Bubble nucleation and gravitational wave from holography

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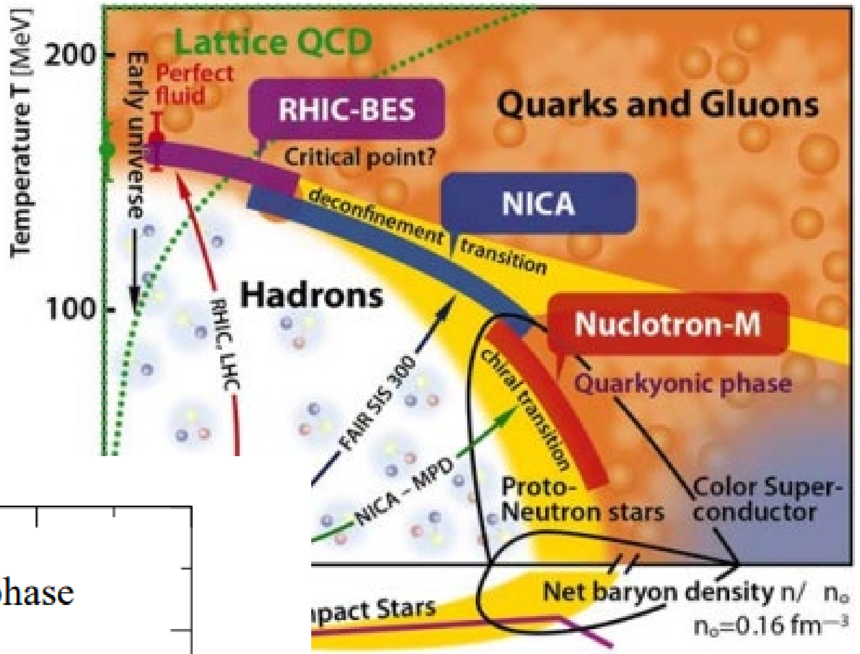
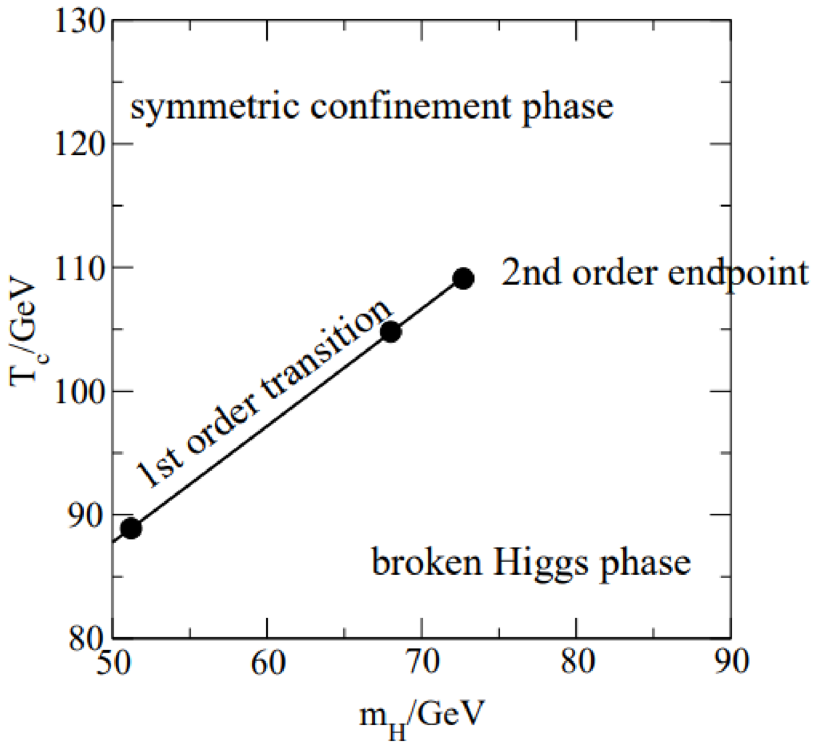
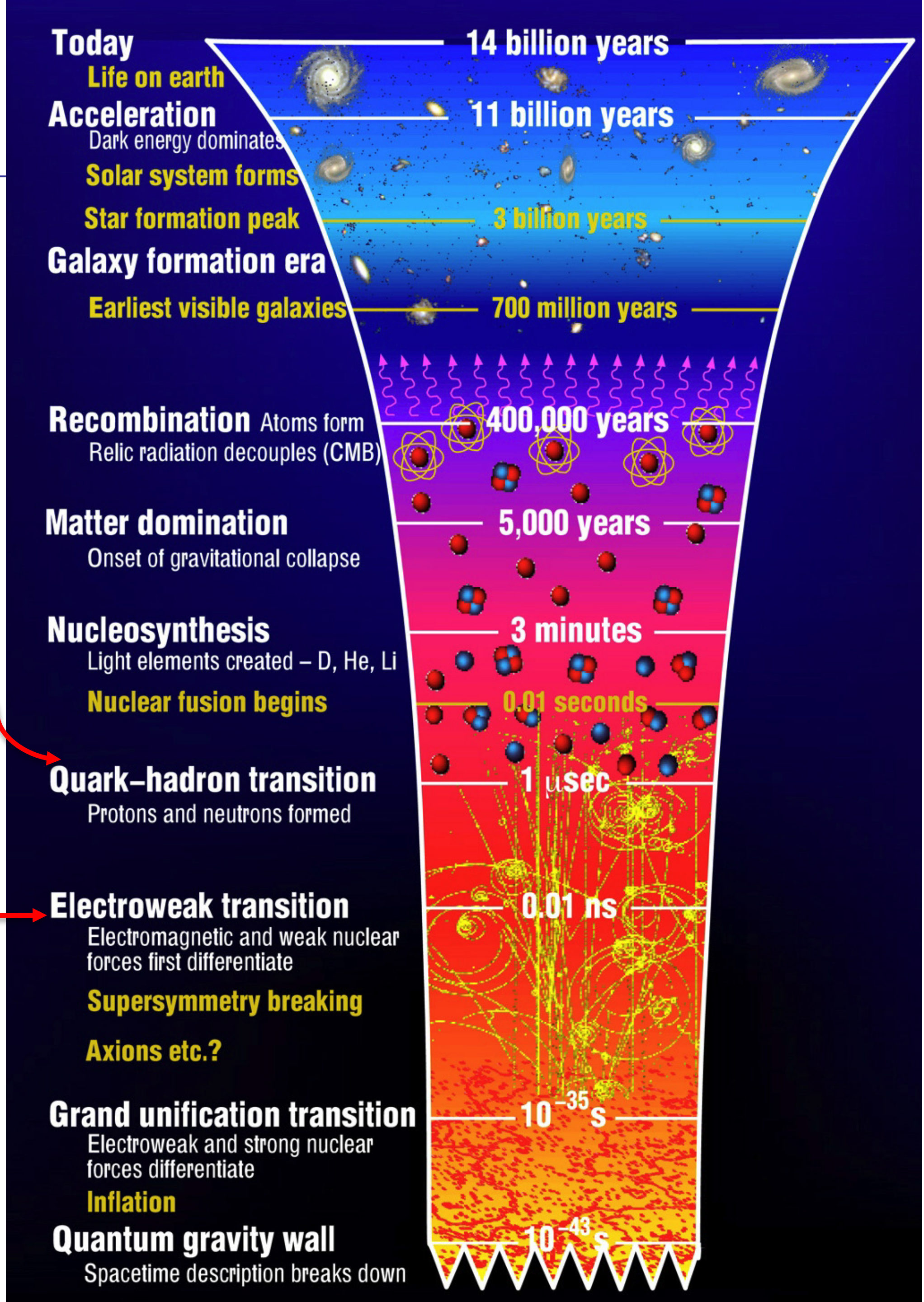
I. Introduction

Cosmic phase transition







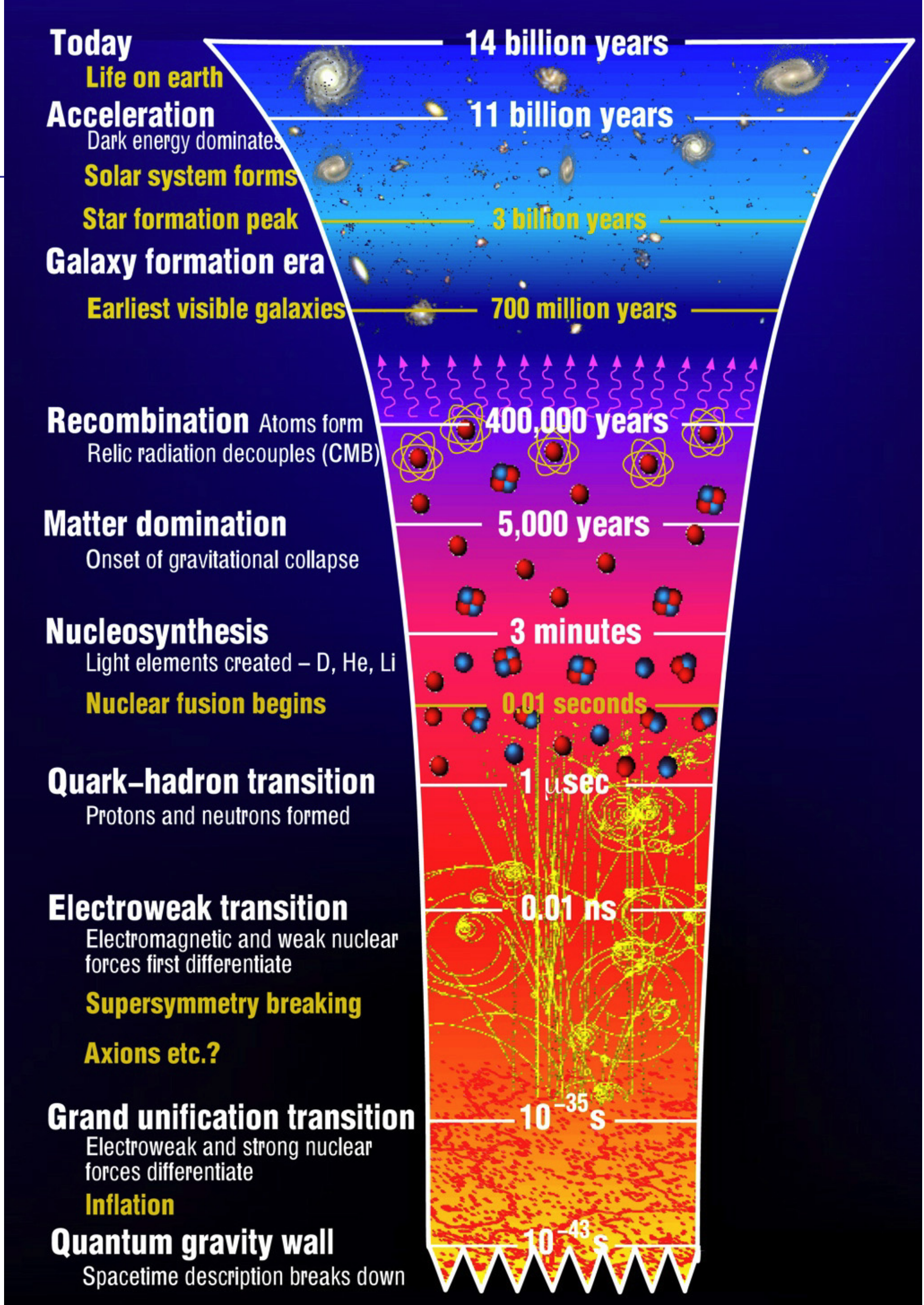
Baryon asymmetry puzzle

Sakharov conditions: **A.D. Sakharov, Pisma Zh. Eksp. Teor. Fiz.(1967)**

1) Baryon number B violation.
2) C-symmetry and CP-symmetry violation.
3) Interactions out of thermal equilibrium.

