强耦合暗物质的相变与引力波

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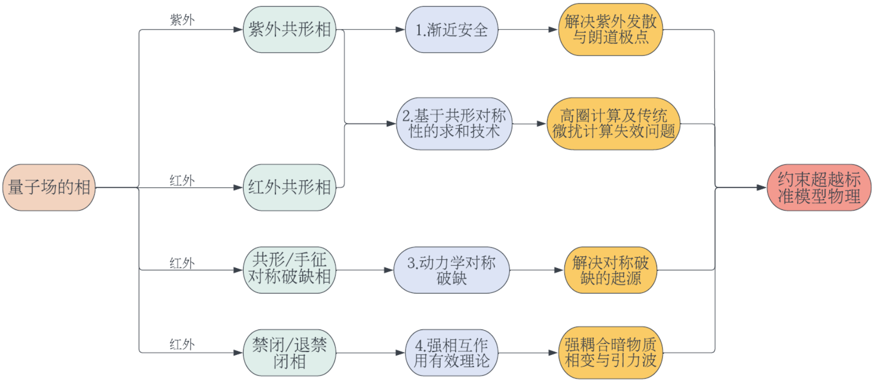
国科大高能核物理课题组前沿讲座

A Landscape of Phases in QFT and its Relation to BSM Physics



Motivations and what we do

- (Dark) composite dynamics: non perturbative physics, dynamical symmetry breaking, UV completion, naturalness
- (Dark) composite dynamics face challenges to be explored both theoretically and via experiments and thus any extra test is important
- We unify first principle lattice simulations and gravitational wave astronomy to constrain the dark sector

What composes the strongly coupled sector?

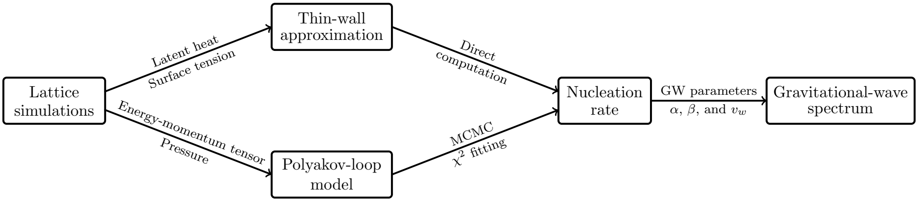
- Dark Yang-Mills theories
- Pure gluons ⇒ confinement-deconfinement phase transition
- Gluons + Fermions
  - Fermions in fundamental representation ⇒ chiral phase transition
  - Fermions in adjoint rep. ⇒ confinement & chiral phase transition
  - Fermions in 2-index symmetric rep. ⇒ confinement & chiral phase transition
- Gluons + Fermions + Scalars (not explored yet)

How to describe the strongly coupled sector?

- Pure gluons
  - Polyakov loop model (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005; Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060)
  - Matrix Model (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
  - Holographic QCD model (Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD **105** (2022) 066020; Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRL **128** (2022) 131101)
- Gluons + Fermions
  - Polyakov loop improved Nambu-Jona-Lasinio model (Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003; Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - linear sigma model (Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
  - Polyakov Quark Meson model (Schaefer, Pawlowski, Wambach, PRD **76** (2007) 074023)

Procedure of pure gluon case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005



Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB 72 (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) \, \mathrm{d}\tau \right]$$

  The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- Convenient to define the trace of the Polyakov loop as an order parameter for the $Z(N)$ symmetry

$$\ell\left(\vec{x}\right) = \frac{1}{N}\mathrm{Tr}_c[\mathbf{L}]\,,$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \to e^{i\phi}\ell, \qquad \phi = \frac{2\pi j}{N}, \qquad j = 0, 1, \cdots, (N-1)$$

- The expectation value of ℓ i.e. $< \ell >$ has the important property:

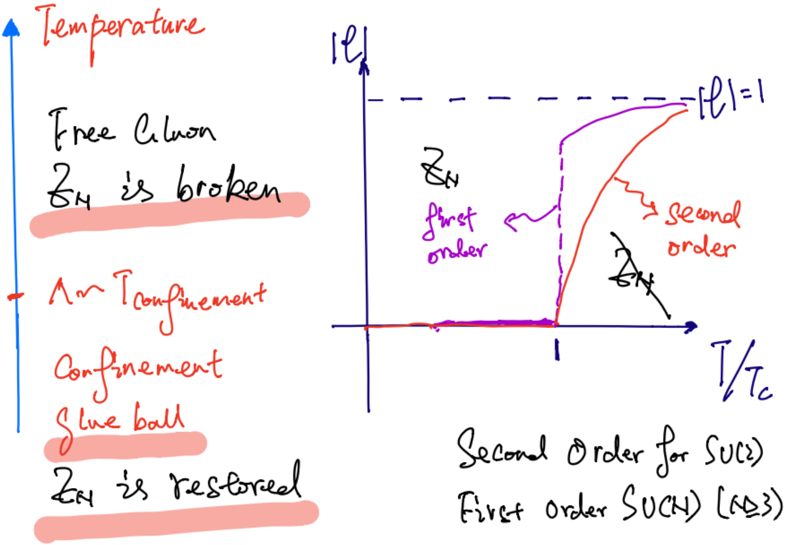
$$\langle \ell \rangle = 0 \quad (T < T_c,\ \mathrm{Confined})\,; \qquad \langle \ell \rangle > 0 \quad (T > T_c,\ \mathrm{Deconfined})$$

- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle \ell \rangle = \exp\left(i\frac{2\pi j}{N}\right)\ell_0, \qquad j = 0, 1, \cdots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \to 1$ as $T \to \infty$

Summary of Pure Gluon Facts



Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\rm PLM}^{\rm (poly)} = T^4\left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 + \cdots - b_3\left(\ell^N + \ell^{*N}\right)\right)$$

$$\text{where}\quad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 + a_4\left(\frac{T_0}{T}\right)^4$$

"$\cdots$" represent any required lower dimension operator than ℓ^N i.e. $(\ell\ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

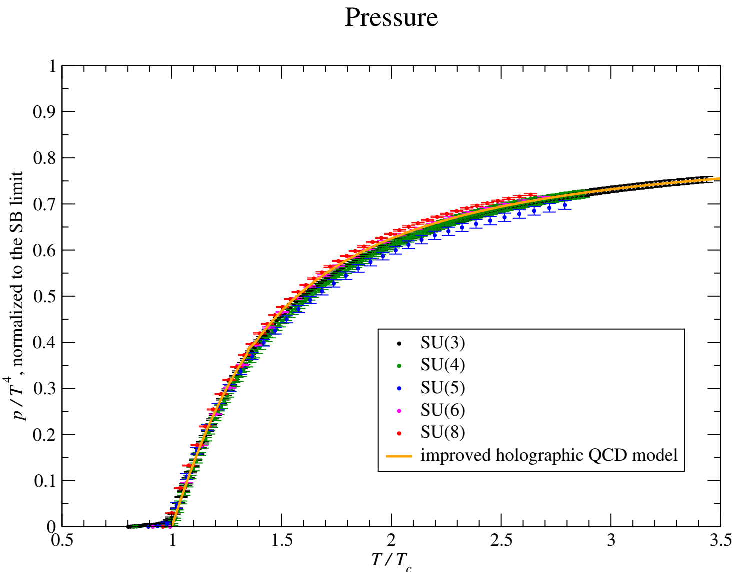
$$V_{\rm PLM}^{\rm (3log)} = T^4\left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right)\right)$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

- The a_i, b_i coefficients in $V_{\rm PLM}^{\rm (poly)}$ and $V_{\rm PLM}^{\rm (3log)}$ are determined by fitting the lattice results

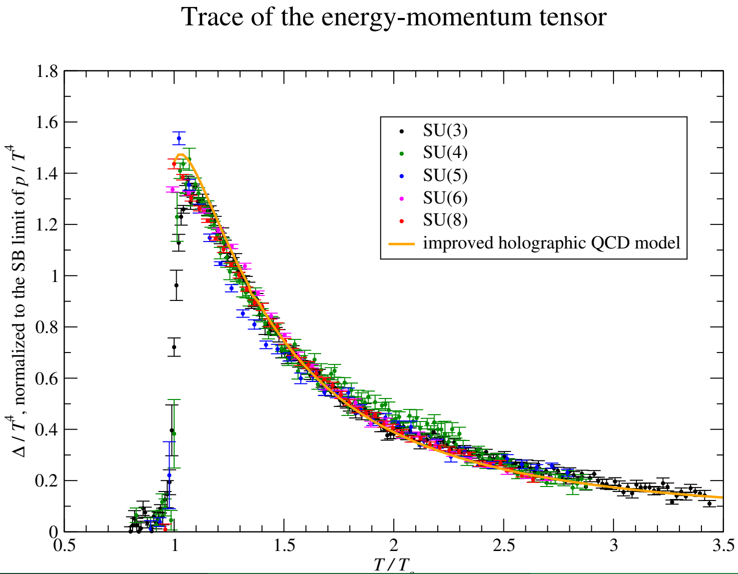
Fitting the Coefficients Using the Lattice Results: I

