Holographic Kibble-Zurek Mechanism and Beyond

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Based on: 1912.08332 , 2111.05568 , 2111.15230 , 2207.10995

Holographic QCD seminar
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- Brief review of Kibble-Zurek Mechanism (KZM)
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Brief review of KZM

History of KZM

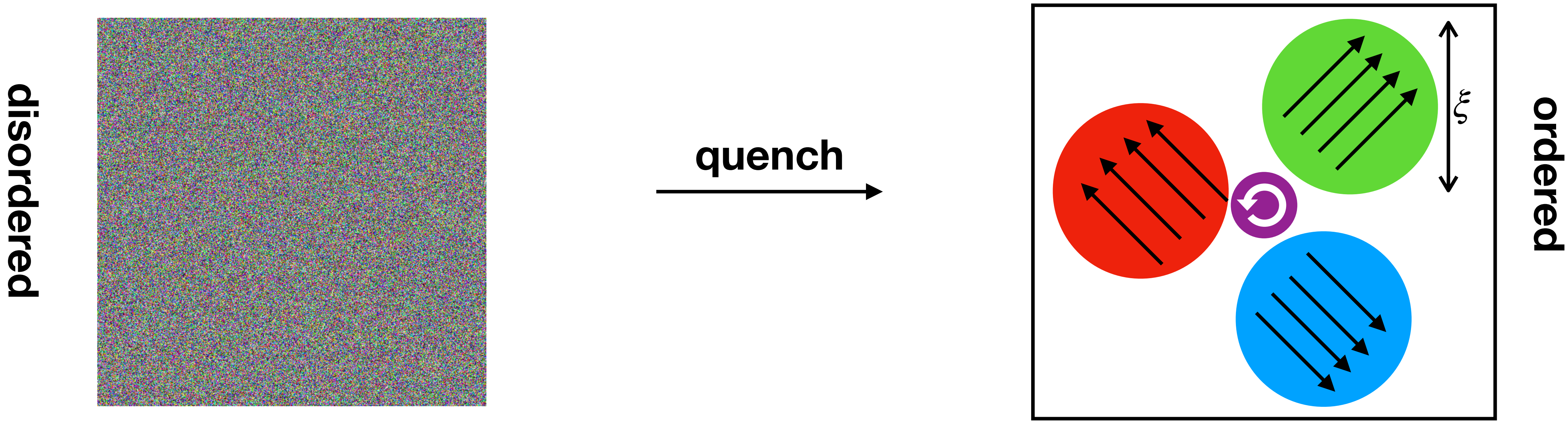
- KZM was first proposed in cosmology by Kibble in 1976.
- Cooling of the early universe will finally result in *topological defects*, such as cosmic string, monopoles, vortices, domain walls …
- However, not found to date.

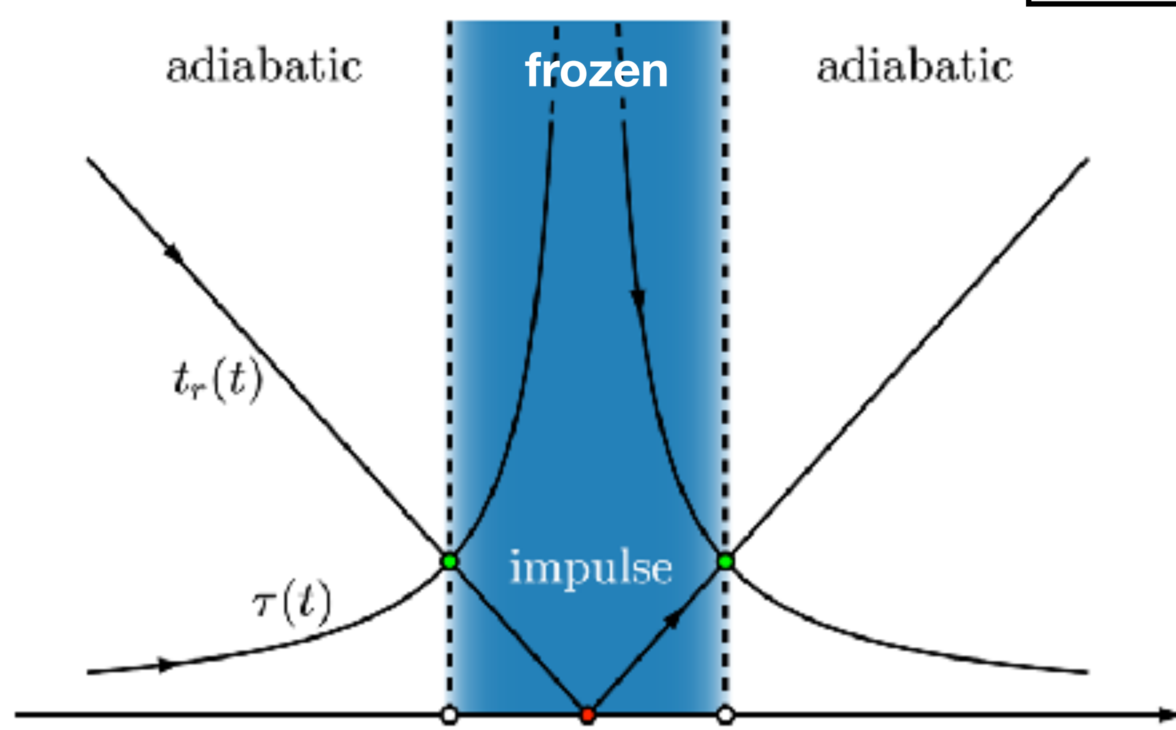
Tom W.B. Kibble (1932-2016)

Wojciech H. Zurek

- Zurek extended this idea into superfluid in 1985.
- Phase transition from normal fluid helium to superfluid helium will induce vortices or vortex lines.
- Confirmed by various experiments.

What is KZM?


disordered
quench
ordered
ξ


adiabatic
frozen
adiabatic
$t_r(t)$
$\tau(t)$
impulse

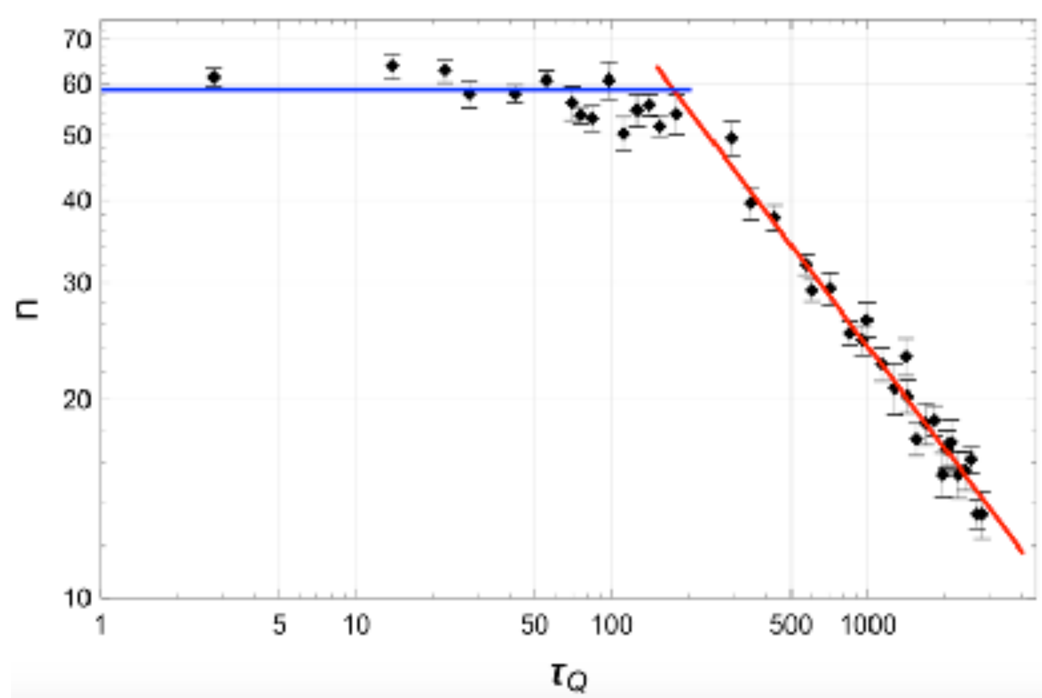
- KZM requires continuous phase transition

$$\xi \propto |\epsilon|^{-\nu}, \qquad \tau \propto |\epsilon|^{-z\nu}. \qquad \text{where } \epsilon = 1 - T/T_c$$

coherence length — relaxation time

- KZM predicts a power law relation between the number density of topological defects and the quench rate

$$n \propto \left(\tau_Q\right)^{\frac{-d\nu}{1+z\nu}}$$



Confirmed by various experiments

- Liquid crystals: Chuang, et.al., Science 251 (1991) 1336; Bowick, et.al.,Science 263 (1994) 943; Digal, et.al., PRL 83 (1999) 5030

- He3 superfluids: Baeuerle,et.al.,Nature 382 (1996) 332; Ruutu et al. ,Nature 382 (1996) 334

- Thin-film superconductors: Maniv,et.al., PRL 91 (2003) 197001; PRL 104, 247002 (2010).

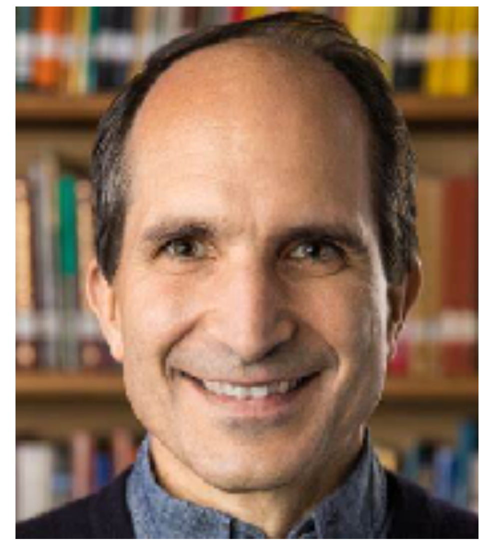
- Quantum optics: Xu, et.al., PRL,112, 035701(2014)

…

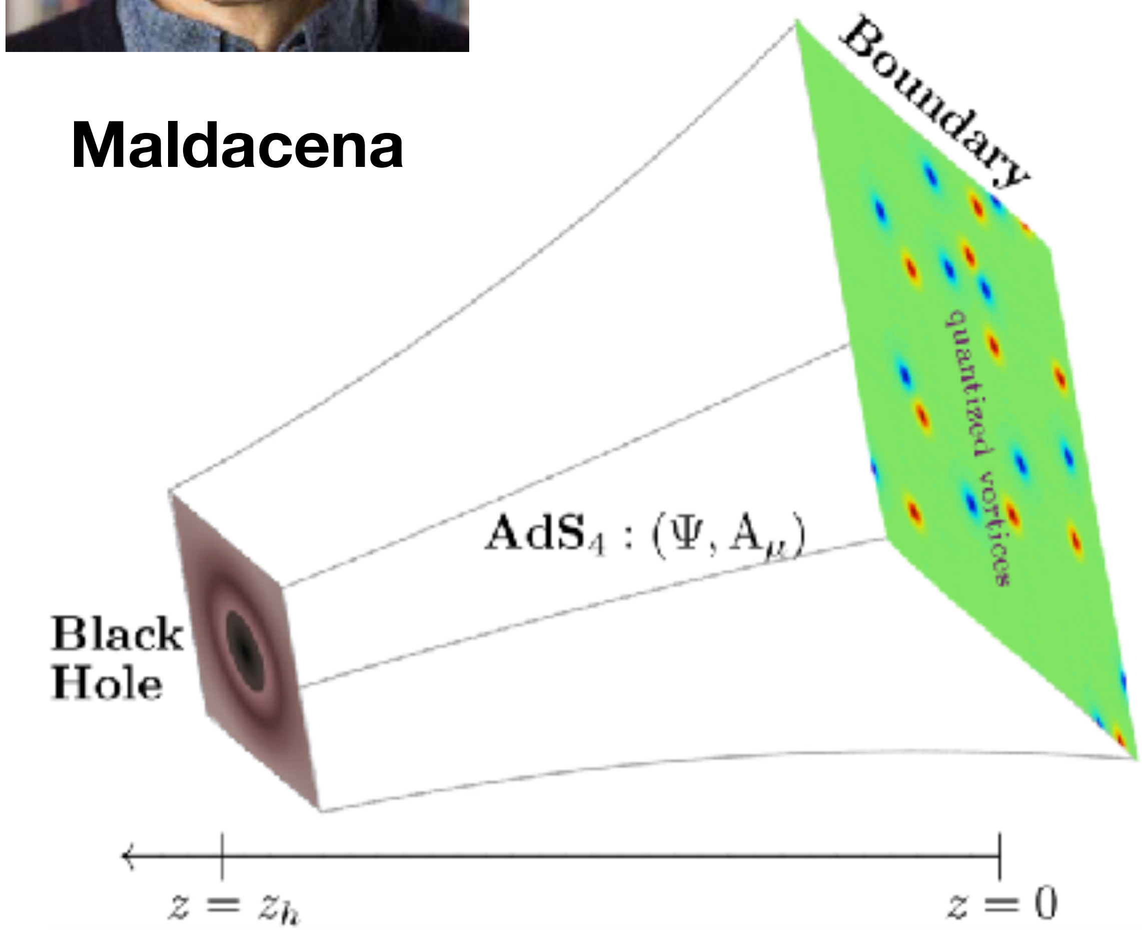
Holographic KZM with continuous symmetry breaking

Gauge-Gravity Duality

- Maldacena proposed a correspondence between supergravity and its boundary SYM field theory in 1998.
- The boundary field theory is strongly coupled; while the gravity side is weakly coupled.



Maldacena



Holographic Setup

- Lagrangian:

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |\partial\Psi - iA\Psi|^2 - m^2|\Psi|^2 .$$

- Background in the probe limit (Eddington):

$$ds^2 = \frac{L^2}{z^2}(-f(z)dt^2 - 2dtdz + dx^2 + dy^2), \quad \text{where} \quad f(z) = 1 - z^3 .$$

With ansatz

$$\Psi = z\Phi(t,z,x,y), A_t = A_t(t,z,x,y), A_x = A_x(t,z,x,y), A_y = A_y(t,z,x,y) \text{ and } A_z = 0$$

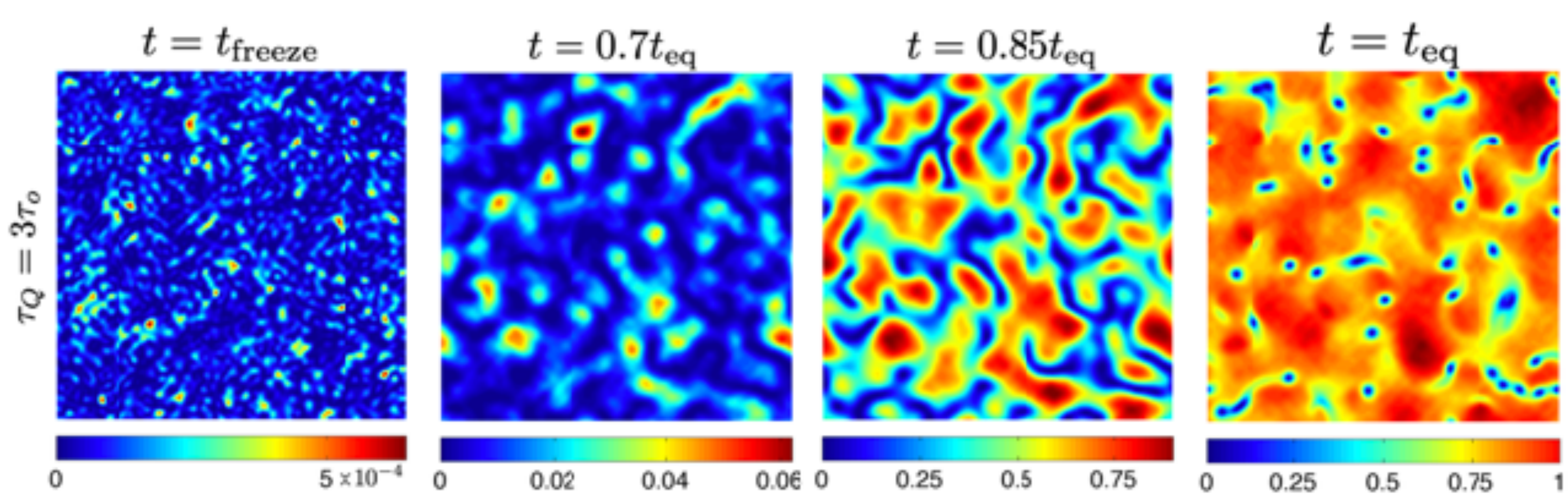
Holographic KZM in superfluid

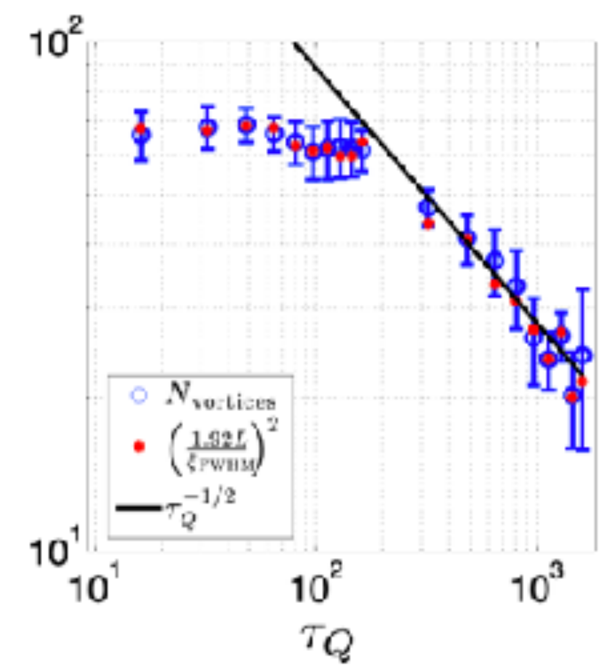
- The expansion of gauge fields near z=0

$$A_\mu(t,x^i) \approx a_\mu(t,x^i) + b_\mu(t,x^i)\ z + \cdots$$

Set $a_i = 0 \quad (i = x, y)$ results in holographic superfluid in the boundary field theory