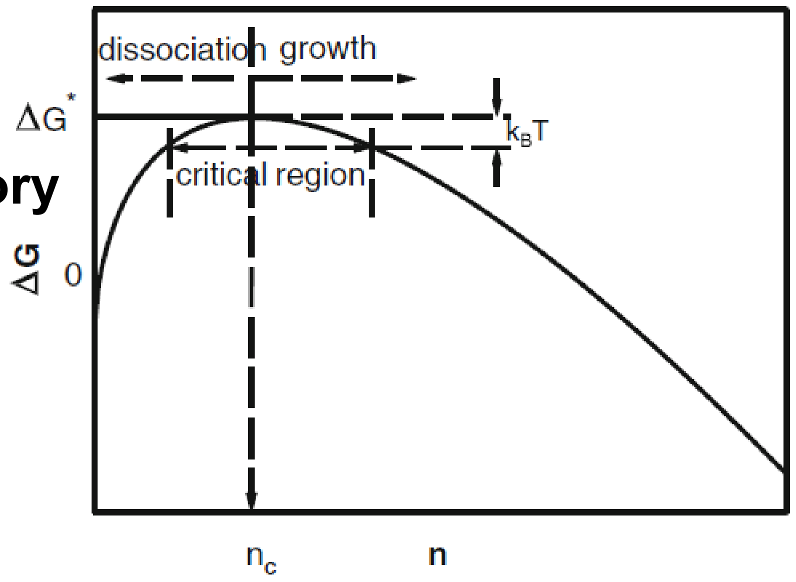
First-Order Phase Transition

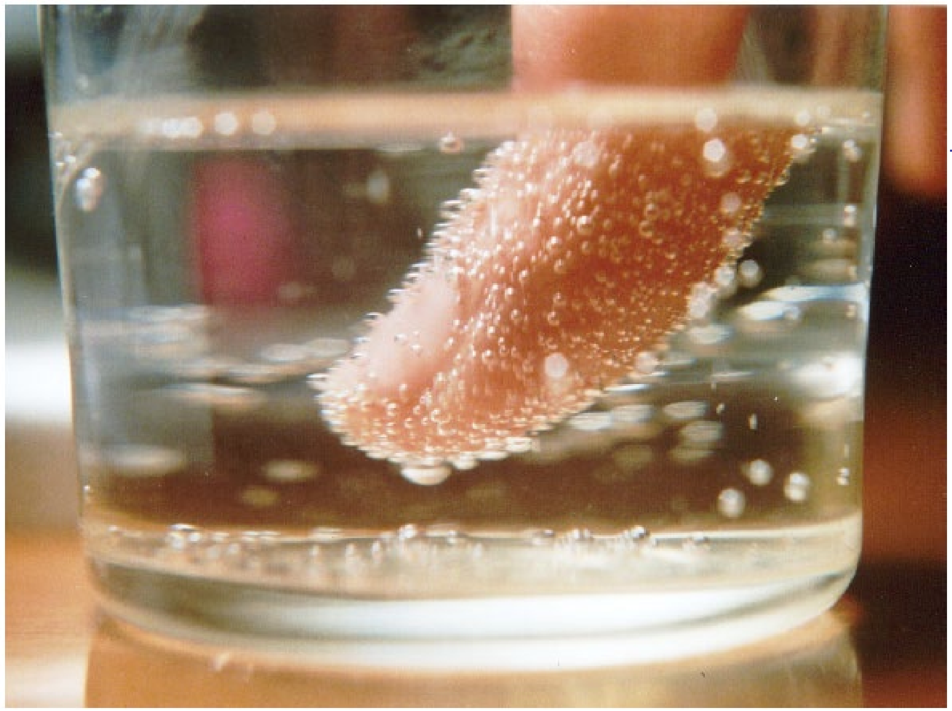
GWs, primordial magnetic field, primordial black holes, dark matter…

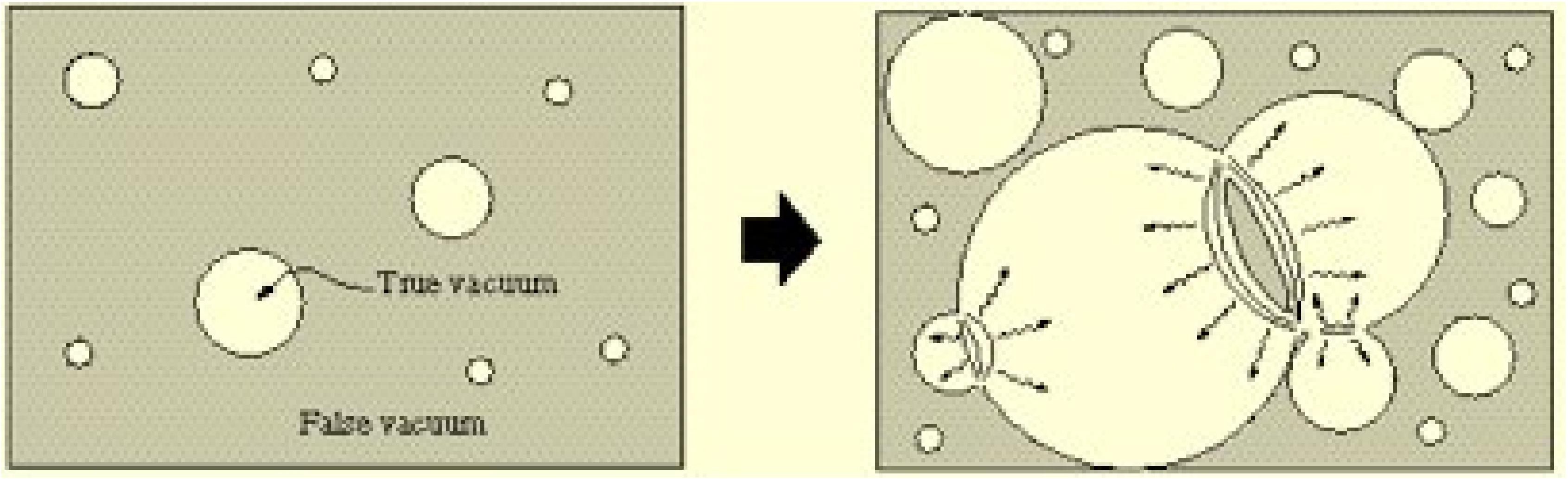


Nonequilibrium first-order transition

Classical Nucleation Theory



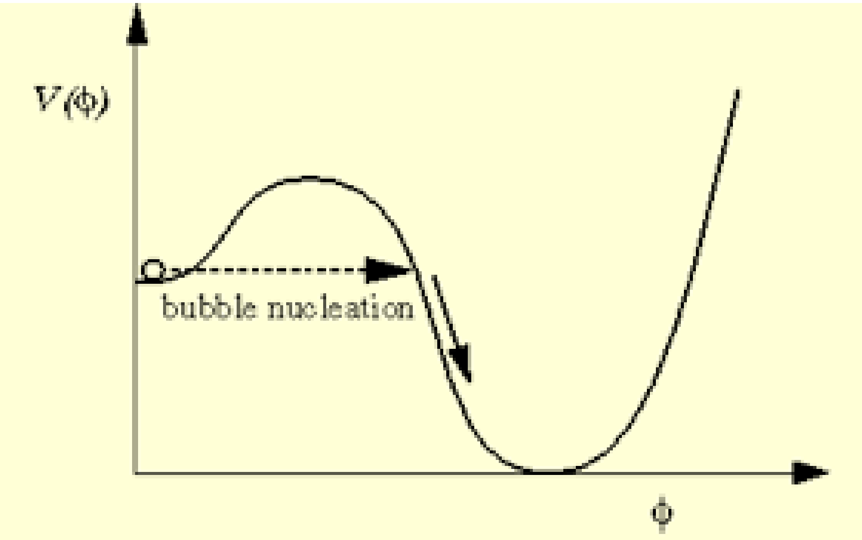




S. Coleman, Phys. Rev. D 15, 2929 (1977); ($T = 0$)
A. D. Linde, Nucl. Phys. B216, 421 (1983). ($T \neq 0$)

Quantum Nucleation Theory

Bounce Solution



At sufficiently high temperatures, the least-action solution has O(3) symmetry [Linde, 1983]. The O(3)-symmetric bubble is governed by

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{dV_T(\phi,T)}{d\phi} = V_T'(\phi,T)$$

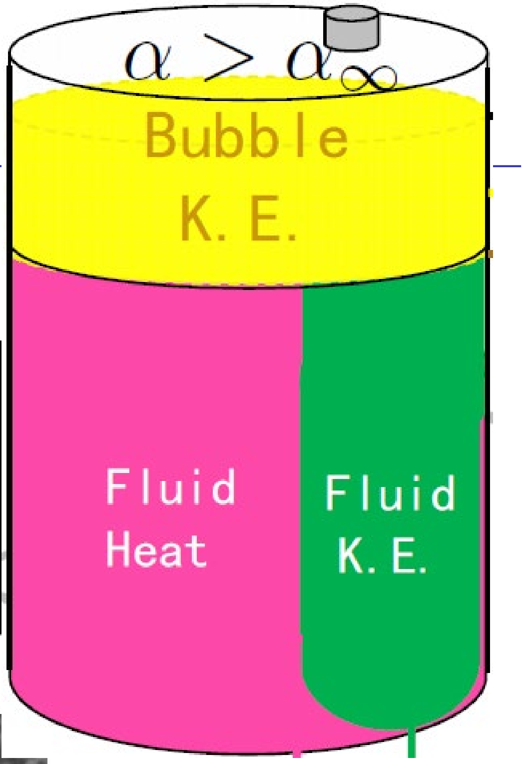
with boundary conditions

$$\begin{cases} \phi \to \phi_F, & r \to \infty \\ \dfrac{d\phi}{dr} = 0, & r \to 0 \end{cases}$$

J.I.Kapusta and C.Gale, Finite-temperature field theory: Principles and applications, Cambridge University Press, 2011

Stochastic gravitational waves

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_{\rm coll} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb},$$



