Recent progress on applications of holographic gravity

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Outlines

I. Some general remarks on gauge/gravity duality

II. Transport property and universal bound

III. Doubly holographic setup

IV. Deep learning and holographic gravity

The key issue:

How to construct the bulk geometry to reproduce the phenomenon observed in lab?

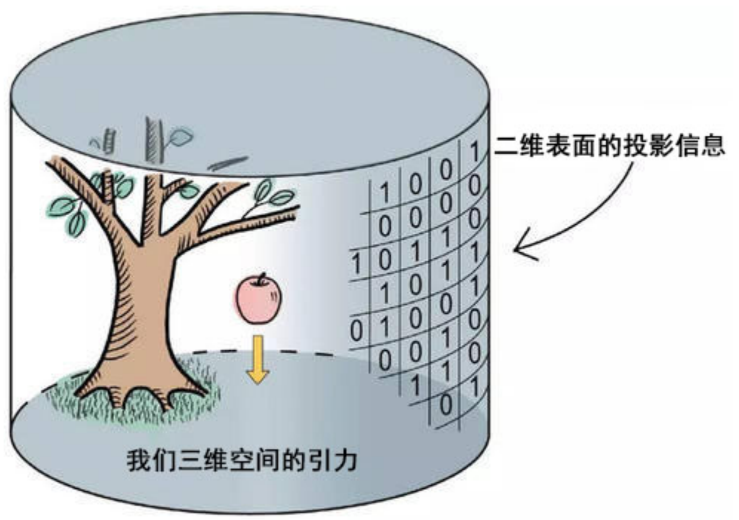
References:

1. Yi Ling, Zhuo-Yu Xian, Zhenhua Zhou. JHEP 1611 (2016) 007.
   Holographic Shear Viscosity in Hyperscaling Violating Theories without Translational Invariance.
2. Yi Ling, Zhuo-Yu Xian, Zhenhua Zhou. Chinese Physics C 41 (2017) 2, 023104
   Power Law of Shear Viscosity in Einstein-Maxwell-Dilaton-Axion model.
3. Yi Ling and Zhuo-Yu Xian. JHEP 1709 (2017) 003.
   Holographic Butterfly Effect and Diffusion in Quantum Critical Region.
4. Yi Ling, Yuxuan Liu, Zhuo-yu Xian. JHEP 03 (2021) 251.
   Island in Charged Black Holes.
5. Yi Ling, P. Liu, Y. Liu, C. Niu, Z. Xian, C. Zhang. JHEP 02 (2022) 037.
   Reflected Entropy in Double Holography.
6. Y. Liu, Z. Xian, C. Peng, Yi Ling. JHEP 09 (2022) 179.
   Black holes entangled by radiation.
7. Kai Li, Yi Ling, Peng Liu, Meng-He Wu. arXiv: 2209.05203.
   Learning the black hole metric from holographic conductivity.

Some general remarks on gauge/gravity duality

- Holographic principle in quantum gravity

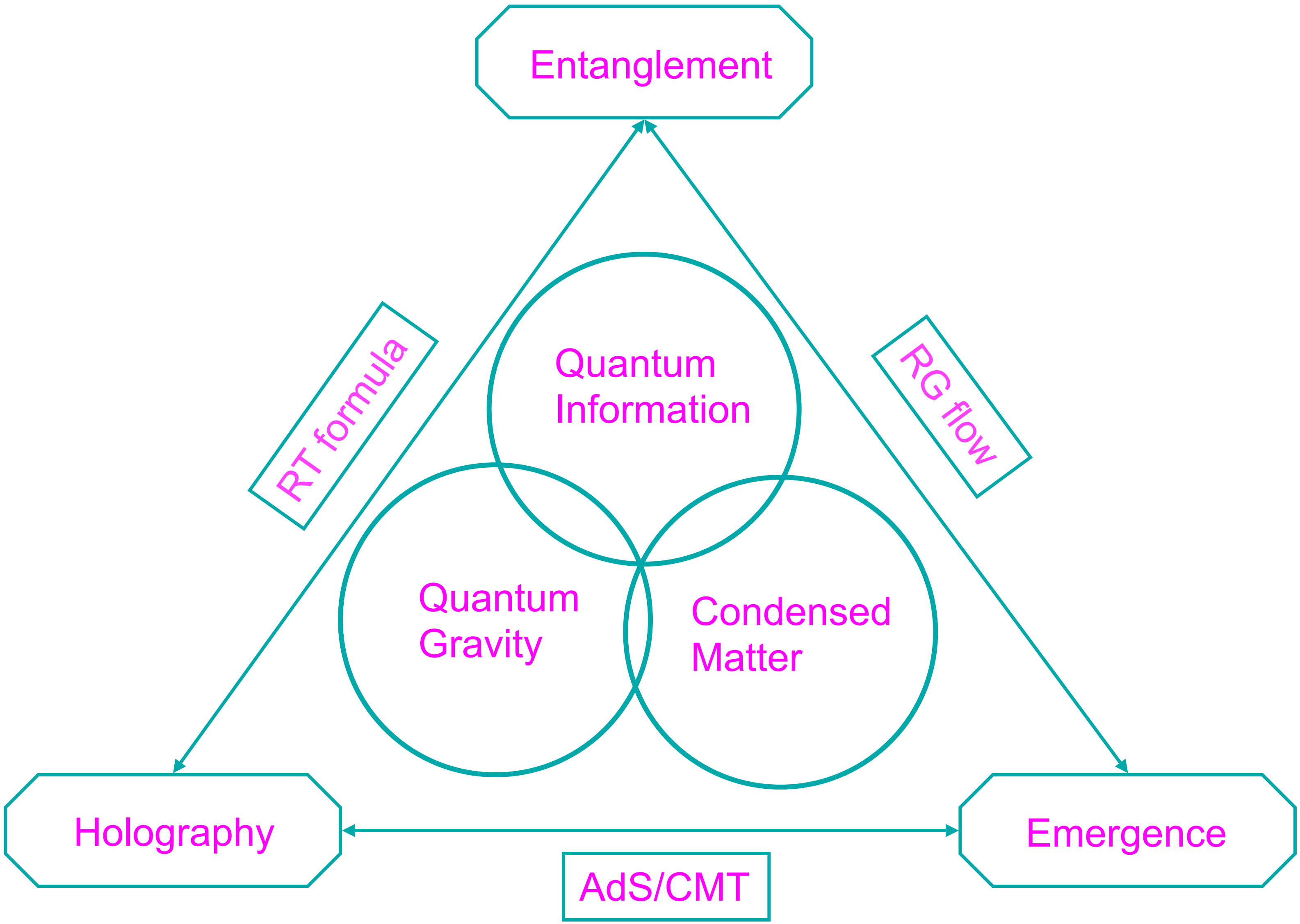
d=D+1 QG ⟷ **d=D QFT**



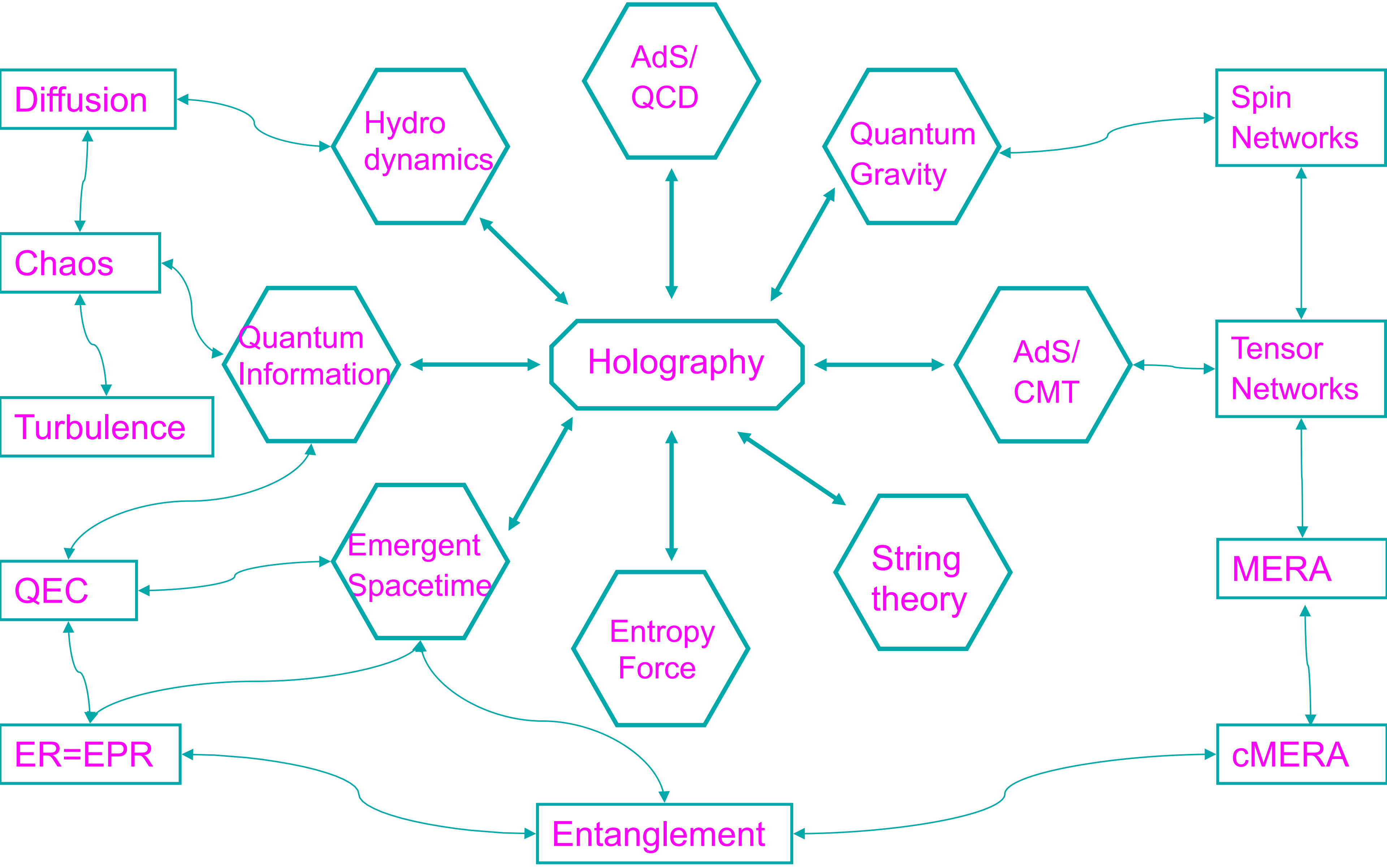
$$Z_b[J] = Z_{QG} \xrightarrow{N \gg 1} \int D\phi \, \mathrm{e}^{iS[\phi_{cl}] + i\int d^d x J o}$$