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

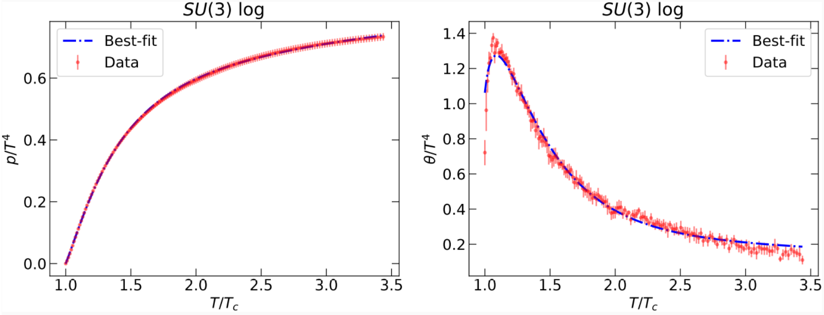
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: III

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Fitted to lattice data of pressure and the trace of energy momentum tensor.



Fitting the Coefficients Using the Lattice Results: IV

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

表: The parameters for the best-fit points.

N	3	3 log	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

Include Fermions

(K. Fukushima, PLB **591** (2004) 277; Ratti, Thaler Weise, PRD **73** (2006) 014019)

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.

- The Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model is used to describe phase-transition dynamics in dark gauge-fermion sectors
- The finite-temperature grand potential of the PNJL models can be generically written as

$$V_{\mathrm{PNJL}} = V_{\mathrm{PLM}}[\ell, \ell^*] + V_{\mathrm{cond}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\mathrm{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$$

- $V_{\mathrm{PLM}}[\ell, \ell^*]$ is the Polyakov loop model potential (discussed above)
- $V_{\mathrm{cond}}\left[\langle\bar{\psi}\psi\rangle\right]$ represents the condensate energy
- $V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right]$ denotes the fermion zero-point energy
- The medium potential $V_{\mathrm{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$ encodes the interactions between the chiral and gauge sector which arises from an integration over the quark fields coupled to a background gauge field

The PNJL model Lagrangian (Fukushima, Skokov, PPNP **96** (2017) 154)

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\rm PNJL} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{\rm 4F} + \mathcal{L}_{\rm 6F} + \mathcal{L}_k$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{\rm 4F}$ is the four-quark interaction which reads:

$$\mathcal{L}_{\rm 4F} = G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma^5\lambda^a\psi)^2], \qquad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{\rm 6F}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{\rm 6F} = G_D[\det(\bar{\psi}_{Li}\psi_{Rj}) + \det(\bar{\psi}_{Ri}\psi_{Lj})]$$

The Condensate Energy (Fukushima, Skokov, PPNP **96** (2017) 154)

- In $\mathcal{L}_{4\mathrm{F}}$, the condensate energy then comes from the combination

$$(\bar{\psi}\lambda^0\psi)^2 + (\bar{\psi}\lambda^3\psi)^2 + (\bar{\psi}\lambda^8\psi)^2 = 2(\bar{u}u)^2 + 2(\bar{d}d)^2 + 2(\bar{s}s)^2$$

- We use the trick is to rewrite $(\bar{u}u)^2$ as

$$\begin{aligned}(\bar{u}u)^2 &= [(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle]^2 = (\bar{u}u - \langle\bar{u}u\rangle)^2 + 2\langle\bar{u}u\rangle\,(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle^2 \\ &\simeq -\langle\bar{u}u\rangle^2 + 2\langle\bar{u}u\rangle\bar{u}u\,,\end{aligned}$$

  where the $(\bar{u}u - \langle\bar{u}u\rangle)^2$ term is dropped in the spirit of the mean-field approximation.