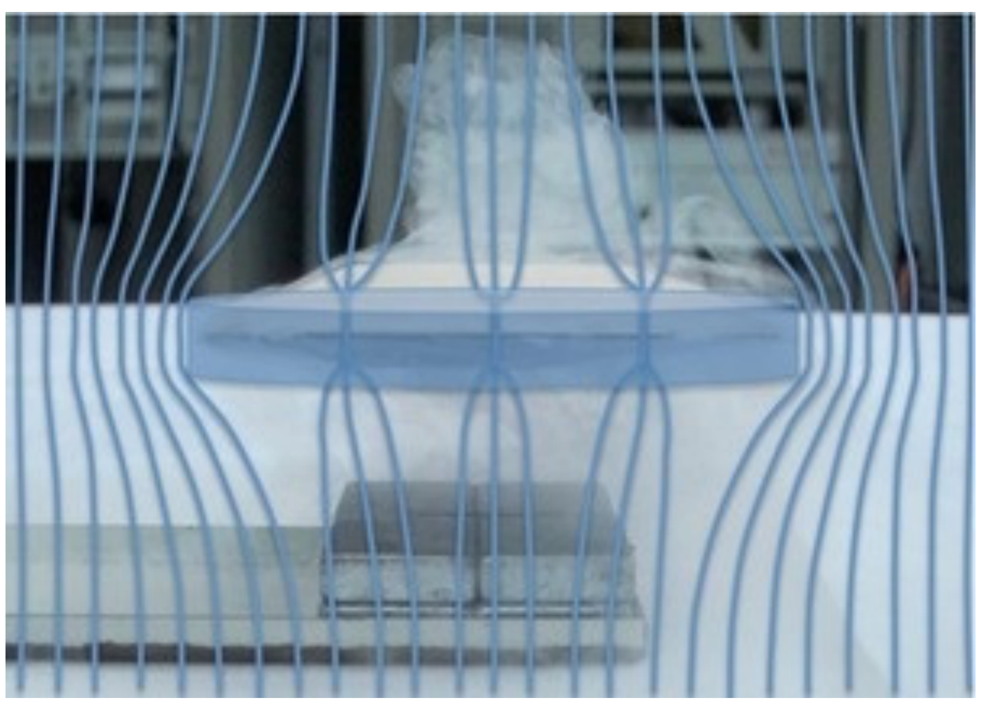
Chesler, Garcia-Garcia and Liu, 1407.1862





Holographic KZM in superconductor

- The superconductor vortices are stabler due to the flux pinning effect



Pinning of a type II superconductor

Shanghai Maglev train

- In order to get magnetic fluxons, we need dynamical gauge fields on the boundary, impose Neumann boundary conditions. Witten, hep-th/0307041 , Domenech, Montull, Pomarol, Salvio and Silva, 1005.1776

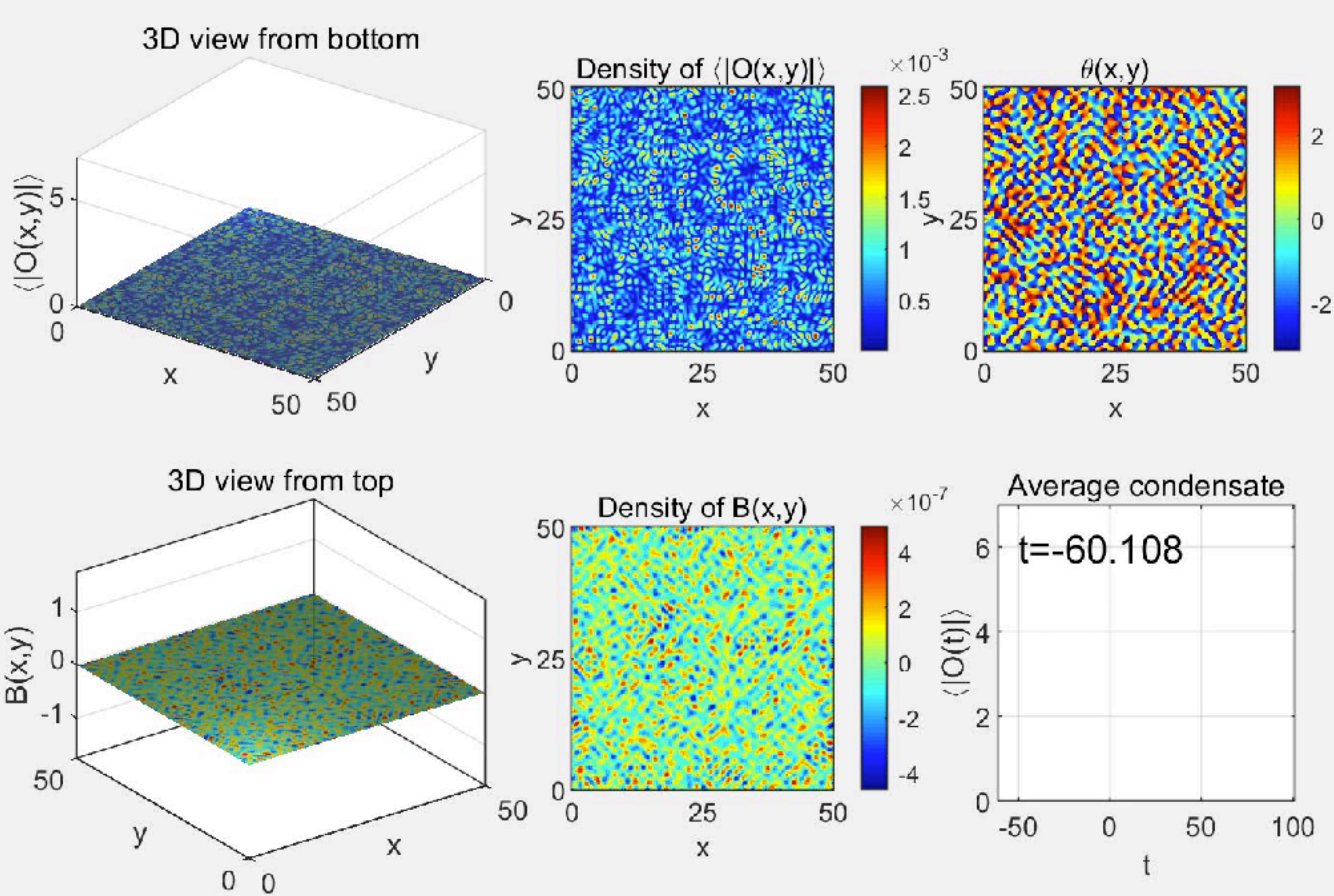
- Expansions of gauge fields near boundary

$$A_\mu(t, x^i) \approx a_\mu(t, x^i) + b_\mu(t, x^i)\ z + \cdots$$

- z-component of Maxwell equation implies the conservation equation on the boundary

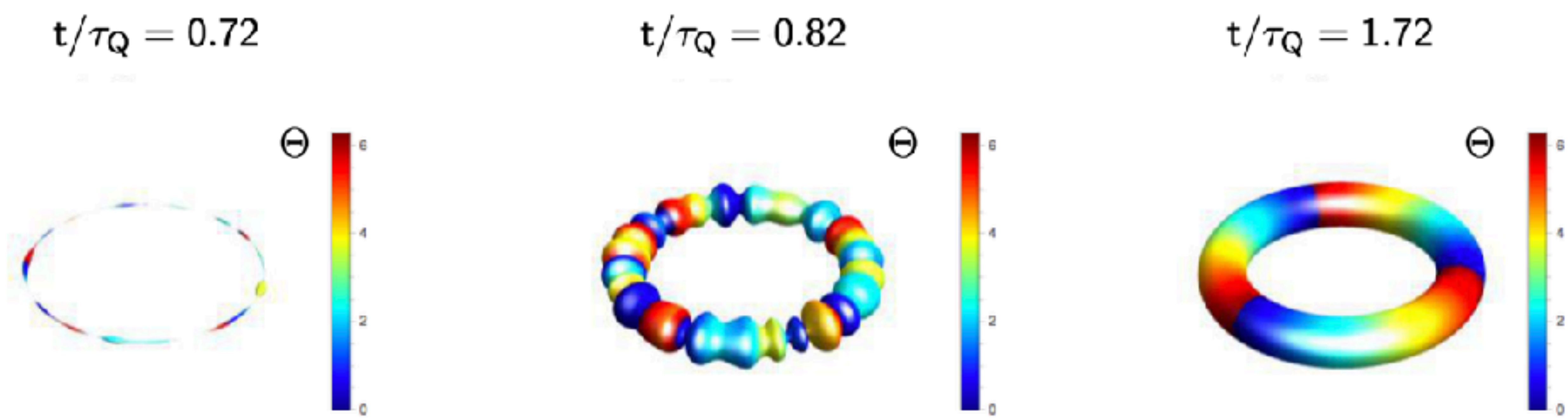
$$\partial_t b_t + \partial_i J^i = 0$$

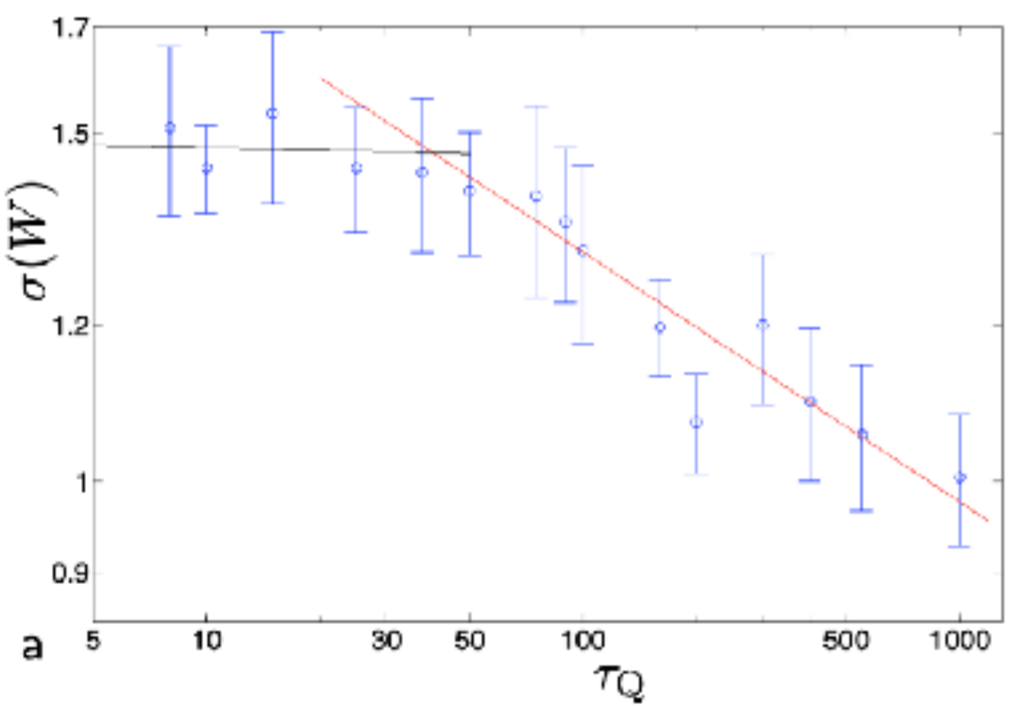
$$J^i = -b_i + (\partial_t a_i - \partial_i a_t) = -b_i + E_i$$



