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Holographic QCD Model For $N_f = 4$

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Outline

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Introduction

- AdS/ CFT correspondence claims: Strongly coupled 4-dimensional gauge theory = Gravitational theory in 5-dimensional AdS space time.

- Gauge theory at zero temperature ⇔ AdS_5 spacetime.
- Gauge theory at finite temperature ⇔ AdS_5 black hole.

- Top-down approach to AdS/QCD: start from a string theory in high dimensions and try to construct a low dimensional theory with features similar to QCD.

- Bottom-up approach to AdS/QCD: start from known QCD phenomenology and try to understand which feature its high dimension gravitational dual should have.

AdS/CFT dictionary

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4D theory	5D theory
Gauge invariant QCD operator $O(x)$	Field $\Psi(x,z)$
Hadrons with quantum numbers of O	Normalizable modes $\Psi_n(x,z)$
Hadron mass square	Eigenvalue of a 5D wave eq.
Conformal dimension Δ	5D mass m_5: $m_5^2R^2 = (\Delta - p)(\Delta + p - 4)$
UV regime	Small $z \rightarrow \varepsilon$
IR regime	Large $z \rightarrow z_m$
Decay constant $< 0\|O\|hadron >$	$\left.\frac{\Psi'(z)}{z^n}\right\|_{z\rightarrow 0}$

$$dS^2 = e^{2A_s}\left(-dt^2 + dz^2 + dx^i dx_i\right)$$
$$A_s = -\log(z), \quad \Phi = \mu^2 z^2$$

Formalism

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- The AdS/ CFT dictionary dictates: local symmetries in 5D ⇨ global symmetries in 4D. The chiral symmetry: $SU_L(N_f) \times SU_R(N_f)$[hep-ph/0501128, hep-ph/0602229]. Let is consider the number of flavor N_f=4 [2007.02273, 1702.08417, 2110.08215].

$$S_M = -\int_\epsilon^{z_m} d^5x\sqrt{-g}\, e^{-\phi}\, \mathrm{Tr}\Big\{(D^M X)^\dagger (D_M X) + m_5^2|X|^2 + \frac{1}{4g_5^2}(L^{MN}L_{MN} + R^{MN}R_{MN}) + (D^M H)^\dagger (D_M H) + m_5^2|H|^2\Big\},$$

$D_M X = \partial_M X - iL_M X + iXR_M$ and $D_M H = \partial_M H - iV_M^{15}H - iHV_M^{15}$ → 2110.08215

$$L_{MN} = \partial_M L_N - \partial_N L_M - i\,[L_M, L_N],$$
$$R_{MN} = \partial_M R_N - \partial_N R_M - i\,[R_M, R_N],$$

the coupling constant $g_5^2 = 12\pi^2/N_c$

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	−3

$$X = e^{i\pi^a t^a}\, X_0\, e^{i\pi^b t^b},$$

$$X_0 = \mathrm{diag}[v_l(z), v_l(z), v_s(z), v_c(z)]$$

$L_M = V_M + A_M$ and $R_M = V_M - A_M$.

$H = \mathrm{diag}[0, 0, 0, h_c(z)]$

Formalism

To obtain the meson masses as well as the three and four point coupling constants, the action is expanded to quartic order [2110.08215], which is given as

$$S = S^{(0)} + S^{(2)} + S^{(3)} + S^{(4)},$$

$$S^{(0)} = -\int_0^{z_m} d^5x \left\{ \frac{e^{-\phi(z)}}{z^3} \Big(2v_l'(z)v_l'(z) + v_s'(z)v_s'(z) + v_c'(z)v_c'(z) \Big) + \frac{e^{-\phi(z)}}{z^5} m_5^2 \Big(2v_l(z)^2 + v_s(z)^2 + v_c(z)^2 \Big) \right.$$
$$\left. + \frac{e^{-\phi(z)}}{z^3} \Big(h_c'(z)h_c'(z) \Big) + \frac{e^{-\phi(z)}}{z^5} m_5^2 h_c(z)^2 \right\},$$

