Meson-meson interactions and the $D_{s0}^{*}(2317)$

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Outline

Introduction

- Meson-meson interaction is important, e.g., some XYZ states [X(3872)$\cdots$] lie very close to the two-meson thresholds.
- ChPT has been widely used to study low-energy hadronic processes and achieved many successes N. Fettes 2000; D. R. Entem 2015. Similar to the HBChPT in the light flavor meson-baryon and baryon-baryon interaction, one can use HMChPT to deal with the charmed mesons.
- Detailed calculations of light pseudoscalar meson-heavy meson scattering lengths up to order p^4 in SU(3) HMChPT Y.-R. Liu 2009; B.-L. Huang 2022. Present calculations concern the partial-wave phase shifts and the nature of the $D^*_{s0}(2317)$.
- In this talk: the light pseudoscalar meson and heavy meson interaction based on the paper:
    - B.-L. Huang, Z.-Y. Lin, K. Chen and S.-L. Zhu, Eur. Phys. J. C (2023) 83: 76.

Outline

Chiral Lagrangian

Eight light meson fields in terms of the traceless hermitian 3×3 matrices Φ:

$$\Phi = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

with the SU(3) matrix $U = \xi^2 = \exp(i\Phi/f)$.

$$P = (D^0, D^+, D_s^+), \quad P^*_\mu = (D^{0*}, D^{+*}, D_s^{+*})_\mu,$$

$$H = \frac{1+\not{v}}{2}(P^*_\mu \gamma^\mu + iP\gamma_5), \quad \bar{H} = (P^{*\dagger}_\mu \gamma^\mu + iP^\dagger \gamma_5)\frac{1+\not{v}}{2}.$$

Chiral Lagrangian

The effective chiral Lagrangian:

$$\mathcal{L}_{\phi\phi}^{(2)} = f^2 \mathrm{tr}(u_\mu u^\mu + \frac{\chi_+}{4}),$$

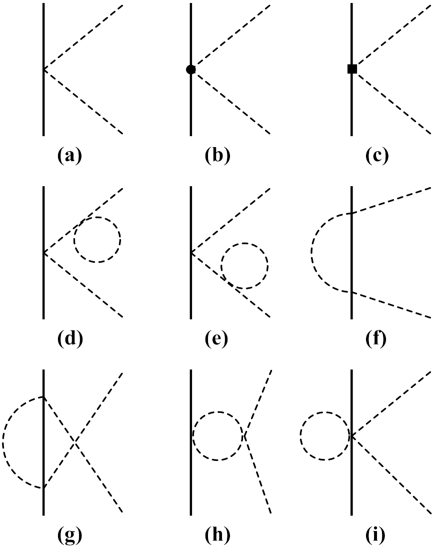
$$\mathcal{L}_{H\phi}^{(1)} = -\left\langle (iv\cdot\partial H)\bar{H}\right\rangle + \left\langle Hv\cdot\Gamma\bar{H}\right\rangle + g\left\langle Hu_\mu\gamma^\mu\gamma_5\bar{H}\right\rangle,$$

$$\begin{aligned}\mathcal{L}_{H\phi}^{(2)} =& c_0\left\langle H\bar{H}\right\rangle \mathrm{tr}(\chi_+) + c_1\left\langle H\chi_+\bar{H}\right\rangle - c_2\left\langle H\bar{H}\right\rangle \mathrm{tr}(v\cdot u\, v\cdot u)\\ &- c_3\left\langle Hv\cdot u\, v\cdot u\bar{H}\right\rangle - c_4\left\langle H\bar{H}\right\rangle \mathrm{tr}(u^\mu u_\mu) - c_5\left\langle Hu^\mu u_\mu\bar{H}\right\rangle,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{H\phi}^{(3)} =& \kappa_1\left\langle H[\chi_-, v\cdot u]\bar{H}\right\rangle + i\kappa_2\left\langle H[v\cdot u,[v\cdot\partial, v\cdot u]]\bar{H}\right\rangle\\ &+ i\kappa_3\left\langle H[u^\mu,[v\cdot\partial, u_\mu]]\bar{H}\right\rangle.\end{aligned}$$

