Strongly coupled first-order phase transitions in the early Universe

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Holographic QCD Seminar X
2023-03-17 09:30 am

2011.11451 (JCAP) "Effective picture of bubble expansion"
2205.02492 (PRD) "Hydrodynamic backreaction force of cosmological bubble expansion"
2210.14094 "Gravitational Waves and Primordial Black Hole Productions from Gluodynamics"
2302.10042 "Bubble expansion at strong coupling"

Outline

1. Cosmological first-order phase transitions
    - 1.1 Stochastic gravitational-wave background
    - 1.2 Bubble wall velocity from weakly coupled FOPT
2. Strongly-coupled first-order phase transitions
    - 2.1 Holographic QCD models
    - 2.2 Holographic numerical simulations
3. Bubble wall velocity for strongly-coupled FOPT
    - 3.1 Perfect fluid hydrodynamics
    - 3.2 Non-relativistic wall velocity

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    2.2 Holographic numerical simulations

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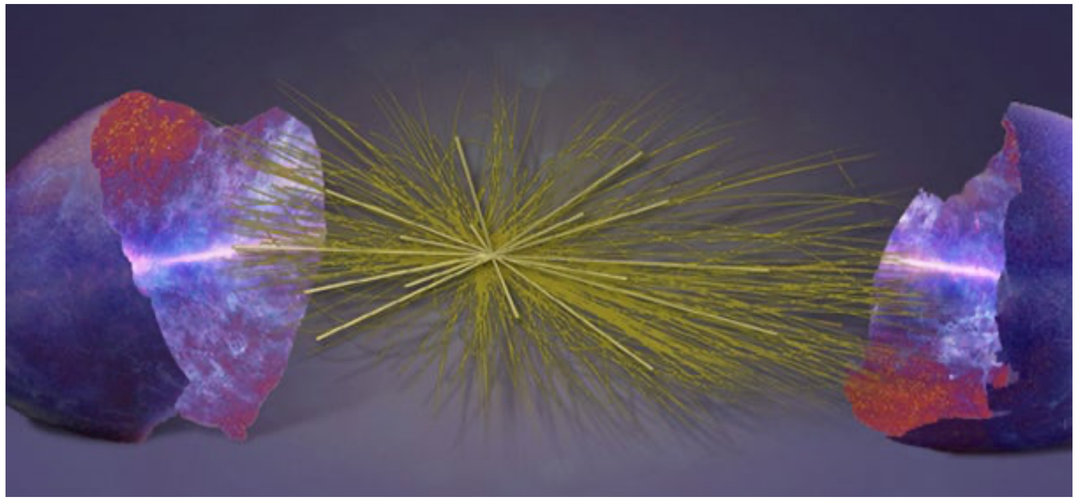
    3.1 Perfect fluid hydrodynamics

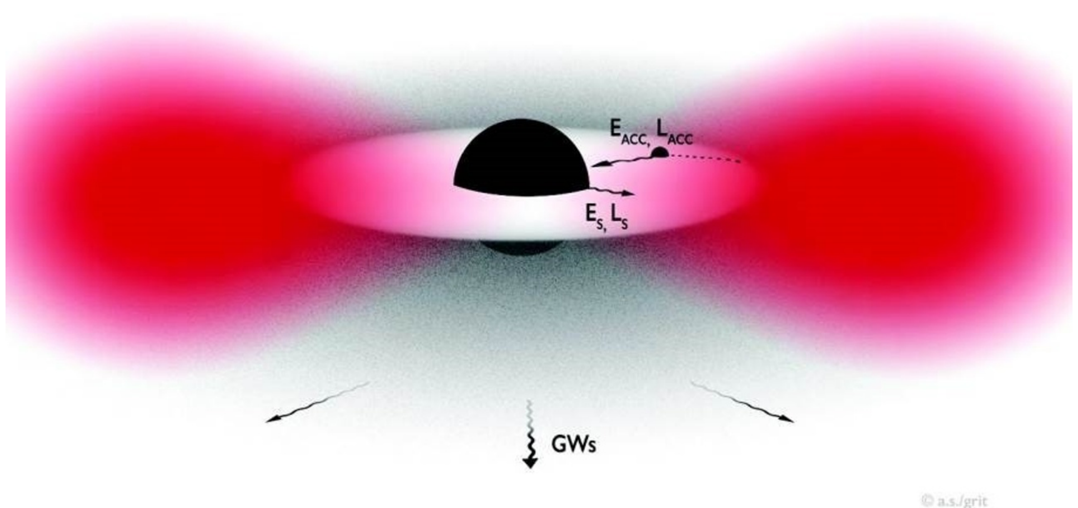
    3.2 Non-relativistic wall velocity

Where to go for BSM?



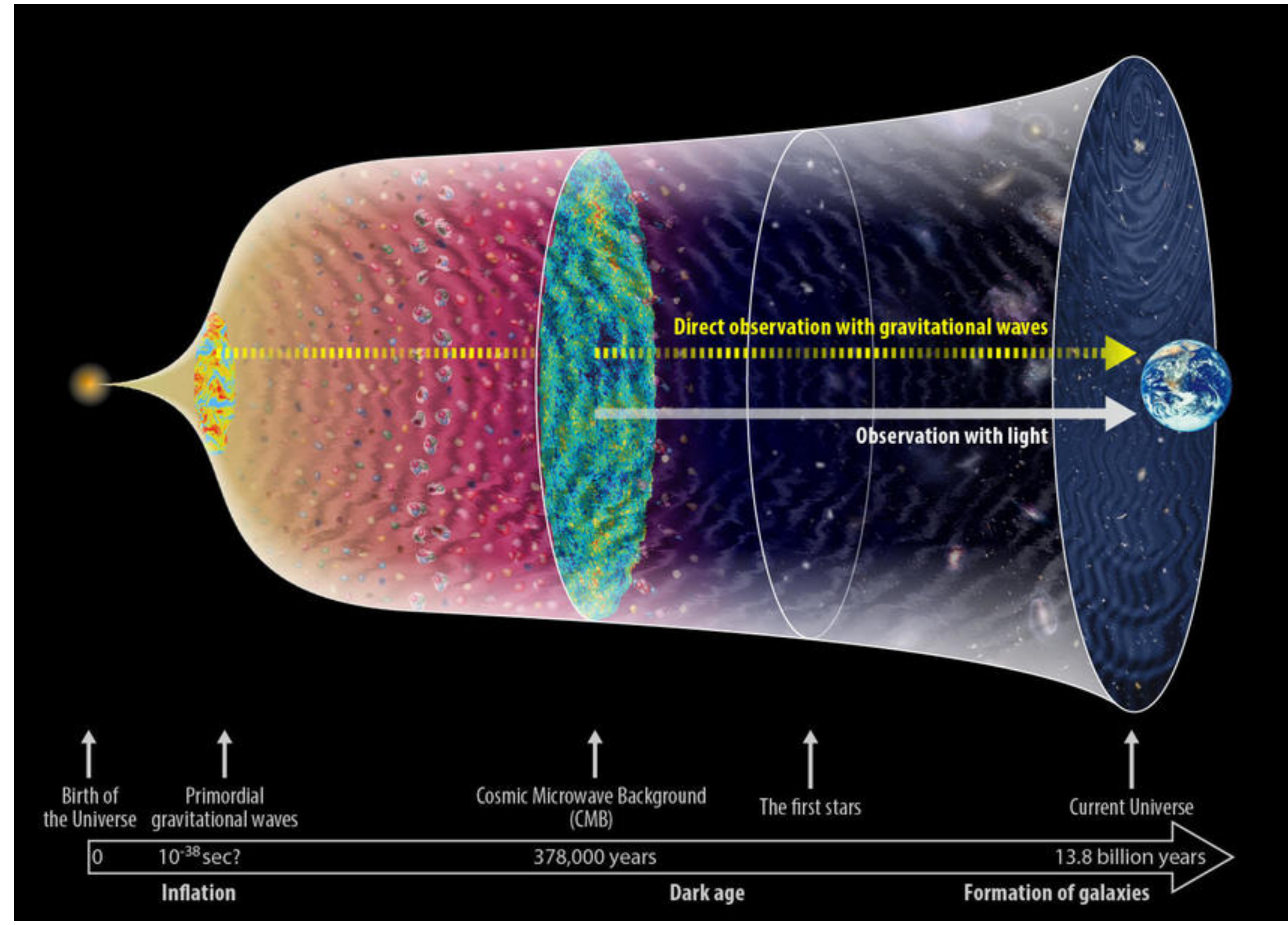
Cosmological collider

BSM@inflation: imprint @CMB/LSS



Gravitational atom

Black hole+orbital clouds: GW/EM

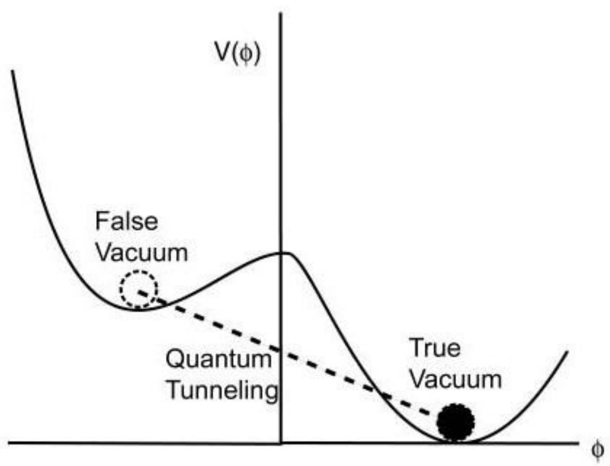


Stochastic gravitational-wave background

Primordial GWs: inflation at large scales
Induced GWs: inflation at small scales
GWs from preheating: inflaton-SM couplings
GWs from FOPT: BSM with FOPT (this talk)
GWs from MBH, SMBH, EMRI: DM properties

Cosmological FOPT

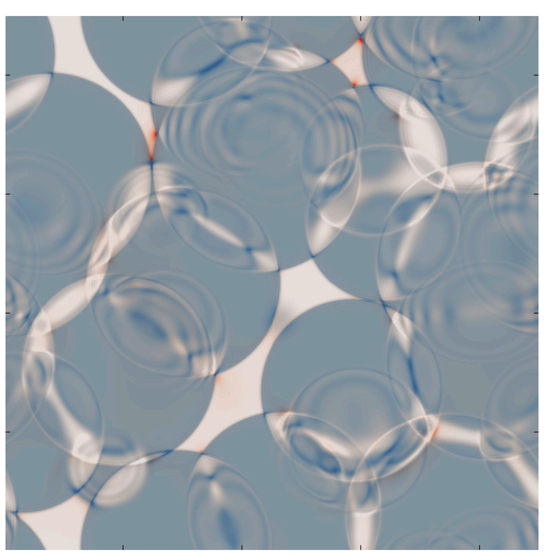
Bubble nucleations



Vacuum decay dynamics $\Gamma(t) = A(t)e^{-B(t)}$

Phase transition dynamics $F(t) = e^{-\int_{t_i}^{t} \mathrm{d}t'\Gamma(t')a(t')^3 V(t,t')}$

Bubble collisions

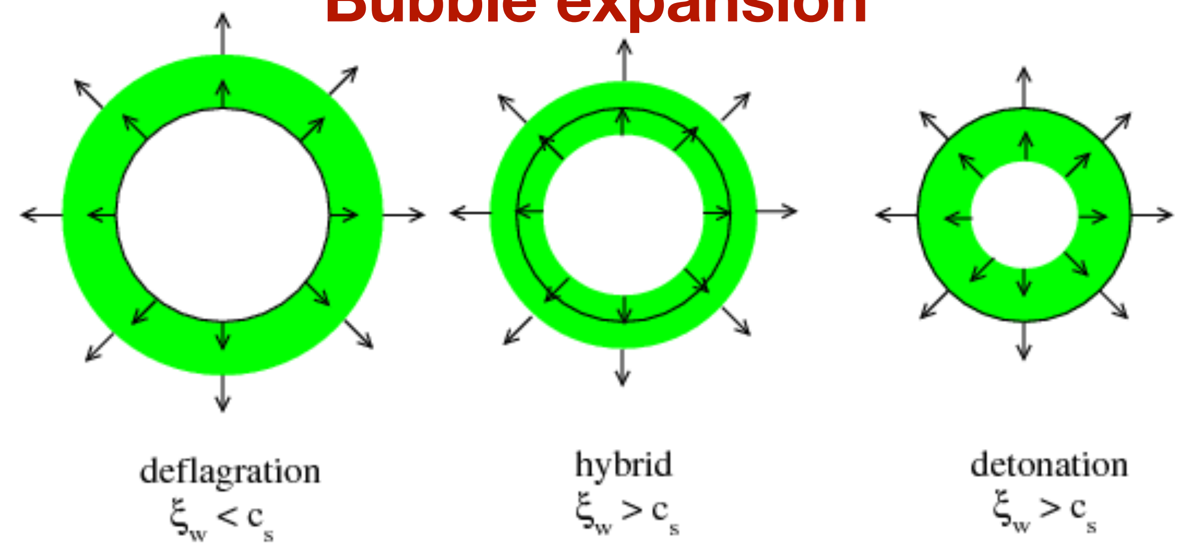


Auxiliary modeling

Numerical simulations

$$h^2\Omega_{\mathrm{GW}}\left(\alpha, \frac{\beta}{H_*}, v_w, \kappa_v\right)$$

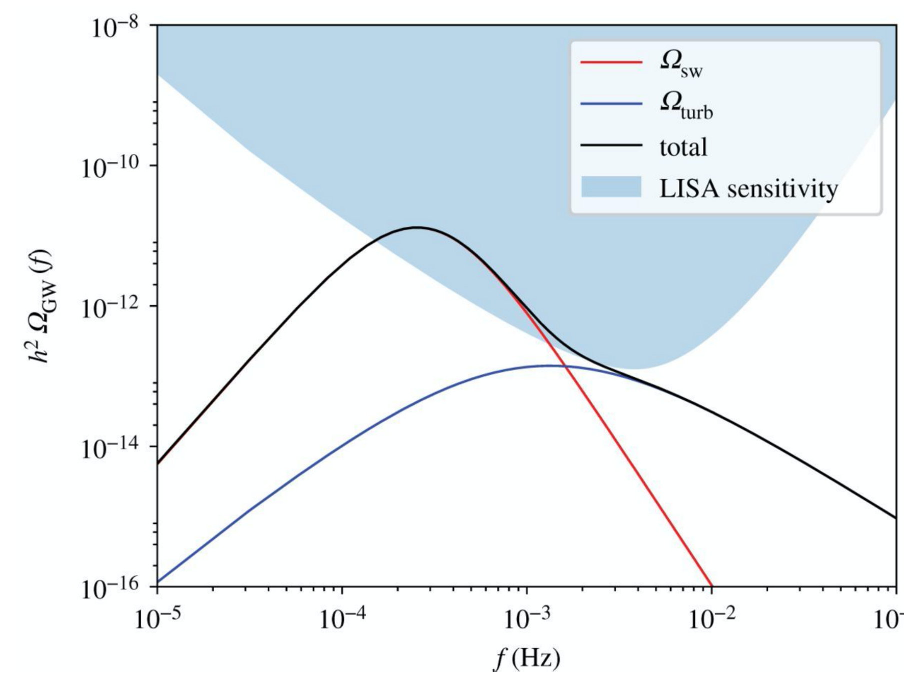
Bubble expansion



Microscopic non-equilibrium v_w

Macroscopic hydrodynamics $\kappa_v(\alpha, v_w)$

Stochastic GW backgrounds



Model buildings

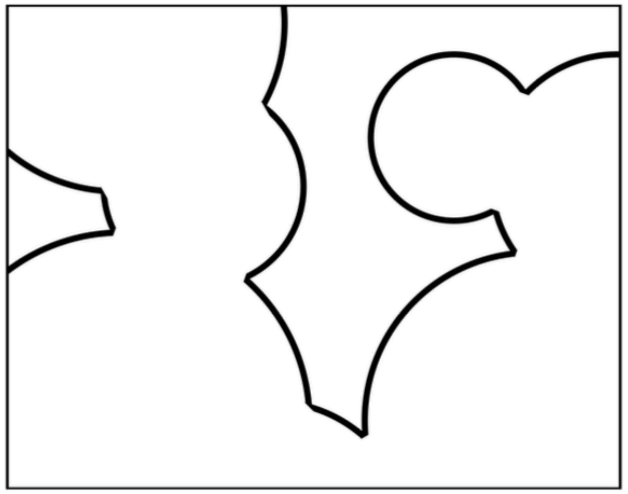
Model selection

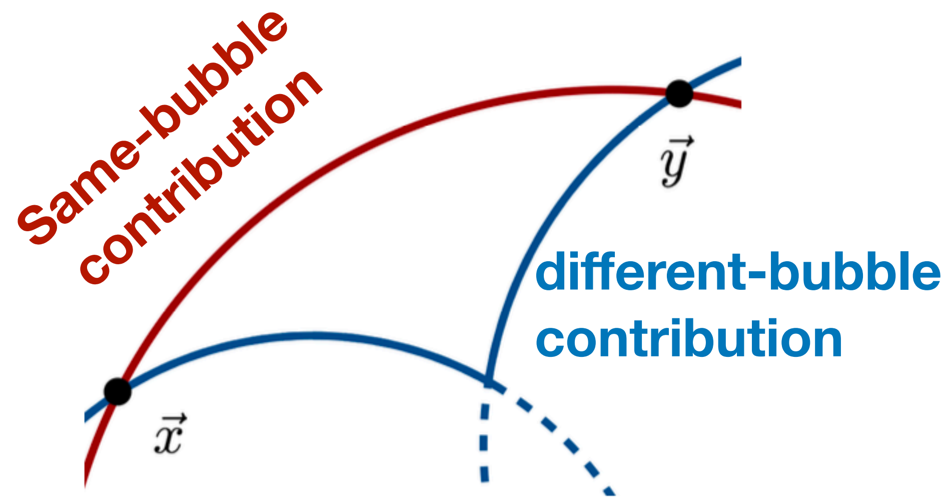
$$\left(\alpha, \frac{\beta}{H_*}, v_w\right)(V_{\mathrm{eff}})$$