- The $2\langle\bar{u}u\rangle\bar{u}u$ term contributes to the constituent quark mass of u
- The $-\langle\bar{u}u\rangle^2$ term leads to a contribution to the condensate energy
- Similar procedures can be applied to $(\bar{d}d)^2$ and $(\bar{s}s)^2$, and to $\mathcal{L}_{6\mathrm{F}}$ gives $\langle\bar{u}u\rangle^3$ and we obtain the total condensate energy:

$$V_{\mathrm{cond}} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3\,, \qquad \sigma \equiv \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle = \frac{1}{3}\langle\bar{\psi}\psi\rangle$$

The Constituent Quark Mass and Zero Point Energy: I

(Fukushima, Skokov, PPNP **96** (2017) 154)

- In $\mathcal{L}_{6\mathrm{F}}$, there is also $\langle\bar{u}u\rangle^2\bar{u}u$ term contributes to the constituent quark mass of u
- The total constituent quark mass from $\mathcal{L}_{4\mathrm{F}}$ and $\mathcal{L}_{6\mathrm{F}}$ is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\mathrm{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\mathrm{dim}(\mathrm{R})\, 2N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} E_p\,, \qquad E_p = \sqrt{\vec{p}^{\,2} + M^2}$$

  E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$
- The above integration diverges and a regularization is required. We choose a sharp three-momentum cutoff Λ entering the expression for observables and thus also a parameter of the theory.
- Parameters: G_S, G_D, Λ; Observables: M, f_π, m_σ

The Constituent Quark Mass and Zero Point Energy: II

(Fukushima, Skokov, PPNP **96** (2017) 154)

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\frac{\dim(\mathrm{R})N_f\Lambda^4}{8\pi^2}\left[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right],$$

in which $\xi \equiv \frac{M}{\Lambda}$.

Medium Potential: Finite Temperature Contribution

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\text{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(\ln\left[1 + e^{-\beta(E-\mu)}\right] + \ln\left[1 + e^{-\beta(E+\mu)}\right] \right)$$

$$\to -2T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_c \left\{ \left(\ln\left[1 + \mathbf{L}\, e^{-\beta(E-\mu)}\right] + \ln\left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)}\right] \right) \right\}$$

- $\mathbf{L}$ is the Polyakov loop:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp\left[i \int_0^{1/T} A_4(\vec{x}, \tau)\, \mathrm{d}\tau \right]$$

- In this work, we consider chemical potential $\mu = 0$.

Second Part: Bubble Nucleation and Gravitatioanl Wave

Bubble Nucleation: Generic Discussion

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

- The generic three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\rho}{\mathrm{d}r} \right)^2 + V_{\text{eff}}(\rho, T) \right] ,$$

where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$

Bubble Nucleation: Confinement Phase Transition

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

- Confinement phase transition occurs for pure gluon and adjoint fermions
- $[\ell] = 0$ dimensionless while $[\rho] = 1$, we rewrite ρ as $\rho = \ell\, T$ and convert the radius into a dimensionless quantity $r' = r\, T$:

$$S_3(T) = 4\pi T \int_0^\infty \mathrm{d}r'\, r'^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\ell}{\mathrm{d}r'} \right)^2 + V'_{\text{eff}}(\ell, T) \right] ,$$

which has the same form as the above generic equation.
- The bubble profile (instanton solution) is obtained by solving the E.O.M. of the $S_3(T)$

$$\frac{\mathrm{d}^2\ell(r')}{\mathrm{d}r'^2} + \frac{2}{r'} \frac{\mathrm{d}\ell(r')}{\mathrm{d}r'} - \frac{\partial V'_{\text{eff}}(\ell, T)}{\partial \ell} = 0$$

- The boundary conditions (deconfinement $\to$ confinement) are

$$\frac{\mathrm{d}\ell(r'=0, T)}{\mathrm{d}r'} = 0\,, \qquad \lim_{r'\to 0} \ell(r', T) = 0$$

- We used the method of overshooting/undershooting (`Python` package)

