}}\frac{\delta S_{\mathrm{MCS}}}{\delta g^{MN}}$$

No coupling constant

$$= (\mathrm{F}^{\mathrm{V}})_M^{\ L}(\mathrm{F}^{\mathrm{V}})_{NL} - \frac{1}{4}g_{MN}(\mathrm{F}^{\mathrm{V}})_{KL}(\mathrm{F}^{\mathrm{V}})^{KL} + (\mathrm{F}^{\mathrm{A}})_M^{\ L}(\mathrm{F}^{\mathrm{A}})_{NL} - \frac{1}{4}g_{MN}(\mathrm{F}^{\mathrm{A}})_{KL}(\mathrm{F}^{\mathrm{A}})^{KL}\,.$$

1. Freedman, D. Z., Mathur, S. D., Matusis, A., & Rastelli, L. (1999). *Nuclear Physics B*, *546*(1-2), 96-118.
2. Policastro, G., Son, D. T., & Starinets, A. O. (2002). *Journal of High Energy Physics*, *2002*(09), 043.

Troublesome Back-reaction

$\frac{\kappa_M}{\kappa_{EH}} = \mathcal{O}(1)$ → Dynamics of gauge fields trigger the back-reaction of metric field

Beyond the AdS/CFT Correspondence, the condition of large N_c limit can be relaxed

General Gauge/Gravity Duality: $$R_{MN} - \frac{1}{2}g_{MN} - \Lambda g_{MN} = -\frac{\kappa_M}{\kappa_{EH}} T_{MN} \approx 0$$

A fixed Schwarzschild-AdS geometry and fluctuated gauge fields are obtained in this way

Advantage of keeping the large N_c condition

Strong-cooling limit of QCD with temperatures ⟷ Super Yang-Mills AdS/CFT Correspondence

引力和规范场的退耦合“窗口” | No Probe Limit !

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\,,\ \mathrm{A}_\mu = \bar{\mathrm{A}}_\mu + \mathbb{A}_\mu\,,\ \mathrm{V}_\mu = \bar{\mathrm{V}}_\mu + \mathbb{V}_\mu$$

Chern-Simons term $S_{\rm CS} \sim \varepsilon^{\cdots} AFF$ 必然会有规范场的二级涨落贡献：

CME“退耦合”的要点是任何规范场的二级涨落不影响 bulk 时空几何

背景有矢量规范场的power-structure： e.g. $V_0 \neq 0$

$$\nabla_N[(\mathrm{F^V})^{NM}\sqrt{-g}] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{\rm CS}}{\kappa_{\rm M}}\cdot\epsilon^{MNOPQ}(\mathrm{F^A})_{NO}(\mathrm{F^V})_{PQ}$$

$$\nabla_N[(\mathrm{F^A})^{NM}\sqrt{-g}] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{\rm CS}}{\kappa_{\rm M}}\cdot\frac{1}{2}\epsilon^{MNOPQ}\Big[(\mathrm{F^V})_{NO}(\mathrm{F^V})_{PQ} + (\mathrm{F^A})_{NO}(\mathrm{F^A})_{PQ}\Big]$$

Vector gauge field : $\mathcal{O}(\mathbb{V}) + \mathcal{O}(h\mathbb{V}) = \mathcal{O}(\mathbb{A}\mathbb{V})$

Axial gauge field : $\mathcal{O}(\mathbb{A}) + \mathcal{O}(h\mathbb{A}) = \mathcal{O}(\mathbb{A}^2) + \mathcal{O}(\mathbb{V}^2)$

Metric field : $\mathcal{O}(h) = \mathcal{O}(\mathbb{V})$

$$R_{MN} - \frac{1}{2}Rg_{MN} - \Lambda\, g_{MN} = -\frac{\kappa_{\rm M}}{\kappa_{\rm EH}}T_{MN}$$

引力场一级涨落 h 和背景规范场的一级涨落 $\mathbb{V}$ 同阶！ ➡ CME模型引起时空反馈

引力和规范场的退耦合“窗口” | No Probe Limit !

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\,,\ \mathrm{A}_\mu = \bar{\mathrm{A}}_\mu + \mathbb{A}_\mu\,,\ \mathrm{V}_\mu = \bar{\mathrm{V}}_\mu + \mathbb{V}_\mu$$

Chern-Simons term $S_{\rm CS} \sim \varepsilon^{\cdots} AFF$ 必然会有规范场的二级涨落贡献：

背景无任何规范场的 power-structure：$(\bar{A} = \bar{V} \equiv 0)$

Vector gauge field :	$\mathcal{O}(\mathbb{V}) + \mathcal{O}(h\mathbb{V}) = \mathcal{O}(\mathbb{A}\mathbb{V})$
Axial gauge field :	$\mathcal{O}(\mathbb{A}) + \mathcal{O}(h\mathbb{A}) = \mathcal{O}(\mathbb{A}^2) + \mathcal{O}(\mathbb{V}^2)$
Metric field :	$\mathcal{O}(h) = \mathcal{O}(\mathbb{V}^2) + \mathcal{O}(\mathbb{A}^2)$

只要体理论背景解没有规范场，那么度规场的涨落只和规范场的三级涨落发生backreaction！

度规场的一级涨落和规范场的二级涨落同阶！

非线性涨落规范场的领头项行为和引力场退耦合 → CME 问题有望“可解”

CME Holographic Set-up | Leading term of CS-Sector

$$S_{\rm MCS} \equiv \kappa_{\rm M}\int d^5X\sqrt{-g}\left[-\frac{1}{4}F_V^2-\frac{1}{4}F_A^2+\frac{\kappa_{\rm CS}}{4\sqrt{2}\kappa_{\rm M}\sqrt{-g}}\varepsilon^{MNOPQ}\left(3A_MF^V_{NO}F^V_{PQ}+A_MF^A_{NO}F^A_{PQ}\right)\right]$$