GWs from wall collisions

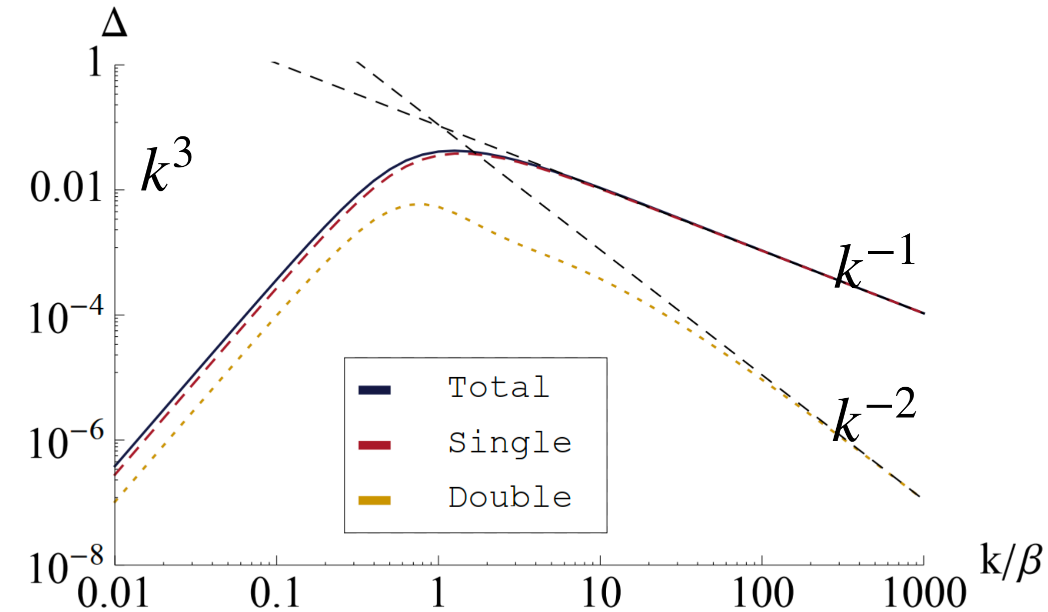
Envelope approximation

Jinno et.al. PRD 95 (2017) 024009



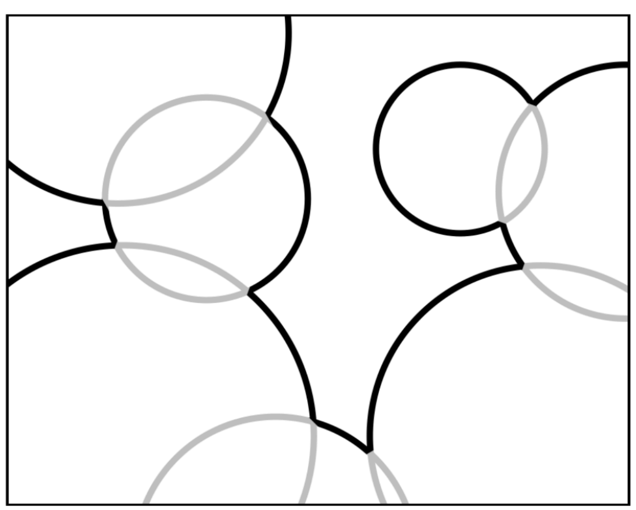


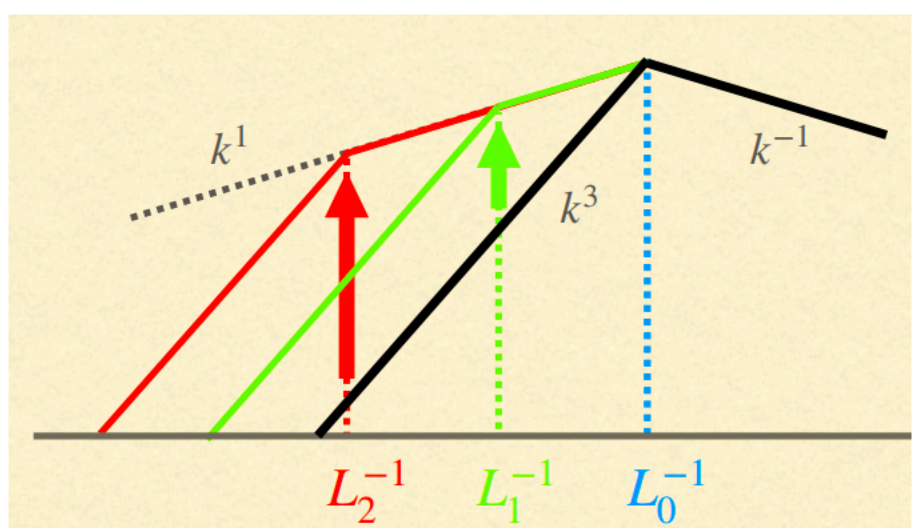
Analytic auxiliary modeling



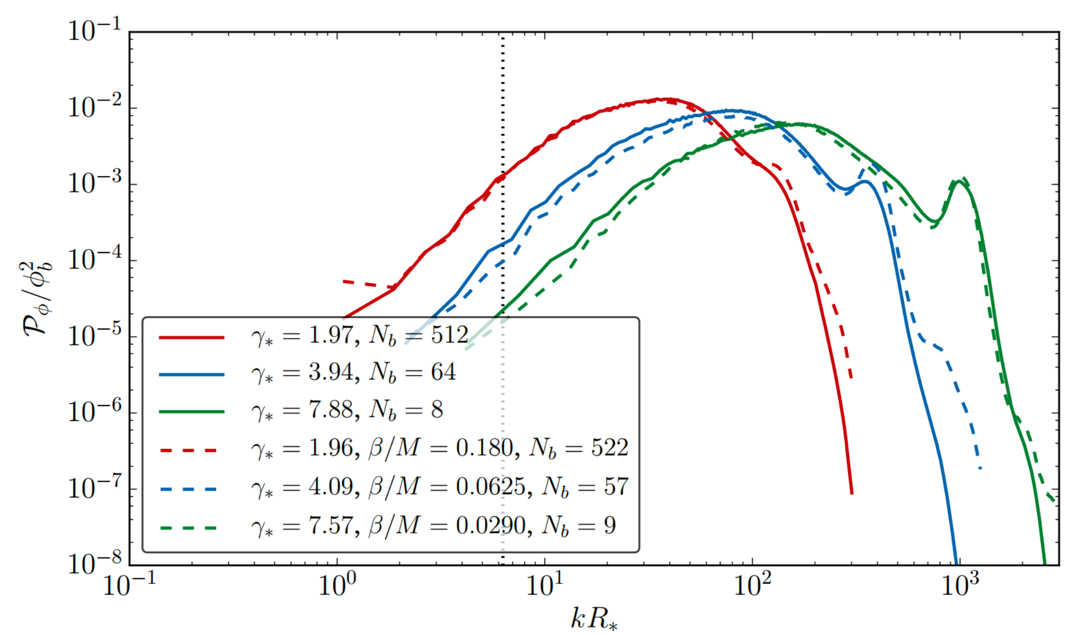
Without envelope approximation

Jinno et.al. JCAP 01 (2019) 060





Numerical simulation



$$h^2\Omega_{\rm env} = 1.67\times10^{-5}\left(\frac{100}{g_{\rm dof}}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\frac{0.48v_w^3}{1+5.3v_w^2+5v_w^4}S_{\rm env}(f)$$

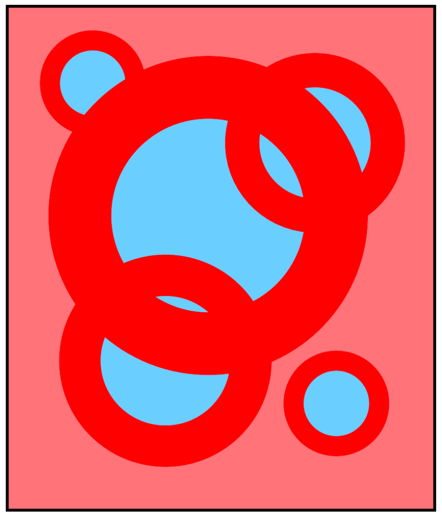
$$S_{\rm env}(f) = \left[c_l\left(\frac{f}{f_{\rm env}}\right)^{-3}+(1-c_l-c_h)\left(\frac{f}{f_{\rm env}}\right)^{-1}+c_h\left(\frac{f}{f_{\rm env}}\right)\right]^{-1}$$

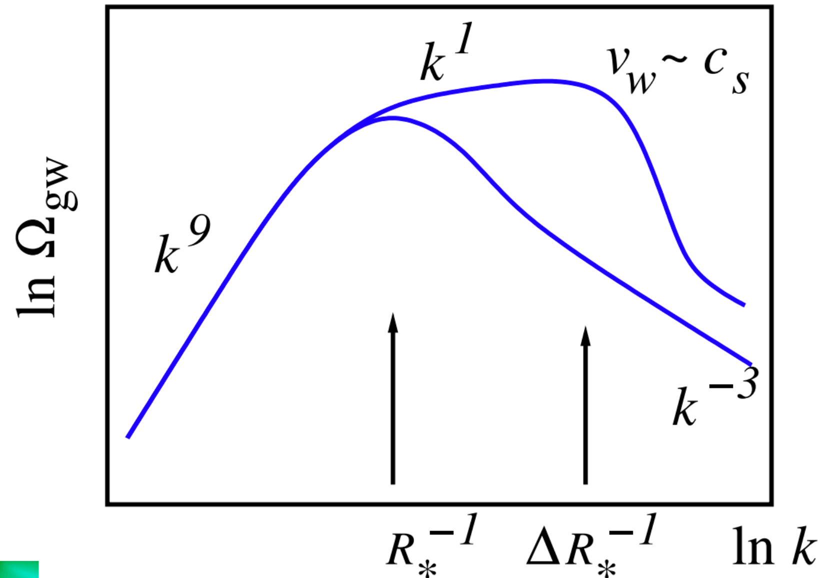
$$\frac{f_{\rm env}}{\rm Hz} = 1.65\times10^{-5}\left(\frac{g_{\rm dof}}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,{\rm GeV}}\right)\frac{0.35(\beta/H_*)}{1+0.069v_w+0.69v_w^4}$$

GWs from fluid motions

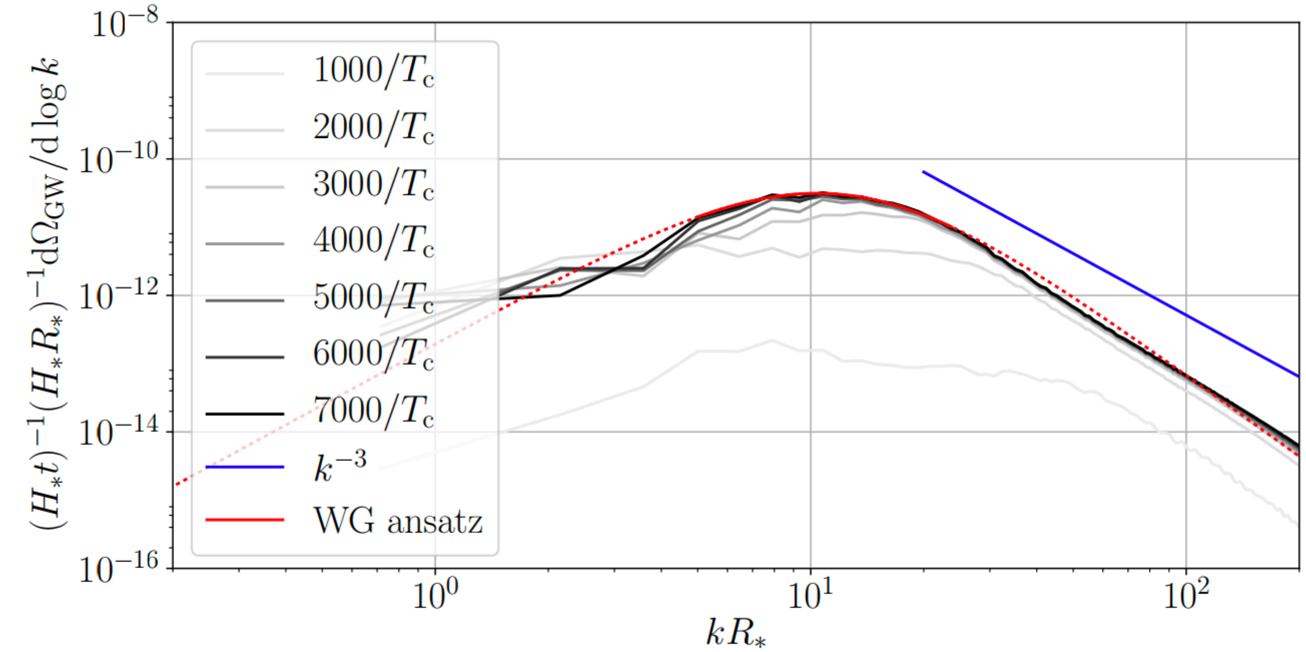
Analytic auxiliary modeling

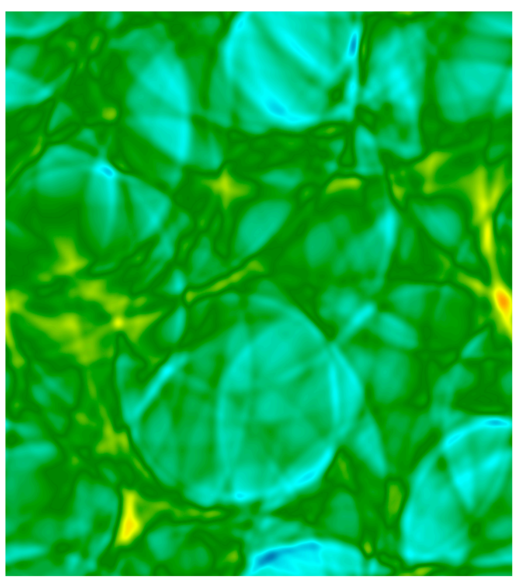
Sound shell model Hindmarsh 18, Hindmarsh & Hijazi 19