$$S^{(2)} = -\int_0^{z_m} d^5x \left\{ \eta^{mn} \frac{e^{-\phi(z)}}{z^3} \Big((\partial_m\pi^a - A_m^a)(\partial_n\pi^b - A_n^b) M_A^{ab} - V_m^a V_n^b M_V^{ab} + V_m^{15} V_n^{15} m_V^{15,15} \Big) \right.$$
$$\left. + \frac{e^{-\phi(z)}}{4g_5^2 z} \eta^{mp}\eta^{nq} \Big(V_{mn}V_{pq} + A_{mn}A_{pq} \Big) \right\},$$

$$S^{(3)} = -\int_0^{z_m} d^5x \left\{ \eta^{mn} \frac{e^{-\phi(z)}}{z^3} \Big(2(A_m^a - \partial_m\pi^a) V_n^b \pi^c g^{abc} + V_m^a (\partial_n(\pi^b\pi^c) - 2A_n^b\pi^c) h^{abc} \Big) \right.$$
$$\left. + \frac{e^{-\phi(z)}}{2g_5^2 z} \eta^{mp}\eta^{nq} (V_{mn}^a V_p^b V_q^c + V_{mn}^a A_p^b A_q^c + A_{mn}^a V_p^b A_q^c + A_{mn}^a A_p^b V_q^c) f^{bca} \right\},$$

$$S^{(4)} = -\int_0^{z_m} d^5x \left\{ \eta^{mn} \frac{e^{-\phi(z)}}{z^3} \Big([\partial_m\pi^a - A_m^a][A_n^b\pi^c\pi^d - \frac{1}{3}\partial_n(\pi^b\pi^c\pi^d)] l^{abcd} + V_m^a V_n^b \pi^c\pi^d (h^{abcd} - g^{acbd}) \right.$$
$$+ [\frac{1}{2}\partial_m(\pi^a\pi^b) - A_m^a\pi^b][\frac{1}{2}\partial_n(\pi^c\pi^d) - A_n^c\pi^d] k^{abcd} \Big) + \frac{e^{-\phi(z)}}{4g_5^2 z} \eta^{mp}\eta^{nq} (V_m^a V_n^b V_p^c V_q^d + A_m^a A_n^b V_p^c V_q^d + V_m^a V_n^b A_p^c A_q^d$$
$$\left. + A_m^a A_n^b A_p^c A_q^d + 2V_m^a A_n^b V_p^c A_q^d + 2A_m^a V_n^b V_p^c A_q^d) f^{abcd} \right\}.$$

Scalar Fields

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The EOMs for the scalar vacuum expectation value $v_{l,s,c}(z)$:

$$-\frac{z^3}{e^{-\phi(z)}}\partial_z \frac{e^{-\phi}}{z^3}\partial_z v_q(z) + \frac{m_5^2}{z^2} v_q(z) = 0$$

With the analytical solutions: $v_q(z) = C_1(q)\ z\ \sqrt{\pi}\ U(\frac{1}{2}, 0, \phi) - C_2(q)\ z\ L(-\frac{1}{2}, -1, \phi)$

Expanding $v_q(z)$ at the UV boundary:

$$v_q(z)|_{z\to 0} = 2\ C_1(q)\ z + \left(C_2(q)\ \mu^2 + C_1(q)\left[-\mu^2 + 2\gamma_E\mu^2 + 2\mu^2 \mathrm{Log}\ z + 2\mu^2 \mathrm{Log}\ \mu + \mu^2\Psi(\frac{3}{2})\right]\right) z^3$$

$$v_{l,s,c}(z) \to M_{l,s,c} z + \Sigma_{l,s,c} z^3$$

The auxiliary field $h_c(z)$ has similar solution,

$$h_c(z) = D_1\ z\ \sqrt{\pi}\ U(\frac{1}{2}, 0, \phi) - D_2\ z\ L(-\frac{1}{2}, -1, \phi)$$

$$h_c(z) \to m_c z$$

Vector Fields

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The EOMs for the transverse component of the vector fields:

$$\left(-\frac{z}{e^{-\phi}}\partial_z\frac{e^{-\phi}}{z}\partial_z+\frac{2g_5^2(m_V^{ab}-M_V^{ab})}{z^2}\right)V_{\mu\perp}^a(q,z)=-q^2V_{\mu\perp}^a(q,z),$$