τ_Q=200, T_i=1.30T_c, T_f=0.64T_c

Holographic KZM in 1+1 dim





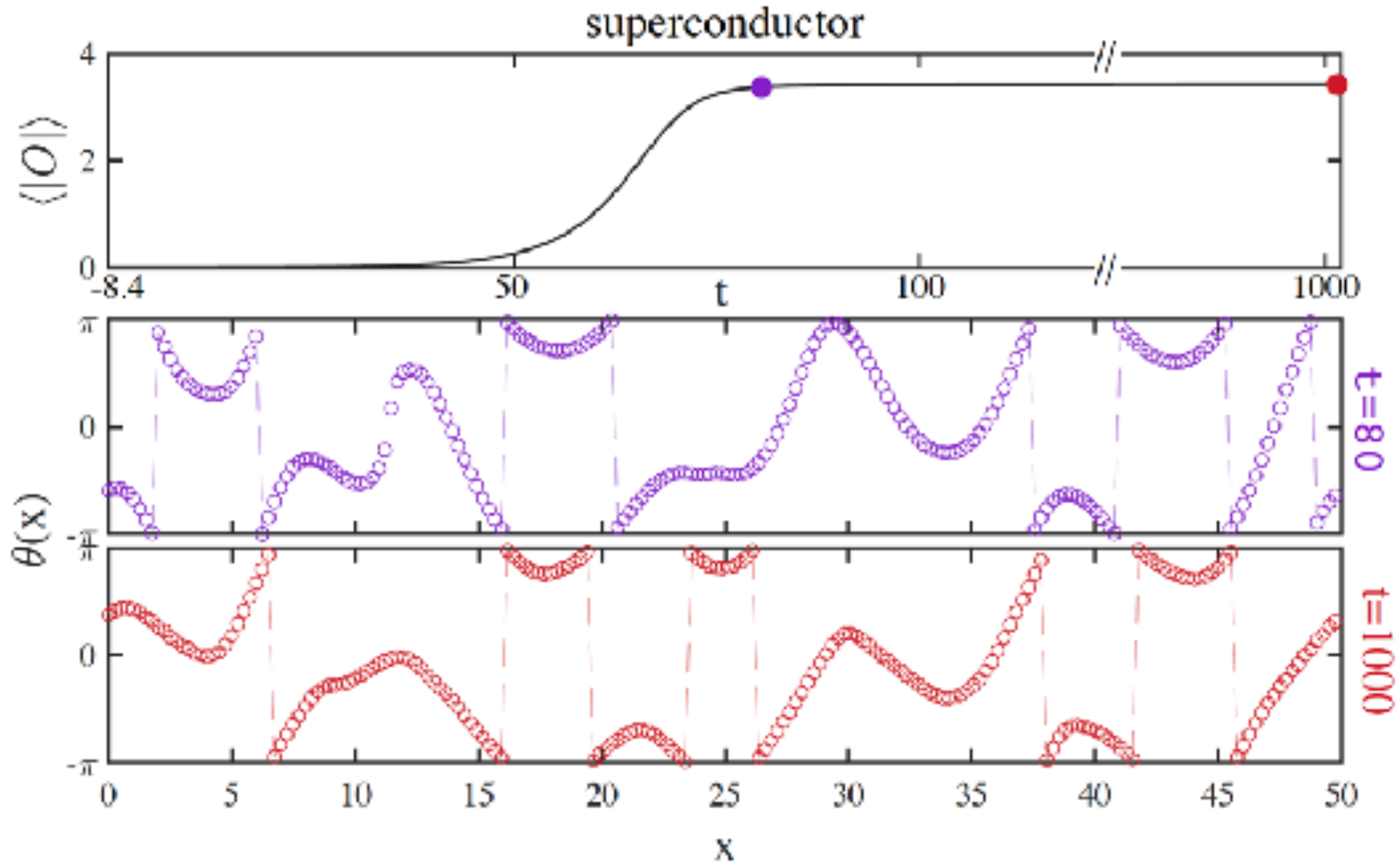
$$\Psi = |\Psi| e^{i\theta}$$

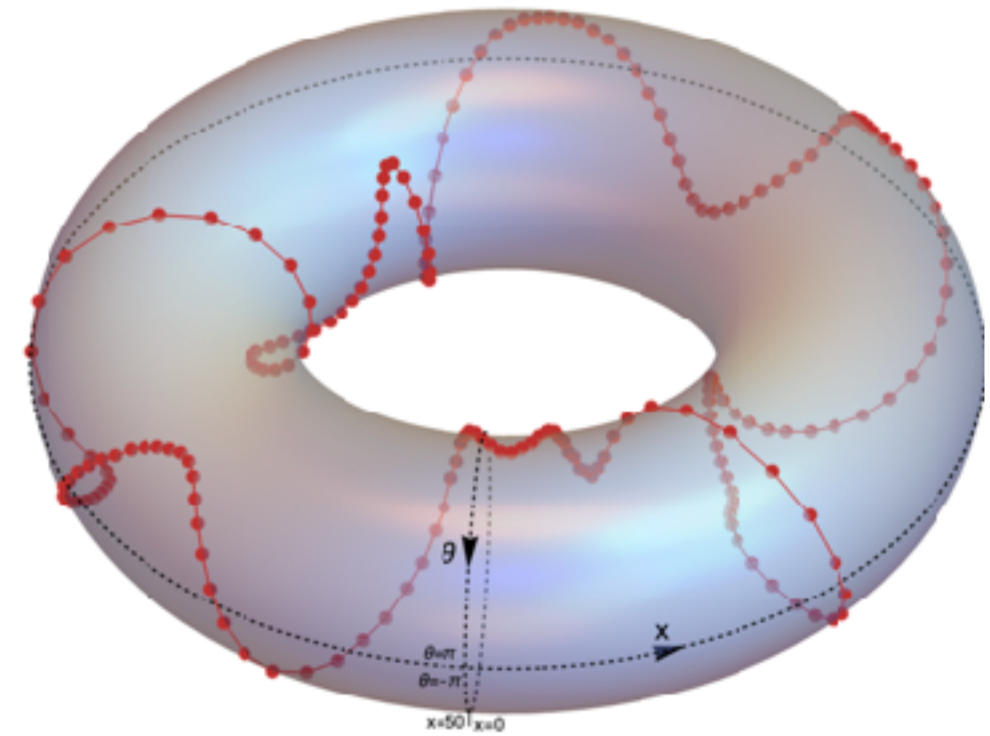
$$W = \frac{1}{2\pi} \oint_C d\theta$$

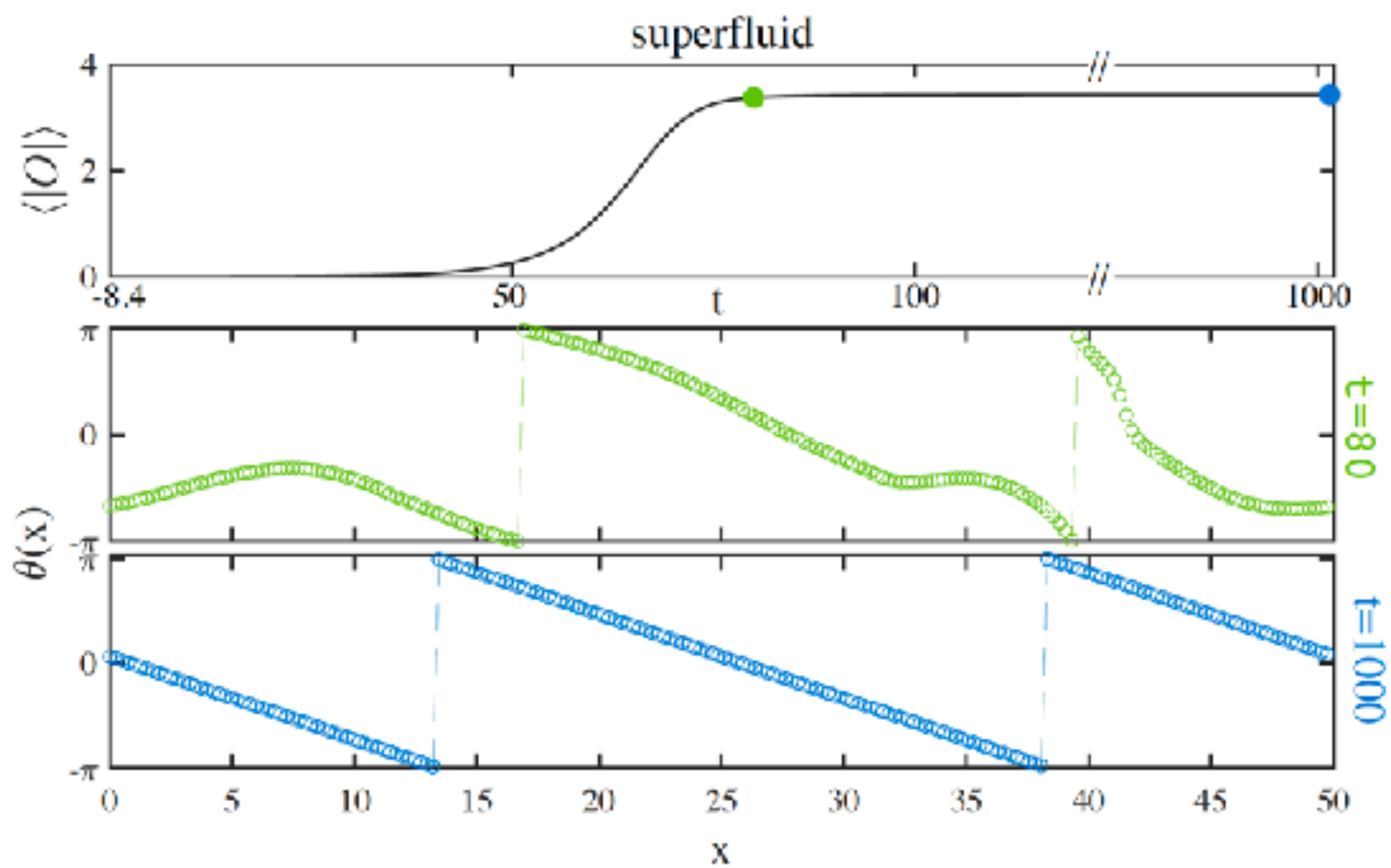
Sonner, del Campo and Zurek, 1406.2329

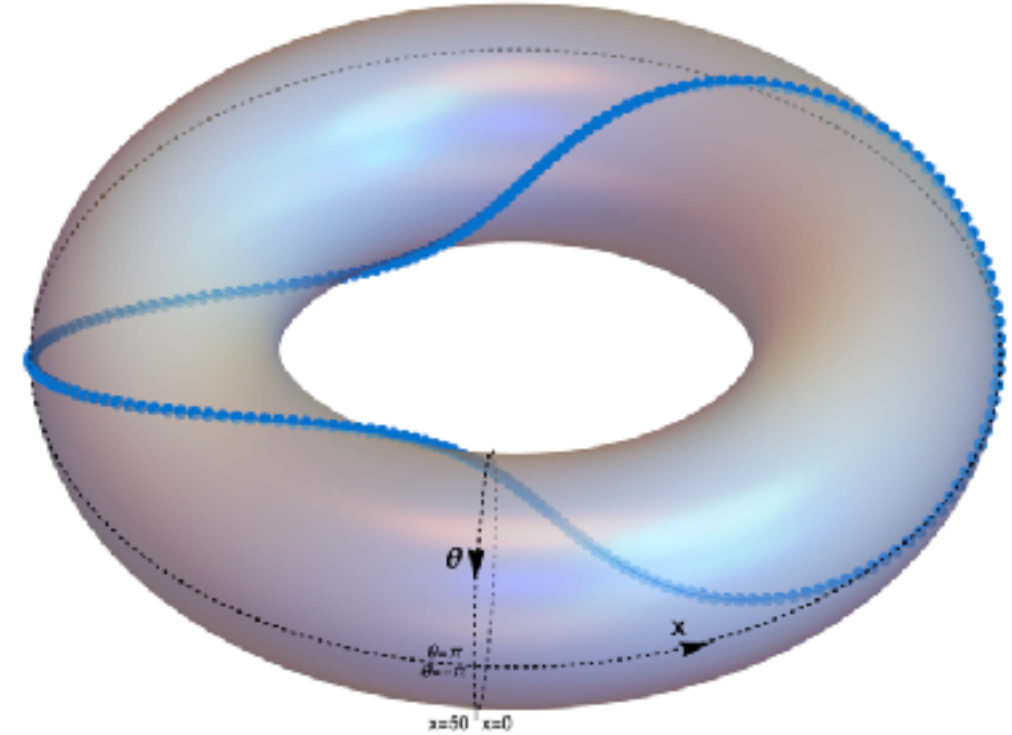
Z.H. Li, H.Q. Shi and HQZ, 2111.15230

$$|D_\mu \Psi|^2 = |(\partial_\mu - i\tilde{e}A_\mu)\psi e^{i\theta}|^2$$









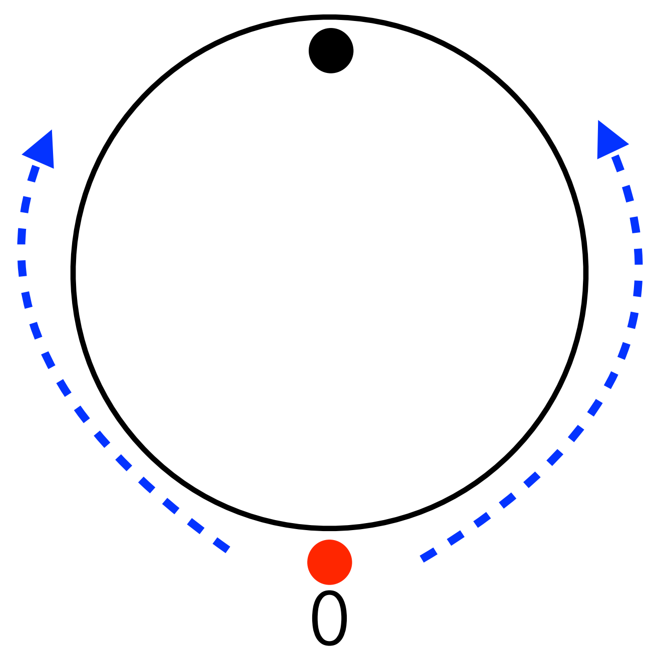
- Two-point functions of the order parameter operator

$$G(x-y) \equiv \langle O(x)O(y)\rangle = \langle O(x)O(y)\rangle_c + \langle O(x)\rangle\langle O(y)\rangle$$

- Since the boundary is a large N field theory,

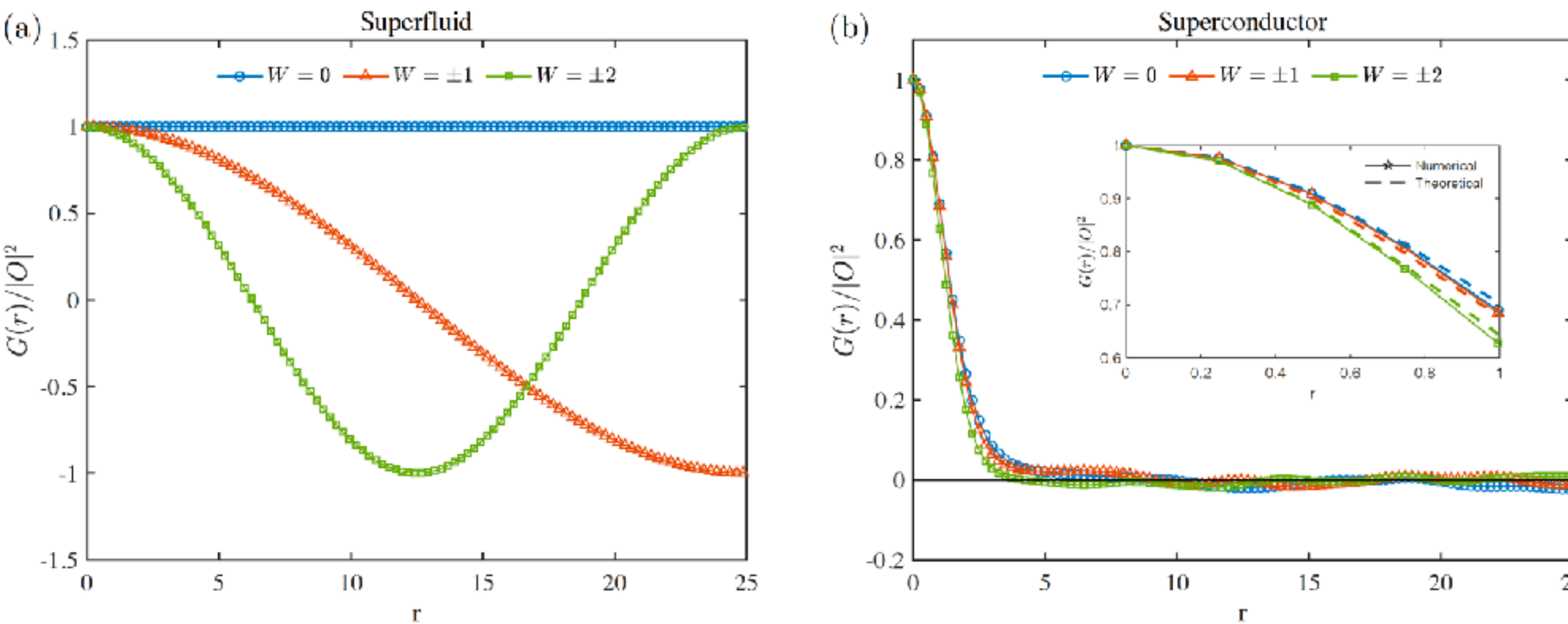
$$G(x-y) = \langle O(x)\rangle\langle O(y)\rangle + \mathcal{O}(N^{-2})$$

$$\therefore, \qquad G(r) \xrightarrow{N\to\infty} \langle O^{\dagger}(r)\rangle\langle O(0)\rangle$$



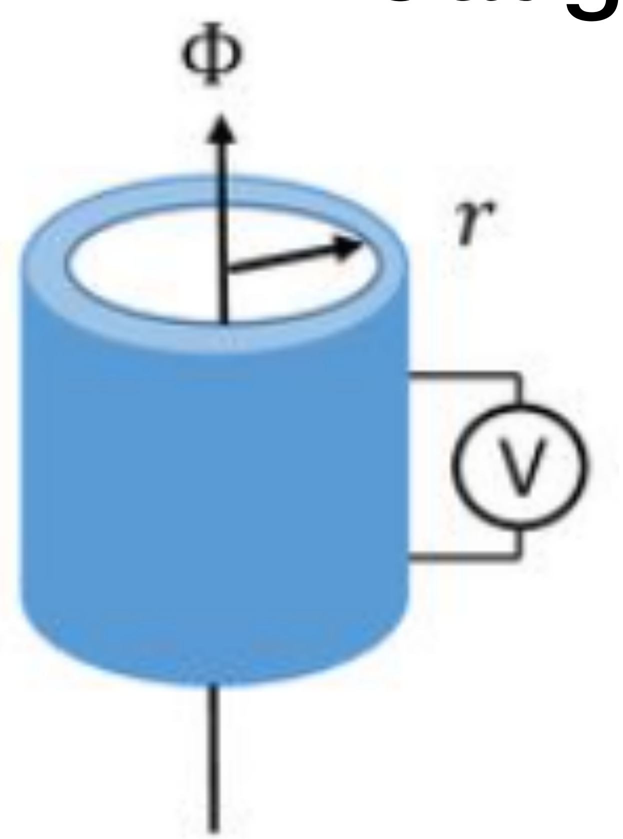
$$\frac{G(r)}{|O|^2} \sim \langle \cos(2\pi W r/L) \rangle$$

$$\frac{G(r)}{|O|^2} \sim \langle \cos(\nabla \theta(r_0) r) \rangle \qquad \text{as } r_0 \to 0$$

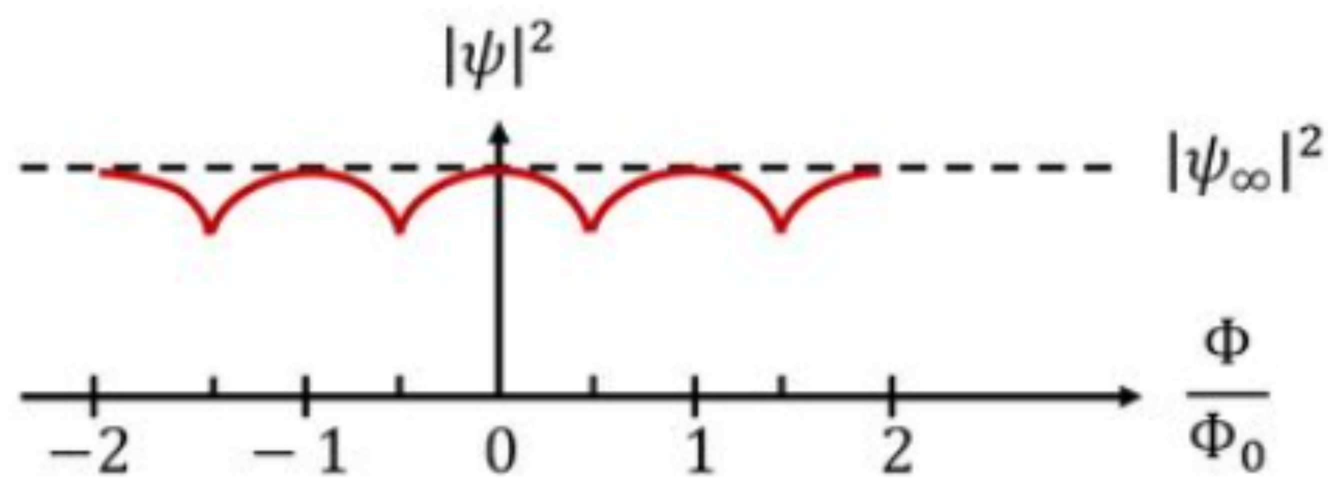


- Little-Parks (LP) experiment is a hallmark of demonstrating the pairing of electrons in the Bardeen-Cooper-Schrieffer (BCS) superconductors.