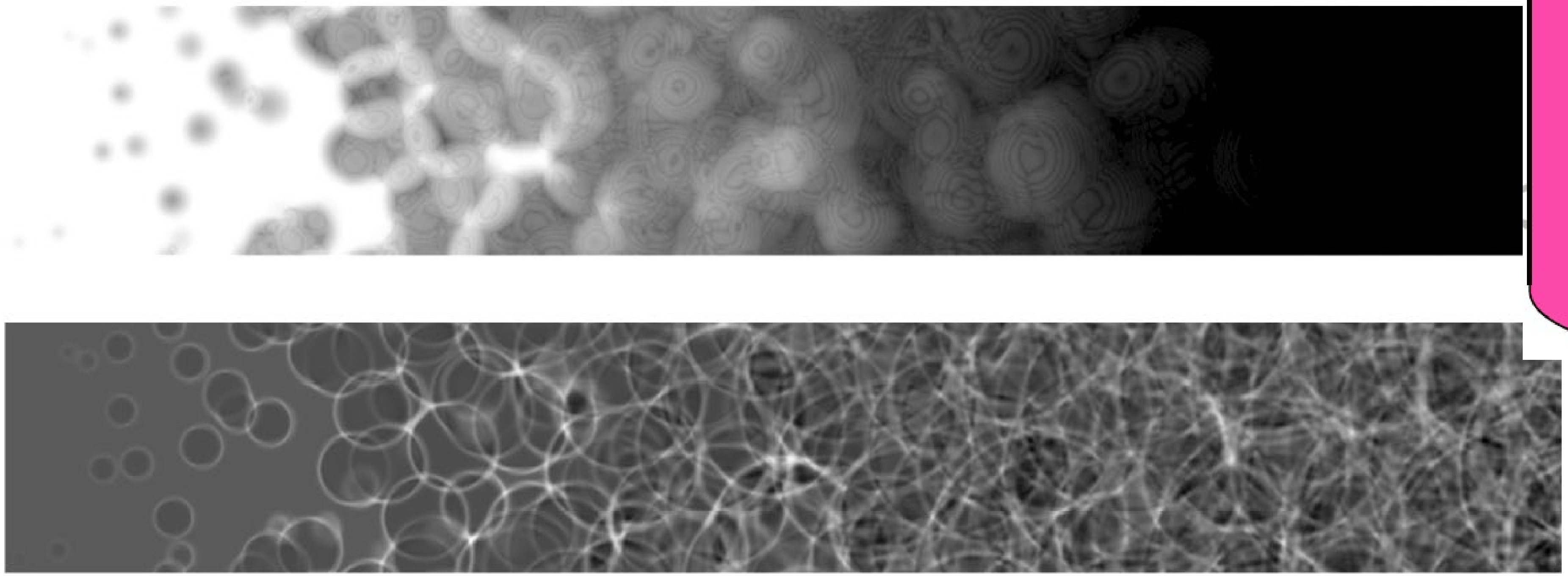


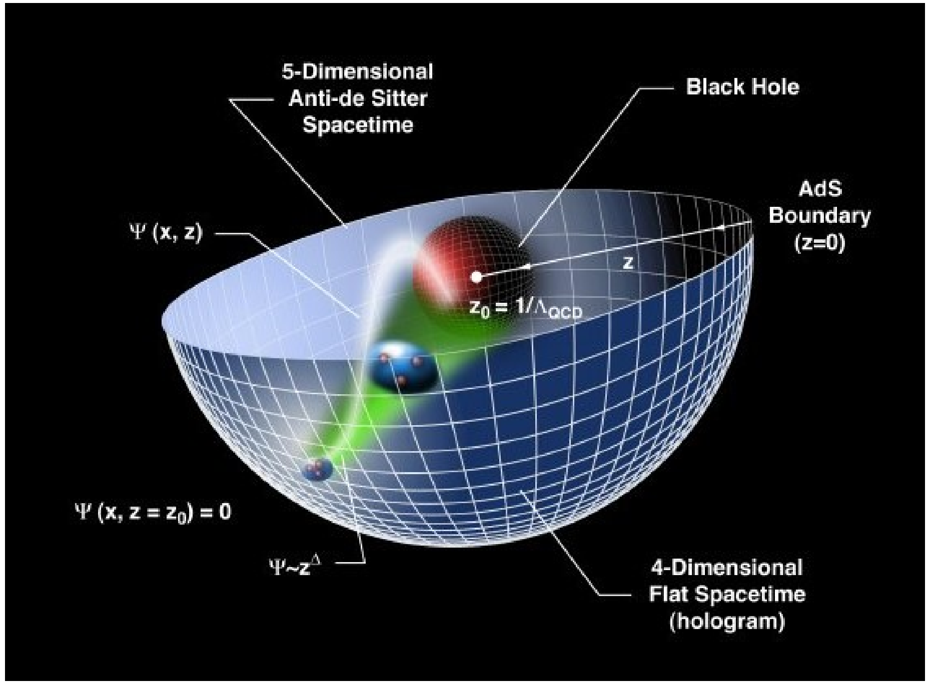
Figure: **Giblin and Mertens [arXiv:1405.4005]** The vertical axis, y, is a one-dimensional slice through the box, and the horizontal axis is a space-like hyper surface, $x + c_f t$. The upper panel shows a scaled version of the field value, roughly $ln|\psi - \psi_-|$. The lower panel shows the logarithm of the energy density of the fluid $\ln(\epsilon)$.

AdS/CFT: original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

relates gravity theories on anti-de Sitter spacetimes to conformal field theories