$\kappa_{\rm CS}$ dimensionless

Chern-Simons Sector

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}\,,\ A_\mu=\bar{A}_\mu+\mathbb{A}_\mu\,,\ V_\mu=\bar{V}_\mu+\mathbb{V}_\mu \qquad \bar{V}=\bar{A}\equiv 0$$

Schwarzschild-AdS$_5$ 时空

Horizon $u=1$

AdS-Boundary $u=0$

$$ds_5^2=\bar{g}_{MN}dx^Mdx^N=\frac{(\pi LT)^2}{u}\left(-f(u)\,dt^2+\sum_{i=1}^{3}d(x^i)^2\right)+\frac{1}{4u^2f(u)}du^2\,,\ f(u)=1-u^2$$

$$\partial_N\left[\bar{g}^{NP}\bar{g}^{MQ}(\mathbb{F}^V)_{PQ}\sqrt{-\bar{g}}\right]=-\frac{3}{2\sqrt{2}}\frac{\kappa_{\rm CS}}{\kappa_{\rm M}}\cdot\epsilon^{MNOPQ}(\mathbb{F}^A)_{NO}(\mathbb{F}^V)_{PQ}$$

$$\partial_N\left[\bar{g}^{NP}\bar{g}^{MQ}(\mathbb{F}^A)_{PQ}\sqrt{-\bar{g}}\right]=-\frac{3}{2\sqrt{2}}\frac{\kappa_{\rm CS}}{\kappa_{\rm M}}\cdot\frac{1}{2}\epsilon^{MNOPQ}\left[(\mathbb{F}^V)_{NO}(\mathbb{F}^V)_{PQ}+(\mathbb{F}^A)_{NO}(\mathbb{F}^A)_{PQ}\right]$$

采用的AdS 边界条件:

$$\lim_{u\to 0}\mathbb{V}_\mu(q\,|\,u)=\left(0,\vec{\mathbb{V}}(q)\right);\qquad \lim_{u\to 0}\mathbb{A}_\mu(q\,|\,u)=\left(\mathbb{A}_0(q),0,0,0\right)$$

$$q=(\omega,\vec{q})$$

EM non-equilibrium: $\vec{\mathbb{V}}(q)$

Chiral non-equilibrium: $\mathbb{A}_0(q)$

CME Holographic Set-up | Orders of CS-Sector

$$\lim_{u\to 0} \mathbb{V}_\mu(q\,|\,u) = \left(0, \overrightarrow{\mathbb{V}}(q)\right); \quad \lim_{u\to 0} \mathbb{A}_\mu(q\,|\,u) = \left(\mathbb{A}_0(q), 0, 0, 0\right)$$

Horizon Conditions : in-falling boundary condition & Regularity of Action

Zeroth Order | First Order

$$\mathbb{A} = \mathscr{A} + \mathcal{O}(\kappa_{CS}) = \mathtt{A} + \mathcal{O}(\kappa_{CS}^2)$$

$$\mathbb{V} = \mathscr{V} + \mathcal{O}(\kappa_{CS}) = \mathtt{V} + \mathcal{O}(\kappa_{CS}^2)$$

Zeroth Order of CS-sector,
满足齐次Maxwell Eqn.

$$\partial_N \left[\bar{g}^{NP}\bar{g}^{MQ}(\mathcal{F}^V)_{PQ}\sqrt{-\bar{g}}\right] = 0$$

$$\partial_N \left[\bar{g}^{NP}\bar{g}^{MQ}(\mathcal{F}^A)_{PQ}\sqrt{-\bar{g}}\right] = 0$$

A Special CME current: $\mathscr{A}_0(q\,|\,u)\big|_{\vec{q}=0}$

$$\mathrm{J}^\mu_{\mathrm{CME}}(x) = \left[-\kappa_{\mathrm{M}}(\mathtt{F}_\mathtt{V})^{5\mu}\sqrt{-g} + \frac{3\,\kappa_{CS}}{\sqrt{2}}\epsilon^{\mu\nu\rho\sigma}\mathscr{A}_\nu(\mathtt{F}_\mathtt{V})_{\rho\sigma}\right]\Bigg|_{u\to 0}$$

$$\partial_0\partial_5\,\mathscr{A}_0 = 0;\ \partial_5^2\mathscr{A}_0 = 0 \quad \Rightarrow \quad \mathscr{A}_0 = au + b$$

a time-independent ; b time-dependent.

CME Holographic Set-up | Axial Chemical Potential

Zeroth Order First Order

$$\mathbb{A} = \mathscr{A} + \mathcal{O}(\kappa_{\rm CS}) = \mathtt{A} + \mathcal{O}(\kappa_{\rm CS}^2)$$

$$\mathbb{V} = \mathscr{V} + \mathcal{O}(\kappa_{\rm CS}) = \mathtt{V} + \mathcal{O}(\kappa_{\rm CS}^2)$$

A Special CME current: $\mathscr{A}_0(q\,|\,u)\,|_{\vec{q}\to 0}$

$$\mathrm{J}^{\mu}_{\rm CME}(x) = \left[-\kappa_{\rm M} (\mathtt{F}_{\rm V})^{5\mu}\sqrt{-g} + \frac{3\,\kappa_{\rm CS}}{\sqrt{2}}\,\epsilon^{\mu\nu\rho\sigma}\,\mathscr{A}_{\nu}(\mathtt{F}_{\rm V})_{\rho\sigma} \right]\Bigg|_{u\to 0}$$

$$\partial_0\partial_5\,\mathscr{A}_0(X) = 0;\ \partial_5^2\mathscr{A}_0(X) = 0 \quad \Rightarrow \quad \mathscr{A}_0 = au + b$$

a time-independent ;
b time-dependent.