Gauge invariant term: $|D_\mu \Psi|^2 = |(\partial_\mu - i\tilde{e}A_\mu)\psi e^{i\theta}|^2$



Flux quantum: $\Phi_0 = \dfrac{2\pi}{\tilde{e}} = \dfrac{2\pi}{2e}$



W.A. Little, R.D. Parks, PRL. **9**, 9 (1962)

- Holographic LP effect in holography

M. Montull, O. Pujolas, A. Salvio and P. J. Silva, PRL. 107, 9 (2011), 181601; JHEP 04 (2012), 135;

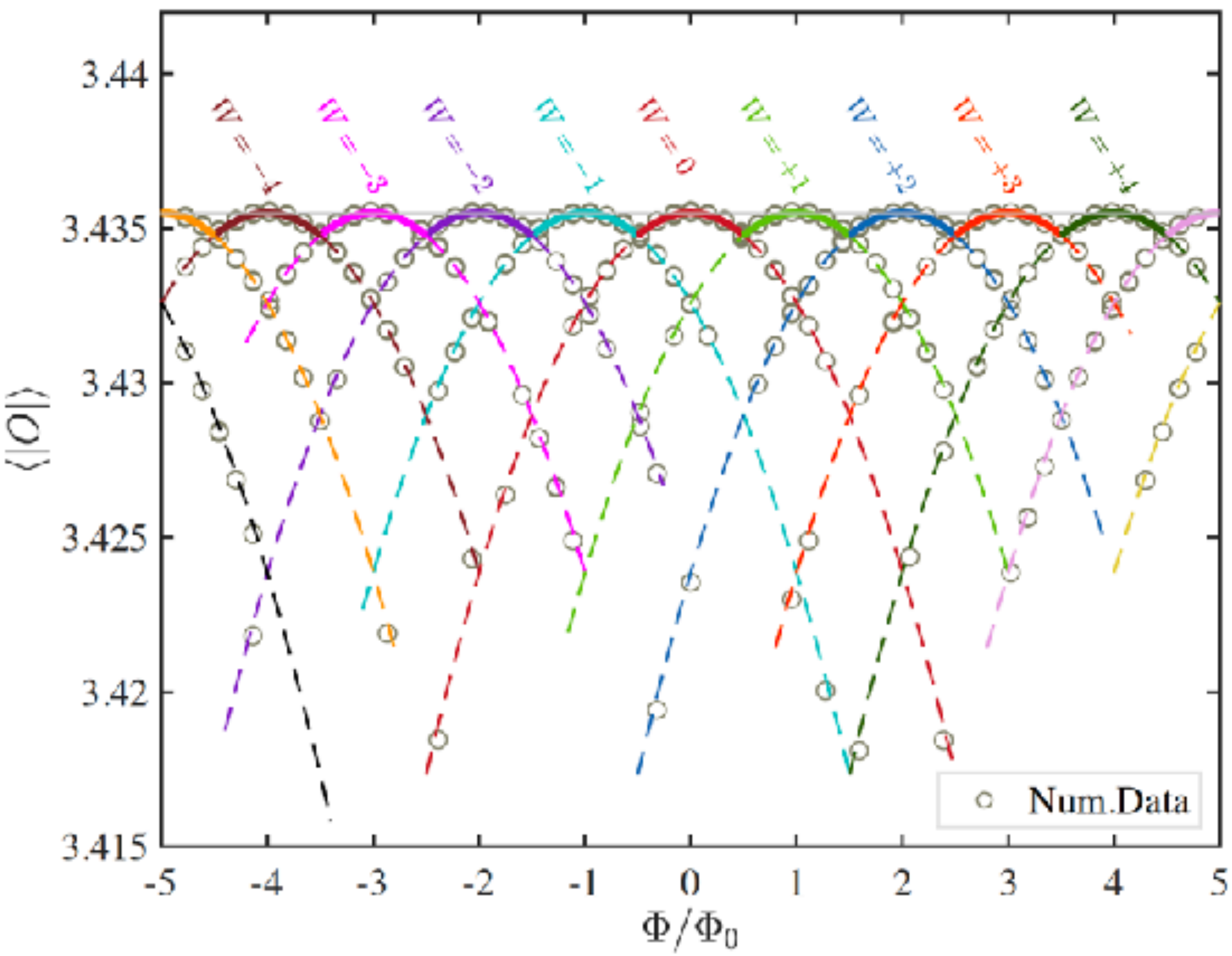
R. G. Cai, Li Li, Li-Fang Li, HQZ, Yun-Long Zhang, PRD. 87, 9 (2013) no.2, 026002;

These works were done in static case and the winding numbers of order parameter were brought in by hand.

- We found that KZM can alternatively realize the LP effect dynamically and statistically.

Z.H. Li and HQZ, 2111.05568

- Holographic LP periodicities of average condensate



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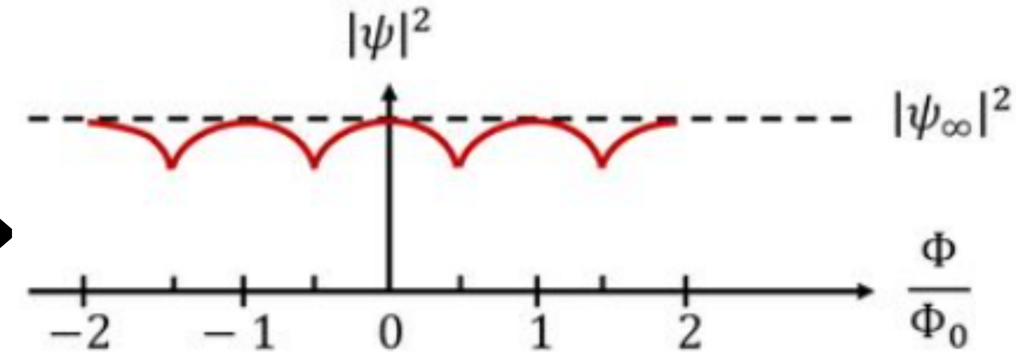
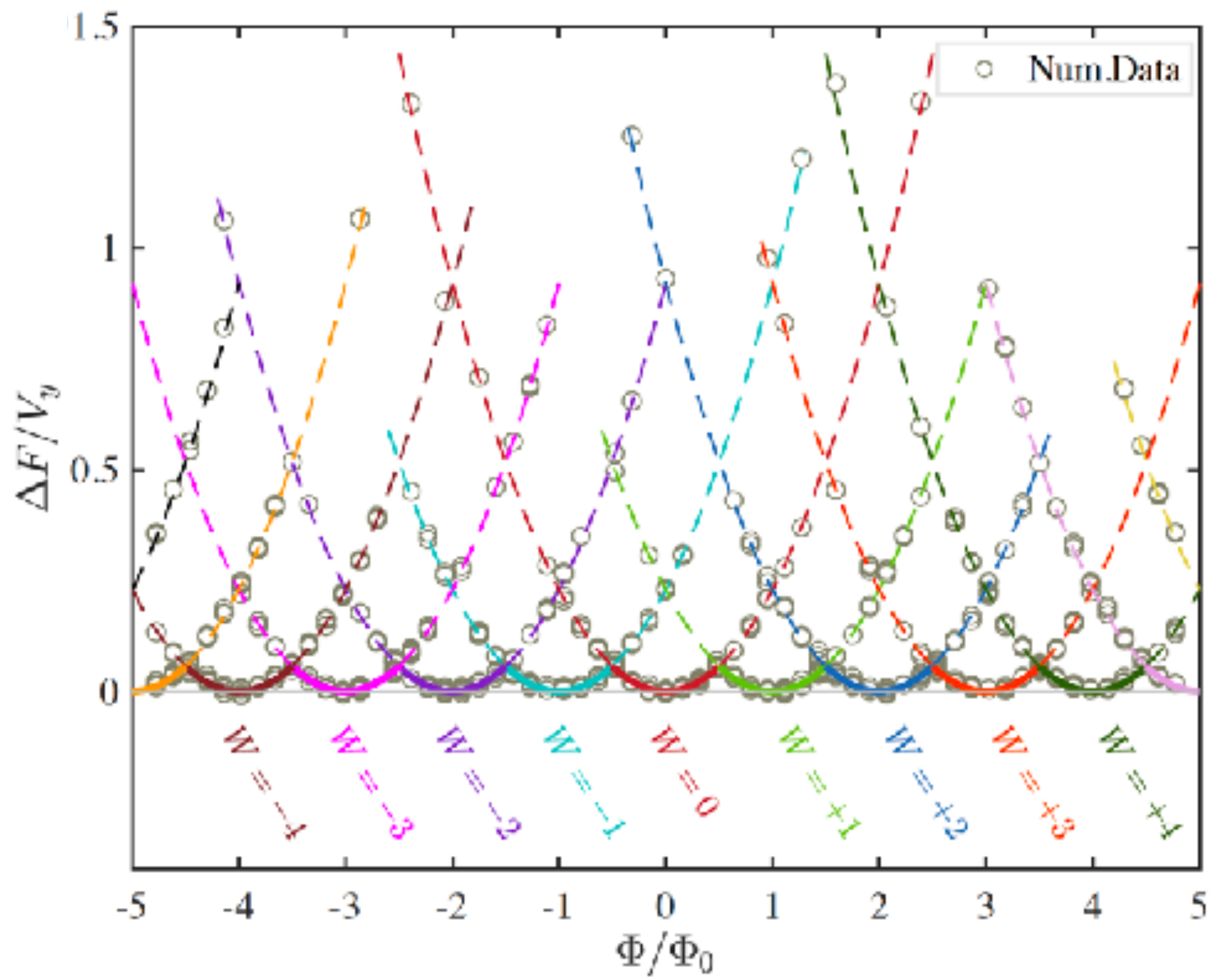


- Little-Parks periodicities of free energy

$$\Delta F = F - F_{W=0,\Phi/\Phi_0=0}$$

$$V_y = \int dy$$

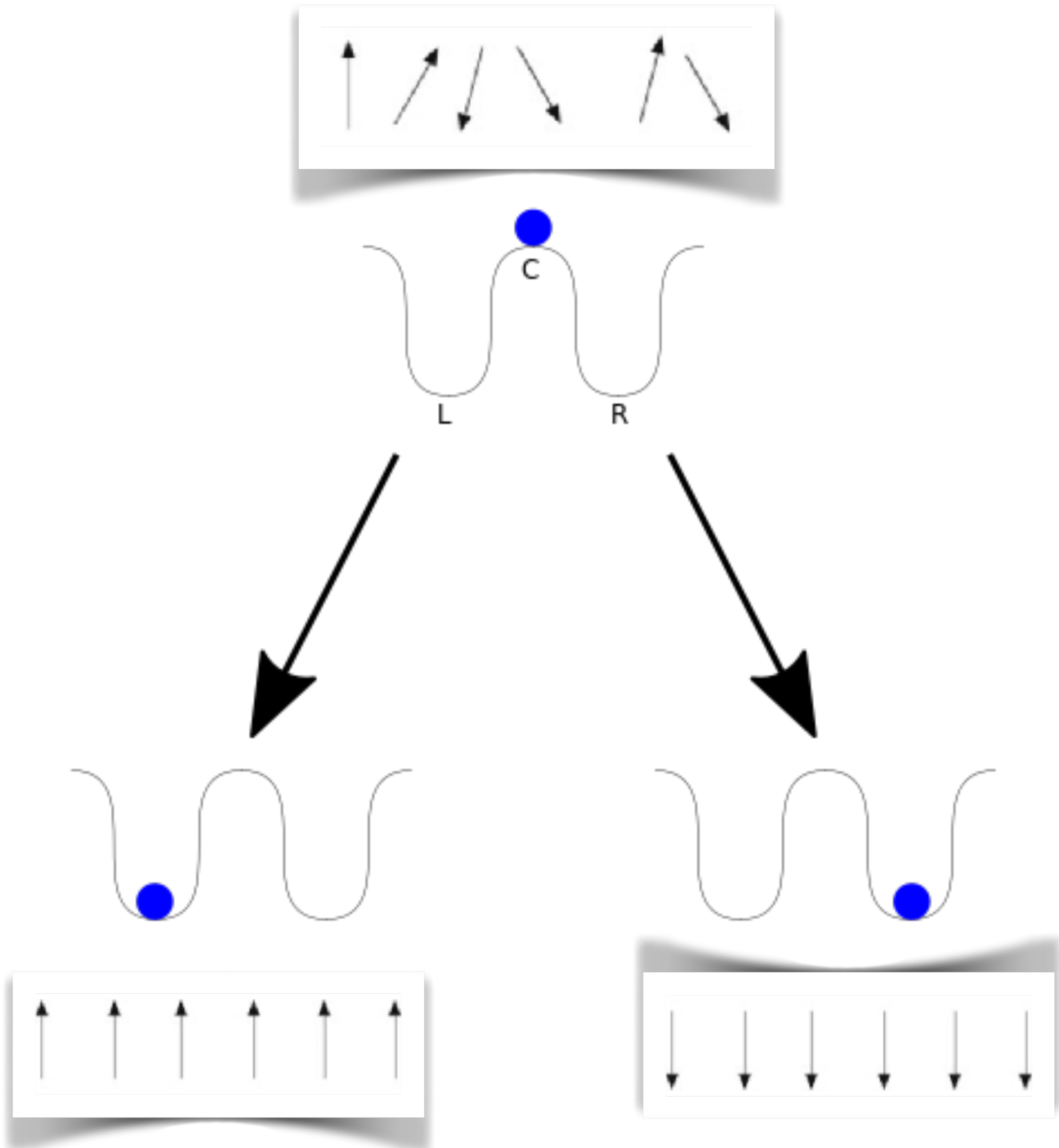
- 1st order phase transition at half-integers of Φ/Φ_0, i.e., $\Phi/\Phi_0 = \mathbb{Z} \pm 1/2$.
- Lowest free energy states incline to appear most frequently.