General Gauge/Gravity duality

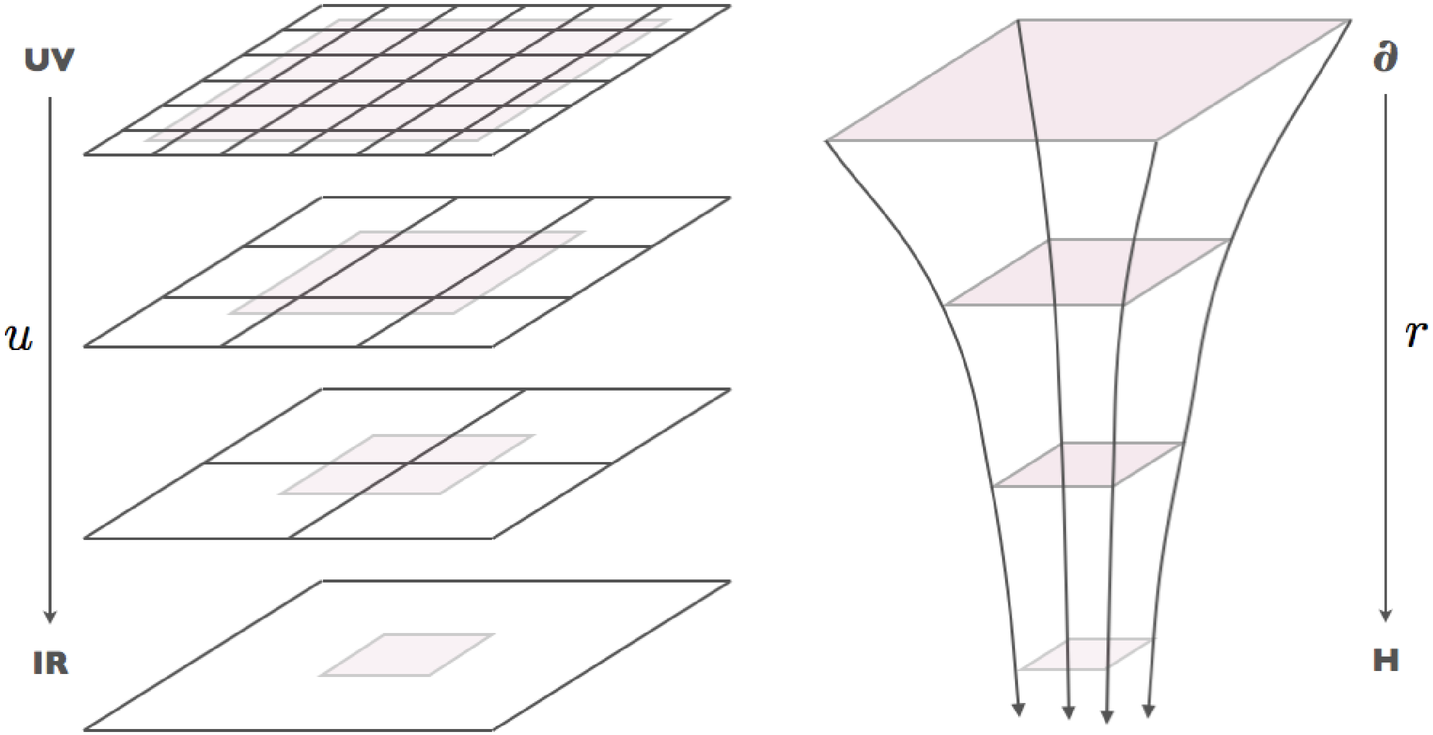


Holographic Duality & RG flow

QFT on lattice
RG scale
Coupling constant

Gravitational problem
Extra spatial dimension
Dynamical field

$$J_i|_{UV} = \Phi_i|_\partial$$



arXiv:1205.5180

QCD

Effective field theories and models

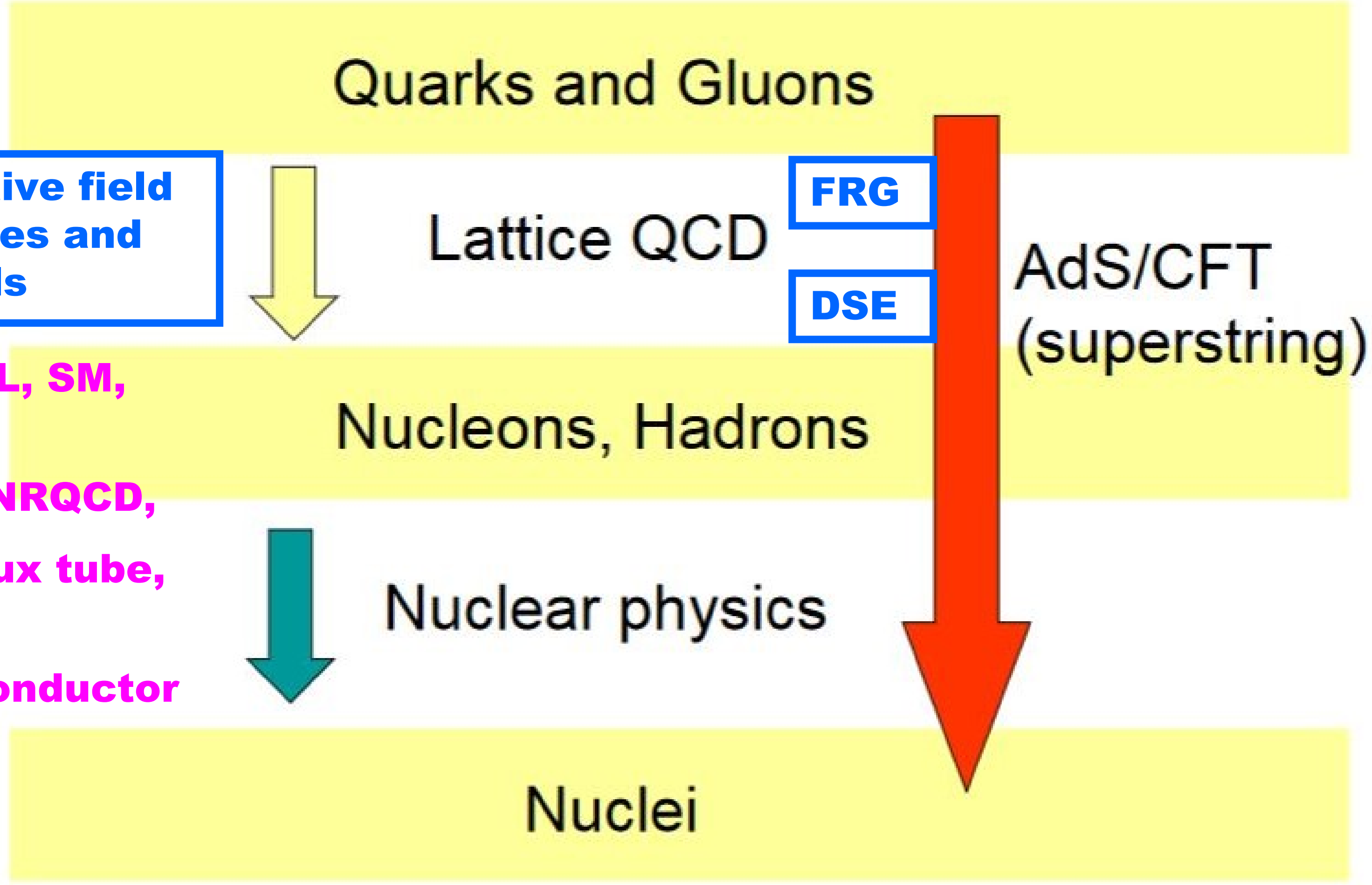
QM, NJL, SM, HLS,

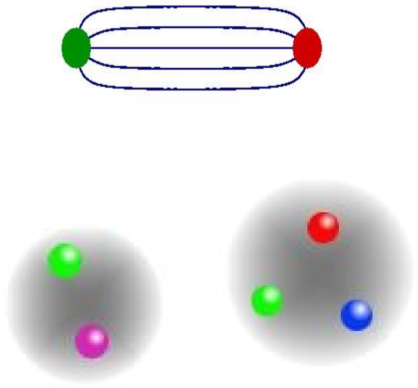
CHPT, NRQCD,

color flux tube,

Dual superconductor

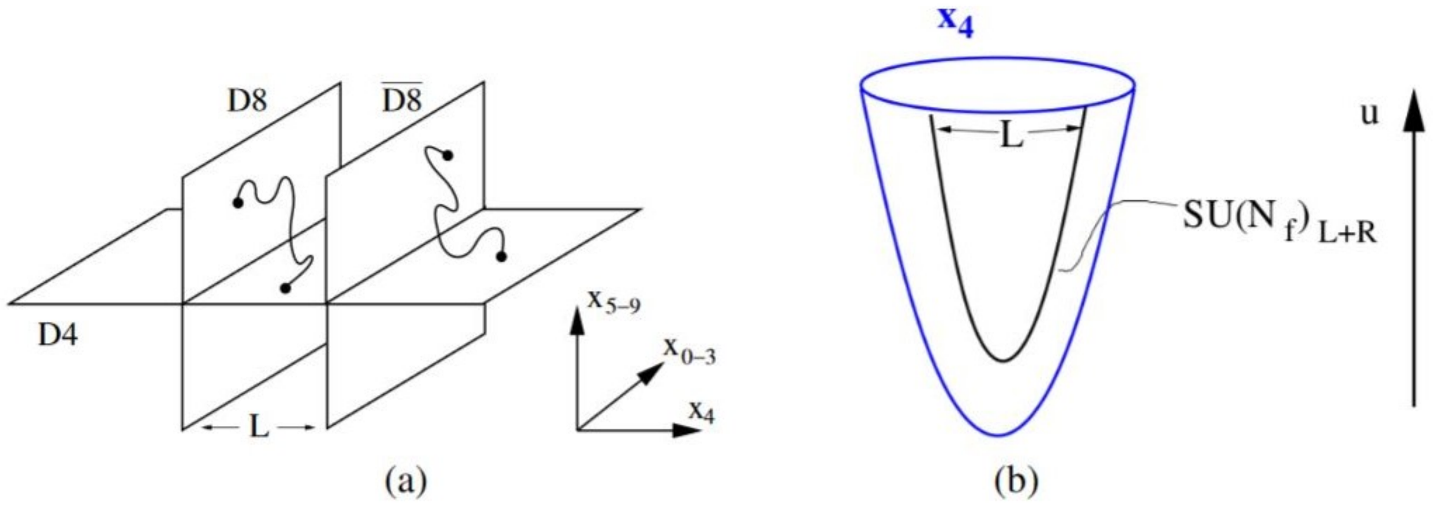
……





Top-down vs Bottom-up

Top-down： based on string theory and D-brane



D3-D7,
D4-D6,
D4-D8,
STU model
…

Bottom-up： from symmetry of QCD

hard-wall, soft-wall, Gubser model, improved holographic QCD, dynamical holographic QCD …

1) **5D effective action**
2) **IR cutoff or dilaton to realize confinement**

Hadron spectra, chiral symmetry breaking & linear confinement, phase transitions, equation of state, transport properties

Holographic dictionary

Boundary QFT	Gravity
Local operator $\mathcal{O}_i(x)$	**Field** $\Phi_i(x,r)$
conformal dimension Δ, p-form field p	$(\Delta - p)(\Delta + p - d) = m_5^2 L^2$
Strongly coupled	**Weakly coupled semi-classical**

$$\boxed{Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]}$$

$$Z_{\mathrm{QFT}}[J] \;\simeq\; e^{-I_{\mathrm{GR}}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\mathrm{GR}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$

KKSS model (soft-wall model)

Karch et al., arXiv: 0602229[hep-ph]

5D action

$$I=\int d^5x\, e^{-\Phi(z)}\sqrt{g}\left\{-|DX|^2+3|X|^2-\frac{1}{4g_5^2}(F_L^2+F_R^2)\right\}$$

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

e.o.m

$$-\psi_n''+V(z)\psi_n=m_n^2\psi_n,$$

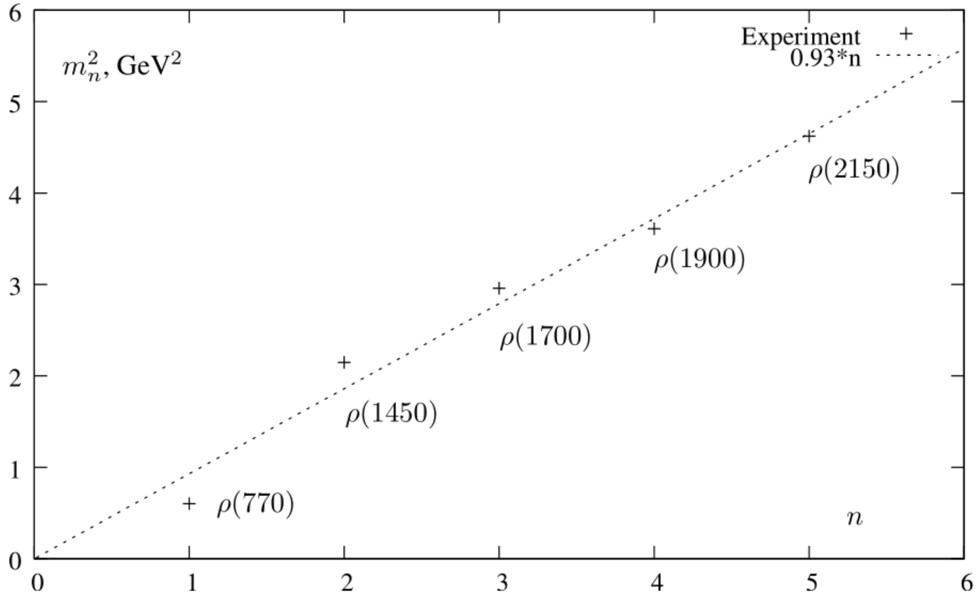
$$V=z^2+3/(4z^2)$$

Dilaton profile

$$\Phi(z)=\mu_G^2 z^2$$

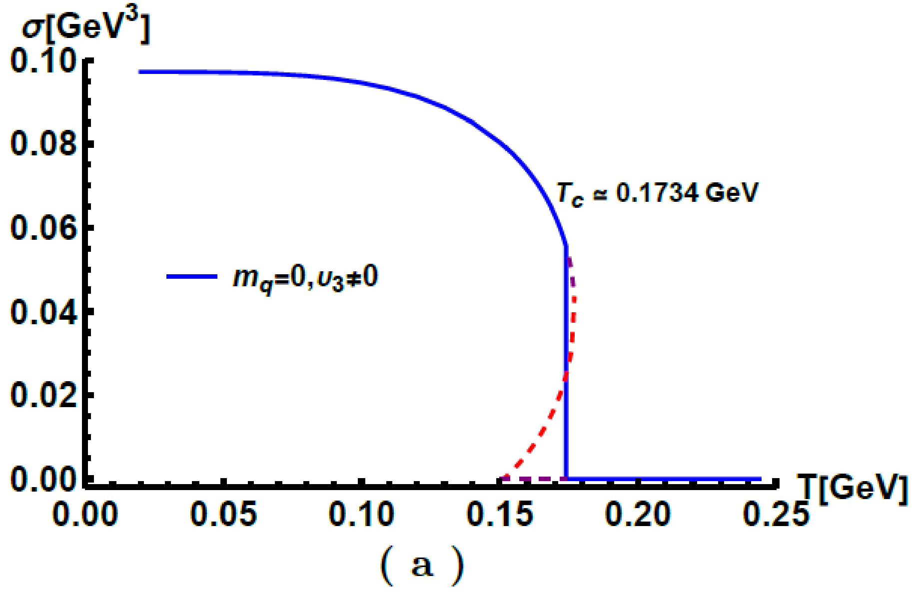
Conformal AdS$_5$

$$ds^2=\frac{L^2}{z^2}(dz^2+\eta_{\mu\nu}dx^\mu dx^\nu),$$

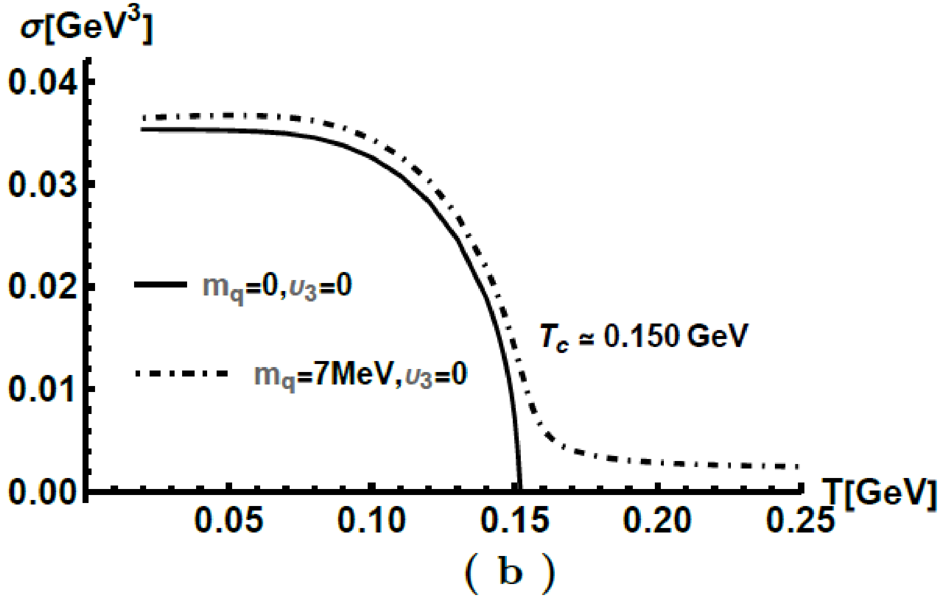


Chiral condensate

Chelabi et al., Danning Li, arXiv: 1511.02721
Chelabi et al., Danning Li, arXiv: 1512.06493



(a)



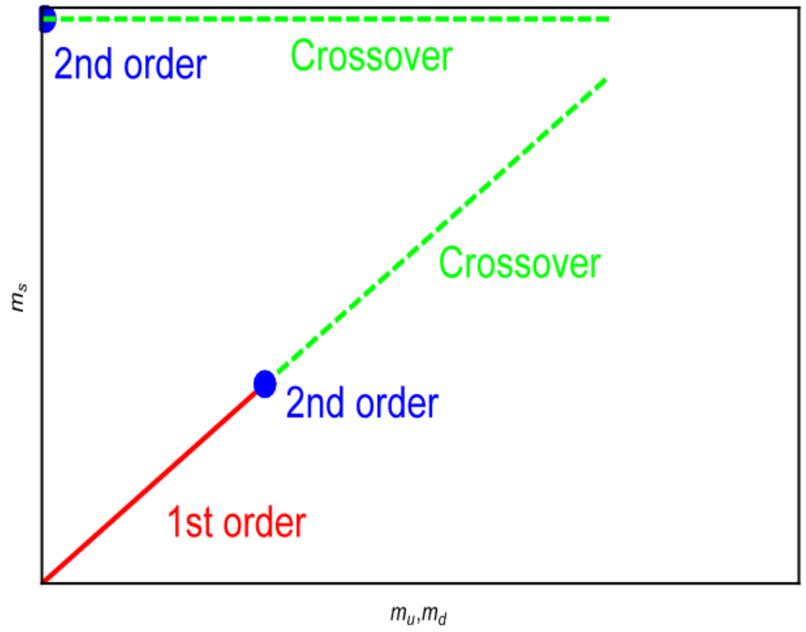
(b)

5D action

$$S = -\int d^5x\sqrt{-g}e^{-\Phi}Tr(D_m X^+ D^m X + V_X(|X|)).$$

$$V(\chi) \equiv Tr(V_X(|X|)) = -\frac{3}{2}\chi^2 + v_3\chi^3 + v_4\chi^4.$$

$$\Phi(z) = -\mu_1 z^2 + (\mu_1 + \mu_0)z^2 \tanh(\mu_2 z^2),$$



II. 5D Setup, bounce solution and bubble velocity

5D Model

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

Φ denotes the dilaton field and the complex scalar field X corresponds to the quark condensation or fermionic condensate of BSMs.

