



# Hyperon electromagnetic form factors in VMD model

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# Outline

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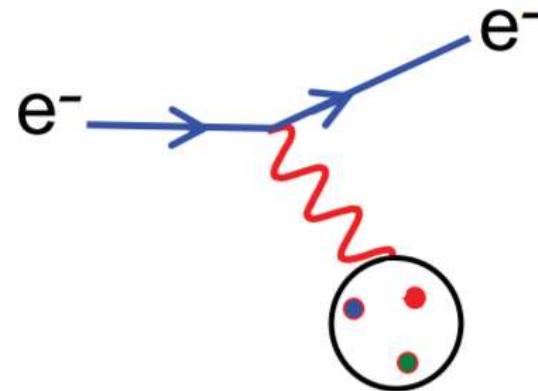
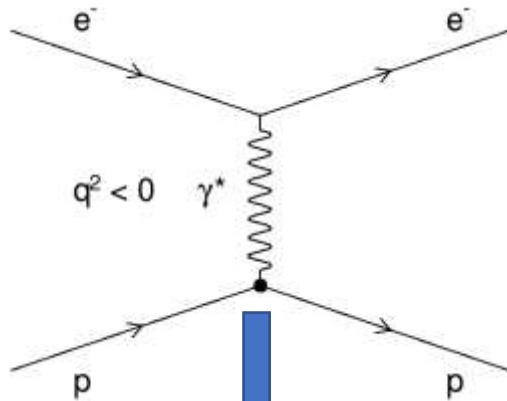
Introduction: electromagnetic form factors

The model: Vector Meson Dominance

Hyperon electromagnetic form factors

Summary

# Electromagnetic form factors (space-like)



$$\langle p_f | \hat{J}^\mu(0) | p_i \rangle = \bar{u}(p_f) \left[ F_1(q^2) \gamma^\mu - F_2(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2M} \right] u(p_i)$$

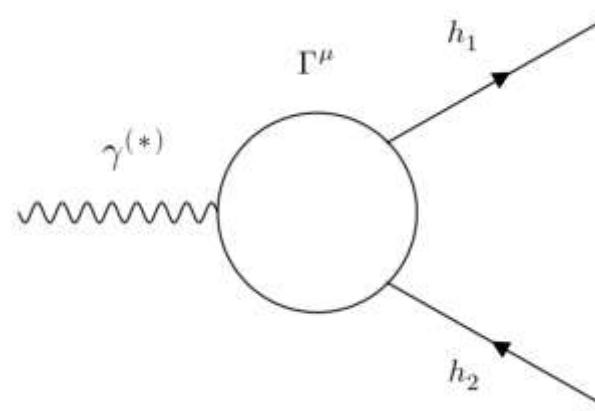
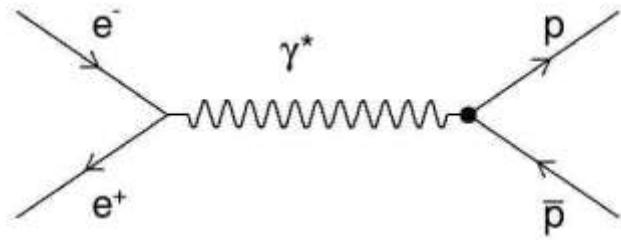
$$\Gamma^\mu(q^2) = \gamma^\mu F_1^p(q^2) + i \frac{F_2^p(q^2)}{2M_p} \sigma^{\mu\nu} q_\nu \quad F_1^N : \text{Dirac form factor} \\ F_2^N : \text{Pauli form factor}$$

$$G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2), \quad G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2), \quad \tau = \frac{Q^2}{4M_N^2}$$

$$F_1^p(0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) = \kappa_p, \quad F_2^n(0) = \kappa_n$$

S. Pacetti, R. Baldini Ferroli and E. Tomasi-Gustafsson, "Proton electromagnetic form factors: Basic notions, present achievements and future perspectives," **Phys. Rept. 550-551, 1-103 (2015)**.

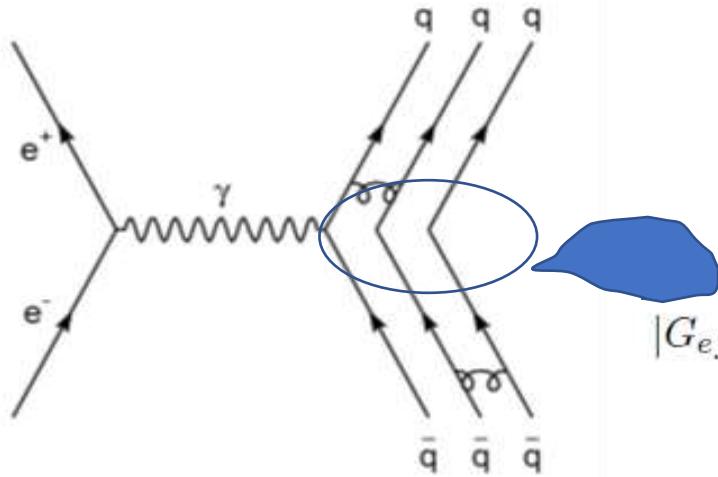
# EMFFs (time-like)



$$\left( \frac{d\sigma}{d\Omega} \right)_{e^+e^- \rightarrow N\bar{N}}^{th} = \frac{\alpha^2 \beta}{4q^2} C_N(q^2) \left\{ |G_M^N(q^2)|^2 (1 + \cos^2 \theta) + |G_E^N(q^2)|^2 \frac{1}{\tau} \sin^2 \theta \right\}$$

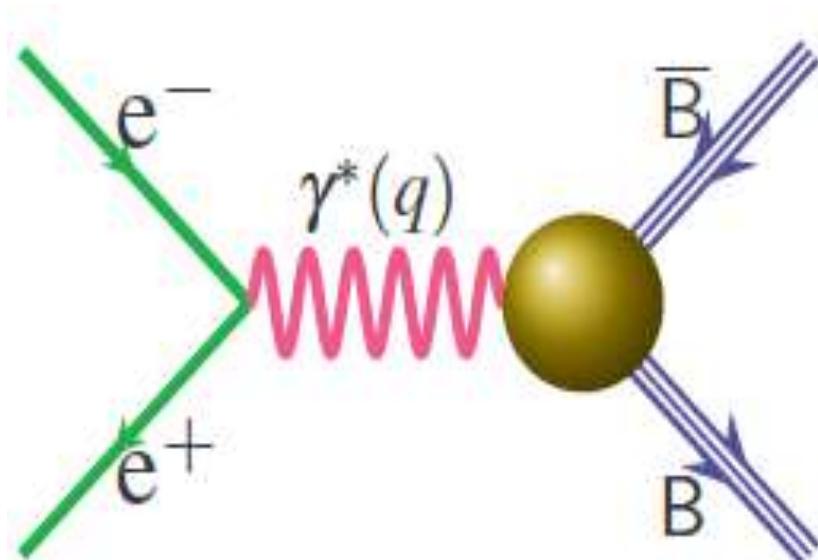
$$\sigma_{e^+e^- \rightarrow N\bar{N}}^{th} = \frac{\alpha^2 \beta}{4q^2} C_N(q^2) \int d\Omega \left[ |G_M^N(q^2)|^2 (1 + \cos^2 \theta) + |G_E^N(q^2)|^2 \frac{\sin^2 \theta}{\tau} \right]$$

$$= \frac{4\pi \alpha^2 \beta}{3q^2} C_N(q^2) \left[ |G_M^N(q^2)|^2 + \frac{|G_E^N(q^2)|^2}{2\tau} \right].$$

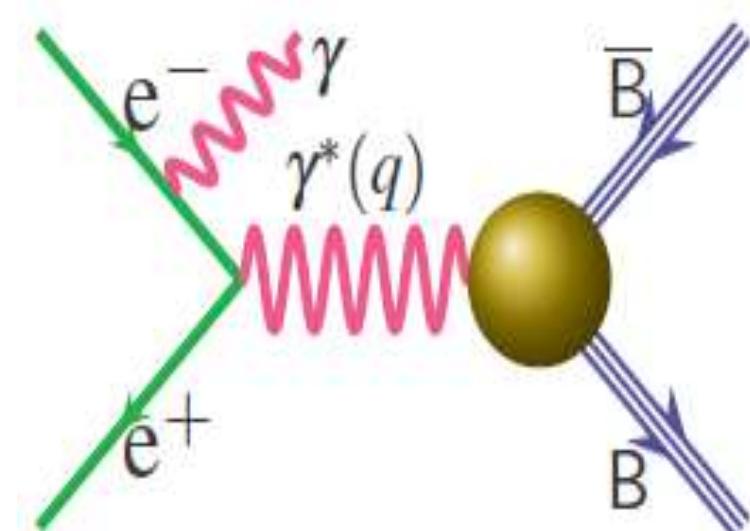


$$|G_{eff}(q^2)| = \sqrt{\frac{\sigma(q^2)}{\sigma_{point}(q^2)}} = \sqrt{\frac{|G_M(s)|^2 + \frac{2M^2}{s}|G_E(s)|^2}{1 + \frac{2M^2}{s}}}$$

# Experimental measurements (time-like)



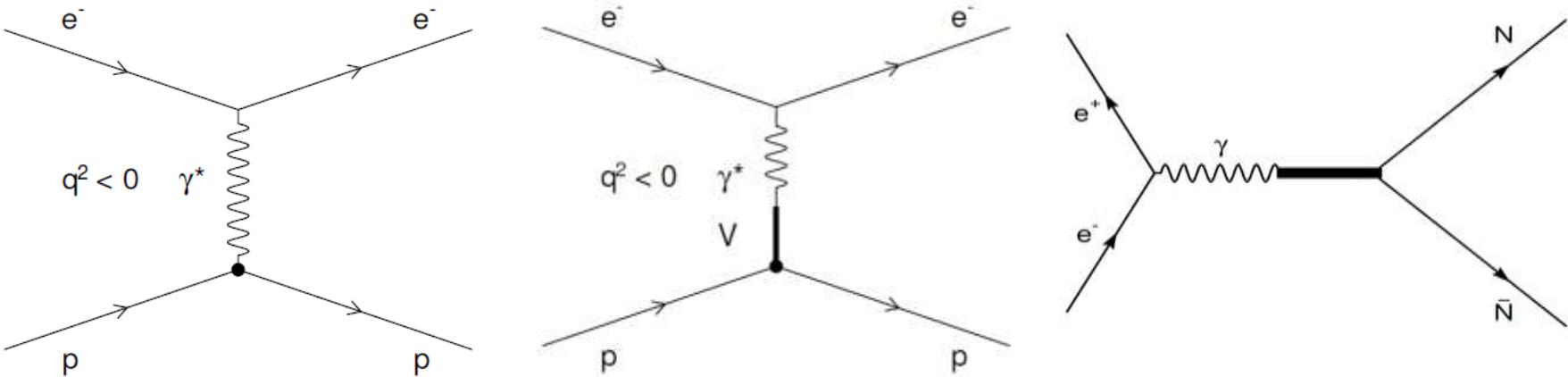
Energy Scan



Initial State Radiation

Both techniques can be used  
at BESIII.