Gauge fixing: $V^{z,a}=0$, and $\partial_\mu V_\perp^{\mu,a}=0$

Boundary Conditions: $V_{\mu\perp}^{(n)a}(z)|_{q,z\to 0}=0$ and $\partial_z V_{\mu\perp}^{(n)a}(z)|_{z\to z_m}=0.$

Decay Constant:

$$\int_x e^{iqx}\langle J_{V,\mu}^a(x)J_{V,\mu}^b(0)\rangle=\delta^{ab}(q_\mu q_\nu-q^2g_{\mu\nu})\Pi_V(q^2),$$

$$\Pi_V(q^2)=-\frac{e^{-\phi(z)}}{g_5^2q^2}\frac{\partial_z V(q,z)}{z}|_{z=\epsilon\to 0},\quad V(z')=\frac{e^{-\phi(z)}}{z}\sum_n\frac{\psi'_{V^n}(\epsilon)\psi_{V^n}(z')}{q^2-m_n^2},$$

$$\Pi_V(q^2)=-\frac{1}{g_5^2q^2}\sum_n\frac{[e^{A(\epsilon)-\phi(\epsilon)}\psi'_{V^n}(\epsilon)]^2}{q^2-m_n^2}.$$

Considering the definition of the decay constant: $\langle 0|J_V^\mu|V(p)\rangle=i\,F_V\,p^\mu$

$$F_{V^n}^2=\frac{[e^{A(\epsilon)-\phi(\epsilon)}\psi'_{V^n}(\epsilon)]^2}{g_5^2}|_{\epsilon\to 0}.$$

Pseudoscalar Fields

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The longitudinal part of the axial vector fields and the pseudoscalar fields have mixing, and their EOMs can be written as:

Method I:

$$q^2\partial_z\varphi^a(q,z) + \frac{2g_5^2 M_A^{ab}}{z^2}\partial_z\pi^a(q,z) = 0\,,$$

$$\frac{z}{e^{-\phi}}\partial_z\left(\frac{e^{-\phi}}{z}\partial_z\varphi^a(q,z)\right) - \frac{2g_5^2 M_A^{ab}}{z^2}\left(\varphi^a(q,z) - \pi^a(q,z)\right) = 0\,,$$

With boundary condition $\pi^{(n)a}(q,z)|_{z\to 0} = \varphi^{(n)a}(q,z)|_{z\to 0} = 0$ and $\partial_z\varphi^{(n)a}(q,z)|_{z\to z_m} = 0$

There are different combination of the φ *and* π fields to solve the EOMs,

Method II: $y(q^2,z) = \frac{e^{-\phi}}{z}\partial_z\Phi, \Rightarrow -\frac{e^{-\phi}}{z}\partial_z\frac{z}{\beta(z)e^{-\phi}}\partial_z y(q^2,z) + \beta(z)y(q^2,z) = -q^2 y(q^2,z),\ \beta(z) = \frac{2g_5^2 M_A^{ab}}{z^2}$

Method III: $\tilde{\pi} = \partial_z\pi, \Rightarrow -\partial_z\frac{z^3}{e^{-\phi}}\partial_z\left(\frac{e^{-\phi}}{z^3}\tilde{\pi}\right) + \beta(z)\tilde{\pi} = -q^2\tilde{\pi},\ \beta(z) = \frac{2g_5^2 M_A^{ab}}{z^2}$

Since $\Pi_A(q^2) \to -f_\pi^2 q^2$ with $q^2 \to 0$, the decay constant of the pseudoscalar meson is

$$f_\pi^2 = -\frac{e^{A(\epsilon)-\phi(\epsilon)}\partial_z A(0,\epsilon)}{g_5^2}\Big|_{\epsilon\to 0}$$