Numerical simulation (sound waves)





Helsinki-Sussex: 2013, 2015, 2017, 2019

$$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta\gamma(\dot{\phi} + v^i\partial_i\phi)$$

$$E = \gamma\epsilon : \quad \dot{E} + \partial_i(Ev^i) + p[\dot{\gamma} + \partial_i(\gamma v^i)] - \frac{\partial V}{\partial \phi}\gamma(\dot{\phi} + v^i\partial_i\phi) = \eta\gamma(\dot{\phi} + v^i\partial_i\phi)$$

$$P_i = \gamma(\epsilon + p)U_i : \quad \dot{P}_i + \partial_j(P_i v^j) + \partial_i p + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta\gamma(\dot{\phi} + v^j\partial_j\phi)\partial_i\phi$$

Friction ansatz

$$h^2\Omega_{\rm sw} = 2.65\times 10^{-6}\left(\frac{100}{g_{\rm dof}}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w \frac{7^{7/2}(f/f_{\rm sw})^3}{(4+3(f/f_{\rm sw})^2)^{7/2}}\Upsilon$$

$$\frac{f_{\rm sw}}{\rm Hz} = 1.9\times 10^{-5}\left(\frac{g_{\rm dof}}{100}\right)^{\frac{1}{6}}\left(\frac{T_*}{100\,\rm GeV}\right)\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_*}\right)$$

$$\Upsilon \equiv 1 - (1 + 2\tau_{\rm sw}H_*)^{-1/2} \qquad \tau_{\rm sw}H_* \approx (8\pi)^{1/3}v_w/(\beta/H_*)/\bar{U}_f \qquad \bar{U}_f^2 = 3\kappa_{\rm sw}\alpha/[4(1+\alpha)]$$

Guo et al. 20

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Microscopic dynamics

$$T_\phi^{\mu\nu} = \nabla^\mu\phi\,\nabla^\nu\phi + g^{\mu\nu}\left[-\frac{1}{2}(\nabla\phi)^2 - V_0(\phi)\right]$$

$$T_f^{\mu\nu} = \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{k^\mu k^\nu}{k^0}\Bigg|_{k^0=E_i(\mathbf{k})} f_i(\mathbf{x},\mathbf{k})$$

Scalar EoM

$$\nabla_\mu T_\phi^{\mu\nu} \equiv [\nabla_\mu\nabla^\mu\phi - V_0'(\phi)]\,\nabla^\nu\phi = +f^\nu$$

Plasma EoM

$$\nabla_\mu T_f^{\mu\nu} \equiv \sum_{i=\mathrm{B,F}} g_i \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{k^\mu k^\nu}{E_i(\mathbf{k})}\nabla_\mu f_i = -f^\nu$$

$$f^\nu = \nabla^\nu\phi \sum_{i=\mathrm{B,F}} g_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f_i}{2E_i}$$

$$\frac{\partial V_T^{1-\mathrm{loop}}}{\partial\phi} = \sum_{i=\mathrm{B,F}} g_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f_i^{\mathrm{eq}}}{2E_i}$$

$$f_i = f_i^{\mathrm{eq}} + \Delta f_i^{\mathrm{eq}} + \delta f_i \qquad f_i^{\mathrm{eq}} = \frac{1}{e^{E_i/T}\mp 1}$$

$$f^\nu = \nabla^\nu\phi\left(\frac{\partial V_T^{1-\mathrm{loop}}}{\partial\phi} + \frac{\partial\Delta V_T}{\partial\phi} - \frac{\partial p_{\delta f}}{\partial\phi}\right)$$

Out-of-equilibrium effect

$$\nabla_\mu\nabla^\mu\phi - \frac{\partial V_{\mathrm{eff}}}{\partial\phi} = -\frac{\partial p_{\delta f}}{\partial\phi} \qquad \nabla_\mu T_f^{\mu\nu} + \nabla^\nu\phi\frac{\partial V_T}{\partial\phi} = \nabla^\nu\phi\frac{\partial p_{\delta f}}{\partial\phi}$$

Microscopic dynamics

Microscopic scalar-dynamics

Integrated scalar EOM $$\int d\xi \frac{d\phi}{d\xi} \nabla^2 \phi = \int d\xi \frac{d\phi}{d\xi} \left(\frac{\partial V_{\rm eff}}{\partial \phi} - \frac{\partial p_{\delta f}}{\partial \phi} \right)$$ **Self-similar Coordinate** $\xi \equiv r/t$

$$\int d\xi \frac{d\phi}{d\xi} \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial t^2} \right) = \Delta V_{\rm eff} - \left(\int d\xi \frac{dT}{d\xi} \frac{\partial V_{\rm eff}}{\partial T} + \int d\xi \frac{d\phi}{d\xi} \frac{\partial p_{\delta f}}{\partial \phi} \right)$$

$$\phi(t, r) = \phi(\gamma_{\rm w}(t)[r - r_{\rm w}(t)]) \equiv \phi(r') \qquad \gamma_w(t) = (1 - \dot{r}_w^2(t))^{-1/2} = (1 - v_w^2(t))^{-1/2}$$

$$t' = \gamma_{\rm w}(t)[t - v_{\rm w}(t) r] \quad r' = \gamma_{\rm w}(t)[r - r_{\rm w}(t)] \qquad \phi(r' = -\infty) = \phi_- \quad \phi(r' = +\infty) = \phi_+$$

$$\sigma \gamma_w^3 \ddot{r}_w + \frac{2\sigma\gamma_w}{r_w} \equiv p_{\rm driving} - (p_{\rm thermal} + p_{\rm friction}) = p_{\rm driving} - p_{\rm backreaction}$$

$$\left(\sigma + \frac{r_w}{3} \frac{dp_{\rm br}}{d\gamma_w} \right) \frac{d\gamma_w}{dr_w} + \frac{2\sigma\gamma_w}{r_w} = p_{\rm dr} - p_{\rm br} \qquad p_{\rm br} = \Delta p_{\rm LO} + f(\gamma_w) \Delta p_{NLO}$$

$$\frac{f(\gamma_w) - f(1)}{f(\gamma_w^{\rm eq}) - f(1)} + \frac{3\gamma_w}{2r_w} = 1 + \frac{1}{2r_w^3} \qquad f(\gamma_w^{\rm eq}) = \frac{p_{\rm dr} - \Delta p_{\rm LO}}{\Delta p_{NLO}}$$

Cai & SJW 2020

Microscopic dynamics

Microscopic scalar-dynamics

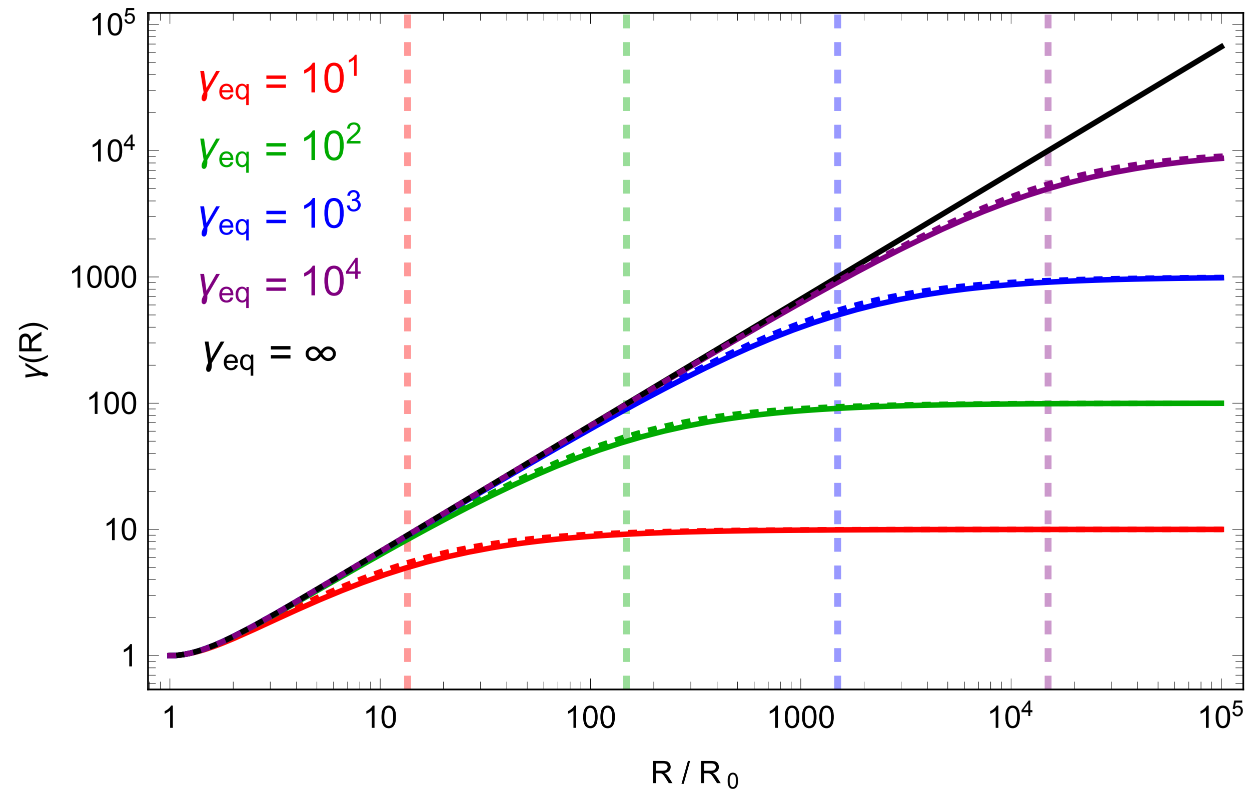
Cai & SJW 2020

Example $p_{\rm br} = \Delta p_{\rm LO} + f(\gamma_w)\Delta p_{NLO}$

$$\alpha = \frac{\Delta V_{\rm eff}}{\rho_R}, \quad \alpha_\infty = \frac{\Delta p_{\rm LO}}{\rho_R}, \quad f(\gamma_w) = \gamma_w$$

Solution $\gamma(R) = \dfrac{2\gamma_{\rm eq}R^3 + \gamma_{\rm eq} - 1}{2R^3 + 3(\gamma_{\rm eq}-1)R^2}$

Plateau scale $R_\sigma \equiv \dfrac{3}{2}(\gamma_{\rm eq}-1)R_0$



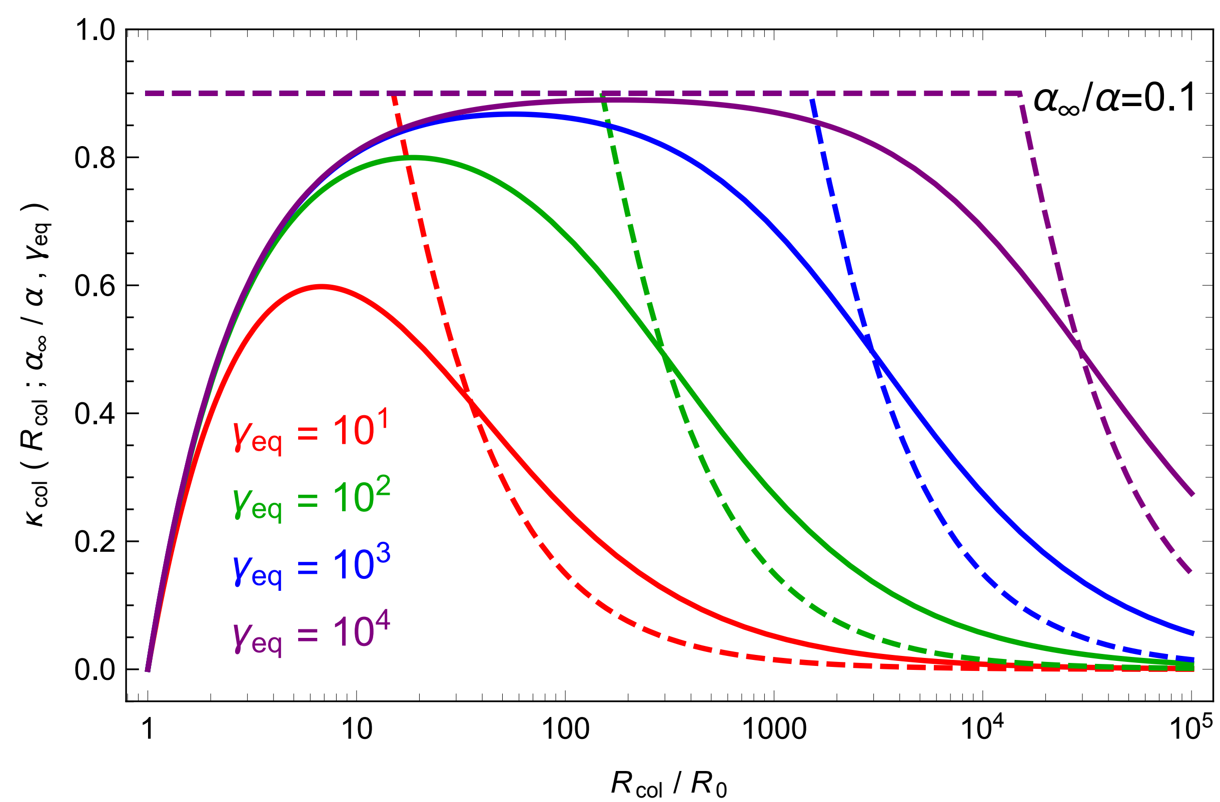
Efficiency factor of wall collisions

$$\kappa_\phi = \frac{E_{\rm wall}(R_*)}{\frac{4}{3}\pi R_*^3 \Delta V_{\rm eff}} = \int_{R_0}^{R_*} \frac{{\rm d}R}{R_*}\frac{\Delta V_{\rm eff} - \Delta p_{\rm br}}{\Delta V_{\rm eff}}$$

$$= \left(1 - \frac{\alpha_\infty}{\alpha}\right)\int_{R_0}^{R_*}\frac{{\rm d}R}{R_*}\left[1 - \frac{f(\gamma(R))}{f(\gamma_{\rm eq})}\right]$$

$$= \frac{1-\alpha_\infty/\alpha}{27\gamma_*^2\gamma_{\rm eq}(\gamma_{\rm eq}-1)}\left[\gamma_*(3\gamma_{\rm eq}-4)(3\gamma_{\rm eq}-1)^2\log\frac{3(\gamma_{\rm eq}+\gamma_*-1)}{3\gamma_{\rm eq}-1} - 4\gamma_*\log\frac{2}{3\gamma_*} - 2(\gamma_{\rm eq}-1)(3\gamma_*-2)\right]$$

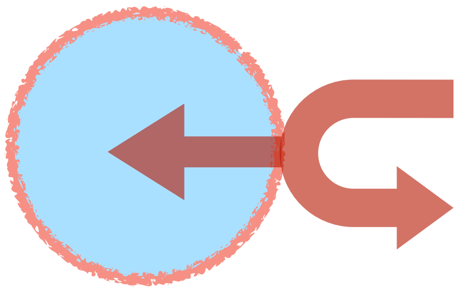
$$\gamma_* \approx \frac{2R_*}{3R_0}$$



Microscopic dynamics

Microscopic plasma-dynamics

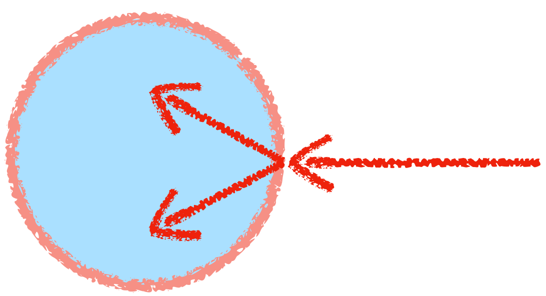
Particle transmission and reflection Bodeker & Moore 09 → **Run-away**



$$P_{1\to1} \approx \frac{\Delta m^2 T^2}{24} \equiv \Delta p_{\rm LO} \qquad \Delta m_i^2 \equiv m_i^2(\phi_-) - m_i^2(\phi_+)$$

$$\Delta m^2 \equiv \sum_i c_i g_i \Delta m_i^2 \begin{cases} c_i = 1, & i = \text{Boson} \\ c_i = 1/2, & i = \text{Fermion} \end{cases}$$