Axial chemical potential with gauge invariance

$$\mu_A \to \int_{\text{Boundary: } u=0}^{\text{Horizon: } u=1} \partial_u A_0\, \mathrm{d}u \equiv A_0\Big|_{u=1} - A_0\Big|_{u=0}$$

$$\mathscr{A}_0(q\,|\,u)\Big|_{\mathbf{q}\equiv 0} = \mu_A u$$

1. Gynther, A., Landsteiner, K., Pena-Benitez, F., & Rebhan, A. (2011). *Journal of High Energy Physics, 2011*(2), 1-17.
2. Rubakov, V. A. (2010). *arXiv preprint arXiv:1005.1888.*

CME Holographic Set-up | Axial Chemical Potential

A Special CME current: $\mathscr{A}_0(q\,|\,u)\,|_{\vec{q}\to 0}$

$$\mathrm{J}^{\mu}_{\mathrm{CME}} = \left[-\kappa_{_\mathrm{M}} (\mathrm{F_V})^{5\mu}\sqrt{-g} + \frac{3\,\kappa_{_\mathrm{CS}}}{\sqrt{2}}\,\epsilon^{\mu\nu\rho\sigma}\,\mathscr{A}_{\nu}(\mathrm{F_V})_{\rho\sigma} \right]\Bigg|_{u\to 0}$$

Zeroth Order　First Order

$$\mathbb{A} = \mathscr{A} + \mathscr{O}(\kappa_{_\mathrm{CS}}) = \mathtt{A} + \mathscr{O}(\kappa^2_{_\mathrm{CS}})$$

$$\mathbb{V} = \mathscr{V} + \mathscr{O}(\kappa_{_\mathrm{CS}}) = \mathtt{V} + \mathscr{O}(\kappa^2_{_\mathrm{CS}})$$

Axial chemical potential with gauge invariance

Fourier 形式：

不受 $q = (\omega, \vec{q})$ 影响

$$\mathscr{A}_0(q\,|\,u)\Big|_{\vec{q}=0} = (2\pi)^4\delta^4(q)\,\mu_A\,u \;\Rightarrow\; \mathscr{A}_0(q\,|\,0)\Big|_{\vec{q}=0} \equiv \mathscr{A}_0(q)\,|_{\vec{q}=0} = 0$$

$$\mathrm{J}^{\mu}(q) = \left[-\kappa_{_\mathrm{M}} (\mathrm{F_V})^{5\mu}\sqrt{-g} \right]\Bigg|_{u\to 0} = \mu_a\, K(q)\,\overrightarrow{\mathrm{B}}(q)$$

Chiral Equilibrium Emerges !

First-order 包含 Zeroth-order, 所以这一项有 μ_A

此时，CME电流要不为零，磁场不能是“常量”

CME Holographic Set-up | Two Kinds of Current

$$\lim_{u\to 0} \mathbb{V}_\mu(q|u) = (0, \vec{\mathbb{V}}(q)); \quad \lim_{u\to 0} \mathbb{A}_\mu(q|u) = (\mathbb{A}_0(q),0,0,0)$$

Zeroth Order　　First Order

$$\mathbb{A} = \mathscr{A} + \mathcal{O}(\kappa_{CS}) = \mathrm{A} + \mathcal{O}(\kappa_{CS}^2)$$

$$\mathbb{V} = \mathscr{V} + \mathcal{O}(\kappa_{CS}) = \mathrm{V} + \mathcal{O}(\kappa_{CS}^2)$$

CME stems from First-order of CS-sector：

$$\partial_N[\bar{g}^{NP}\bar{g}^{MQ}(\mathrm{F}^V)_{PQ}\sqrt{-\bar{g}}] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{CS}}{\kappa_M}\cdot \epsilon^{MNOPQ}(\mathrm{F}^A)_{NO}(\mathrm{F}^V)_{PQ}$$

$$\partial_N[\bar{g}^{NP}\bar{g}^{MQ}(\mathrm{F}^A)_{PQ}\sqrt{-\bar{g}}] = -\frac{3}{2\sqrt{2}}\frac{\kappa_{CS}}{\kappa_M}\cdot \frac{1}{2}\epsilon^{MNOPQ}\left[(\mathrm{F}^V)_{NO}(\mathrm{F}^V)_{PQ} + (\mathrm{F}^A)_{NO}(\mathrm{F}^A)_{PQ}\right]$$

$$\mathrm{J}^\mu_{\mathrm{CME}} = \left[-\kappa_M(\mathrm{F}_V)^{5\mu}\sqrt{-g} + \frac{3\kappa_{CS}}{\sqrt{2}}\epsilon^{\mu\nu\rho\sigma}\mathscr{A}_\nu(\mathrm{F}_V)_{\rho\sigma}\right]\Big|_{u\to 0}$$

在有μ_A时,C-S sector的贡献，由运动方程给出

$$\mathrm{J}^\mu(q) = \left[-\kappa_M(\mathrm{F}_V)^{5\mu}\sqrt{-g}\right]\Big|_{u\to 0} = \mu_A K(q)\vec{\mathrm{B}}(q)$$

