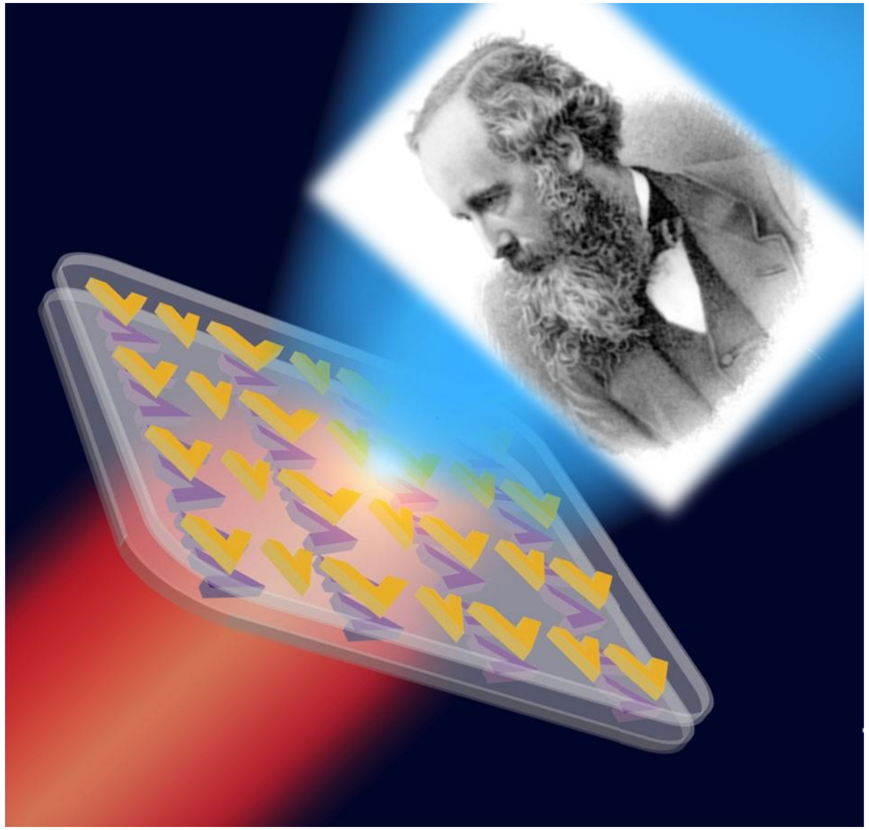
How to sit Maxwell on the boundary of AdS



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Based on

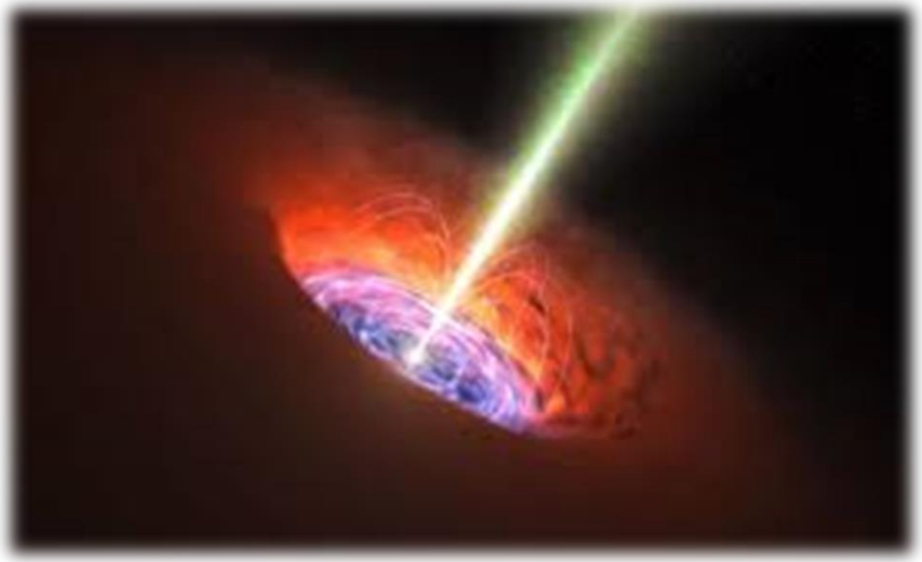
Holography and magnetohydrodynamics with dynamical gauge fields

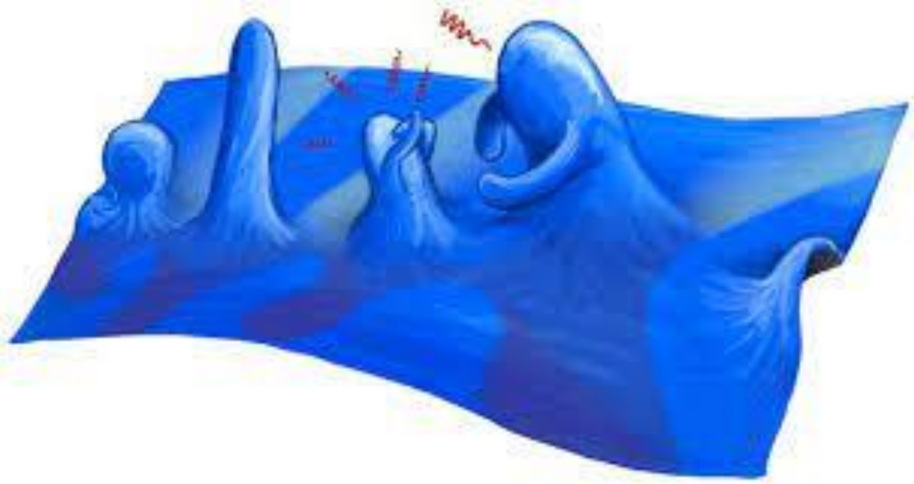
Yongjun Ahn, Matteo Baggioli, Kyoung-Bum Huh, Hyun-Sik Jeong, Keun-Young Kim, Ya-Wen Sun

Within the framework of holography, the Einstein-Maxwell action with Dirichlet boundary conditions corresponds to a dual conformal field theory in presence of an external gauge field. Nevertheless, in many real-world applications, e.g., magnetohydrodynamics, plasma physics, superconductors, etc. dynamical gauge fields and Coulomb interactions are fundamental. In this work, we consider bottom-up holographic models at finite magnetic field and (free) charge density in presence of dynamical boundary gauge fields which are introduced using mixed boundary conditions. We numerically study the spectrum of the lowest quasi-normal modes and successfully compare the obtained results to magnetohydrodynamics theory in $2+1$ dimensions. Surprisingly, as far as the electromagnetic coupling is small enough, we find perfect agreement even in the large magnetic field limit. Our results prove that a holographic description of magnetohydrodynamics does not necessarily need higher-form bulk fields but can be consistently derived using mixed boundary conditions for standard gauge fields.

Comments: 54 pages, 22 figures

Holography





Gravitational bulk

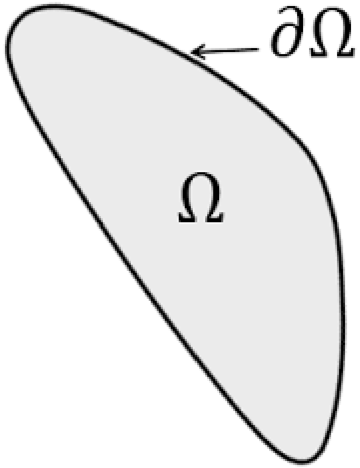
Boundary QFT

Holography

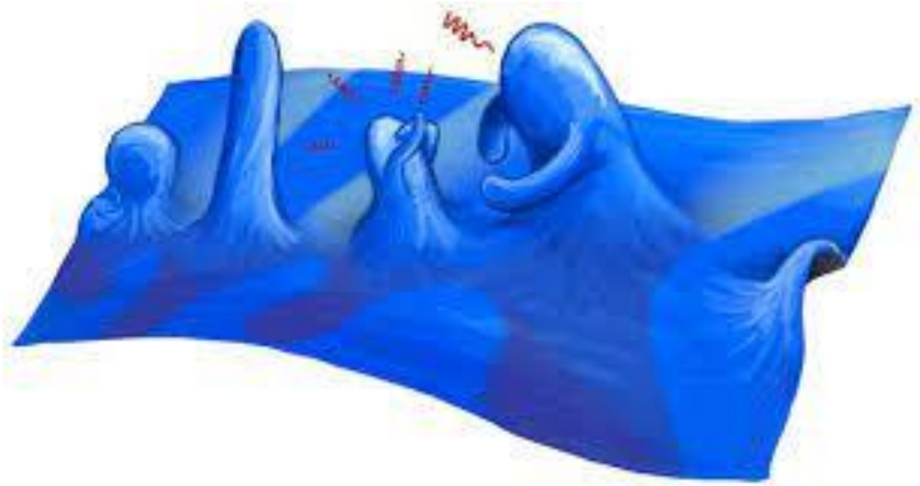
Gravitational bulk action

$$S = \int d^{d+1}x\sqrt{-g}\,\left[R - 2\Lambda + \mathcal{L}\left(F_{\mu\nu}\right) + \ldots\right]$$

Boundary conditions



$$A_\mu\left(r,t,\vec{x}\right) \underset{r\to\infty}{\sim} A_\mu^{(0)}\left(t,\vec{x}\right) + A_\mu^{(1)}\left(t,\vec{x}\right)\, r^{1-D}$$



Boundary QFT

A typical example

- Temperature
- Charge
- Magnetic field

BULK :

$$S = \int \mathrm{d}^4x \sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right), \qquad F = \mathrm{d}A,$$

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2(\mathrm{d}x^2 + \mathrm{d}y^2), \qquad A = A_t(r)\,\mathrm{d}t - \frac{B}{2}y\,\mathrm{d}x + \frac{B}{2}x\,\mathrm{d}y,$$

$$A_\mu(r,t,\vec{x}) \underset{r\to\infty}{\sim} A_\mu^{(0)}(t,\vec{x}) + A_\mu^{(1)}(t,\vec{x})\, r^{1-D}$$

BOUNDARY

Assuming standard quantization

Dirichlet boundary conditions

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\Phi \exp\left[iS_0(\Phi) + i\int d^3x \left(A_\mu J^\mu(\Phi) + \frac{1}{2}g_{\mu\nu}T^{\mu\nu}(\Phi)\right)\right]$$

What do we get on the other side?

A QFT with a conserved U(1) current in presence of an external gauge field

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\mu} J_\mu \\ \partial_\mu J^\mu &= 0\,. \end{aligned}$$

Diffusion of conserved charge

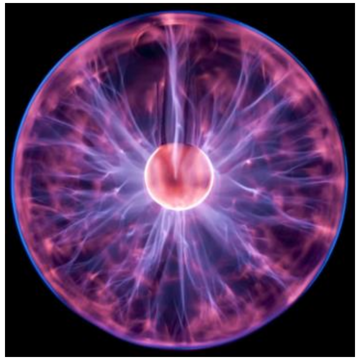
Hydrodynamics of the gauge sector

$$\omega = -iDk^2$$

Is this everything we can do?

Unsatisfactory in many physical situations → **DYNAMICAL GAUGE FIELDS**
COULOMB INTERACTIONS

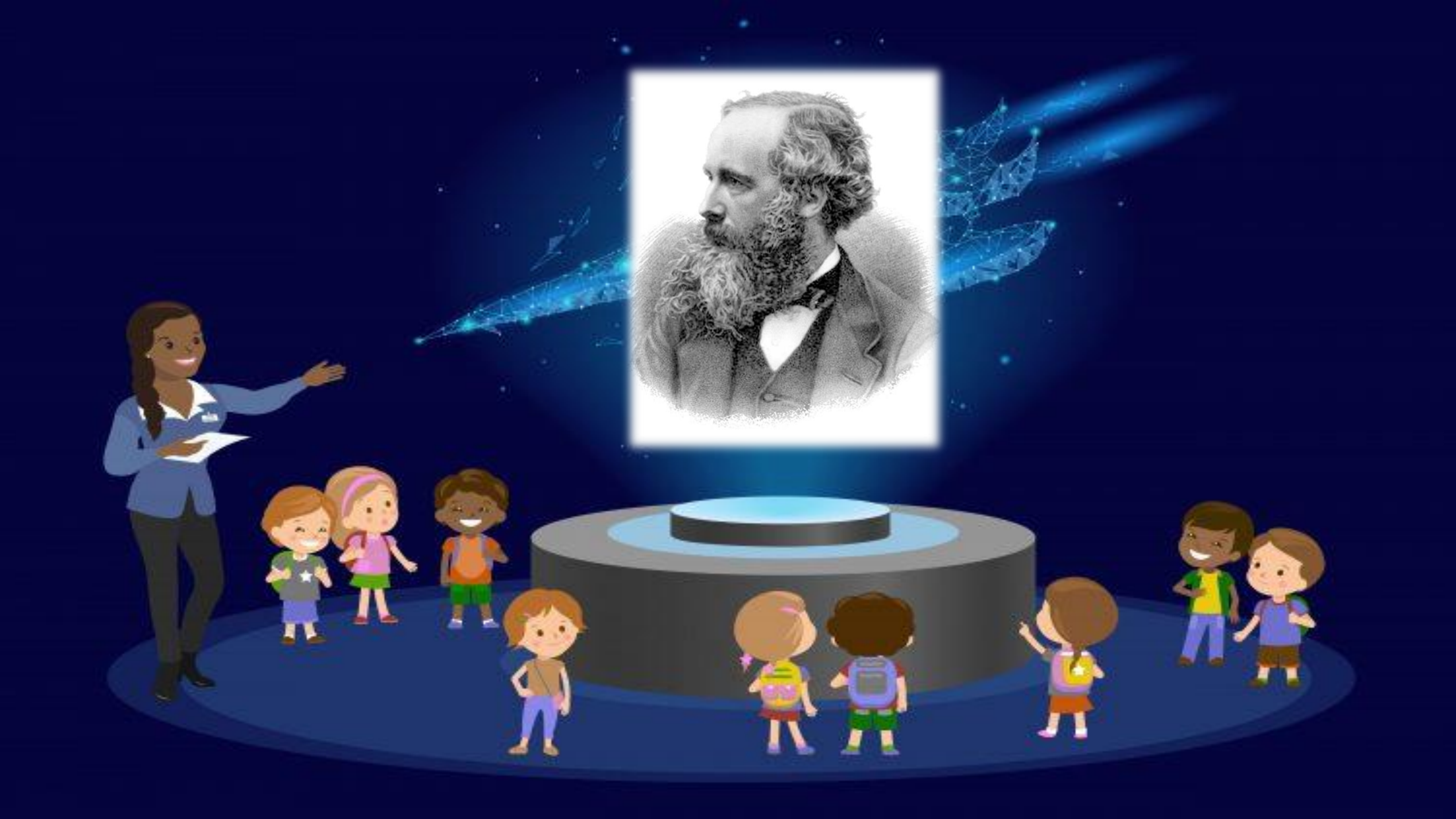
electromagnetism



plasmas

plasmons

superconductors



Fancy Maxwell's equations

Generalized global symmetries and dissipative magnetohydrodynamics

Sašo Grozdanov, Diego M. Hofman, Nabil Iqbal

Electromagnetism without gauge symmetries

The true global symmetry of $U(1)$ electrodynamics is actually something different. Consider the following antisymmetric tensor

$$J^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}. \tag{1.2}$$

It is immediately clear from the Bianchi identity (i.e. the absence of magnetic monopoles) that $\nabla_\mu J^{\mu\nu} = 0$. This is not related to the conservation of electric charge, but rather states that magnetic field lines cannot end.