# VMD: vector meson dominance model



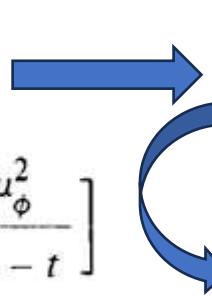
Dirac and Pauli isoscalar and isovector form factors

are

$$F_1^S(t) = \frac{e}{2} g(t) \left[ (1 - \beta_\omega - \beta_\phi) + \beta_\omega \frac{\mu_\omega^2}{\mu_\omega^2 - t} + \beta_\phi \frac{\mu_\phi^2}{\mu_\phi^2 - t} \right] \quad F_2^V(t) = \frac{e}{2} g(t) \left[ 3.706 \frac{\mu_\rho^2}{\mu_\rho^2 - t} \right]$$

$$F_1^V(t) = \frac{e}{2} g(t) \left[ (1 - \beta_\rho) + \beta_\rho \frac{\mu_\rho^2}{\mu_\rho^2 - t} \right]$$

$$F_2^S(t) = \frac{e}{2} g(t) \left[ (-0.120 - \alpha_\phi) \frac{\mu_\omega^2}{\mu_\omega^2 - t} + \alpha_\phi \frac{\mu_\phi^2}{\mu_\phi^2 - t} \right]$$



$$F_1 = F_1^S + F_1^V$$

$$F_2 = F_2^S + F_2^V$$

$$G_E = F_1 - \tau F_2$$

$$G_M = F_1 + F_2$$

## SEMI-PHENOMENOLOGICAL FITS TO NUCLEON ELECTROMAGNETIC FORM FACTORS

F. IACHELLO\* and A.D. JACKSON\*\*

*The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark 2100*

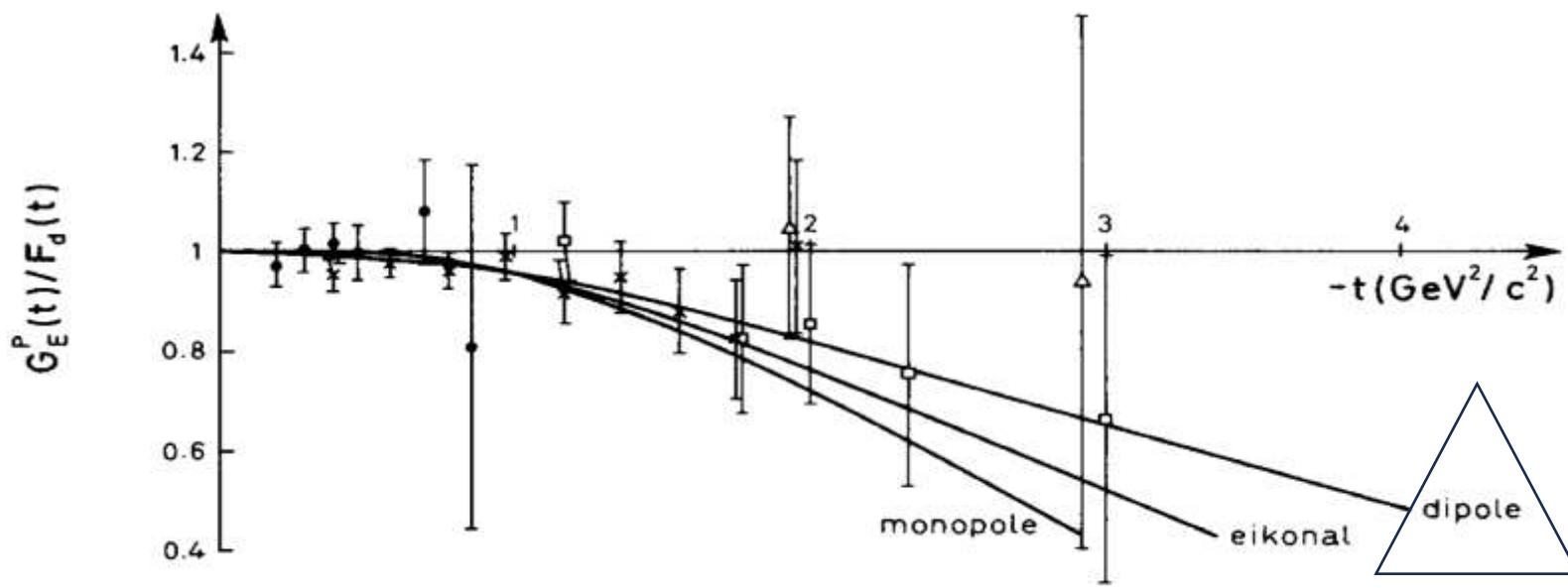
and

A. LANDE

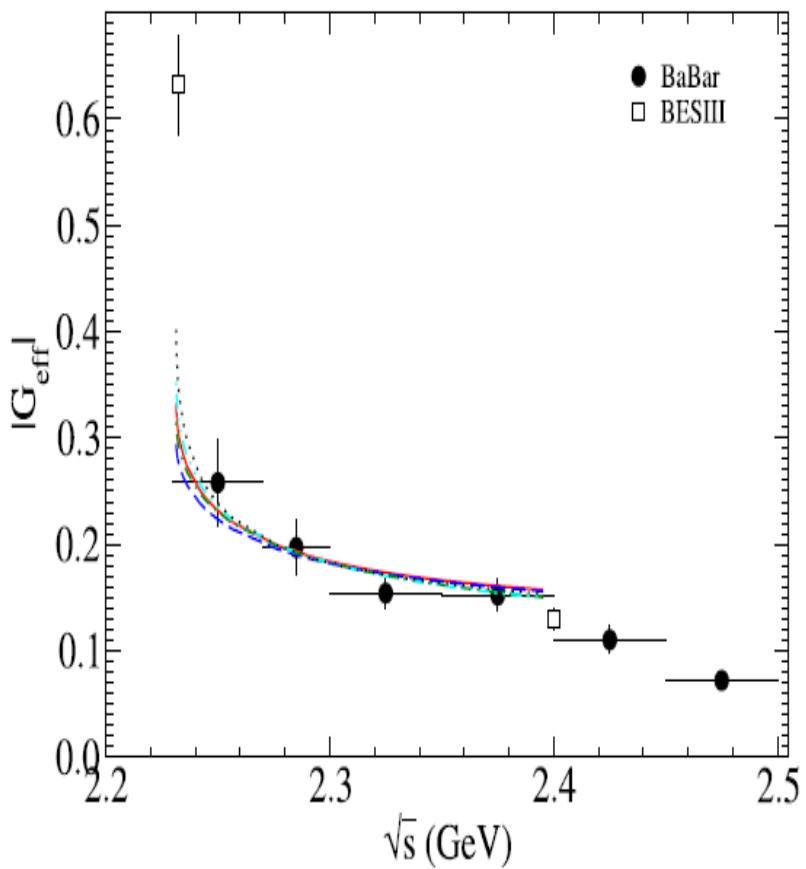
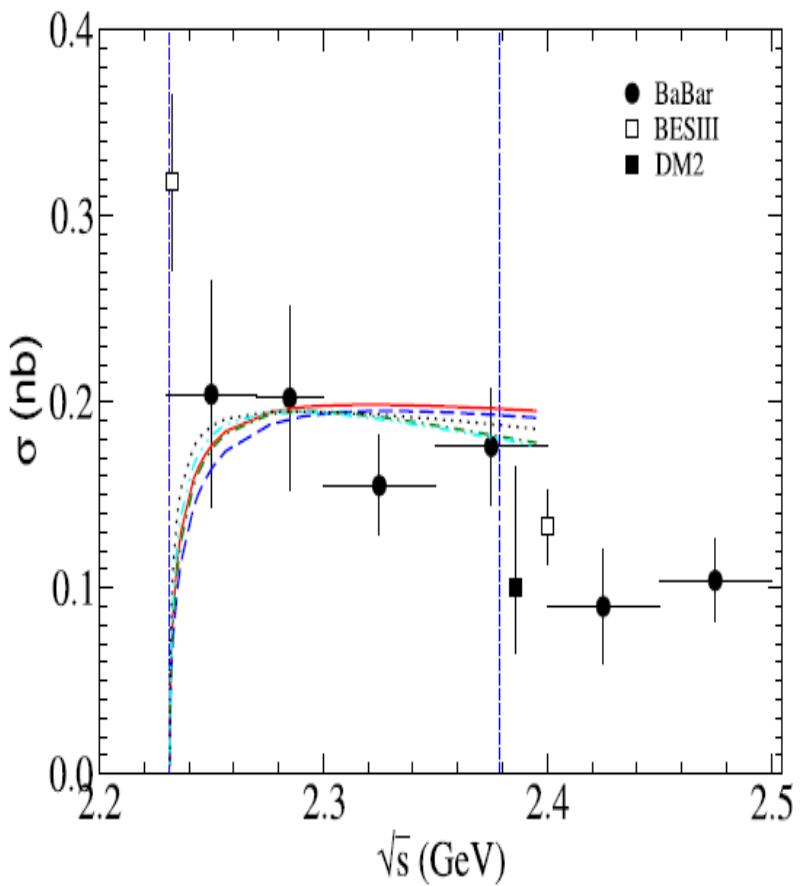
*Institute for Theoretical Physics, University of Groningen, Groningen, The Netherlands*