轴化学势的贡献　　非平衡态的贡献

$$J^\mu_{\mathrm{CME}}(q) = \mathrm{J}^\mu(q) + J^\mu_{\mathrm{AVV}}(q)$$

CME Holographic Set-up | Chiral Non-equilibrium

Axial Field

$\vec{q} = 0$ Spatial Homogeneity: Chiral Equilibrium

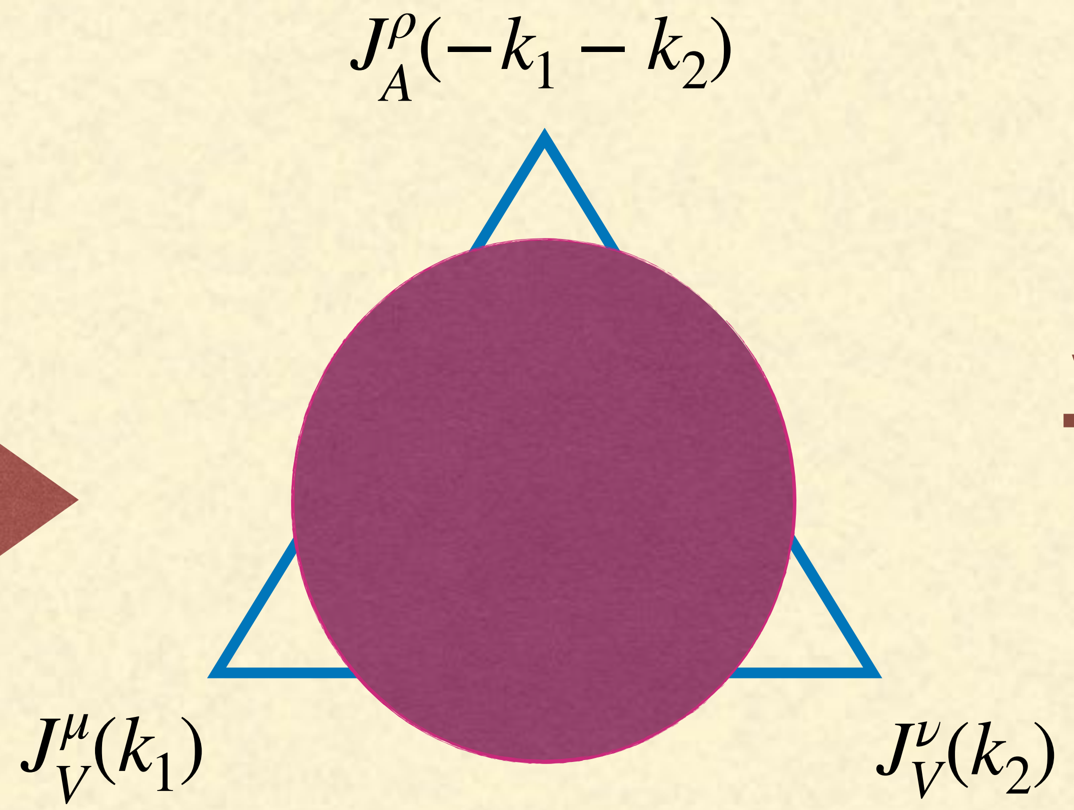
$$\mathscr{A}_0(q\,|\,u)\Big|_{\vec{q}=0} = (2\pi)^4\delta^4(q)\,\mu_A\,u$$

$\vec{q} \neq 0$ Spatial Imhomogeneity: Chiral Non-Equilibrium

轴化学势的贡献

非平衡态的贡献

$$J^\mu_{\rm CME}(q) = \mathrm{J}^\mu(q) + J^\mu_{\rm AVV}(q)$$



手征非平衡态的CME电流由三点关联函数给出

$$J^\mu_{\rm AVV}(q) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4}\frac{\mathrm{d}^4 q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1+q_2-q)\,\mathscr{J}^\mu(q_1,q_2)$$

电磁场 $\vec{\mathrm{E}}(q_1)$ & $\vec{\mathrm{B}}(q_1)$ 时空动力学

手征不均衡的时空依赖 $\mathrm{A}_0(q_2)$

Equations in Bulk

轴化学势的贡献　　　非平衡态的贡献

$$J^{\mu}_{\mathrm{CME}}(q) = \mathrm{J}^{\mu}(q) + J^{\mu}_{\mathrm{AVV}}(q)$$

Zeroth Order of κ_{CS}

$$\mathfrak{w}\mathcal{A}_0' + f\,(\mathfrak{q}\cdot\mathcal{A})' = 0$$
$$\mathfrak{w}\mathcal{V}_0' + f\,(\mathfrak{q}\cdot\mathcal{V})' = 0$$
$$\mathcal{A}_0'' - \frac{1}{uf}\left[\mathfrak{q}^2\mathcal{A}_0 + \mathfrak{w}\,(\mathfrak{q}\cdot\mathcal{A})\right] = 0$$
$$\mathcal{V}_0'' - \frac{1}{uf}\left[\mathfrak{q}^2\mathcal{V}_0 + \mathfrak{w}\,(\mathfrak{q}\cdot\mathcal{V})\right] = 0$$
$$\mathcal{A}_k'' + \frac{f'}{f}\mathcal{A}_k' + \frac{1}{uf^2}\left[\mathfrak{w}^2\mathcal{A}_k + \mathfrak{w}\mathfrak{q}_k\,\mathcal{A}_0\right] - \frac{1}{uf}\left[|\mathfrak{q}|^2\mathcal{A}_k - \mathfrak{q}_k\,(\mathfrak{q}\cdot\mathcal{A})\right] = 0$$
$$\mathcal{V}_k'' + \frac{f'}{f}\mathcal{V}_k' + \frac{1}{uf^2}\left[\mathfrak{w}^2\mathcal{V}_k + \mathfrak{w}\mathfrak{q}_k\,\mathcal{V}_0\right] - \frac{1}{uf}\left[|\mathfrak{q}|^2\mathcal{V}_k - \mathfrak{q}_k\,(\mathfrak{q}\cdot\mathcal{V})\right] = 0$$