$$\nabla_\mu J^{\mu\nu} = 0.$$

$$\nabla_\mu T^{\mu\nu} = H^{\nu}{}_{\rho\sigma} J^{\rho\sigma},$$

Only global symmetries

Better for hydrodynamics, etc…

Magnetohydrodynamics from holography

Generalised global symmetries in holography: magnetohydrodynamic waves in a strongly interacting plasma

Sašo Grozdanov, Napat Poovuttikul

$$S = \frac{N_c^2}{8\pi^2}\left[\int d^5x\sqrt{-G}\left(R + \frac{12}{L^2} - \frac{1}{3e_H^2}H_{abc}H^{abc}\right)\right.$$

$$H = dB, \quad B \to B + d\lambda$$

$$B_{\mu\nu} \longrightarrow J^{\mu\nu}$$

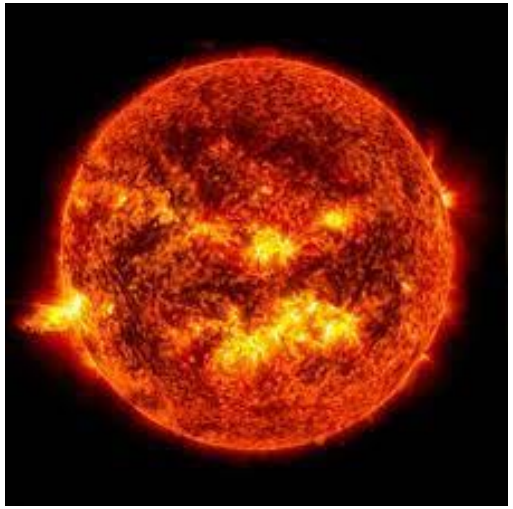
Short summary:
the standard thing but with
a higher-form ☺

What do they get?

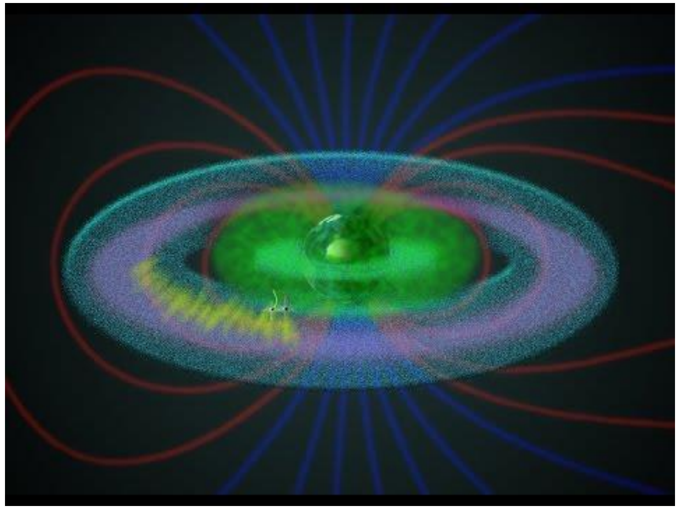
Relativistic magnetohydrodynamics

Juan Hernandez, Pavel Kovtun

A QFT with a dynamical gauge field (dynamical E, B)



Alfven waves



Magnetosonic waves

Our questions/results

1. Where is the trick ?

2. Do we really need this fanciness ?

3. Is there another solution ?

4. Magnetohydrodynamics from holography (no dress code)

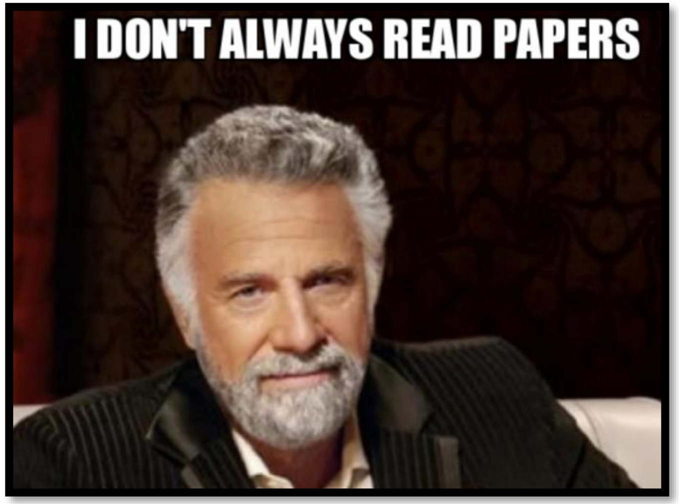
The story of today …

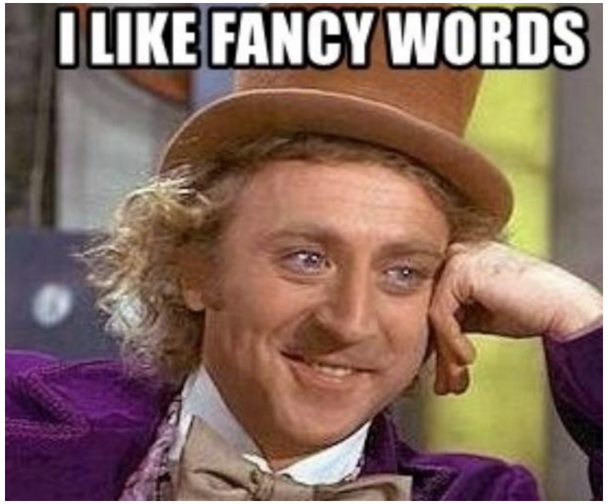
Disclaimers

- The trick has been already revealed (we did not discover it ☹)
- The other solution is known since more than 10 years (☺)

But …

[provocative]





Field theory break I

Let us start by considering the generating functional $Z[g_{\mu\nu}, A_\mu]$:

$$Z[g_{\mu\nu}, A_\mu] = \int \mathcal{D}\Phi \exp\left[iS_0(\Phi) + i\int d^3x \left(A_\mu J^\mu(\Phi) + \frac{1}{2} g_{\mu\nu} T^{\mu\nu}(\Phi)\right)\right],$$

$$S[g_{\mu\nu}, A_\mu] := -i \ln Z[g_{\mu\nu}, A_\mu]$$

$$\langle J^\mu \rangle, \quad \langle J^\mu J^\nu \rangle$$

$$\langle T^{\mu\nu} \rangle, \quad \langle T^{\mu\nu} T^{\sigma\lambda} \rangle$$

$$\langle J^\mu T^{\sigma\lambda} \rangle, \quad \langle \ldots\ldots \rangle$$

Field theory break II

$$S_{\text{tot}} = S_{\text{m}}[g_{\mu\nu}, A_\mu] + \int \mathrm{d}^3x\sqrt{-g}\left[\underline{-\frac{1}{4\lambda}F^2} + \underline{A_\mu\, J^\mu_{\text{ext}}}\right].$$

Maxwell kinetic term + Legendre transform (coupling to external current)

$$\delta_{A_\mu} S_{\text{tot}} = \int \mathrm{d}^3x\,\sqrt{-g}\,\boxed{\left[J^\mu_{\text{m}} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\text{ext}}\right]}\delta A_\mu\,.$$

MAXWELL EQUATIONS

Field theory break III

$$\nabla_\mu \left(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm EM}\right) \;=\; F^{\lambda\nu} J_{{\rm ext}\lambda}\,, \qquad \nabla_\mu J^\mu_{\rm m} \;=\; 0\,,$$

$$J^\mu_{\rm m} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\rm ext} = 0\,, \qquad \epsilon^{\alpha\beta\gamma}\,\nabla_\alpha F_{\beta\gamma} \;=\; 0\,,$$

Standard electromagnetism

$$T^{\mu\nu}_{\mathbf{EM}} = \frac{1}{\lambda}F^{\mu\sigma}F^{\nu}{}_{\sigma} - \frac{1}{4\lambda}F^2 g^{\mu\nu}$$

Into holography

Holographic Jump!

Song by The Movement

The Movement - Holographic Jump! - YouTube

https://www.youtube.com › watch

Let us forget about higher form symmetries and do our homework as from textbook electrodynamics

[more details later]

Old but gold

[Witten, Marolf-Ross, ...]

Boundary asymptotics : $$A_\mu(r,t,\vec{x}) \underset{r\to\infty}{\sim} A_\mu^{(0)}(t,\vec{x}) + A_\mu^{(1)}(t,\vec{x})\, r^{1-D},$$

$$\boxed{\alpha\, A_\mu^{(0)}(t,\vec{x}) + \beta\, A_\mu^{(1)}(t,\vec{x}) = \text{fixed}\,,}$$

MIXED BOUNDARY CONDITIONS

$$\beta = 0$$

standard

$$\longrightarrow \mathcal{L}_{CFT} + \int d^d x\, A_\mu^{(0)} J^\mu$$

First application

Building a Holographic ~~Superconductor~~

Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz
Phys. Rev. Lett. **101**, 031601 – Published 14 July 2008

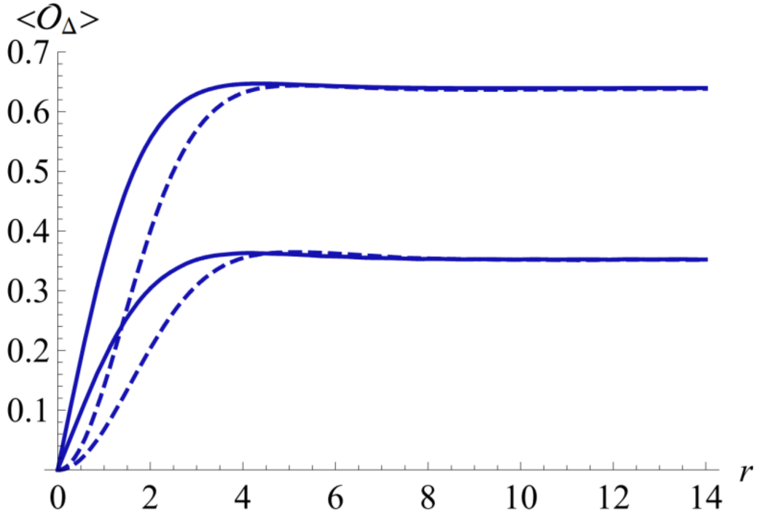
Making a real holographic superconductor (2009)

[Many authors …]

Example.

Holographic Superconductor Vortices

Marc Montull, Alex Pomarol, and Pedro J. Silva
Phys. Rev. Lett. **103**, 091601 – Published 26 August 2009



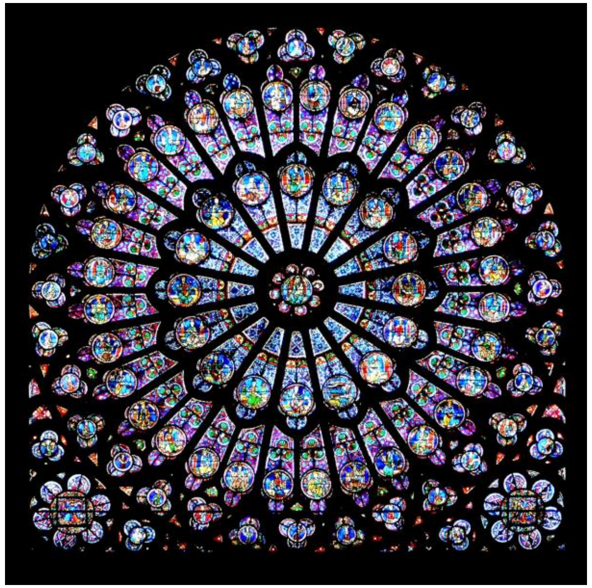
Recent application

High Energy Physics - Theory

[Submitted on 15 Dec 2017 (v1), last revised 26 Nov 2018 (this version, v4)]