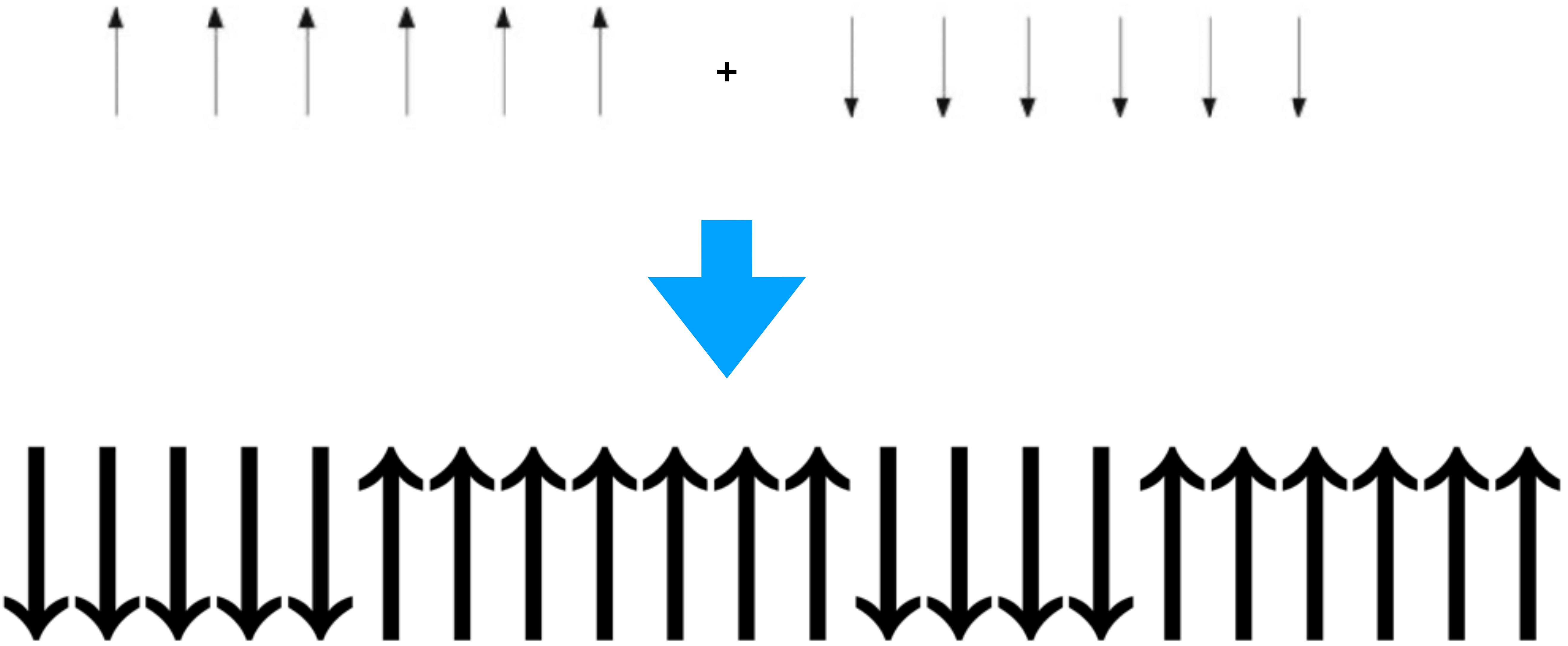
- Significance: Lowest energy configurations can be probed by dynamics and statistics.

Holographic KZM with discrete symmetry breaking

Parity symmetry breaking (Z_2 symmetry breaking)



Kink formation in a spin chain



- Action

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\tilde{\Psi}|^2 - m^2|\tilde{\Psi}|^2$$

- Make the gauge transformation

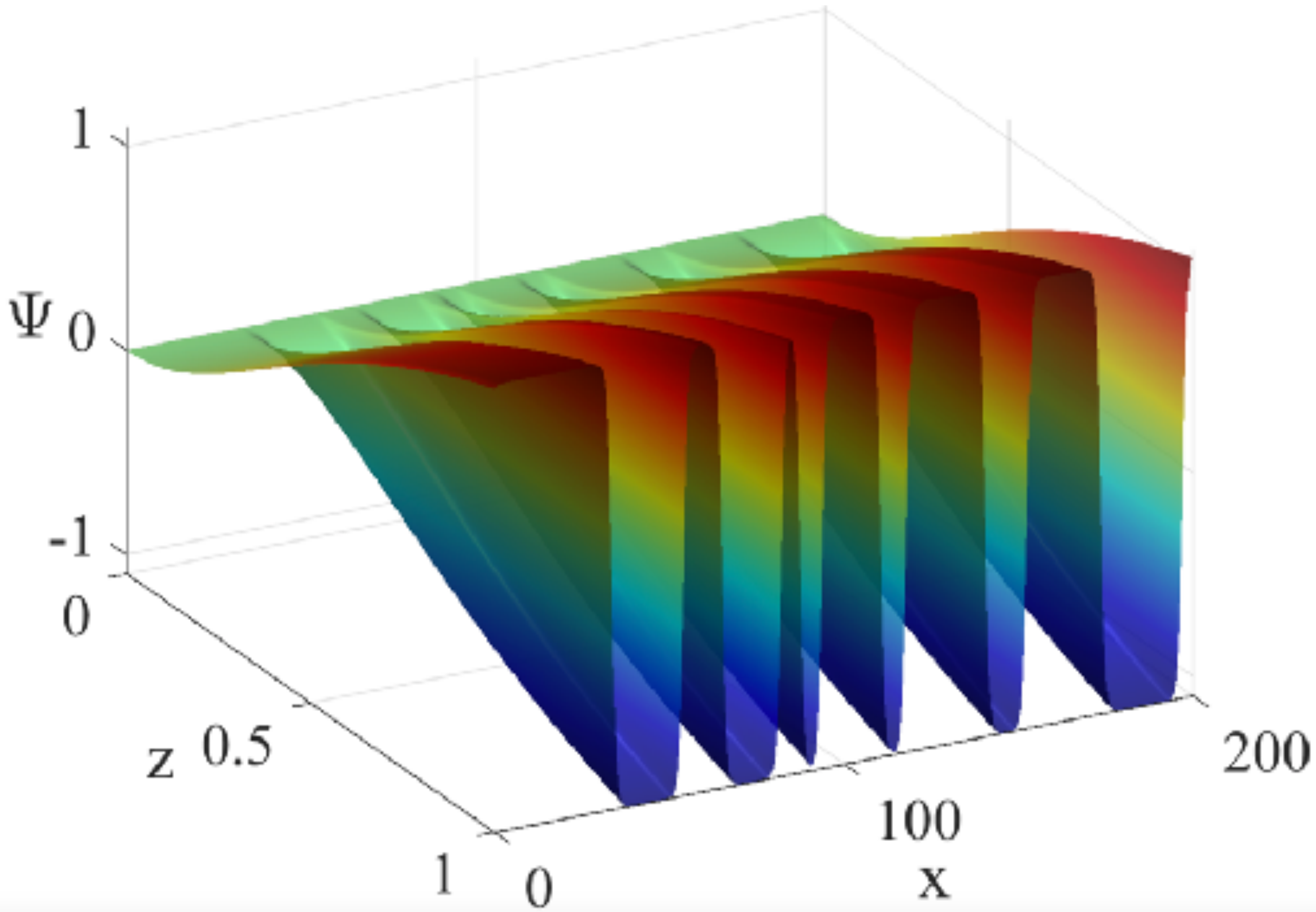
$$\tilde{\Psi} = \Psi e^{i\lambda} \text{ and } A_\mu = M_\mu + \partial_\mu\lambda$$

- EoMs of real functions

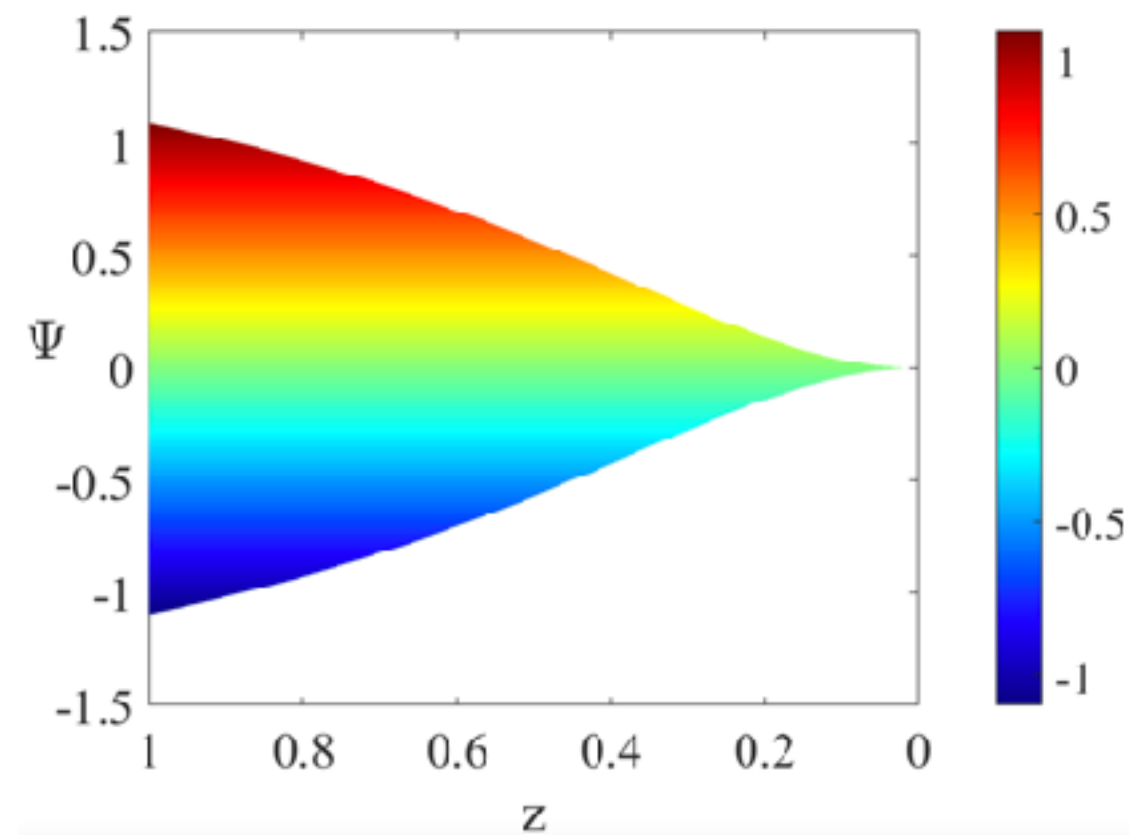
$$(\nabla_\mu - iM_\mu)(\nabla^\mu - iM^\mu)\Psi - m^2\Psi = 0, \qquad \nabla_\mu F^{\mu\nu} = 2M^\nu\Psi^2.$$