First Order of κ_{CS}

$$\mathfrak{w}\mathrm{V}_0' + f\,(\mathfrak{q}\cdot\mathrm{V})' = \frac{\kappa_{\mathrm{CS}}}{\kappa_{\mathrm{M}}}G^5_{\mathrm{V}}(q|u)$$
$$\mathrm{V}_0'' - \frac{1}{uf}\left[|\mathfrak{q}|^2\mathrm{V}_0 + \mathfrak{w}\,(\mathfrak{q}\cdot\mathrm{V})\right] = \frac{\kappa_{\mathrm{CS}}}{\kappa_{\mathrm{M}}}G^0_{\mathrm{V}}(q|u)$$
$$\mathrm{V}_k'' + \frac{f'}{f}\mathrm{V}_k' + \frac{1}{uf^2}\left[\mathfrak{w}^2\mathrm{V}_k + \mathfrak{w}\mathfrak{q}_k\,\mathrm{V}_0\right] - \frac{1}{uf}\left[|\mathfrak{q}|^2\mathrm{V}_k - \mathfrak{q}_k\,(\mathfrak{q}\cdot\mathrm{V})\right] = \frac{\kappa_{\mathrm{CS}}}{\kappa_{\mathrm{M}}}G^k_{\mathrm{V}}(q|u)\,.$$

轴化学势的贡献

两点函数的计算

$$\boldsymbol{G}_{\mathrm{V}}(q|u) = -\frac{3}{\sqrt{2}}\frac{\mu_A}{(\pi T)^2 L f}\boldsymbol{\mathcal{B}}(q|u)\,; \qquad G^0_{\mathrm{V}}(q|u) = G^5_{\mathrm{V}}(q|u) = 0$$

非平衡态的贡献

三点函数的计算

$$G^M_{\mathrm{V}}(q|u) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4}\frac{\mathrm{d}^4 q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1+q_2-q)\;\mathcal{G}^M_{\mathrm{V}}(q_1,q_2|u)$$

$$\mathcal{G}^5_{\mathrm{V}}(q_1,q_2|u) = -\frac{3}{\sqrt{2}}\frac{uf}{(\pi T)^2 L}\frac{1}{|\mathfrak{q}_2|^2}\mathcal{A}_0''(q_2|u)\;(\mathfrak{q}_2\cdot\boldsymbol{\mathcal{B}}(q_1|u))\,;$$
$$\mathcal{G}^0_{\mathrm{V}}(q_1,q_2|u) = \frac{3}{\sqrt{2}}\frac{1}{(\pi T)^2 L\,f}\frac{\mathfrak{w}_2}{|\mathfrak{q}_2|^2}\mathcal{A}_0'(q_2|u)\;(\mathfrak{q}_2\cdot\boldsymbol{\mathcal{B}}(q_1|u))\,;$$
$$\boldsymbol{\mathcal{G}}^V(q_1,q_2|u) = -\frac{3}{\sqrt{2}}\frac{1}{(\pi T)^2 L f}\Big[\mathcal{A}_0'(q_2|u)\boldsymbol{\mathcal{B}}(q_1|u) - \frac{\mathfrak{w}_2}{f\,|\mathfrak{q}_2|^2}\mathcal{A}_0'(q_2|u)\;\mathfrak{q}_2\times\boldsymbol{\mathcal{E}}(q_1|u) - \mathrm{i}\,uf\frac{2\pi T}{|\mathfrak{q}_2|^2}\mathcal{A}_0''(q_2|u)\;\mathfrak{q}_2\times\mathcal{V}'(q_1|u)\Big]$$

三点函数下，卷积“分化”出 **EM 和 Chiral Imbalance** 各自的时空动力学

CME Current | Chiral Equilibrium

Homogeneous but non-static magnetic field condition:

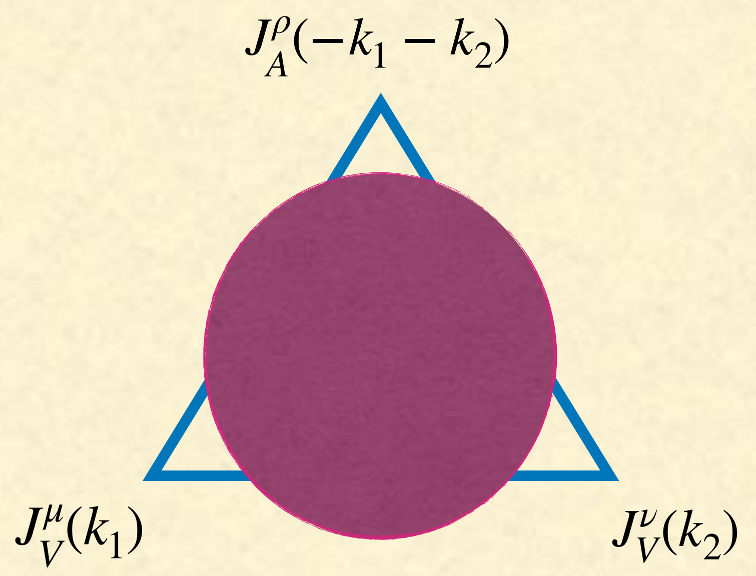
若chiral equilibrium 和 EM equilibrium 同时成立，那CME 电流将不存在