Bubble Profile of Confinement Phase Transition

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

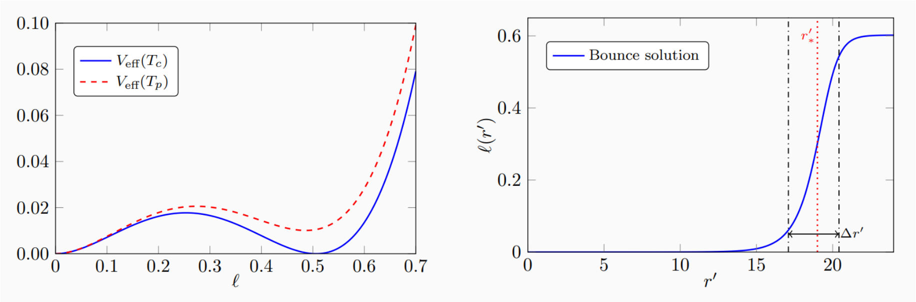


图: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$) lying the confinement phase, the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$) lying the deconfinement phase, the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Bubble Nucleation: Chiral Phase Transition

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552)

- Chiral phase transition occurs when including fermions
- $\bar{\sigma}$ is classically nonpropagating in PNJL and it's kinetic term is induced only via quantum fluctuations
- We thus include its wave-function renormalization Z_σ with

$$Z_\sigma^{-1} = -\frac{\mathrm{d}\Gamma_{\sigma\sigma}(q^0, \mathbf{q}, \bar{\sigma})}{\mathrm{d}\mathbf{q}^2}\bigg|_{q^0=0, \mathbf{q}^2=0}, \qquad \Gamma_{\sigma\sigma} = -i \sum 2\,\text{point 1PI}\,\sigma\sigma\,\text{graph}$$

- The three-dimensional Euclidean action and E.O.M. are modified to:

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{Z_\sigma^{-1}}{2}\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r}\right)^2 + V_{\text{eff}}(\bar{\sigma}, T)\right]$$

$$\frac{\mathrm{d}^2\bar{\sigma}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} - \frac{1}{2}\frac{\partial \log Z_\sigma}{\partial \bar{\sigma}}\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r}\right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{\mathrm{d}\bar{\sigma}(r=0,T)}{\mathrm{d}r} = 0\,, \qquad \lim_{r\to\infty} \bar{\sigma}(r,T) = 0$$

Gravitational Wave Parameters: Inverse Duration Time

- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use percolation temperature T_p: the temperature at which 34% of false vacuum is converted
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*)e^{\beta(t-t_*)}$$

- The inverse duration time then follows as

$$\beta = -\frac{\mathrm{d}}{\mathrm{d}t}\frac{S_3(T)}{T}\bigg|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T\frac{\mathrm{d}}{\mathrm{d}T}\frac{S_3(T)}{T}\bigg|_{T=T_*},$$

where we used that $\mathrm{d}T/\mathrm{d}t = -H(T)T$.

Gravitational Wave Parameters: Strength Parameter I

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022)003, arXiv:2109.11552.)

- We define the strength parameter α from the trace of the energy-momentum tensor θ weighted by the enthalpy

$$\alpha = \frac{1}{3}\frac{\Delta\theta}{w_+} = \frac{1}{3}\frac{\Delta e \; - \; 3\Delta p}{w_+}\,, \qquad \Delta X = X^{(+)} - X^{(-)},\ \text{for}\ X = (\theta, e, p)$$

$(+)$ denotes the meta-stable phase (outside of the bubble) while $(-)$ denotes the stable phase (inside of the bubble).

- The relations between enthalpy w, pressure p, and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}\,, \qquad e = \frac{\partial p}{\partial \ln T} - p\,,$$

which are extracted from the effective potential with

$$p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

Gravitational Wave Parameters: Strength Parameter II

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

- α is thus given by

$$\alpha = \frac{1}{3}\frac{4\Delta V_{\text{eff}} - T\frac{\partial \Delta V_{\text{eff}}}{\partial T}}{-T\frac{\partial V_{\text{eff}}^{(+)}}{\partial T}},$$