5D action

$$S = -\int d^5x\sqrt{-g}e^{-\Phi}\mathrm{Tr}[(D^M X)^\dagger(D_M X) + V_X(|X|)].$$

Dilaton profile

$$\Phi(z) = -\mu_1 z^2 + (\mu_1 + \mu_0)z^2 \tanh(\mu_2 z^2).$$

Metric

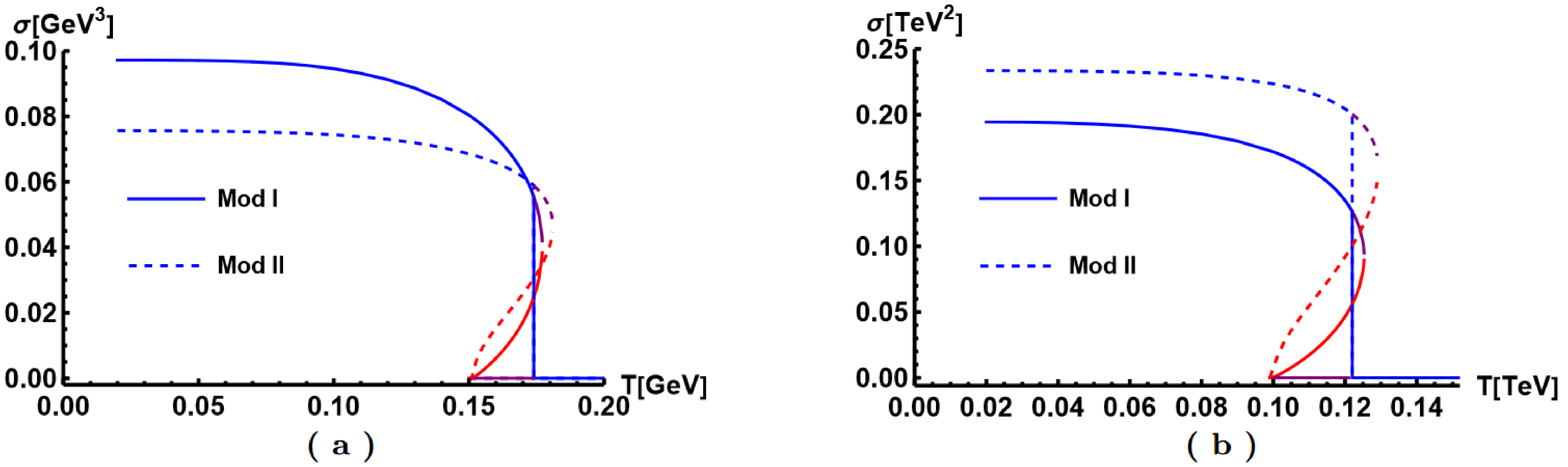
$$ds^2 = \frac{1}{z^2}[-f(z)dt^2 + \frac{1}{f(z)}dz^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2], \qquad f(z) = 1 - (\frac{z}{z_h})^4$$

Potential

$$V(\chi) \equiv \mathrm{Tr}[V_X(|X|)] = \frac{M_5^2}{2}\chi^2 + v_3\chi^3 + v_4\chi^4 + v_6\chi^6,$$

5D Model

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591



Potential $$V(\chi) \equiv \mathrm{Tr}[V_X(|X|)] = \frac{M_5^2}{2}\chi^2 + v_3\chi^3 + v_4\chi^4 + v_6\chi^6,$$

Model I: $v_3 \neq 0, v_6 = 0$

Model II: $v_3 = 0, v_6 \neq 0$

Bounce Solution

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

the equations of motion of the scalar field X can be obtained from action

$$\partial_r^2\chi(z,r)+\frac{2\partial_r\chi(z,r)}{r}+f(z)\partial_z^2\chi(z,r)+\left(f'(z)-f(z)\Phi'(z)-\frac{3f(z)}{z}\right)\partial_z\chi(z,r)-\frac{\partial_\chi V(\chi)}{z^2}=0.$$

$$\frac{d^2\phi}{dr^2}+\frac{2}{r}\frac{d\phi}{dr}=\frac{dV_T(\phi,T)}{d\phi}=V_T'(\phi,T)$$

The solution requires the following boundary conditions

$$\lim_{r\to\infty}\chi(z,r)=\chi_f,$$

$$\left.\frac{d\chi(z,r)}{dr}\right|_{r=0}=0.$$

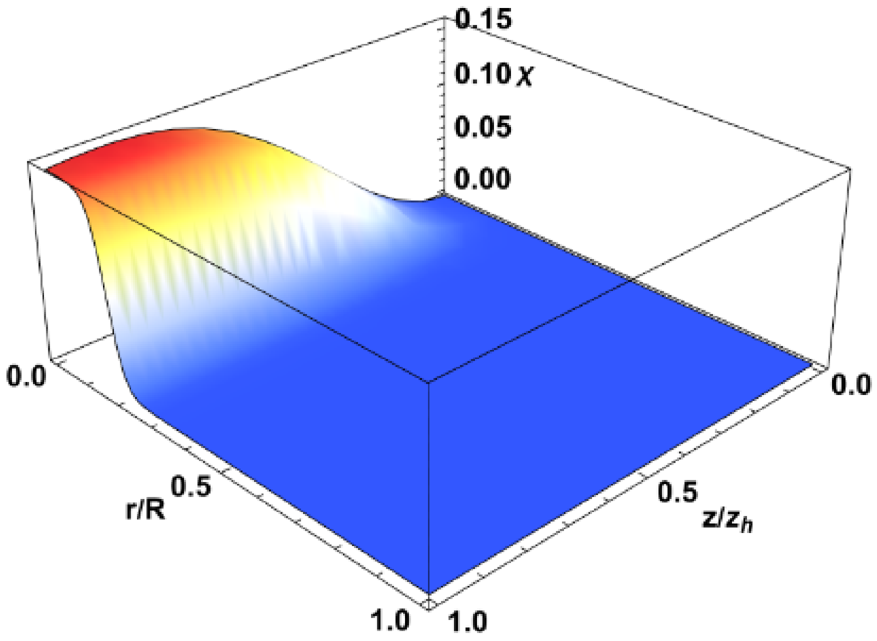
$$\chi_f|_{z\to 0}=\ldots+\sigma_f\frac{z^3}{\zeta}+\ldots$$

$$\Longleftrightarrow$$

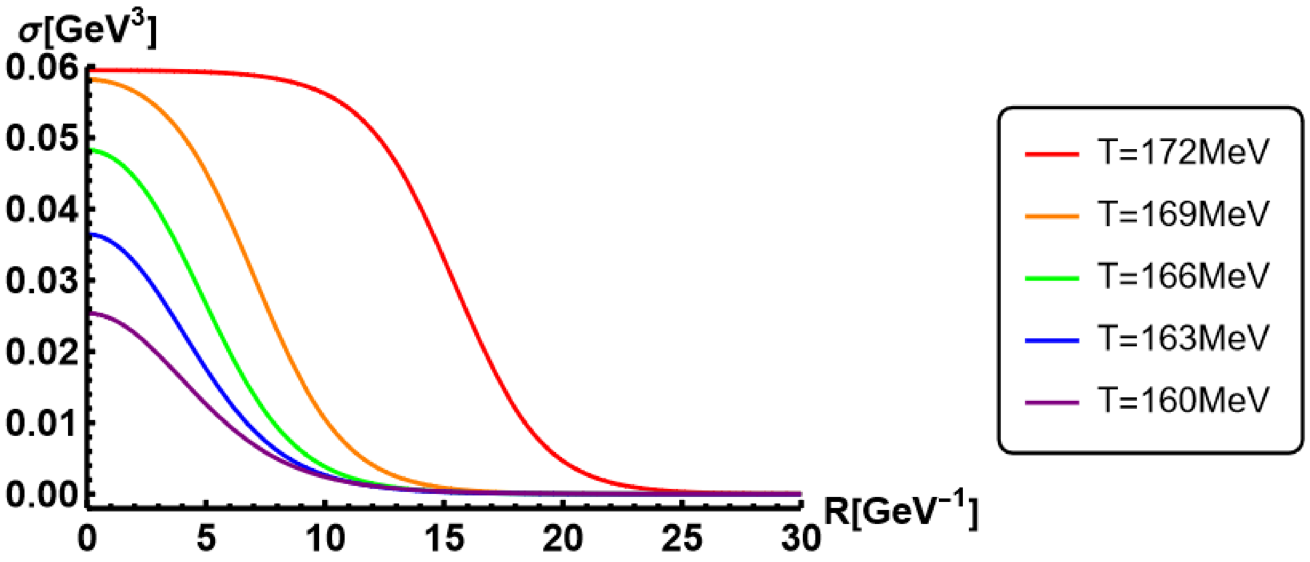
$$\begin{cases}\phi\to\phi_F, & r\to\infty\\ \dfrac{d\phi}{dr}=0, & r\to 0\end{cases}$$

Bounce Solution

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591



(a)



(b)

the bounce solution of the scalar field χ as a function of the fifth dimensional coordinate z and the radial coordinate r

the condensate σ as a function of the radial coordinate r at different temperatures.