(Semi-)classical gravity ⟷ **Strongly coupled system**

Some general remarks on gauge/gravity duality



Some general remarks on gauge/gravity duality



Transport property and universal bound

Kovtun-Son-Starinets (KSS) bound

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

A bound on Chaos

$$\lambda_L \leq 2\pi k_B T$$

Maldacena, arXiv:1503.01409

A bound on Diffusion constant

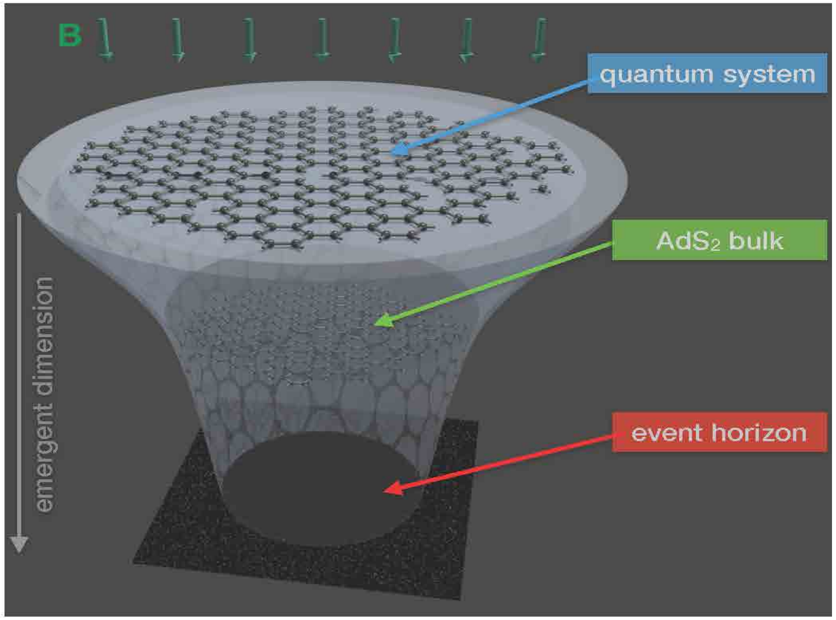
$$D \geq \frac{\mathrm{h} v_B^2}{k_B T}$$

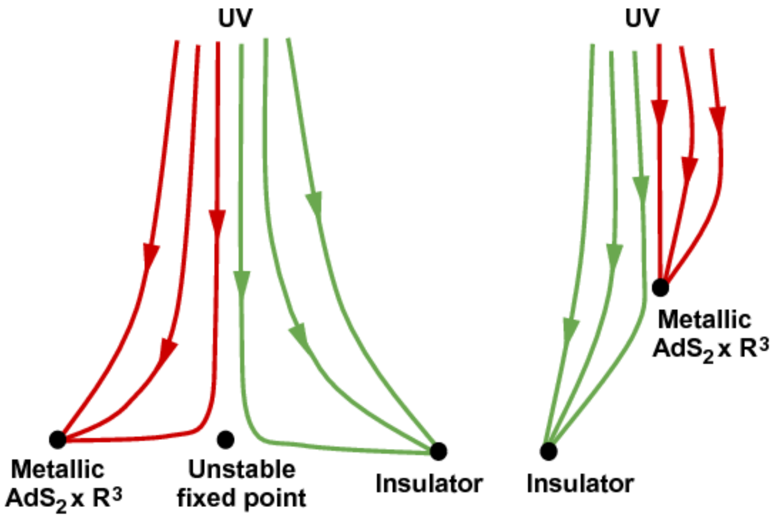
Blake, arXiv:1603.08510

Transport property and universal bound

Gauge/Gravity Duality

Donos and Hartnoll, Nature Phys.9, 649 (2013).





$$\omega \to 0 \qquad \frac{\eta}{s}, \quad \sigma_D, \quad D, \quad v_B$$

Infrared (IR) physics ⟷ **Horizon formula** (Scaling symmetry)

$$AdS_4, \quad AdS_2, \quad HL\ldots$$

Transport property and universal bound

1. Kovtun-Son-Starinets (KSS) bound

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

$$\eta = \lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} G^R_{\hat{T}^{xy}\hat{T}^{xy}}(\omega, k=0)$$

2. Translational invariance is broken

$$\frac{\eta}{s} \sim T^{\kappa} \qquad 0 \leq \kappa \leq 2 \qquad T \to 0$$

the (weaker) horizon formula

$$\frac{\eta}{s} = \frac{1}{4\pi} h_0(r_+)^2$$

Hartnoll, Ramirez and Santos, JHEP 1603, 170 (2016)

Transport property and universal bound

3. In general holographic models with hyperscaling violation

Scaling dimensions: (d,θ,z)

The dimension of space: d

Dynamical critical exponent : z

Hyperscaling violation exponent: θ (Subject to NEC)

$[x]=-1,\ [t]=-z$

$$ds^2 = L^2 r^{\frac{2\theta}{d}}\left(-\frac{dt^2}{r^{2z}}+\frac{dr^2+\sum_{i=1}^{d}dx_i^2}{r^2}\right)$$

scaling transformation

$$x\to\lambda x, r\to\lambda r, t\to\lambda^z t \longrightarrow ds\to\lambda^{\theta/d}ds \qquad x\sim r\sim t^{1/z}\sim ds^{(d/\theta)}$$

Black holes with finite temperature

$$ds^2 = L^2 r^{\frac{2\theta}{d}}\left(-\frac{f(r)dt^2}{r^{2z}}+\frac{dr^2}{r^2 f(r)}+\frac{\sum_{i=1}^{d}dx_i^2}{r^2}\right) \qquad f(r)=1-\left(\frac{r}{r_+}\right)^{\delta_0},\quad \delta_0=d+z-\theta$$

Transport property and universal bound

Universal Formula

$$\frac{\eta}{s} \sim T^{\kappa}$$

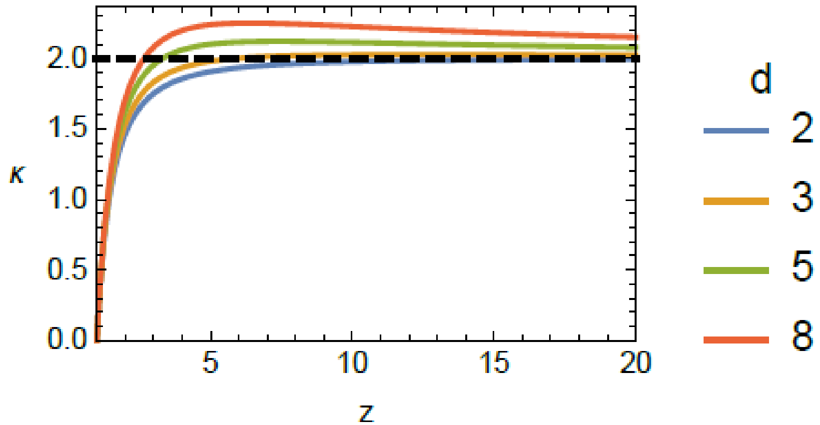
$$\kappa = \frac{d+z-\theta}{z}\left(-1+\sqrt{\frac{8(z-1)}{(d+z-\theta)(1+e^2)}+1}\right)$$

$$e^2 \equiv g^{tt}\frac{\delta L_m}{\delta g^{tt}} \Bigg/ \left(-g^{xx}\frac{\delta L_m}{\delta g^{xx}}\right) \sim F^2 \geq 0$$

