Hadron structures in 3D

How hydrodynamics helps us understand hadron structures

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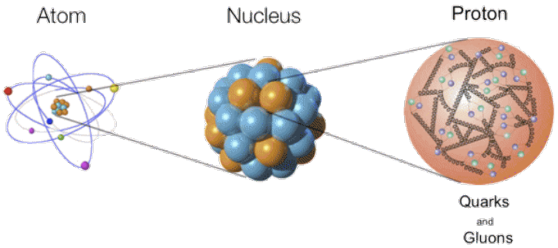
The fundamental structure of matter



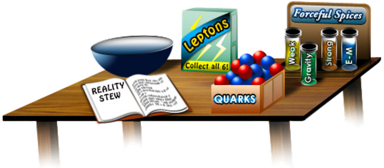
It boils down to the structure of the hadrons (proton, nucleon, pion, ...)

How to "cook" a hadron?

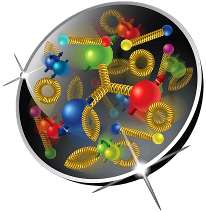
Ingredients:

Cuisine:

SM particles



Hadrons



The secret recipe of hadrons

A brief (and biased) history of hadron structure:

- Elastic scattering of proton
- Quark model
- Deep inelastic scattering
- Quantum chromodynamics

Nobel Prize, 1961

Nobel Prize, 1969

Nobel Prize, 1990

Nobel Prize, 1999 & 2004

Image credits: The Nobel Foundation

Many puzzles remain:

- Confinement
- Origin of mass
- Exotic hadrons
- Finite temperature/density
- Intrinsic charm

⋮

Let us start from the simplest ones: density distribution

Nuclear structure

In principle, the nuclear many-body wave functions provide the full information.

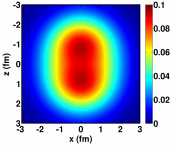
$\downarrow$ too much ~~winning~~ info?

One-body densities,

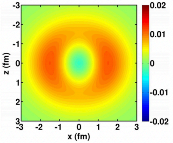
$$\rho_{p,n}(\vec{r}) = \langle\Psi|\sum_i e_{i|p,n}\delta^3(\underline{\vec{r}_i} - \vec{r})|\Psi\rangle$$

Experimentally, the Fourier transform of these densities (aka. form factors) can be measured through appropriate probes in elastic scattering $\sigma_{\text{el}} \propto \left|F(\vec{q}^2)\right|^2$,

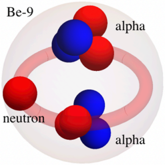
$$F_{p,n}(q^2) = \int \mathrm{d}^3r\, e^{i\vec{q}\cdot\vec{r}}\rho_{p,n}(\vec{r}).$$



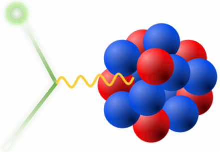
proton density



n-p density



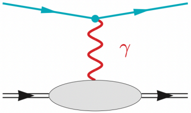
[Maris:2012ee, Cockrell:2012hfi]



Hadron structure

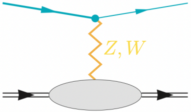
Electromagnetic probe:

$$\langle p', s' | J^\mu(0) | p, s \rangle = \overline{u}_{s'}(p') \Big[\frac{M P^\mu}{P^2} G_E(q^2) + \frac{i\varepsilon^{\mu\nu\rho\sigma} q_\nu P_\rho \gamma_\sigma \gamma_5}{2P^2} G_M(q^2) \Big] u_s(p)$$



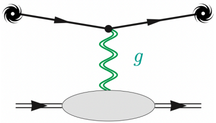
Weak probe:

$$\langle p', s' | J_5^{a\mu}(0) | p, s \rangle = \overline{u}_{s'}(p') \Big[\gamma^\mu \gamma_5 G_A(q^2) + \frac{q^\mu}{2M} G_P(q^2) \gamma_5 \Big] \frac{\tau^a}{2} u_s(p)$$



Gravitational probe:

$$\langle p', s' | T^{\mu\nu}(0) | p, s \rangle = \overline{u}_{s'}(p') \Big[\frac{P^\mu P^\nu}{M} A(q^2) + \frac{i P^{\{\mu} \sigma^{\nu\}\rho} q_\rho}{2M} J(q^2) + \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{4M} D(q^2) \Big] u_s(p)$$



Here, $P = \frac{1}{2}(p + p')$, $q = p' - p$

Global properties

- Form factor at $q^2 = 0$: global properties

$$F(0) = \int \mathrm{d}^3 r \, \rho(\vec{r})$$

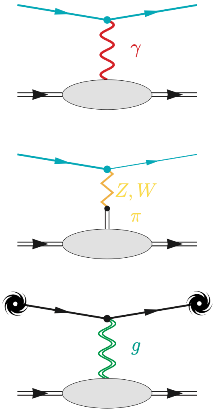
- Constrained by symmetries, e.g. $G_E(0) = 1$, $A(0) = 1$, $G_A(0) = g_A$, $J(0) = 1/2$
  [Teryaev:1999su]