Received 31 August 1972

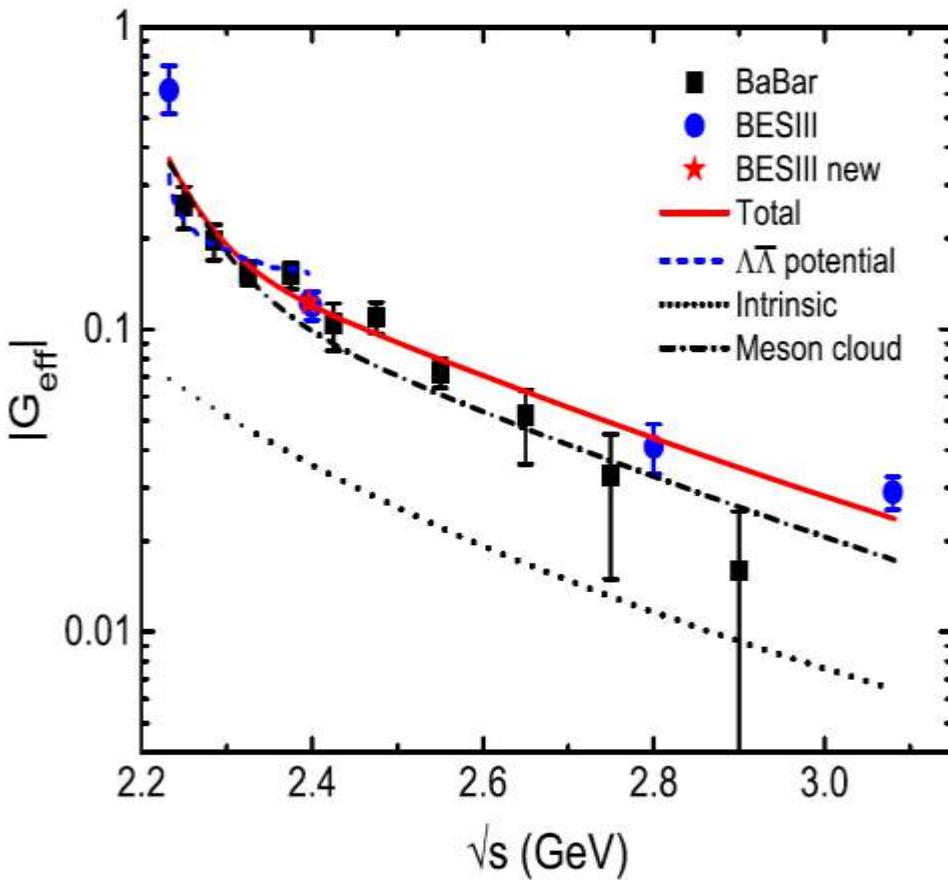
Several theoretically interesting forms of the nucleon EM form factor have been considered and found to provide quantitative descriptions of available data with as few as three adjustable parameters.



# $\Lambda$



J. Haidenbauer and U. G. Meißner, Phys. Lett. B 761, 456-461(2016).

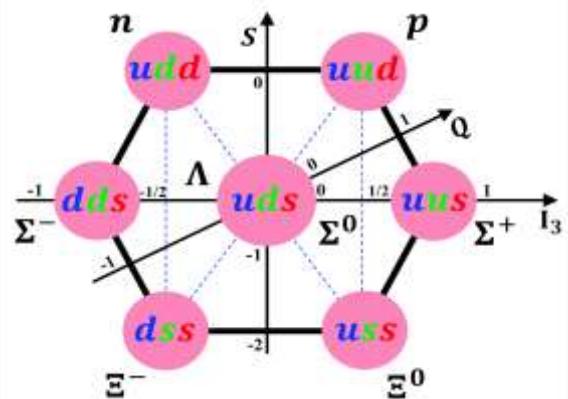


State	Mass	Width	State	Mass	Width
$\omega(782)$ [55]	782	8.1	$\phi(1020)$ [56]	1019	4.2
$\omega(1420)$ [57]	1418	104	$\phi(1680)$ [57]	1674	165
$\omega(1650)$ [57]	1679	121	$\phi(2170)$ [58]	2171	128

fit. In the present scenario, there are 16 experimental data and 10 free parameters. The value of intrinsic parameter  $\gamma$  is fitted to be  $0.336 \text{ GeV}^{-2}$  and the other parameters are summarized in Table II. It should be noticed that  $g(q^2)$

$$g(q^2) = \frac{1}{(1 - \gamma q^2)^2}$$

$$\gamma_N = \frac{1}{0.71 \text{ GeV}^2} = 1.408 \text{ GeV}^{-2}$$



Y. Yang, D. Y. Chen and Z. Lu,  
Phys. Rev. D 100, 073007 (2019).

# $\Lambda$ EMFFs in VMD (New proposal)

$$F_1(Q^2) = g(Q^2) \left[ -\beta_\omega - \beta_\phi + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} + \beta_x \frac{m_x^2}{m_x^2 + Q^2} \right]$$

$$F_2(Q^2) = g(Q^2) \left[ (\mu_\Lambda - \alpha_\phi) \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} + \alpha_x \frac{m_x^2}{m_x^2 + Q^2} \right]$$

$$g(Q^2) = 1/(1 + \gamma Q^2)^2$$

$Q^2 \rightarrow -q^2$

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

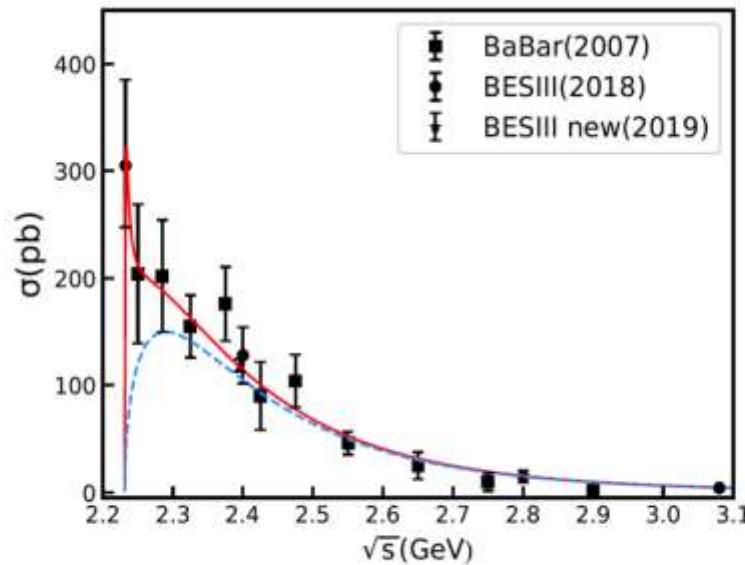
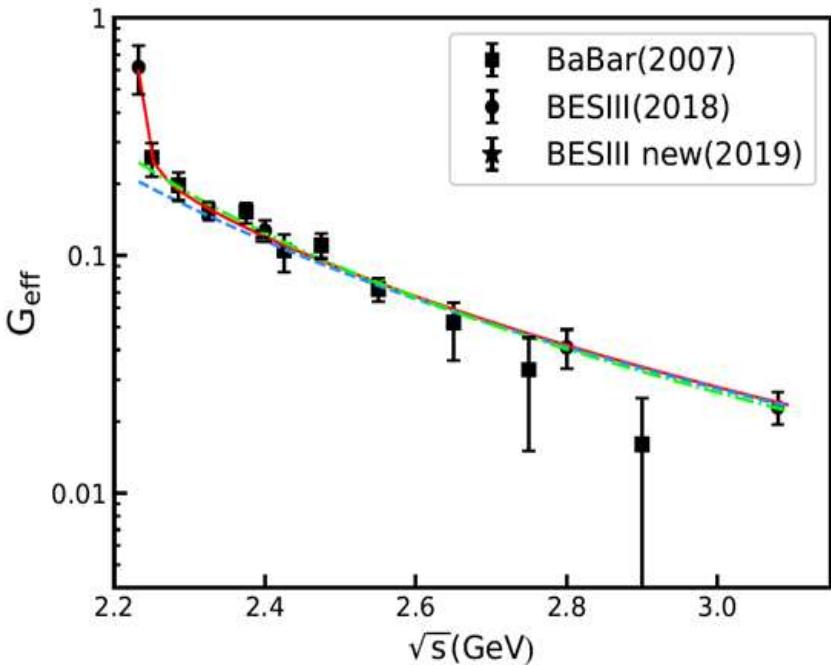


Figure: Cross section of the reaction  $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ .

Z. Y. Li, A. X. Dai and J. J. Xie,  
Chin. Phys. Lett. 39, 011201 (2022).



The red solid curve represents the total contributions from  $\omega$ ,  $\phi$  and  $X(2231)$ , while the blue dashed curve stands for the results without the contribution from the new  $X(2231)$  state. The green-dash-dotted curve stands for the fitted results with the effective form factor as in [1].

$$G_{\text{eff}} = C_0 g(q^2) = \frac{C_0}{(1 - \gamma q^2)^2}$$

Table: Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
$\gamma (\text{GeV}^{-2})$	0.43	$\beta_\omega$	-1.13
$\beta_\phi$	1.35	$\alpha_\phi$	-0.40
$\beta_x$	0.0015	$m_x$ (MeV)	2230.9
$\Gamma_x$ (MeV)	4.7		

New state  
 $X(2231)$ ?

# Flatte function

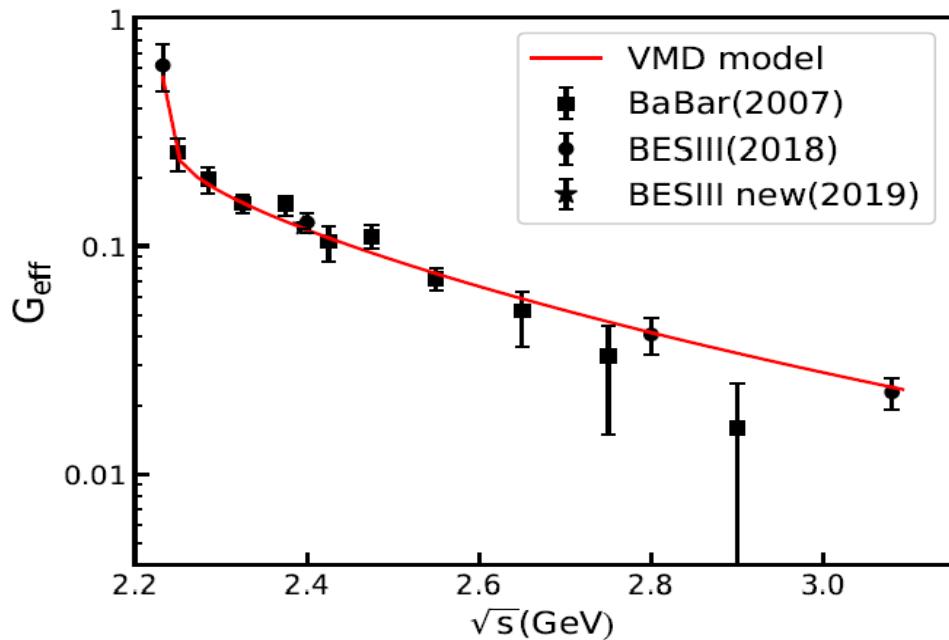


Figure: Fitting result of  $|G_{eff}|$  with Flatte.

