

Axial meson exchange and the molecular $Z_c(3900)$ and $Z_{cs}(3985)$ resonances



中国科学院理论物理研究所

2023年7月25-30日

第八届 XYZ 粒子研讨会

Outline

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- X(3872) and $Z_c(3900)$ in $D^*\bar{D}$ scattering
- Z_{cs}(3985) generation

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Summary

Observations on $Z_c(3900)$ and $Z_{cs}(3985)$

Observations on $Z_c(3900/4020)$

•
$$e^+e^- \to J/\psi\pi^+\pi$$
, $Z_c(3900) \to J/\psi\pi^{\pm}$,
 $M_R - i\Gamma = 3899.0 \pm 8.5 - i46 \pm 10.8 \,\mathrm{MeV}$ PRL 110,252001(2013), BESIII
 $D^*\bar{D}$ threshold: 3875.80 MeV
• $e^+e^- \to h_c\pi^+\pi$, $Z_c(4020) \to h_c\pi^{\pm}$ and $Z_c(3900) \to h_c\pi^{\pm}$ not seen
 $M_R - i\Gamma = 4022.9 \pm 3.5 - i7.9 \pm 5.3 \,\mathrm{MeV}$ PRL 111,242001(2013), BESIII
 $D^*\bar{D}^*$ threshold: 4017 11 MeV

Observation on $Z_{cs}(3985)$

- $e^+e^- \rightarrow K^+ Z_{cs}(3985) \rightarrow K^+ (D_s^- D^{*0} + D_s^{*-} D^0)$ $3982.5^{+1.8}_{-2.6} - i12.8^{+5.3}_{-4.4} \text{ MeV}$ prl 126,102001(2021), Prl 129,112003 (2022), BESIII $D_s^- D^{*0} - D_s^{*-} D^0$ thresholds: 3975.19-3977.03 MeV
- A broad $Z_{\it cs}(4000) o J/\psi K$, prl 127 8, 082001(2021), LHCb
- $Z_{cs}^{\prime}(4123) \rightarrow D_{s}^{*-} D^{*0}$ (2.1 σ), CPC 47 (2023) 3, 033001, Besiii

Models on the molecular $Z_c(3900)$

π exchange

• π exchange generates $Z_{c}(3900)$ 2306.12406, G.J. Wang et al

σ and π exchange

• π and σ exchange generate $Z_{c}(4020)$, EPJC,61,(2009),Xiang Liu et al;PRD 99, (2019),

M.Z. Liu et al

Two pions exchange

• Two-pion exchange generates $Z_c(3900/4020)$, PRD 90, (2014), E. Oset et al

Charmonium exchange

• Charmonium exchange generate $Z_c(3900/4020)$, PRD 90,(2014) J. He; Prog. Phys.

41. 65 (2021) X.K. Dong et al

Coupled scattering

• Probing $Z_c(3900)$ in $J/\psi\pi-D^*ar{D}$, 2307.12283, Lin-Wan Yan et al

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The $L\sigma M$

L^{NσL}_{int} = g ψ_N(φ₀ + i γ₅ τ · φ)ψ_N, where ψ_N is the relativistic nucleon field and g a coupling constant.
 (⟨φ₀⟩ = f_π/√2) which also provides the nucleons with mass.
 The σ field is defined as a perturbation of the φ₀ field around its vacuum expectation value (φ₀ = f_π/√2 + σ).

•
$$g = g_{\sigma NN} = g_{\pi NN} = \sqrt{2} M_N / f_{\pi} = 10.2.$$

$\pi - a_1$ mixing

• $I^{G}(J^{PC}) = 1^{-}(1^{++})$ can mix with the pions, $\partial_{\mu}\pi \rightarrow \partial_{\mu}\pi + \lambda_{1} m_{a1} a_{1\mu}$ where π^{a} and $a_{1\mu}$ are the pion and axial meson fields and λ_{1} a proportionality constant, which we expect to be in the $\lambda_{1} \sim (1.6 - 2.1)$ range.