- Dictated by dynamics, e.g. $G_M(0) = g/2$

- $D(0)$ is still unknown and a bit mysterious

[Polyakov:2018zvc]

em:	$\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N' \vert J^\mu_{\mathrm{em}} \vert N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\mathrm{C}$
				$\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\mathbf{weak}} \vert N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$
				$g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\mathbf{grav}} \vert N \rangle$	$\longrightarrow$	$m = 938.272013(23)\,\mathrm{MeV}/c^2$
				$J = \frac{1}{2}$
				$\boldsymbol{D} = \boldsymbol{?}$



Gravitational form factor D: interpretation I

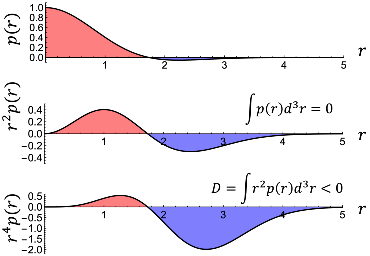
- D is related to the mechanical properties of hadrons, eps. stability of matter [Perevalova:2016dln]

$$0 = \int \mathrm{d}^3 r \, p(r) \qquad \text{(von Laue condition} \Leftarrow \partial_\nu T^{\mu\nu} = 0\text{)},$$

$$D = M \int \mathrm{d}^3 r \, r^2 p(r) \overset{\text{conj.}}{<} 0$$

where, $p(r)$ is the pressure, $T^{ij}(\vec{r}) = [\hat{r}^i \hat{r}^j - (1/3)\delta^{ij}] s(r) + \delta^{ij} p(r)$,

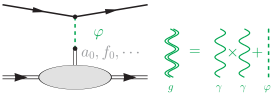
- Stability of matter requires a negative D



Gravitational form factor D: interpretation II

- D is related to the Weyl anomaly of QCD:

$$T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s} F^{\mu\nu a} F^{a}_{\mu\nu} + O(m_q)$$



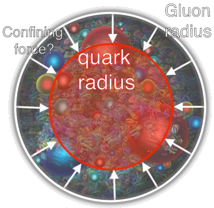
[Guo:2021ibg; Kharzeev:2021qkd]

- The trace is a gluonic scalar:

$$\langle p', s' | T^{\mu}_{\mu}(0) | p, s \rangle = \bar{u}_{s'}(p') G(q^2) u_s(p),$$

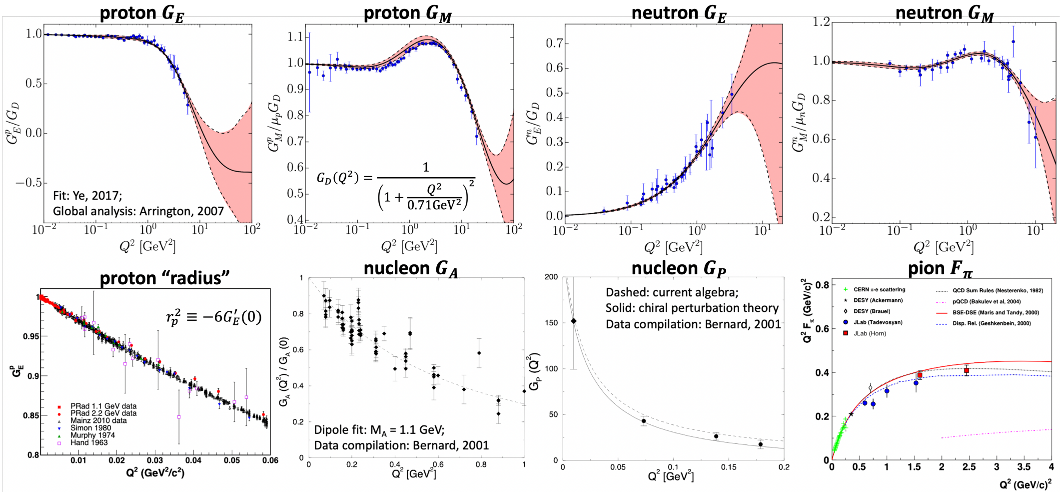
$$D = \frac{2M^2}{9} \Big[\underbrace{6A'(0)}_{\substack{r^2_M \\ \text{matter radius}}} - \underbrace{6G'(0)}_{\substack{r^2_S \\ \text{gluon radius}}} \Big]$$

- Confinement requires a negative D



Form factors: experiments

- Nucleon e.m. form factors are extensively measured in eN scattering up to $10\,\mathrm{GeV}^2$.
- Data for other form factors as well as other hadrons are scarce
- At low-Q^2, these form factors can be well approximated by dipole ansatz G_D



Generalized parton distributions

The GPDs (and their analytic continuation GDA) are defined as the hadron matrix elements of light-like bilocal operators.