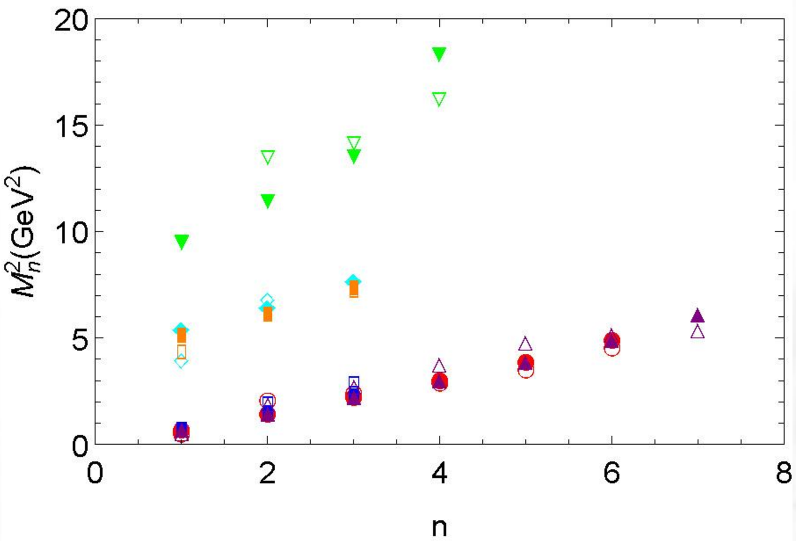
Free parameters

There are several free parameters in the model which can be fixed by the masses of different mesons.

- $\mu = 0.44\ GeV$: Use the Regge slope: $m_\rho^2 = 4\mu^2 n$.
- $C_1(l) = 0.069\ GeV$: $-\frac{z}{e^{-\phi}}\partial_z\left(\frac{e^{-\phi}}{z}\partial_z A^{(n)a}(q,z)\right) + \frac{2g_5^2(2v_l^2(z))}{z^2}A^{(n)a}(q,z) = m_{a_1}^{(n)2}A^{(n)a}(q,z)$ for $n=1$, $a=3$ and $m_{a_1} = 1.230$ GeV.
- $C_1(s) = 0.090\ GeV$: $-\frac{z}{e^{-\phi}}\partial_z\left(\frac{e^{-\phi}}{z}\partial_z V^{(n)a}(q,z)\right) + \frac{2g_5^2(-\frac{1}{2}(v_l(z)-v_s(z))^2}{z^2}V^{(n)a}(q,z) = m_{K^*}^{(n)2}V^{(n)a}(q,z)$ for $n=1$, $a=4$ and $m_{K^*} = 0.892$ GeV.
- $C_1(c) = 0.614\ GeV$: $\left(-\frac{z}{e^{-\phi}}\partial_z\frac{e^{-\phi}}{z}\partial_z + \frac{2g_5^2\frac{1}{6}(2v_l^2(z)+v_s(z)^2+9v_c(z)^2)}{z^2}\right)A^{(n)a}(q,z) = m_{\chi_{c1}}^{(n)2}A^{(n)a}(q,z)$ for $n=1$, $a=15$ and $m_{\chi_{c1}} = 3.511$ GeV.
- $D_1(c) = 0.509\ GeV$: $\left(-\frac{z}{e^{-\phi}}\partial_z\frac{e^{-\phi}}{z}\partial_z + \frac{2g_5^2(\frac{3}{2}h_c^2(z))}{z^2}\right)V^{(n)a}(q,z) = m_{J/\Psi}^{(n)2}V^{(n)a}(q,z)$ for $n=1$, $a=15$ and $m_{J/\Psi} = 3.097$ GeV.
- $M_l = 138\ MeV, M_s = 180\ MeV, M_c = 1228\ MeV, m_c = 1018\ MeV$.
- $\Sigma_l = (137\ MeV)^3, \Sigma_s = (149\ MeV)^3, \Sigma_l = (283\ MeV)^3, \sigma_c = (262\ MeV)^3$