$$\Gamma_x = \Gamma_0 + \Gamma_{\Lambda\bar{\Lambda}}(s) \quad \Gamma_{\Lambda\bar{\Lambda}} = \frac{g^2}{4\pi} \sqrt{\frac{s}{4} - M_\Lambda^2}$$

$$\frac{d\sigma_i}{dm} = C \left| \frac{m_R \sqrt{\Gamma_o \Gamma_i}}{m_R^2 - m^2 - im_R(\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})} \right|^2$$

$$\Gamma_{\pi\eta} = g_\eta q_\eta$$

$$\Gamma_{K\bar{K}} = \begin{cases} g_K \sqrt{(1/4)m^2 - m_K^2} & \text{above threshold} \\ ig_K \sqrt{m_K^2 - (1/4)m^2} & \text{below threshold} \end{cases}$$

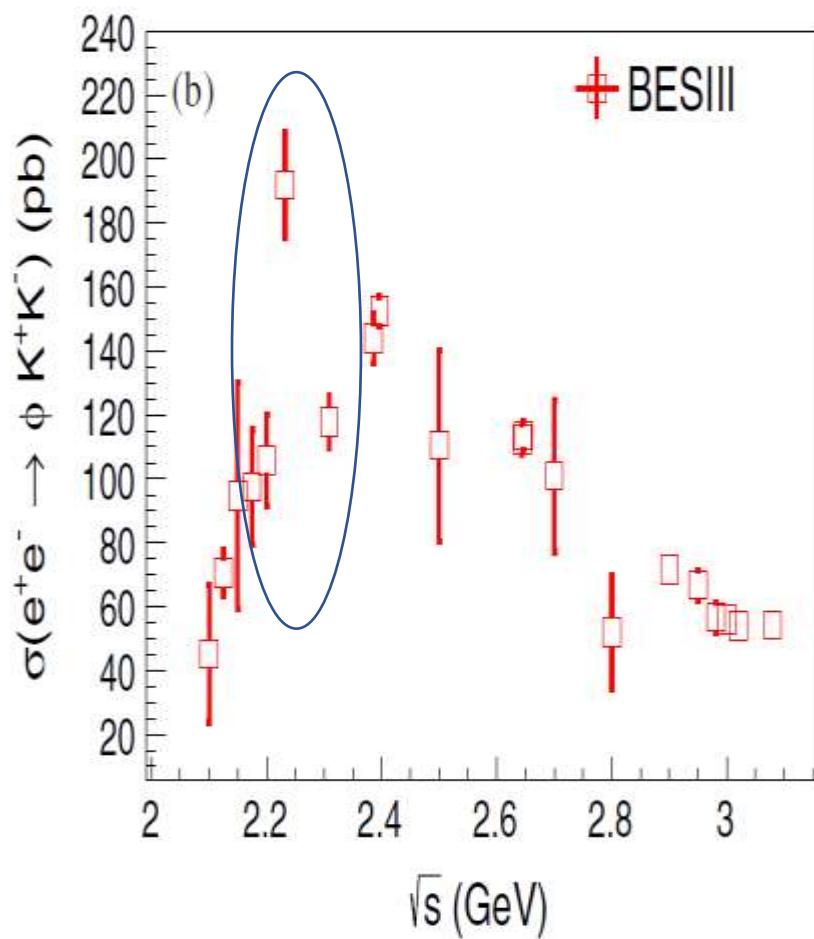
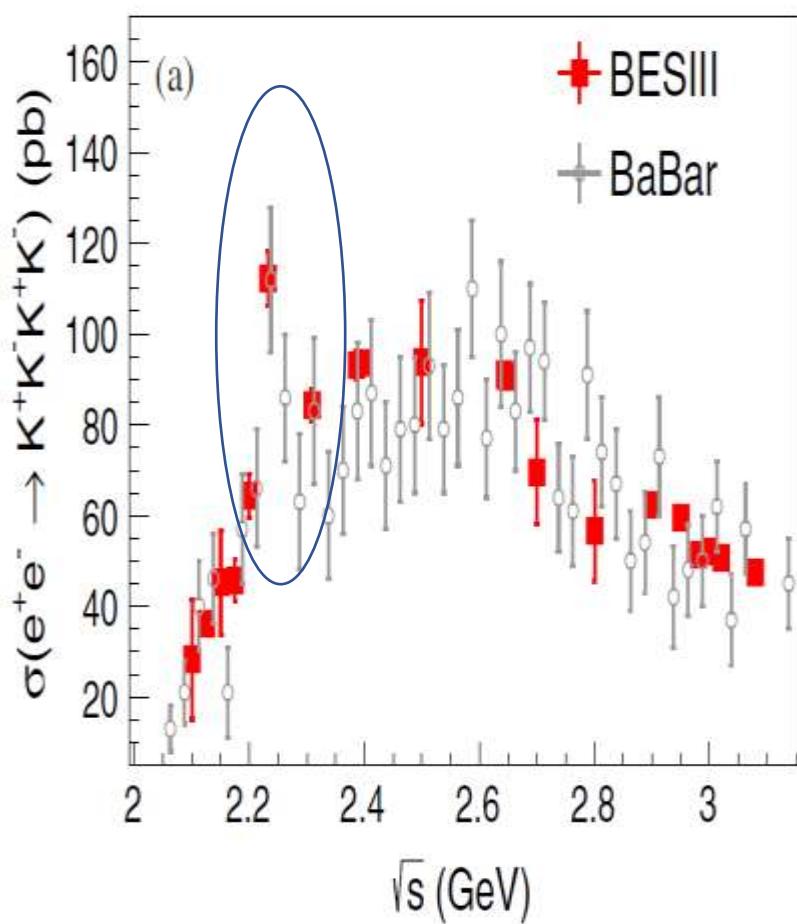
S.M. Flatte, Phys. Lett. B 63, 224-227 (1976).

On the other hand, if one takes a Flatté form for the total decay width of  $\omega(1420)$ ,  $\omega(1650)$ ,  $\phi(1680)$ , and  $\phi(2170)$ , the experimental data can also be well reproduced with a strong coupling of these resonances to the  $\Lambda\bar{\Lambda}$  channel.

Parameter	Value	Parameter	Value
$\gamma$ (GeV $^{-2}$ )	$0.57 \pm 0.21$	$\beta_{\omega\phi}$	$-0.3 \pm 0.31$
$\beta_x$	$-0.03 \pm 0.09$	$m_x$ (MeV)	$2237.7 \pm 50.2$
$\Gamma_0$ (MeV)	$8.8^{+75.9}_{-8.8}$	$g_{\Lambda\bar{\Lambda}}$	$3.0 \pm 1.9$

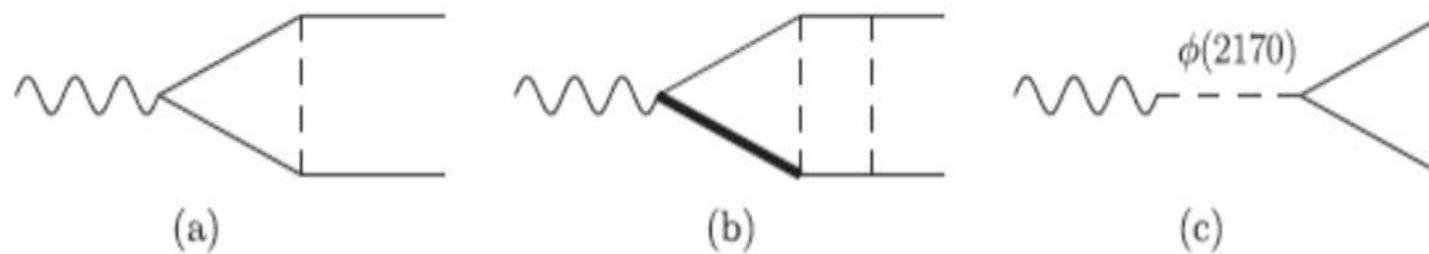
Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

# Where is X(2231)?



M. Ablikim, et al., Phys. Rev. D 100, 032009(2019).

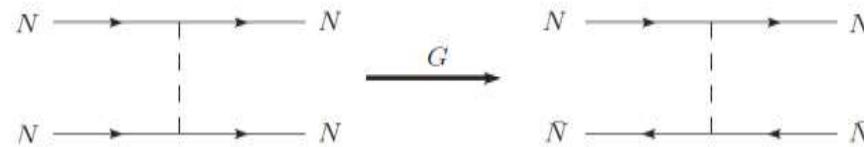
# Threshold enhancement: final state interaction



*FSI-meson exchange*

*$N^*$ -excitation ( $\Delta$ )*

*First flavorless vector meson*



$$V_{\bar{N}N}^{\text{OBE}} = (-1)^I V_{NN}^{\text{OBE}}, \quad V_{\bar{N}N}^{\text{TBE}} = (-1)^{I_1 + I_2} V_{NN}^{\text{TBE}}$$

$$T_{L''L'}(p'', p'; E_k) = V_{L''L'}(p'', p')$$

$$+ \sum_L \int \frac{dpp'^2}{(2\pi)^3} V_{L''L}(p'', p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p, p'; E_k)$$