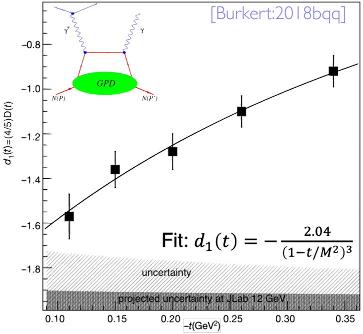
GPDs can be measured from deeply virtual Compton scattering (DVCS) experiments at large-Q^2. They provide an alternative way to access the FFs including the GFFs.

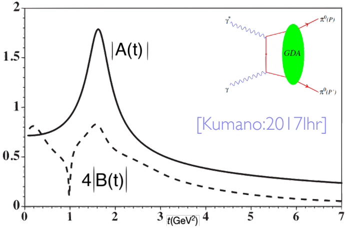
$$\int_{-1}^{+1} \mathrm{d}x\; x\; H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t),$$

$$\int_{-1}^{+1} \mathrm{d}x\; x\; E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t).$$

Another advantage of the GPD formalism is that it provides the access to each individual species (u,d,s,g), which sparks interests in the so-called ``decomposition". [Ji:1996nm]

Low-Q^2: near threshold VM production [Kharzeev:2021qkd]





Relativistic charge distribution

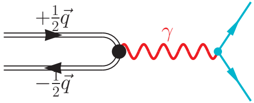
Knowing the e.m. (weak, gravitational) form factors, what is the charge (axial charge, matter) distribution of a hadron?

Textbook answer, e.g. Landau-Lifshitz:

[Ernst:1960zza, Sachs:1962zzc]

$$\rho_E^{(\text{Sach})}(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} G_E(-\vec{q}^2) e^{-i\vec{q}\cdot\vec{r}}.$$

- Interpretation: F.T. in Breit frame $\vec{P} \equiv \frac{1}{2}(\vec{p}+\vec{p}\,') = 0$, $q^0 = 0$
- Ambiguities: G_E vs F_1 vs $G_E/\sqrt{1+\tau}$
- Unphysical singularities



[e.g. Kelly:2002if, Miller:2009qu]

Is the Sachs distribution a true density admitting probabilistic interpretations?

$$\rho \sim \sum_i e_i P_i \sim \langle \Psi | \hat{O} | \Psi \rangle$$

Gerald Miller et al.'s criticism

[Miller:2018ybm; Freese:2021czn; Freese:2021mzg; Lorce:2020onh]

- Pathologies in the reconstructed densities using the experimental data
- Not a physical observable

$$\overline{O}_\Psi = \langle\Psi|O|\Psi\rangle, \quad \rho(x)_\Psi = \langle\Psi|\sum_i e_i N_i|\Psi\rangle$$

- Frame & wavepacket dependence

$$r^2_\Psi \equiv \int \mathrm{d}^3 r\, r^2\, \rho_\Psi(r) = 6F'(0) + 3R^2_\Psi \xrightarrow{\text{pl. wv.}} \infty$$

Galilean boosts (kinematical) in non-relativistic dynamics allow factorization of the c.m. motion, whereas Lorentz boosts (dynamical) don't $\langle P - \frac{1}{2}q|\rho(x)|P+\frac{1}{2}q\rangle \neq \langle -\frac{1}{2}q|\rho(x)|+\frac{1}{2}q\rangle$

$$j^\mu(x)_\Psi \equiv \langle\Psi|J^\mu(x)|\Psi\rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 2p^0} \int \frac{\mathrm{d}^3 p'}{(2\pi)^3 2p'^0} \widetilde{\Psi}^*(\vec{p}\,')\widetilde{\Psi}(\vec{p})\langle p'|J^\mu(x)|p\rangle$$

$$= \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \widetilde{\Psi}^*(\vec{P} + \tfrac{1}{2}\vec{q})\widetilde{\Psi}(\vec{P} - \tfrac{1}{2}\vec{q}) \frac{P^\mu}{2p^0 p'^0} F_{\text{ch}}(q^2) e^{iq\cdot x}.$$

where, $\widetilde{\Psi}(\vec{p}) = \langle p|\Psi\rangle$ is the momentum space wavepacket of the hadron.