The essence of CME is non-equilibrium

Heun Equation Changes into Hypergeometric Equation

$$\vec{\mathrm{J}}(\mathfrak{w}) = 3\sqrt{2}\kappa_{cs}\,\mu_A\,\mathrm{B}(\mathfrak{w})\frac{\Gamma^2(\frac{1-i\mathfrak{w}}{2})\Gamma^2(\frac{3-i\mathfrak{w}}{2})}{\Gamma^2(1-i\mathfrak{w})}\int_0^1 \mathrm{d}u\left[\left(\frac{1-u}{1+u}\right)^{-i\frac{\mathfrak{w}}{2}} F\left(\frac{1-i}{2}\mathfrak{w}, -\frac{1+i}{2}\mathfrak{w}; 1-i\mathfrak{w}; \frac{1-u}{1+u}\right)\right]^2$$

$$\mathbf{J} \propto C_A\,\mathbf{B}$$



CME Current | Chiral Non-Equilibrium

$$J^\mu_{\text{AVV}}(q) = \int \frac{d^4 q_1}{(2\pi)^4}\frac{d^4 q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1+q_2-q)\,\mathcal{J}^\mu(q_1,q_2)$$

$$\mathcal{G}^5_{\text{V}}(q_1,q_2|u) = -\frac{3}{\sqrt{2}}\frac{uf}{(\pi T)^2 L}\frac{1}{|\mathbf{q}_2|^2}\mathcal{A}''_0(q_2|u)\,(\mathbf{q}_2\cdot\boldsymbol{\mathcal{B}}(q_1|u))\,;$$

$$\mathcal{G}^0_{\text{V}}(q_1,q_2|u) = \frac{3}{\sqrt{2}}\frac{1}{(\pi T)^2 L\, f}\frac{\mathfrak{w}_2}{|\mathbf{q}_2|^2}\mathcal{A}'_0(q_2|u)\,(\mathbf{q}_2\cdot\boldsymbol{\mathcal{B}}(q_1|u))\,;$$

$$\boldsymbol{\mathcal{G}}^V(q_1,q_2|u) = -\frac{3}{\sqrt{2}}\frac{1}{(\pi T)^2 L f}\Big[\mathcal{A}'_0(q_2|u)\boldsymbol{\mathcal{B}}(q_1|u) - \frac{\mathfrak{w}_2}{f\,|\mathbf{q}_2|^2}\mathcal{A}'_0(q_2|u)\,\mathbf{q}_2\times\boldsymbol{\mathcal{E}}(q_1|u) - \mathrm{i}\,uf\frac{2\pi T}{|\mathbf{q}_2|^2}\mathcal{A}''_0(q_2|u)\,\mathbf{q}_2\times\boldsymbol{\mathcal{V}}'(q_1|u)\Big]$$

考虑小动量$\{q_1;q_2\}$展开：

手征非平衡态CME电流的时间分量：

$$\mathcal{J}^0_{(0)}(q_1,q_2) = 3\sqrt{2}\kappa_{\text{CS}}\frac{\mathsf{A}_0(q_2)\,(\mathbf{q}_2\cdot\mathbf{B}(q_1))}{D_{(0)}(q)D_{(0)}(q_2)}\mathfrak{w}_2$$

手征非平衡态CME电流的空间分量：

$$\boldsymbol{\mathcal{J}}_{(0)}(q_1,q_2) = -3\sqrt{2}\kappa_{\text{CS}}\frac{\mathsf{A}_0(q_2)}{D_{(0)}(q_2)}\,i\mathfrak{w}_2\left[\mathbf{B}(q_1) + \frac{\mathbf{q}}{D_{(0)}(q)}(\mathbf{q}_2\cdot\mathbf{B}(q_1))\right]$$

Diffusive Structure

$$D_{(0)}(q) = i\mathfrak{w} - |\mathbf{q}|^2$$

→ 非平凡红外行为

Non-local Response

CME Current | Infrared Behaviours

$\mathfrak{q}_2 = \{\mathfrak{w}_2, \vec{\mathfrak{q}}_2\}$ proxies the dynamics of chiral imbalance in spacetime

For the case: $|\mathbf{q}_2|^2 \ll \mathfrak{w}_2 \ll 1$

$$\mathcal{J}_{(0)}(q_1, q_2) \simeq -3\sqrt{2}\kappa_{\text{CS}} \mathrm{A}_0(q_2)\mathbf{B}(q_1)$$

The opposite case: $\mathfrak{w}_2 \ll |\mathbf{q}_2|^2 \ll 1$

$$\mathcal{J}_{(0)}(q_1, q_2) \simeq 0$$

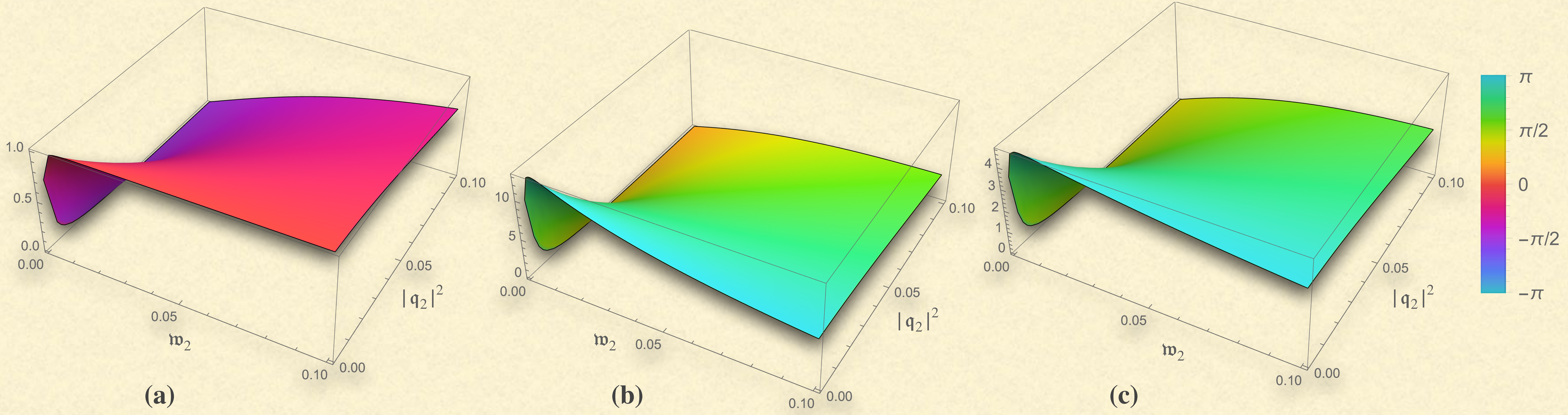
Match the weakly-coupled Field Theory — Hou, D. F., Liu, H., & Ren, H. C. (2011). *Journal of High Energy Physics, 2011*(5), 1-24.