Saturated potential: scalar and axial meson

Scalar meson:

$$egin{array}{rcl} \mathcal{L}_{\mathcal{S}} &=& g_{\sigma 1} \, q_L^\dagger \sigma q_L \,, \ \mathcal{V}_{\sigma}(ec{q}) &=& -rac{g_{\sigma 1}^2}{ec{q}^2+m_\sigma^2} \,, \end{array}$$

Axial meson:

$$\mathcal{L} = rac{\mathcal{g}_1}{\sqrt{2} f_\pi} q_L^\dagger \, ec{\sigma}_L \cdot ec{a} \, q_L \, ,$$

with $\vec{\sigma}_L$ the spin operators (Pauli matrices) as applied to the light-quark spin.

$$V_{\pi}(\vec{q}) = -\zeta \, rac{g_1^2}{6 f_{\pi}^2} \, ec{ au_1} \cdot ec{ au_2} \, rac{ec{\sigma}_{L1} \cdot ec{\sigma}_{L2} \, ec{q}^2}{ec{q}^2 + m_{\pi}^2}$$

$$V_{a_1}(\vec{q}) = -\zeta \,\lambda_1^2 \frac{g_1^2 m_{a_1}^2}{2f_{\pi}^2} \,\vec{\tau}_1 \cdot \vec{\tau}_2 \, \frac{\vec{\sigma}_{L1} \cdot \vec{\sigma}_{L2}}{\vec{q}^2 + m_{a_1}^2} \, (1 + \frac{\vec{q}^2}{3m_{a_1}^2}) + \dots$$

Saturated potential: vector meson exchange

Lagrangian (HHV) reads

$$\mathcal{L}_{V} = \mathcal{L}_{E0} + \mathcal{L}_{M1} = q_{L}^{\dagger} \left[g_{V1} V^{0} + \frac{f_{V1}}{2M} \epsilon_{ijk} \sigma_{Li} \partial_{j} V_{k} \right] q_{L},$$

with g_{V1} and $f_{V1} = g_{V1}\kappa_{V1}$.

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with g_{V1} and $f_{V1}=g_{V1}\kappa_{V1}$.The potential reads

$$V_V(\vec{q}) = V_{E0}(\vec{q}) + V_{M1}(\vec{q}),$$

which read

$$\begin{split} V_{E0}(\vec{q}) &= +\frac{g_{V1}^2}{\vec{q}^2 + m_V^2} \,, \\ V_{M1}(\vec{q}) &= -\frac{f_{V1}^2}{4M^2} \frac{(\vec{\sigma}_{L1} \times \vec{q}) \cdot (\vec{\sigma}_{L1} \times \vec{q})}{\vec{q}^2 + m_V^2} \\ &= -\frac{f_{V1}^2}{6M^2} \, \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L1} \, \frac{\vec{q}^2}{\vec{q}^2 + m_V^2} + \dots \,, \end{split}$$

X(3872) and $Z_c(3900)$ generation in $D^*\overline{D}$ scattering

Assuming X(3872) and $Z_c(3900)$ are molecules

- V_X and V_Z are determined by light meson saturation
- Regularizing the potential in a form of

$$V_{\mathcal{M}}(\vec{q}) \rightarrow f_{\mathcal{M}}^2(\vec{q}) V_{\mathcal{M}}(\vec{q})$$
.

with

$$f_M(q) = \left(rac{\Lambda^2 - m^2}{\Lambda^2 - q^2}
ight)^{n_P},$$

and $n_P = 2$.

- After Fourier transformation and solving the Schrödinger equation, Λ is a parameter to produce the binding energies.
- Generation on X(3872) and $Z_c(3900)$ correspond to binding energies in $D^*\bar{D}$ scattering, $BE_X = 4 \text{ MeV}$ and $BE_Z = 0 \text{ MeV}$

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X(3872) and $Z_c(3900)$ generation in $D^*\overline{D}$ scattering

$m_{\sigma} = 550, \, 450 - 600 \, { m MeV}$, $\Lambda_X = 1.0 \, { m GeV}$

Iight meson saturation:

$$\frac{\Lambda(Z_c)}{\Lambda(X)} = 1.33 \,(1.08 - 1.41) \,.$$

Within HQSS uncertainty

$$\frac{\Lambda_Z}{\Lambda_X} = 1 \pm 0.15 = (0.85 - 1.15) \,,$$

Polarity (n_P)	$R(Z_c/X)$	$R^{\not =_1}(Z_c/X)$
2	1.33 (1.08-1.41)	1.45 (1.08-1.69)
3	1.33 (1.09-1.42)	1.48 (1.10-1.72)
4	1.39 (1.15-1.48)	1.65 (1.18-1.98)

X(3872) and $Z_c(3900)$ in $D^*\overline{D}$ scattering



- *a*₁ exchange provides additional attractive interaction in *Z_c* spectrum in blue.
- w/o a_1 exchange, isovector $D\bar{D}$ generates a virtual pole.