Holographic Plasmons

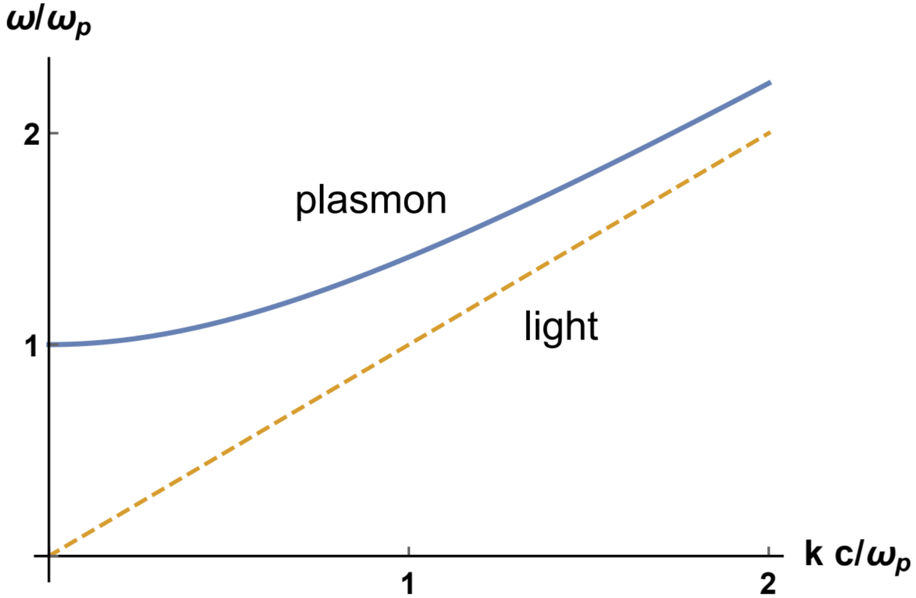
Ulf Gran, Marcus Tornsö, Tobias Zingg



Maxwell equation
Continuity equation
(Drude model)

$$\epsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$

$$\boxed{\omega^2 = \omega_p^2 + c^2 k^2}$$



See also

Holographic Meissner Effect

Makoto Natsuume, Takashi Okamura

The holographic superconductor is the holographic dual of superconductivity, but there is no Meissner effect in the standard holographic superconductor. This is because the boundary Maxwell field is added as an external source and is not dynamical. We show the Meissner effect analytically by imposing the semiclassical Maxwell equation on the AdS boundary. Unlike in the Ginzburg-Landau (GL) theory, the extreme Type I limit cannot be reached even in the $e \to \infty$ limit where e is the $U(1)$ coupling of the boundary Maxwell field. This is due to the bound current which is present even in the pure bulk Maxwell theory. In the bulk 5-dimensional case, the GL parameter and the dual GL theory are obtained analytically for the order parameter of scaling dimension 2.

$$e^2 \mathcal{J}_i = -\frac{1}{\lambda^2} \mathcal{A}_i \ ,$$

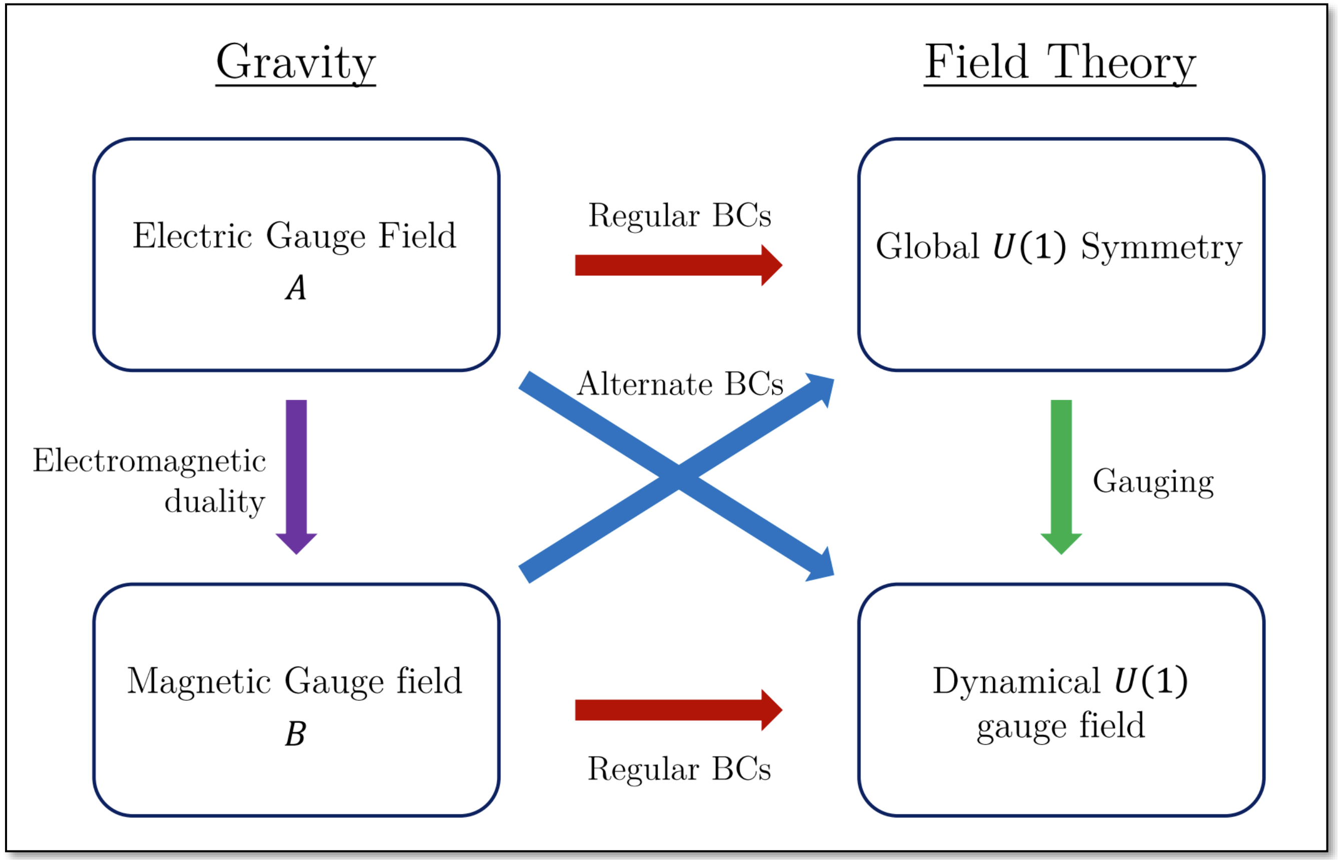
$$\partial_j \mathcal{F}^{ij} = e^2 \mathcal{J}^i \ .$$

Back to the higher-form trick

arXiv:2010.06594 [pdf, other] hep-th doi 10.1103/PhysRevD.103.026011

Generalized symmetries and 2-groups via electromagnetic duality in AdS/CFT

Authors: Oliver DeWolfe, Kenneth Higginbotham



Hodge dual in the bulk does not preserve the boundary conditions !!

It changes them from Dirichlet to mixed

Higher-forms are just a fancy way to implement mixed boundary conditions

On the shoulders of giants

$$\Pi^{\mu} - \frac{1}{\lambda}\partial_{\nu}F^{\mu\nu} + J^{\mu}_{\rm ext} = 0\,, \qquad \Pi^{\mu} = \frac{\delta S_{\text{on-shell}}}{\delta A_{\mu}} = -\sqrt{-g}\,F^{r\mu}\,,$$

Our mixed boundary conditions

$$\delta J^{x\,(L)}_{\rm ext} = Z^{(L)}_{A_1} + \frac{\lambda}{\omega^2 - k^2}\,Z^{(S)}_{A_1}\,,$$

$$\delta J^{y\,(L)}_{\rm ext} = (\omega^2 - k^2)\,Z^{(L)}_{A_2} + \lambda\,Z^{(S)}_{A_2}\,.$$

- Lambda parametrizes the EM coupling
- Notice the factors of (w^2-k^2)

  [back to this later]

Re-discovering the known

$$\epsilon = 2r_h^3 + \frac{\mu^2 r_h}{2} + \frac{B^2}{2r_h} + \frac{B^2}{2\lambda}\,, \qquad p = r_h^3 + \frac{\mu^2 r_h}{4} - \frac{3B^2}{4r_h} - \frac{B^2}{2\lambda}$$

$$T_{xx} = \left(p + \frac{B^2}{\mu_m}\right) \neq p$$

Thermodynamic and mechanical pressure are not equal

$$T^{\mu}{}_{\mu} = \frac{1}{4\lambda} F^2$$

Maxwell theory in 2+1 is scale invariant but not conformal invariant !!

What Maxwell Theory in D<>4 teaches us about scale and conformal invariance

Sheer El-Showk, Yu Nakayama, Slava Rychkov

Magnetohydrodynamics

Step 1: EOMs

$$\nabla_\mu \left(T^{\mu\nu}_{\rm m} + T^{\mu\nu}_{\rm EM}\right) = F^{\lambda\nu} J_{{\rm ext}\lambda}\,, \qquad \nabla_\mu J^\mu_{\rm m} = 0\,,$$

$$J^\mu_{\rm m} - \frac{1}{\lambda}\nabla_\nu F^{\mu\nu} + J^\mu_{\rm ext} = 0\,, \qquad \epsilon^{\alpha\beta\gamma}\,\nabla_\alpha F_{\beta\gamma} = 0\,,$$

Step 2: constitutive relations

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\,\Delta^{\mu\nu} + \mathcal{H}^{\mu\gamma}\, F^\nu_{\ \gamma} + \Pi^{\mu\nu}\,,$$

$$J^\mu = \rho\, u^\mu - \nabla_\nu \mathcal{H}^{\mu\nu} + \nu^\mu\,, \quad \mathcal{H}^{\mu\nu} := \frac{1}{\lambda} F^{\mu\nu} - M^{\mu\nu}_{\mathbf{m}}\,,$$

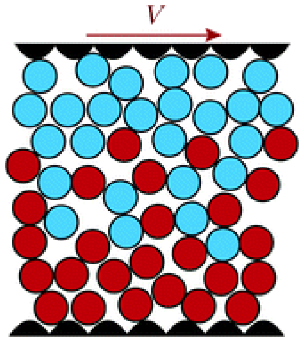
Step 3: dynamical matrix and QNMs

$$\mathcal{M}(\omega,k)\cdot s_A = 0\,, \quad \det \mathcal{M}(\omega,k) = 0\,.$$

$$s_A = \{\delta T, \delta u^{i=x,y}, \delta E^{i=x,y}, \delta B\}$$

Zero density and zero B

$$\omega = \pm v_s\, k - i\frac{\Gamma_s}{2}\,k^2\,, \qquad \omega = -i\frac{\eta}{\epsilon + p}\,k^2$$



where $v_s^2 = \partial p/\partial\epsilon$ and $\Gamma_s = \eta/(\epsilon + p)$.

The stress tensor sector is "trivial" since decouples

Zero density and zero B

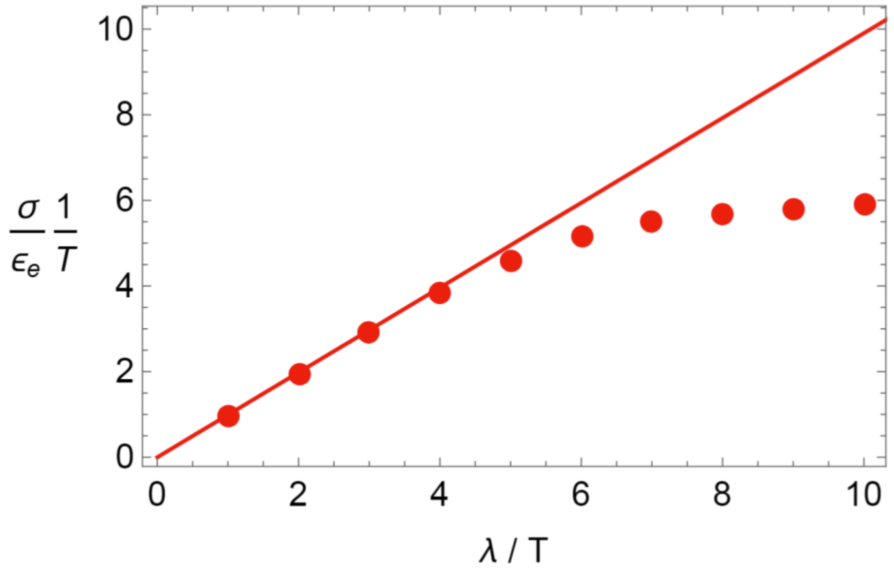
$$\omega\left(\omega + i\,\frac{\sigma}{\epsilon_{\rm e}}\right) = \frac{k^2}{\epsilon_{\rm e}\,\mu_{\rm m}}$$

"EM waves"

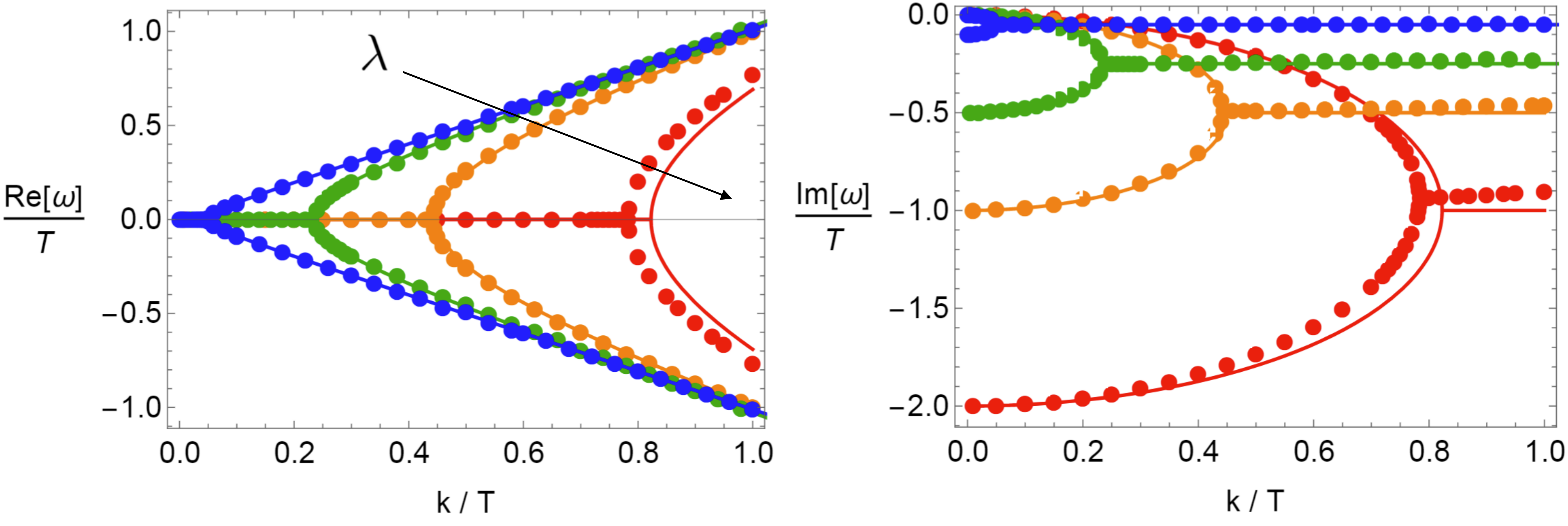
$$\omega = -i\,\frac{\sigma}{\epsilon_{\rm e}} - i\,\frac{\sigma}{\partial\rho/\partial\mu}\,k^2$$