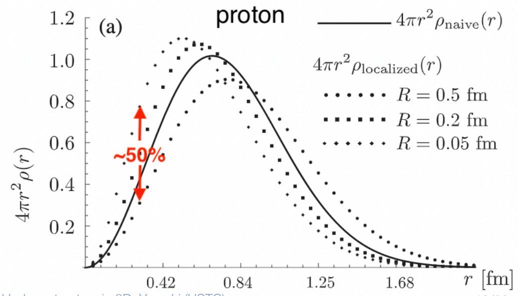
Robert Jaffe's criticism

[Jaffe:2020ebz; Lorce:2020onh; Belitsky:2003nz; Ji:2004gf; Belitsky:2005qn]

- Sachs distribution is a non-relativistic approximation. It is valid provided,

$$r_{\text{hadron}} \gg Q_{\max}^{-1} \gg R_w \gg M^{-1} \quad \Rightarrow \quad r_{\text{hadron}} \gg \lambda_\gamma \gg \lambda_{\text{hadron}} \geq \lambda_{\text{C}}$$

  - $r_{\text{hadron}} \gg Q_{\max}^{-1}$: resolving hadron structure
  - $r_{\text{hadron}} \gg R_w$: the effect of the wavepacket is small
  - $R_w \gg M^{-1}$: non-localization in relativistic quantum mechanics
- The above inequalities cannot be satisfied for hadrons, since $r_{\text{hadron}} M \sim 1$.
- Hadrons are intrinsically relativistic!



	r	M	rM
pion	0.67 fm	0.14 GeV	0.5
charmonium	0.25--0.4 fm	3.0 GeV	3.8--6
proton	0.87 fm	0.94 GeV	4
nuclei	1.3 $A^{\frac{1}{3}}$ fm	0.94A GeV	6$A^{\frac{4}{3}}$

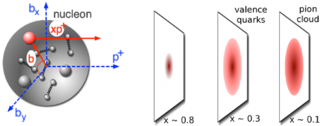
Transverse charge density on the light-front

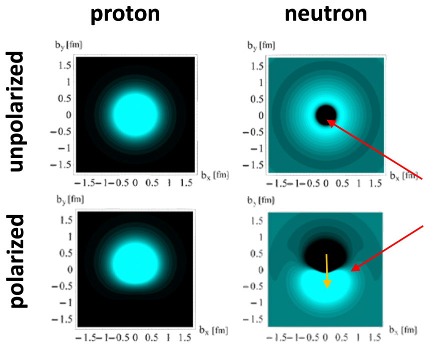
[Miller:2010nz, Freese:2022fat; cf. Burkardt:2000za]

Miller et al.'s solution: $J^+ = \sum_i e_i N_i = \bar{q}_i \gamma^+ q_i$ and light-front dynamics in Drell-Yan frame $q^+ = 0$

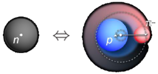
$$\rho_{\rm LF}(\vec{b}_\perp) = \delta(b_\parallel) \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} G_E(-\vec{q}_\perp^{\,2}) e^{-i\vec{q}_\perp \cdot \vec{b}_\perp}$$

- Light-front boosts are Galilean - c.m. motion factorizes
- Direct interpretation from light-front quantization
- Related to GPDs in coordinate space



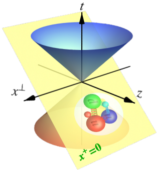


Proton-pion fluctuation

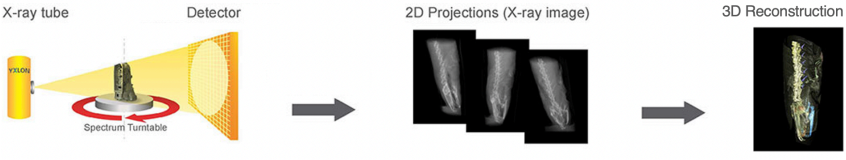


Negative neutron core?

Neutron EDM?



From 2D to 3D: Abel-Radon transformation [Panteleeva:2021iip]



- An Abel-Radon signal at angle $\hat{n}$ and distance s is obtained by the line integral along the path of the X ray.

$$\mathcal{R}f(s,\hat{n}) = \int \mathrm{d}^3 r\, f(\vec{r})\delta(s - \hat{n}\cdot\vec{r})$$

- Inversion problem
- Panteleeva & Polyakov showed with some reasonable assumptions the 3D Sachs and the 2D light-front distributions can be related by the Abel-Radon transformation [Panteleeva:2021iip]

Phase space approach

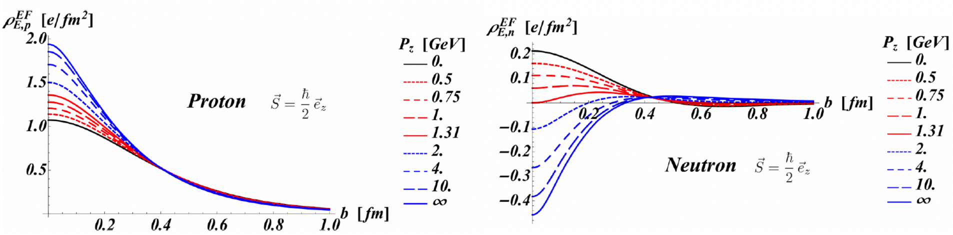
[Lorce:2020onh, Chen:2022smg]