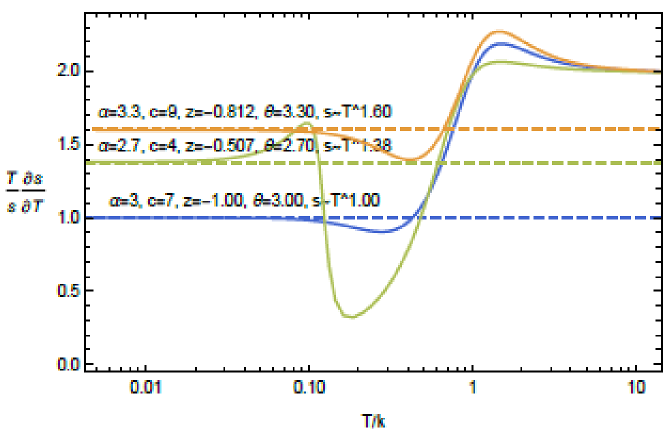
Transport property and universal bound

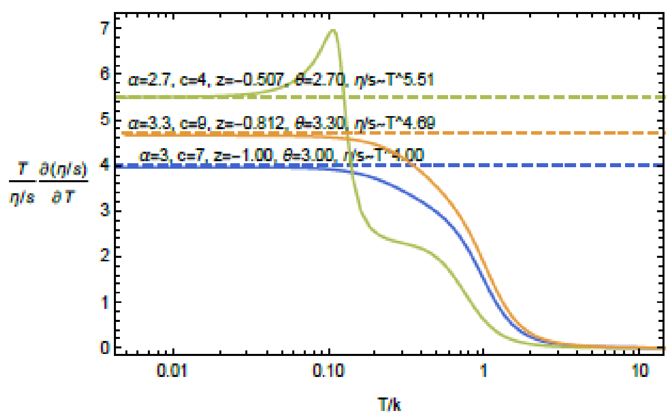
- Neutral black holes $(e^2 = 0)$

$$(d > 2, \theta = 0, z \geq 1)$$



$$(d = 2, \theta \neq 0)$$

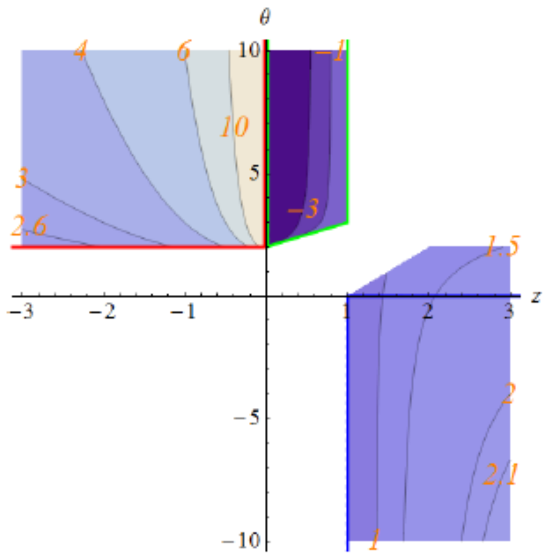




Transport property and universal bound

- Neutral black holes $(e^2 = 0)$

$d = 2$

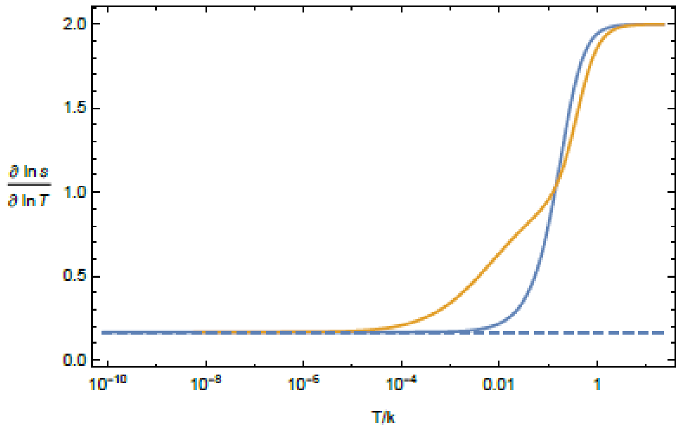


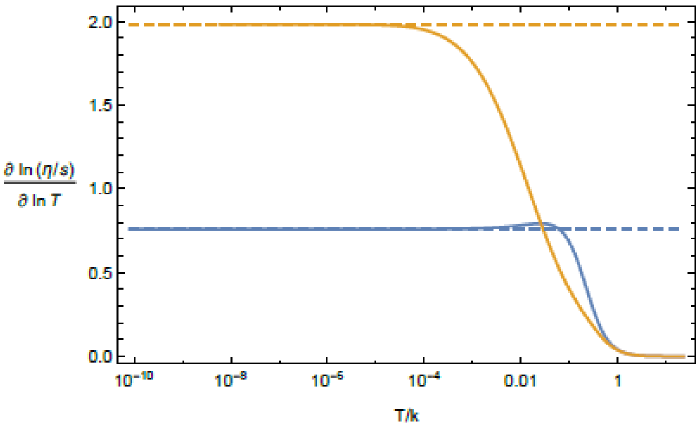
Regions	IR	Limit of T	(z, θ)	(α, c)	κ
Region A	$r \xrightarrow{IR} 0$	$T \to 0$	$z < 0 \wedge \theta > 2$	$2 < \alpha < \sqrt{4+c}$	$\kappa > 2$
Region B	$r \xrightarrow{IR} 0$	$T \to +\infty$	$0 < z \leq 1 \wedge \theta > z + 2$	$(2 < \alpha \leq 3 \wedge -\alpha^2 + 8\alpha - 12 < c < \alpha^2 - 4)$ $\vee(\alpha > 3 \wedge \alpha^2 - 2\alpha \leq c < \alpha^2 - 4)$	$\kappa \leq 0$
Region C	$r \xrightarrow{IR} +\infty$	$T \to 0$	$\theta < 0 \wedge z \geq 1$	$\alpha < 0 \wedge c \geq \alpha^2 - 2\alpha$	$0 \leq \kappa < 4$

Transport property and universal bound

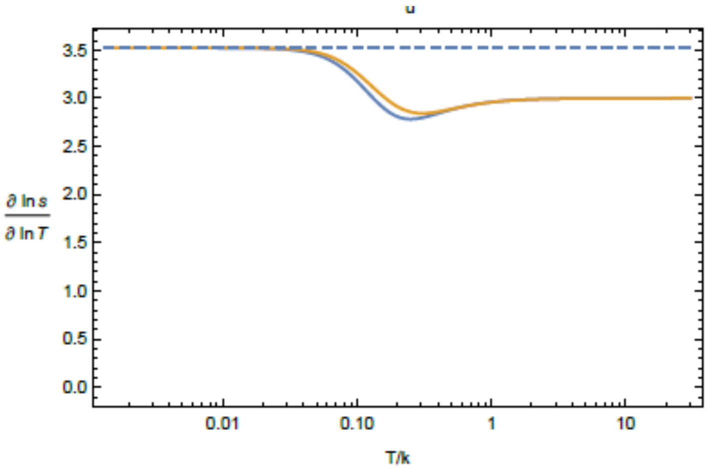
- Charged black holes $(e^2 \neq 0)$

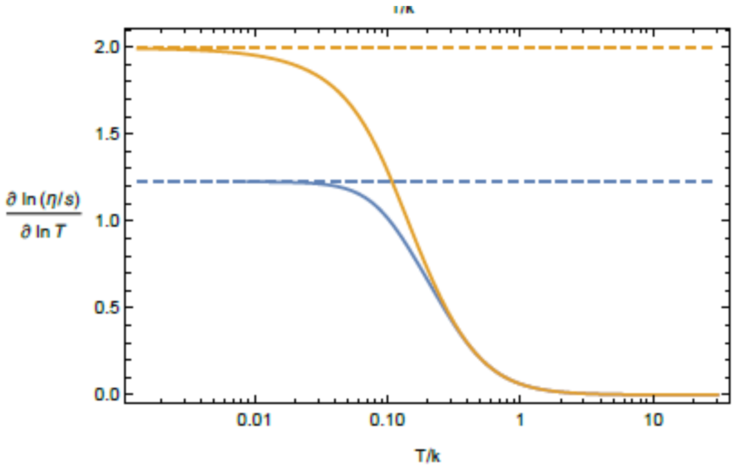
$$d = 2, \theta = 0$$





$$d = 3, \theta < 0$$



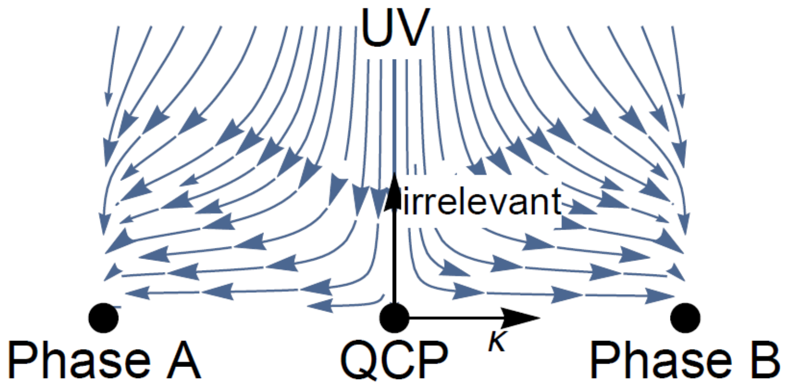


Transport property and universal bound

Quantum Critical Phenomenon

Relevant operator deformations

$$S_{QPT} = S_{QCP} + \kappa \int W(\mathrm{O}) d^d x dt$$



RG flow of QPT

Scaling dimensions:

$$[x] = -1,\ [t] = -z,\ [\kappa] \equiv \Delta_- \equiv 1/\nu,$$

$$[W] = d + z - \Delta_- \equiv \Delta_+,$$

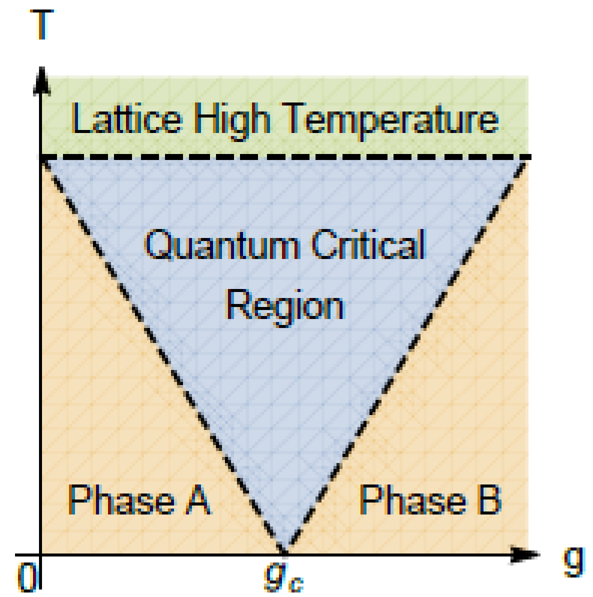
Transport property and universal bound

Two important scales:

Temperature T

Source $\kappa \sim g - g_c$

(Energy gap $\Delta_E \sim |\kappa|^{z\nu}$)



Phase diagram of the second order QPT

Quantum critical region: $T \gg |\kappa|^{z\nu}$

Phase A and B: $T = |\kappa|^{z\nu}$ (Non-quantum critical region)

Lattice high temperature: $T \gg \Lambda_{UV}$

Transport property and universal bound

$$ds^2 = -E(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}^2, \quad \phi = \phi(r)$$

$$v_B{}^2 = \frac{E'(r_h)}{dC'(r_h)}$$

Roberts and Swingle, arXiv:1603.09298

Scaling formula: $[v_B] = [x] - [t] = -1 + z$

Butterfly velocity:

$$v_B{}^2 = T^{2-\frac{2}{z}} \Phi\left(\kappa / T^{\frac{1}{z\nu}}\right)$$

Diffusion constant:

$$\frac{D\lambda_L}{v_B{}^2} = \Psi\left(\kappa / T^{\frac{1}{z\nu}}\right)$$

For an AdS fixed point with scalar (single trace) deformation (z=1) :

$$v_B^2 = \frac{d+1}{2d}\left(1 - \gamma \frac{\kappa^2}{T^{2/\nu}} + \ldots\right), \qquad T \gg |\kappa|^\nu$$

γ : Non-negative

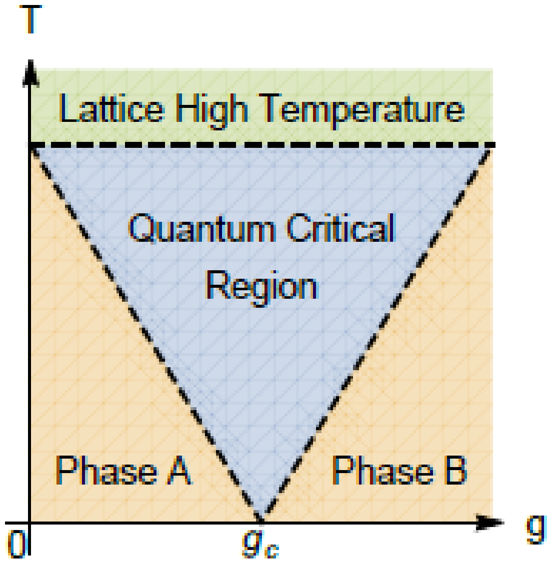
constant derermined by (d, ν)

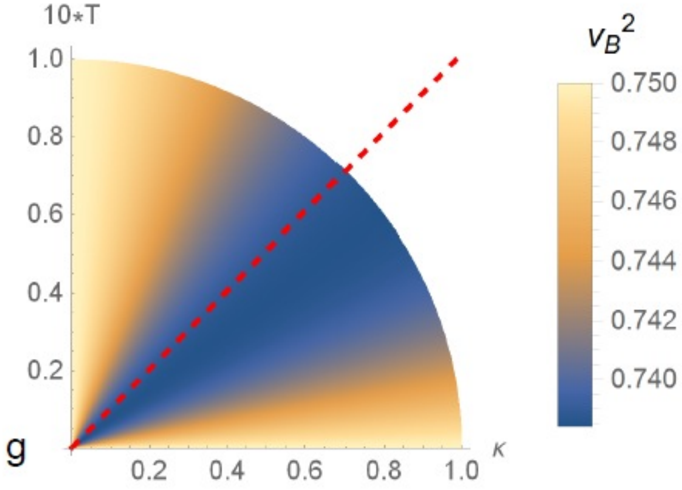
$$\frac{D\lambda_L}{v_B^2} = \frac{d}{d-1}\left(1 + \eta \frac{\kappa^2}{T^{2/\nu}} + \ldots\right), \qquad T \gg |\kappa|^\nu$$

$\eta \geq 0$

Transport property and universal bound

- Numerical check:



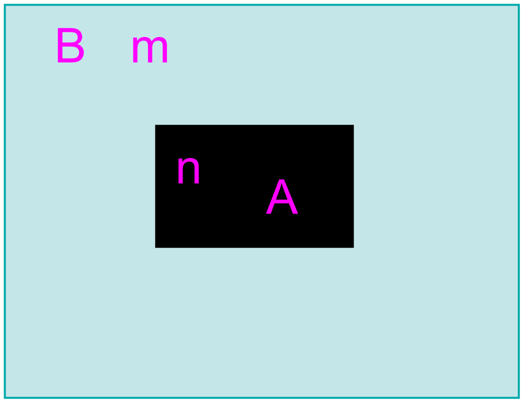


AdS-AdS domain wall

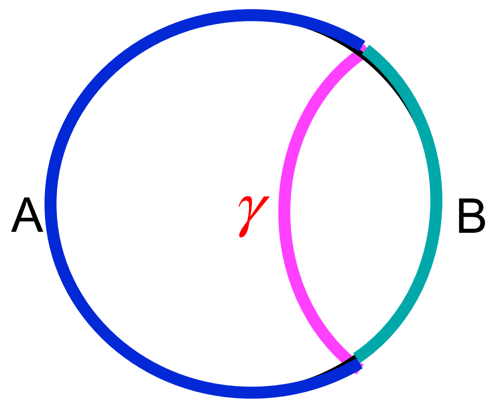
Doubly holographic setup

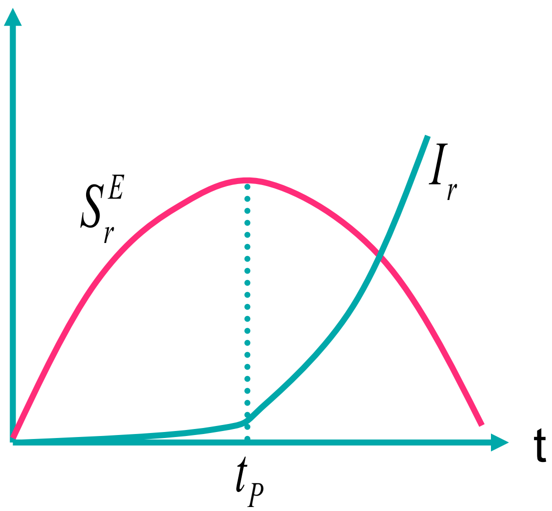
Background:

- **The geometric description of entanglement entropy sheds light on the black hole information loss paradox**
- **Doubly holographic setup provides new framework for the measure of entanglement entropy of the radiation.**
- **Black hole information loss paradox is attacked.**



$$S = \frac{A(\gamma)}{4G}$$





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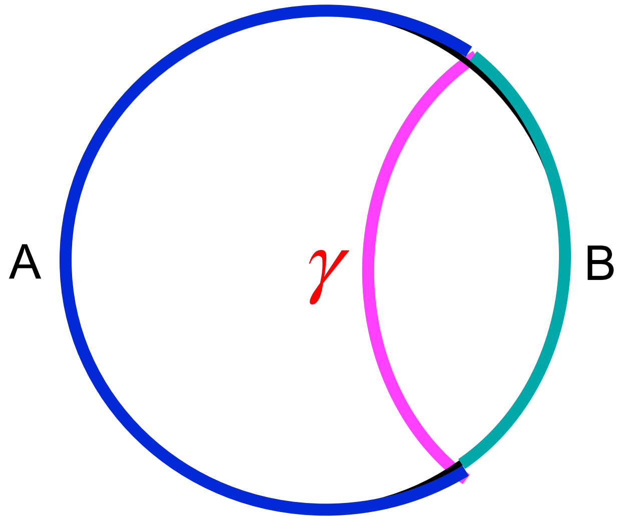
Doubly holographic setup