Outline

Feynman diagrams



Outline

T matrices

The T matrix with the three orders read

$$
\begin{aligned}
T_{\pi D}^{(1/2)} =& \frac{2w_\pi}{f_\pi^2} + \frac{1}{f_\pi^2}[8c_0 m_\pi^2 + 4c_1 m_\pi^2 + 2c_2 w_\pi^2 + c_3 w_\pi^2 + 2c_4(w_\pi^2 - q^2 z) \\
& + c_5(w_\pi^2 - q^2 z)] + \frac{1}{f_\pi^2}[16\bar{\kappa}_1 m_\pi^2 w_\pi + 4\bar{\kappa}_2 w_\pi^3 + 4\bar{\kappa}_3 w_\pi(w_\pi^2 - q^2 z)] \\
& - \frac{w_\pi}{12 f_\pi^4}\{3w_\pi[3J_0(-w_\pi, m_K) + 4J_0(-w_\pi, m_\pi) - 9J_0(w_\pi, m_K) \\
& - 17J_0(w_\pi, m_\pi)] + 12 I_2(t, m_K) + 16 I_2(t, m_\pi)\},
\end{aligned}
$$

T matrices

$$
\begin{aligned}
T_{KD}^{(0)} =& \frac{2w_K}{f_K^2} + \frac{1}{f_K^2}[8c_0 m_K^2 + 8c_1 m_K^2 + 2c_2 w_K^2 + 2c_3 w_K^2 + 2c_4(w_K^2 - q^2 z) \\
&+ 2c_5(w_K^2 - q^2 z)] + \frac{1}{f_K^2}[16\bar{\kappa}_1 m_K^2 w_K + 4\bar{\kappa}_2 w_K^3 + 4\bar{\kappa}_3 w_K(w_K^2 - q^2 z)] \\
&- \frac{w_K}{12 f_K^4}\{3w_K[2J_0(-w_K, m_K) + 24J_0(w_K, m_\eta) + 22J_0(w_K, m_K) \\
&+ J_0(w_K, m_\pi)] - 10I_2(t, m_K) - 36I_2(t, m_\pi)\},
\end{aligned}
$$

$\cdots$ omitting many more equations $\cdots$

Outline

Phase shifts and scattering lengths

The partial-wave amplitudes $f_l^{(I)}(q)$ are obtained by a projection:

$$f_l^{(I)}(q) = \frac{M_H}{16\pi\sqrt{s}} \int_{-1}^{+1} dz\, [T_{\phi H}^{(I)} P_l(z)],$$

The phase shifts $\delta_l^{(I)}(q)$ are calculated as (N. Fettes 1998)

$$\delta_l^{(I)}(q) = \arctan[q\mathrm{Re} f_l^{(I)}(q)].$$

The scattering lengths for the S waves and the scattering volumes for P waves are obtained by

$$a_l^{(I)} = \lim_{q\to 0} q^{-2l} f_l^{(I)}(q).$$

Outline

Bound states of nonlocal potentials

The nonlocal potentials can be given by (N. Kaiser 1995)

$$V(q,q^{'}) = -\frac{1}{\pi\sqrt{w(q)w(q^{'})}} f_l^{(l)}(q,q^{'}) e^{-(q+q^{'})/\Lambda}.$$

The momentum-space Schrödinger equation:(Rubin H. Landau Computational Physics)

$$\frac{q^2}{2w(q)}\psi(q) + \int_0^\infty dq^{'} q^{'2} V(q,q^{'})\psi(q^{'}) = E\psi(q).$$

The coordinate-space wave function via the Bessel transform

$$\psi(r) = \int_0^\infty dq\psi(q)\frac{\sin(qr)}{qr} q^2.$$

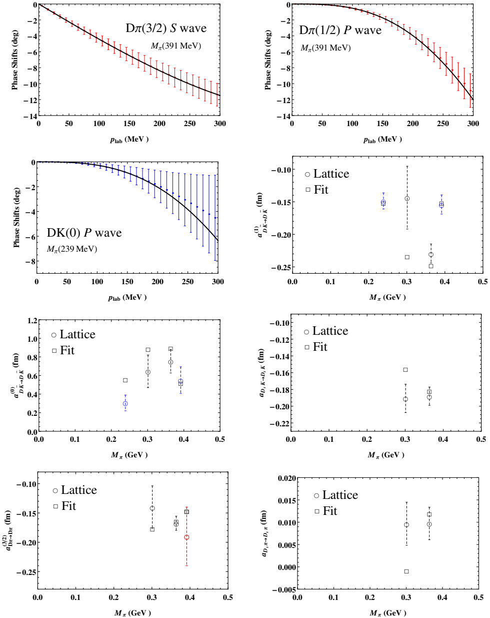
Outline

Fit to the phase shifts and scattering lengths.