Damped charge diffusion

We see the effects of polarization and screening (dynamical EM in matter)



Screened EM waves

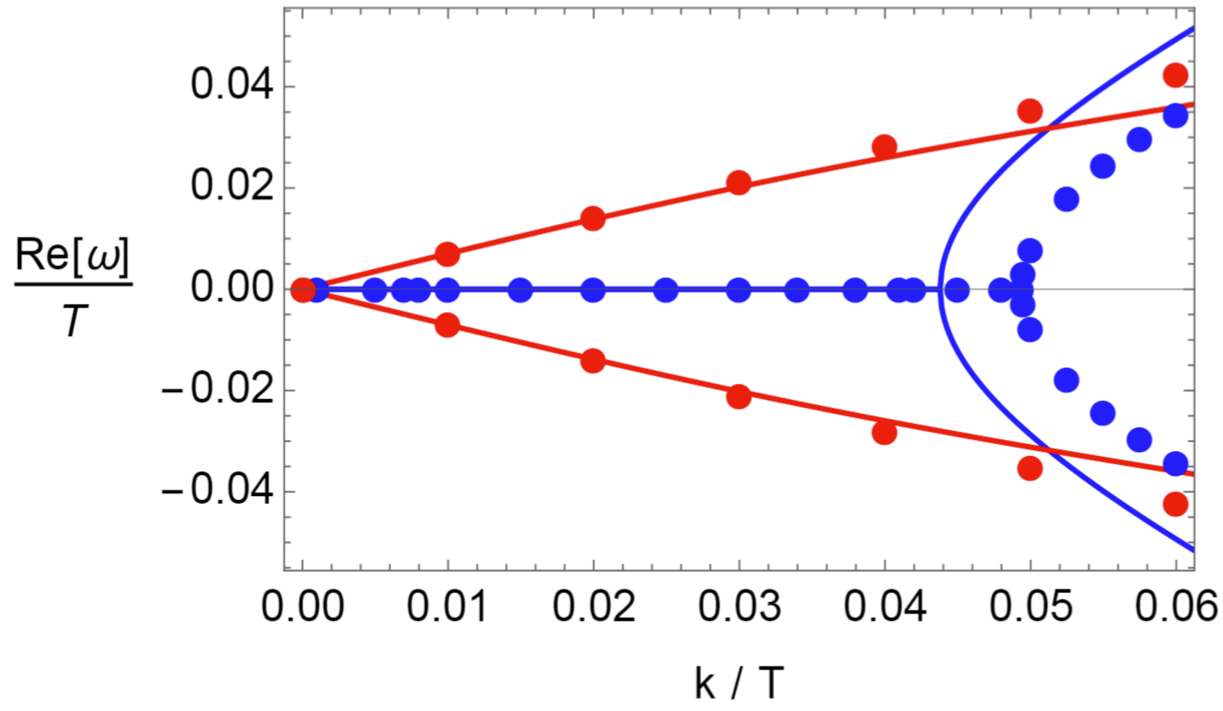


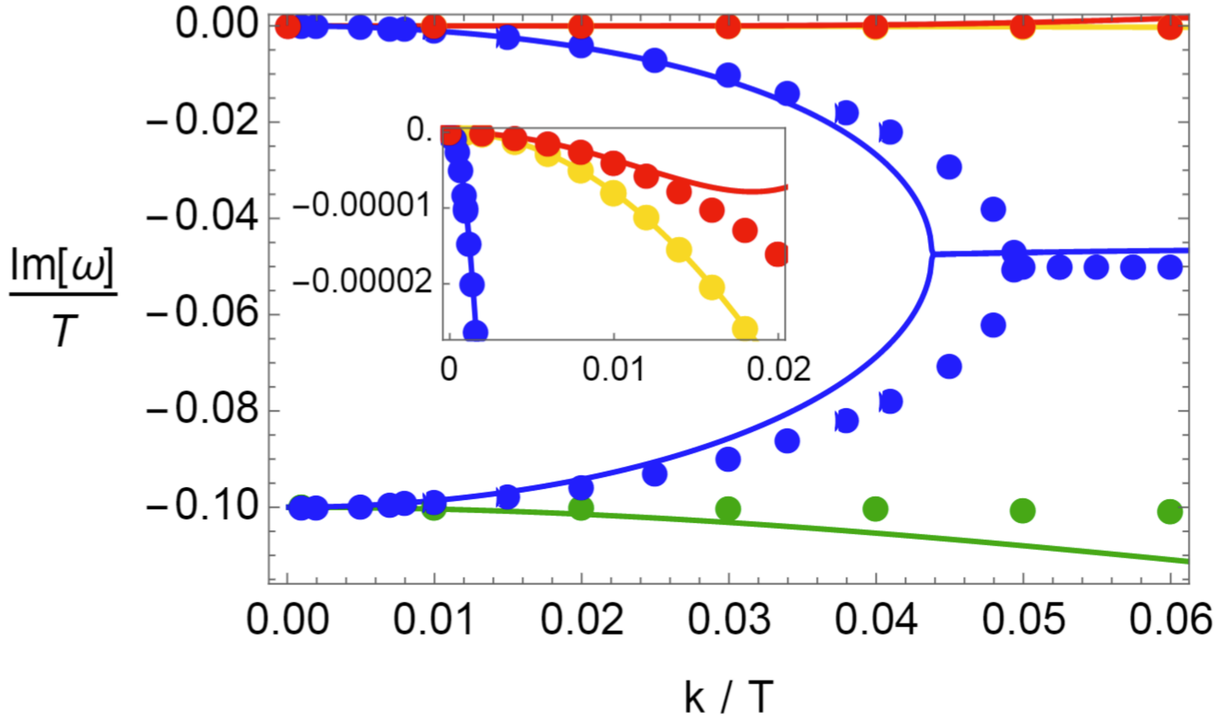
The photon is screened (skin effect).
Just solve it for real w and complex k
→ standard textbooks

Numerical data (QNMs)

1st order hydro

Zero density and zero B





sound waves

EM waves

shear diffusion

damped charge diffusion

Zero density and finite B

All dispersion relations remain of the same type but the coefficients are strongly modified by B^2

$$\omega = \pm v_{\mathrm{ms}}\, k - i\frac{\Gamma_{\mathrm{ms}}}{2}\, k^2$$

Magnetosonic waves

[sound mode carries magnetic flux now]

Zero density and finite B

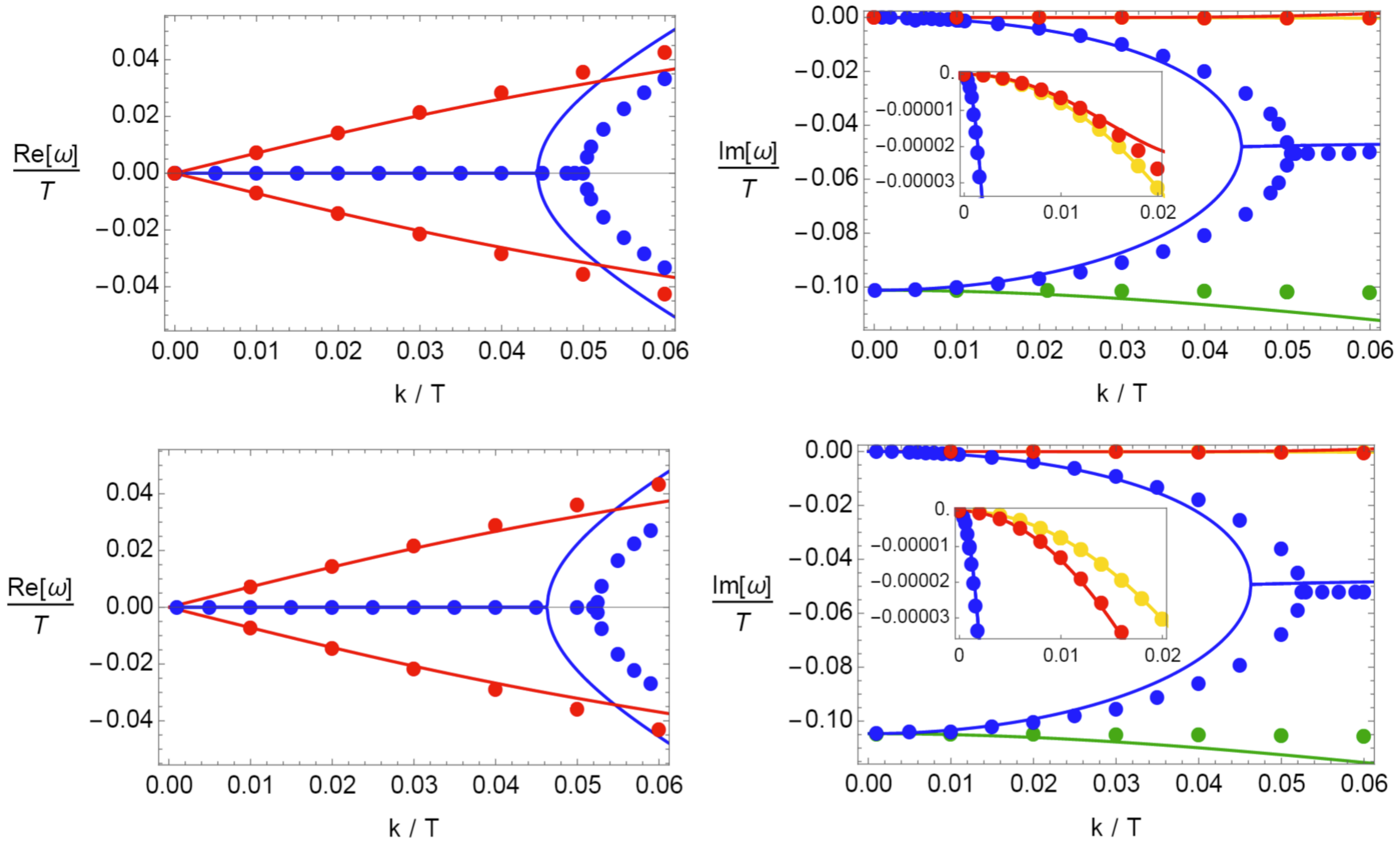


Figure 2. Dispersion relations of the lowest QNMs at zero density ($\mu/T = 0$) and $B/T^2 \neq 0$. Top and bottom panels are respectively for $B/T^2 = 0.5, 1$.

Finite density and finite B

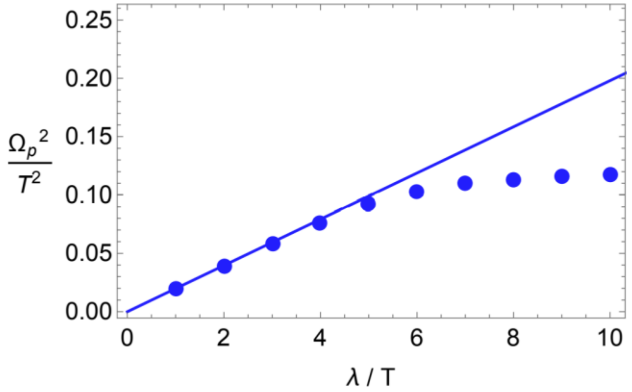
$$\omega = -i\,\frac{\left(\frac{\partial\rho}{\partial\mu}\right)_{T,B}(\epsilon+p)^2\;\sigma}{T\left[\left(\frac{\partial\epsilon}{\partial T}\right)_{\mu,B}\left(\frac{\partial\rho}{\partial\mu}\right)_{T,B}-\left(\frac{\partial\epsilon}{\partial\mu}\right)_{T,B}\left(\frac{\partial\rho}{\partial T}\right)_{\mu,B}\right](\rho^2+B^2\sigma)}\,k^2\,,\qquad \omega=-i\frac{\eta}{\mu_{\rm m}\,\rho^2}\,k^4\,.$$

- Longitudinal diffusive mode
- Shear diffusion becomes sub-diffusive
- EM waves and sound modes couple together (4 non-hydro modes)

$$\left[\omega\left(\omega+i\,\frac{\sigma}{\epsilon_{\rm e}}\right)-\Omega_p^2\right]^2=\frac{B^2}{\epsilon_{\rm e}^2\,\mu_{\rm m}^2(\epsilon+p)^2}\left[\rho^2-\mu_{\rm m}^2\sigma^2(\rho^2-B^2)+\omega^2\left(2(\epsilon+p)(\sigma-i\epsilon_{\rm e}\omega)\right)\right]\,,$$

where Ω_p is the plasma frequency

$$\Omega_p^2\;:=\;\frac{\rho^2}{\epsilon_{\rm e}(\epsilon+p)}\,.$$



Finite density and zero B

$$\left[\omega\left(\omega + i\,\frac{\sigma}{\epsilon_{\mathrm{e}}}\right) - \Omega_p^2\right]^2 = 0$$

$$\left(\sigma^2/\epsilon_{\mathrm{e}}^2 \gg 4\,\Omega_p^2\ ;\ \text{small density}\right):\quad \omega = -\,i\,\frac{\epsilon_{\mathrm{e}}}{\sigma}\,\Omega_p^2\,,\qquad \omega = -i\,\frac{\sigma}{\epsilon_{\mathrm{e}}} + i\,\frac{\epsilon_{\mathrm{e}}}{\sigma}\,\Omega_p^2\,,$$

$$\left(\sigma^2/\epsilon_{\mathrm{e}}^2 \ll 4\,\Omega_p^2\ ;\ \text{large density}\right):\quad \omega = \pm\,\Omega_p - i\,\frac{\sigma}{2\epsilon_{\mathrm{e}}}\,.$$