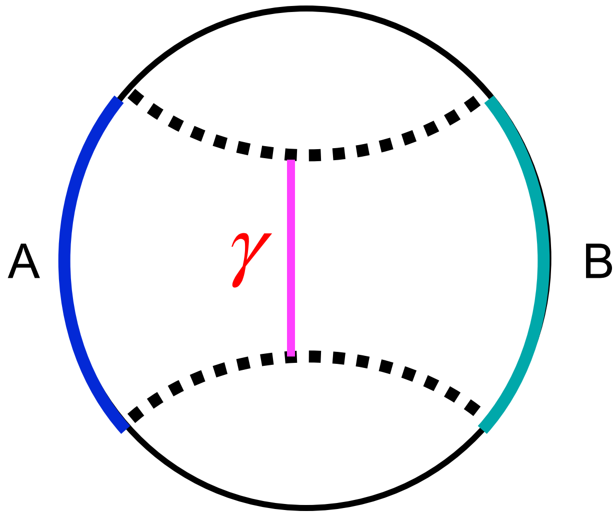
- **HEE:**

$$S = \frac{A(\gamma)}{4G}$$



- **PHEE**

$$S = \frac{A(\gamma)}{4G}$$

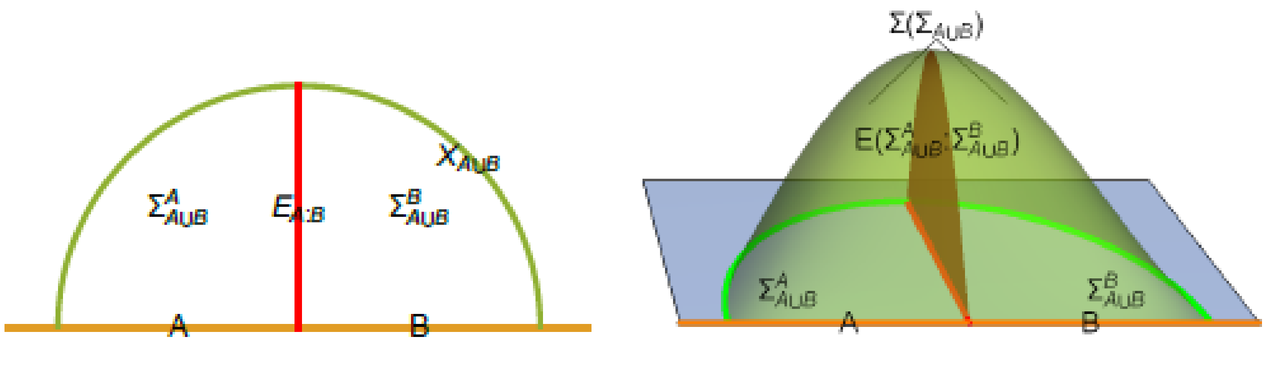


Takayanagi and Umemoto, Nature Phys. 14, 573{577 (2018), arXiv:1708.09393

Doubly holographic setup

- **PHEE in Doubly holographic setup:**

Ling,Liu,et.al., arXiv: 2109.09243



(a) (b)

$$S_r(A:B)=\min_{E_{A:B}}\left\{\frac{A[E_{A:B}]}{4G^{(d)}}+S^r(\Sigma^A_{A\cup B}:\Sigma^B_{A\cup B})\right\}$$

⬇

$$S_r(A:B)=\min_{E_{A:B}}\left\{\frac{A[E_{A:B}]}{4G^{(d)}}+\frac{Area[E(\Sigma^A_{A\cup B}:\Sigma^B_{A\cup B})]}{4G^{(d+1)}}\right\}$$

Quantum Extremal Surface (QES)

Doubly holographic setup

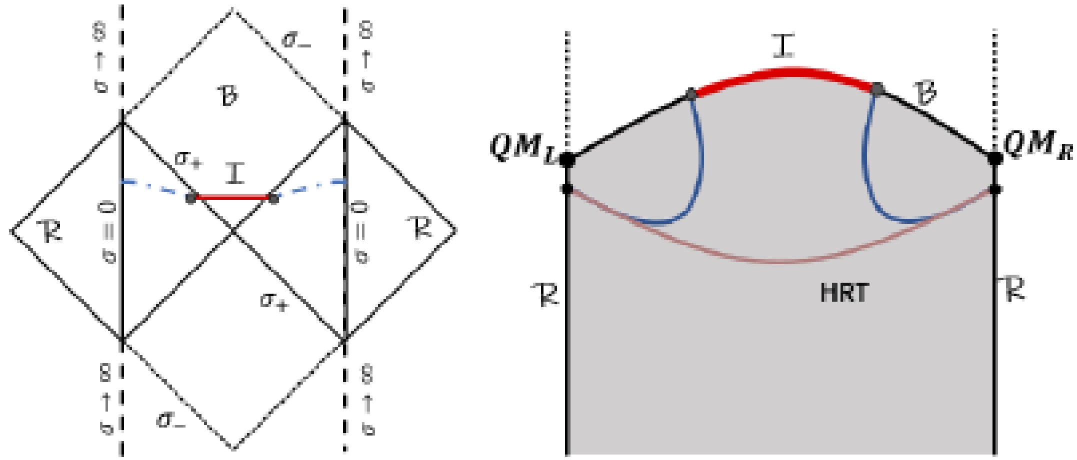
- **Island in doubly holographic setup**

Ling,Liu,Xian, **JHEP** 03 (2021) 251

$$S_r = \min_I \left\{ \underset{I}{ext} \left[S_e[R \cup I] + \frac{A[\partial I]}{4G} \right] \right\}$$

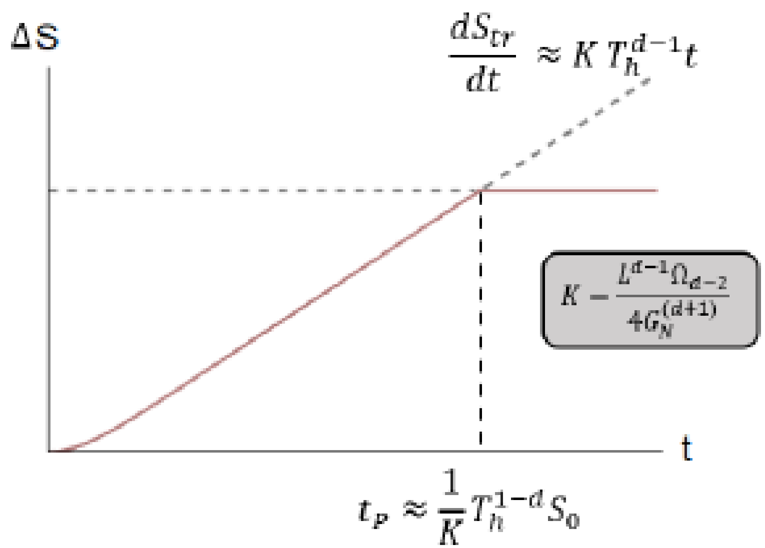
⬇

$$S_r = \min_I \left\{ \frac{A[\gamma_{R\cup I}]}{4G^{(d+1)}} + \frac{A[\partial I]}{4G^{(d)}} \right\}$$



(a) (b)

- **Page-like curve:**

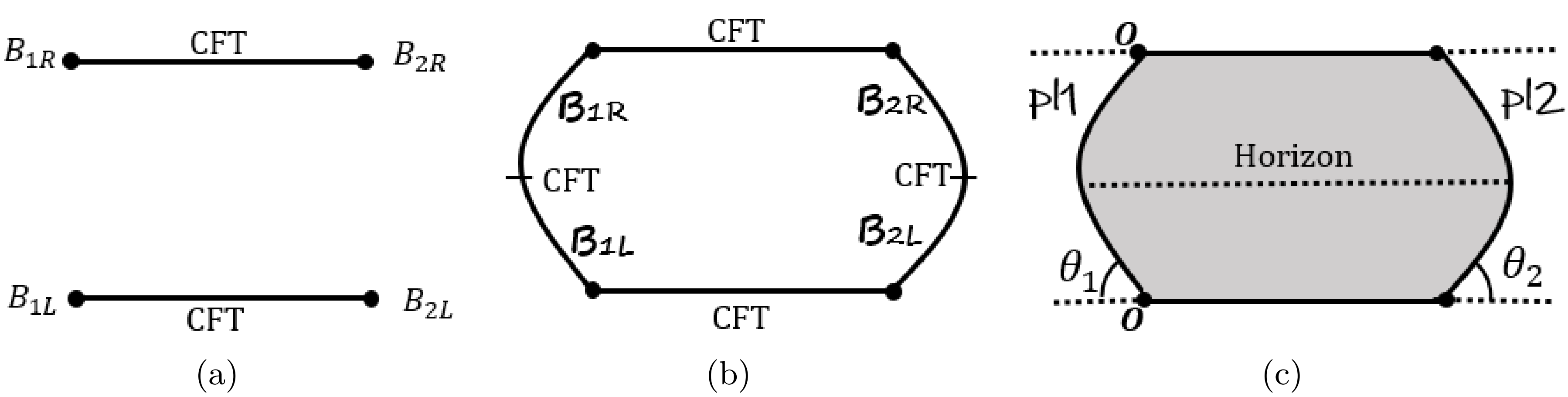


Doubly holographic setup

- **Radiation between two black holes:**

Liu, Xian, Peng, Ling, JHEP 09 (2022) 179

$$
\begin{aligned}
I =& \frac{1}{16\pi G_N^{(d+1)}}\left[\int d^{d+1}x\sqrt{-g}\left(R+\frac{d(d-1)}{L^2}\right)+2\int_\partial d^dx\sqrt{-h_\partial}K_\partial\right. \\
&\left. -\int d^{d+1}x\sqrt{-g}\frac{1}{2}F^2+2\sum_{i=1}^{2}\left(\int_{pl_i} d^dx\sqrt{-h_i}\,(K_i-\alpha_i)-\int_{pl_i\cap\partial} d^{d-1}x\sqrt{-\Sigma_i}\,\theta_i\right)\right] \\
&+\sum_{i=1}^{2}\left[\frac{1}{16\pi G_{b,i}^{(d)}}\int d^dx\sqrt{-h_i}R_{h_i}+\frac{1}{8\pi G_{b,i}^{(d)}}\int_{pl_i\cap\partial} d^{d-1}x\sqrt{-\Sigma_i}\,k_i\right]. \qquad (2
\end{aligned}
$$