Bubble velocity

To investigate the real-time evolution, we transform the framework to the ingoing Eddington-Finkelstein coordinate, then the metric becomes

$$ds^2 = \frac{1}{z^2}[-f(z)dt^2 - 2dtdz + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2].$$

Under the coordinate transformation, the scalar field χ is invariant, and its equation of motion becomes as follows

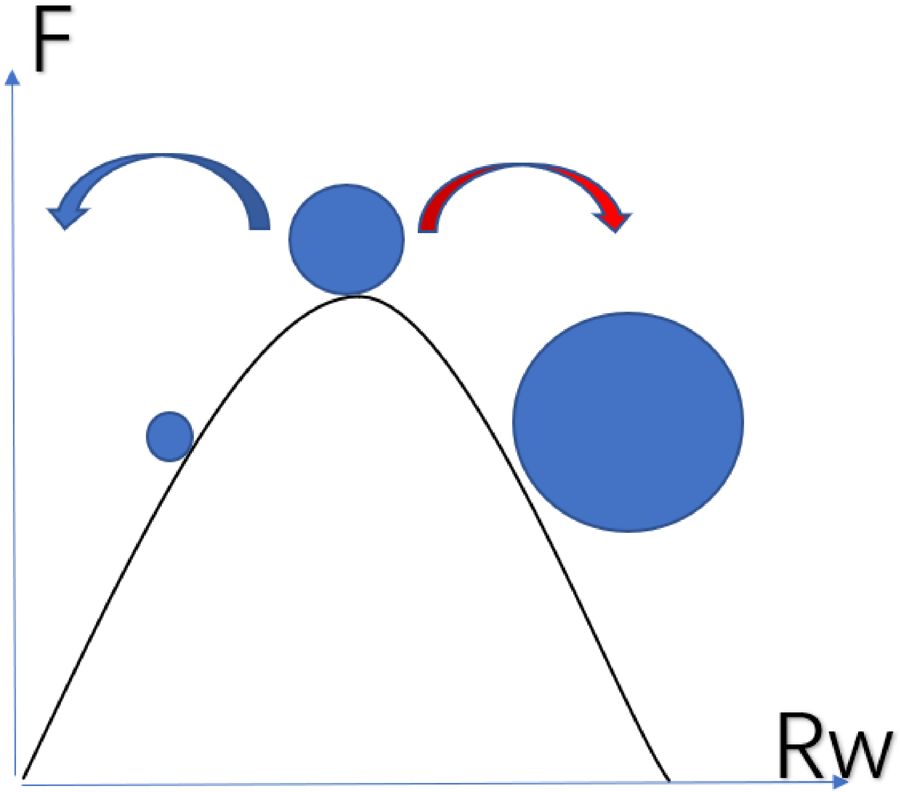
$$\partial_r^2\chi(t,z,r) + \frac{2\partial_r\chi(t,z,r)}{r} + f(z)\partial_z^2\chi(t,z,r) + \left(f'(z) - f(z)\Phi'(z) - \frac{3f(z)}{z}\right)\partial_z\chi(t,z,r)$$
$$-\frac{\partial_\chi V(\chi)}{z^2} + \left(\frac{3}{z} + \Phi'(z)\right)\partial_t\chi(t,z,r) - 2\partial_t\partial_z\chi(t,z,r) = 0.$$

We choose the following boundary conditions

$$\partial_t\partial_r\chi|_{r=0} = \partial_t\partial_r\chi|_{r=R} = 0. \qquad \partial_t\partial_z\chi|_{z=0} = 0.$$

bounce solution=critical bubble solution

$$\partial_t \chi_b = 0$$



$$\chi_{ini} = \chi_b + \delta\chi$$

$$\begin{cases} \delta\chi > 0, & expand \\ \delta\chi < 0, & shrink \end{cases}$$

Bubble velocity

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

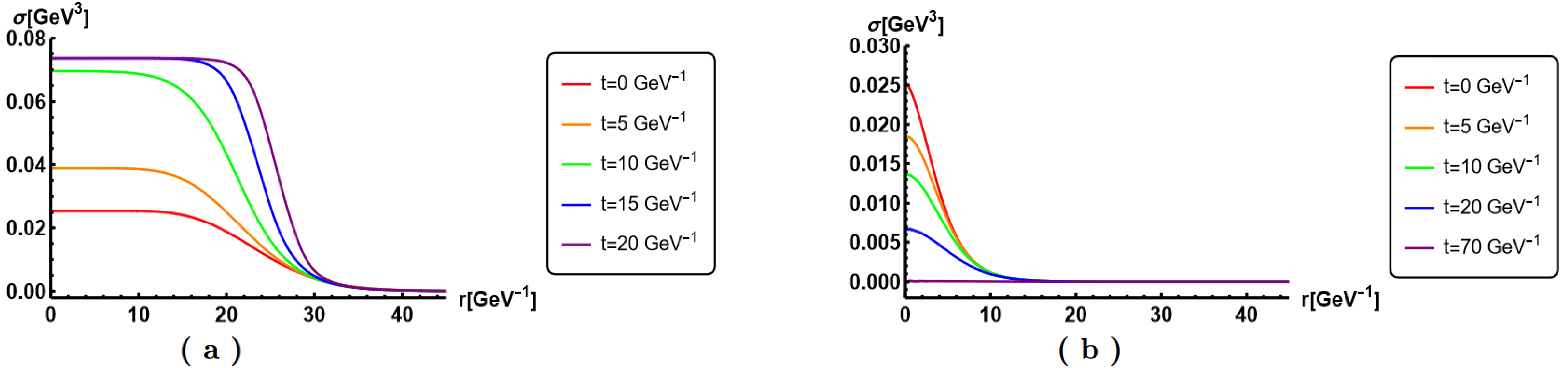


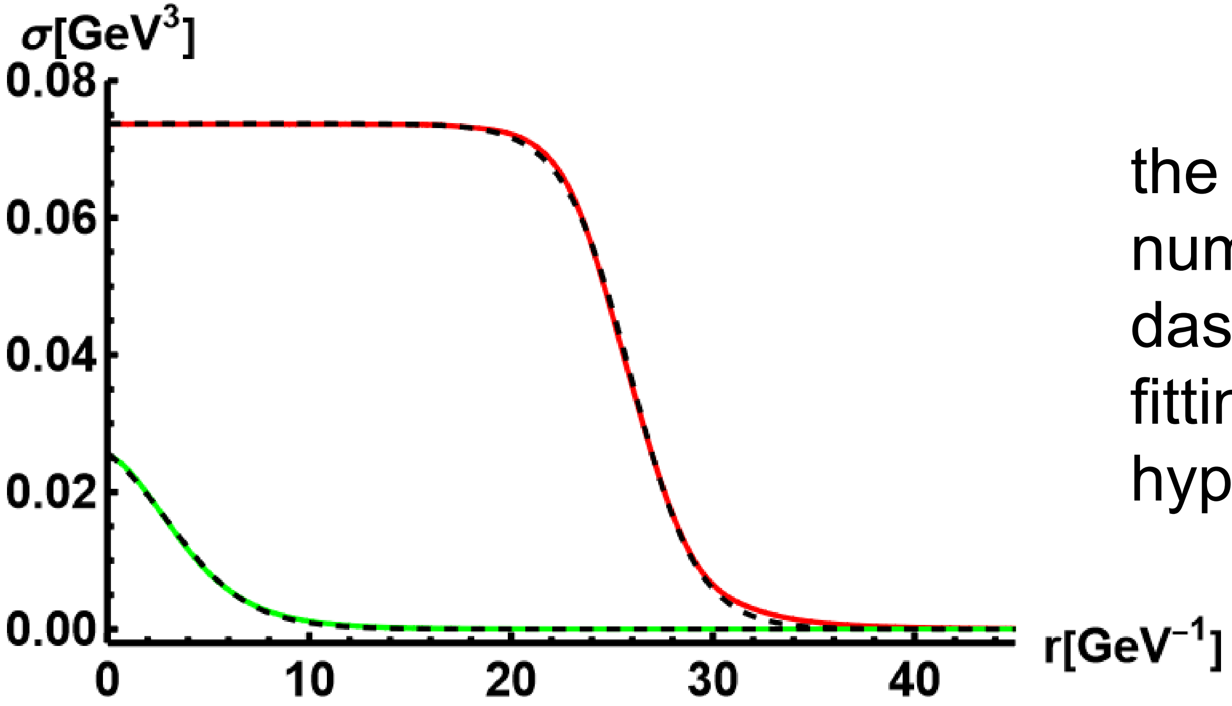
Figure 3. The panels (a) and (b) show the profiles of the bubbles at different times with $T = 160$ MeV when positive and negative perturbations are added, respectively.

Bubble velocity

To obtain the bubble wall velocity, we used the function

$$\sigma(r) = \frac{\sigma_0(t)}{2}[-\tanh(\frac{r(t)-r_w(t)}{l_w(t)}) + 1]$$

we can define the velocity of the bubble wall by $$v(t) = \frac{dr_w(t)}{dt}$$

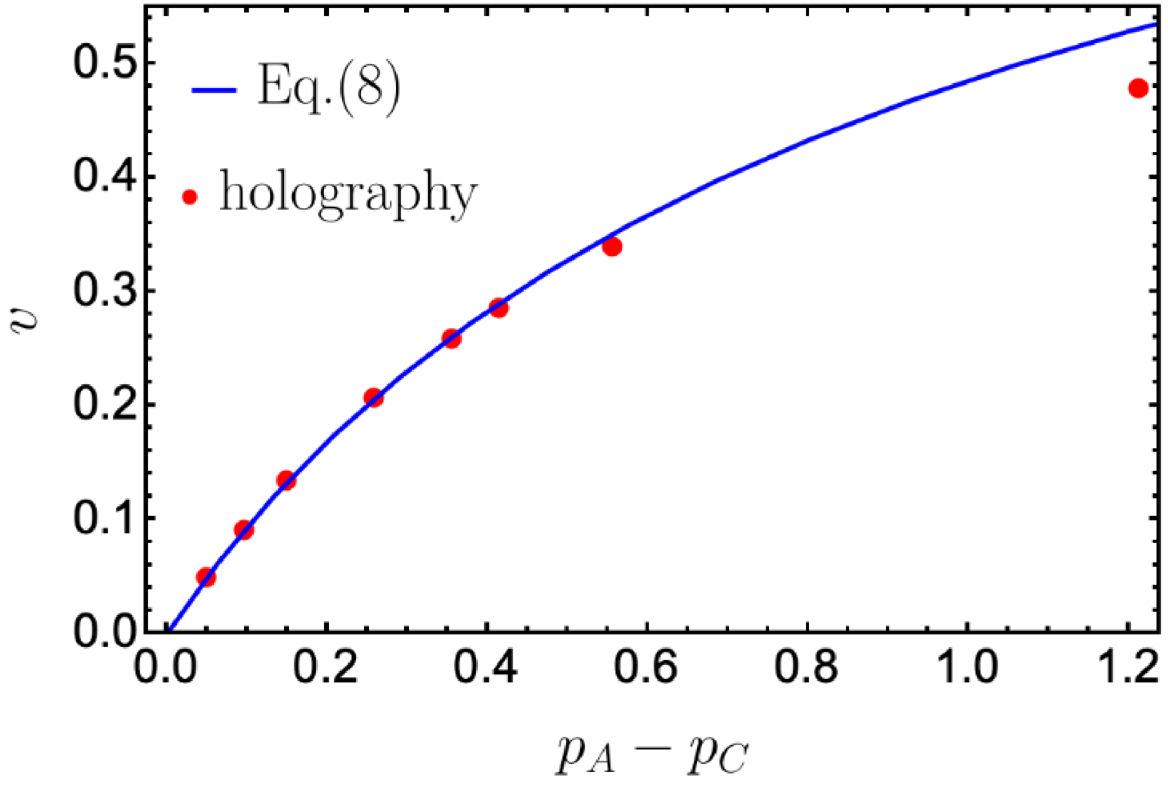


the solid line is the numerical solution and the dashed line represents the fitting results of the hyperbolic tangent function.