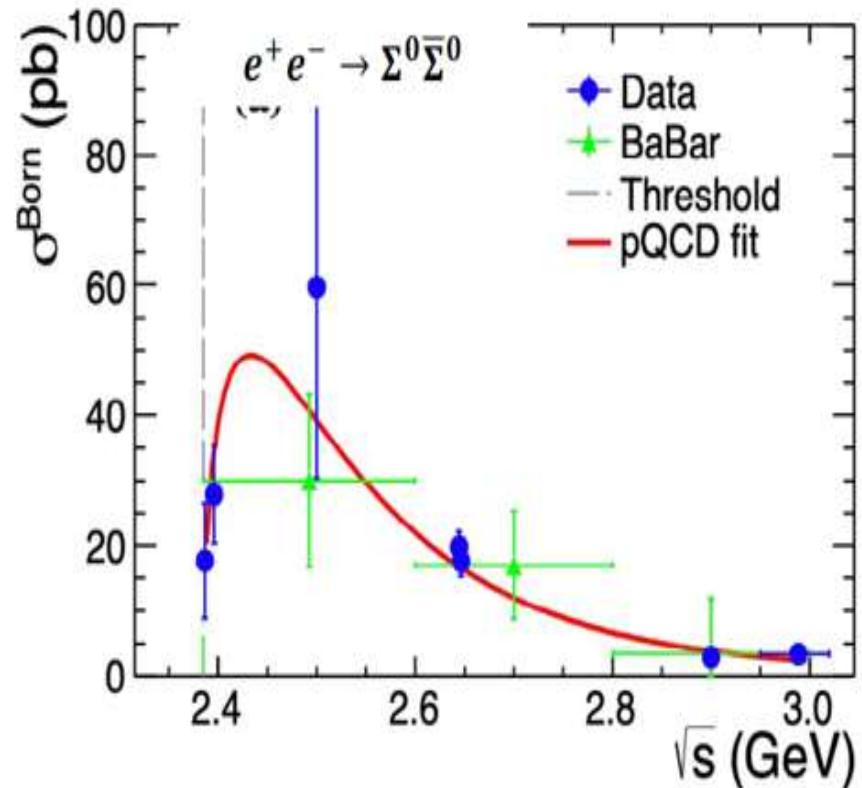
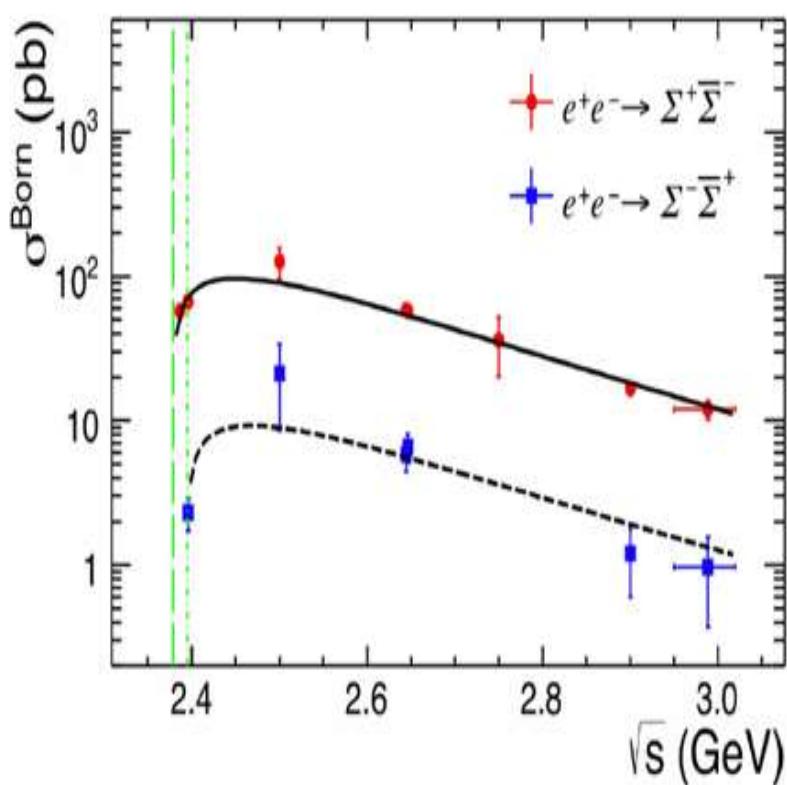
I.T. Lorenz, H.W. Hammer and U.G. Meissner, **Phys. Rev. D92, 034018 (2015)**.

Q.H. Yang, L.Y. Dai, D. Guo, J. Haidenbauer, X.W. Kang and U.G. Meissner, arXiv:2206.01494.

$\Sigma$

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$$\sigma(s) = \frac{4\pi\alpha^2\beta}{3s} C(s) \left[ 1 + \frac{2M^2}{s} \right] |G_{eff}(s)|^2 = \sigma_{point}(s) |G_{eff}(s)|^2.$$



BESIII, Phys. Lett. B 814, 136110 (2021); Phys. Lett. B 831, 137187 (2022).

The ratio  $\Sigma^+\bar{\Sigma}^- : \Sigma^0\bar{\Sigma}^0 : \Sigma^-\bar{\Sigma}^+$  is about  $9.7 \pm 1.3 : 3.3 \pm 0.7 : 1$ .

# $\Sigma^+$ and $\Sigma^-$ EMFFs (VMD)

$$|\Sigma^+ \bar{\Sigma}^-\rangle = \frac{1}{\sqrt{2}} |1, 0\rangle + \frac{1}{\sqrt{3}} |0, 0\rangle + \frac{1}{\sqrt{6}} |2, 0\rangle$$

$$|\Sigma^- \bar{\Sigma}^+\rangle = -\frac{1}{\sqrt{2}} |1, 0\rangle + \frac{1}{\sqrt{3}} |0, 0\rangle + \frac{1}{\sqrt{6}} |2, 0\rangle$$

$$|\Sigma^0 \bar{\Sigma}^0\rangle = -\frac{1}{\sqrt{3}} |0, 0\rangle + \sqrt{\frac{2}{3}} |2, 0\rangle$$



$$F_1^{\Sigma^+} = g(q^2)(f_1^{\Sigma^+} + \frac{\beta_\rho}{\sqrt{2}} B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}} B_{\omega\phi}),$$

$$F_2^{\Sigma^+} = g(q^2)(f_2^{\Sigma^+} B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}} B_{\omega\phi}),$$

$$F_1^{\Sigma^-} = g(q^2)(f_1^{\Sigma^-} - \frac{\beta_\rho}{\sqrt{2}} B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}} B_{\omega\phi}),$$

$$F_2^{\Sigma^-} = g(q^2)(f_2^{\Sigma^-} B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}} B_{\omega\phi}),$$

$$F_1^{\Sigma^0} = g(q^2)(\frac{\beta_{\omega\phi}}{\sqrt{3}} - \frac{\beta_{\omega\phi}}{\sqrt{3}} B_{\omega\phi}),$$

$$F_2^{\Sigma^0} = g(q^2) \mu_{\Sigma^0} B_{\omega\phi},$$

Isospin  
decomposition

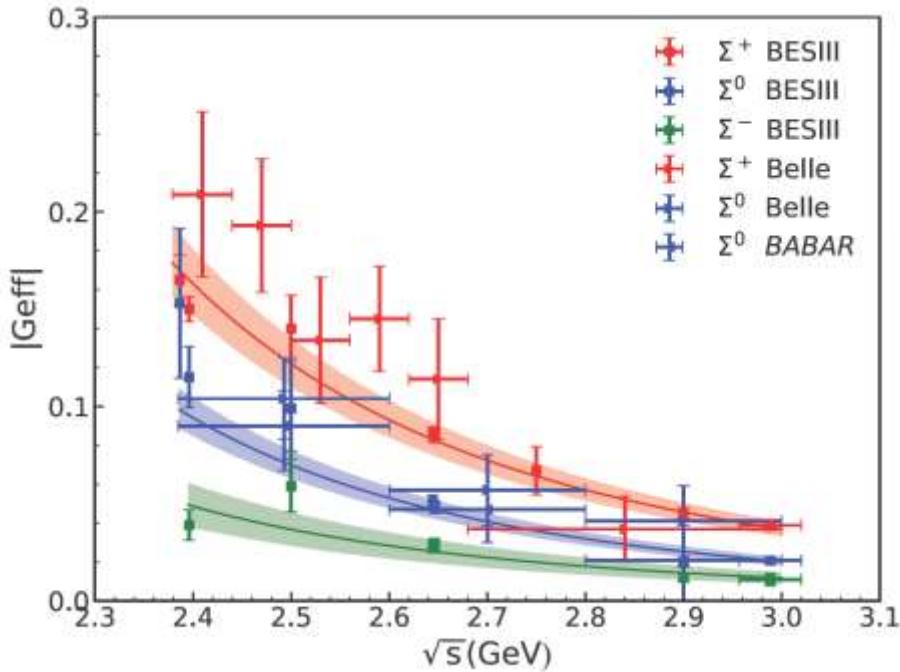
$$B_\rho = \frac{m_\rho^2}{m_\rho^2 - q^2 - im_\rho \Gamma_\rho},$$

$$B_{\omega\phi} = \frac{m_{\omega\phi}^2}{m_{\omega\phi}^2 - q^2 - im_{\omega\phi} \Gamma_{\omega\phi}},$$

$$f_1^{\Sigma^+} = 1 - \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^+} = 2.112 + \frac{\alpha_{\omega\phi}}{\sqrt{3}},$$

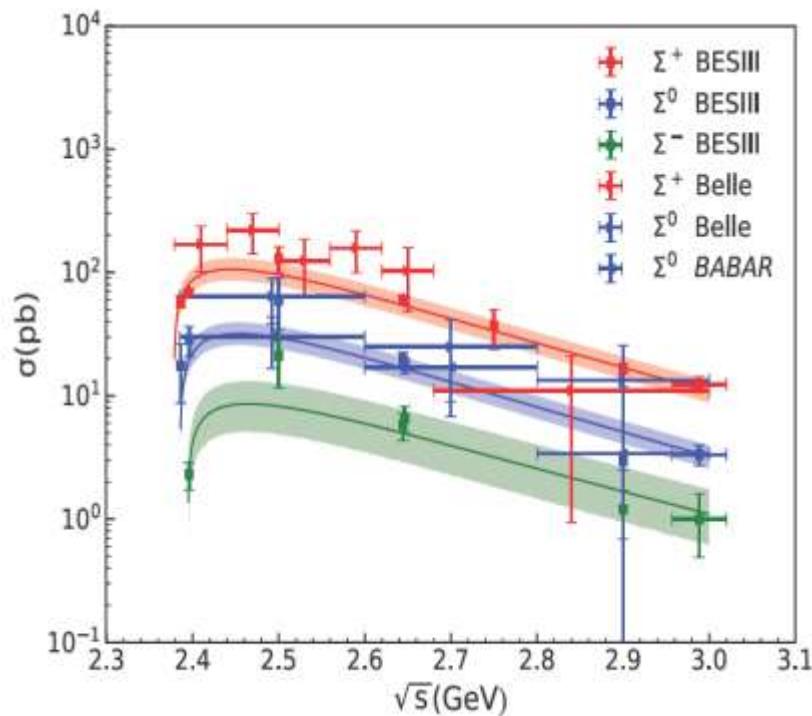
$$f_1^{\Sigma^-} = -1 + \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^-} = -0.479 + \frac{\alpha_{\omega\phi}}{\sqrt{3}}$$