Doubly holographic setup

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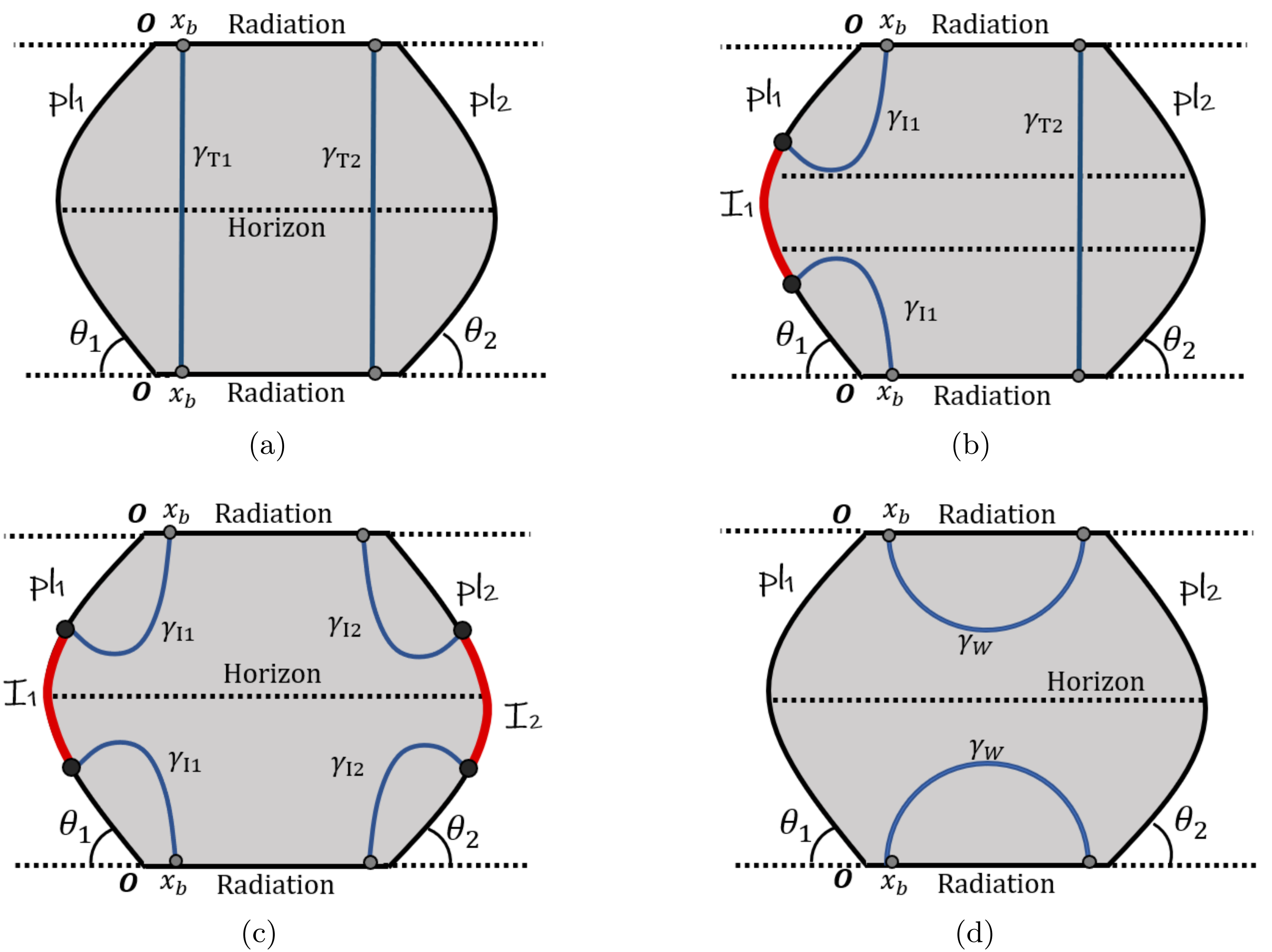

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K. Hashimoto, et.al. [arXiv:1802.08313].

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J. Tan and C. B. Chen, [arXiv:1908.01470].

Y. K. Yan, S. F. Wu, X. H. Ge and Y. Tian, [arXiv:2004.12112].

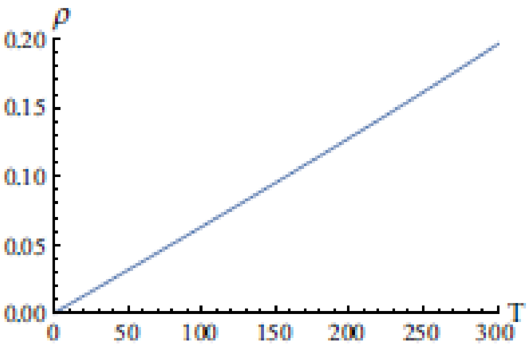
T. Akutagawa, K. Hashimoto and T. Sumimoto. [arXiv:2005.02636].

K. Hashimoto, K. Ohashi and T. Sumimoto, [arXiv:2108.08091].

Universal behavior of strange metals

1、Linear resistivity

$$\rho_{DC} = \sigma_{DC}^{-1} \sim T$$



2、Hall angle

$$\theta_H^{-1} \sim T^2$$

Gauge/Gravity duality → ? → **Non-fermi liquid theory**

Deep learning and holographic gravity

- **The background:**

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R+\frac{6}{L^2}-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)$$

$$ds^2=\frac{1}{z^2}\left[-f(z)dt^2+\frac{dz^2}{f(z)}+dx^2+dy^2\right] \qquad A=\mu(1-z)dt$$

$$f(z)=1-z^3-\frac{\mu^2z^3}{4}+\frac{\mu^2z^4}{4}$$

- **The perturbations:**

$$\delta A_x=a_x(x,z)e^{-i\omega t}$$

$$a_x(x,z)=a_x^{(0)}(x)+a_x^{(1)}(x)z+\ldots$$

$$\sigma=\frac{1}{i\omega}G^R(\omega)=-i\frac{a_x^{(1)}(x)}{\omega a_x^{(0)}(x)}=-i\frac{a_x^{(1)}(x)}{\omega}$$

- **Inverse problem:** Given σ $\quad f(z)=?$

$$\delta A_x = A_x(z)e^{-i\omega t}$$

$$z^4 A_x''(z) + 2z^3 A_x'(z) - z^2 A_x'(z)B(z) + C(z)A_x = 0$$

Generalization of conductivity:

$$\sigma(z,\omega) = \frac{\partial_z A_x(z)}{i\omega A_x(z)}$$

$$\boxed{z^4(i\omega\sigma'(z) - \omega^2\sigma^2) + i\omega\sigma(2z^3 - z^2 B(z)) + C(z) = 0}$$

Discretizing the equation:

$$\Delta z = \frac{z_h - z_b}{N-1}, \quad z(n) = z_b + n\Delta z$$

$$z(0) = z_b, \qquad z(N-1) = z_h$$

Deep learning and holographic gravity

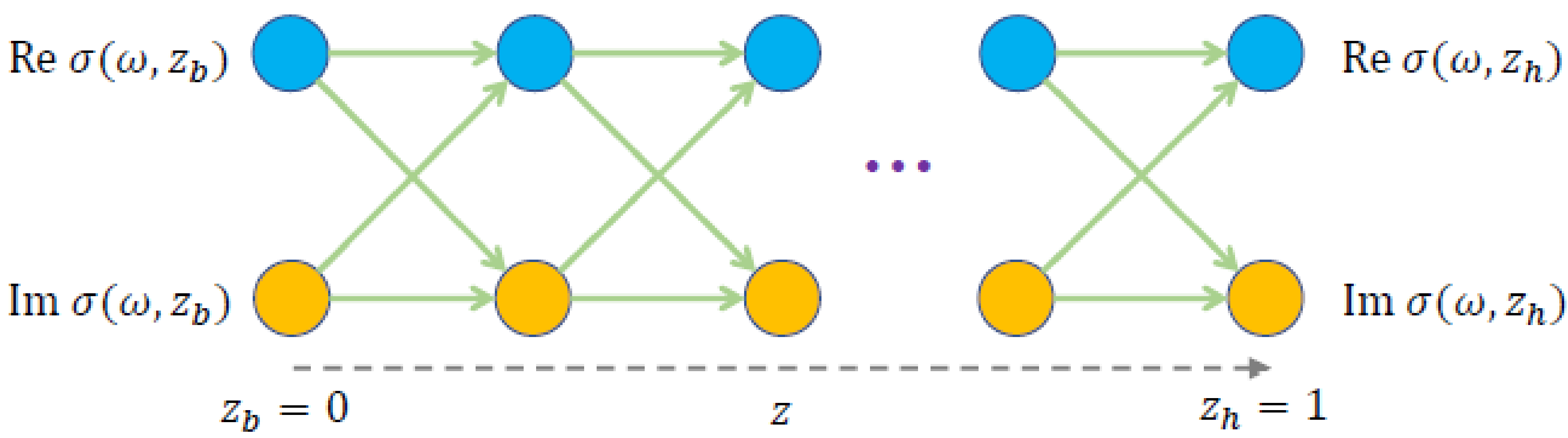
$$\begin{pmatrix} \mathrm{Re}\sigma_r(z+\Delta z) \\ \mathrm{Im}\sigma_r(z+\Delta z) \end{pmatrix} = \mathbf{W}_{2\times 2} \begin{pmatrix} \mathrm{Re}\sigma_r(z) \\ \mathrm{Im}\sigma_r(z) \end{pmatrix} + \vec{\mathrm{b}}.$$