Small density → overdamped modes
Large density → underdamped modes

Finally, setting all the dissipative coefficients (e.g., $\sigma = 0$) to zero, one finds

sound waves → $\omega^2 = \Omega_p^2 + v_s^2\,k^2\,,\qquad \omega^2 = \Omega_p^2 + \dfrac{k^2}{\epsilon_{\mathrm{e}}\,\mu_{\mathrm{m}}}\,.$ ← EM waves

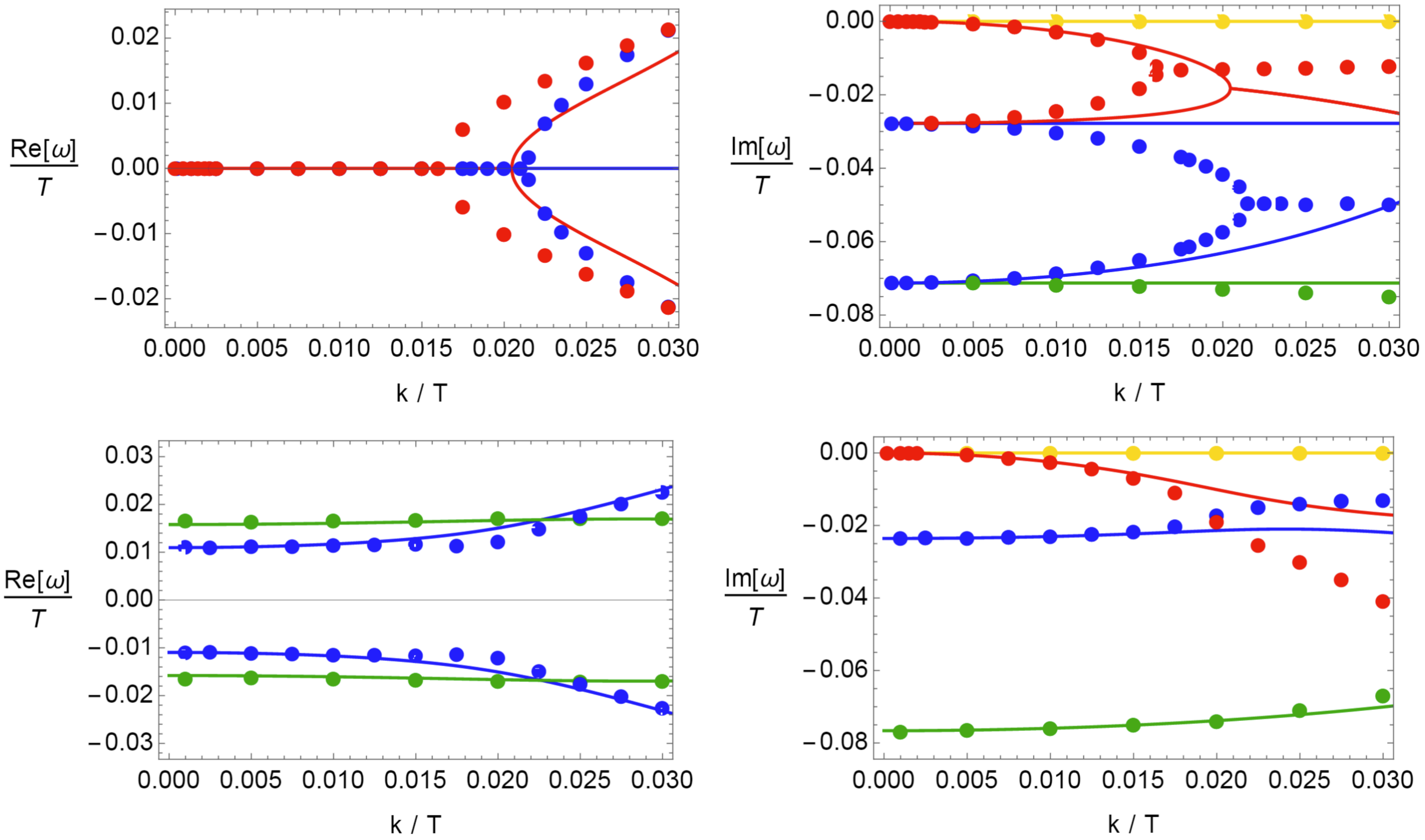


Figure 3. Dispersion relations of the lowest QNMs at finite density ($\mu/T = 0.5$). Top and bottom panels refer respectively to $B/T^2 = 0, 0.5$.

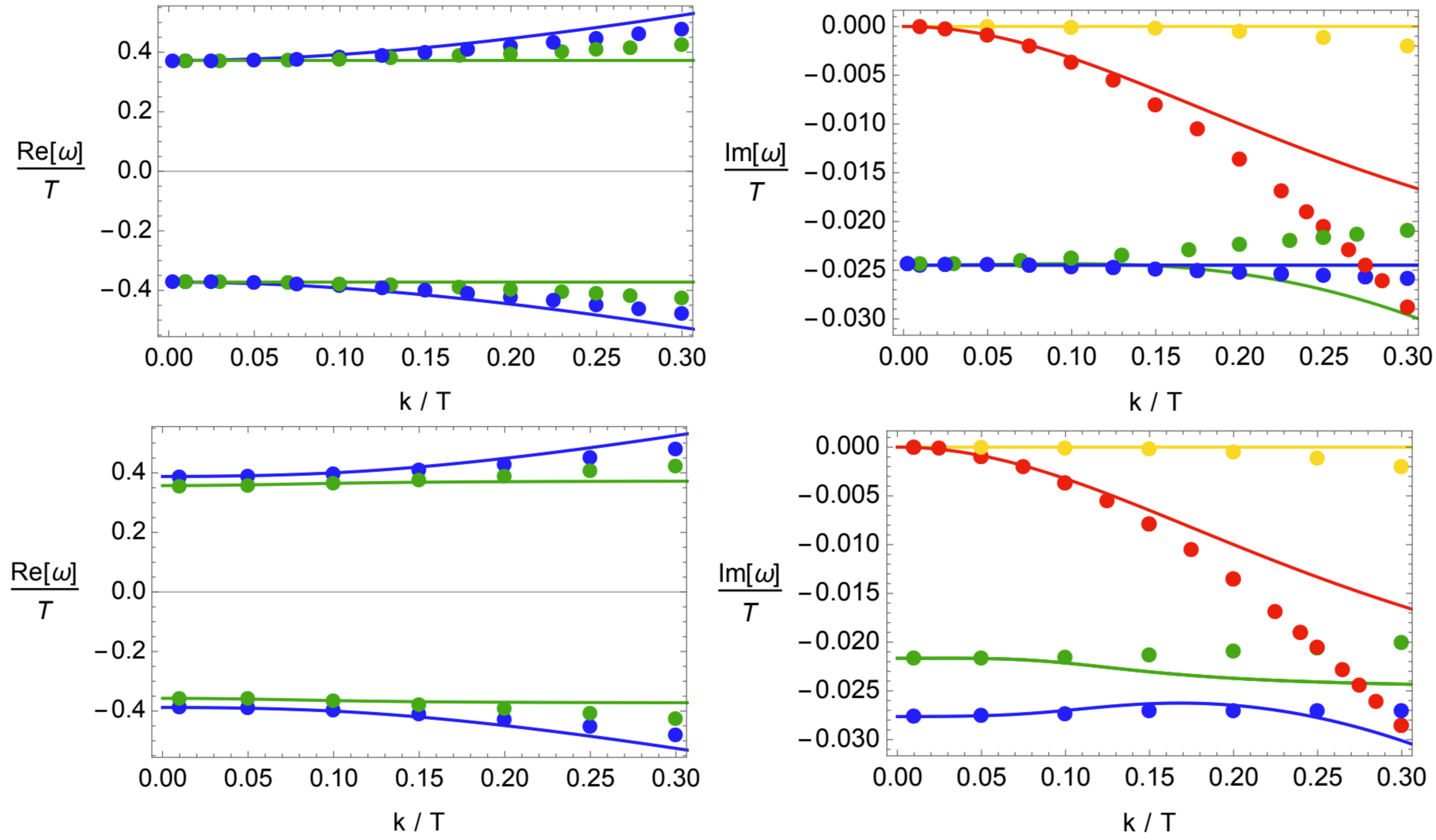


Figure 4. Lowest QNMs at finite density ($\mu/T = 5$). Top and bottom panels refer respectively to $B/T^2 = 0, 0.5$.

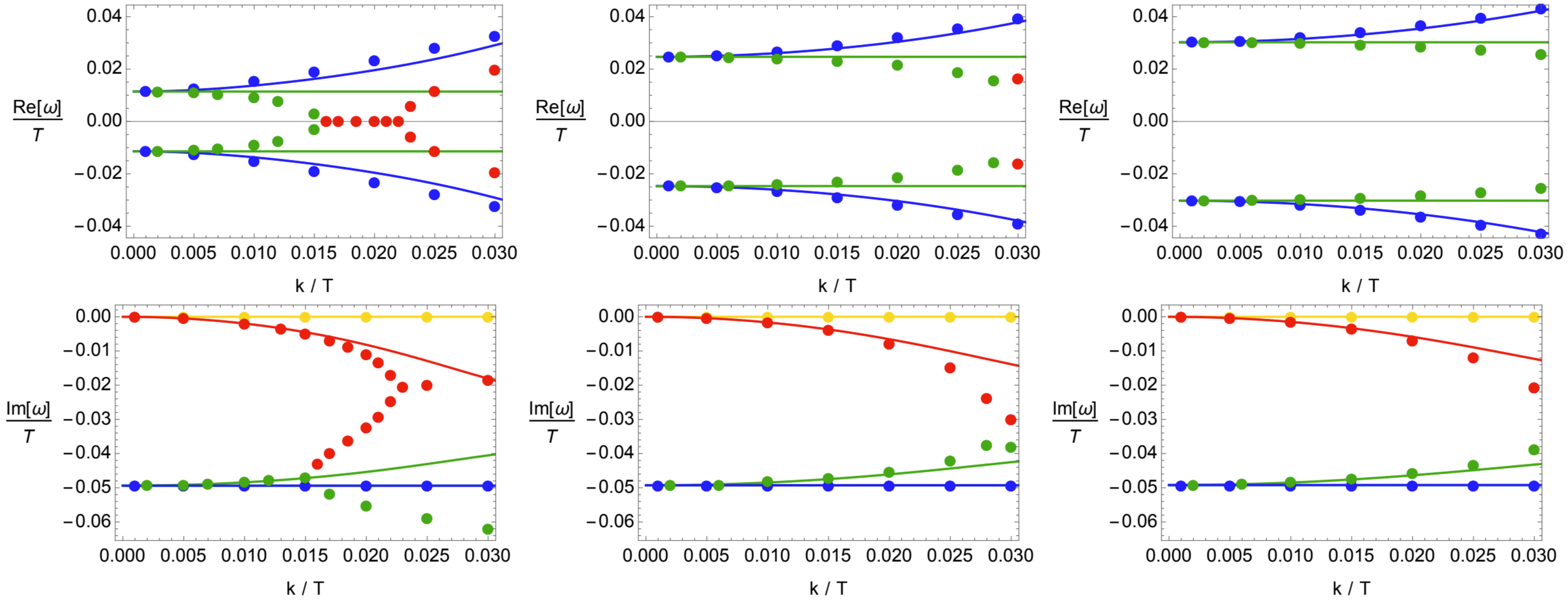


Figure 6. Dispersion relation of the lowest QNMs. Left, center and right panels correspond respectively to $\mu/T = 0.57, 0.62, 0.65$.

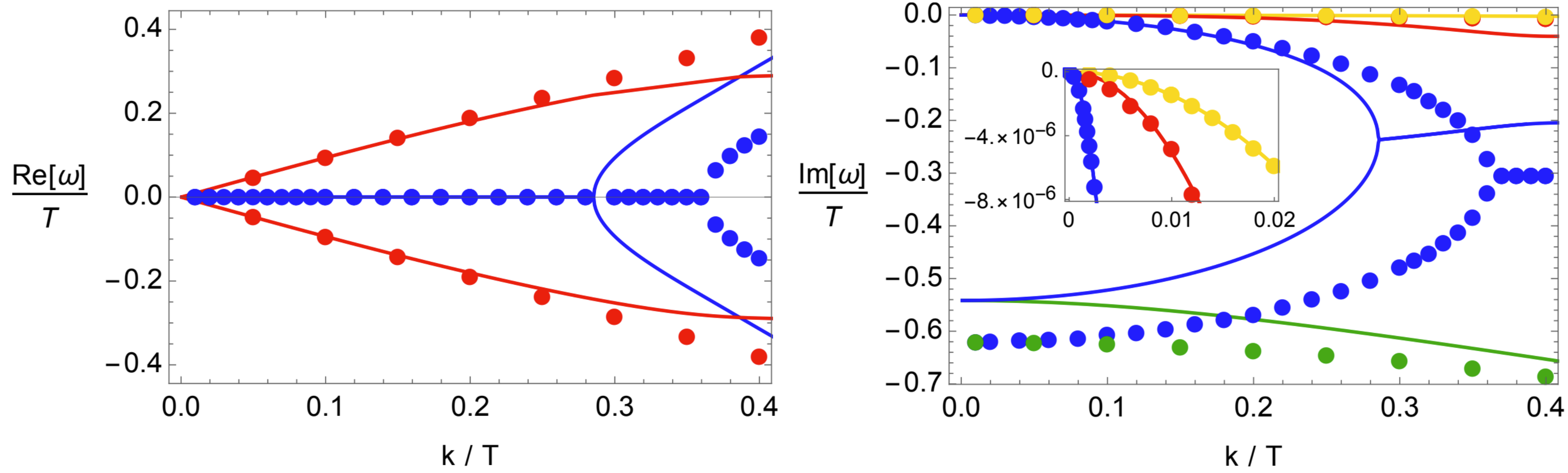
	External gauge fields	**Dynamical gauge fields**
Gappless modes	Energy diffusion mode (2.43), Subdiffusive mode (2.43),	Magnetosonic waves (2.31), Shear diffusion mode (2.31), Magnetic diffusion mode (2.33),
Gapped modes	Cyclotron mode (2.44).	Damped diffusion mode (2.33), Damped charge diffusion mode (2.34).