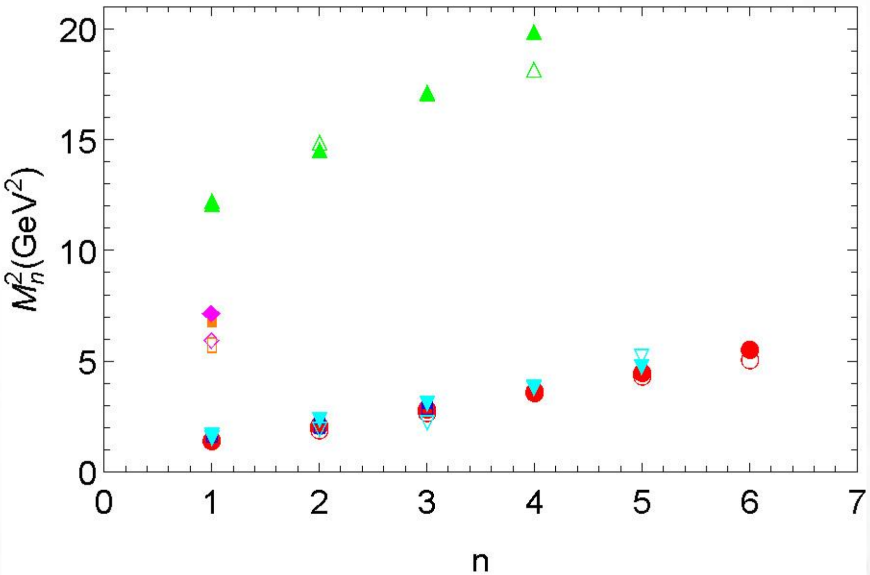
Vector Meson Spectra

n	ρ (MeV)		K^*(MeV)		ω(MeV)		D^*(MeV)		D_s^*(MeV)		J/ψ(MeV)	
	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model
1	775	880	892	892	782	880	2007	2341	2112	2293	3097	3100
2	1465	1245	1414	1251	1410	1245	2627	2551		2500	3686	3394
3	1570	1525	1718	1529	1670	1525	2781	2789	2714	2733	3773	3692
4	1720	1764			1960	1764					4040	4289
5	1900	1993			2205	1993						
6	2150	2233			2290	2233						
7					2330	2490						



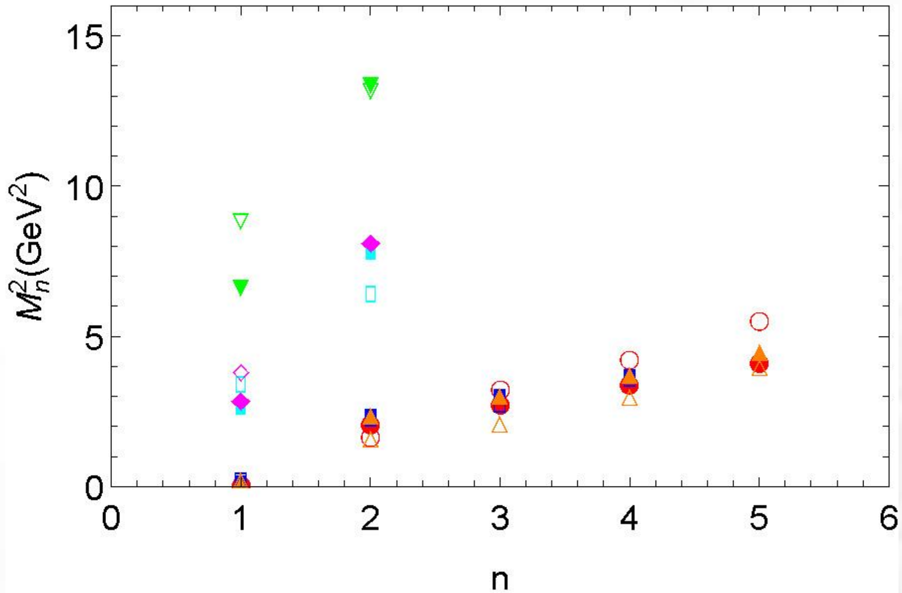
Axial Vector Meson Spectra

n	a_1 (MeV)		f_1 (MeV)		K_1(MeV)		D_1 (MeV)		D_{s1}(MeV)		χ_{c1}(MeV)	
	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model	Exp.	Model
1	1230	1232	1282	1335	1253	1306	2422	2649	2460	2695	3511	3493
2	1411	1487	1426	1573	1403	1548					3872	3828
3	1655	1710	1518	1784	1672	1763					4147	4153
4	1930	1919	1971	1987							4274	4472
5	2096	2135	2310	2201								
6	2270	2368										