Match A Simple Model — Rubakov, V. A. (2010). arXiv preprint arXiv:1005.1888.

当手征系统 $\mathrm{A}_0(x)$ 在空间中变化缓慢，则 $\mathrm{A}_0(x) \sim \mu_A$,三点函数的贡献和 μ_A 的贡献相当

当$\mathrm{A}_0(x)$ 在空间的分布改变剧烈；在时间的演化涨落急速，则 $|\mathrm{A}_0(x)| \gg |\mu_A|$ ，手征非平衡态对CME电流的贡献巨大。

CME Current | Infrared Behaviours



(a) is the transverse , (b);(c) are the longitudinal component with two different $\mathfrak{q}_1$

Longitudinal Component	$\vec{\mathfrak{q}} \times \vec{J}_{\text{AVV}}(\mathfrak{q}_1, \mathfrak{q}_2)$
Transverse Component	$\vec{\mathfrak{q}} \cdot \vec{J}_{\text{AVV}}(\mathfrak{q}_1, \mathfrak{q}_2)$

4 momenta identity : $\mathfrak{q} = \mathfrak{q}_1 + \mathfrak{q}_2$

$$J^{\mu}_{\text{AVV}}(q) = \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} (2\pi)^4 \delta^4(q_1 + q_2 - q)\, \mathcal{J}^{\mu}(q_1, q_2)$$

CME Current | Subleading Order of $q_{1,2} = \{\mathfrak{w}_{1,2}, \vec{\mathfrak{q}}_{1,2}\}$

$$\mathcal{J}(q_1,q_2) = 2(\pi T)^2 L\kappa_{\rm CS}\left[\int_0^1 du\,(1-u^2)\mathcal{G}_\perp^V(q_1,q_2|u)\,\psi(q|u) + \frac{\mathfrak{w}\,\mathbf{q}}{|\mathbf{q}|^2 D(q)}\int_0^1 du\,u(1-u^2)\mathcal{M}(q_1,q_2|u)\,\phi(q|u)\right] + 3\sqrt{2}\kappa_{\rm CS}\frac{\mathbf{q}}{|\mathbf{q}|^2}\mathrm{A}_0(q_2)(\mathbf{q}_2\cdot\mathbf{B}(q_1)) - 3\sqrt{2}\kappa_{\rm CS}\,\mathrm{A}_0(q_2)\mathbf{B}(q_1)$$

Diffusive Structure

$$D(q) := i\mathfrak{w} - |\vec{\mathfrak{q}}|^2 + \mathcal{O}(\mathfrak{w}^2; \mathfrak{w}|\vec{\mathfrak{q}}|^2; |\vec{\mathfrak{q}}^4)$$

All terms involving $\dfrac{\vec{\mathfrak{q}}_1\cdot\vec{\mathfrak{q}}_2}{|\vec{\mathfrak{q}}|^2}$ or $\dfrac{1}{|\vec{\mathfrak{q}}|^2}$ are cancelled !

$$\begin{aligned}\mathcal{J}(q_1,q_2) &\simeq \mathcal{J}_{(0)}(q_1,q_2) + \mathcal{J}_{(1)}(q_1,q_2)\\
&= -3\sqrt{2}\kappa_{\rm CS}\frac{\mathrm{A}_0(q_2)}{D_{(0)}(q_2)}\Bigg\{ i\mathfrak{w}_2\left[\mathbf{B}(q_1) + \frac{\mathbf{q}}{D_{(0)}(q)}\big(\mathbf{q}_2\cdot\mathbf{B}(q_1)\big)\right]\\
&\quad - \left[\left(2\,\mathfrak{w}_1\mathfrak{w}_2 + \mathfrak{w}_2^2 + 2i\,\mathfrak{w}_1|\mathbf{q}_2|^2\right)\frac{\ln 2}{D_{(0)}(q_2)} + \frac{\pi^2}{12}\left(|\mathbf{q}_1|^2 + |\mathbf{q}|^2\right)\right]|\mathbf{q}_2|^2\,\mathbf{B}(q_1)\\
&\quad - \Bigg[\frac{\pi^2}{12}|\mathbf{q}_2|^2 + \left(\frac{\mathfrak{w}^2}{2} + \frac{i\mathfrak{w}}{2}|\mathbf{q}|^2 - (|\mathbf{q}|^2)^2\right)\frac{\ln 2}{D_{(0)}(q)} + \left(\frac{\mathfrak{w}_2^2}{2} + \frac{i\mathfrak{w}_2}{2}|\mathbf{q}_2|^2 - (|\mathbf{q}_2|^2)^2\right)\frac{\ln 2}{D_{(0)}(q_2)}\\
&\qquad + \left(\frac{1}{2}\mathfrak{w}_1\mathfrak{w}_2\ln 2 + i\mathfrak{w}_2|\mathbf{q}_2|^2\ln 2 - \mathfrak{w}|\mathbf{q}|^2\ln 2 + \frac{\pi^2}{8}\,i\mathfrak{w}_1|\mathbf{q}_1|^2\right)\Bigg]\frac{\mathbf{q}}{D_{(0)}(q)}\big(\mathbf{q}_2\cdot\mathbf{B}(q_1)\big)\\
&\quad - \left(-\frac{\pi^2}{8}i\mathfrak{w}_2 + \frac{\pi^2}{12}|\mathbf{q}_2|^2\right)\,\mathbf{q}_2\times\big(\mathbf{q}_1\times\mathbf{B}(q_1)\big) - i\ln 2\,|\mathbf{q}_2|^2\big(\mathbf{q}_2\times\mathbf{E}(q_1)\big)\Bigg\}\,.\end{aligned}$$