# $\Sigma^+$ , $\Sigma^-$ , and $\Sigma^0$ EMFFs (VMD)

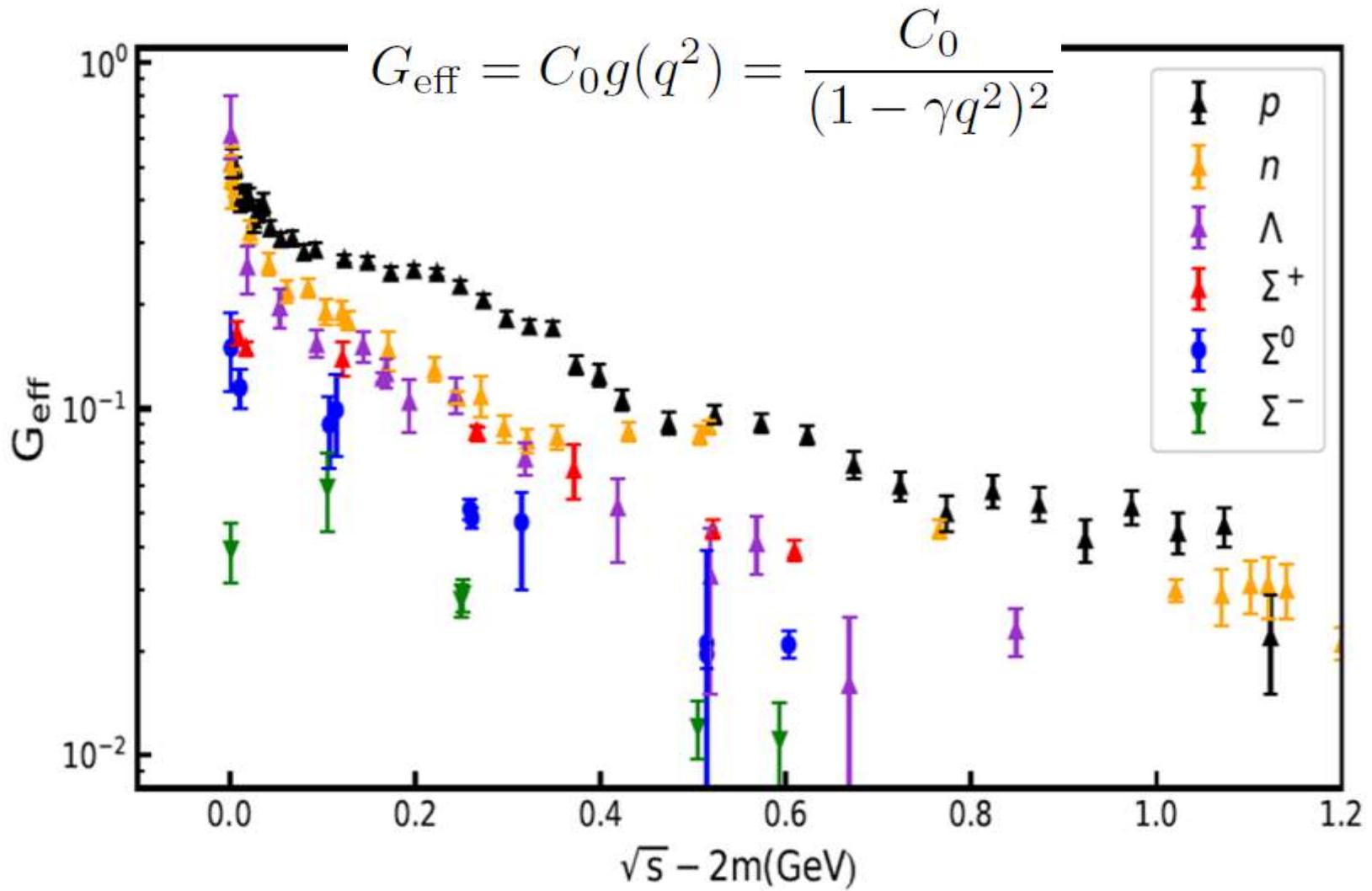


With one  $\gamma$ , we can describe all the current experimental data on  $\Sigma^+$ ,  $\Sigma^-$ , and  $\Sigma^0$  EMFFs .

Parameter	Value	Parameter	Value
$\gamma (\text{GeV}^{-2})$	$0.527 \pm 0.024$	$\alpha_{\omega\phi}$	$-3.18 \pm 0.77$
$\beta_{\omega\phi}$	$-0.08 \pm 0.06$	$\beta_\rho$	$1.63 \pm 0.07$

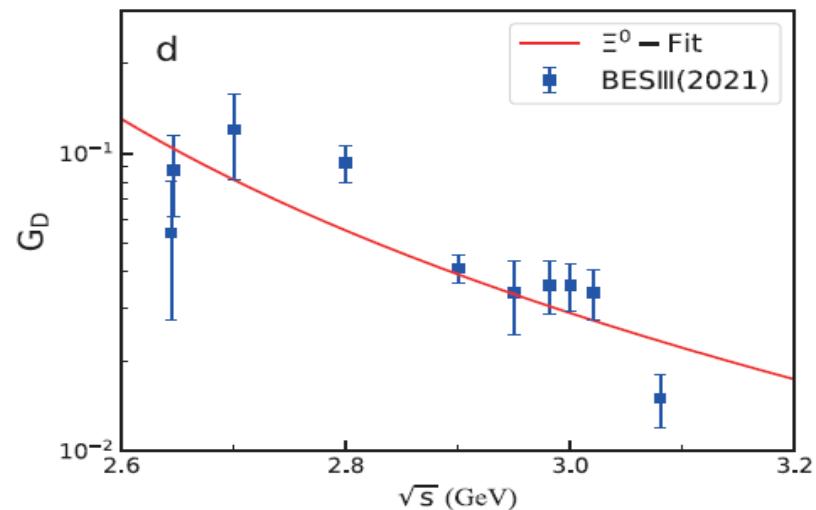
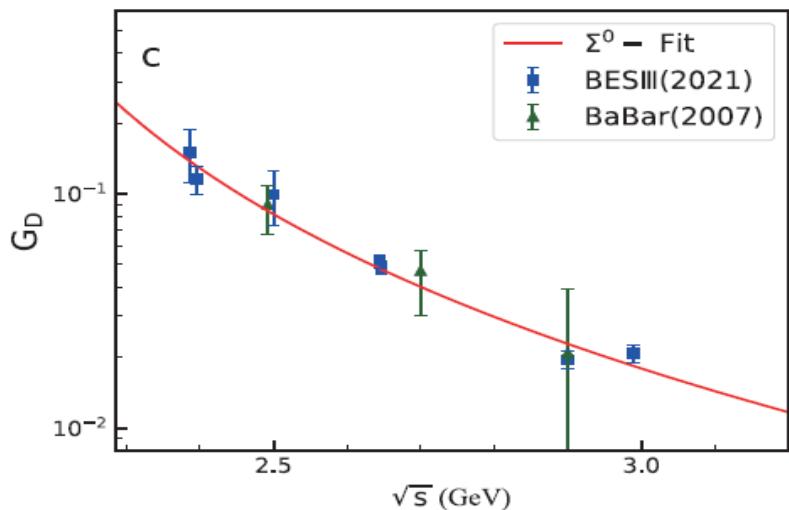
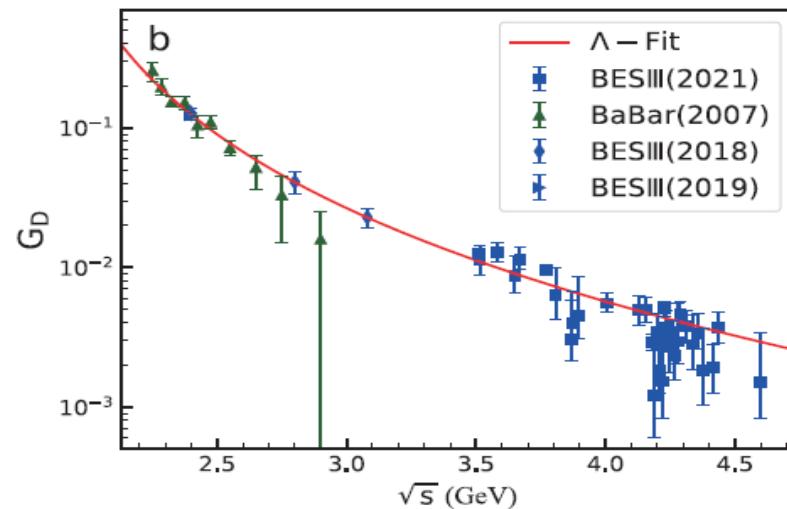
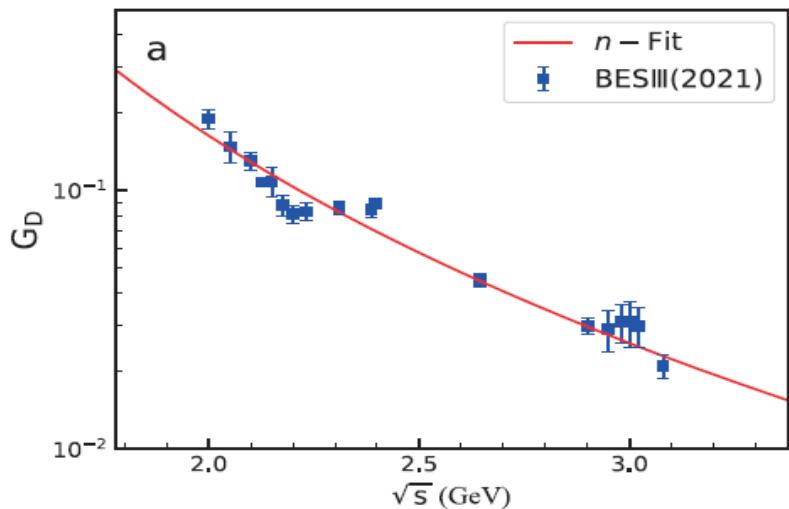


# Dipole behavior of baryon effective form factors



$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

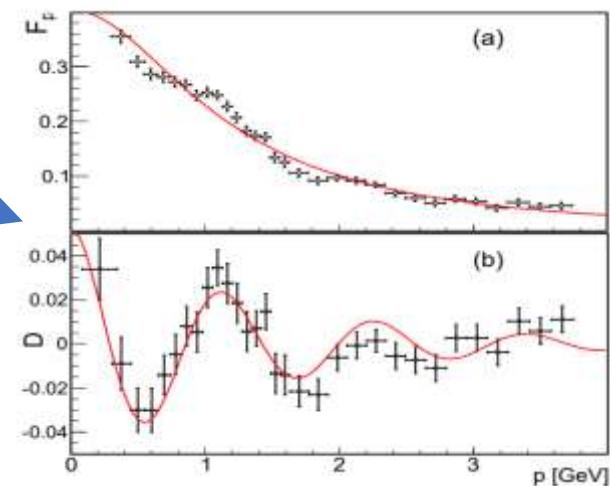
Parameter	$n$	$\Lambda$	$\Sigma^0$	$\Xi^0$
$\gamma$	1.41 (fixed)	$0.34 \pm 0.08$	$0.26 \pm 0.01$	$0.21 \pm 0.02$
$c_0$	$3.48 \pm 0.06$	$0.11 \pm 0.01$	$0.033 \pm 0.007$	$0.023 \pm 0.008$
$\chi^2/\text{dof}$	4.3	2.4	1.1	3.0