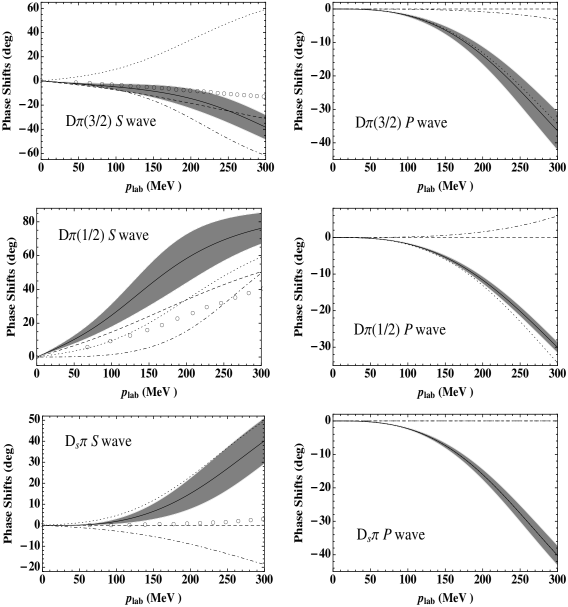
	Values	c_0	c_1	c_2	c_3	c_4	c_5	$\bar{\kappa}_1$	$\bar{\kappa}_2$	$\bar{\kappa}_3$
c_0 (GeV^{-1})	-0.77 ± 0.39	1.00	0.99	-0.86	-0.88	0.00	0.00	0.02	-0.92	0.00
c_1 (GeV^{-1})	-0.64 ± 0.35		1.00	-0.86	-0.89	0.00	0.00	0.01	-0.93	0.00
c_2 (GeV^{-1})	-5.04 ± 1.83			1.00	0.95	-0.51	-0.41	-0.01	0.61	0.50
c_3 (GeV^{-1})	5.47 ± 1.57				1.00	-0.36	-0.45	0.10	0.67	0.40
c_4 (GeV^{-1})	8.99 ± 0.93					1.00	0.80	0.00	0.37	-0.98
c_5 (GeV^{-1})	-3.08 ± 0.70						1.00	0.00	0.34	-0.90
$\bar{\kappa}_1$ (GeV^{-2})	0.21 ± 0.04							1.00	0.01	0.00
$\bar{\kappa}_2$ (GeV^{-2})	7.81 ± 3.88								1.00	-0.38
$\bar{\kappa}_3$ (GeV^{-2})	-1.87 ± 1.47									1.00
χ^2/d.o.f.	$\frac{34.85}{195-9} = 0.19$									

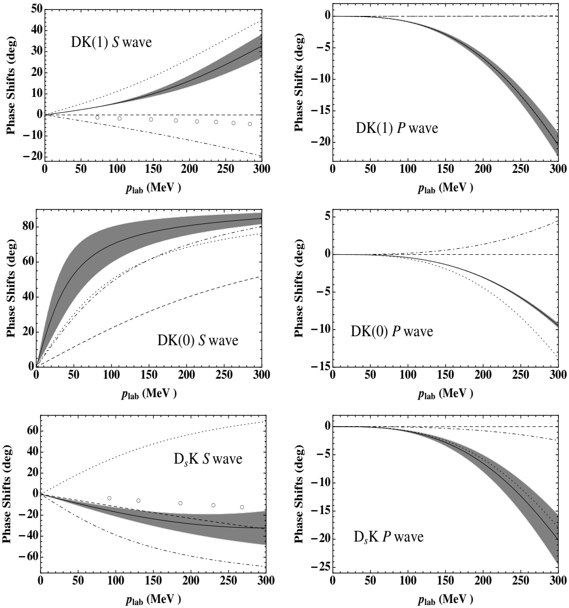
Using the linear combinations of the LECs