- For confinement phase transition: $\alpha \approx 1/3$ (ΔV_{eff} is negligible since $e_+ \gg p_+$ and $e_- \sim p_- \sim 0$ in PLM potential)
- For chiral phase transition: we find smaller values, $\alpha \sim \mathcal{O}(10^{-2})$, due to the fact that more relativistic d.o.f.s participate in the phase transition
- Relativistic SM d.o.f.s do not contribute to our definition of α since they are fully decoupled from the phase transition but these d.o.f.s will play a role to dilute the GW signals

GW parameters α, β and PNJL observables

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

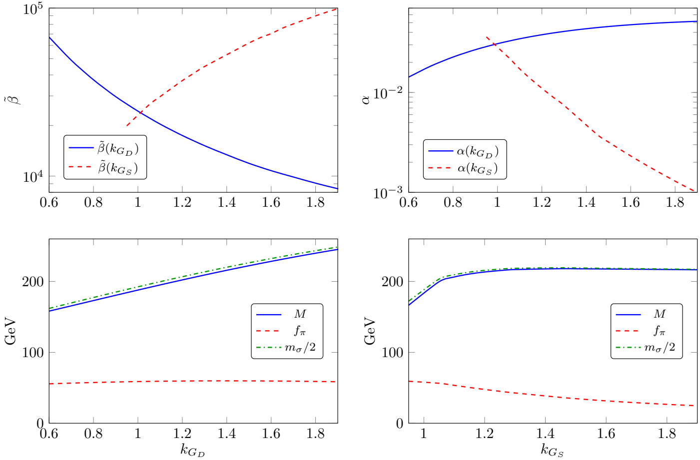


图: The GW parameters $\tilde{\beta}$, α with the observables M, f_π, and m_σ as a function of $G_S = k_{G_S} \cdot 4.6\,\text{GeV}^{-2}$ and $G_D = k_{G_D} \cdot (-743\,\text{GeV}^{-5})$. We use $T_c = 100\,\text{GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S,\text{crit}} = 0.882$, no chiral symmetry breaking occurs.

Gravitational-wave spectrum

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

- Contributions from bubble collision and turbulence are subleading
- The GW spectrum from sound waves is given by

$$h^2\Omega_{\mathrm{GW}}(f) = h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \left(\frac{f}{f_{\mathrm{peak}}}\right)^3 \left[\frac{4}{7} + \frac{3}{7}\left(\frac{f}{f_{\mathrm{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\mathrm{peak}} \simeq 1.9 \cdot 10^{-5}\,\mathrm{Hz} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \left(\frac{T}{100\,\mathrm{GeV}}\right) \left(\frac{\tilde{\beta}}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \simeq 2.65 \cdot 10^{-6} \left(\frac{v_w}{\tilde{\beta}}\right) \left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \Omega_{\mathrm{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.

$$\Omega_{\mathrm{dark}} = \frac{\rho_{\mathrm{rad,dark}}}{\rho_{\mathrm{rad,tot}}} = \frac{g_{*,\mathrm{dark}}}{g_{*,\mathrm{dark}} + g_{*,\mathrm{SM}}}$$

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{sw} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\text{sw}} = \sqrt{\tau_{\text{sw}}}\,\kappa_v$$

- τ_{sw} is dimensionless and measured in units of the Hubble time (H.-K. Guo, Sinha, Vagie and White, JCAP **01** (2021) 001)

$$\tau_{\text{sw}} = 1 - 1/\sqrt{1 + 2\frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f}} \Rightarrow \tau_{\text{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \quad \text{for } \beta >> 1$$

where $\bar{U}_f$ is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} \mathrm{d}\xi\, \xi^2 \frac{v(\xi)^2}{1 - v(\xi)^2} \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w. At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

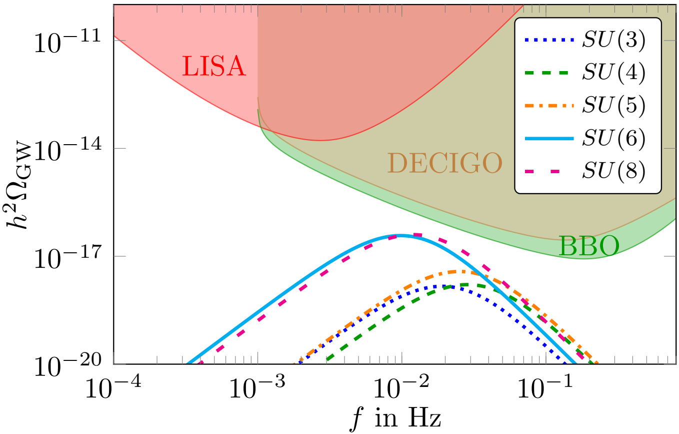


图: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with Tc$= 1\,\mathrm{GeV}$.