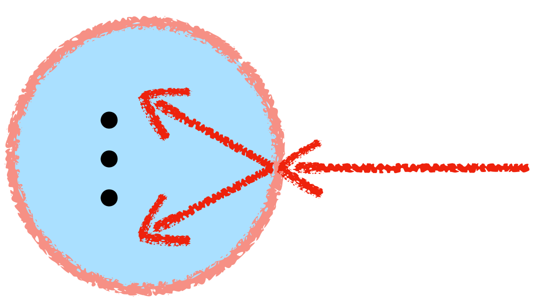
Transition splitting of a fermion emitting a soft vector boson Bodeker & Moore 17 → **No run-away**



$$P_{1\to2} \approx \gamma g^2 \Delta m_V T^3 \equiv \gamma \Delta p_{\rm NLO} \qquad \Delta m_i \equiv m_i(\phi_-) - m_i(\phi_+)$$

$$g^2 \Delta m_V = \sum_i g_i \lambda_i^2 \Delta m_i, \qquad i =$$ **gauge boson with gauge coupling**

Re-summing multiple soft gauge bosons scattering to all orders Hoche et al 20 → **No run-away**



$$P_{1\to N} \approx 0.005 \gamma^2 g^2 T^4 \equiv \gamma^2 \Delta p_{N\rm LO}$$

$$g^2 = \sum_i g_i \lambda_i^2, \qquad i =$$ **gauge bosons to which the scalar field couples with coupling** λ_i

Re-summing real and virtual gauge emissions at all leading-log orders Gouttenoire et al 21

$$\Delta V_{\rm eff} = P_{\rm LO} + P_{\rm LL} \qquad P_{\rm LL} = \mathcal{O}(1) \times g^2 \gamma m_{c,h} T_{\rm nuc}^3 \log \frac{m_{c,h}}{\mu}$$ → **No run-away**

Macroscopic hydrodynamics

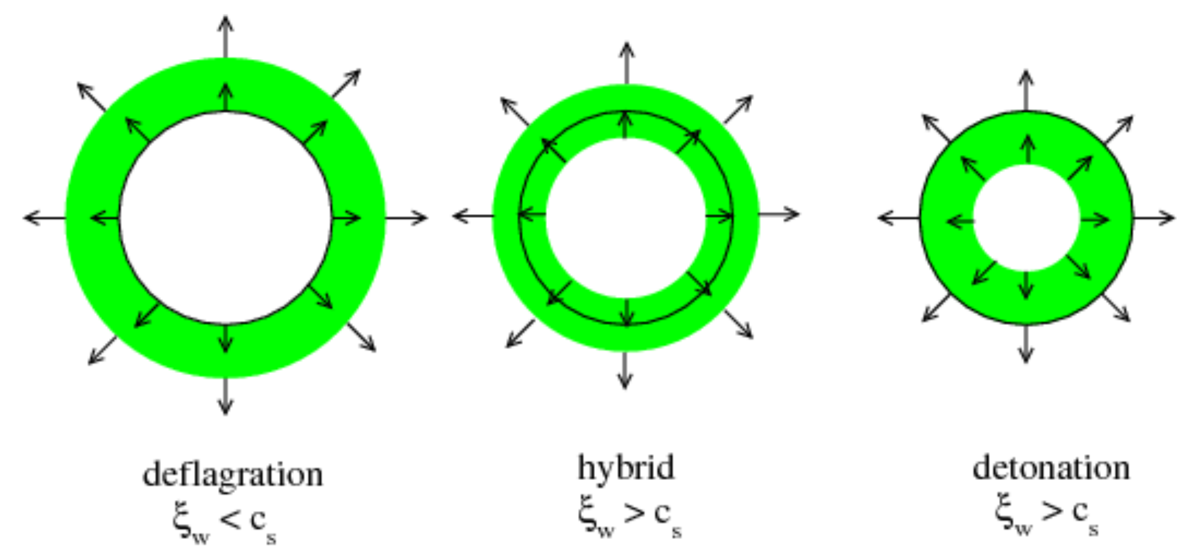
Assumptions: thin-wall steady-state bubble in a flat background of thermal plasma without shear and bulk viscosity

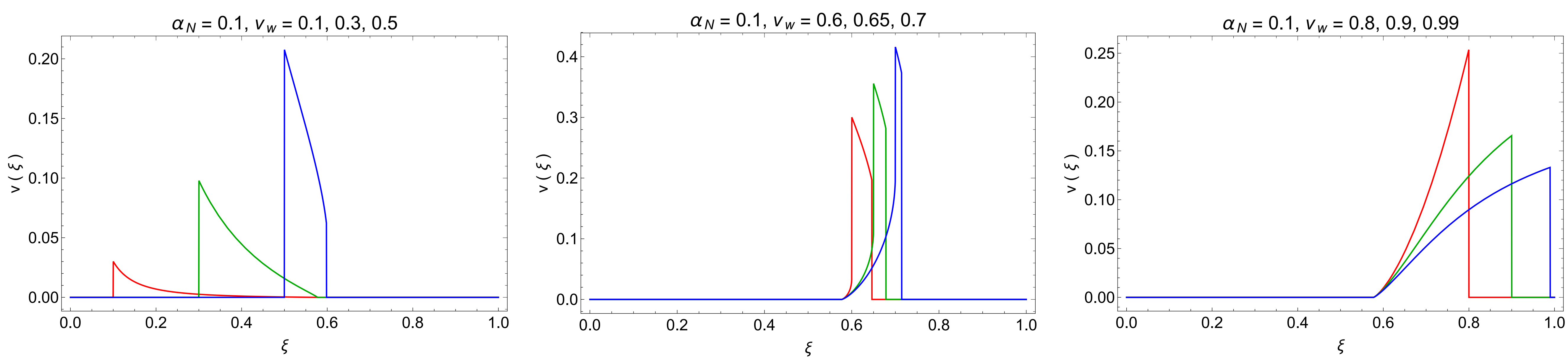
$$T^{\mu\nu} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu} \qquad u^\mu = \begin{cases} \gamma(v)(1,v,0,0), & \textbf{plasma frame} \\ \bar{\gamma}(1,-\bar{v},0,0), & \textbf{wall frame} \end{cases}$$

Fluid EOMs

$$D\frac{v}{\xi} = \gamma(v)^2(1-\xi v)\left(\frac{\mu(\xi,v)^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

$$\frac{\mathrm{d}\ln w}{\mathrm{d}\xi} = \gamma(v)^2\mu(\xi,v)\left(\frac{1}{c_s^2} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$





See, eg. Espinosa et al. 10

Macroscopic hydrodynamics

Bag equation of state

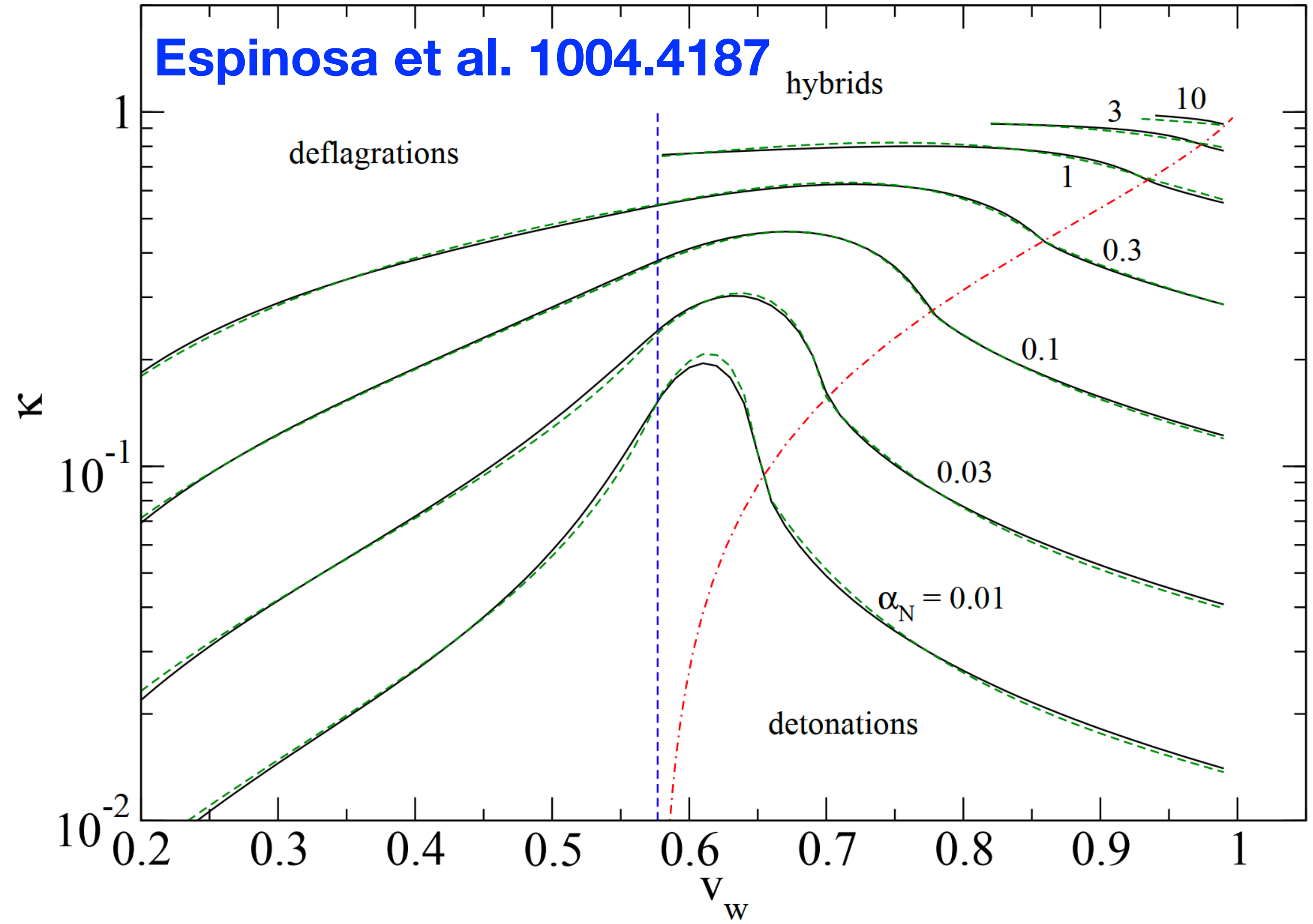
$$p_\pm = c_s^2 a_\pm T^4 - \epsilon_\pm, \quad \rho_\pm = a_\pm T^4 + \epsilon_\pm$$

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3}, \quad \epsilon = V_0(\phi_\pm)$$

$$a = \frac{\pi^2}{30}\left(\sum_{\text{light B}} g_i + \frac{7}{8}\sum_{\text{light F}} g_j\right)$$

$$\kappa_v = \frac{3}{\xi_w^3 \Delta\epsilon}\int w(\xi) v^2 \gamma^2 \xi^2 \mathrm{d}\xi$$

Espinosa et al. 1004.4187



Beyond bag equation of state — constant sound velocity : ν − model

$$p_\pm = c_{s,\pm}^2 a_\pm T^{\nu_\pm} - \epsilon_\pm$$

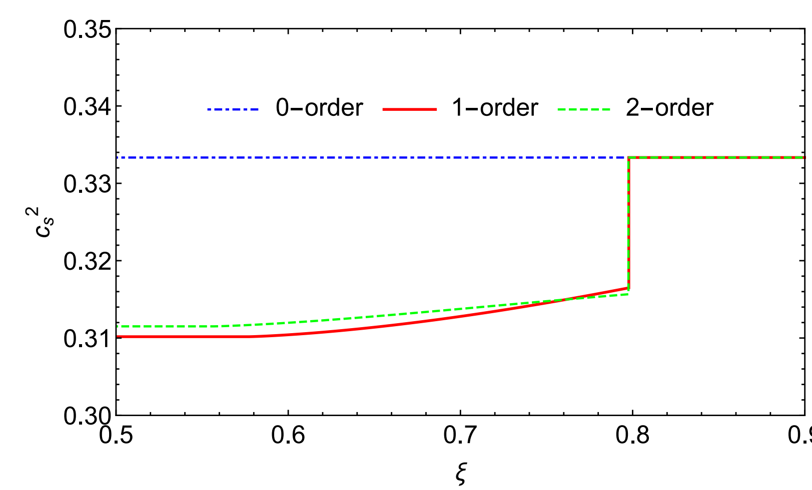
$$\rho_\pm = a_\pm T^{\nu_\pm} + \epsilon_\pm$$

$$c_{s,\pm}^2 = \frac{\partial p_\pm}{\partial \rho_\pm} = \frac{1}{\nu_\pm - 1}$$

Leitao & Megevand 1410.3875

Giese et al. 2004.06995, 2010.09744

Wang et al. 2010.13770, 2112.14650



Beyond bag equation of state — varying sound velocity : SJW & Yuwen 22

Micro-Macro-Relation

Microscopic plasma-dynamics

$$u_\nu \nabla_\mu (w u^\mu u^\nu + p_f \eta^{\mu\nu}) + u_\nu \nabla^\nu \phi \frac{\partial V_T}{\partial \phi} = u_\nu \nabla^\nu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

$$u_\nu u^\nu = -1, \quad u_\nu \nabla_\mu u^\nu = 0$$

$$\nabla_\mu p_f = -\nabla_\mu V_T = -\nabla_\mu T \frac{\partial V_T}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi} = -\nabla_\mu T \frac{\partial V_{\rm eff}}{\partial T} - \nabla_\mu \phi \frac{\partial V_T}{\partial \phi}$$

Entropy flow $$T \nabla_\mu (s u^\mu) = -u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

Enthalpy flow $$-\nabla_\mu (w u^\mu) = u^\mu \nabla_\mu T \frac{\partial V_{\rm eff}}{\partial T} + u^\mu \nabla_\mu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

SJW & Yuwen 22

Thermal force $$p_{\rm thermal} \equiv \int {\rm d}\xi \frac{{\rm d}T}{{\rm d}\xi} \frac{\partial V_{\rm eff}}{\partial T} = \int {\rm d}\xi \frac{{\rm d}T}{{\rm d}\xi} \left(-\frac{4}{3} a(T) T^3 \right)$$

Friction force $$p_{\rm friction} = \int {\rm d}\xi \frac{{\rm d}\phi}{{\rm d}\xi} \frac{\partial p_{\delta f}}{\partial \phi} = \int {\rm d}r \frac{T \nabla_\mu (s u^\mu)}{\gamma(\xi - v)} = \int_0^1 {\rm d}\xi \left(-T \frac{{\rm d}s}{{\rm d}\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu} \frac{{\rm d}v}{{\rm d}\xi} \right)$$

Backreaction force $$p_{\rm backreaction} = \int {\rm d}r \frac{\nabla_\mu (w u^\mu)}{\gamma(\xi - v)} = \int_0^1 {\rm d}\xi \left(-\frac{{\rm d}w}{{\rm d}\xi} + \frac{2wv}{\xi(\xi - v)} + \frac{w\gamma^2}{\mu} \frac{{\rm d}v}{{\rm d}\xi} \right)$$