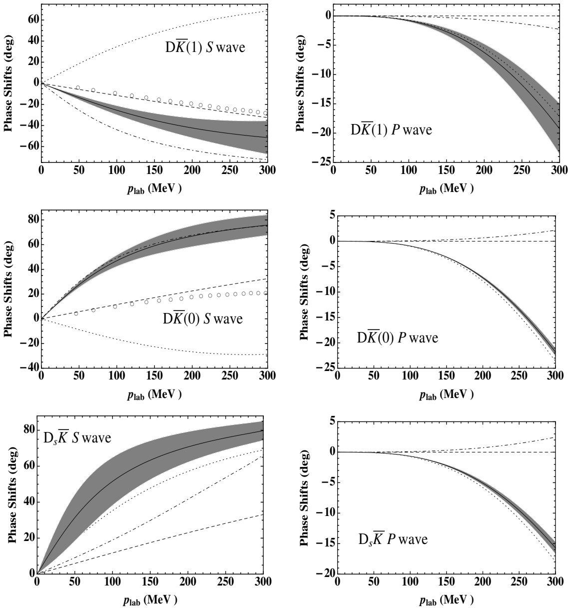
	Values	c_0+c_1	c_2+c_4	c_3+c_5	$\bar{\kappa}_1$	$\bar{\kappa}_2$	$\bar{\kappa}_3$
c_0+c_1 (GeV^{-1})	0.04 ± 0.02	1.00	0.01	-0.47	-0.22	-0.79	0.00
c_2+c_4 (GeV^{-1})	0.89 ± 0.02		1.00	-0.09	0.01	0.01	0.00
c_3+c_5 (GeV^{-1})	-0.32 ± 0.04			1.00	0.95	0.77	0.00
$\bar{\kappa}_1$ (GeV^{-2})	0.21 ± 0.07				1.00	0.60	0.00
$\bar{\kappa}_2$ (GeV^{-2})	-7.13 ± 0.34					1.00	-0.39
$\bar{\kappa}_3$ (GeV^{-2})	6.11 ± 0.14						1.00
χ^2/d.o.f.	$\frac{147}{195-6} = 0.78$						

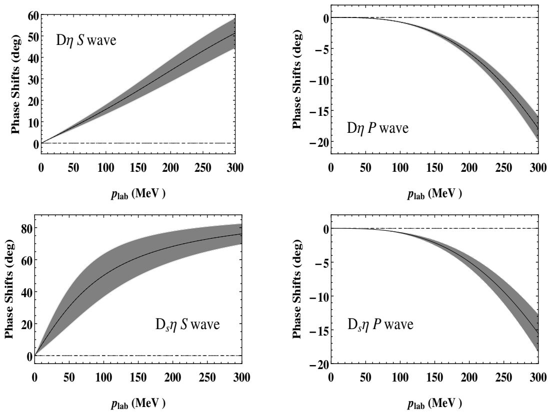

Dπ(3/2) S wave
M_π(391 MeV)
Dπ(1/2) P wave
M_π(391 MeV)
DK(0) P wave
M_π(239 MeV)
Phase Shifts (deg)
$p_{\rm lab}$ (MeV)
Lattice
Fit
M_π (GeV)

Outline


Dπ(3/2) S wave
Dπ(3/2) P wave
Dπ(1/2) S wave
Dπ(1/2) P wave
Dsπ S wave
Dsπ P wave
Phase Shifts (deg)
plab (MeV)


DK(1) S wave
DK(1) P wave
DK(0) S wave
DK(0) P wave
D_sK S wave
D_sK P wave
Phase Shifts (deg)
p_lab (MeV)


DK̄(1) S wave
DK̄(1) P wave
DK̄(0) S wave
DK̄(0) P wave
D_sK̄ S wave
D_sK̄ P wave
Phase Shifts (deg)
p_lab (MeV)


Dη S wave
Dη P wave
D_sη S wave
D_sη P wave
Phase Shifts (deg)
p_lab (MeV)

Outline

Scattering lengths and scattering volumes

Sca. Len.	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Liu2013	Guo2019
$a^{(3/2)}_{(0,D\pi)}$	-0.24	0.23	-0.16	$-0.17(6)$	$-0.100(2)$	$-0.103^{+0.003}_{-0.003}$
$a^{(1/2)}_{(0,D\pi)}$	0.48	0.23	0.00	0.71(16)	$0.37^{+0.03}_{-0.02}$	$0.40^{+0.03}_{-0.02}$
$a_{(0,D_s\pi)}$	0.00	0.06	-0.08	$-0.02(0)$	$-0.002(1)$	$0.012^{+0.003}_{-0.003}$
$a^{(1)}_{(0,DK)}$	0.00	0.45	-0.24	0.21(1)	$0.07^{+0.03}_{-0.03} + i0.17^{+0.02}_{-0.01}$	$-0.01^{+0.05}_{-0.03} + i0.39^{+0.04}_{-0.04}$
$a^{(0)}_{(0,DK)}$	1.01	2.90	2.65	6.57(318)	$-0.84^{+0.17}_{-0.22}$	$-1.51^{+0.72}_{-2.35}$
$a_{(0,D_sK)}$	-0.51	1.69	-1.96	$-0.78(19)$	$-0.18(1)$	$-0.20^{+0.01}_{-0.01}$
$a^{(1)}_{(0,D\bar{K})}$	-0.51	1.67	-2.38	$-1.21(19)$	$-0.20(1)$	$-0.20^{+0.01}_{-0.01}$
$a^{(0)}_{(0,D\bar{K})}$	0.51	-0.78	2.86	2.58(19)	0.84(15)	21.9
$a_{(0,D_s\bar{K})}$	0.51	1.69	0.78	2.98(161)	$-0.09^{+0.06}_{-0.05} + i0.44^{+0.05}_{-0.05}$	$-0.57^{+0.06}_{-0.04} + i0.35^{+0.08}_{-0.07}$
$a_{(0,D\eta)}$	0.00	0.67	0.00	0.67(9)		$0.29^{+0.15}_{-0.22} + i0.61^{+0.30}_{-0.26}$
$a_{(0,D_s\eta)}$	0.00	2.97	0.00	2.97(139)		$-0.39^{+0.05}_{-0.03} + i0.06^{+0.02}_{-0.02}$