The weight matrix:

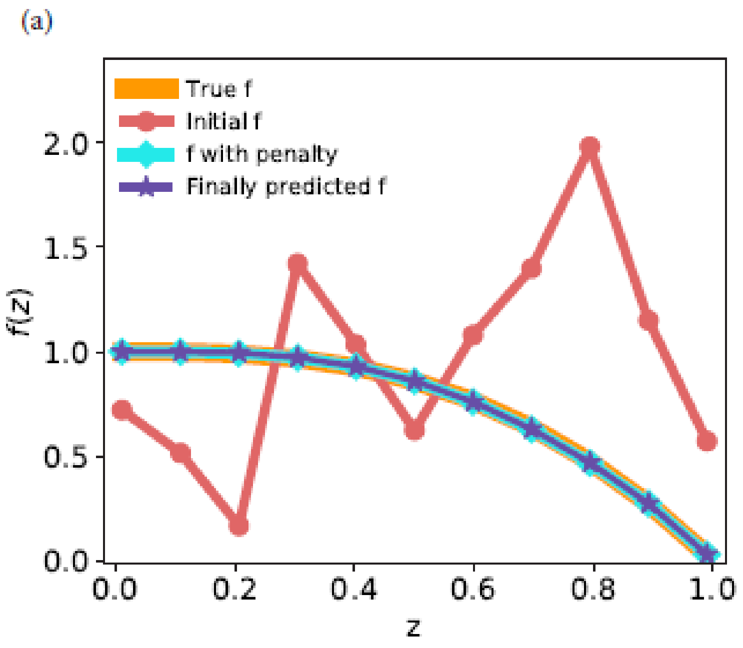
$$\mathbf{W}_{2\times 2} = \begin{pmatrix} 1-\Delta z\frac{f'(z)}{f(z)} & \Delta z\frac{2\omega}{4\pi T(1-z)} \\ -\Delta z\frac{2\omega}{4\pi T(1-z)} & 1-\Delta z\frac{f'(z)}{f(z)} \end{pmatrix}$$

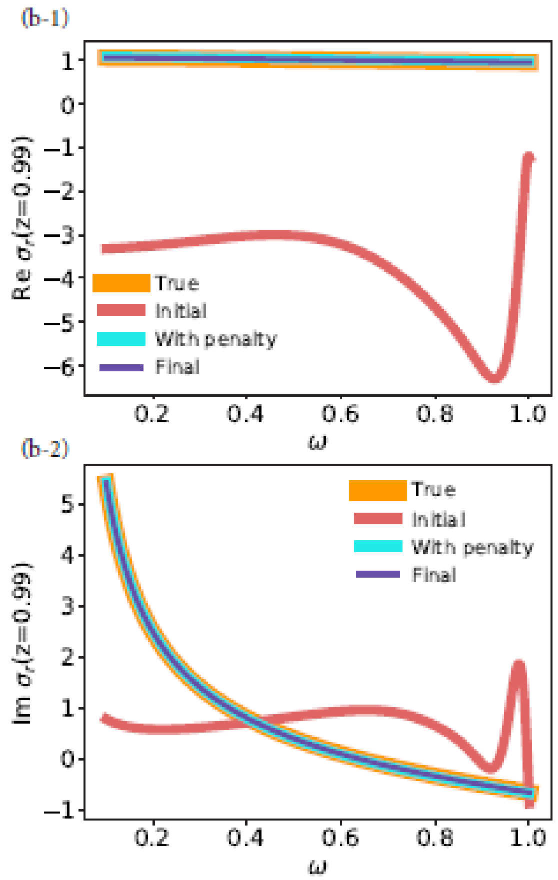
The bias term:

$$\vec{\mathrm{b}} = \Delta z \begin{pmatrix} -\frac{f'(z)}{f(z)}\frac{1}{4\pi T(1-z)} - \frac{1}{4\pi T(1-z)^2} + 2\omega\mathrm{Im}\sigma_r(z)\mathrm{Re}\sigma_r(z) \\ -\frac{\omega}{(4\pi T)^2(1-z)^2} + \frac{\omega}{f^2(z)} - \frac{\mu^2 z^2}{\omega f(z)} + \omega(\mathrm{Im}\sigma_r(z))^2 - \omega(\mathrm{Re}\sigma_r(z))^2 \end{pmatrix}$$



- **The training result:**





$$f(z)=1-z^3-\frac{\mu^2 z^3}{4}+\frac{\mu^2 z^4}{4}$$

Summary

- The scaling symmetry near the horizon plays a significant role in controlling the universal behavior of transport quantities in holography.

- Doubly holographic setup is powerful to take the quantum entanglement of matter in the bulk into account.

- Deep learning would be helpful for us to construct the appropriate holographic model for the specific phenomenon observed in strongly coupled system.

Appendices

Holographic Butterfly Effect

Classical Butterfly Effect

- **Lyapunov exponent:** λ_L

Early perturbations increase with time exponentially

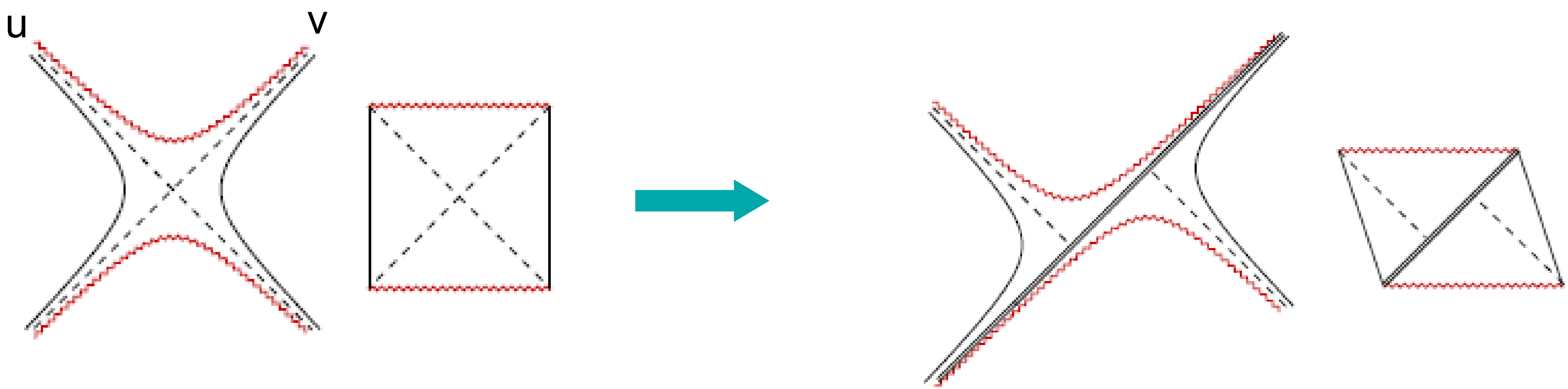
$$\delta a(t) \cong \delta a(0) e^{\lambda_L t}$$

The criteria for butterfly effect

$$\lambda_L > 0$$

Holographic Butterfly Effect

Holographic Butterfly Effect



- **A dimensionless effect**

$$\frac{E}{M} e^{2\pi t_w / \beta} \qquad \beta = \frac{2\pi l^2}{r_H}$$

Fast Scrambling time

$$t_w \to t_* : \frac{\beta}{2\pi} \log \frac{M}{E} \xrightarrow{E \sim T_H} \frac{\beta}{2\pi} \log S$$