These functions satisfy a symmetry of $+\Psi \leftrightarrow -\Psi$

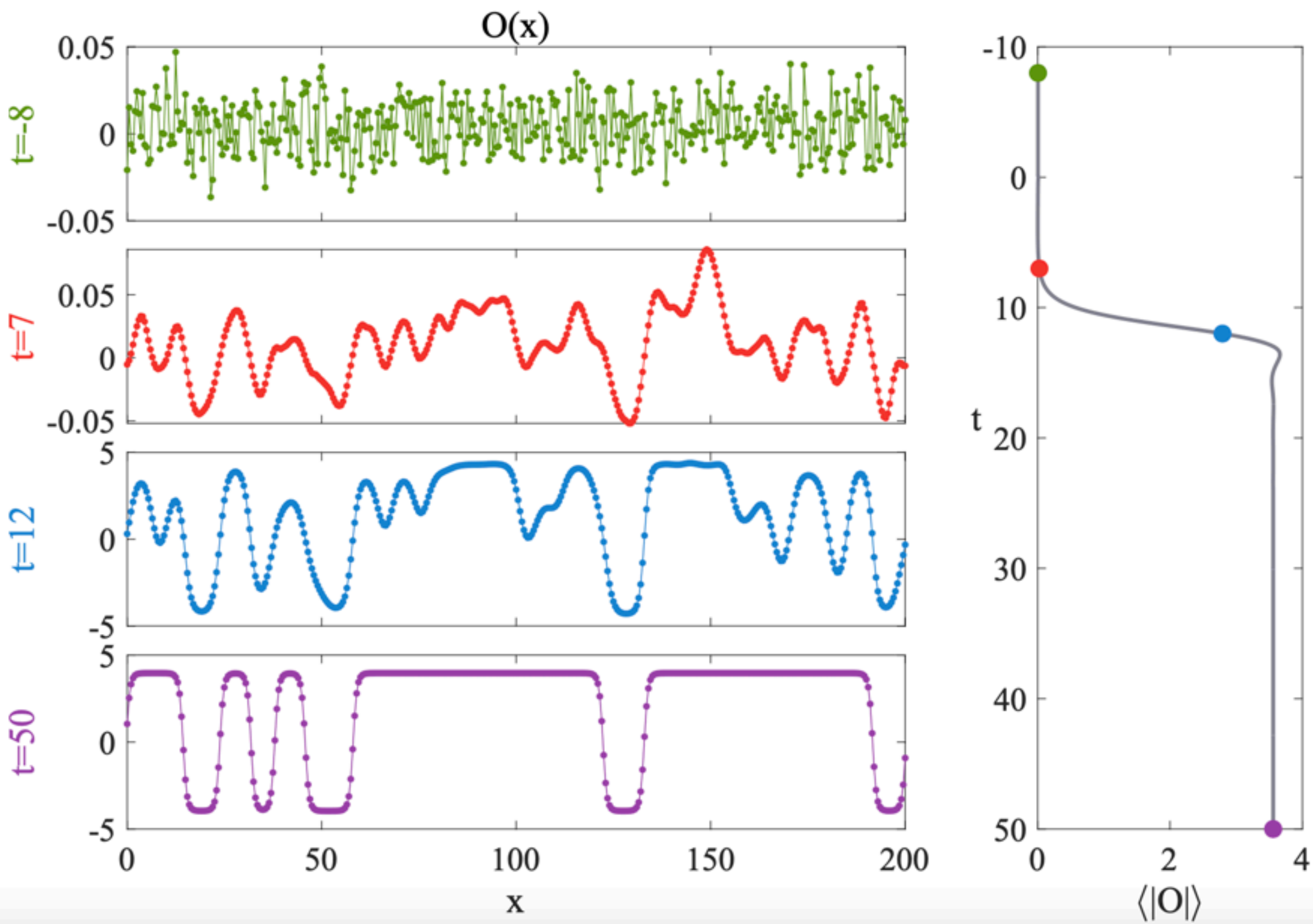
Holographic kink formation in 1+1 dim



Kink hairs in the bulk

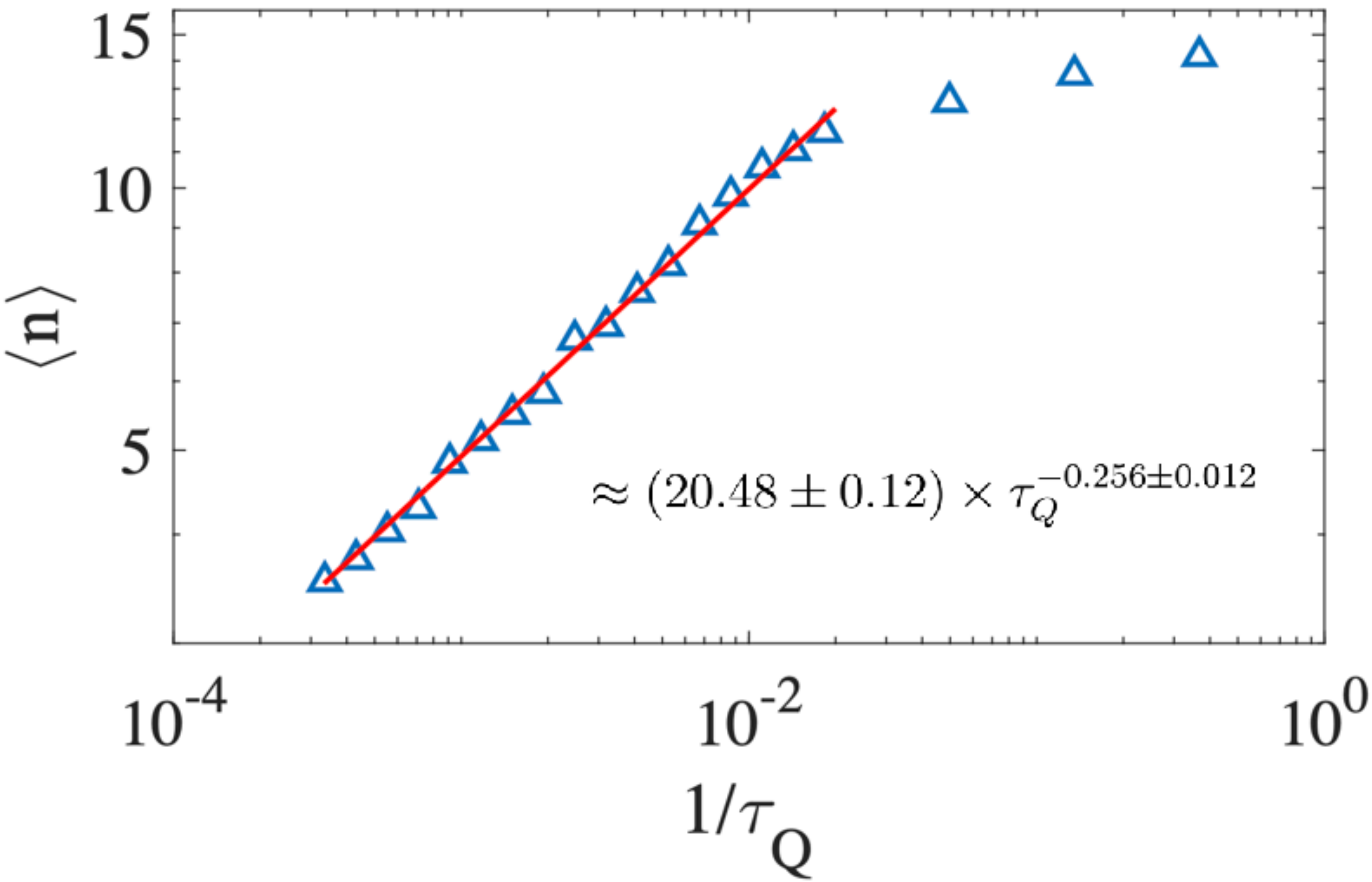


View along x-direction



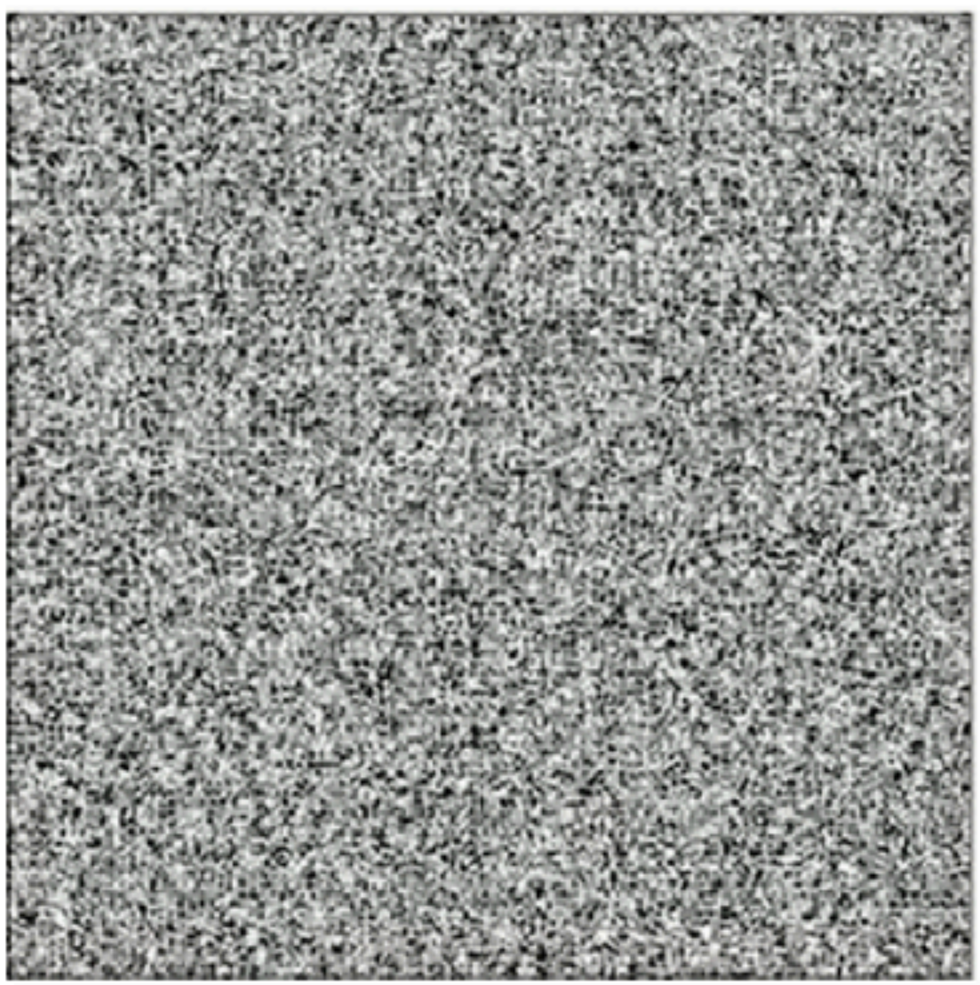
Kink formation in the boundary

Satisfy the KZM relation $\langle n \rangle \propto \left(\frac{1}{\tau_Q}\right)^{\frac{d\nu}{1+z\nu}}$



Holographic domain wall formation in 2+1dim (in progress)

- Monte Carlo simulation of the domain wall in an Ising model (from Wikipedia)

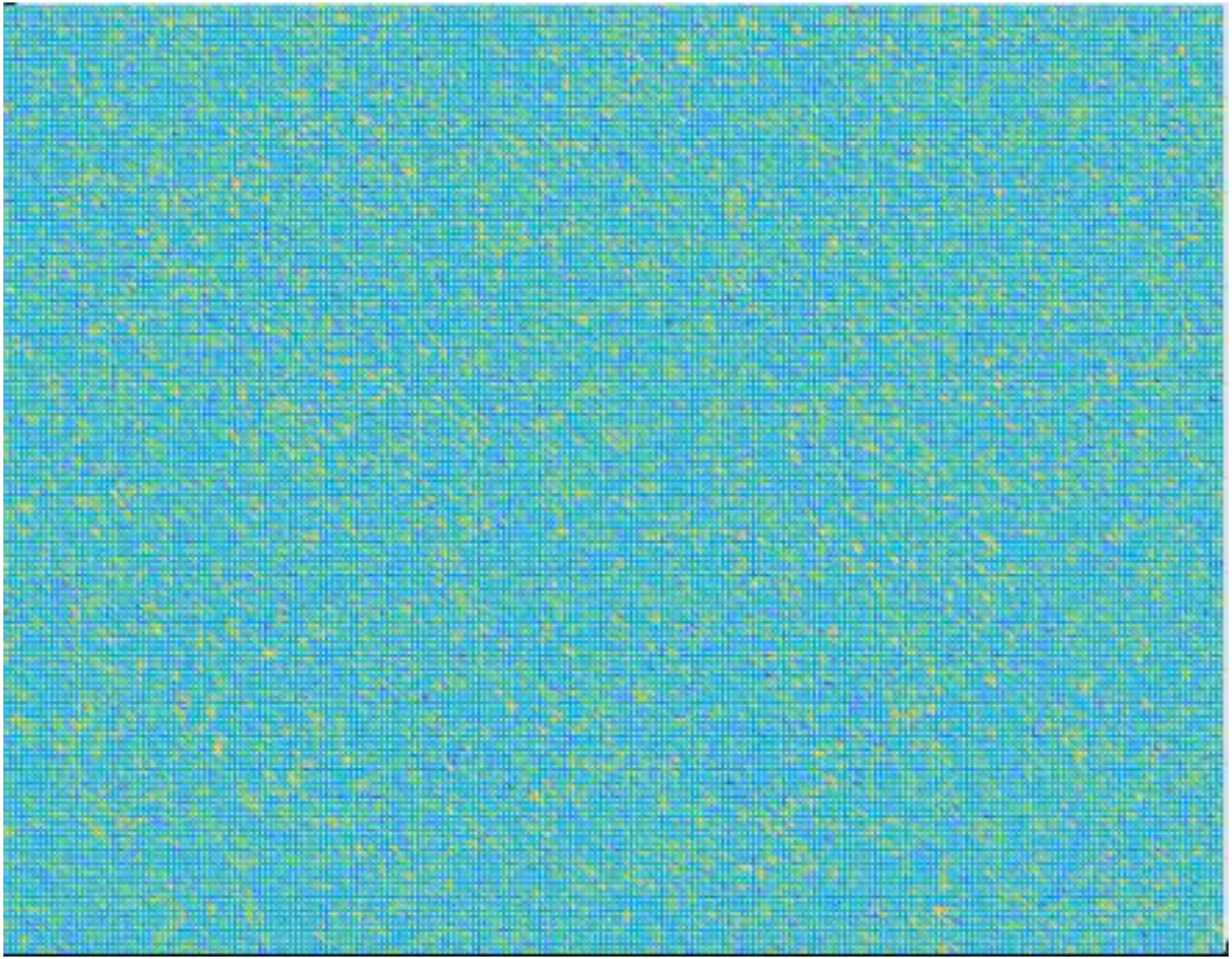


Initial state

Equilibrium after quench

- Holographic simulations similar to the Ising model

T.C. Ma, H.Q. Shi and HQZ, to appear

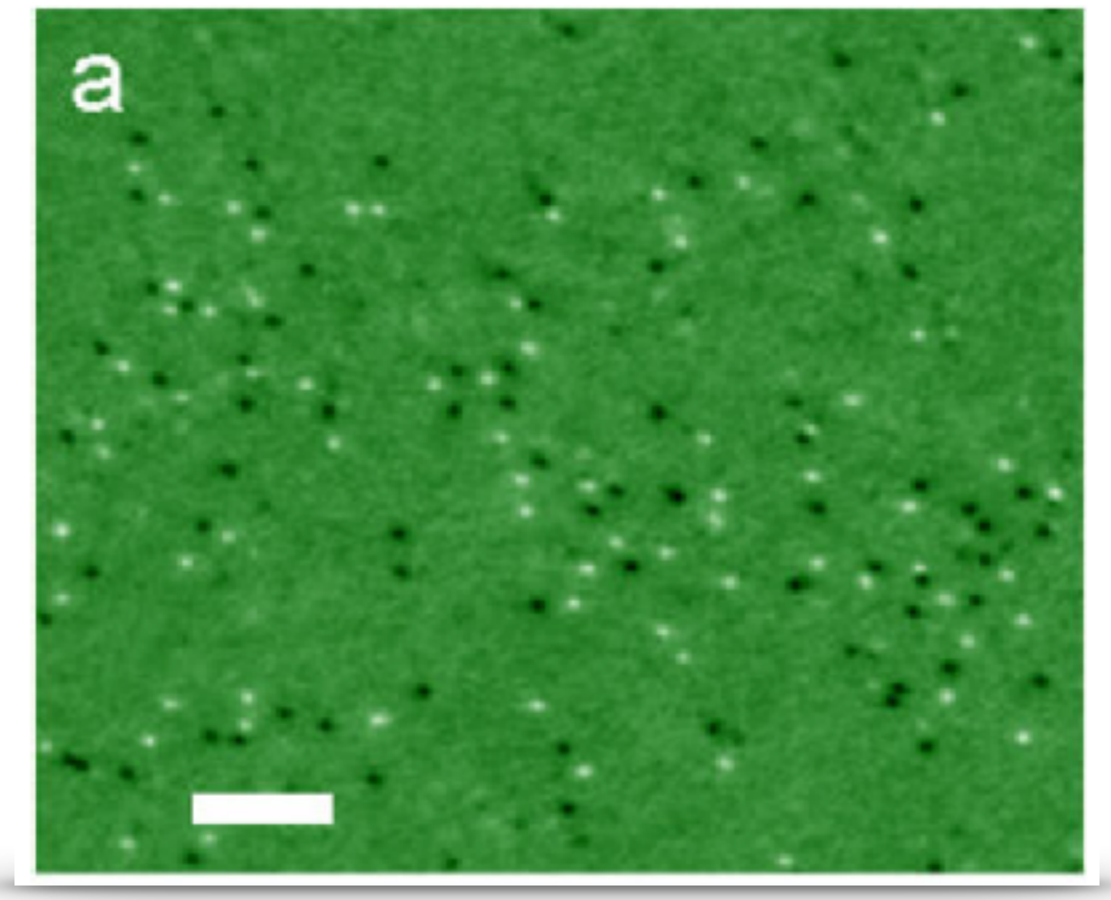


Holographic flux-trapping mechanism beyond KZM

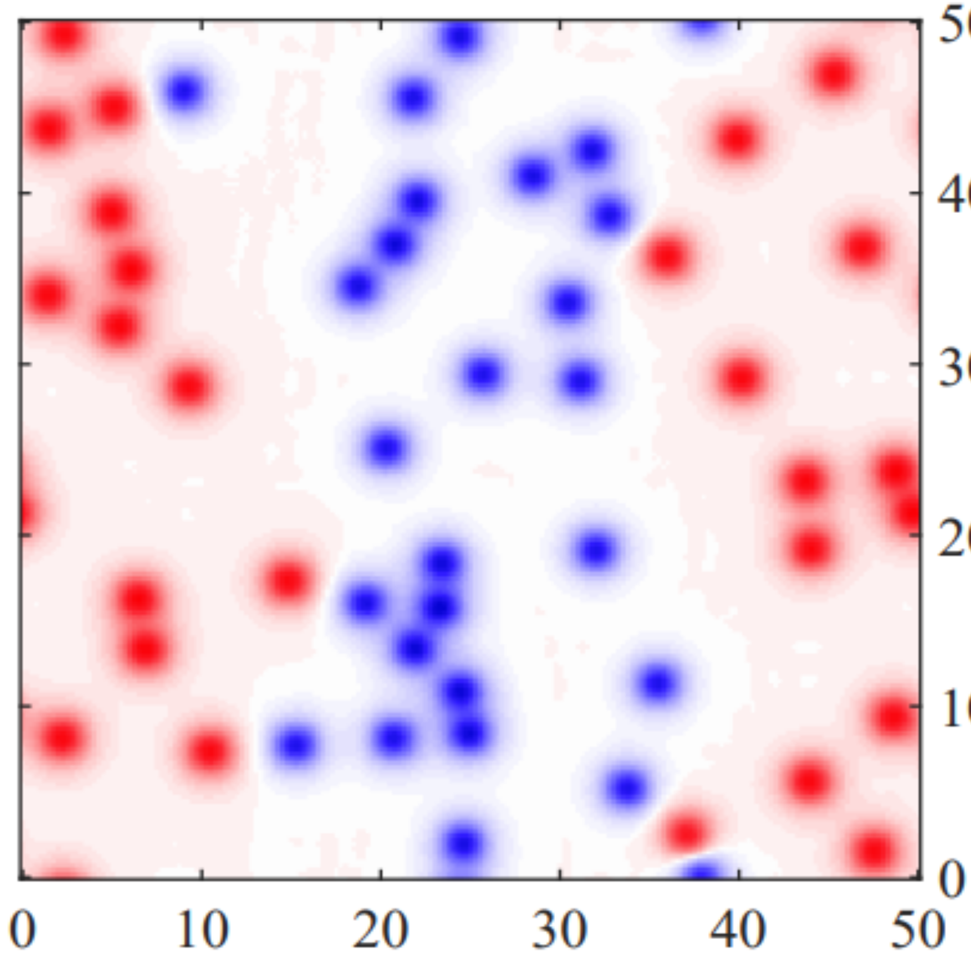
If local gauge symmetry plays an important role in the vortex formation, things will change: clusters of equal-sign vortices appear

We call it “Flux-trapping mechanism” (FTM)

The direct result is that:
KZM generates vortices which are anti-correlated;
FTM generates vortices with positive correlations.



KZM



FTM

Hindmarsh-Rajantie [PRL 85,4660(2000)] proposed the FTM:

1. At first, random gauge fields have large amplitudes, besides vanishing order parameter

2. Quench the system. Fluctuations with short wave length (large momentum k) will decay rapidly; Only long wave length of gauge fields exist. The exponential decay is roughly $e^{-\gamma t}$ with $\gamma \approx k^2$

3. However, due to the quench in a finite time, the decay will stop as order parameter grows. The magnetic field survives.