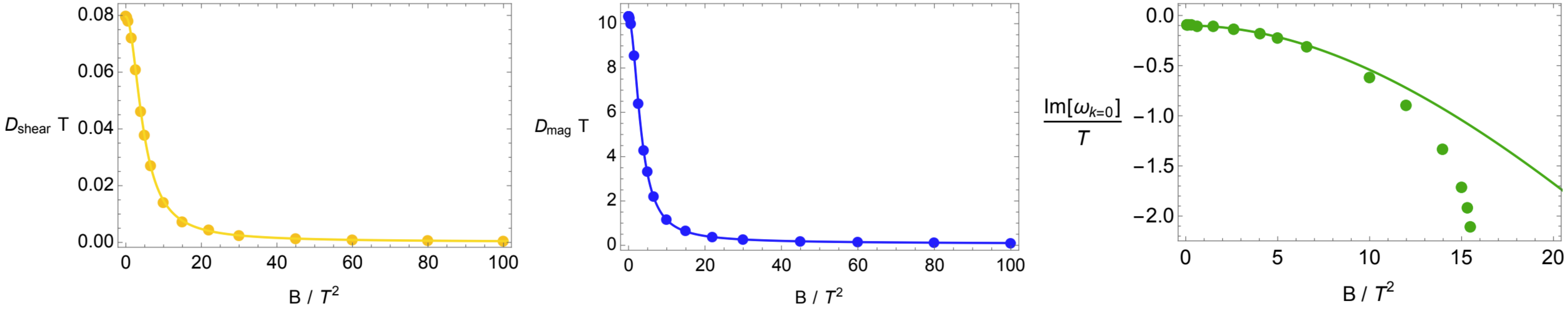
Table 1. The low energy modes at zero density and finite magnetic field.

	External gauge fields	**Dynamical gauge fields**
Gappless modes	Diffusion mode (2.45), Subdiffusive mode (2.45),	Diffusion mode (2.35), Subdiffusive mode (2.35),
Gapped modes	Cyclotron mode (2.46).	Gapped plasma modes (2.38).

Table 2. The low energy modes at finite density and finite magnetic field.

Large B limit





Large B limit

Magnetohydrodynamics still works well

+ interesting features

There seems to be an emergent gapless mode with dispersion w=k !

An emergent photon???

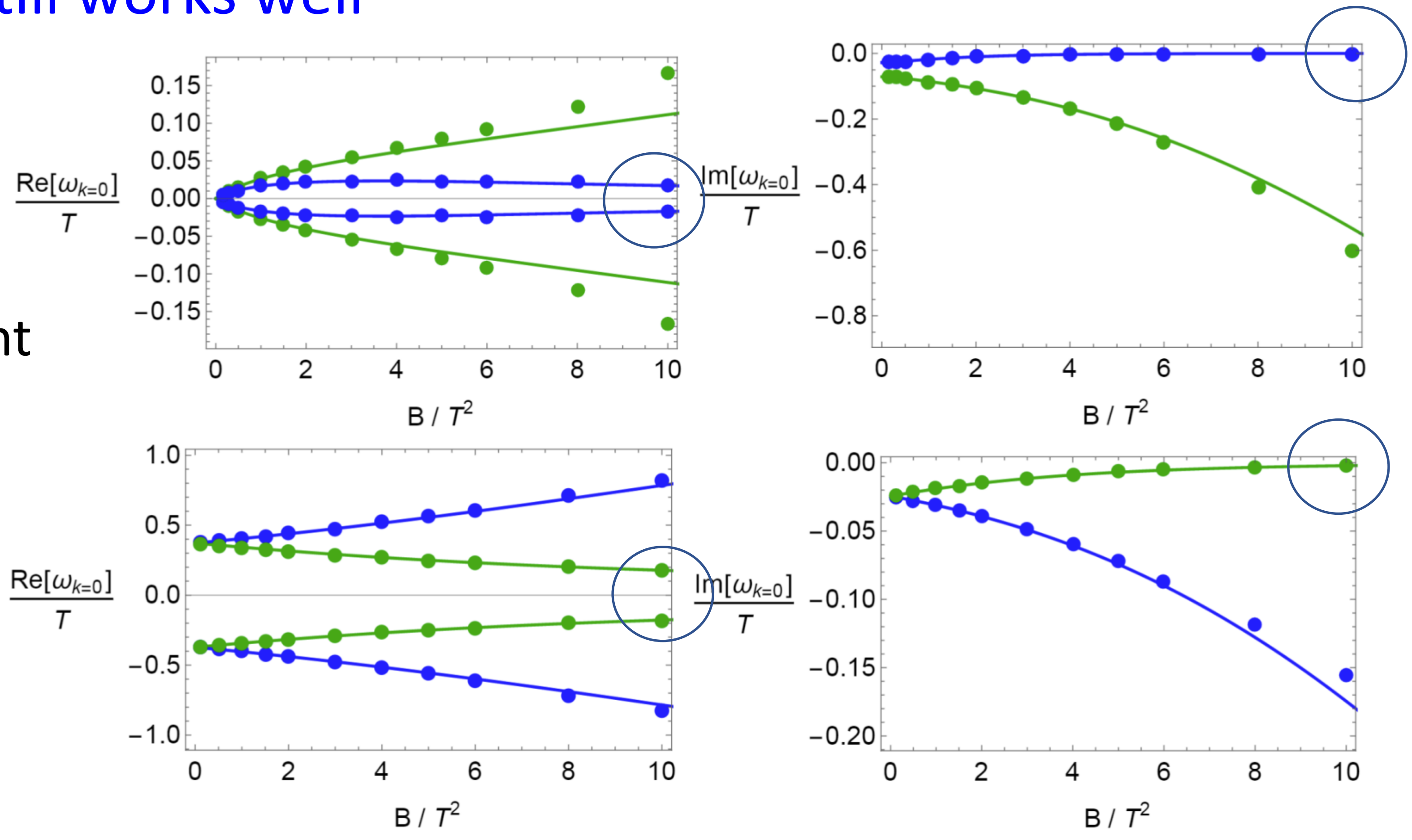


Figure 13. The real and imaginary part of the first non-hydrodynamic modes at density $\mu/T =$ $(0.5, 5)$ (upper panels, lower panels).

Unscreened photon

$$\lambda \to 0 \; :$$

$$Z^{(L)}_{A_1} = 0$$

$$(\omega^2 - k^2) Z^{(L)}_{A_2} = 0$$

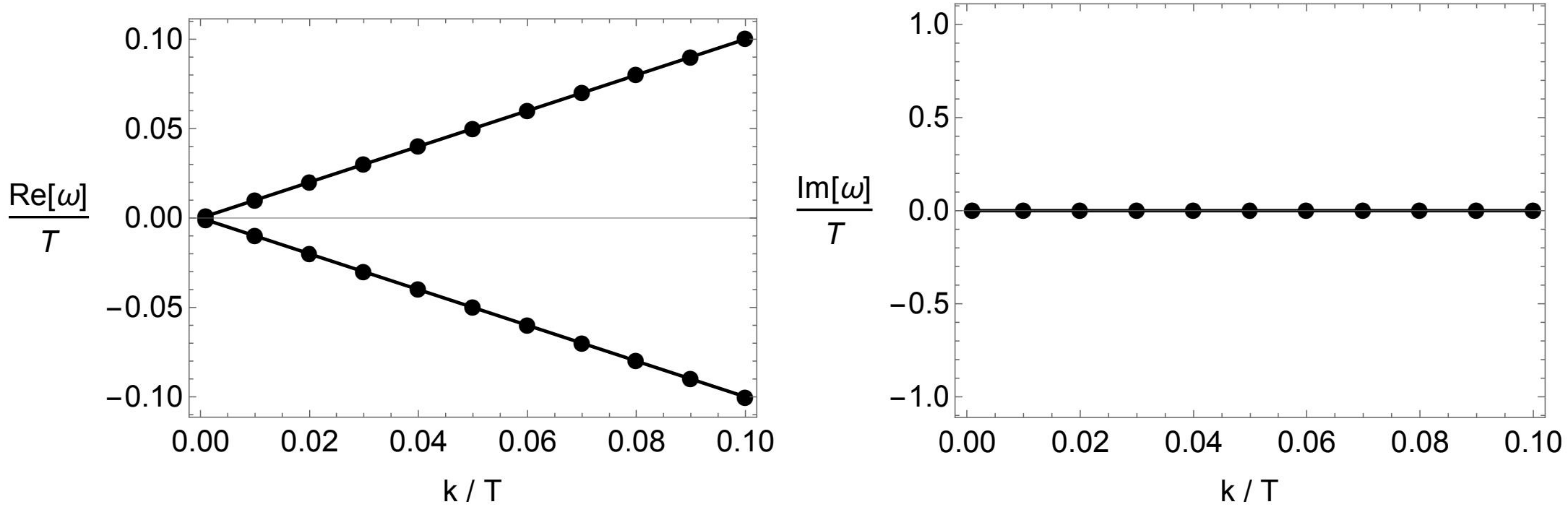


Figure 15. The emergent propagating photon at zero density, zero magnetic field and zero EM coupling $\lambda/T = 0$.

In this limit we are decoupling the photon from matter [no screening, no polarization] → emergent photon

This can be derived analytically [see paper]

Hydrodynamics breakdown

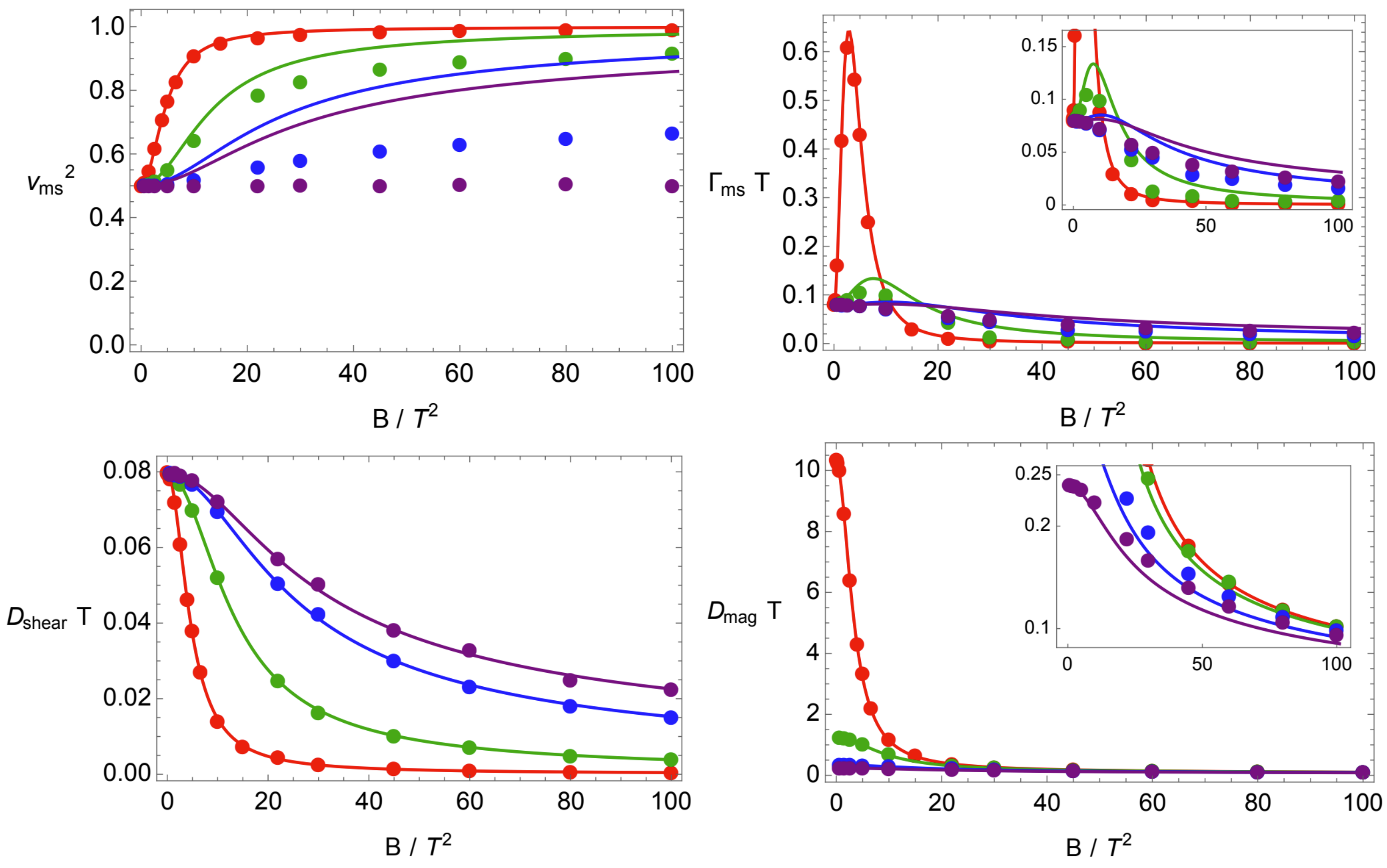


Figure 16. **Top:** Speed and attenuation constant of magnetosonic waves. **Bottom:** The diffusion constants of shear and magnetic diffusion. The colors correspond to $\lambda/T = (0.1, 1, 10, 1000)$ (red, green, blue, purple). The insets are a zoom in the low magnetic field regime.