$Z_{cs}(3985)$ generation

For the $D^*\bar{D}_s$ - $D\bar{D}_s^*$ molecules, even if we consider these two channels to be degenerate (which we do here), the structure of the potential is still better understood as a coupled channel problem, i.e.

$$V(Z_{cs}) = \begin{pmatrix} V_a^{(1)} & V_b^{(1)} \\ V_b^{(1)} & V_a^{(1)} \end{pmatrix} ,$$

where $V_a^{(1)}$ and $V_b^{(1)}$ are the central and spin-dependent parts of the potential.

$$\begin{array}{lll} V(\tilde{Z}_{cs}) & = & V_a^{(1)} + V_b^{(1)} \, , \\ V(Z_{cs}) & = & V_a^{(1)} - V_b^{(1)} \, , \end{array}$$

which would be the strange counterparts of the 1^{++} and 1^{+-} isovector configurations

While the a_0 and K_0^* are pure octets, the σ and $f_0(980)$ are a mixture of singlet and octet, i.e.

$$\begin{pmatrix} f_0(500)\\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} S_1\\ S_8 \end{pmatrix} ,$$

with $\theta = 19 \pm 5^{\circ}$.

The second problem is the singlet and octet mixing: if we consider that the isoscalar partners of the a_1 are the $f_1(1285)$ and $f_1(1420)$, they will be a non-trivial mixture of a singlet and octet axial meson

$$\begin{pmatrix} f_1(1285)\\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1\\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} f_1^1\\ f_1^8 \end{pmatrix},$$

where f_1^1 and f_1^8 are the singlet and octet components of the two f_1 's.

$Z_{cs}(3985): m_{\sigma} = 550, 450 - 600 \,\mathrm{MeV}$

$$\begin{split} & \frac{\Lambda^{\sigma(\text{NS})}(Z_{cs})}{\Lambda^{\sigma(\text{NS})}(Z_{c})}\Big|_{\theta_{1}^{+}} &= 3.69 \left(3.35 - 4.92\right), \\ & \frac{\Lambda^{\sigma(\text{NS})}(Z_{cs})}{\Lambda^{\sigma(\text{NS})}(Z_{c})}\Big|_{\theta_{1}^{-}} &= 3.57 \left(3.24 - 4.74\right), \\ & \frac{\Lambda^{\sigma(\text{FS})}(Z_{cs})}{\Lambda^{\sigma(\text{FS})}(Z_{c})}\Big|_{\theta_{1}^{+}} &= 1.04 \left(1.00 - 1.06\right), \\ & \frac{\Lambda^{\sigma(\text{FS})}(Z_{cs})}{\Lambda^{\sigma(\text{FS})}(Z_{c})}\Big|_{\theta_{1}^{-}} &= 1.06 \left(1.00 - 1.09\right), \end{split}$$

with $\theta_1^{\pm} = 35.3^{\circ} \pm (31^{\circ} \pm 2^{\circ})$. SU(3)-flavor symmetry,

$$\frac{\Lambda(Z_{\rm cs})}{\Lambda(Z_{\rm c})}\simeq 1.0\,. \label{eq:cs}$$

More thinking on σ exchange

σ coupling

- $g_{\sigma} = 2.4$, a virtual pole ($Z_R = 3860 \text{ MeV}$) corresponding to X(3872).
- $D_s^- \Lambda_c$ binds as $P_{us}^{\Lambda}(4255)$, which is suggested to be a virtual pole by the fit. 2211.08211, Z. Y. Yang et al; PRD 108 (2023) L011501, Satoshi et al
- $\mathcal{M} = V_p/(1/(a_1 + ia_2) ik)$ in $J/\psi \Lambda D_s^- \Lambda_c^+$ scattering with $a^{eff} = (0.24 \pm 0.06) + i(0.35 \pm 0.04)$ fm.



More thinking on σ exchange

σ exchange in P_{ψ}^{N} and $P_{\psi s}^{\Lambda}$

- P_{ψ}^{N} : $\mathcal{M} = P_{1}t_{11} + P_{2}t_{21}$, 5 pars
- $P_{\psi s}^{\Lambda}$: $\mathcal{M} = P_1 t_{11} + P_2 t_{21} + P_3 t_{31}$ with HQSS, 5 pars
- A virtual pole $(4211, 48)\,{
 m MeV}$ w.r.t $ar{D}\Lambda_{c}$, 2305.01449, D. Winney et al.
- ω -exchange providing repulsive interaction in $D\Lambda_c$ scatttering.