Sca. Vol.	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total
$a^{(3/2)}_{(1,D\pi)}$	-0.33	-0.01	$-0.34(6)$
$a^{(1/2)}_{(1,D\pi)}$	-0.33	0.02	$-0.31(4)$
$a_{(1,D_s\pi)}$	-0.40	0.00	$-0.40(4)$
$a^{(1)}_{(1,DK)}$	-0.24	0.00	$-0.24(2)$
$a^{(0)}_{(1,DK)}$	-0.16	0.05	$-0.11(0)$
$a_{(1,D_sK)}$	-0.20	-0.02	$-0.22(5)$
$a^{(1)}_{(1,D\bar{K})}$	-0.20	-0.02	$-0.22(5)$
$a^{(0)}_{(1,D\bar{K})}$	-0.28	0.02	$-0.26(1)$
$a_{(1,D_s\bar{K})}$	-0.20	0.02	$-0.18(1)$
$a_{(1,D\eta)}$	-0.22	0.00	$-0.22(3)$
$a_{(1,D_s\eta)}$	-0.18	0.00	$-0.18(3)$

Outline

Explicitly include $D_{s0}^*(2317)$ in $DK(I=0)$

$$\mathcal{L}_{D_R} = \bar{D}_R(iv\cdot\partial - M_R + M_D)D_R + (g_R \left\langle \bar{D}_R v\cdot A\, D\right\rangle + \text{h.c.})$$

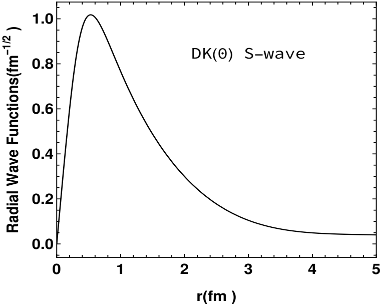
$$T_{KD}^{(0)} = \frac{2w_K^2 g_R^2}{f_K^2}\Big(\frac{1}{M_R - M_D - w_K} + \frac{1}{M_R - M_D + w_K}\Big).$$

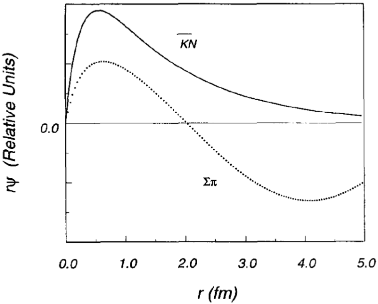
$g_R^2 = 0.81 \pm 0.04$ by using the lattice scattering length $-1.33(20)$ fm.

$\Lambda(1405) \to \bar{g}_{\Lambda_R}^2 = 0.15$.

Bound state of the $DK(I = 0)$

$\Lambda = 340$ MeV
$\rightarrow$ binding energy $\Delta E \simeq -45.8$ MeV, mass $M \simeq 2317.6$ MeV.





(N. Kaiser 1995)

Root mean square radius: 1.1 fm (DK) $\longleftrightarrow$ 1.3 fm ($N\bar{K}$)
$D_{s0}^*(2317)$ and $\Lambda(1405)$ are probably molecular states.

Summary and Outlook

- T-matrices to the third order for light meson and heavy meson scattering in SU(3) HMChPT.
- Physical quantity for light meson and heavy meson scattering.
- The nature of the $D_{s0}^*(2317)$.
- Outlook
  - Higher-order calculations.
  - Including the D^* meson.
  - Combining with more Lattice QCD data.
  - Finding the more bound states or resonances.

Thank you for your attention!