We see a failure of the hydro description only at large magnetic field together with large EM coupling.

We suspect the problem is the EM coupling

Two more experiments

(1) "Alternative quantization" $$\Pi^\mu - \frac{1}{\lambda}\partial_\nu F^{\mu\nu} + J^\mu_{\text{ext}} = 0\,, \qquad \lambda \to \infty$$

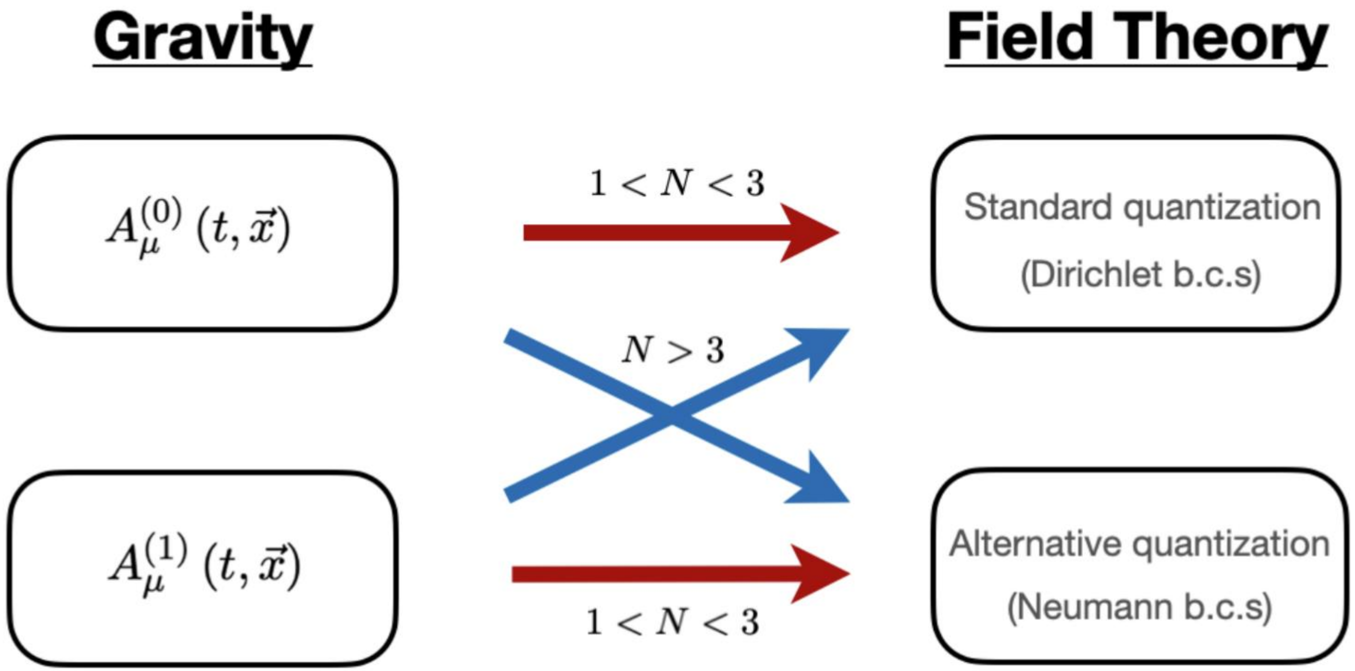
(2) "bulk experiment" $$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}\left(F^2\right)^{N/2}\right]$$

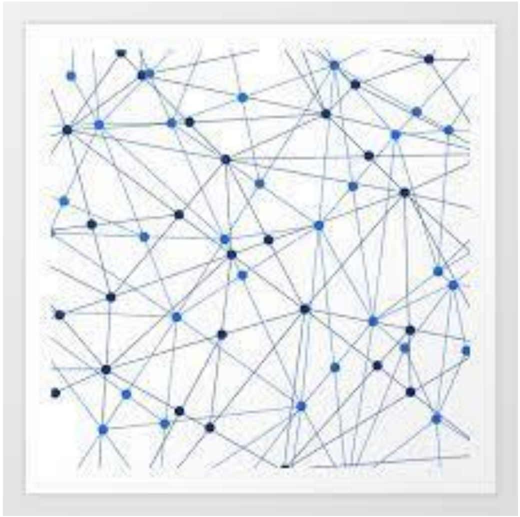
- Are (1) and (2) the same thing?
- What is the dual field theory picture?

The idea

[inspiration from holographic models with broken translations]

$$A_\mu\left(r,t,\vec{x}\right) \underset{r\to\infty}{\sim} A_\mu^{(0)}\left(t,\vec{x}\right) + A_\mu^{(1)}\left(t,\vec{x}\right)\, r^{1-D}$$





Hydrodynamic modes

1. Sound wave

$$\omega = \pm\sqrt{\frac{\partial \bar{P}}{\partial \epsilon}}\, k - i \frac{\eta}{2\left(\epsilon + \bar{P}\right)}\, k^2$$

2. Shear diffusion

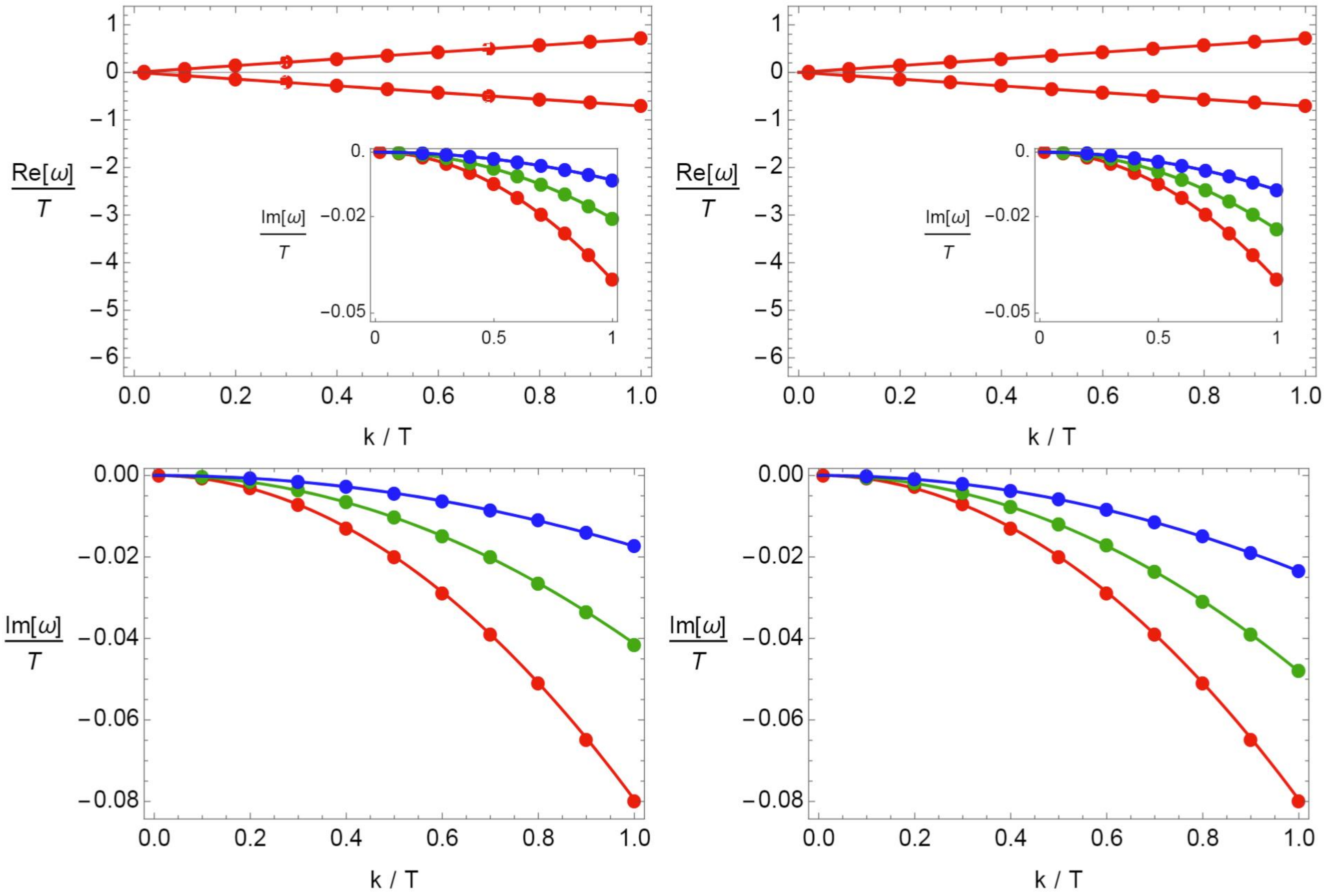
$$\omega = -i \frac{\eta}{\epsilon + \bar{P}}\, k^2$$

3. Magnetic diffusion

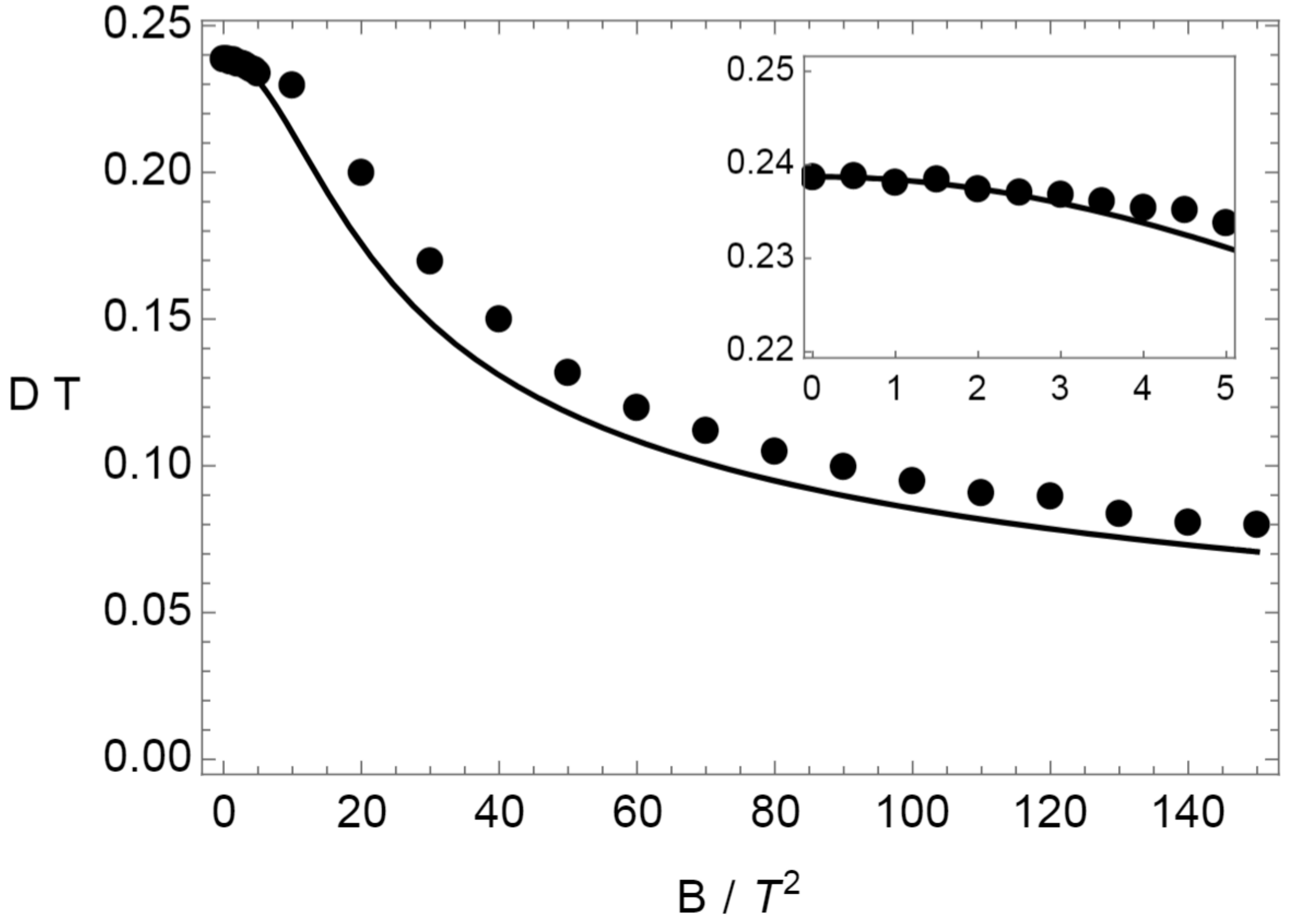
$$\omega = -i D_{\text{mag}}\, k^2$$

$$D_{\text{mag}} = -\frac{\chi_{BB}}{\sigma} - \frac{\chi_{BB}^2}{\sigma(\epsilon + \bar{P})} B^2 + \mathcal{O}(B)^3$$