# Oscillation of baryon effective form factors

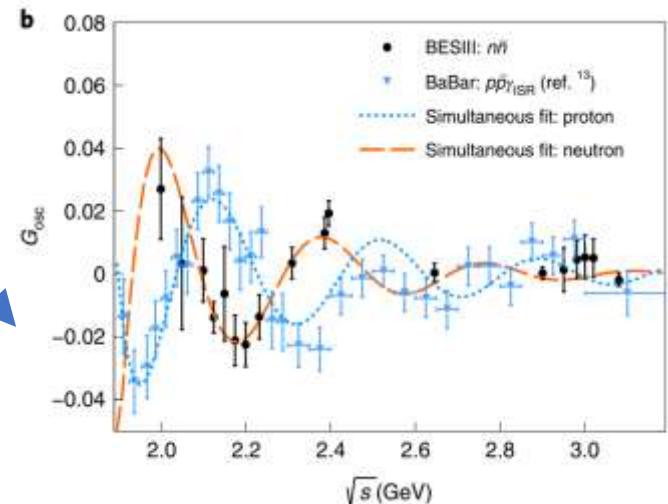
2015, Andrea Bianconi et al., Phys. Rev. Lett., 2015, 114(23): 232301.

$$F_{3p}(s) = \frac{F_0}{\left(1 + \frac{s}{m_a^2}\right)\left(1 - \frac{s}{m_0^2}\right)^2},$$
$$F_{\text{osc}}(p(s)) = A e^{-Bp} \cos(Cp + D).$$



2021, BESIII Collaboration, Nature Phys., 2021, 17(11): 1200-1204.

$$F_{\text{osc}}^{n,p} = A^{n,p} \exp(-B^{n,p} p) \cos(Cp + D^{n,p})$$



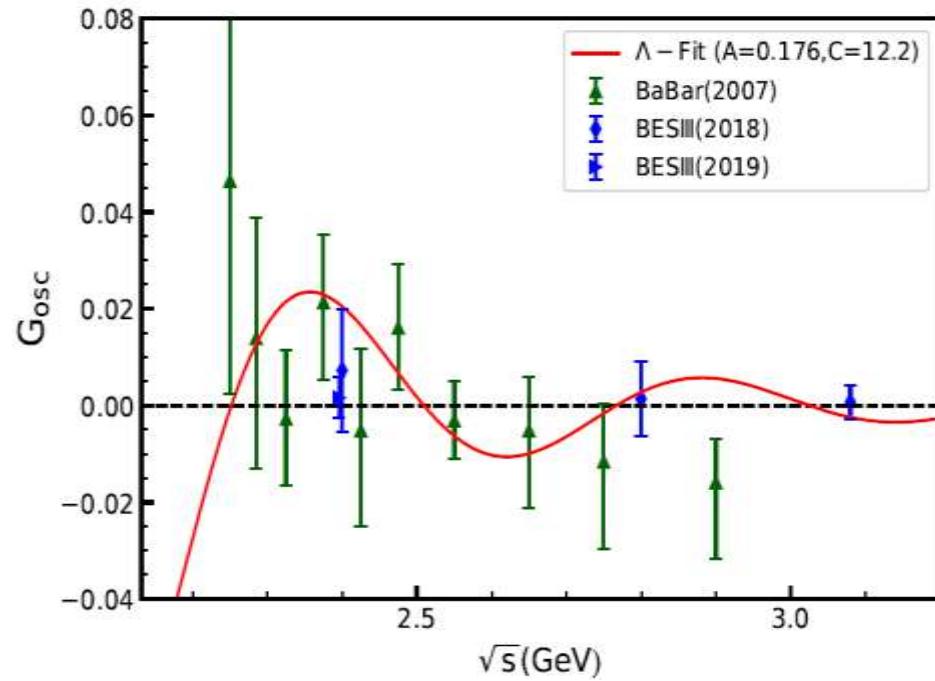
# New parametrization

$$G_{osc} = A \cdot \frac{c_0}{(1 - \gamma \cdot s)^2} \cdot \cos(C \cdot \sqrt{s} + D)$$

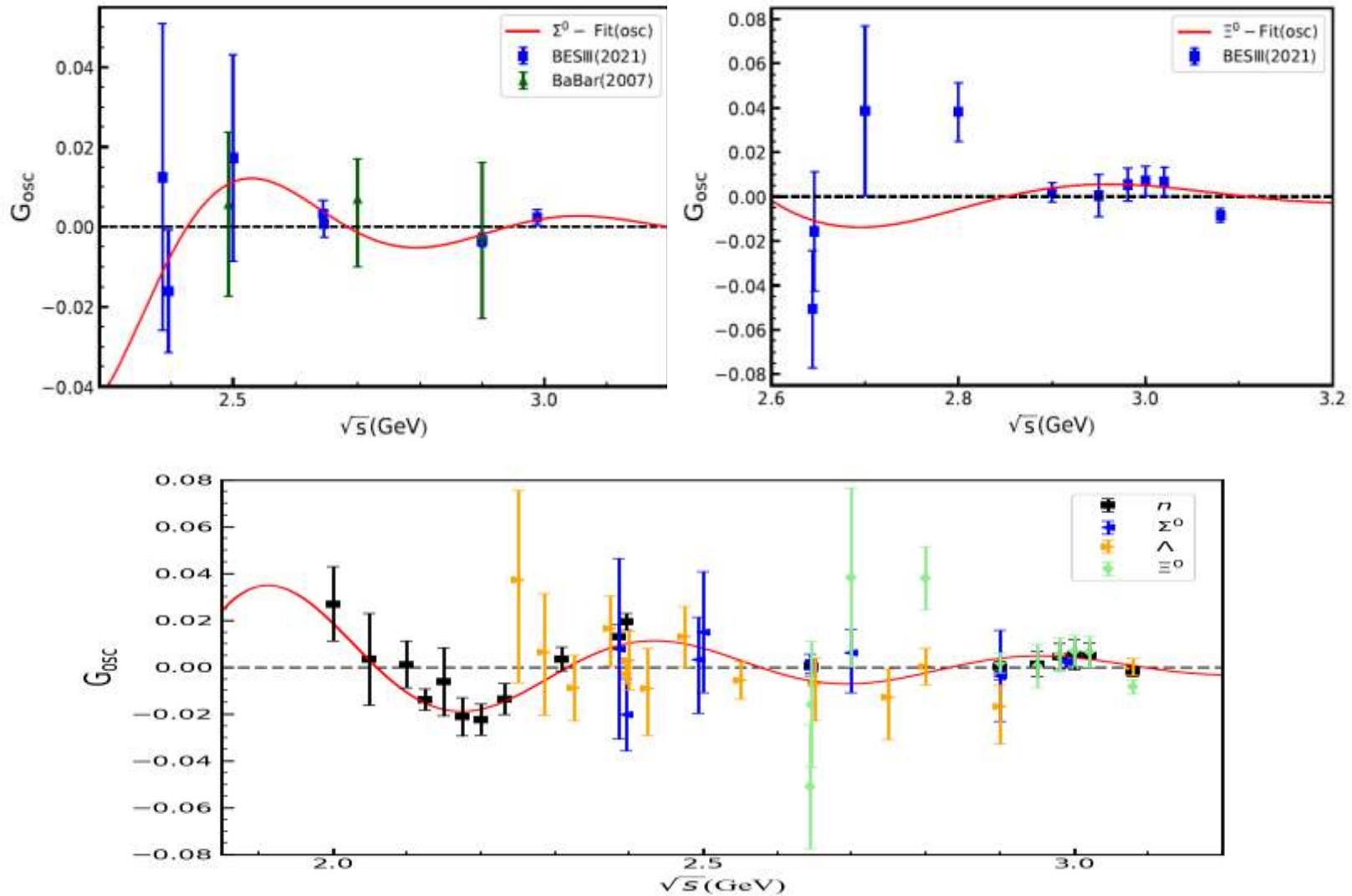
$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

$$\begin{aligned} G_{\text{eff}}(s) &= G_D(s) + G_{\text{osc}}(s) \\ &= \frac{c_0}{(1 - \gamma s)^2} \left( 1 + A \cos(C \sqrt{s} + D) \right) \end{aligned}$$

$$\begin{aligned} \text{data} &= G_{\text{eff}} = G_D + G_{\text{osc}} \\ \rightarrow G_{\text{osc}} &= \text{data} - G_D \end{aligned}$$



# Numerical results



## Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

Xu Cao<sup>1,2,\*</sup>, Jian-Ping Dai<sup>3,†</sup>, and Horst Lenske<sup>4,‡</sup>

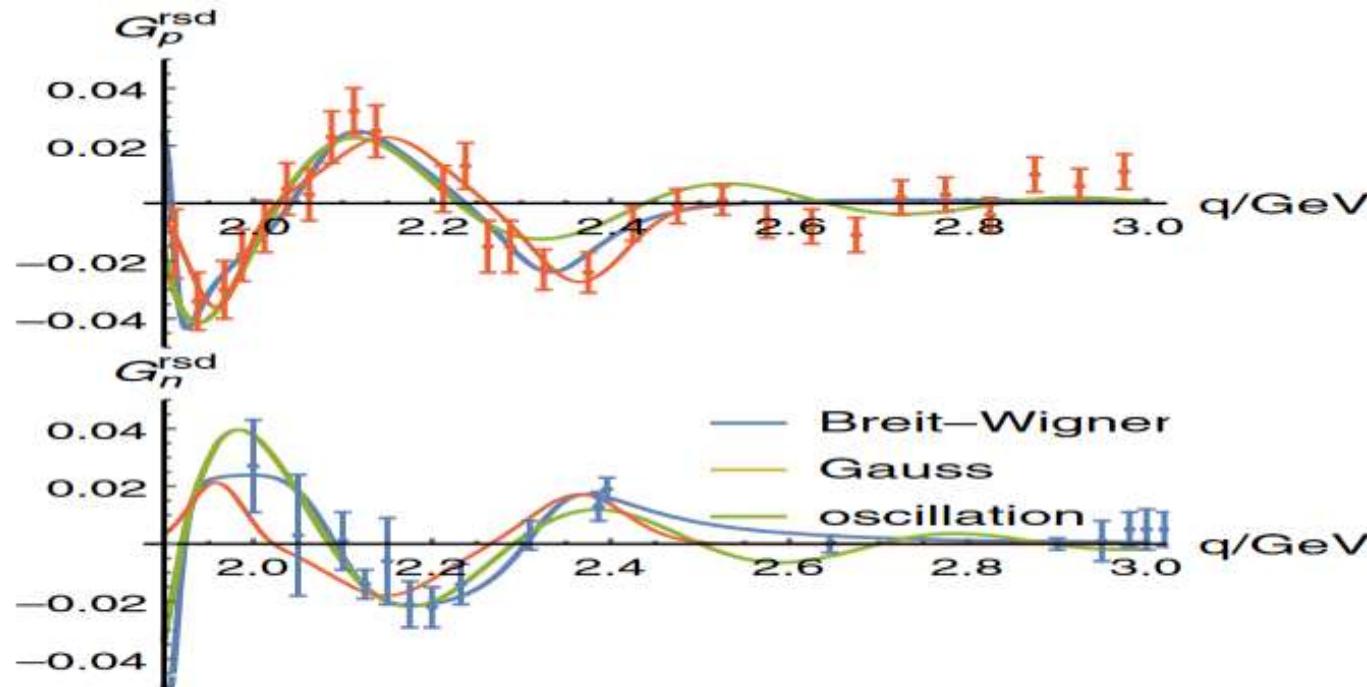


FIG. 1. The fit of the Breit–Wigner distribution and the Gauss distribution to three local structures below 2.5 GeV in comparison with BESIII data [16,18].