No "direction Singularity"

No "Action at a distance"

Summary

1. 从 Power Structure 的分析可发现 Large N_C limit 限制了 without back-reaction 的调节方式，建立了 SYM 框架内研究手征反常系统的方法。

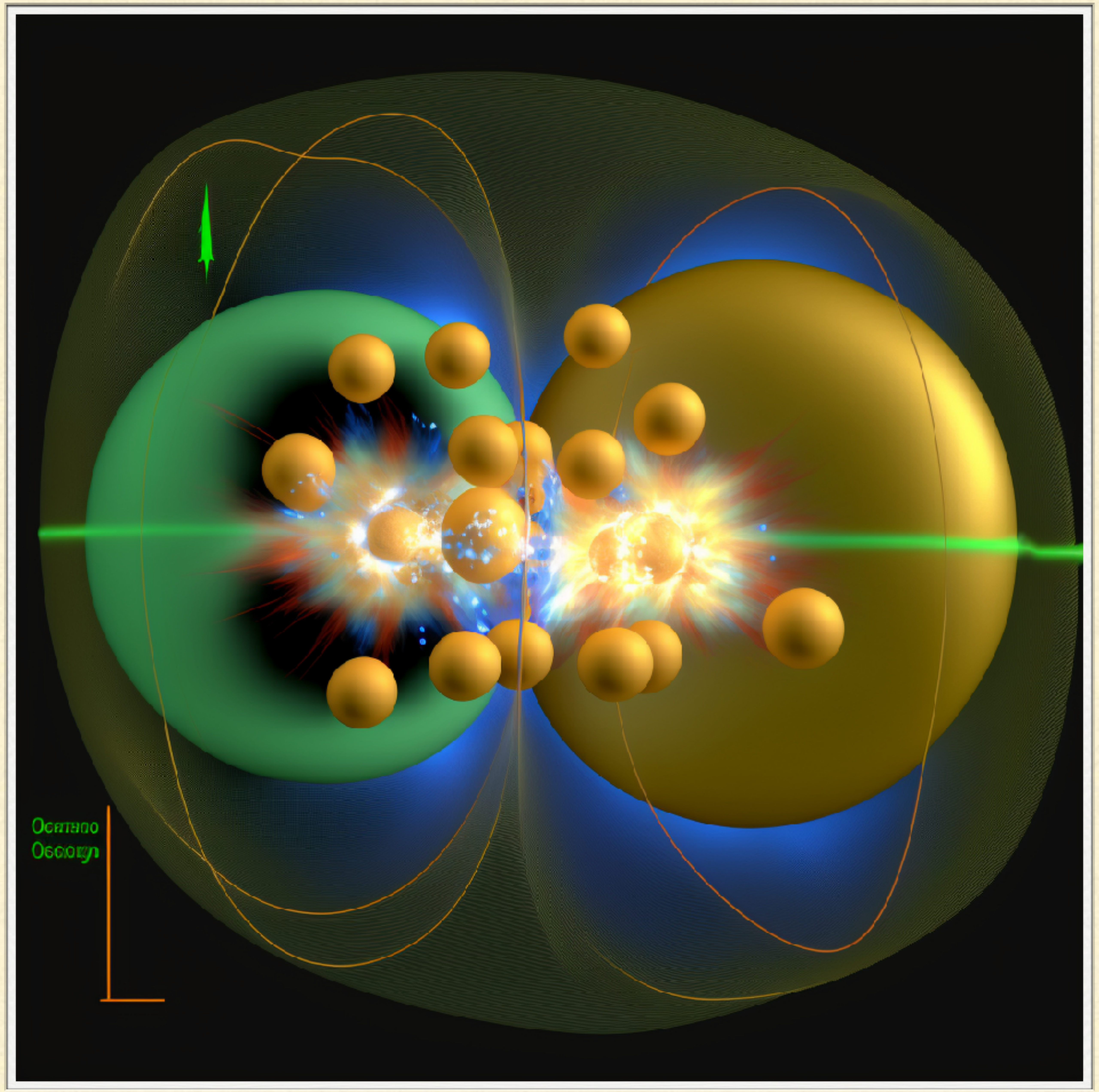
2. 空间均匀的轴矢量场 $\mathscr{A}_0(X)$ 在 horizon 的取值代表了轴化学式，这是手征平衡态对 CME 电流的贡献。

3. 手征非平衡态和电磁非平衡态产生的 CME 电流由三点函数贡献。

4. 强耦合 CME 电流也有非平凡红外极限次序行为，轴化学式不是简单取静态和均匀极限下的轴矢量场。

5. 计算了CME 电流的三点函数小动量展开.......

谢谢



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