Sound and shear diffusion



Magnetic diffusion

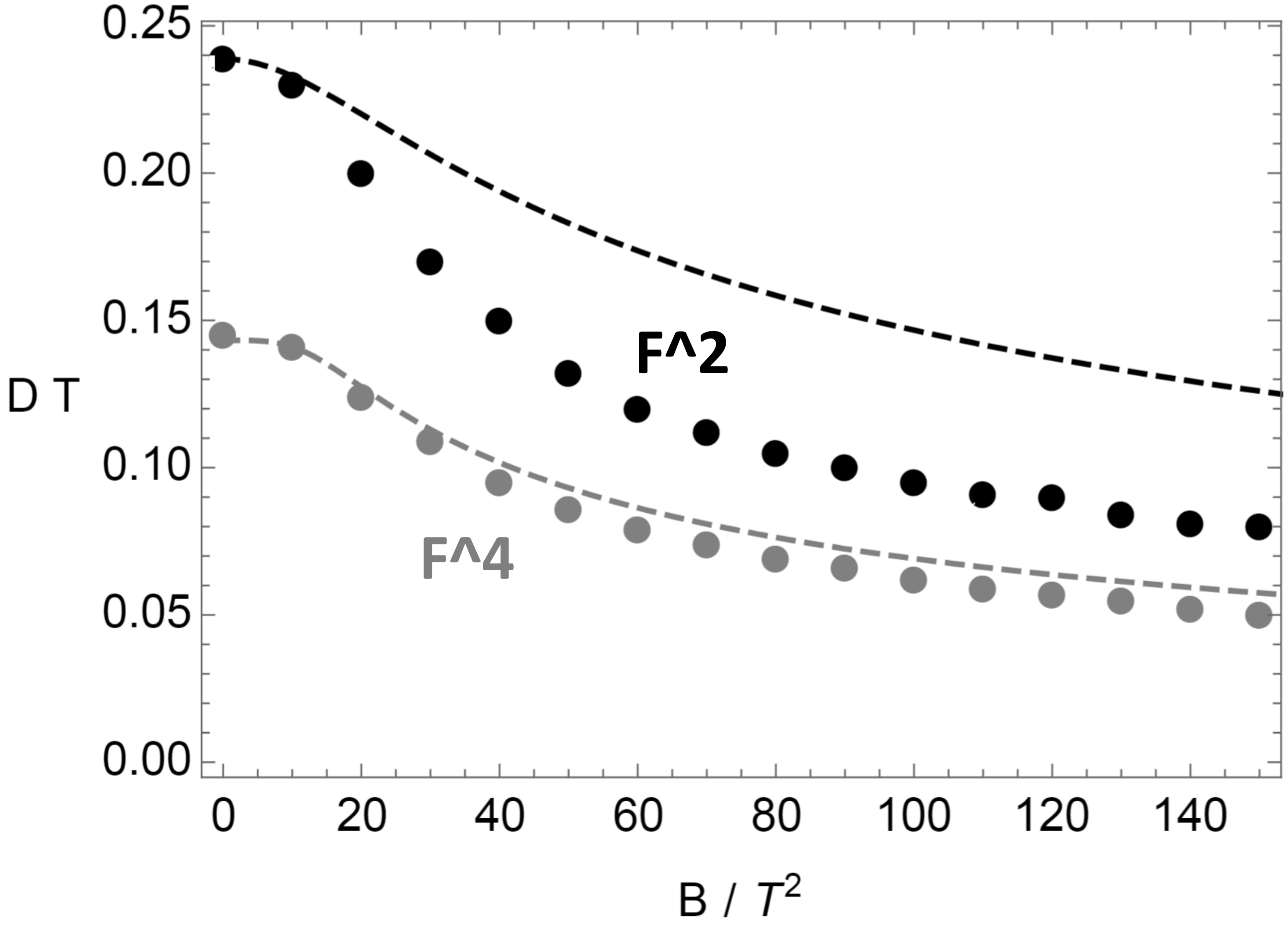


For F^2 with alternative b.c.s. the hydro formula works very well at small B

No surprise at large B it does not since we are both at large magnetic field and infinite EM coupling

For F^4 we do not know how to compute the transport coefficients

Magnetic diffusion from the bulk

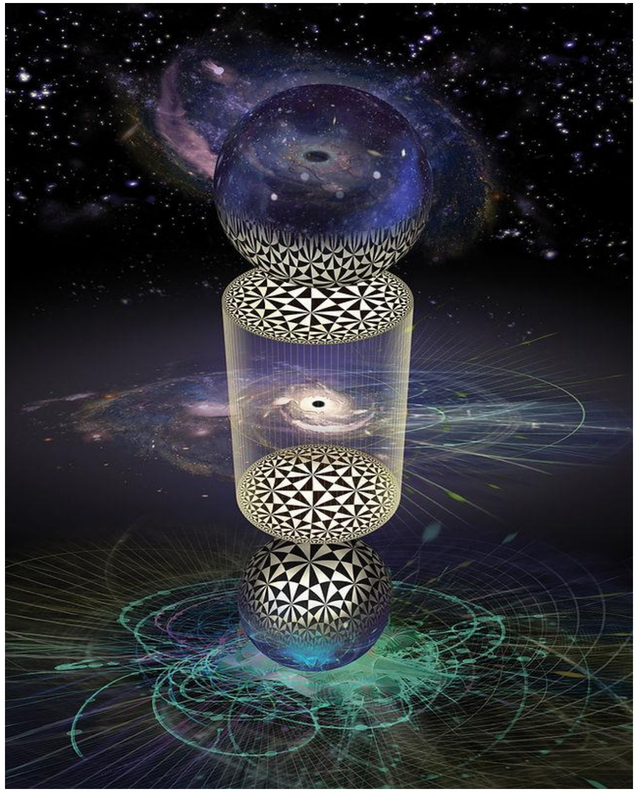


$$D_{B\to 0} = \frac{N-1}{2N-3}\, r_h^{-1} + \mathcal{O}\left(B\right)$$

Perturbative bulk computation works well for both models

Summary

Maxwell + mixed b.c.s. = **magnetohydrodynamics**



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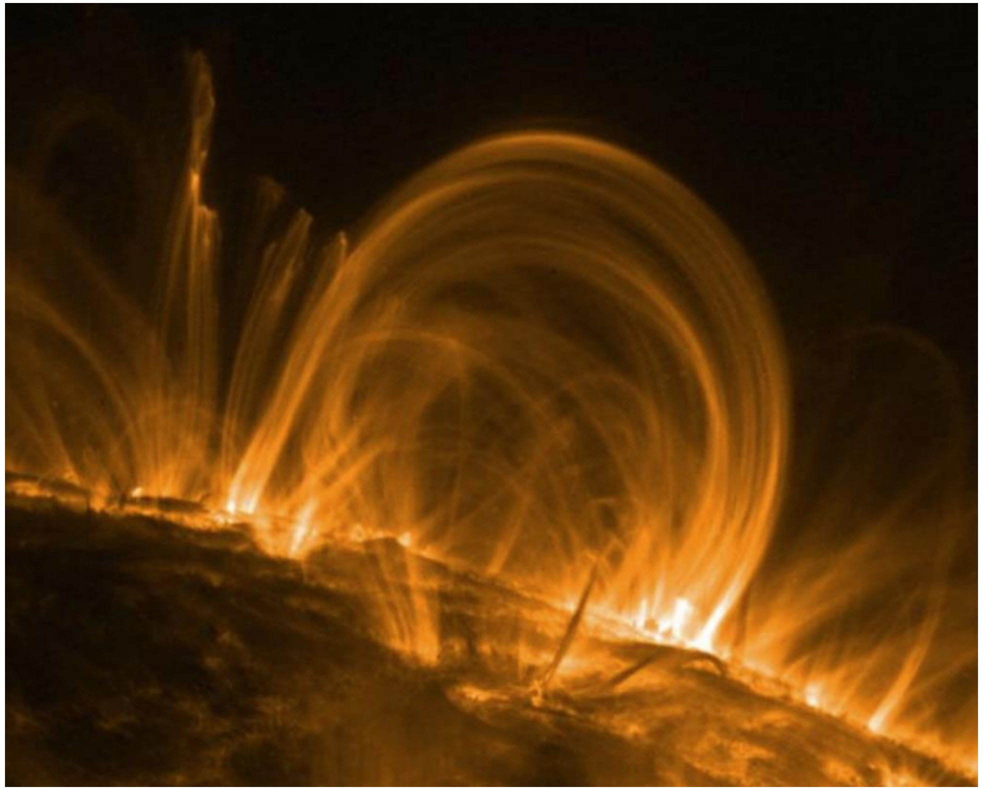


[no need of fancy higher forms, sorry]

Future

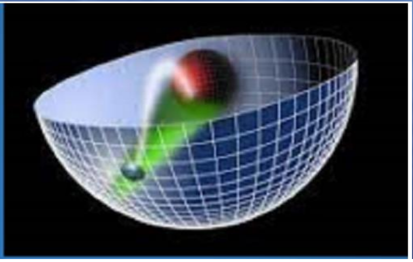
- What is the emergent physics at infinite electromagnetic coupling in $2+1$ dimensions and how can that be described (see [23] for earlier discussions on this point)?
- Can we understand better the large B limit and in particular test the recent claims made in [96] about magnetic diffusion?
- Is there an emergent photon in the strong B regime?
- What is the correct dual field theory interpretation of the higher-derivative F^{2N} bulk model? Which are the corresponding transport properties?
- Are the modified boundary conditions giving the correct phenomenology of superconductors once the U(1) symmetry is spontaneously broken [97] (see for example [30])?

Coming (hopefully) soon

How to sit  on the boundary of AdS

with Hyun-Sik Jeong, Ya Wen Sun and Keung Young Kim


THANKS A LOT !!



$$\delta_{g_{\mu\nu}} S_{\mathbf{m}} = \frac{1}{2}\int \mathrm{d}^3x\,\sqrt{-g}\,T^{\mu\nu}_{\mathbf{m}}\,\delta g_{\mu\nu}\,, \qquad \delta_{A_\mu} S_{\mathbf{m}} = \int \mathrm{d}^3x\,\sqrt{-g}\,J^{\mu}_{\mathbf{m}}\,\delta A_\mu\,.$$

$$\begin{aligned} T^{\mu\nu}_{\mathrm{m}} &= T^{\mu\nu} - T^{\mu\nu}_{\mathrm{EM}} = \epsilon_{\mathrm{m}}\,u^\mu u^\nu + p_{\mathrm{m}}\,\Delta^{\mu\nu} - M^{\mu\gamma}_{\mathrm{m}}\,F^{\nu}_{\ \gamma} + \Pi^{\mu\nu} \\ J^{\mu}_{\mathrm{m}} &= J^{\mu} - J^{\mu}_{\mathrm{EM}} = \rho_{\mathrm{m}}\,u^\mu + \nabla_\nu M^{\mu\nu}_{\mathrm{m}} + \nu^\mu\,, \end{aligned}$$

$$\begin{aligned} \left(\frac{\partial p}{\partial B}\right)_{T,\mu} &= \left(\frac{\partial p_{\mathrm{m}}}{\partial B}\right)_{T,\mu} - \frac{B}{\lambda} = \chi_{BB}B - \frac{B}{\lambda}\,, \\ \left(\frac{\partial \epsilon}{\partial B}\right)_{T,\mu} &= \left(\frac{\partial \epsilon_{\mathrm{m}}}{\partial B}\right)_{T,\mu} + \frac{B}{\lambda} = -2\chi_{BB}B + \frac{B}{\lambda}\,, \end{aligned}$$

$$\frac{1}{\lambda}\nabla_\nu F^{\mu\nu} = J^\mu_{\rm free} + J^\mu_{\rm bound} + J^\mu_{\rm ext}\,, \tag{2.18}$$

in which $J^\mu_{\rm free} := \rho_{\rm m} u^\mu + \nu^\mu$ and $J^\mu_{\rm bound} := \nabla_\nu M^{\mu\nu}_{\rm m}$. $J^\mu_{\rm free}$ refers to the current of free charges while $J^\mu_{\rm bound}$ incorporates the polarization effects. **We can decompose the polarization tensor $M^{\mu\nu}$ and $\mathcal{H}^{\mu\nu}$ with respect to fluid velocity as**

$$\begin{aligned} M^{\mu\nu}_{\rm m} &= P^\mu u^\nu - P^\nu u^\mu - \epsilon^{\mu\nu\rho} u_\rho M\,, \\ \mathcal{H}^{\mu\nu} &= u^\mu D^\nu - u^\nu D^\mu - \epsilon^{\mu\nu\rho} u_\rho H\,, \end{aligned} \tag{2.19}$$

and can also be identified with $M^{\mu\nu}_{\rm m} = 2\partial p_{\rm m}/\partial F_{\mu\nu}$, $\mathcal{H}^{\mu\nu} = 2\partial p/\partial F_{\mu\nu}$. In (3+1) dimensions [50], the magnetization M in Eq.(2.19) becomes the magnetic polarization vector M^μ. The electric polarization vector P^μ and the magnetization M are associated with the electric field E^μ and magnetic field B via the susceptibilities $(\chi_{EE}\,, \chi_{BB})$, i.e.,

$$P^\mu = \chi_{EE} E^\mu\,, \qquad M = \chi_{BB} B\,, \tag{2.20}$$

with

$$\chi_{EE} = 2\frac{\partial p_{\rm m}}{\partial E^2}\,, \qquad \chi_{BB} = 2\frac{\partial p_{\rm m}}{\partial B^2}\,. \tag{2.21}$$

The physical meaning of D^μ and H are the electric displacement vector and the magnetic H-field. This can be seen by re-writing Eq. (2.18) in terms of $\mathcal{H}^{\mu\nu}$

$$\nabla_\nu \mathcal{H}^{\mu\nu} = J^\mu_{\rm free} + J^\mu_{\rm ext}\,. \tag{2.22}$$

Eq.(2.20) implies that D^μ and H are also proportional to the electric and magnetic field E^μ and B via the following relations

$$D^\mu = \frac{1}{\lambda} E^\mu + P^\mu = \epsilon_{\rm e} E^\mu\,, \qquad H = \frac{1}{\lambda} B - M = \frac{1}{\mu_{\rm m}} B\,, \tag{2.23}$$

in which we have defined the electric permittivity $\epsilon_{\rm e}$ and the magnetic permeability $\mu_{\rm m}$. Using all the previous identities and definitions, we finally arrive at the following identities

$$\chi_{EE} = \epsilon_{\rm e} - \frac{1}{\lambda}\,, \qquad \chi_{BB} = \frac{1}{\lambda} - \frac{1}{\mu_{\rm m}}\,, \tag{2.24}$$