A holographic Bottom-up approach to the light nuclide spectrum

Holographic QCD Seminar

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Motivation: from holographic hadrons to holographic nuclides

Why nuclides?

- Nuclides are a strongly coupled system of nucleons (protons and neutrons).
- Therefore, we can use holographic tools to describe them!
- What observables can we study? In principle, we will start with the mass spectrum.

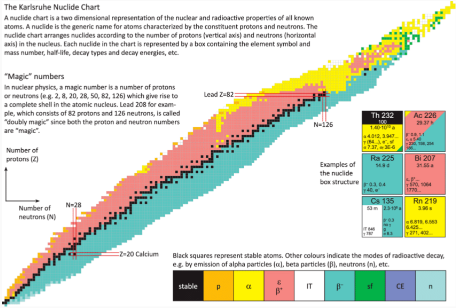

Figure 1: Nuclide table

How do we define nuclides in holography?

- We will follow the AdS/QCD lead:
  - At the boundary: A nuclide is a collection of strong-interacting SU(2) symmetric nucleons.
  - At the bulk: Nuclides are dual to eigenstates coming from a holographic potential.
  - Corner stone: as in bottom-up AdS/QCD, the dilaton field will capture the strong-interaction phenomenology.

Let's see how we define hadrons in bottom-up holography!!

How we describe hadronic spectrum in bottom-up holography

- Hadrons are organized in taxonomic structures called Regge Trajectories (RT).
    - These trajectories define the hadronic mass in terms of hadronic numbers, such as spin, angular momentum, or excitation number.
    - Each hadron is a point in these trajectories.
    - RTs are a consequence of confinement.

- Holographic hadrons are dual to eigenstates coming from holographic potentials:

$$V(z) = \frac{1}{4}B'^2 - \frac{1}{2}B'' + M_5^2\, e^{2\,A(z)}$$

  With $A(z)$ the warp factor, $B(z) = \phi(z) - \beta\, A(z)$, β related to hadronic spin, $\phi(z)$ the dilaton field, and M_5 the bulk mass associated with the bulk fields dual to hadrons.
- $\phi(z)$ induces confinement.

Hadronic Identity: how we construct hadrons in bottom-up models

- From the holographic dictionary: Bulk field conformal dimension $\Delta \Longleftrightarrow$ dimension of the operator creating hadrons, i.e., $\Delta = \dim \mathcal{O}_h = \tau + L$.
- This information is encoded into the bulk mass M_5:

$$
\begin{aligned}
M_5^2\, R^2 &= (\Delta - p)\,(\Delta - p - 1 + \beta) \\
&= (\Delta - p)\,(\Delta + p - 4)\,.
\end{aligned}
$$

Conclusion: the hadronic information is codified into the bulk mass!

Example: Soft wall model for light vector mesons

Light Vector Mesons (Phys. Rev. D 74 (2006) 015005)

AdS Bulk

$$
\begin{aligned}
dS^2 &= \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right) \\
I_{\mathrm{H}} &= \int d^5x\, \sqrt{-g}e^{-\kappa^2 z^2}\, \mathcal{L}_{\mathrm{M}} \\
\mathcal{L}_{\mathrm{M}} &= -\frac{1}{4\, g_v^2} F_{mn}\, F^{mn}
\end{aligned}
$$

Holographic Meson

$$
-\phi''(z) + V_{\mathrm{SWM}}(z)\,\phi(z) = M_n^2\,\phi(z)
$$

$$
V_{\mathrm{SWM}}(z) = \frac{4\,L^2 - 1}{4\,z^2} + \kappa^2\, z^2
$$

$$
M_n^2 = 4\kappa(n + \tfrac{L+1}{2}),\ \ n, L > 0,
$$

Example: Soft wall model for light vector mesons

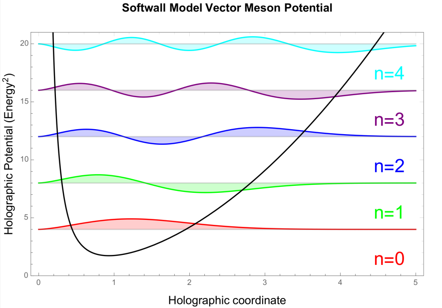

Figure 2: Holographic potential for vector mesons with $\kappa = L = 1$, with the ground state and the first excited states.