Weakly/Strongly coupled FOPT

Weakly coupled FOPT **Bubble wall velocity is in principle calculable (but in practice difficult)**

$$\left(k^\mu \partial_\mu + m_i F_i^\mu \frac{\partial}{\partial k^\mu}\right)\Theta(k^0)\delta(k^2+m_i^2)f_i(x,k) = C[f_i] \quad \Rightarrow \quad f^\nu = \nabla^\nu \phi \left(\frac{\partial V_T^{1-\text{loop}}}{\partial\phi} + \frac{\partial \Delta V_T}{\partial \phi} - \frac{\partial p_{\delta f}}{\partial \phi}\right)$$

$$\Downarrow$$

$$f(\gamma_w^{\text{eq}}) = \frac{p_{\text{dr}} - \Delta p_{\text{LO}}}{\Delta p_{NLO}} \quad \Leftarrow \quad p_{\text{br}} = \Delta p_{\text{LO}} + f(\gamma_w)\Delta p_{NLO} \quad \Leftarrow \quad p_{\text{br}} = \left(\int \mathrm{d}\xi \frac{\mathrm{d}T}{\mathrm{d}\xi}\frac{\partial V_{\text{eff}}}{\partial T} + \int \mathrm{d}\xi \frac{\mathrm{d}\phi}{\mathrm{d}\xi}\frac{\partial p_{\delta f}}{\partial \phi}\right)$$

Strongly coupled FOPT

Effective potential cannot be perturbatively defined

Holographic QCD models **Cai et al. 2201.02004** **He et al. 2210.14094**

Bounce action from holography **Ares et al. 2109.13784, 2110.14442**

Collision terms cannot be known exactly/approximately

Holographic numerical simulations **Bea et al. 2104.05708, 2202.10503**

Outline

1. Cosmological first-order phase transitions

    1.1 Stochastic gravitational-wave background

    1.2 Bubble wall velocity from weakly coupled FOPT

2. Strongly-coupled first-order phase transitions

    2.1 Holographic QCD models

    2.2 Holographic numerical simulations

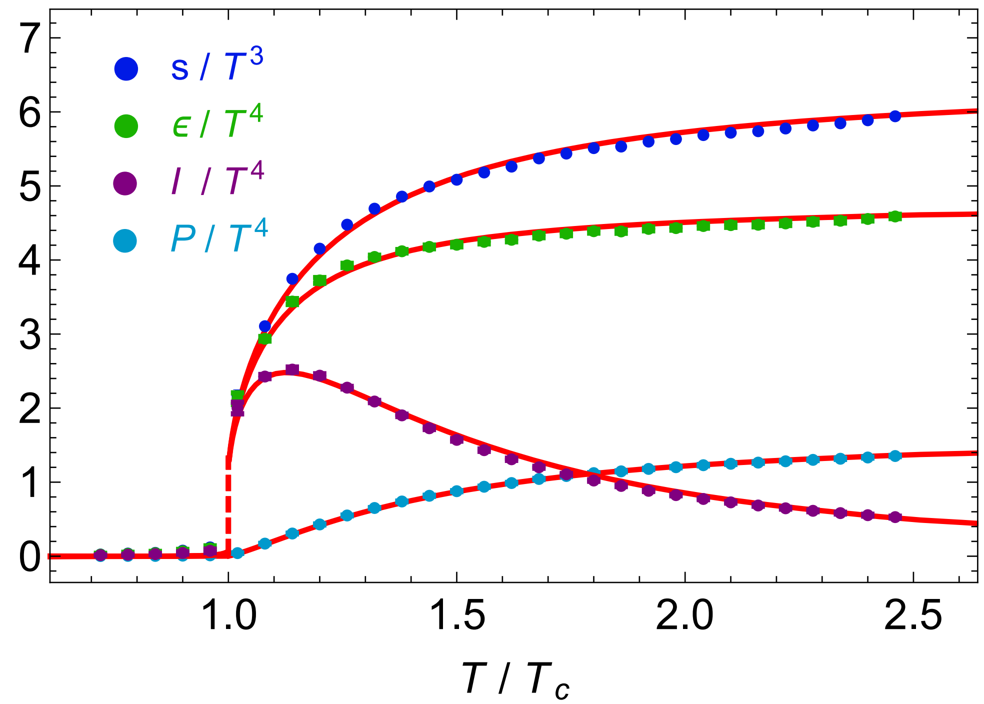
3. Bubble wall velocity for strongly-coupled FOPT