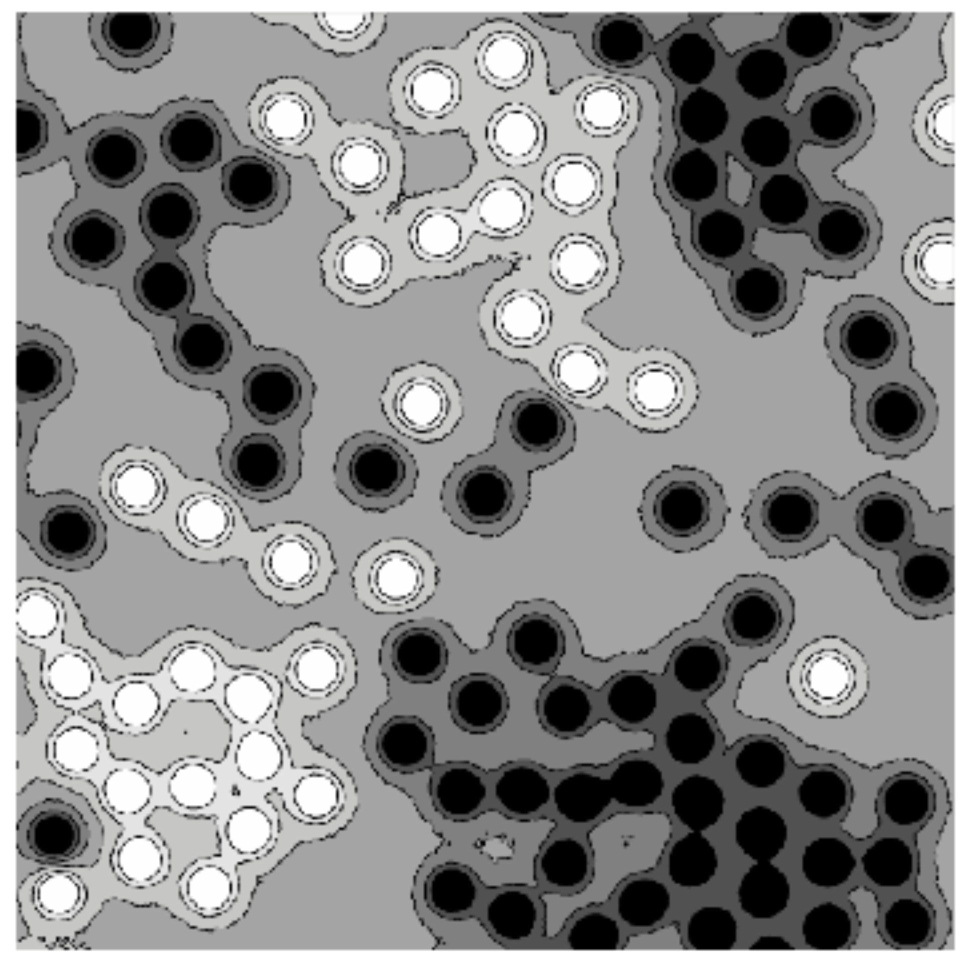
4. Meissner effect will prevent the existence of the magnetic field. Therefore, the only way the magnetic field can do is to form vortices, each of which carries one flux quantum (fluxon)

5. Since long wave magnetic field exists, which extends in a large area. If the total flux in this area can support the fluxons, there will be a mount of fluxons appear in this area.

6. Therefore, this FTM predicts the clusters of vortices with equal sign, i.e., vortices are positively correlated.

However, experiments are few. The reason is that the initial random large amplitude of magnetic field is hard to prepare; besides, the overdamping decay will finally make magnetic field really tiny, which cannot support the clusters of equal-sign fluxons.

Stephens, et.al., PRL,88,137004 (2002)

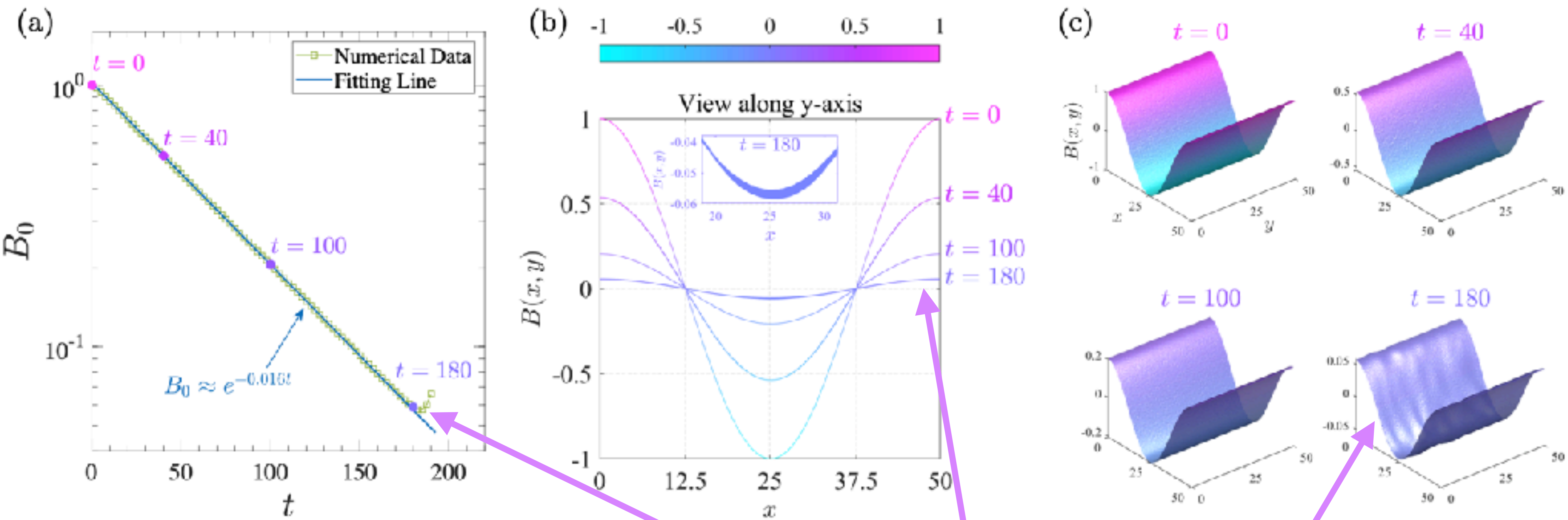


Our idea is simple: to have a toy model with plane-wave magnetic fields in the initial state, which can be easily prepared in experiment

To realize it in AdS/CFT, which implies the strongly coupled vortices. Since equal-sign vortices repel each other, thus, the existence of the equal-sign clusters of the vortices with strongly coupled interactions seem more natural than in weakly coupled theory.

Early stage exponential decay

Z.H. Li, C.Y. Xia, H. B. Zeng, HQZ, 2103.01485

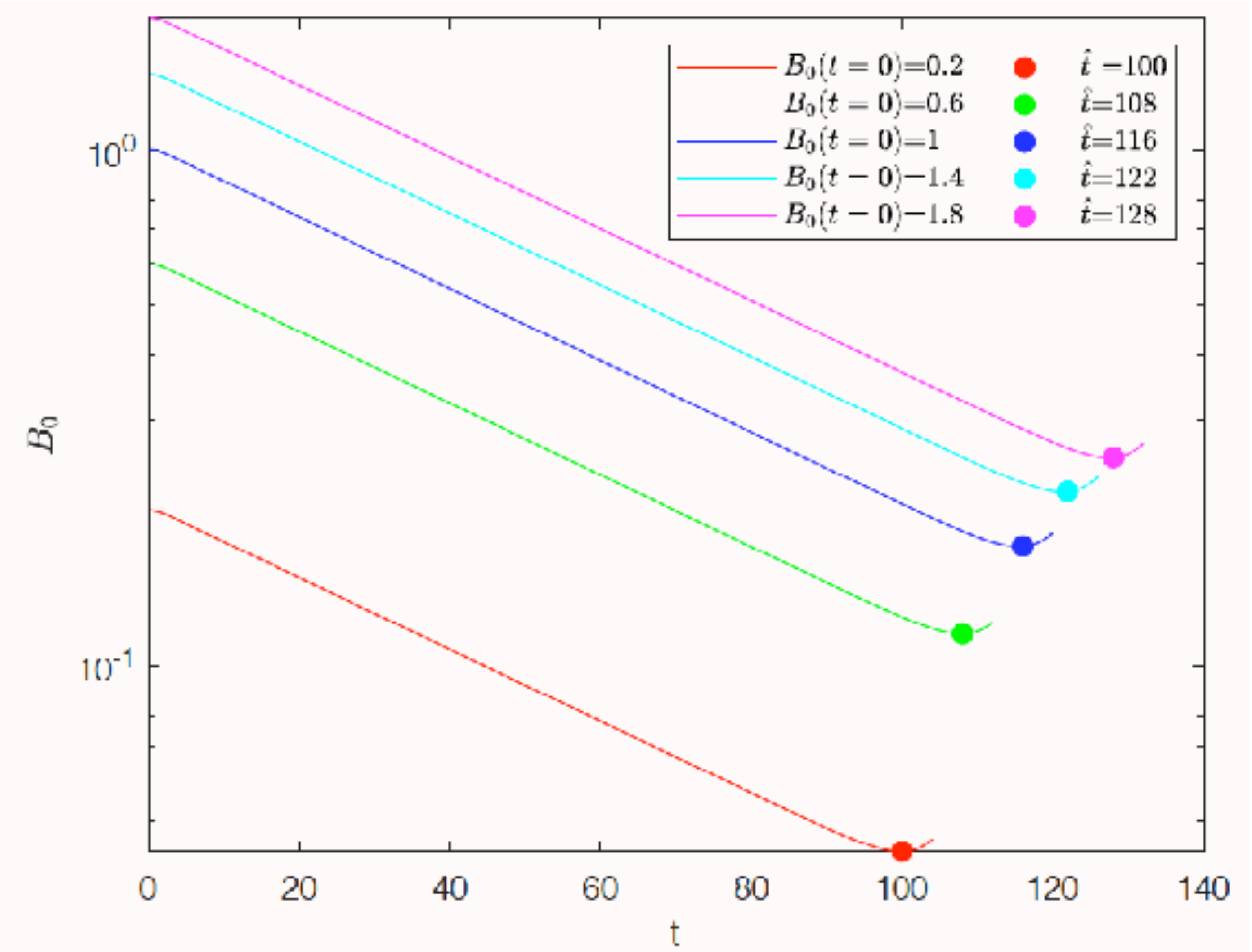


$$B(x,y)\big|_{t=t_i} = B_0 \cos(kx)$$

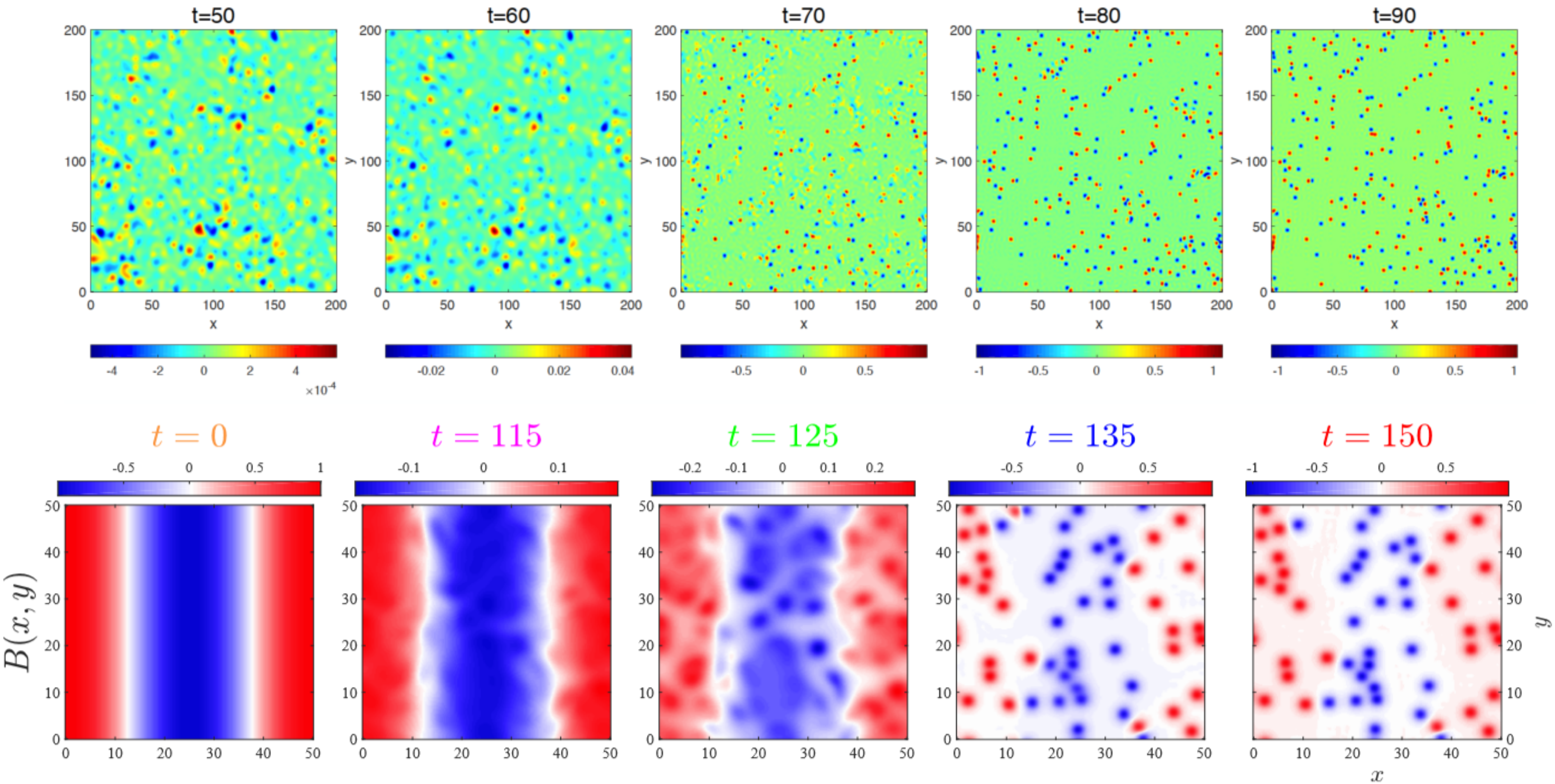
Flux trapping time

Decay rate $\gamma \approx k^2 = (2\pi/50)^2 \approx 0.016$

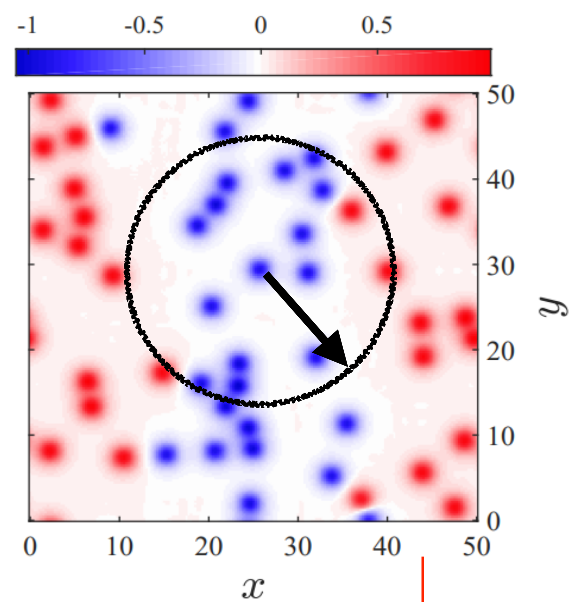
Early stage decay rate is independent of the initial amplitudes of the magnetic field. They all have $\gamma \approx k^2$