Cedric Lorcé interpreted the F.T. as the Weyl function of the current operator,

$$\rho_W(\vec{r},\vec{P}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \langle \vec{P}+\tfrac{1}{2}\vec{q}|J^0|\vec{P}-\tfrac{1}{2}\vec{q}\rangle$$

- Weyl quantization $O \leftrightarrow O_W(\vec{r},\vec{p})$, eps. the Wigner function $\varrho \leftrightarrow W(\vec{r},\vec{p})$
- Sachs distribution: $\vec{P}=0$; Light-front distribution: $\vec{P}\to\infty$ (IMF)
- Quasi-probabilistic
- Still need a special frame, $q^0 = 0$ (elastic frame)

$$\overline{O}_\Psi = \int d^3r \int \frac{d^3p}{(2\pi)^3} \times O_W(\vec{r},\vec{p}) W(\vec{r},\vec{p})$$

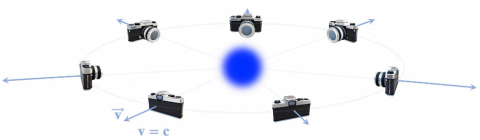


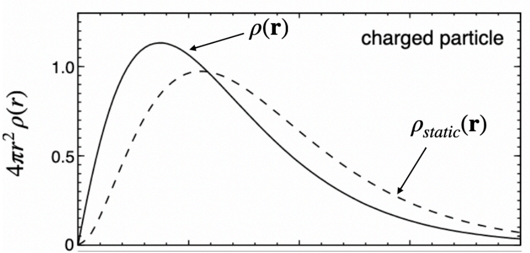
Sharply localized wavepacket [Epelbaum:2022fjc, Panteleeva:2022khw, Panteleeva:2022uii, Carlson:2022eps]

Epelbaum et al. proposed to use sharply localized wavepacket $R_w \to 0$ with spherical symmetry,

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \int_{-1}^{+1} \mathrm{d}\alpha \frac{1}{2} F_{\mathrm{ch}}\Big[-(1-\alpha^2)\vec{q}^2\Big]$$

- Angle-averaged light-front density
- Particle localization in QFTs



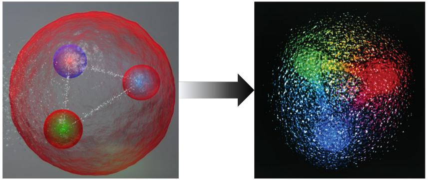


A hadron is a de Broglie wave

[de Broglie, 1924]

resolving a non-relativistic particle: $r_{\text{hadron}} \gg \lambda_\gamma \gg \lambda_{\text{hadron}} \geq \lambda_{\text{C}}$

resolving a relativistic hadron: $\lambda_{\text{hadron}} \gtrsim r_{\text{hadron}} \sim \lambda_{\text{C}} \gg \lambda_\gamma$



For the resolving photon, the hadron does not appear as a particle, instead, it appears as wave -- a de Broglie wave -- a medium!

The relevant theory is the relativistic theory of macroscopic electromagnetism

Minkowski's lost course

- The relativistic theory of macroscopic electromagnetism for moving bodies is established by Minkowski (1908), Einstein and Laub (1909), as a sequel of relativity -- "electrodynamics of moving bodies" (1905)
  - Electron structure, $r_e^{\text{cl.}} m_e = \alpha_{\text{em}} \ll 1$, alas electron is structureless - right theory for a wrong problem
  - Modern applications: cosmology, black hole merger, inertial fusion, QGP, ..., hadrons
  - The expression appears identical to Maxwell's macroscopic electromagnetism (but different physical contents!)
  - Rediscovered several times in applied physics

$$j^\beta = j_f^\beta - \partial_\alpha M^{\alpha\beta} \;\Rightarrow\; \partial_\alpha H^{\alpha\beta} = j_f^\beta, \qquad (H^{\alpha\beta} \equiv F^{\alpha\beta} - M^{\alpha\beta})$$

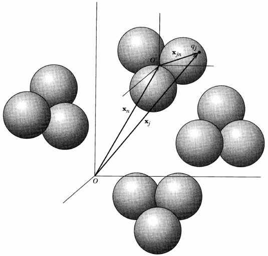
- The free current j_f^α, consists of the convective current ϱu^α and the conducting current j_{cd}^α (Ohm's law)
- Bianchi identity $\partial_\alpha \widetilde{F}^{\alpha\beta} = 0$ remains the same
- Components of H, field induction tensor: $H^{0i} = D^i, H^{ij} = \epsilon^{ijk} H^k$
- Media in motion: interpretation of $\vec{D}, \vec{H}$ are different, e.g. $\vec{D} \neq \epsilon \vec{E}$, $\vec{H} \neq \mu^{-1} \vec{B}$