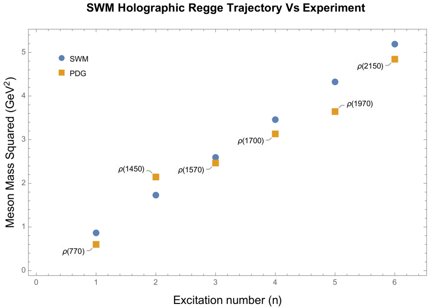

Figure 3: ρ meson trajectory with $M_n^2 = 4\,\kappa^2(n+1)$, and $\kappa = 0.465$ GeV y $L = 1$. Experimental data are taken from Particle Data Group.

We will use the bulk mass M_5 to define a given nuclide via the *conformal dimension*, as in the case of hadrons.

Holographic nuclides

Main idea and starting points

- The nuclear interactions in the nuclear realm can be understood as *residual effects* coming from strong interactions. Thus we will use a dilaton field $\Phi(\zeta)$ to model strong force in the nuclide.
- A Holographic nuclide, with spin p, having A nucleons: $\Longleftrightarrow$ bulk field with spin p.
- Twist operator $\tau \Longleftrightarrow$ Nuclide mass number A.
- For simplicity, we will have symmetric nuclides, i.e., $A = 2Z$, in S-wave ($L = 0$). $\Longrightarrow \Delta = A = 2Z$.
- The bulk mass will be defined as a function of the atomic number Z of a given nuclide: $M_5 = M_5(Z)$.

Geometric Setup

We start from

$$I_{\text{Bulk}} = \int d^5x \sqrt{-g}\, e^{-\Phi(\zeta)}\, \mathcal{L}_{\text{Nuclide}},$$

with

$$\mathcal{L}_{\text{Nuclide}} = \frac{(-1)^p}{2} \times \left[\frac{1}{g_p^2}\, g^{mn}\, g^{m_1\, n_1} \ldots g^{m_p\, n_p} \nabla_m A_{m_1 \ldots m_p} \nabla_n A_{n_1 \ldots n_p} - M_5^2\, g^{m_1\, n_1} \ldots g^{m_p\, n_p} A_{m \ldots m_p} A_{n \ldots n_p}\right].$$

p-form A_p will be associated with the nuclide at the boundary. These fields live in the usual AdS Poincare patch:

$$dS^2 = \frac{R^2}{\zeta^2}\left(d\zeta^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right).$$

Dilaton Engineering

General Equations of motion

$$\partial_\zeta \left[e^{-B(\zeta)}\, \psi'(\zeta) \right] + M_0^2\, e^{-B(\zeta)}\, \psi(\zeta) - \frac{M_5^2\, R^2}{\zeta^2}\, e^{-B(\zeta)}\, \psi(\zeta) = 0,$$

Holographic Potential

Constructed from the geometry and the dilaton, it will carry the information about the nuclide mass spectrum

$$V(\zeta, \Delta) = \frac{15 - 16\Delta + 4\Delta^2}{4\,\zeta^2} + \frac{(3 - 2\,p)}{2\,\zeta}\,\Phi' + \frac{1}{4}\,\Phi'^2 - \frac{\Phi''}{2}.$$

The nuclide spectrum will be the ground state of $V(\zeta, \Delta)$.

We will consider three possible static dilaton profiles: $\Phi = 0$ with a hard cutoff, $\Phi \propto \zeta^2$ and a WS-like one.

Let’s see how each model works!

Hard Wall Approach

Hard Wall nuclide

- Inspired by the hadronic HW [See EPJC 32, 529–533 (2004) and Phys. Rev. Lett 95, 261602, (2005)].
- Defined by imposing $\Phi(\zeta) = 0$ and a hard cutoff as $\Lambda_N = \frac{M_{^4_2\text{He}}}{\alpha_{2,1}} = 0.7794$ u.
- First kind Bessel zeroes give the mass spectrum: $M_0(\Delta) = \Lambda_N \, \alpha_{\Delta-2,1}$.