Bubble velocity

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

The final velocity of the bubble wall v_w is defined as

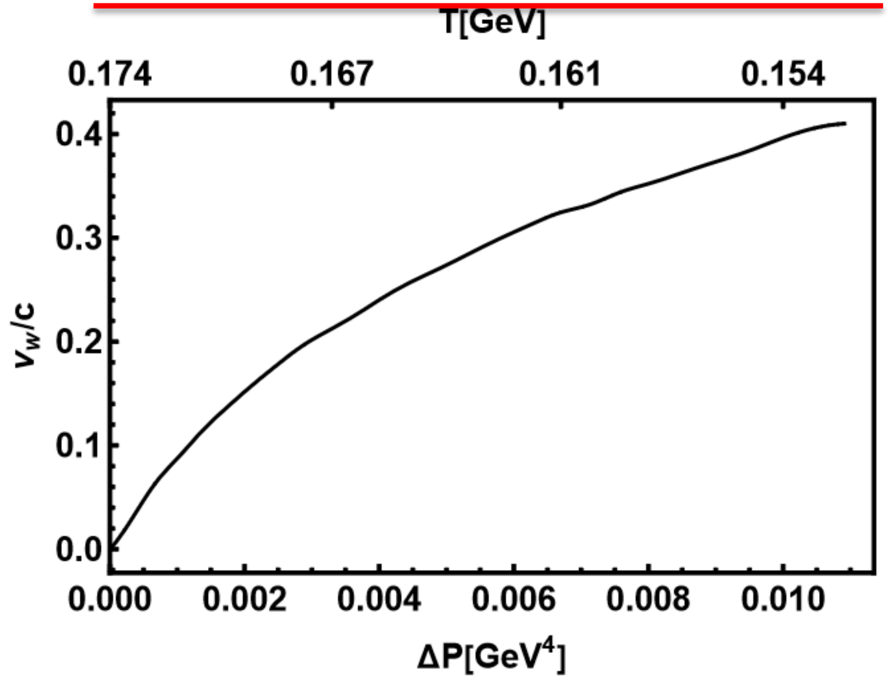


R. A. Janik et al., Phys Rev Lett(2022)

(a)

$v(t)$

$$c_s = \frac{1}{\sqrt{3}}$$



(b)

Figure 5. The panel (a) shows the bubble wall velocity as a function of time. The panel (b) shows the final velocity of the bubble wall as a function of the pressure difference ΔP between the inside and outside or the temperature T.

III. Bubbles and thin wall approximation

Characteristic temperatures

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

To calculate GW spectra, several different temperatures that characterize the phase transition in an expanding universe are relevant:

1) The T_c is the critical temperature, at which both minima of the effective potential are degenerate
2) The T_n is the nucleation temperature, which is determined by comparing the decay rate $\Gamma(t)$ of the false vacuum to the expansion rate of the universe, described by the Hubble parameter $H(t)$.
3) The percolation temperature T_p is defined as the temperature at which the probability to have the false vacuum is about 0.7.

Bounce solution

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

the tunneling rate of the stochastically generated bubbles is

$$\Gamma(T) = A(T)e^{-\frac{S_b}{T}},$$

where Sb is the Euclidean action evaluated on the bounce solution and the factor is $A(T) = T^4\left(\frac{S_b}{2\pi T}\right)^{3/2}$.

The nucleation temperature Tn can be given by relation $\frac{S_b(T_n)}{T_n} \sim 180$ for QCDPT ($\frac{S_b(T_n)}{T_n} \sim 140$ for EWPT).

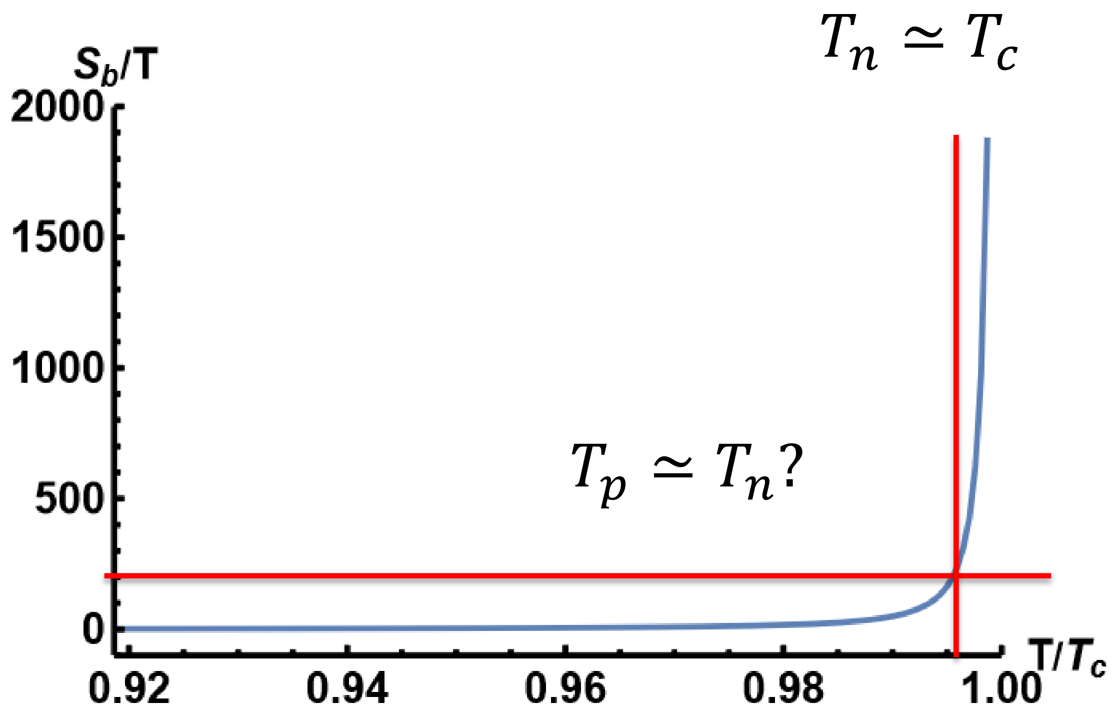


Figure 6. The Euclidean action $\frac{S_b}{T}$ as a function of temperature.

X. Chen, Danning Li and M. Huang arXiv: 2212.06591

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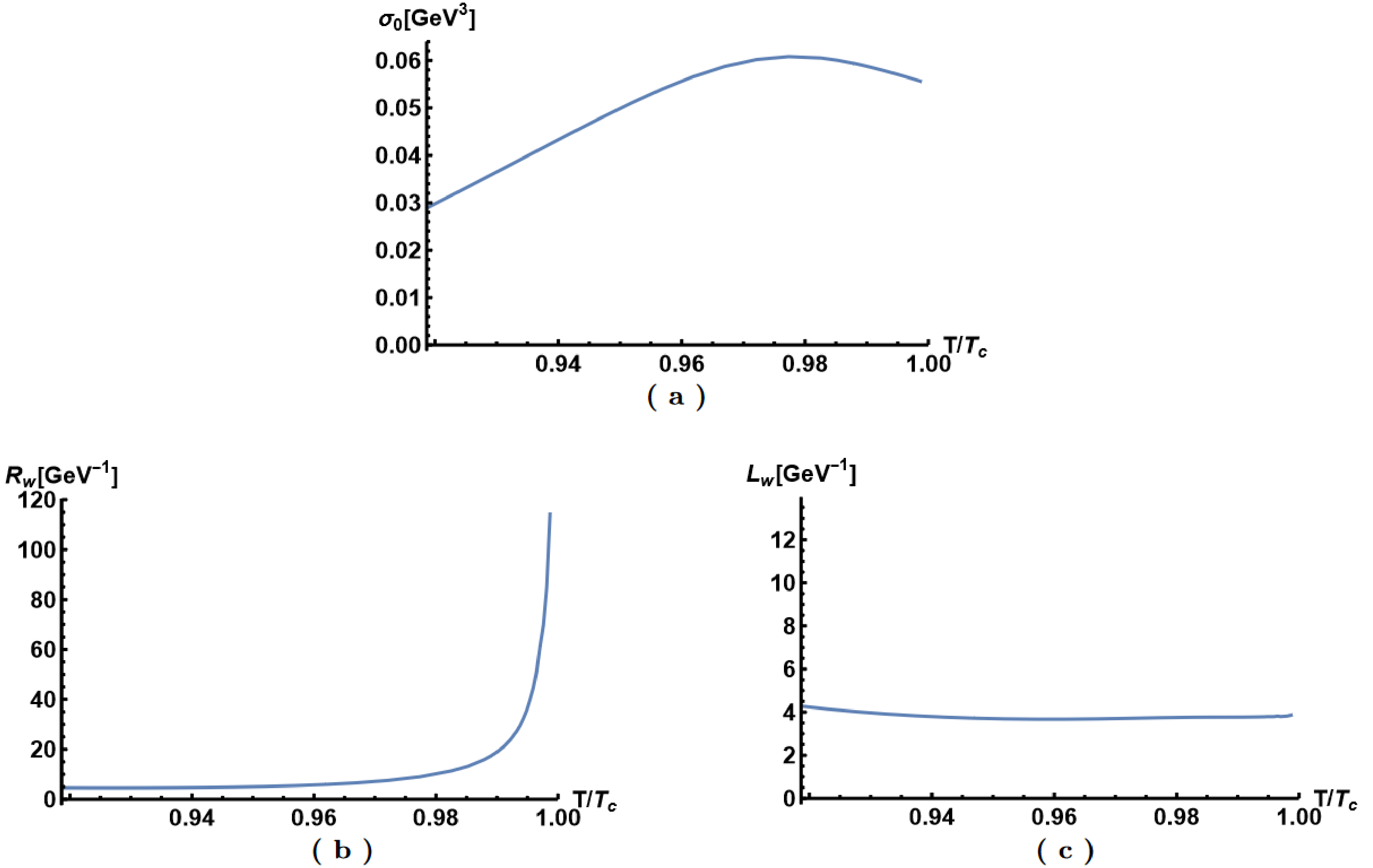
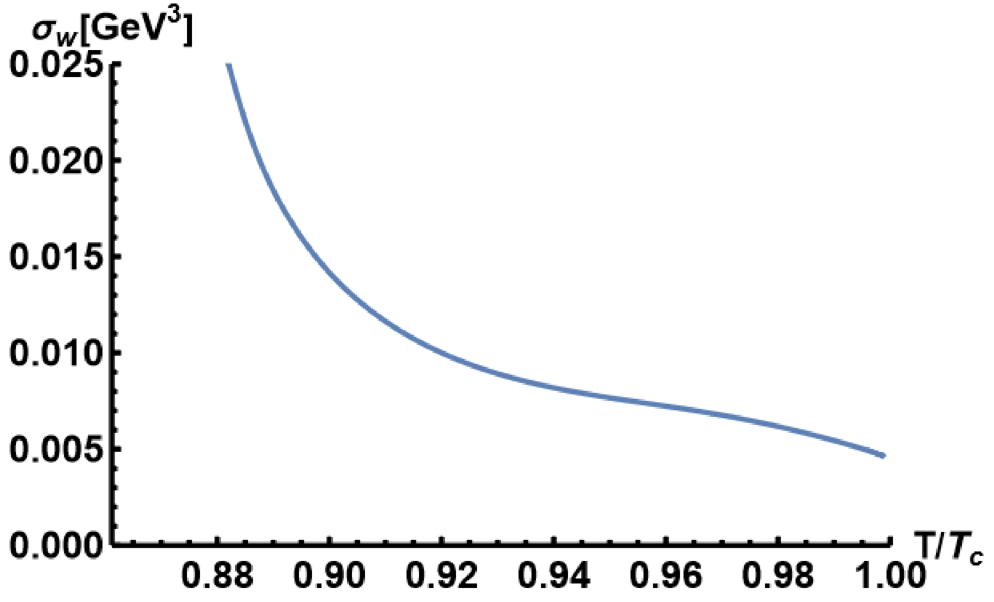
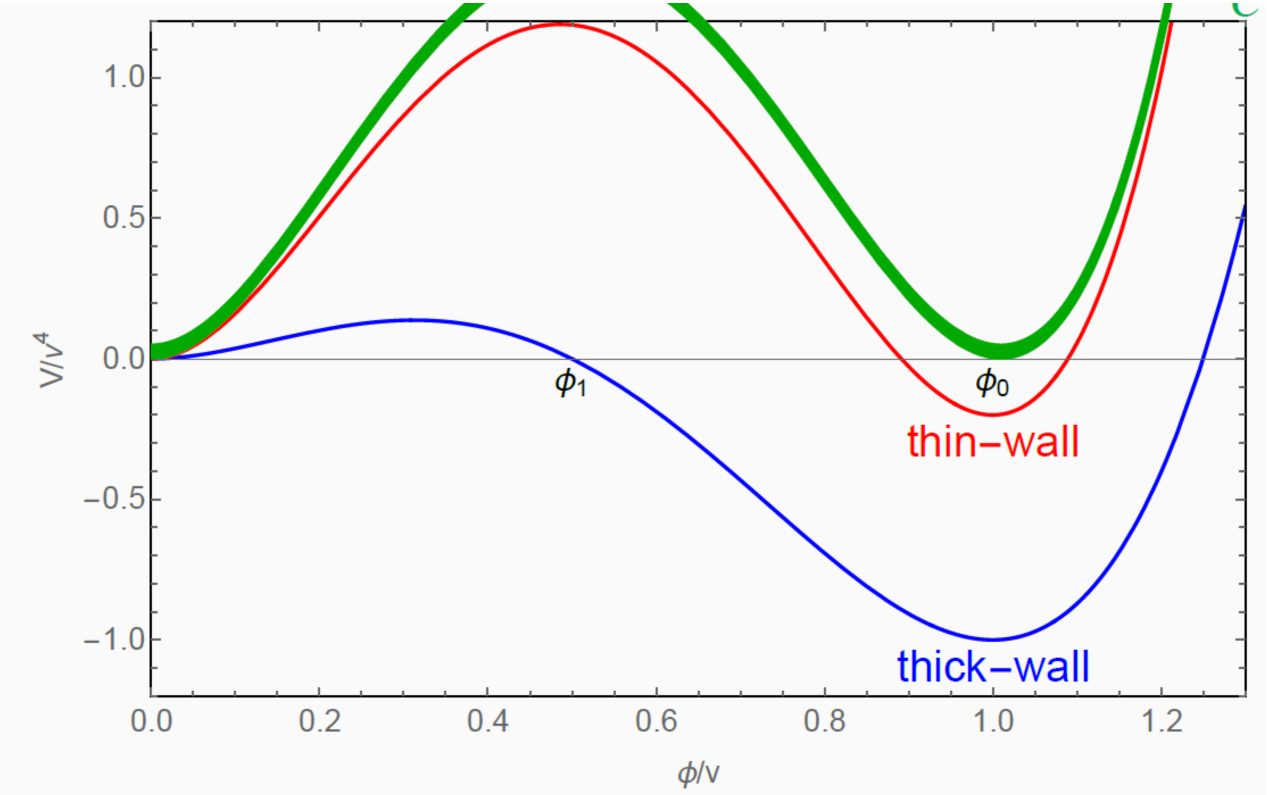