Macroscopic densities

[Jackson; Pauli; de Groot]

Medium polarization tensor $M^{\alpha\beta}$:

- Polarization & magnetization vector densities: $P^i = M^{0i}, M^i = \frac{1}{2}\epsilon^{ijk}M^{jk}$
  - Polarization charge density: $\rho_{\text{pol}} = -\nabla\cdot\vec{P}$, effective magnetic charge density $\rho_{\text{mag}} = \nabla\cdot\vec{M}$ and magnetized current density $\vec{j}_{\text{mag}} = \nabla\times\vec{M}$
- Co-moving vector densities: $M^{\alpha\beta} = u^\alpha\mathcal{P}^\beta - u^\beta\mathcal{P}^\alpha + \varepsilon^{\alpha\beta\kappa\lambda}u_\kappa\mathcal{M}_\lambda$
  - Co-moving polarization/effective magnetic charge density: $\varrho_{\text{pol}} = -\partial_\alpha\mathcal{P}^\alpha$, $\varrho_{\text{mag}} = -\partial_\alpha\mathcal{M}^\alpha$.
- Depend on the choice of the fluid velocity u^α, e.g. Landau-Lifshitz vs. Eckart frames [Eckart:1940te]

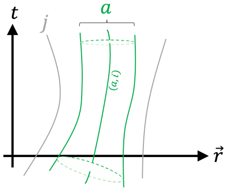


In practice, media comprise of composite particles, e.g. atoms, molecules, ...

$$j^\alpha(x) = \int \mathrm{d}^3r\,\mathcal{D}(\vec{x}-\vec{r})j_f^\alpha(\vec{r},t)$$

$\mathcal{D}(\vec{x})$: atomic density, multipole expansion

Classical theory



- Consider a current of microscopic origin,

$$j^\mu(x) = \sum_{a,i} e_{ai} \int \mathrm{d}\tau_{ai} \dot{X}^\mu_{ai}(\tau_{ai}) \delta^4(X_{ai}(\tau_{ai}) - x)$$

where a and i enumerate the atoms and their constituents, respectively.

- We can choose a privileged worldline for each atom $X_a(\tau_a)$ and define $r_{ai}(\tau_a) = X_{ai}(\tau_a) - X_a(\tau_a)$. Then, the full current can be written as,

$$j^\mu(x) = \sum_{a\in A} \int \tau_a \sum_{i\in a} e_{ai} \left[\dot{X}^\mu_a(\tau_a) + \dot{r}^\mu_{ai}(\tau_a) \right] \sum_{n=0}^{\infty} (r_{ai}\cdot\partial)^n \delta^4(X_a(\tau_a) - x).$$

- The $n = 0$ term does not depend on the microscopic details of the atoms, which is the free current

$$j^\mu_f(x) = \sum_a \int \mathrm{d}\tau_a e_a \dot{X}^\mu_a(\tau_a) \delta^4(X_a(\tau_a) - x).$$

$$M^{\alpha\beta} = \sum_{a\in A, i\in a} e_{ai} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \int \mathrm{d}\tau_a \Big\{ r^\alpha_{ai} u^\beta_a - u^\alpha_a r^\beta_{ai} + \frac{n}{n+1} \Big(r^\alpha_{ai} \dot{r}^\beta_{ai} - \dot{r}^\alpha_{ai} r^\beta_{ai} \Big) \Big\} (r_{ai}\cdot\partial)^{n-1} \delta^4(X_a - x).$$

- Weyl quantization, Wigner-Newton position operator & particle localization

Quantum theory

[Li:2022ldb; cf. similar developments in LF coordinates: Freese:2022fat]

$$j^\mu(x)_\Psi \equiv \langle\Psi|J^\mu(x)|\Psi\rangle = \int\frac{\mathrm{d}^3P}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\widetilde{\Psi}^*(\vec{P}+\tfrac{1}{2}\vec{q})\widetilde{\Psi}(\vec{P}-\tfrac{1}{2}\vec{q})\frac{P^\mu}{2p^0p'^0}F_{\text{ch}}(q^2)e^{iq\cdot x}.$$

- For hadrons, the microscopic current is exactly parametrized by the covariant form factors
- To compare with the classical theory, introduce the ``coordinate space wave function'' as the F.T.,

$$\Psi(x) = \int\frac{\mathrm{d}^3p}{(2\pi)^3 2p^0}\widetilde{\Psi}(\vec{p})e^{ip\cdot x}.$$