Pseudoscalar Meson Spectra

n	π(MeV)		K^0 (MeV)		η(MeV)		D^0 (MeV)		$D_s^{\pm}$ (MeV)		η_c(MeV)	
	Exp.	**Model**	**Exp.**	**Model**	**Exp.**	**Model**	**Exp.**	**Model**	**Exp.**	**Model**	**Exp.**	**Model**
1	135	337	498	388	548	408	1865	1658	1968	1709	2984	2586
2	1300	1461	1482	1533	1294	1560	2549	2811			3637	3668
3	1810	1671	1629	1737	1475	1762						
4	2070	1861	1874	1923	1751	1947						
5	2360	2048			2010	2130						



Different Methods To Obtain The Pseudoscalar Eigenvalues

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π(MeV)

n	Exp.	Method I	Method II	Method III
1	135	337	345	-
2	1300	1461	1433	1513
3	1810	1671	1637	1728
4	2070	1861	1941	1923
5	2360	2048	2010	2112

Decay Constants

Comparison between the predicted ratios of the decay constants with experiment and lattice QCD.

Observable	Exp./LQCD	Model
f_K/f_π	1.196	1.150
f_{D_s}/f_D	1.180	1.030
f_{η_c}/f_{D_s}	1.576	1.448
$f_{K^*}^{1/2}/f_\rho^{1/2}$	1.005	0.990
$f_{D_s^*}^{1/2}/f_{D^*}^{1/2}$	0.954	1.595
$f_{K_1}^{1/2}/f_{a_1}^{1/2}$	1.085	0.799
$f_{D_{s1}}^{1/2}/f_{D_1}^{1/2}$	-	0.386

Coupling Constants

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The three-point interaction of mesons can be obtained by the cubic order terms of 5D action.

$$S_{VVV} = -\int_0^{z_m} d^5x \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} V^{\mu\nu,a} V_\mu^b V_\nu^c;$$

$$S_{VAA} = -\int_0^{z_m} d^5x \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} \left(V^{\mu\nu,a} A_\mu^b A_\nu^c + 2A^{\mu\nu,a} V_\mu^b A_\nu^c\right),$$

$$S_{VA\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} 2V^{\mu,a} A_\mu^b \pi^c \left(g^{bac} - h^{abc}\right) + \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} \left(V^{\mu\nu,a} A_\mu^b A_\nu^c + 2A^{\mu\nu,a} V_\mu^b A_\nu^c\right)\},$$

$$S_{V\pi\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} \left(2h^{abc} V^{\mu,a} (\pi^b \partial_\mu \pi^c - A_\mu^b \pi^c) - 2g^{abc} (A^{\mu,a} - \partial^\mu \pi^a) V_\mu^b \pi^c)\right)$$
$$+ \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} \left(V^{\mu\nu,a} A_\mu^b A_\nu^c + A^{\mu\nu,a} V_\mu^b A_\nu^c + V^{\mu\nu,a} A_\mu^b V_\nu^c\right)\},$$

$$g_{VVV} = \int_0^{z_m} dz \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} \psi^a_{V(n)} \psi^b_{V(m)} \psi^c_{V(k)},$$

$$g_{VAA} = \int_0^{z_m} dz \frac{e^{-\phi(z)}}{2g_5^2 z} f^{bca} \psi^a_{V(n)} \psi^b_{A(m)} \psi^c_{A(k)},$$

$$g_{VA\pi} = \int_0^{z_m} dz \frac{e^{-\phi(z)}}{z^3} 2\psi^a_{V(m)} \psi^b_{A(m)} \psi^c_{\pi(k)} \left(g^{bac} - h^{abc}\right),$$

$$g_{V\pi\pi} = \int_0^{z_m} dz \frac{e^{-\phi(z)}}{z^3} \{\frac{e^{-\phi(z)}}{z^3} \left(2h^{abc} \psi^a_{V(n)} (\psi^b_{\pi(m)} \psi^c_{\pi(k)} - \psi^b_{\Phi(m)} \psi^c_{\pi(k)}) + 2g^{abc} (\psi^a_{\Phi(n)} - \psi^a_{\pi(n)}) \psi^b_{V(m)} \psi^c_{\pi(k)}\right)$$
$$+ \frac{m_n^2 e^{-\phi(z)}}{2g_5^2 z} f^{bca} \left(\psi^a_{V(n)} \psi^b_{\Phi(m)} \psi^c_{\Phi(k)}\right)\}.$$