A Landscape of GW Signatures with Pure Gluon

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

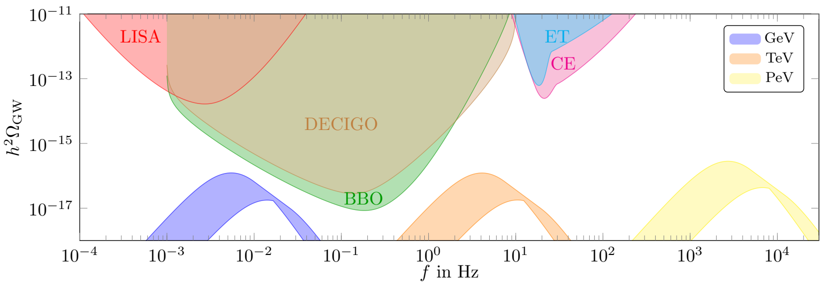


图: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Signal to Noise Ratio

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

$$\mathrm{SNR} = \sqrt{\frac{3\mathrm{year}}{s}\int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} \mathrm{d}f \left(\frac{h^2\Omega_{\mathrm{GW}}}{h^2\Omega_{\mathrm{det}}}\right)^2}$$

$h^2\Omega_{\mathrm{GW}}$ is the GW spectrum while Ω_{det} is the sensitivity curve of the detector.

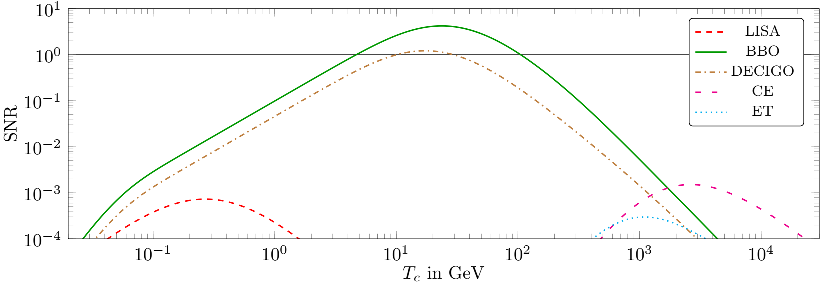


图: We display the SNR for the phase transition in a dark $SU(6)$ sector as a function of the confinement temperature Tc from experiments of LISA, BBO, DECIGO, CE, and ET. We assume an observation time of three years.

Landscape of GW spectrum with three Dirac fermions

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

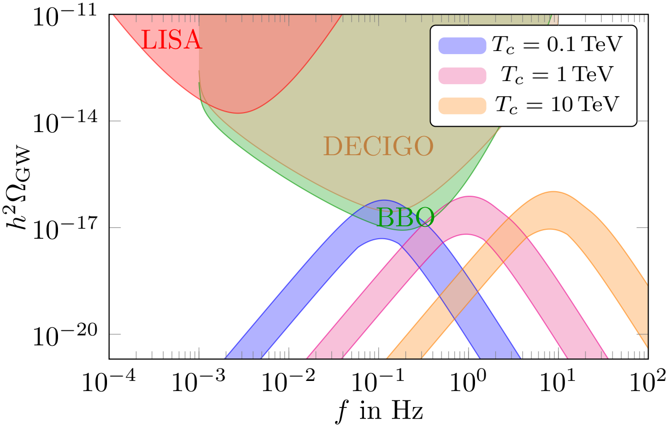


图: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures. The band comes from varying wall velocity $c_s \le v_w \le 1$.

Representation Matters

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

Rep.	flavour	chiral PT	conf.-deconf.
Fund.	3	1st	X
adjoint	1	2nd	1st
2-index Sym.	1	2nd	1st

表: Representations versus different phase transitions.

- Need small N_f to remain below the conformal Banks-Zaks window ($N_f \leq 2$ for adjoint and $N_f \leq 3$ for 2-index symmetric under $SU(3)$).