$\Psi(x)$ satisfies the Klein-Gordan equation -- normalized as a current:

$$\boxed{f\overleftrightarrow{\partial}g = f\partial g - \partial f g}$$

$$\int\mathrm{d}^3x\,\Psi^*(x)i\overleftrightarrow{\partial}_t\Psi(x) = 1.$$

- N.B. Ψ is not a true coordinate-space wave function (nor should such thing exist)

$$\int\mathrm{d}^3x\,\big|\Psi(x)\big|^2 \neq 1.$$

Quantum theory

[Li:2022ldb; cf. similar developments in LF coordinates: Freese:2022fat]

- Free current: convective current $i\overleftrightarrow{\partial}^\alpha \sim P^\alpha$

$$j^\mu_f(x) \equiv \Psi^*(x) i\overleftrightarrow{\partial}^\mu \Psi(x)$$

- Fluid velocity (particle frame): $u^\alpha = P^\alpha/\sqrt{P^2}$, where $P^2 = M^2(1 - \frac{q^2}{4M^2})$ and $u^2 = 1$
- Medium polarization tensor,

$$M^{\mu\nu}(x) = \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \widetilde{\Psi}^*(\vec{P} + \tfrac{1}{2}\vec{q}) \widetilde{\Psi}(\vec{P} - \tfrac{1}{2}\vec{q}) \frac{P^\mu q^\nu - P^\nu q^\mu}{2p^0 p'^0} \frac{F_{\text{ch}}(q^2) - 1}{iq^2} e^{iq\cdot x}$$

- Macroscopic co-moving charge density $\varrho = u_\mu J^\mu$: depending on the wavepacket Ψ

$$\varrho_\Psi(x) = \int \mathrm{d}^3 r\, \Psi^*(\vec{r}, t) \Big\{ \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \sqrt{4M^2 - q^2} F_{\text{ch}}(q^2) e^{-i\vec{q}\cdot(\vec{x}-\vec{r})} \Big\} \Psi(\vec{r}, t)$$

The wavepacket independent part is not factorisable: $q^2 = (q^0)^2 - \vec{q}^2$, where

$$q^0 = \sqrt{(\vec{P} + \frac{1}{2}\vec{q})^2 + M^2} - \sqrt{(\vec{P} - \frac{1}{2}\vec{q})^2 + M^2}$$

and $\vec{P} = (-i/2)\overleftrightarrow{\nabla}_r$.

Mesoscopic density

[Li:2022ldb; cf. similar developments in LF coordinates: Freese:2022fat]

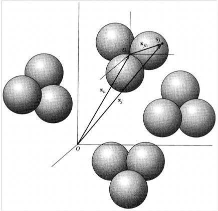
In analogy to the classical case, we can express the full quantum current in terms of a mesoscopic density $\mathcal{D}$,

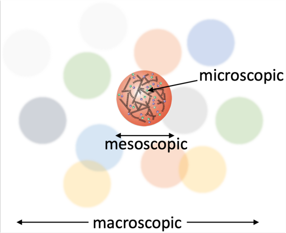
$$j^\alpha(x) = \int \mathrm{d}^3 r\, \Psi(\vec{r},t) i\overleftrightarrow{\partial}^\alpha \mathcal{D}(\vec{b}; \tfrac{-i}{2}\overleftrightarrow{\nabla}_r)\Psi(\vec{r},t),$$

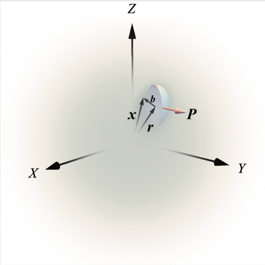
$$\Rightarrow \quad \mathcal{D}(\vec{b};\vec{P}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_{\text{ch}}(q^2) e^{-i\vec{q}\cdot\vec{b}}$$

Where, $\vec{b} = \vec{x} - \vec{r}$, and $q^2 = (q^0)^2 - \vec{q}^2$, $q^0 = \sqrt{(\vec{P}+\frac{1}{2}\vec{q})^2 + M^2} - \sqrt{(\vec{P}-\frac{1}{2}\vec{q})^2 + M^2}$.

- The mesoscopic composite particle smears out due to quantum diffusion of the wavepacket
- For general states, e.g. thermal states, the wavepacket can be replaced by the density matrix







Multipole expansion

[Li:2022ldb; cf. similar developments in LF coordinates: Freese:2022fat]

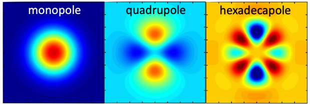
Remaining issue: $\mathcal{D}$ is a function of the differential operator $\vec{P} = (-i/2)\overleftrightarrow{\nabla}$:

- Taylor series around $|\vec{P}| = 0$: multipole expansion

$$\mathcal{D}(\vec{x}) = \sum_{n=0}^{\infty} \frac{(-i)^n}{2^n n!} d_n^{i_1 i_2 \cdots i_n}(\vec{x}) \overleftrightarrow{\nabla}^{i_1} \overleftrightarrow{\nabla}^{i_2} \ldots \overleftrightarrow{\nabla}^{i_n}.$$