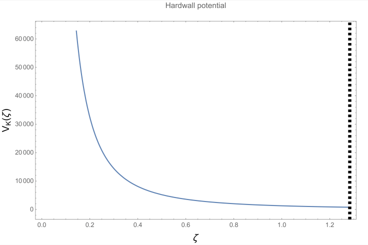

Figure 4: Holographic potential for potassium nuclide, $\Delta = 36, p = 3$.

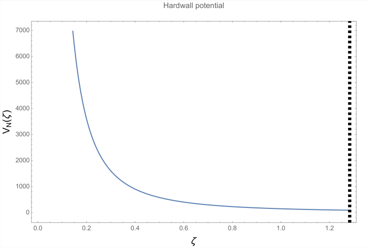

Figure 5: Holographic Potential for Nitrogen nuclide, $\Delta = 14$, $p = 1$.

Light Nuclide mass spectrum (Experimental Data Huang 2021)

Experimental nuclide data			HW-like Model	
Z	Nuclear Spin	M_0^{Exp} (u.)	M_0^{Th} (u)	Rel. Error (%)
2	0	4.00260	4.00260*	0.00
3	1	6.01511	5.91421	1.68
4	0	8.00530	7.74401	3.26
5	3	10.0129	9.52800	4.84
6	0	12.0000	11.2819	5.98
7	1	14.0030	13.0143	7.06
8	0	15.9949	14.7303	7.91
9	1	18.0009	16.4333	8.71
10	0	19.9949	18.1259	9.34
11	3	21.9944	21.4859	9.93
12	0	23.9850	23.1558	10.4

Parameters set: fitted with the Helium data. RMS error fitting 29 nuclides with one parameter: 11.6%.

Soft Wall Approach

Soft Wall Nuclide

- Inspired by the hadronic soft wall model [See Phys. Rev. D 74, 015005 (2006)].
- Defined by imposing $\Phi(\zeta) = \frac{\Delta}{2}\,\kappa_0\,\zeta^2$, where $\kappa_0 = 1.0006$ u, proportional to the proton mass.
- Nuclide Mass spectrum: $M_0^2(\Delta) = \Delta\,\kappa_0^2\,(\Delta - p)\,.$

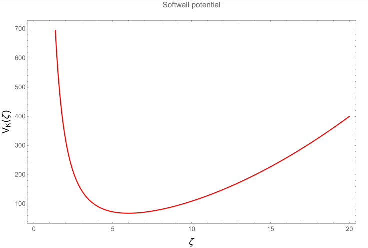

Figure 6: Holographic potential for potassium nuclide, $\Delta = 36, p = 3$.

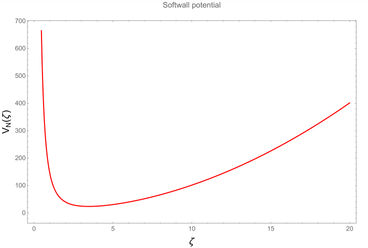

Figure 7: Holographic Potential for Nitrogen nuclide, $\Delta = 14$, $p = 1$.

Light Nuclide mass spectrum (Experimental Data Huang 2021)

Experimental nuclide data			SW-like Model	
Z	Nuclear Spin	M_0^{Exp} (u.)	M_0^{Th} (u)	Rel. Error (%)
2	0	4.00260	4.00260*	0.00
3	1	6.01511	5.480971	8.88
4	0	8.00530	8.005206	0.01
5	3	10.0129	8.372045	16.4
6	0	12.0000	12.00781	3.59
7	1	14.0030	13.49952	3.59
8	0	15.9949	16.01301	0.09
9	1	18.0009	16.01302	2.76
10	0	19.9949	17.50424	0.10
11	3	21.9944	20.01301	6.98
12	0	23.9850	20.45835	0.13

RMS error fitting 29 nuclides with one parameter: 4.4%.