A freely falling particle with

$$\delta T_{uu} \sim E_0 e^{\frac{2\pi}{\beta} t_i} \delta(u) \delta(x, y)$$

- Shock wave solution as a result of backreaction

$$h(x, \tilde{y}) \sim \frac{E_0 e^{\frac{2\pi}{\beta}(t_i - t_*) - m|x|}}{|x|^{1/2}}$$

$$\lambda_L = \frac{2\pi}{\beta}, v_B = \frac{2\pi}{\beta m}$$

Arxiv:1306.0622, Shenker and Stanford

HBE in Quantum Critical Region

- The simplest model with Einstein gravity and scalar field in d+2 dimensions

$$L = R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)$$

The equations of motion

$$R_{\mu\nu} - \frac{1}{d} g_{\mu\nu} V(\phi) - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi = 0$$
$$\nabla^2\phi - V'(\phi) = 0$$

d+2 AdS solution $\quad \phi = \phi_* \;\Rightarrow\; V'(\phi_*) = 0,\; V(\phi_*) < 0$

$$ds^2 = \frac{L^2}{r^2}\left(-dt^2 + dr^2 + d\vec{x}^2\right),\; -V(\phi_*)L^2 = (d+1)d,\;\; \phi = \phi_*$$

d+2 AdS-Schwarzschild black hole with flat horizon

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + f(r)^{-1}dr^2 + d\vec{x}^2\right), f(r) = 1 - \left(\frac{r}{r_h}\right)^{d+1}$$

$$v_B{}^2 = \frac{d+1}{2d},\;\; T = \frac{d+1}{4\pi r_h}$$

HBE in Quantum Critical Region

- Outlines of scalar deformations over AdS BH:

$$V(\phi)=V(\phi_*)+\frac{m^2}{2}(\phi-\phi_*)^2+\dots,$$

From the equation of motion for scalar field

$$\phi=\phi_*+\phi_- r^{\Delta_-}+\dots+\phi_+ r^{\Delta_+}+\dots.$$

$$\Delta_\pm=\frac{1}{2}\left(d+1\pm\sqrt{(d+1)^2+4m^2L^2}\right)$$

Near boundary

$$\theta\equiv\frac{\Delta_-}{d+1}$$

$$\phi_+=\phi_- H(\theta) r_h^{\Delta_- -\Delta_+}+O(\phi_-^2),$$

$$H(\theta)=-\frac{\Gamma(1-\theta)^2\Gamma(2\theta)}{\Gamma(2-2\theta)\Gamma(\theta)^2}$$

The scalar field back-reacts to the metric at order $o(\phi-\phi_*)^2$, and affects the butterfly velocity through the horizon formula

$$v_B^{\ 2}=\frac{d+1}{2d}\left(1-\phi_-^2 r_h^{2\Delta_-} I(\theta)\frac{d+1}{2d}\right)+O(\phi_-^3)$$

$$-\frac{1}{2}<\theta<\frac{1}{2},\quad I(\theta)\geq 0$$

$$I(\theta)=\int_0^1 y^2\phi_1'(y)^2\,dy$$

HBE in Quantum Critical Region

- Appendix: Details of deformation

$$ds^2 = -E(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}^2, \quad \phi = \phi(r)$$

We take the coordinate transformation $\xi = 1 - f(r) = \left(\frac{r}{r_h}\right)^{d+1}$ Boundary: $\xi = 0$ Horizon: $\xi = 1$

We write such deformations into the series expansion

$$\phi(\xi) = \phi_* + \lambda\phi_1(\xi) + \lambda^2\phi_2(\xi) + \ldots,$$
$$E(\xi) = \frac{L^2}{r_h^2}(1-\xi)\xi^{-\frac{2}{d+1}}(1 + \lambda E_1(\xi) + \lambda^2 E_2(\xi) + \ldots),$$
$$B(\xi) = \ldots,$$
$$C(\xi) = \ldots,$$

$$v_B{}^2 = \frac{E'(r_h)}{dC'(r_h)}$$

Roberts and Swingle, arXiv:1603.09298

The solutions of the leading order $E_1 = B_1 = C_1 = 0$

$$(1-\xi)\phi_1'' - \phi_1' + \frac{(1-\theta)\theta}{\xi^2}\phi_1 = 0$$ Gaussian hypergeometric functions

$\xi \to 0$ $$\phi_1 : \xi^\theta + \ldots + H(\theta)\xi^{1-\theta} + \ldots = \left(\frac{r}{r_h}\right)^{\Delta_-} + \ldots + H(\theta)\left(\frac{r}{r_h}\right)^{\Delta_+} + \ldots \quad \lambda = \phi_- r_h^{-\Delta_-} = \phi_+ H(\theta) r_h^{-\Delta_+}$$

$\xi \to 1$

The solutions of the sub-leading order $C_2'(\xi) = E_2'(\xi) = \frac{1}{d+1}B_2'(\xi) = -\frac{1}{d\xi^2}I_1(\xi;\theta)$

$$I_1(\xi;\theta) = \int_0^\xi y^2\phi_1'(y)dy \qquad I_1(1;\theta) = I(\theta)$$

HBE in Quantum Critical Region

- The standard quantization method

$$[O_\phi] = \Delta_+$$

$$\phi_- = \kappa_s W'((\Delta_+ - \Delta_-)\phi_+)$$

For single trace deformation $\kappa_s = \phi_-$

$$W(O_\phi) = O_\phi, \quad 1/v = [\kappa_s] = \Delta_-$$

$$v_B{}^2 = \frac{d+1}{2d}\left(1 - \gamma \frac{\kappa_s^2}{T^{2/v}}\right) + O(\kappa_s^3)$$

$$\gamma = \frac{(d+1)I(\theta)}{2d}\left(\frac{d+1}{4\pi}\right)^{2/v}$$

For double trace deformation

$$W(O_\phi) = \frac{1}{2}O_\phi{}^2, \quad [O_\phi{}^2] = 2\Delta_+ > d+1, \; 1/v = [\kappa_s] = \Delta_- - \Delta_+ < 0$$

Such deformation is irrelevant.

HBE in Quantum Critical Region

- The charge diffusion constant

$$L = R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \frac{1}{4}F^2$$

$$D_c = \frac{\sigma_{DC}}{\chi} = C(r_h)^{\frac{d}{2}-1}\int_0^{r_h} C(r)^{-\frac{d}{2}}\sqrt{B(r)E(r)}dr$$

Blake, arXiv:1603.08510

For single trace deformation

$$\frac{D_c\lambda_L}{v_B^2} = \psi(0)\left(1+\eta\frac{\kappa_s^2}{T^{2/\nu}}\right)+O(\kappa_s^3)$$

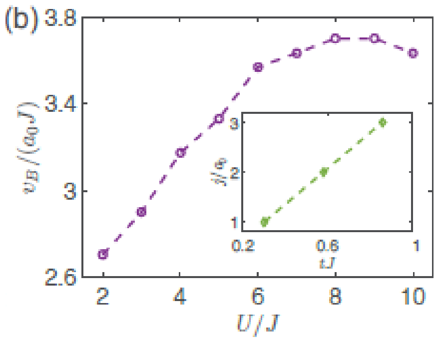
$$\eta = J(\theta,d)\left(\frac{d+1}{4\pi}\right)^{2/\nu} \geq 0, \qquad \psi(0)=\begin{cases}\log(\frac{\Lambda_{UV}}{2\pi T}), & d=1\\ \frac{d}{d-1}, & d>1\end{cases}$$

HBE in Quantum Critical Region

- Comparison with many body system:

(1 + 1) Bose-Hubbard model

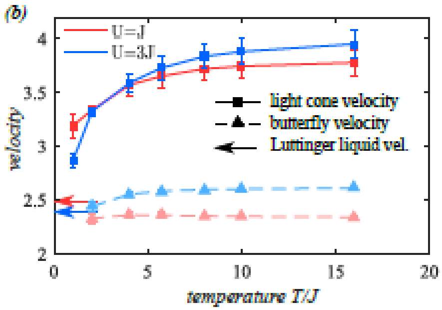
Mott insulator-superfluid transition happens at U/J ≈ 3.4
BKT transition
a peak of λ_L near U/J ≈ 3



(2 + 1) O(N) nonlinear sigma model

well defined quasi-particles [Sachdev:1999];

a QPT with broken-symmetry at g_c.



Finite T dynamic depends on dispersion relation $\varepsilon_k{}^2 = c^2k^2 + \mu^2$,
where thermal mass μ is monotonous with respect to g:

Maximization behavior of $\{\tau_\phi, \tau_L, v_B\}$ may be absent at g ; g_c in such weak chaos system.

附录：格林函数与输运系数

$$Z_b[J] = \left\langle e^{-\int L_J} \right\rangle_{CFT} = Z_{QG}\left[b.c.\ \text{depends on J}\right] \xrightarrow{N?\ 1} e^{-S_{grav}}\Big|_{EOM}$$

$$Z_{boundary}[J] = \int D\phi\, \mathrm{e}^{iS[\phi_{cl}]+i\int d^d x J o}$$

$$\lim_{z\to 0} \frac{\phi_{cl}(z,x)}{z^{\Delta}} = J(x)$$

$$G(x-y) = -i\langle T o(x) o(y)\rangle_{QFT} = -\frac{\delta^2 S[\phi_{cl}]}{\delta J(x)\delta J(y)}\Big|_{\phi(z=0)=J}$$

$$\phi(z,x^{\mu}) = e^{ik_{\mu}x^{\mu}} f_k(z), \quad k_{\mu}x^{\mu} = -\omega t + \vec{k}\cdot\vec{x}$$

$$f_k(z) = A_k z^{\Delta_+} + B_k z^{\Delta_-}$$

附录：引力/流体对偶

- 流体动力学量

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} G^R_{xy,xy}(\omega, 0)$$

$$G^R_{xy,xy}(\omega, 0) = \int dt dx e^{i\omega t} \theta(t) \left\langle \left[T_{xy}(t, x), T_{xy}(0, 0) \right] \right\rangle$$

粘滞系数/熵边界假设：

$$\frac{\eta}{s} \geq \frac{h}{4\pi}$$