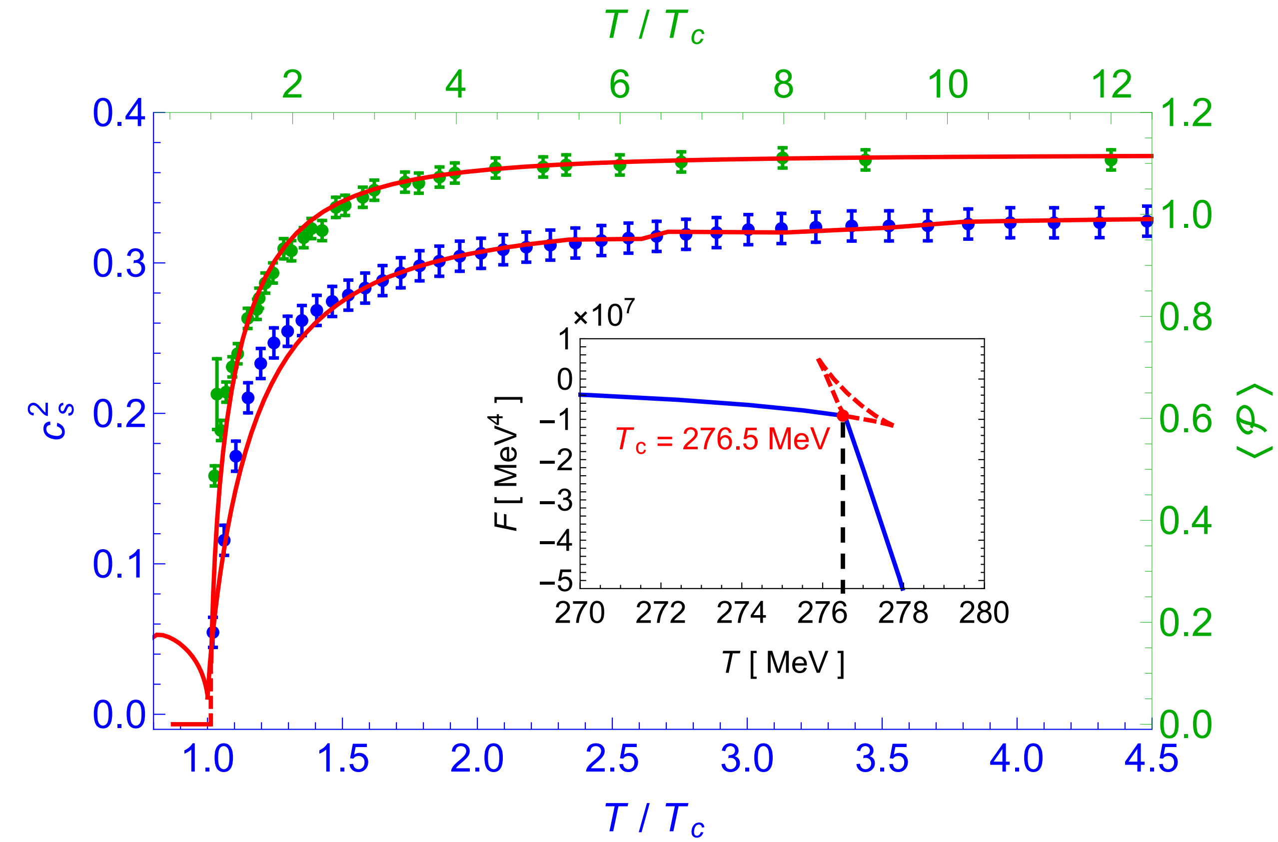
    3.1 Perfect fluid hydrodynamics

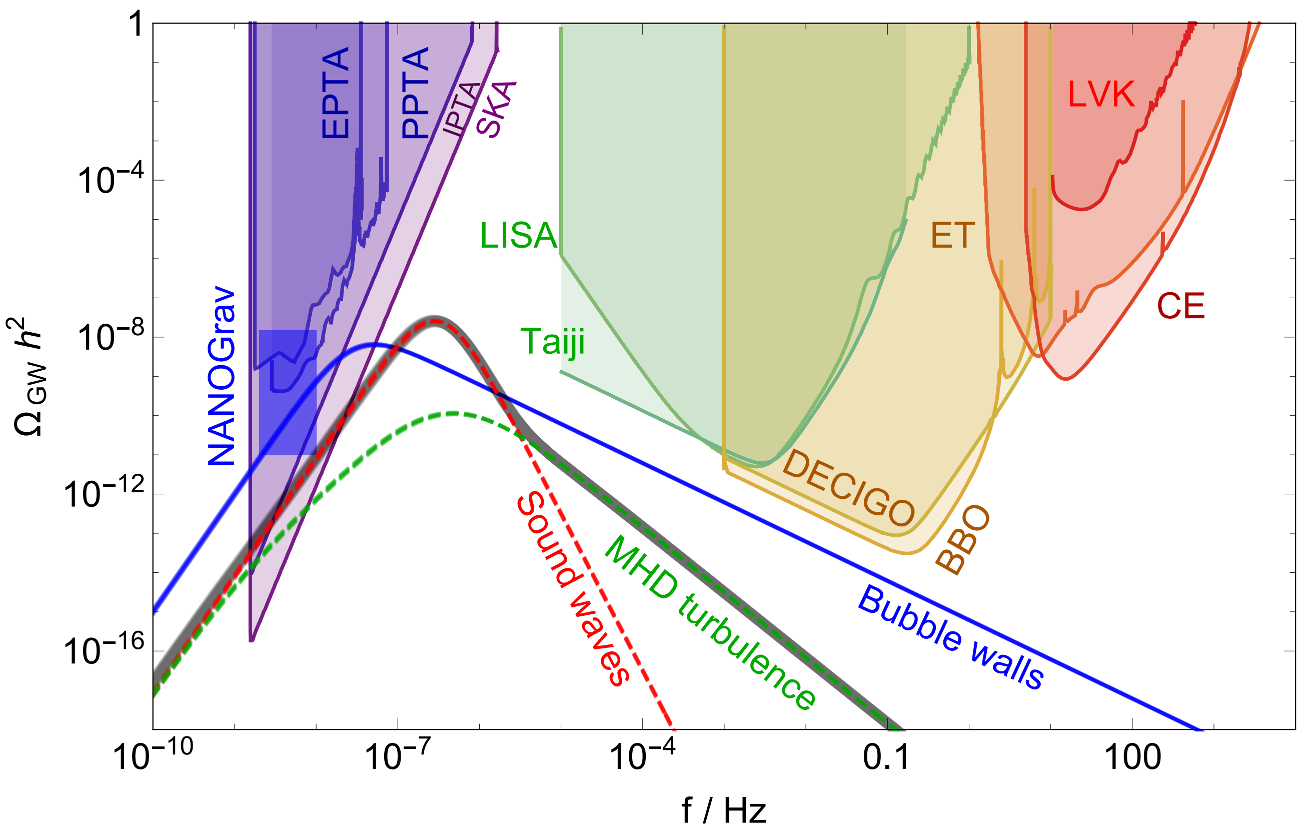
    3.2 Non-relativistic wall velocity

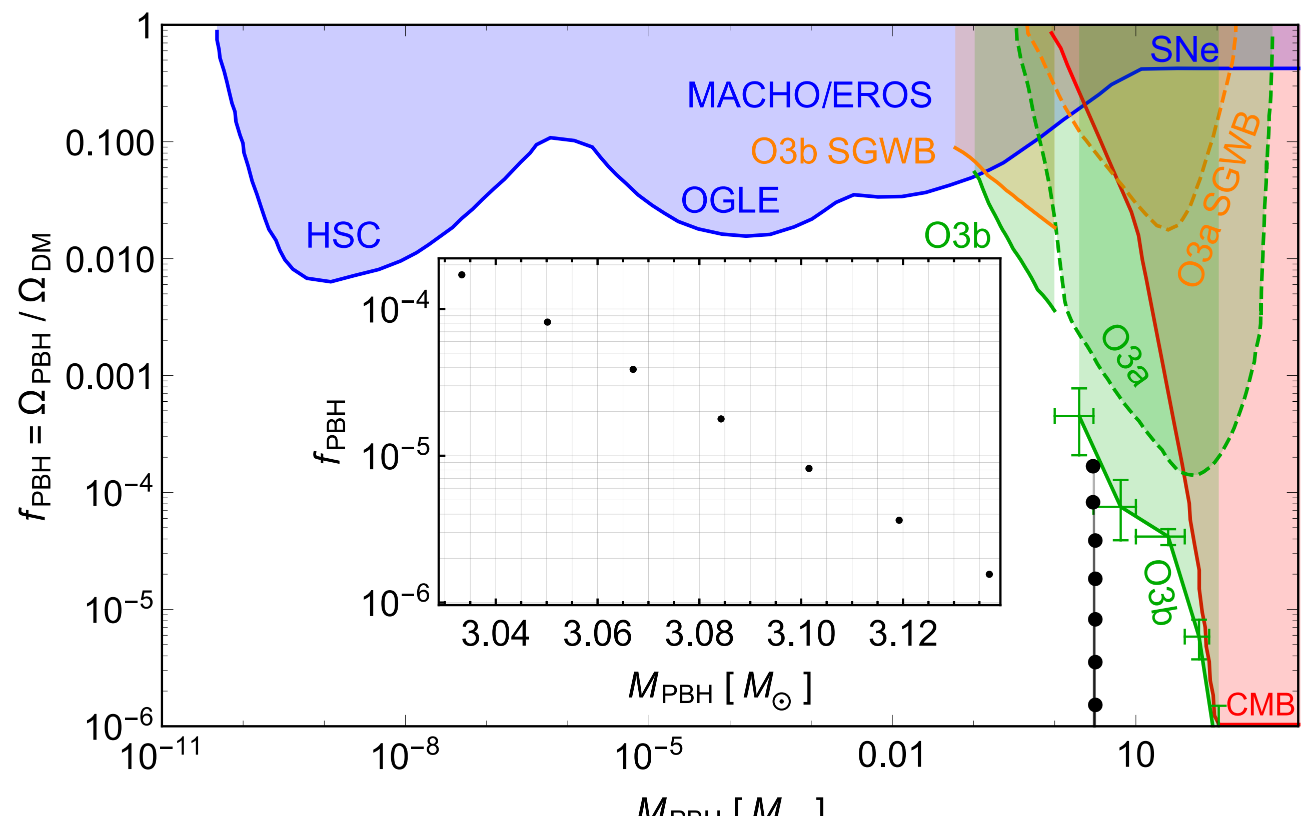
Holographic QCD models

Pure gluon **He, Li, Li, SJW 2210.14094**









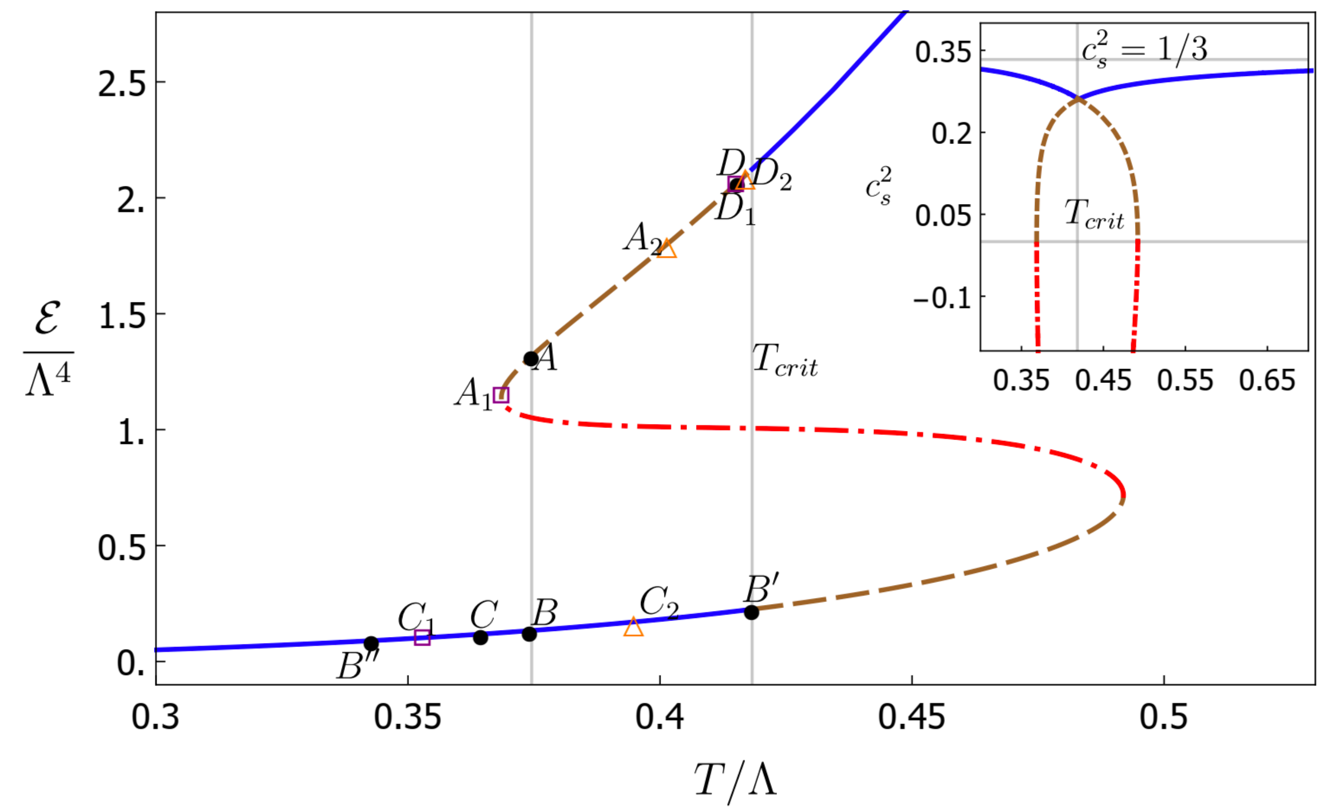
Holographic numerical simulations

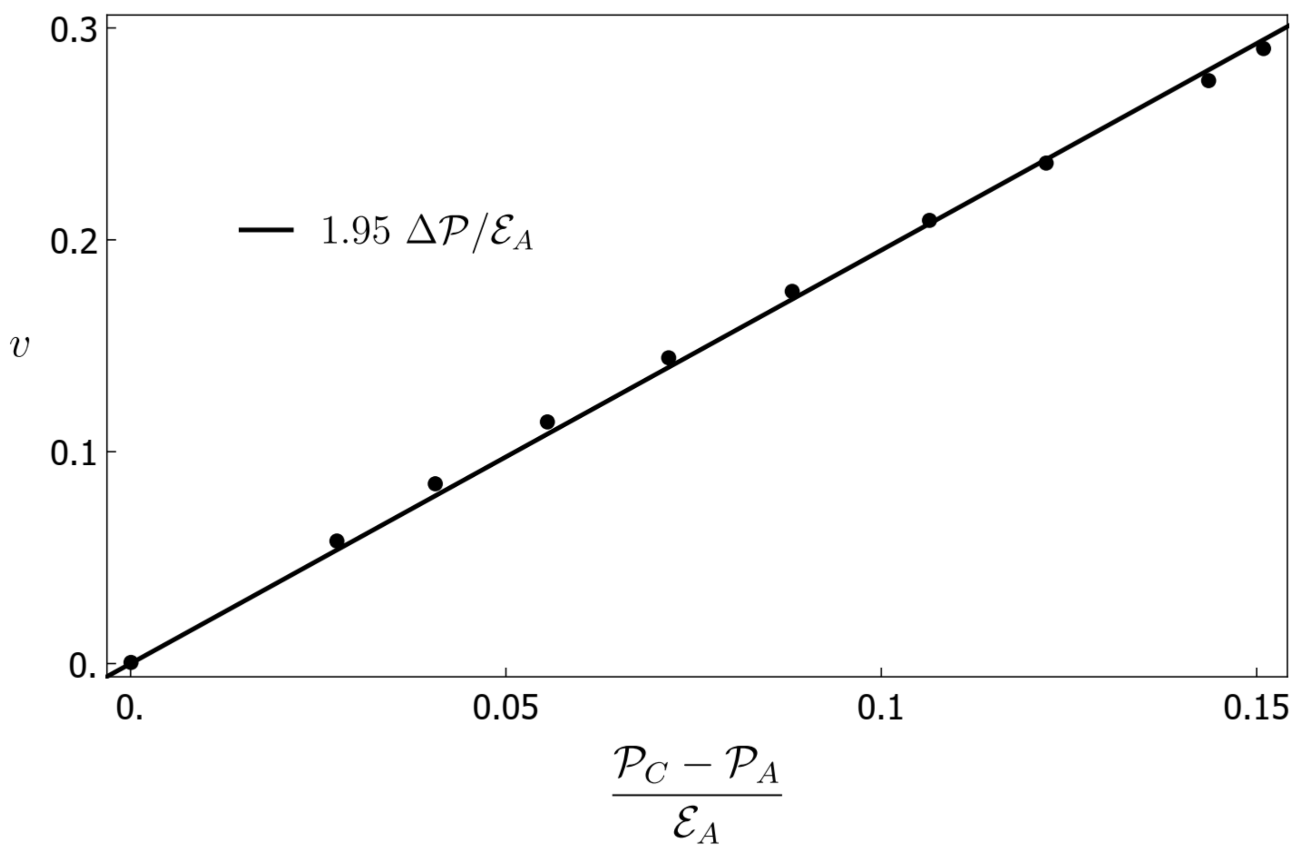
Planar wall **Bea et al. 2104.05708**

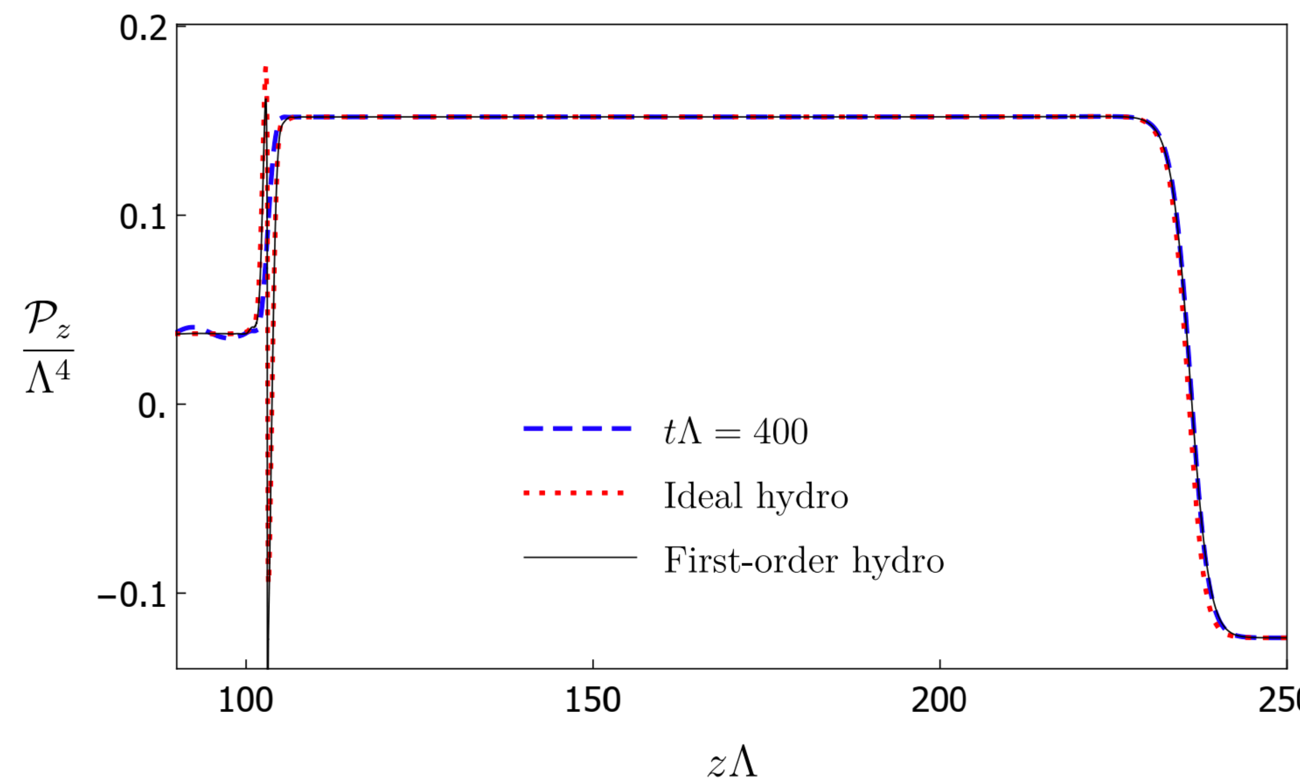
$$S = \frac{2}{\kappa_5^2}\int \mathrm{d}^5x\sqrt{-g}\left[\frac{1}{4}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$V(\phi) = -\frac{4}{3}W(\phi)^2 + \frac{1}{2}W'(\phi)^2$$

$$LW(\phi) = -\frac{3}{2} - \frac{\phi^2}{2} - \frac{\phi^4}{4\phi_M^2} + \frac{\phi^6}{\phi_Q^4}$$







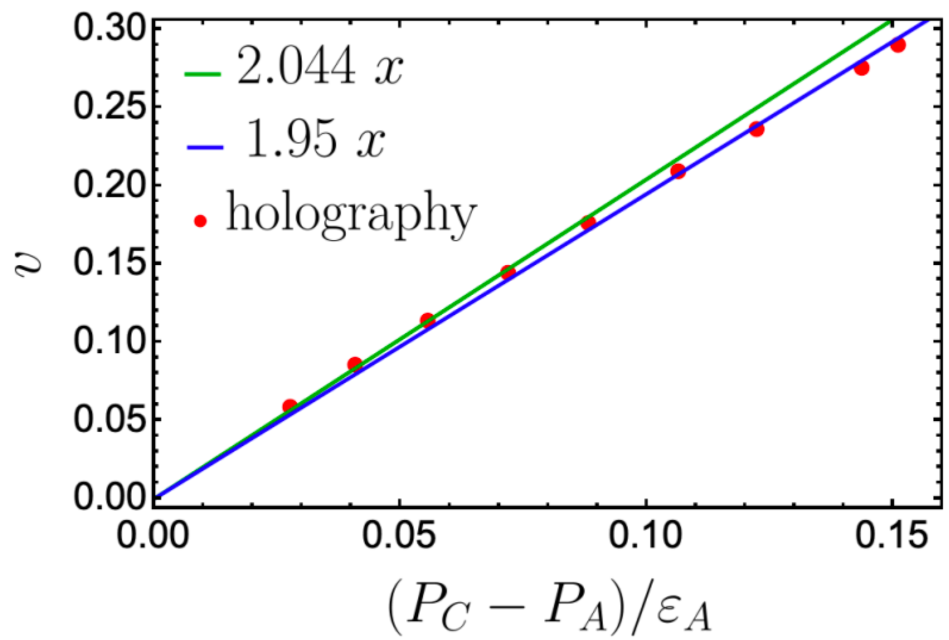
Cylindrical wall **Bea et al. 2202.10503**

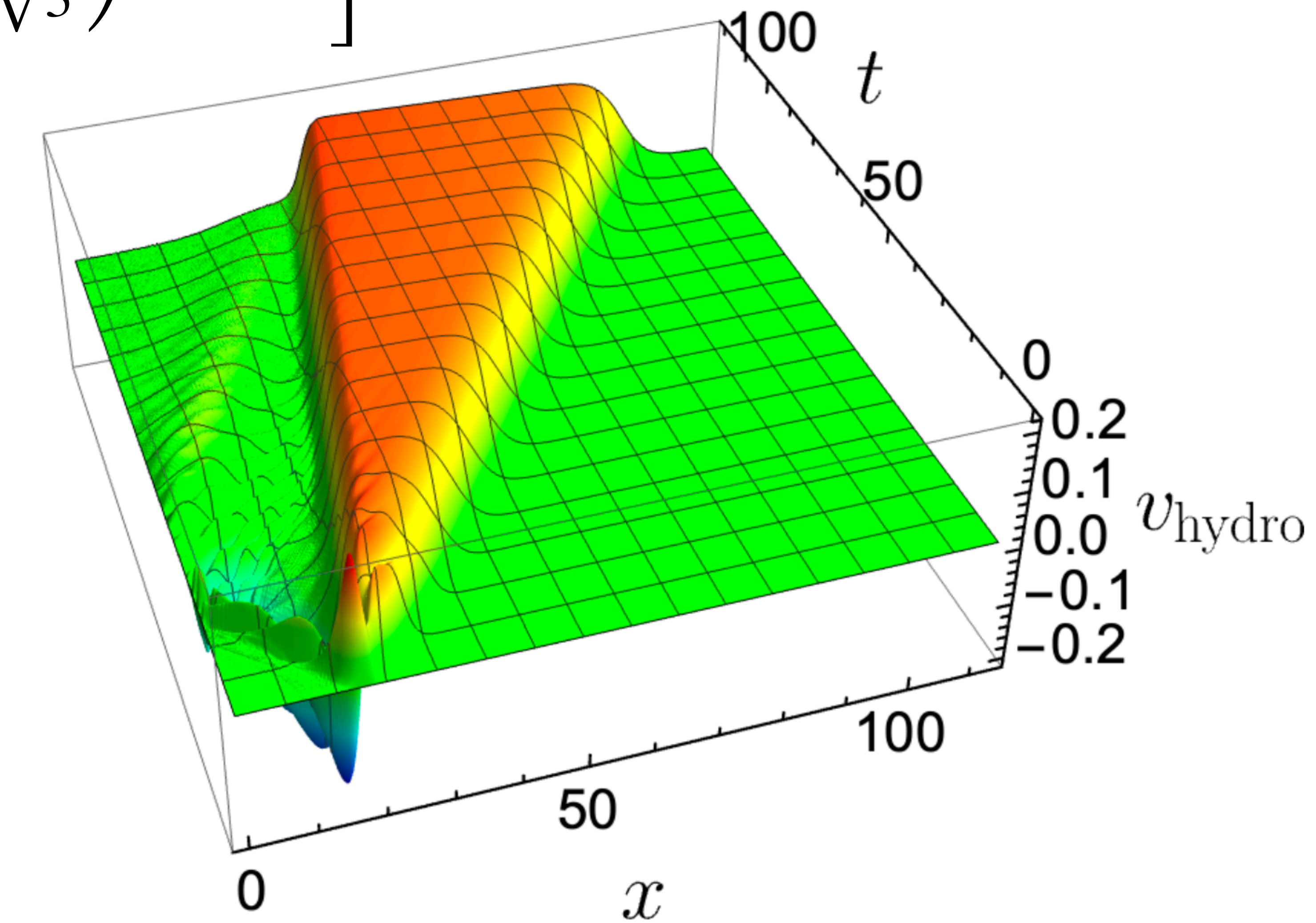
Spherical wall **Not yet, Currently**

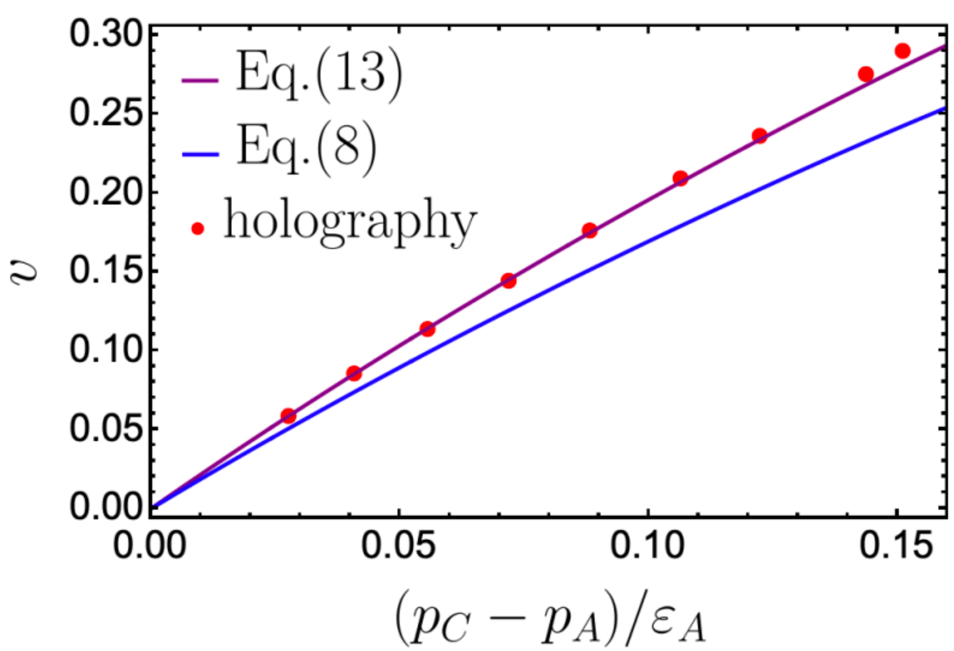
Holographic numerical simulations

Planar wall **Janik et al. 2205.06274**

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + 6\cosh\left(\frac{\Phi}{\sqrt{3}}\right) + 0.2\Phi^4\right]$$