Woods-Saxon Approach

Woods-Saxon-like dilaton

- Inspired by the so-called WS model, we look out for a dilaton that reproduces a WS potential profile in the bulk:

$$V_{WS}(\zeta) = \left[A_1 - \frac{A_2}{1 + \exp\left(\frac{\zeta - B}{\Delta}\right)}\right]\Delta$$

- With the potential, we reconstruct the dilaton using

$$V_{\mathrm{WS}}(\zeta) = \frac{15 - 16\Delta + 4\Delta^2}{4\,\zeta^2} + \frac{(3 - 2\,p)}{2\,\zeta}\,\Phi' + \frac{1}{4}\,\Phi'^2 - \frac{\Phi''}{2}.$$

WS-like dilaton

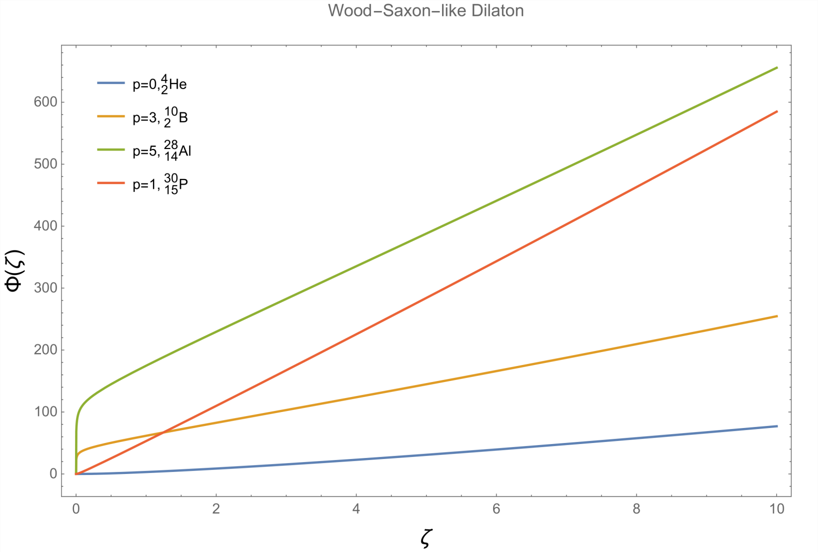

Figure 8: WS dilaton reconstruction for selected nuclides.

Light Nuclide mass spectrum (Experimental Data Huang 2021)

Experimental nuclide data			WS-like Model	
Z	Nuclear Spin	M_0^{Exp} (u.)	M_0^{Th} (u)	Rel. Error (%)
2	0	4.00260	4.00391	0.01
3	1	6.01511	6.10883	2.34
4	0	8.00530	8.16760	2.70
5	3	10.0129	10.2040	2.39
6	0	12.0000	12.2212	2.21
7	1	14.0030	14.2297	1.89
8	0	15.9949	16.2301	1.68
9	1	18.0009	18.2244	1.40
10	0	19.9949	20.2137	1.23
11	3	21.9944	22.1989	1.02
12	0	23.9850	24.1805	0.89

Parameters choice: $A_1 = A_2 = 1.863$u and $B = 2.5$u

RMS Error fitting 29 masses with three parameters: 1.2%

Which one is the best model?

Holographic $^{40}_{20}$Ca excited states								
Hardwall			Softwall			WS		
n	M_n (u)	ΔM_n(u)	n	M_n (u)	ΔM_n(u)	n	M_n (u)	ΔM_n(u)
0	**34.7183**	0	0	**40.02603**	0	0	**39.9523**	0
1	38.8472	4.1289	1	41.0145	0.9884	1	40.0085	0.0562
2	42.4237	7.7054	2	41.9796	1.9536	2	40.0641	0.1117
3	45.7299	11.012	3	42.9231	2.8971	3	40.1187	0.1665
4	48.8703	14.152	4	43.8463	3.8203	4	40.1728	0.2206
5	51.8971	17.179	5	44.7505	4.7244	5	40.2262	0.2740

This table summarizes the $^{40}_{20}$Ca ground state (in bold font) with the first five excited radial states for each holographic model considered.