- Monopole density d_0 is the Sachs distribution

$$d_0(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} F_{\text{ch}}(-\vec{q}^2) e^{-i\vec{q}\cdot\vec{r}}$$



- High-multipole moments are nonzero: Lorentz distortion

$$\overleftrightarrow{d}_2(\vec{r}) = \frac{2}{M^2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{F'_{\text{ch}}(-\vec{q}^2)}{1+\tau} \vec{q}\vec{q} e^{-i\vec{q}\cdot\vec{r}}$$

- No special frame is taken (cf. Breit frame, elastic frame, Drell-Yan-West frame, ...)
- No non-relativistic approximation is taken: no need for $r_{\text{hadron}} \gg \lambda_C$
- The multipole densities are physical quantities in their own rights
- Convergence of the multipole series: $|\vec{P}| \sim \lambda_{\text{hadron}}^{-1}$ -- sufficiently delocalized wavepacket $\rightarrow$ guaranteed convergence using planewave!

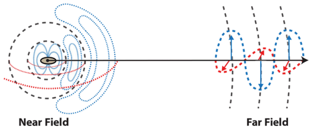
Light-front multipole moments

[Li:2022ldb]

Is the multipole expansion unique? No!

Alternative: Laurent series around $1/|\vec{P}| = 0$ (near field expansion)

- Note that sufficient to take $P_z \to \infty \;\Rightarrow\; |\vec{P}| = \sqrt{\vec{P}_\perp^2 + P_z^2} \to \infty$
- Monopole density gives the 2D light-front distribution
- No infinite momentum frame or Drell-Yan-West $q^+ = 0$ frame invoked for the light-front distribution! $\to$ new definition of light-front densities
- Convergence of the multipole series: $|\vec{P}| \sim \lambda_{\text{hadron}}^{-1}$ -- sufficiently localized wavepacket in at least one dimension, say, $z \to$ guaranteed convergence in the partial z-localization limit
- Multipole densities are frame independent & wavepacket independent
- Not fully explored in classical field theories



near field

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_\ell \frac{4\pi}{2\ell+1}\sum_m (-1)^m \frac{r_<^\ell}{r_>^{\ell+1}} Y_\ell^{m*}(\hat{r})Y_\ell^m(\hat{r}')$$

far field

Extension to energy-momentum tensor (EMT)

Splitting the energy-momentum tensor,

$$t^{\alpha\beta}(x) \equiv \langle\Psi|T^{\mu\nu}(x)|\Psi\rangle = t_f^{\alpha\beta}(x) + \partial_\sigma \chi^{\alpha\beta\sigma}$$

where, the ``free EMT"

$$t_f^{\alpha\beta}(x) = \partial^{\{\alpha}\Psi^*\partial^{\beta\}}\Psi + g^{\alpha\beta}(\partial_\sigma\Psi^*\partial^\sigma\Psi - M^2|\Psi|^2)$$

- $t_f^{\alpha\beta}$ is fluid-like
- Soft-graviton theorem: soft graviton ($\partial \sim q \to 0$) does not distinguish composite particles from pointlike particles $\Rightarrow$ no anomalous gravitomagnetic moment. [Boulware:1974sr, Cho:1976de, cf. Szekeres:1971ss]
- Mesoscopic densities ($\vec{b} = \vec{x} - \vec{r}$),

$$t^{\alpha\beta}(x) = \int \mathrm{d}^3 r\, \Psi^*(\vec{r},t)\Big\{ -\overleftrightarrow{\nabla}^\alpha\overleftrightarrow{\nabla}^\beta A(\vec{b}) + \frac{1}{4M} D^{\alpha\beta}(\vec{b})\Big\}\Psi(\vec{r},t)$$

$$A(\vec{b}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} A(q^2) e^{-i\vec{q}\cdot\vec{b}}, \quad D^{\alpha\beta}(\vec{b}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} (q^\alpha q^\beta - g^{\alpha\beta}q^2) D(q^2) e^{-i\vec{q}\cdot\vec{b}},$$

Note that q^2 depends on $\overleftrightarrow{\nabla}$ as before; hence $A(\vec{b})$ and $D(\vec{b})$ are not factorizable.

Summary

[YL, W. Dong, Y. Yin, Q. Wang, J.P. Vary, arXiv:2206.12903 [hep-ph]]

- Hadrons are unique relativistic systems $r \sim \lambda_C$
- 3D structure of hadrons viewed through the electroweak, gravitational lenses
- Hadrons are de Broglie waves subjected to Minkowski's relativistic theory of macroscopic electromagnetism
- Traditional Sachs charge distribution & light-front charge distribution can be understood as multipole moment densities

Thank you!