Witten-Sakai-Sugimoto model

Bigazzi et al. 2104.12817 $v = C_d^{-1}\frac{T_c}{T_{\mathrm{boost}}}\frac{p_t(T) - p_f(T_{\mathrm{boost}})}{w_f(T_{\mathrm{boost}})}$

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Strongly coupled FOPT

Lesson 1: **Terminal wall velocity seems to prefer a non-relativistic one** $\gamma_w^{\rm eq} \approx 1$

Heuristic proof by contradiction

$$p_{\rm br} \equiv p_{\rm LO} + f(\gamma_w) p_{NLO}$$

$$\frac{f(\gamma_w) - f(1)}{f(\gamma_w^{\rm eq}) - f(1)} + \frac{3\gamma_w}{2r_w} = 1 + \frac{1}{2r_w^3} \qquad f(\gamma_w^{\rm eq}) = \frac{p_{\rm dr} - p_{\rm LO}}{p_{\rm NLO}}$$

Assumption $\gamma_w^{\rm eq} \gtrsim \mathcal{O}(1)$

$$p_{\rm LO} = \Delta m^2 T^2/24 \quad \Delta m^2 \equiv \sum_i c_i g_i \Delta m_i^2 \quad \Delta m_i^2 \equiv m_i^2(\phi_-) - m_i^2(\phi_+)$$

$$p_{\rm dr} - p_{\rm LO} = \Delta V_0 - \frac{1}{4}\Delta w + \mathcal{O}(m_i^3/T^3)$$ **is largely independent of gauge couplings for a bag EoS**

$$\gamma p_{\rm NLO} = \gamma g^2 \Delta m_V T^3 \quad g^2 \Delta m_V = \sum_i g_i \lambda_i^2 \Delta m_i \quad \Delta m_i \equiv m_i(\phi_-) - m_i(\phi_+)$$

Strong gauge coupling limit $\lambda_i \gg 1$ $\quad \gamma_w^{\rm eq} = \dfrac{\Delta V_0 - \frac{1}{4}\Delta w}{T^3 \sum_i g_i \lambda_i^2 \Delta m_i} \ll 1$

Physical argument: as a bubble wall strongly interacting with the thermal plasma, the backreaction force is so rapidly growing that it only takes a very short moment of time for acceleration before the backreaction force had already balanced the driving force

Strongly coupled FOPT

Lesson 2: Hydrodynamics still serves as a good approximation away from the wall

Unlike the relativistic case where the local thermal equilibrium around the bubble wall cannot be sustained as the particles had not had enough time to fully thermalize before the bubble wall swept over, the non-relativistic case could largely retain the local thermal equilibrium and hence the perfect fluid hydrodynamic approximation near (but not right at) the wall interface.

Assumption **MIT bag EoS for strongly coupled FOPT**

$$c_s = 1/\sqrt{3}$$

$$e_\pm = a_\pm T_\pm^4 + V_0^\pm \qquad V_0^\pm \equiv V_0(\phi_\pm) \qquad \alpha_+ \equiv 4\Delta V_0/(3w_+) \qquad \alpha_N \equiv 4\Delta V_0/(3w_N)$$

$$p_\pm = \frac{1}{3}a_\pm T_\pm^4 - V_0^\pm \qquad a_\pm \equiv \frac{\pi^2}{30}g_{\rm eff}^\pm \qquad \Delta V_0 \equiv V_0^+ - V_0^- \qquad w_N = \frac{4}{3}a_+ T_N^4$$

Fluid EoM + junction conditions

$$D\frac{v}{\xi} = \gamma(v)^2(1-\xi v)\left(\frac{\mu(\xi,v)^2}{c_s^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

Bubble wall $w_-\bar{v}_-\bar{\gamma}_-^2 = w_+\bar{v}_+\bar{\gamma}_+^2$

$$\frac{\mathrm{d}\ln w}{\mathrm{d}\xi} = \gamma(v)^2\mu(\xi,v)\left(\frac{1}{c_s^2} + 1\right)\frac{\mathrm{d}v}{\mathrm{d}\xi}$$

Shock front $w_L\tilde{v}_L\tilde{\gamma}_L^2 = w_R\tilde{v}_R\tilde{\gamma}_R^2$

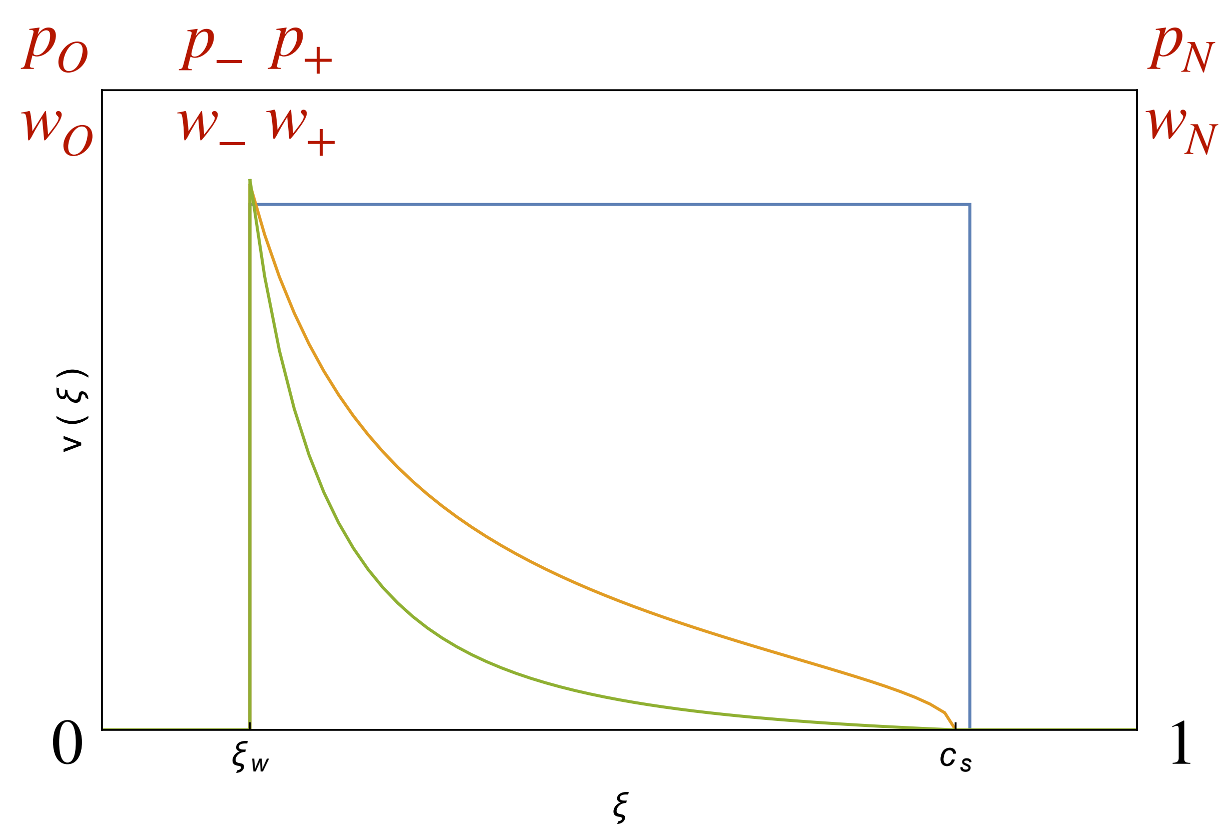
Phase pressure difference

Two kinds of phase pressure difference

Phase pressure difference away from the bubble wall

$$p_{\rm dr} = \Delta V_{\rm eff} = -\Delta p = \Delta\left(-\frac{1}{3}aT^4 + V_0\right) = -\frac{1}{4}\Delta w + \frac{3}{4}\alpha_N w_N, \quad \Delta p \equiv p_N - p_O, \quad \Delta w \equiv w_N - w_O$$

$$\frac{p_- - p_N}{w_N} = \frac{3\xi_w(1+3v_+^2-4v_+\xi_w)}{4v_+(1+2v_+\xi_w-3\xi_w^2)}\alpha_N - \frac{1}{4}$$



Phase pressure difference near the wall

$$\frac{p_+ - p_-}{w_N} = \frac{w_+ - w_-}{w_N} + \frac{v_+}{v_+ - \xi_w}\frac{w_-}{w_N}$$

$$= \frac{3\xi_w(\xi_w - v_+)\alpha_N}{1+2v_+\xi_w-3\xi_w^2}$$

Boring details: $v_+(\xi_w, \alpha_N)$

Junction condition at bubble wall for bag EoS

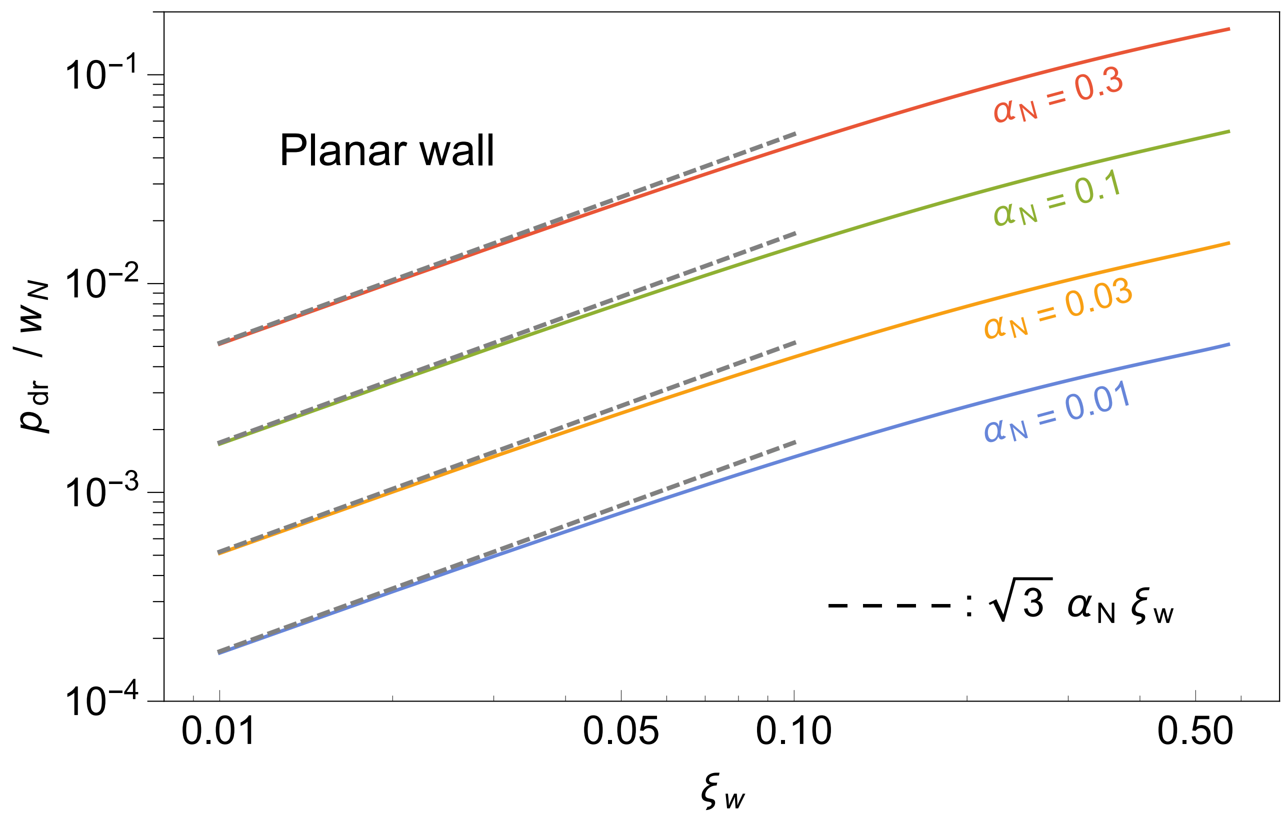
$$w_-\bar{v}_-\bar{\gamma}_-^2 = w_+\bar{v}_+\bar{\gamma}_+^2 \qquad \alpha_+ w_+ = \alpha_N w_N$$

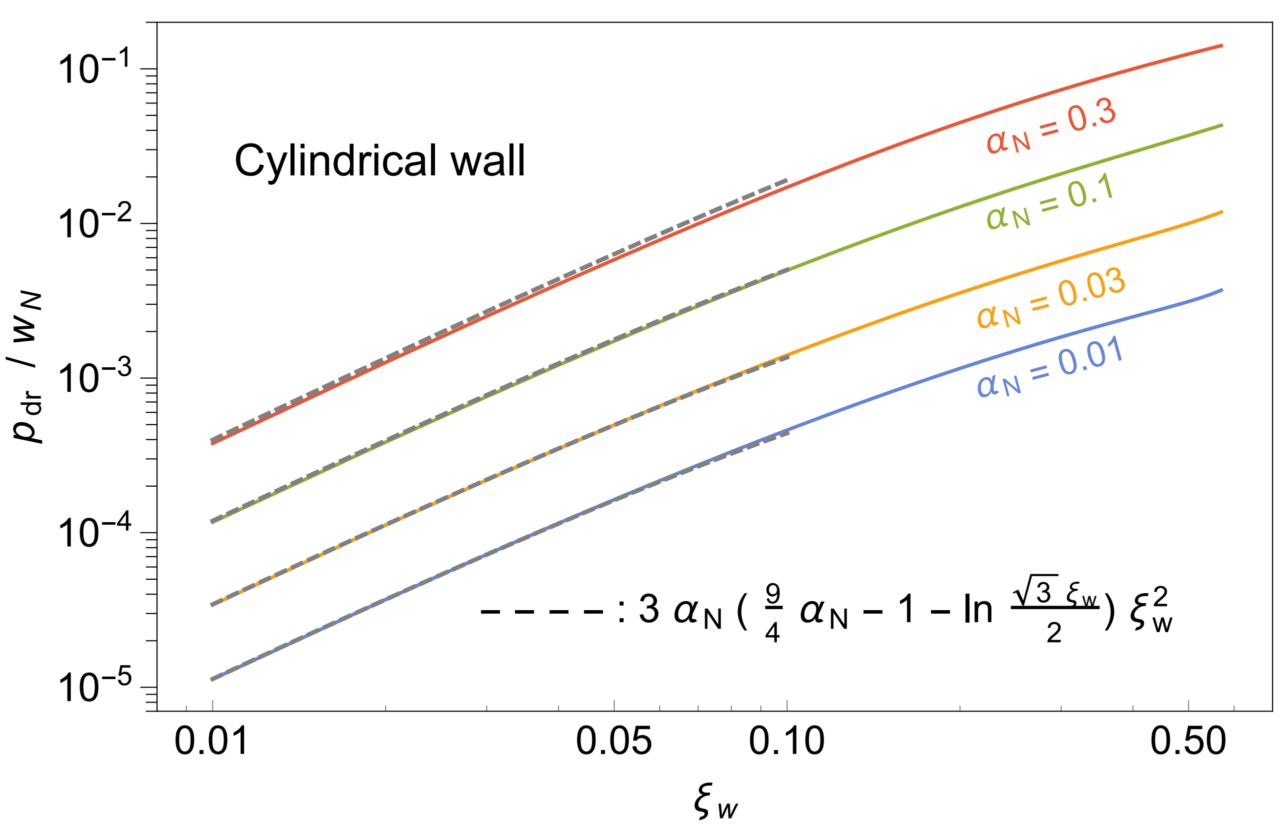
$$\bar{v}_+ = \frac{1}{1+\alpha_+}\left(X_+ \pm \sqrt{X_-^2 + \alpha_+^2 + \frac{2}{3}\alpha_+}\right) \qquad X_\pm \equiv \frac{\bar{v}_-}{2} \pm \frac{1}{6\bar{v}_-} \qquad \Leftrightarrow w_+ = \frac{3(1-v_+^2)\xi_w\alpha_N w_N}{v_+(1+2v_+\xi_w-3\xi_w^2)}$$