Thin-wall approximation

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

As in Ref. **[A. Eichhorn, et al., JCAP (2021)]**, for $L_w \ll R_w$ (thin-wall approximation), we can estimate the surface tension and the parameters α and β/H using the Euclidean action.





Thin-wall approx	QCDPT	EWPT
α	4-6	0.4-0.6
β/H	30000-60000	6000-20000
v_w	0.04	0.1

IV. Gravitational waves

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

Within the linear approximation, the total GW power spectra can be written as

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_{\rm coll} + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb},$$

we have found that for the holographic model, the final velocity of the bubble wall is less than the speed of sound, i.e., the **non-runaway case**. According to Ref. [C. Caprini et al., JCAP(2016)], the GWs generated by collisions in the non-runaway case can be neglected with respect to acoustic waves and magnetohydrodynamic turbulence. Therefore, the total power spectrum is approximated to

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}.$$

The key quantities which determine these power spectra are the following:

1) The percolation temperature T_p is defined as the temperature at which the probability to have the false vacuum is about 0.7.

2) The phase transition strength parameter α, which is related to the scalar potential energy released during the phase transition

3) The bubble wall speed v_w

4) The transition rate parameter β, which can be thought of as the inverse phase transition duration

GW power spectra

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

The nucleation temperature is defined as one bubble per unit Hubble volume and is written as

$$N(T_n) = \int_{T_n}^{T_c} \frac{\mathrm{d}T}{T} \frac{\Gamma(T)}{H(T)^4} = 1,$$

the probability of a false vacuum is defined as

$$P(T) = e^{-I(T)}, \qquad I(T) = \frac{4\pi}{3} \int_{T}^{T_c} \mathrm{d}T' \frac{\Gamma(T')}{H(T')T'^4} \left(\int_{T}^{T'} \mathrm{d}T'' \frac{v_w(T'')}{H(T'')} \right)^3 .$$

The percolation temperature is defined as $I(T_p) \simeq 0.34$, which is the temperature when the probability of false vacuum is about $P(T_p) \simeq 0.7$.

GW power spectra

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

The parameter α is defined as

$$\alpha \equiv \frac{1}{\rho_{\rm rad}}\left(\Delta V_{\rm eff} - \frac{T}{4}\frac{\partial \Delta V_{\rm eff}}{\partial T}\right)\Bigg|_{T=T_p} = -\frac{1}{\rho_{\rm rad}}\left(\Delta F - \frac{T}{4}\frac{\partial \Delta F}{\partial T}\right)\Bigg|_{T=T_p},$$

The inverse of the duration time is defined as

Near-conformal model α~O(1)

$$\frac{\beta}{H} \equiv T\frac{d}{dT}\left(\frac{S_b}{T}\right)\Bigg|_{T=T_p}.$$

	QCDPT		EWPT	
Model	I ($v_3 \neq 0$)	II ($v_6 \neq 0$)	I ($v_3 \neq 0$)	II ($v_6 \neq 0$)
g_*	10		100	
α	4.881	6.142	0.238	0.763
β/H	41151	23276	17198	7238
v_w	0.027	0.041	0.063	0.125
T_c[GeV]	0.1741		122.1	
T_n[GeV]	0.1733	0.1712	120.7	118.1
T_p[GeV]	0.1732	0.1703	120.4	117.6

Strongly coupled theory
β~10000 ?

F.R. Ares et al., Phys.Rev.Lett.(2022);
J. Shao, Mei Huang, Phys.Rev.D(2023)

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

From numerical simulations, the power spectrum from sound waves has the form of

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{H}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w S_{\rm sw}(f),$$

$$S_{\rm sw}(f) = (f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2},$$

$$f_{\rm sw} = 1.9\times 10^{-2}{\rm mHz}\frac{1}{v_w}\left(\frac{\beta}{H}\right)\left(\frac{T_p}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}},$$

The specific form of the factor κv depends on the bubble wall velocity and has the following form in the limits of large and small velocities

$$\kappa_v \simeq \begin{cases} \alpha\left(0.73+0.083\sqrt{\alpha}+\alpha\right)^{-1}, & v_w\sim 1,\\ v_w^{6/5}\, 6.9\alpha\left(1.36-0.037\sqrt{\alpha}+\alpha\right)^{-1}, & v_w\lesssim 0.1.\end{cases}$$

For the power spectrum from the Kolmogorov-type turbulence, numerical simulations show that it can be given as

$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w S_{\rm turb}(f),$$

$$S_{\rm turb}(f) = \frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}(1+8\pi f/h_*)},$$

$$f_{\rm turb} = 2.7\times 10^{-2}{\rm mHz}\frac{1}{v_w}\left(\frac{\beta}{H}\right)\left(\frac{T_p}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}},$$

$$h_* = 16.5\times 10^{-3}{\rm mHz}\left(\frac{T_p}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}.$$

the factor κturb is chosen to be

$$\kappa_{\rm turb} = 0.05\kappa_v.$$

GW power spectra

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

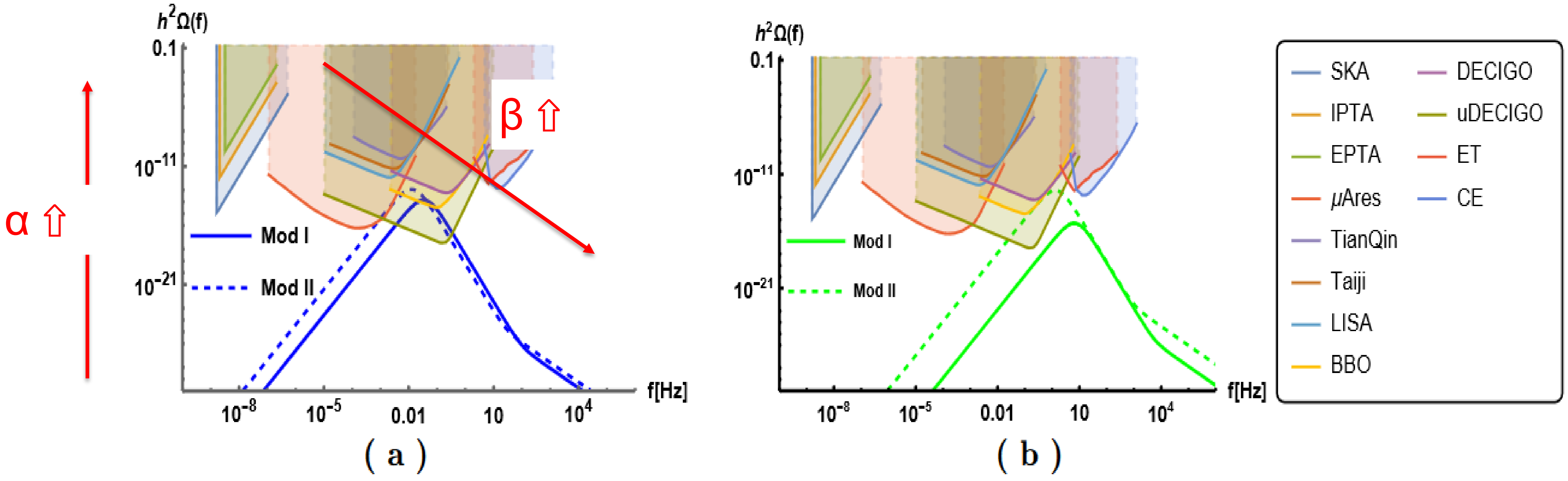


Figure 9. The GW power spectra in different holographic models, where the blue (a) and green (b) lines represent QCDPT and EWPT, respectively.

V. Summary

Summary

Y. Chen, Danning Li and M. Huang arXiv: 2212.06591

1) We investigate the **bounce solution** in the holographic QCD and electroweak models with first-order phase transition. The strength parameter α, inverse duration time β/H and bubble wall velocity vw are calculated by holographic bounce solution.

2) We find the parameter **α is about O(1)** and **β/H is about 10000**. The critical, nucleation and percolation temperatures of the phase transition are close to each other in the holographic model. In addition, the **velocity vw is found to be less than the sound speed**.

Summary

3) For QCD phase transition, the gravitational wave power spectrum can reach 10−13 − 10−14 around the peak frequency of 0.01 Hz, which can be detected by BBO and Ultimate-DECIGO. For electroweak phase transition, the gravitational wave power spectrum can reach 10−12 − 10−16 around the peak frequency 1 − 10 Hz.

4) **The primordial black hole is not favorable for formation due to the large parameter β/H and small velocity vw.**

Thanks!