Summary

- L σ M and πa_1 are introduced to the light meson exchange and explain X(3872) and $Z_c(3900)$ in $D^*\bar{D}$ scattering, where σ and a_1 contribute to attractive potentials, simultaneously.
- $Z_{cs}(3985)$ can be understood as a $D_s^- D^{*\,0} D_s^{*\,-} D^0$ molecule.
- σ may couple to strange quark with identical strength to u/d quark.
- Besides σ exchange, a_1 exchange is non-negligible in $I = I_1 + I_2$ cases.
- σ exchange potential plays a crucial role in $\bar{D}_s \Lambda_c$ and $\bar{D} \Lambda_c$ scattering.

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- Is there a duality of σ and charmonium exchange? s t symmetry in scattering?

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Thanks!

Backup: potential in coordinate space

If we particularize for the ρ meson, we will have to include isospin factors

$$V_{\rho}(\vec{r}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \left[g_{\rho 1}^2 + f_{\rho 1}^2 \frac{m_{\rho}^2}{6M^2} \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L1} \right] \frac{e^{-m_{\rho}r}}{4\pi r} + \dots$$

For the ω no isospin factor is required, but there is a sign coming from the negative G-parity of this meson

$$V_{\omega}(\vec{r}) = \zeta \left[g_{\omega 1}^2 + f_{\omega 1}^2 \frac{m_{\omega}^2}{6M^2} \vec{\sigma}_{L1} \cdot \vec{\sigma}_{L1} \right] \frac{e^{-m_{\omega}r}}{4\pi r} + \dots ,$$

where, as usual, $\zeta = +1$ (-1) for the meson-meson (meson-antimeson) potential.

Coupling	Value	Relevant to meson(s)
g 1	0.60	π , a_1
g_{σ}	3.4	σ
g_{V1}	2.9	$ ho$, ω
κ_{V1}	2.8	$ ho$, ω
λ_1	1.8	a_1

Table 1: Couplings of the light-mesons we are considering in this work (π , σ , ρ , ω and a_1) to the charmed mesons. For the masses of the light-mesons we will use $m_{\pi} = 138 \text{ MeV}$, $m_{\sigma} = 550 \text{ MeV}$, $m_{\rho} = 770 \text{ MeV}$, $m_{\omega} = 780 \text{ MeV}$ and $m_{a_1} = 1230 \text{ MeV}$. For the vector mesons we use the scaling mass M = 938 MeV. For the charmed mesons we will consider their isospin-averaged masses, $m_D = 1867 \text{ MeV}$ and $m_{D^*} = 2009 \text{ MeV}$.

The momentum dependent light meson exchange potential always contains contribution in short distance, which is not important in shallow bound state generation. To remove this ambiguity, a scheme in renormalization is introduced as

$$rac{ec{q^2}}{m^2+ec{q}^2} o 1 - rac{m^2}{m^2+ec{q}^2},$$

where 1 corresponds the δ force and will be dropped in the following calculations.

The basic idea is to substitute the narrow meson propagator by a propagator averaged over the actual mass distribution of the meson

$$\frac{1}{m^2 + \vec{q}^2} \to \int_{m_{\rm th}}^{\infty} \frac{\rho(\mu^2) \, d(\mu^2)}{\mu^2 + \vec{q}^2} \,, \tag{1}$$

where $\rho(\mu^2)$ is the spectral distribution of the wide meson and $m_{\rm th}$ the threshold mass of the particles into which this meson can decay,which amounts to a two-pole approximation of the previous integral

$$\int_{m_{\rm th}}^{\infty} \frac{\rho(\mu^2) \, d(\mu^2)}{\mu^2 + \vec{q}^{\,2}} \approx \frac{\alpha_1}{m_1^2 + \vec{q}^{\,2}} + \frac{\alpha_2}{m_2^2 + \vec{q}^{\,2}} \, .$$

where α_1 and α_2 are positive numerical coefficients such that $\alpha_1 + \alpha_2 = 1$ and m_1 , m_2 are the masses of the two poles, which obey the relation $m_1 < m$ and $m_2 > m$. The two most important of these contributions are the following: close to $mr \sim 1$, the original Yukawa potential is modified into a potential of the type

$$V_{\mathbf{Y}}(\mathbf{r}) pprox - rac{\mathbf{g}^2}{4\pi} \, rac{\mathbf{e}^{-m\mathbf{r}}}{\mathbf{r}} \left(1 - rac{\Gamma \mathbf{r}}{\pi} - rac{\Gamma}{\pi m}\right),$$

with Γ the width of the meson.