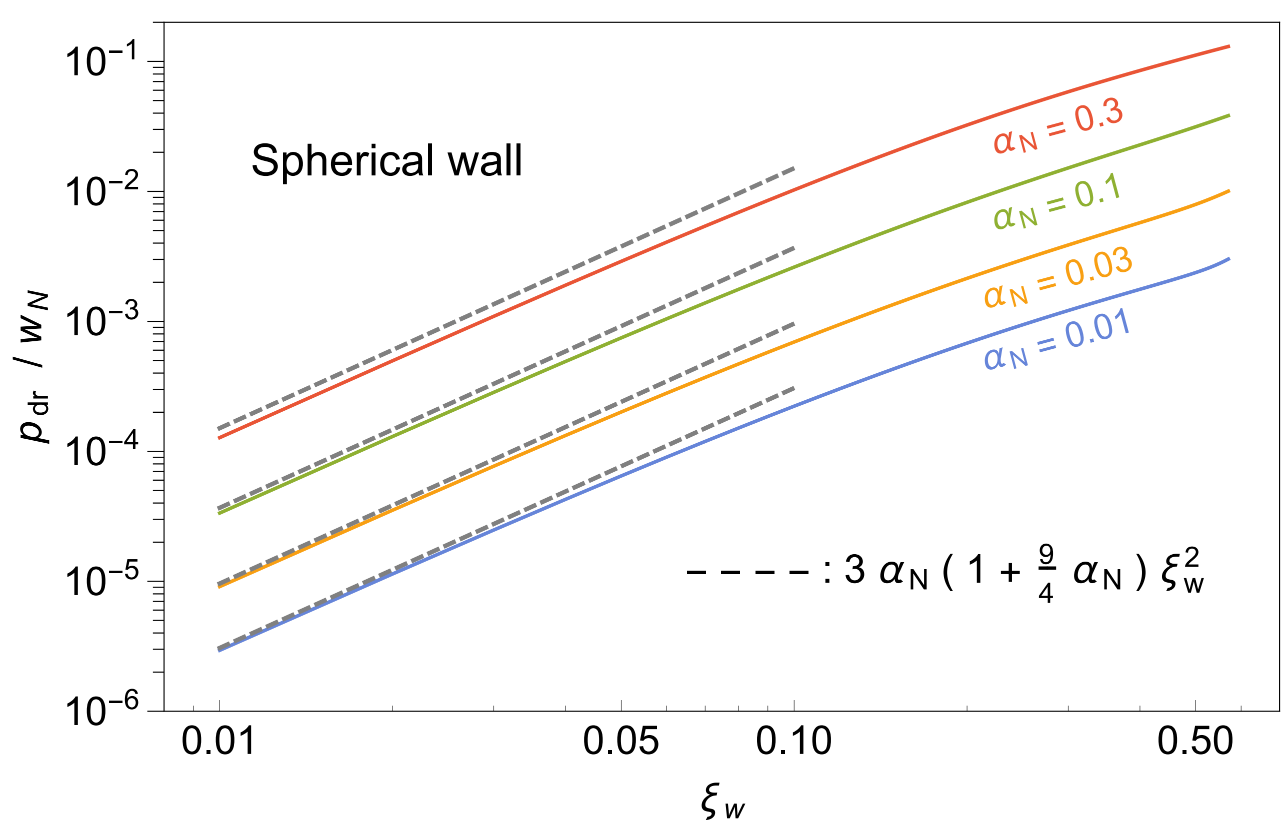
Phase pressure difference

1. We reproduce the novel linear relation from holographic numerical simulation for planar-wall bubble expansion (but cannot be directly compared with as the sound velocity is a varying profile in holography)

2. We predict new relations for the cases of cylindrical and spherical walls, which might be tested in future holographic simulations





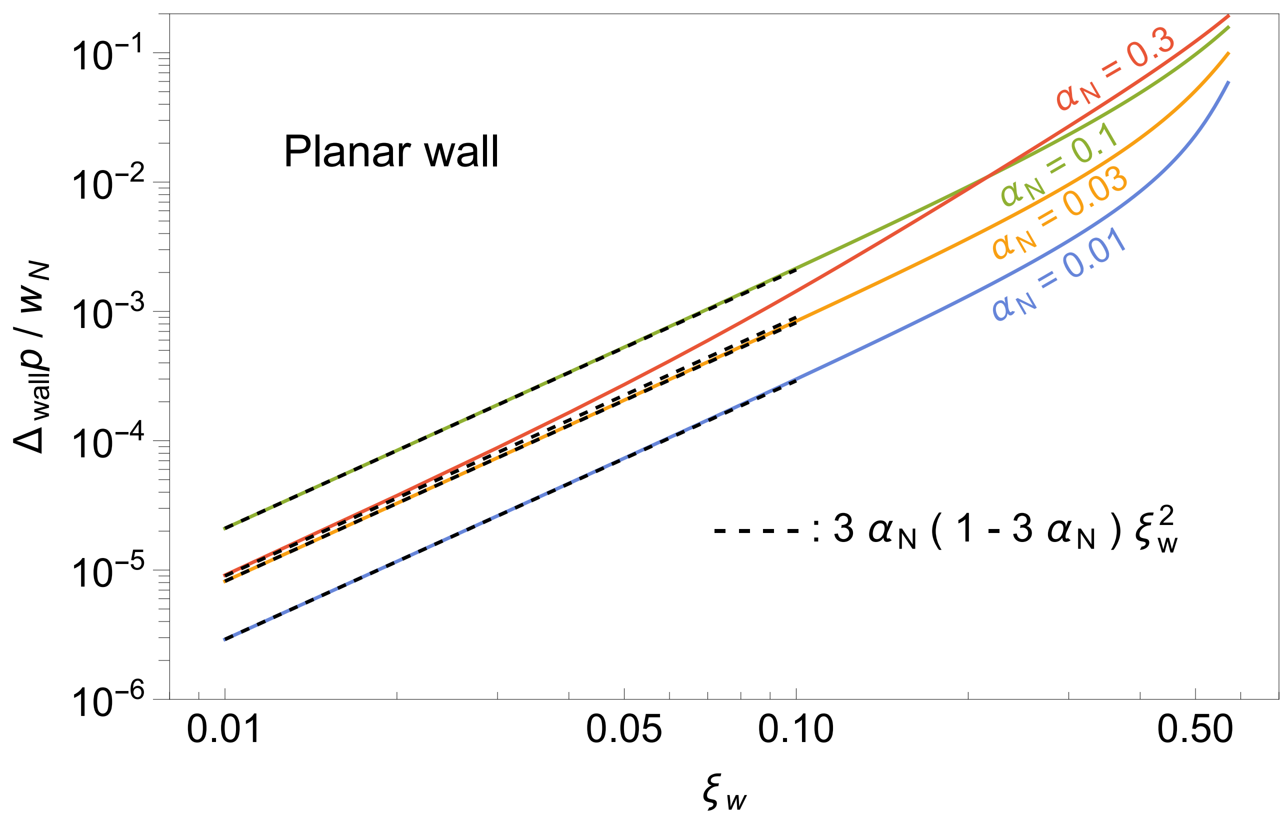


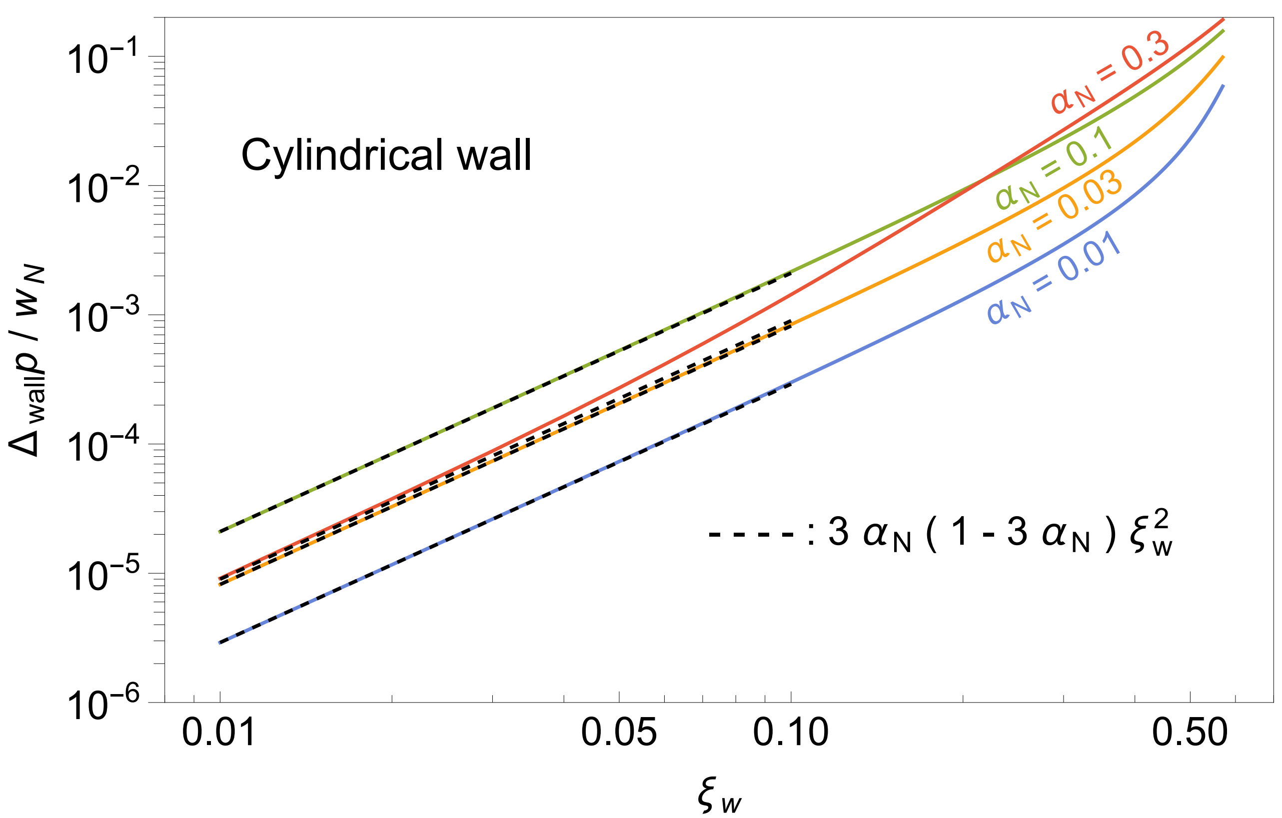
Phase pressure difference

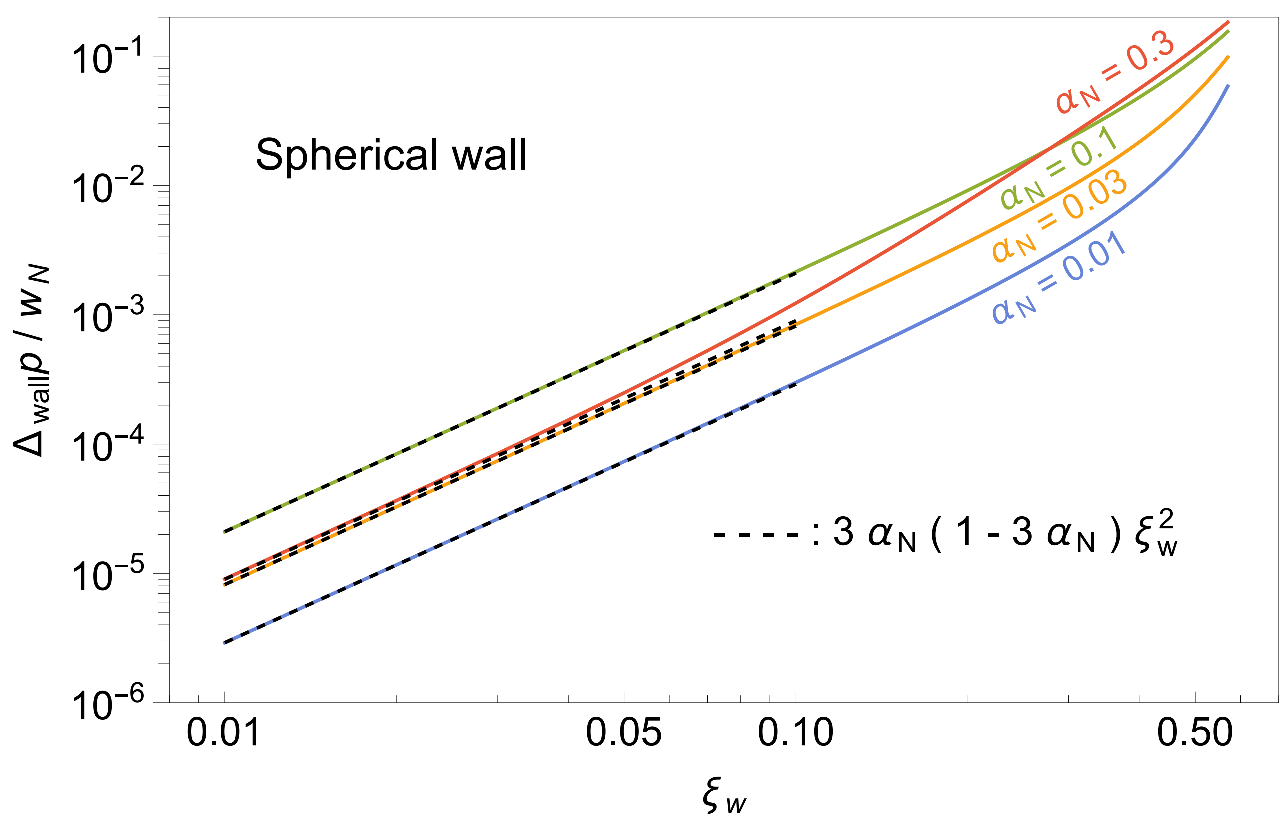
We have found a universal scaling for different wall geometries

$$\frac{p_+ - p_-}{w_N} = 3\alpha_N(1 - 3\alpha_N)\xi_w^2 + \mathcal{O}(\xi_w^3)$$

This can be understood as the phase pressure difference near the wall does not care about the different shapes of the sound shell from different wall geometries







Conclusions and discussions

2302.10042 Li, SJW, Yuwen "Bubble expansion at strong coupling"

We have solved the problem of wall-velocity computation for strongly coupled FOPT

$$\xi_w = \sqrt{\frac{\delta}{4+9\alpha_N}}, \quad \alpha_N = \frac{\Delta V_0}{\rho_R}, \quad \delta = \frac{\Delta V_{\rm eff}}{\Delta V_0} = \left(1 - \frac{1}{4}\frac{\partial \ln \Delta V_{\rm eff}}{\partial \ln T}\right)^{-1}$$

Possible generalizations

Beyond bag EoS: piecewise function of sound velocities inside and outside of the bubble wall

Relaxing thin wall: the bubble wall at the non-relativistic limit is not yet becoming a thin wall

Numerical simulation: holographic numerical simulations for spherical-wall bubble expansion

Thank you