Four-Point Interaction

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$$S_{VVVV} = -\int_0^{z_m} d^5x \frac{e^{-\phi(z)}}{4g_5^2 z} f^{abcd} V^{\mu,a} V^{\nu,b} V_\mu^c V_\nu^d,$$

$$S_{VVAA} = -\int_0^{z_m} d^5x \frac{e^{-\phi(z)}}{4g_5^2 z} \{2V^{\mu,a} V^{\nu,b} A_\mu^c A_\nu^d \left(f^{abcd} + f^{cbad}\right) + 2V^{\mu,a} V_\mu^b A^{\nu,c} A_\nu^d f^{acbd}\},$$

$$S_{AAAA} = -\int_0^{z_m} d^5x \frac{e^{-\phi(z)}}{4g_5^2 z} f^{abcd} A^{\mu,a} A^{\nu,b} A_\mu^c A_\nu^d,$$

$$S_{AA\pi\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} A^{\mu,a} A_\mu^b \pi^c \pi^d (k^{acbd} - l^{abcd}) + \frac{e^{-\phi(z)}}{4g_5^2 z} f^{abcd} A^{\mu,a} A^{\nu,b} A_\mu^c A_\nu^d\},$$

$$S_{VV\pi\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} V^{\mu,a} V_\nu^b \pi^c \pi^d (h^{abcd} - g^{acbd})$$

$$+ \frac{e^{-\phi(z)}}{4g_5^2 z} \{2V^{\mu,a} V^{\nu,b} A_\mu^c A_\nu^d \left(f^{abcd} + f^{cbad}\right) + 2V^{\mu,a} V_\mu^b A^{\nu,c} A_\nu^d f^{acbd}\},$$

$$S_{A\pi\pi\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} l^{bacd} (\partial^\mu \pi^a A_\mu^b \pi^c \pi^d + A^{\mu,a} \partial_\mu \pi^b \pi^c \pi^d - A^{\mu,a} A_\mu^b \pi^c \pi^d)$$

$$-(\frac{e^{-\phi(z)}}{z^3}) k^{abcd} (A^{\mu,a} \pi^b A_\mu^c \pi^d - \partial^\mu \pi^a \pi^b A_\mu^c \pi^d - A^{\mu,a} \pi^b \partial_\mu \pi^c \pi^d) + \frac{e^{-\phi(z)}}{4g_5^2 z} f^{abcd} A^{\mu,a} A^{\nu,b} A_\mu^c A_\nu^d\},$$

$$S_{\pi\pi\pi\pi} = -\int_0^{z_m} d^5x \{\frac{e^{-\phi(z)}}{z^3} \left(\partial^\mu \pi^a \partial_\mu \pi^b \pi^c \pi^d + \partial_z \pi^a \partial_z \pi^b \pi^c \pi^d\right) \left(k^{acbd} - l^{abcd}\right)$$

$$+ \frac{e^{-\phi(z)}}{z^3} l^{bacd} (\partial^\mu \pi^a A_\mu^b \pi^c \pi^d + A^{\mu,a} \partial_\mu \pi^b \pi^c \pi^d - A^{\mu,a} A_\mu^b \pi^c \pi^d)$$

$$- \frac{e^{-\phi(z)}}{z^3} k^{abcd} (A^{\mu,a} \pi^b A_\mu^c \pi^d - \partial^\mu \pi^a \pi^b A_\mu^c \pi^d - A^{\mu,a} \pi^b \partial_\mu \pi^c \pi^d)$$

$$+ \frac{e^{-\phi(z)}}{4g_5^2 z} f^{abcd} A^{\mu,a} A^{\nu,b} A_\mu^c A_\nu^d\}$$