Signal to Noise Ratio for Different Representations

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

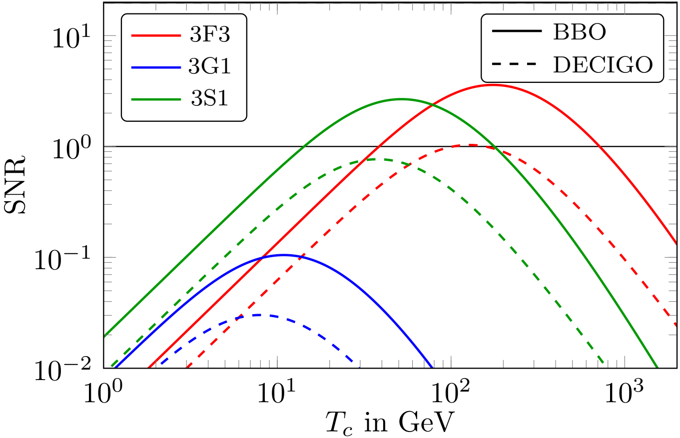


图: Signal-to-noise ratio as a function of the critical temperature for the best-case scenarios of each model at BBO and DECIGO with an observation time of 3 years.

Future Thinking

- The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3} \frac{\sigma(T_c)^3}{L(T_c)^2} \frac{T_c^2}{(T_c - T)^2} \, ,$$

- The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T} \bigg|_{T=T_p} \approx \mathcal{O}(10^3) \frac{T_c^{1/2} L}{\sigma^{3/2}} \, .$$

- $\tilde{\beta}$ stems from the competition between the surface tension and latent heat. $L \sim N^2$ while σ can be either $\sigma \sim N$ or $\sigma \sim N^2$ with limited data up to $SU(8)$
- How to construct models with smaller latent heat and larger surface tension to enhance the gravitational wave signals?

Thank you for your attention!

About Center Symmetry and Confinement

- The standard physical interpretation is that it is related to the free energy of adding an external static color source in the fundamental representation.

$$\ell(\vec{x}) = \exp(-\mathrm{F}\beta)$$

- In the confinement phase, Polyakov loop is zero corresponds to infinity free energy to add a color source and the same time center symmetry is unbroken.

Center Symmetry $Z(N)$ at Nonzero Temperature

- The boundary conditions in imaginary time τ the fields must satisfy are:

$$A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0)\,, \qquad q(\vec{x},\beta) = -q(\vec{x},0)\ ,$$

where gluons as bosons must be periodic in τ while quarks as fermions must be anti-periodic.

- 't Hooft first noticed that one can consider more general gauge transformations which are only periodic up to Ω_c

$$\Omega(\vec{x},\beta) = \Omega_c, \quad \Omega(\vec{x},0) = 1 \qquad \left(\text{here, } \Omega_c = e^{i\phi}I,\ \phi = \frac{2\pi j}{N}\right).$$

- Color adjoint fields are invariant under this transformation, while those in the fundamental representation are not:

$$A^\Omega(\vec{x},\beta) = \Omega_c^\dagger A_\mu(\vec{x},\beta)\,\Omega_c = A_\mu(\vec{x},\beta) = +A_\mu(\vec{x},0)\ ,$$
$$q^\Omega(\vec{x},\beta) = \Omega_c^\dagger q(\vec{x},\beta) = e^{-i\phi} q(\vec{x},\beta) \neq -q(\vec{x},0)\ .$$

- Thermal Wilson line transforms like an adjoint field under local $SU(N)$ gauge transformations:

$$L(x) \to \Omega^\dagger(\vec{x},\beta)\,L(\vec{x})\Omega^\dagger(\vec{x},0)\ .$$

About Thin Wall Approximation

The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3} \frac{\sigma(T_c)^3}{L(T_c)^2} \frac{T_c^2}{(T_c - T)^2} \,,$$

The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale. From this we infer that

$$T_c - T_p \approx \sqrt{\frac{16\pi\sigma^3 T_c}{3L^2 \cdot \mathcal{O}(150)}} \,,$$

and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T} \bigg|_{T=T_p} \approx \mathcal{O}(10^3) \frac{T_c^{1/2} L}{\sigma^{3/2}} \,.$$

$\tilde{\beta}$ stems from the competition between the surface tension and latent heat.