At short distances the potential for a broad meson is weaker than for a narrow one;

A broad meson decaying into two lighter mesons of mass 2M generates an additional attractive longer-range contribution to the potential at distances $2Mr \sim 1$.

Backup: uncertainty from *a*₁ exchange



Figure 1: Cutoff ratios $R(Z_c/X)$ as a function of the mass (left panel) and coupling (right panel) of the axial meson for the OBE model. The mass and coupling of the scalar meson are taken to be $m_{\sigma} = 550 \text{ MeV}$ and $g_{\sigma} = 3.4 \pm 1.0$, where in the case of the coupling we have added a 30% relative uncertainty, which is shown as the error band around the solid line. The $m_{a_1} = 1230 \text{ MeV}$ and $\lambda_1 = 1.8$ ratios for the axial-full theory are highlighted as a round dot. When we vary the axial mass (coupling), we set the axial coupling (mass) to its expected central value, i.e. $\lambda_1 = 1.8 \ (m_{a_1} = 1230 \text{ MeV})$.

(X-like)	IG	J ^{PC}	$B^{\not a_1} / E_V^{\not a_1}$	$M^{ atural}_1$	$B^{a_1} / E_V^{a_1}$	M^{a_1}
DD	0+	0^{++}	$-0.0\substack{+0.1\\-0.7}$	$3734.4_{-0.7}^{+0.0}$	$-0.0^{+0.1}_{-0.8}$	$3734.4_{-0.8}^{+0.0}$
$D^*\bar{D}$	0+	1^{++}	Input	Input	Input	Input
$D^*\bar{D}$	0-	1^{+-}	$-4.0^{+3.6}_{-10.3}$	$3871.8^{+3.6}_{-10.3}$	$-4.2^{+3.9}_{-11.2}$	$3871.6^{+3.9}_{-11.2}$
$D^*\bar{D}^*$	0+	0^{++}	$-^{-}_{-0.3}$	—	$-^{-}_{-0.3}$	—
$D^*\bar{D}^*$	1-	1^{+-}	$-1.0^{+1.0}_{-1.8}$	$4016.2^{+1.0}_{-1.8}$	$-1.0^{+1.0}_{-1.9}$	$4016.2^{+1.0}_{-1.9}$
$D^*\bar{D}^*$	0+	2^{++}	$+3.5^{+0.0}_{-0.1}$	$4013.7_{-0.0}^{+0.1}$	$+3.5^{+0.0}_{-0.1}$	$4013.7_{-0.0}^{+0.1}$
(Z_c-like)	IG	J ^{PC}	$B^{a_1} / E_V^{a_1}$	M [≉] 1	$B^{a_1} / E_V^{a_1}$	M^{a_1}
DD	0^+	0^{++}	$-0.1^{+0.0}_{-\infty}$	$3734.3^{0.0}_{-\infty}$	$-0.6^{+6.9}_{-13.4}$	$3733.8^{+0.6}_{-13.4}$
$D^*\bar{D}$	1+	1^{++}	$-0.0^{+0.0}_{-\infty}$	$3875.8^{+0.0}_{-\infty}$	$-1.3^{+6.0}_{-16.1}$	$3874.5^{+1.3}_{-16.1}$
$D^*\bar{D}$	1-	1^{+-}	Input	Input	Input	Input
$D^*\bar{D}^*$	1-	0^{++}	$+0.0^{+0.0}_{-\infty}$	$4017.2^{+0.0}_{-\infty}$	$+0.3^{+10.2}_{-0.3}$	$4016.9^{+0.3}_{-10.2}$
$D^*\bar{D}^*$	1+	1^{+-}	$+0.0^{+0.0}_{-\infty}$	$4017.2^{+0.0}_{-\infty}$	$+0.0^{+0.0}_{-0.0}$	$4017.2^{+0.0}_{-\infty}$
$D^*\bar{D}^*$	1-	2^{++}	$+0.2^{+0.0}_{-\infty}$	$4017.0^{+0.2}_{-\infty}$	$-0.5^{+0.7}_{-4.7}$	$4016.7_{-4.7}^{+0.5}$