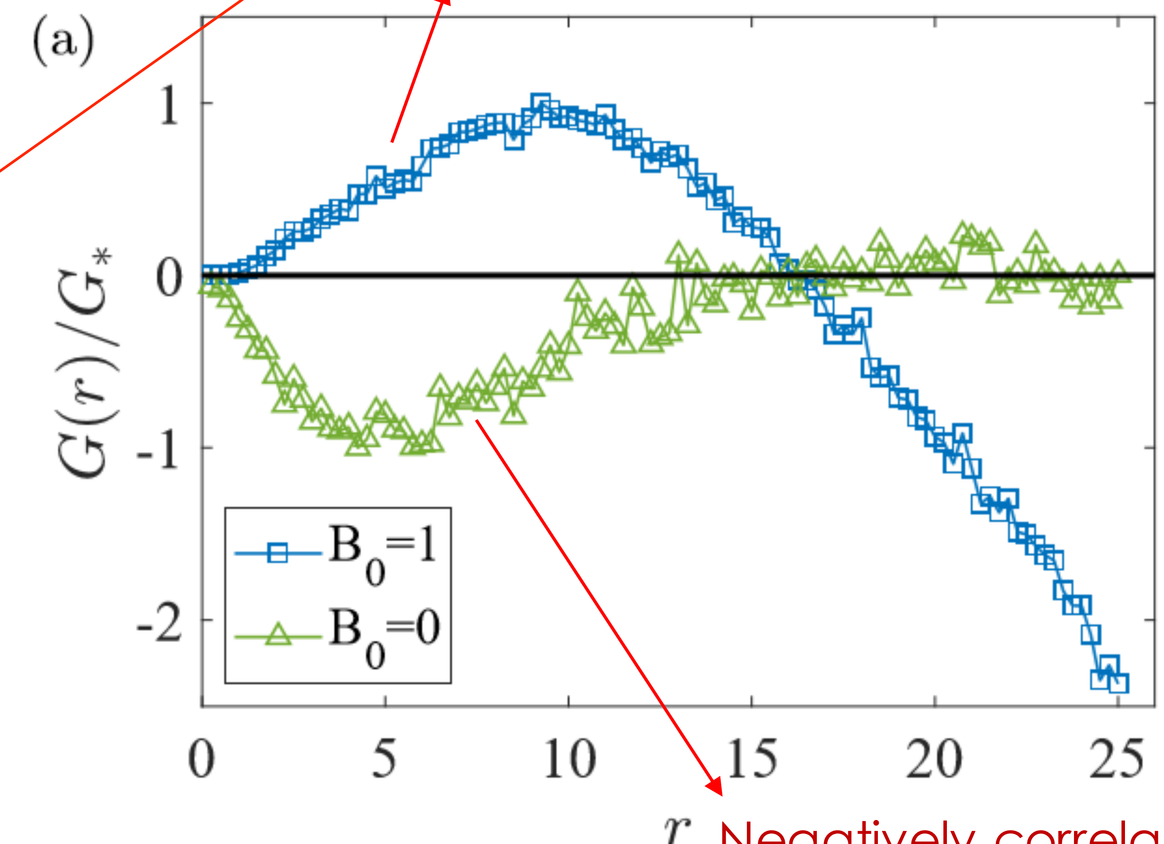
Key difference between KZM and FTM



Vortex spatial correlations & net vorticity

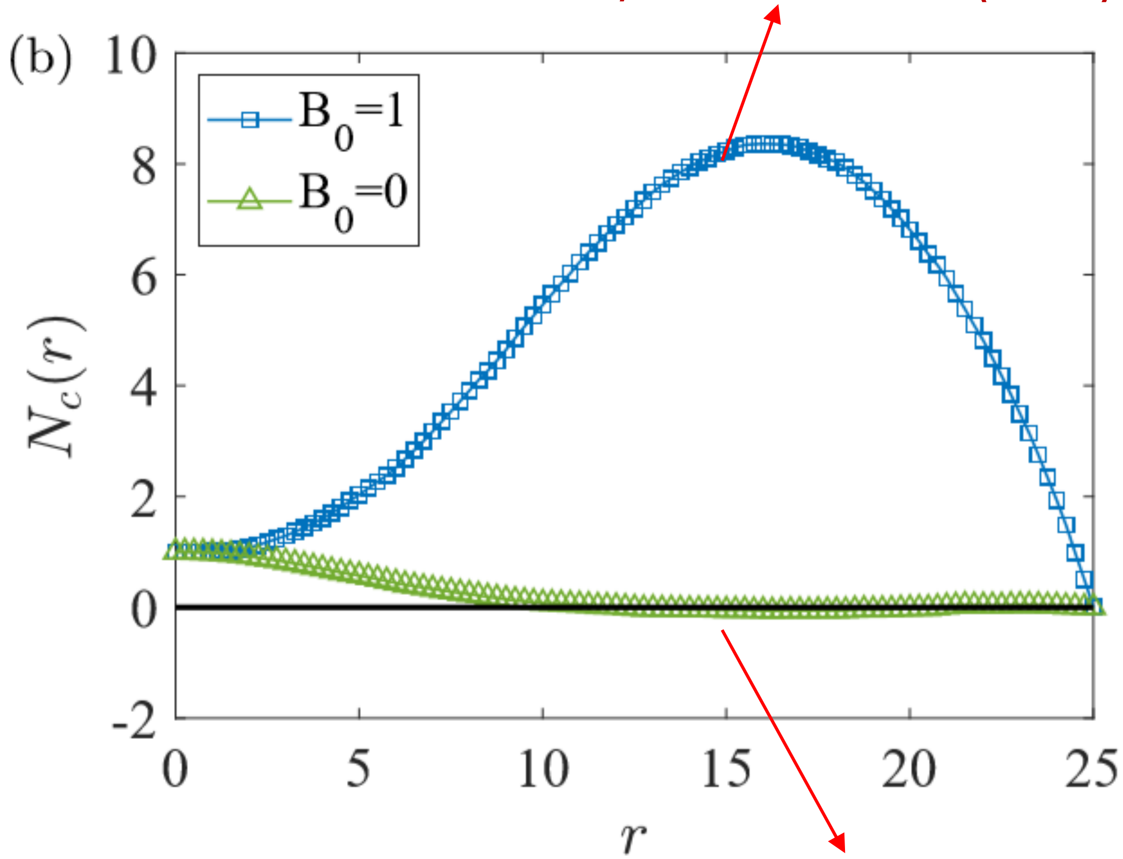


Positively correlated (FTM)

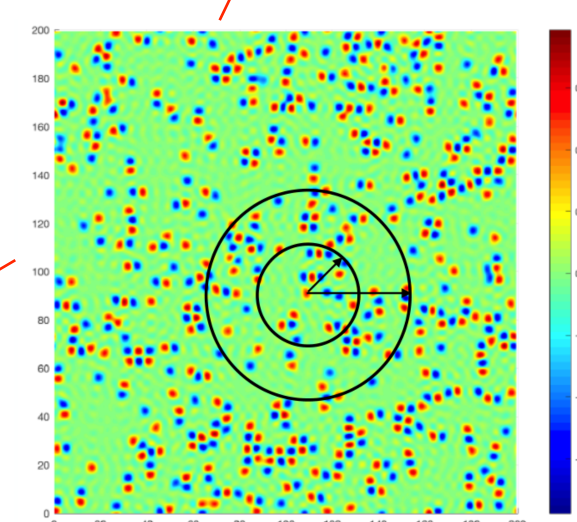


Negatively correlated (KZM)

Positively correlated (FTM)

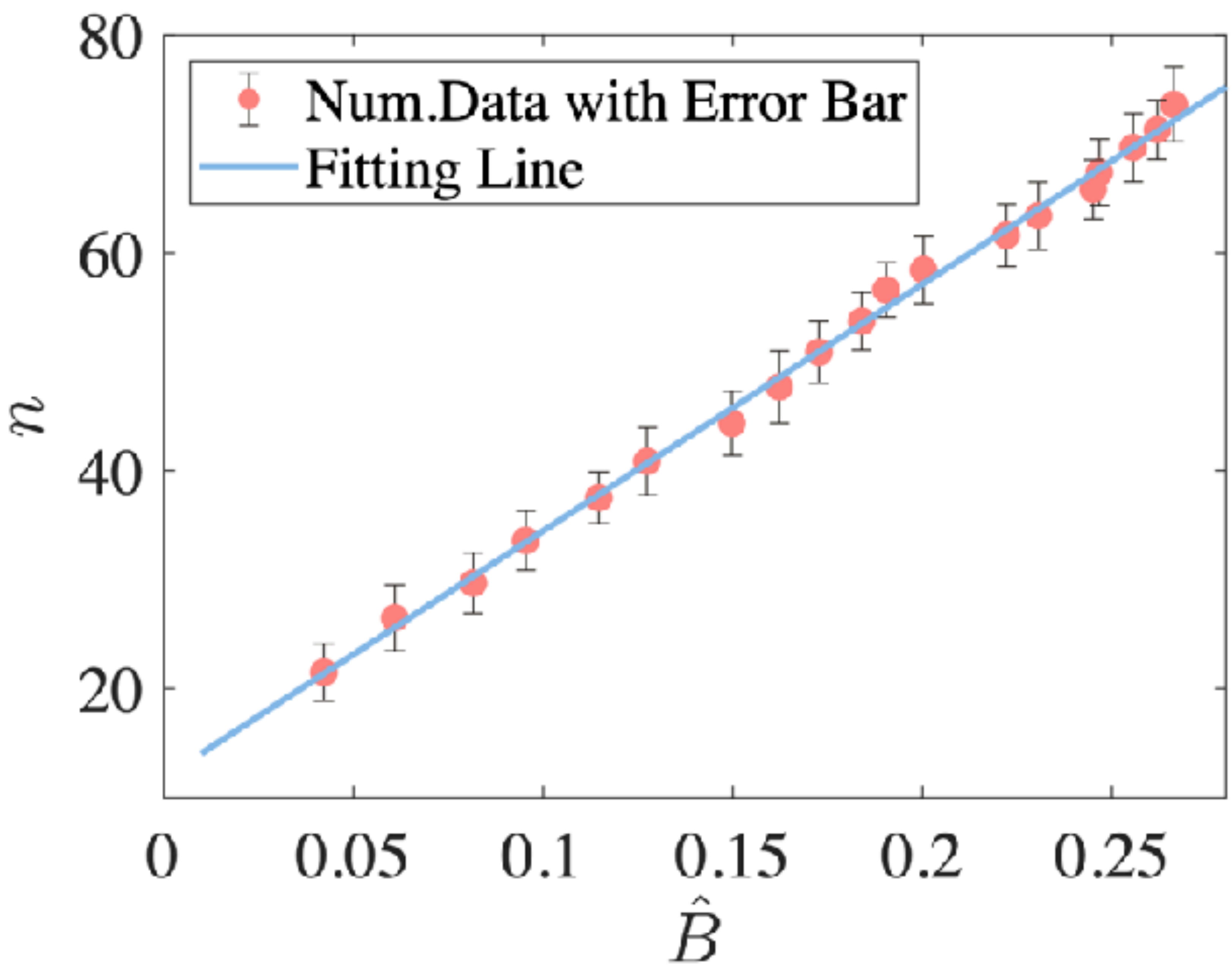


Negatively correlated (KZM)



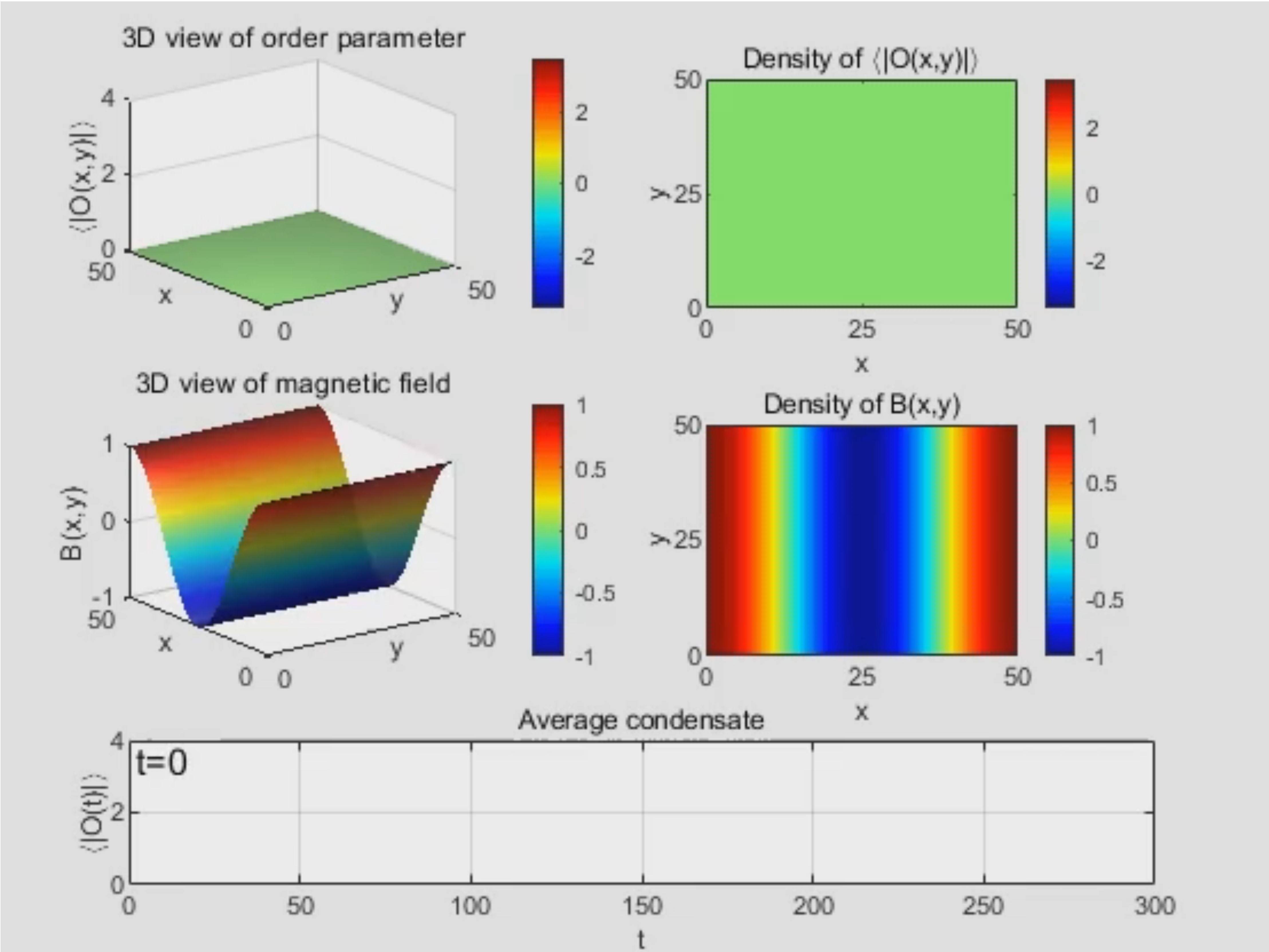
Linear relation between the vortex number and the amplitude of magnetic field at trapping time supports the FTM

$$n \propto |\hat{\Phi}| = \int |\hat{B}\cos(kx)|dxdy \propto \hat{B}.$$



Animations

At late time, pinning effect exists



Summaries

- KZM is key to the pattern formation, especially in condensed matter;

- KZM in holographic superconductor/superfluid can be realized by setting the different boundary condition of Ax;

- The merit of topological defects in holographic superconductor was that they were stabler, which made the counting statistics much easier;

- The behavior of winding numbers for holographic superconductor/superfluid are significantly different;

- For discrete symmetry breaking, we realized the kink hairs in the bulk, and the corresponding kink formation in the boundary field theory;

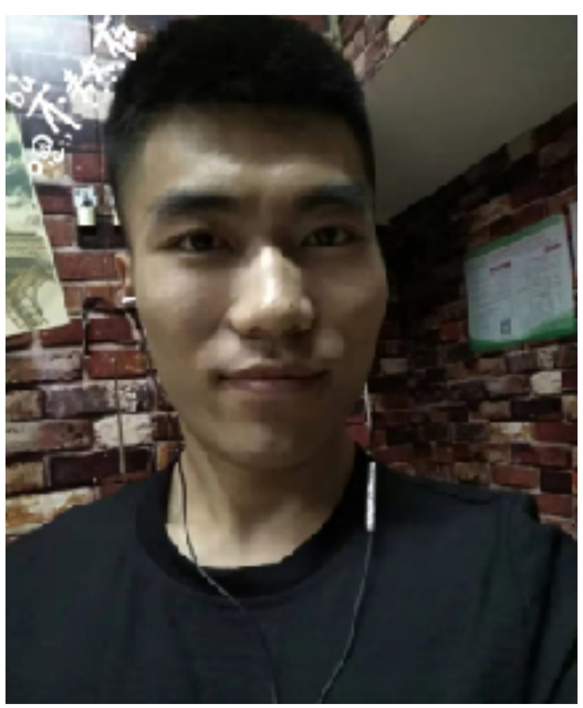
- The FTM is totally different from KZM. FTM can create equal-sign vortices, but KZM cannot.

Acknowledgements

Zhi-Hong Li

Hua-Bi Zeng

Adolfo del Campo



Chuan-Yin Xia

Han-Qing Shi

Tian-Chi Ma

Thank you very much for your attention!