Thus, at least from purely spectroscopic arguments, WS is the best choice!

Configurational entropy

Configurational entropy

- For continuous variables, it is defined as

$$S_C\,[f] = -\int d^d\,k\tilde{f}\,(k)\,\log\tilde{f}\,(k),$$

  where $\tilde{f}\,(k) = f\,(k)\,/f\,(k)_{\mathrm{Max}}$ defines the modal fraction, $f(k)_{\mathrm{Max}}$ is the maximum value assumed by $f(k)$.
- CE measures the relationship between the information content and EQM of a given physical system.
- CE measures (logarithmically) how spatially-localized are these EOM solutions.
- CE is increased when energy or particle number is increased.

To compute the DCE for a given physical system, we must address the following algorithm:

1. Obtain the localized solutions to the equations of motion.
2. Evaluate the on-shell energy density.
3. Transform to momentum space.
4. Calculate the modal fraction.
5. Evaluate the DCE integral expression.

CE for nuclides

- We will compute the differential CE (DCE) from the bulk momentum-energy tensor:

$$T_{mn} = \frac{2}{\sqrt{-g}} \frac{\partial \left[\sqrt{-g}\, \mathcal{L}_{\text{Nuclide}}\right]}{\partial\, g^{mn}},$$

- From this quantity, we compute the energy density as

$$\rho(\zeta) \equiv T_{00} = \frac{e^{-\Phi(\zeta)}}{2} \left(\frac{\zeta^2}{R^2}\right)^p \times \\ \left\{ \left[\frac{1}{g_p^2} \left(M_0^2\, \psi^2 + \psi'^2\right) - \frac{M_5^2\, R^2}{\zeta^2} \psi^2 \right] \right\} \Omega,$$

  Where Ω is a constant carrying plane wave information and polarization contraction factors.

Next, we will Fourier-transform the density ρ and compute DCE!

Numerical Results for DCE

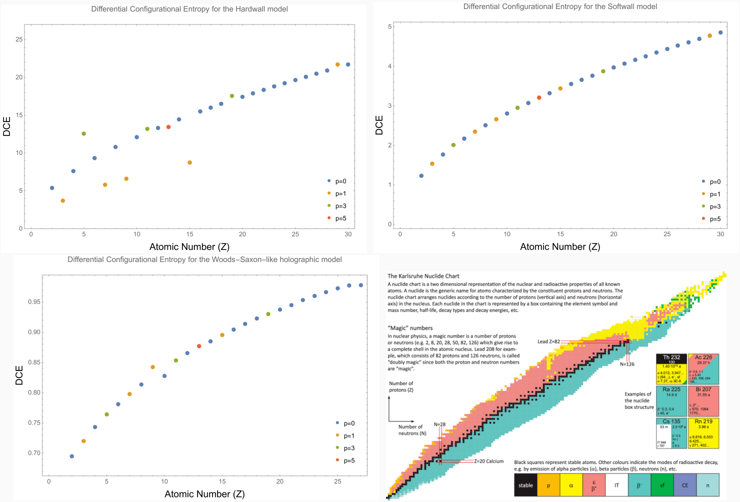

Figure 9: Differential Configurational Entropy (DCE) for holographic models considered as a function of the atomic number Z.

Conclusions

Remarks

- For each nuclide, we have a Holographic potential, defined by Δ and the nuclear spin.
- Each holographic mass is the ground state mass from the associated potential.
- By analyzing for each potential the splitting between the ground state and the excited modes, we can conclude if the ground state and excited modes are related as excited states of the same nuclide or different particles (as in the hadronic Regge Trajectories). For WS, this shifting is less than a proton mass. Thus, the entire spectrum can be associated with a single nuclide.
- WS and SW have a DCE growing with the nucleon content. Thus heavier nuclides are expected to be more unstable than light ones.
- Among the three models studied, the best choice to describe nuclides is WS.

The End

Thank you!