Coupling Constants

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$$g_{VVVV}=\int_0^{z_m}dz\frac{e^{-\phi(z)}}{4g_5^2 z}f^{abcd}\psi^a_{V(n)}\psi^b_{V(m)}\psi^c_{V(k)}\psi^d_{V(j)},$$

$$g_{VVAA}=\int_0^{z_m}dz\frac{e^{-\phi(z)}}{4g_5^2 z}2\psi^a_{V(n)}\psi^b_{V(m)}\psi^c_{A(k)}\psi^d_{A(j)}\left(f^{abcd}+f^{cbad}+f^{acbd}\right),$$

$$g_{AAAA}=\int_0^{z_m}dz\frac{e^{-\phi(z)}}{4g_5^2 z}f^{abcd}\psi^a_{A(n)}\psi^b_{A(m)}\psi^c_{A(k)}\psi^d_{A(j)},$$

$$g_{VV\pi\pi}=\int_0^{z_m}dz\{\frac{e^{-\phi(z)}}{z^3}\psi^a_{V(n)}\psi^b_{V(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}(h^{abcd}-g^{acbd})$$
$$+\frac{m_n^2 e^{-\phi(z)}}{2g_5^2 z}\psi^a_{V(n)}\psi^b_{V(m)}\psi^c_{\Phi(k)}\psi^d_{\Phi(j)}\left(f^{abcd}+f^{cbad}+f^{acbd}\right)\},$$

$$g_{AA\pi\pi}=\int_0^{z_m}dz\{\frac{e^{-\phi(z)}}{z^3}\psi^a_{A(n)}\psi^b_{A(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}(k^{acbd}-l^{abcd})$$
$$+\frac{m_n^2 e^{-\phi(z)}}{2g_5^2 z}\psi^a_{A(n)}\psi^b_{A(m)}\psi^c_{\Phi(k)}\psi^d_{\Phi(j)}\left(f^{abcd}+f^{cbad}+f^{acbd}\right)\},$$

$$g_{A\pi\pi\pi}=\int_0^{z_m}dz\{\frac{e^{-\phi(z)}}{z^3}(\psi^a_{A(n)}\psi^b_{\pi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}-\psi^a_{A(n)}\psi^b_{\Phi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)})(l^{abcd}+l^{bacd}-k^{acbd}-k^{cbad})$$
$$+\frac{m_n^2 e^{-\phi(z)}}{4g_5^2 z}\psi^a_{A(n)}\psi^b_{\Phi(m)}\psi^c_{\Phi(k)}\psi^d_{\Phi(j)}\left(f^{cbad}+f^{acbd}\right)\},$$

$$g_{\pi\pi\pi\pi}=\int_0^{z_m}dz\{\frac{e^{-\phi(z)}}{z^3}[\psi^a_{\pi(n)}\psi^b_{\pi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}(k^{abcd}-l^{abcd})+\psi^a_{\pi(n)}\psi^b_{\Phi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}(l^{abcd}+l^{bacd}-k^{acbd}-k^{bacd})$$
$$-\psi^a_{\Phi(n)}\psi^b_{\Phi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)}(l^{abcd}-k^{acbd})-\partial_z\psi^a_{\pi(n)}\partial_z\psi^b_{\pi(m)}\psi^c_{\pi(k)}\psi^d_{\pi(j)})(l^{abcd}-k^{acbd})]$$
$$+\frac{m_n^2 e^{-\phi(z)}}{4g_5^2 z}\psi^a_{\Phi(n)}\psi^b_{\Phi(m)}\psi^c_{\Phi(k)}\psi^d_{\Phi(j)}f^{abcd}\}$$

Coupling Constants

Observable	Model	2110.08215
$g_{K^*D^*D_s^*}/g_{\rho D^*D^*}$	1.025	1.038
$g_{K^*D\,D_s}/g_{\rho D\,D}$	1.047	0.203
$g_{K_1D_sD^*}/g_{a_1D\,D^*}$	1.520	0.433
$g_{\psi D_s^*DK}/g_{\psi D^*D\pi}$	1.060	0.435

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Future plan

- ✓ Apply the approach to the hard-wall model and include the scalar field in the action.
- ✓ Extend the approach to the Baryonic sector and study the heavy baryons.
- ✓ Adding the $U(1)_A$ axial anomaly to the action and investigate the effect on the heavy mesons (with Mamiya).

谢谢！

THANKS!

二〇二三年〇四月二七日