e-Print: [2206.01494](#) [nucl-th]

## New insights into the oscillation of the nucleon electromagnetic form factors

Qin-He Yang<sup>1,2</sup>, Ling-Yun Dai<sup>1,2</sup>  Di Guo<sup>1,2</sup>, Johann Haidenbauer<sup>3</sup>, Xian-Wei Kang<sup>4,5</sup>, and Ulf-G. Meißner<sup>6,3,7</sup>

PHYSICAL REVIEW D **107**, L091502 (2023)

Letter

### Toy model to understand the oscillatory behavior in timelike nucleon form factors

Ri-Qing Qian,<sup>1,2,3,4,\*</sup> Zhan-Wei Liu, Xu Cao, and Xiang Liu

PHYSICAL REVIEW LETTERS **128**, 052002 (2022)

### New Insights into the Nucleon's Electromagnetic Structure

Yong-Hui Lin,<sup>1</sup> Hans-Werner Hammer,<sup>2,3</sup> and Ulf-G. Meißner<sup>1,4,5</sup>

# New experimental results

Eur. Phys. J. C (2022) 82:761  
https://doi.org/10.1140/epjc/s10052-022-10696-0

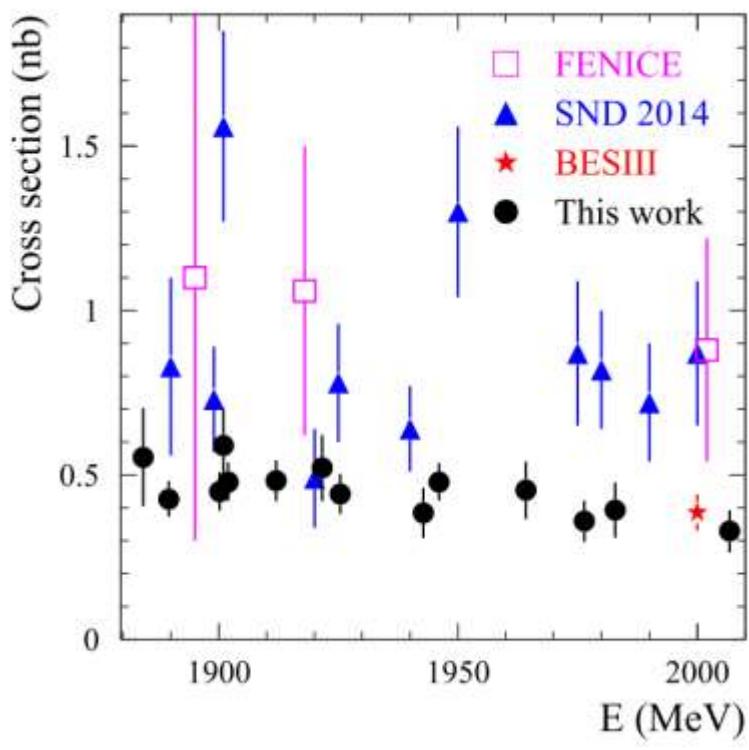
THE EUROPEAN  
PHYSICAL JOURNAL C



Regular Article - Experimental Physics

## Experimental study of the $e^+e^- \rightarrow n\bar{n}$ process at the VEPP-2000 $e^+e^-$ collider with the SND detector

SND Collaboration

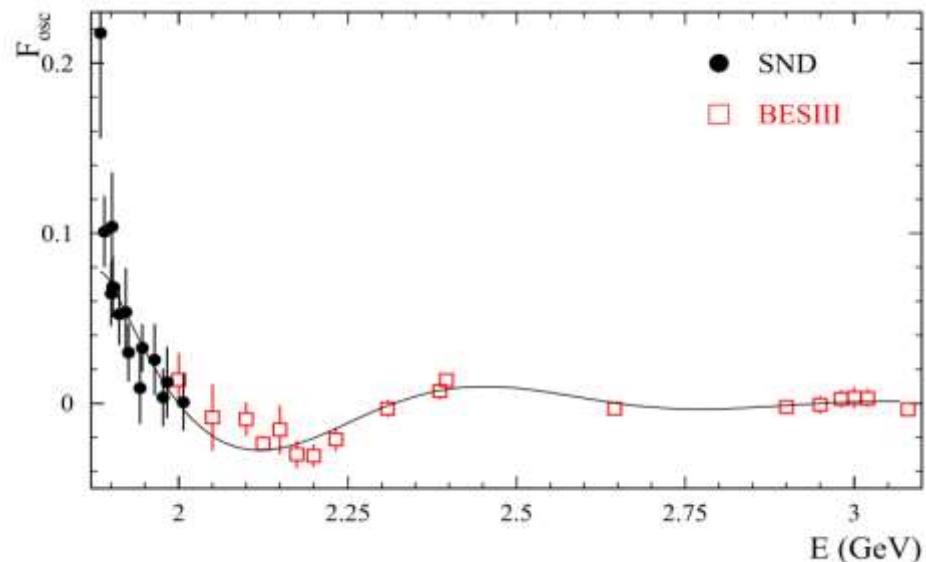


$$F(s) = F_0(s) + F_{\text{osc}}(s),$$

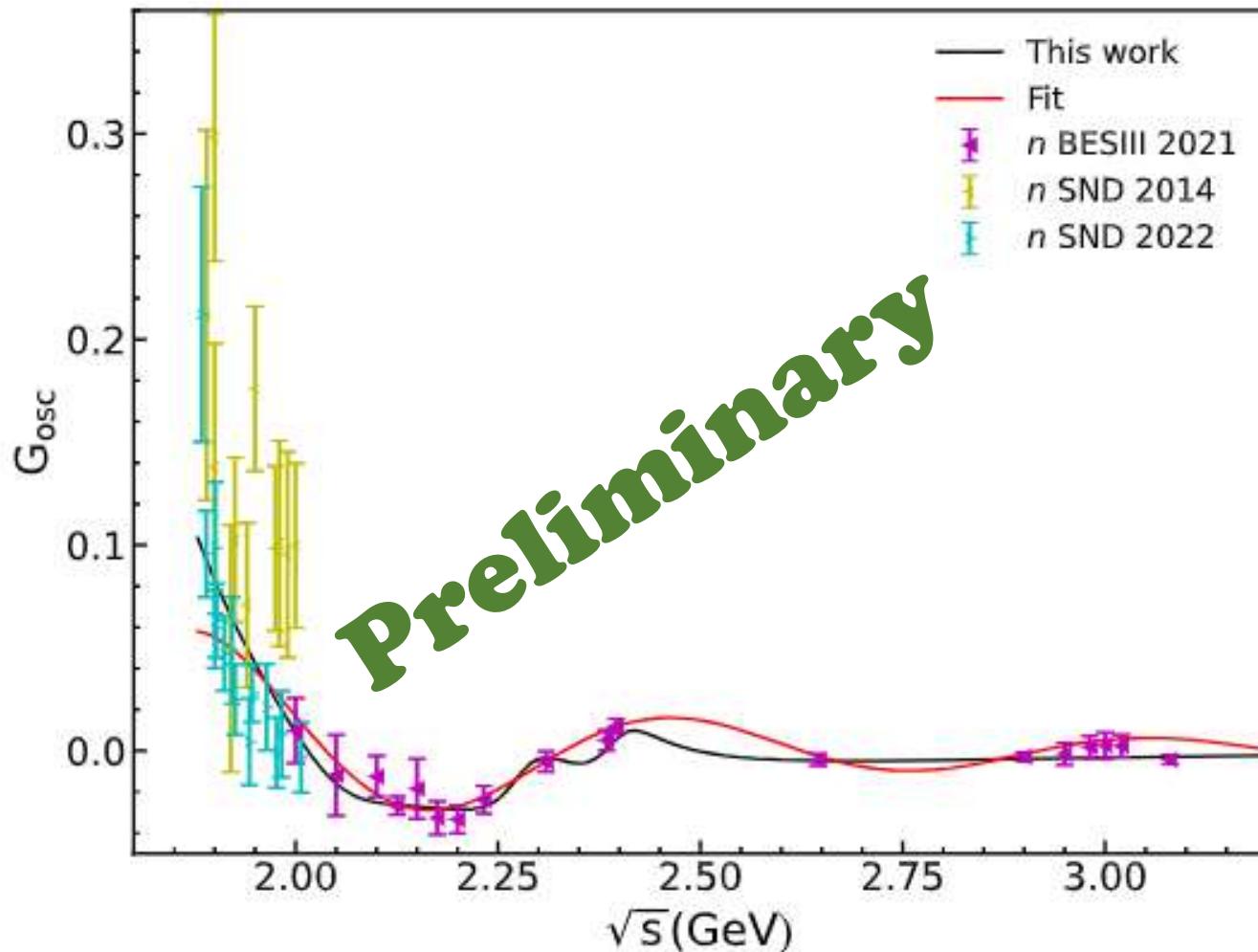
$$F_0(s) = \frac{\mathcal{A}_n}{[1 - s/0.71(\text{GeV}^2)]^2},$$

$$F_{\text{osc}}(s) = A \exp(-Bp) \cos(Cp + D),$$

$$p = \sqrt{(s/2m_n - m_n)^2 - m_n^2}.$$



# Vector meson dominance



# Summary

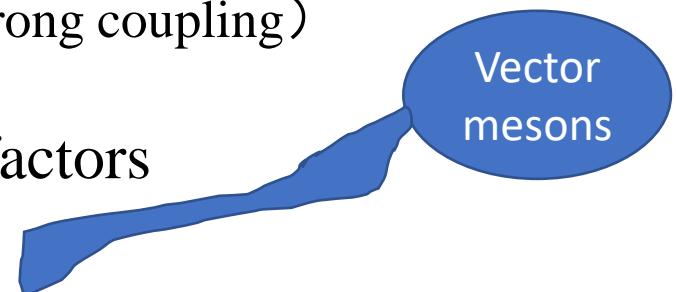
## 1. Threshold enhancement

- a), Final state interaction      b), Flatte (strong coupling)

## 2. Oscillation of baryon effective form factors

- a), Phenomenology      b), Mechanism unknown

$$g(q^2) = \frac{1}{(1 - \gamma q^2)^2}$$



Thank you very much for your attention!