Understanding the nature of $\Omega(2012)$ in a coupled-channel approach

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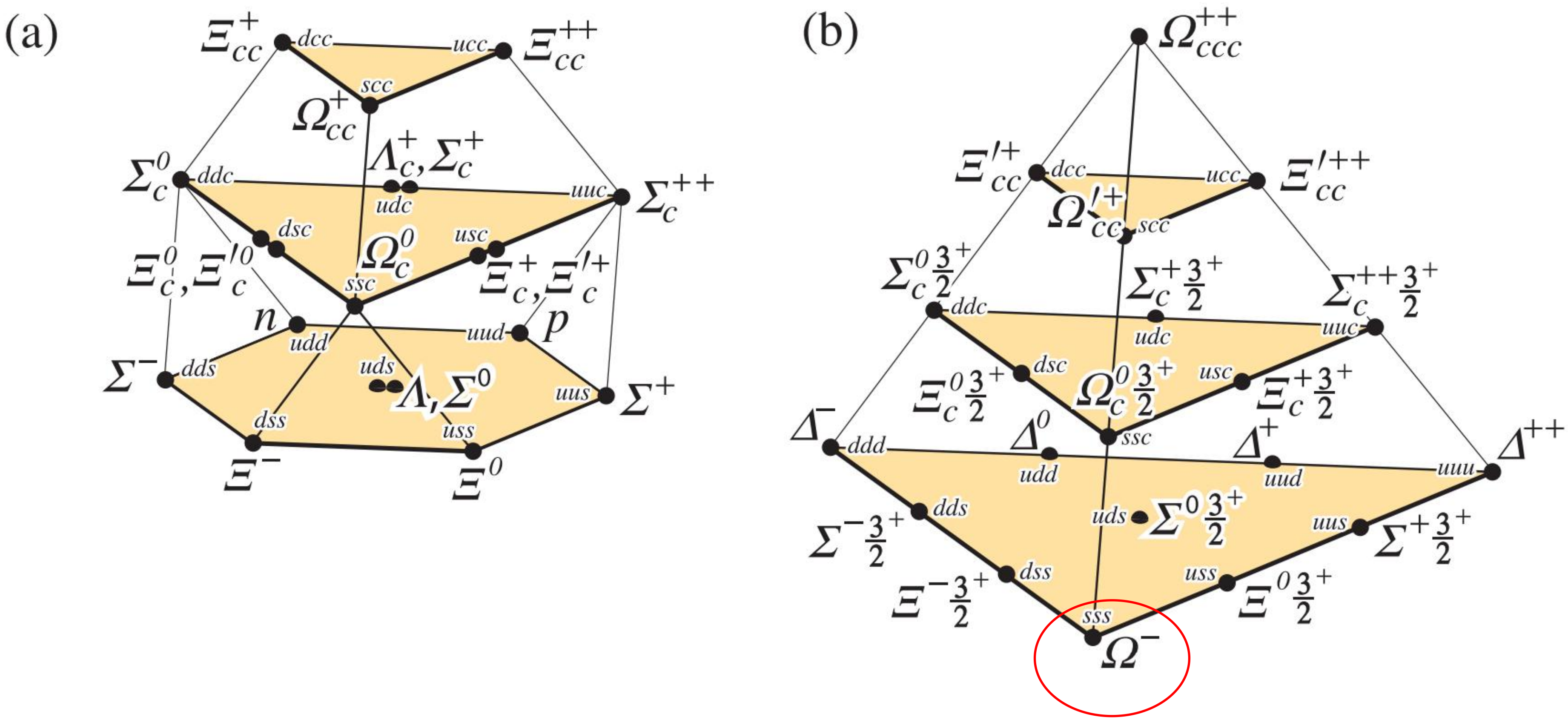
Jilin 2023.07.27

Outline

- Background
- Existing problems
- Coupled-channel approach
- Summary

Conventional baryons

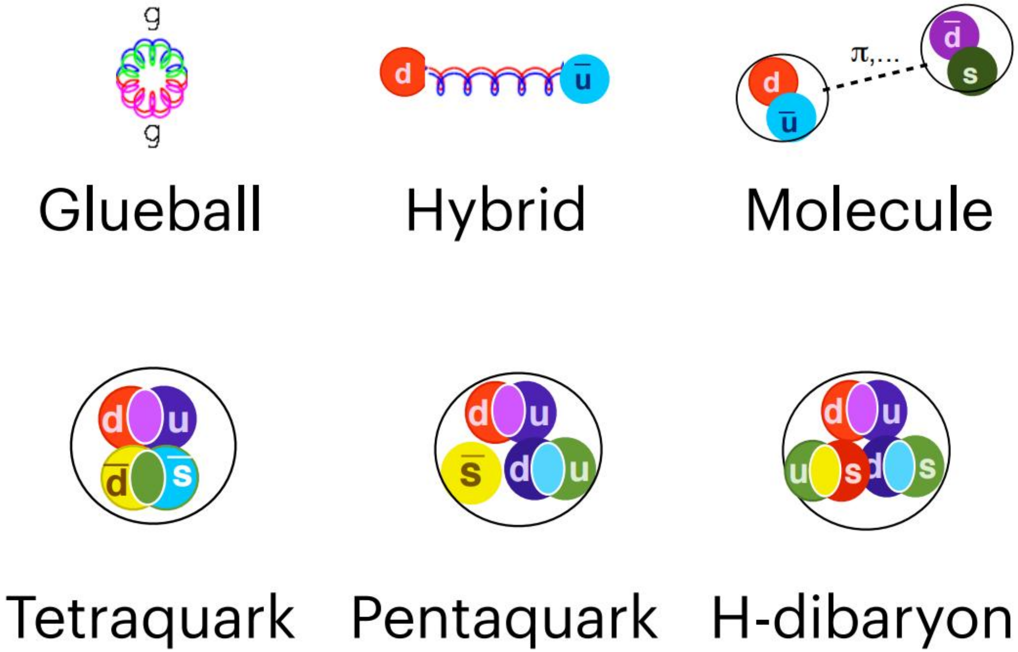


Conventional baryons

R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

Exotic states

In addition to the conventional states, QCD also permits the existence of other types of hadrons, known as exotic states.



Some tetraquark candidates observed recently： X(6900), X(2900), $Z_{cs}(3985)$, $Z_{cs}(4000)$, $T_{cc}(3875)$, $P_{cs}(4459)$ ...

Status of Ω family

Hadrons	J^P	Mass (MeV)	Width (MeV)	Interpretation
Ω	$3/2^+$	1672	×	$1^4S_{3/2+}$
$\Omega(2012)$	$?^-$	2012	$6.4^{+2.5}_{-2.0} \pm 1.6$	In dispute
$\Omega(2250)$	$?^?$	2252	55 ± 18	UN
$\Omega(2380)$	$?^?$	2380	26 ± 23	UN
$\Omega(2470)$	$?^?$	2474	72 ± 33	UN

Ω(2012) observed by Belle Collaboration

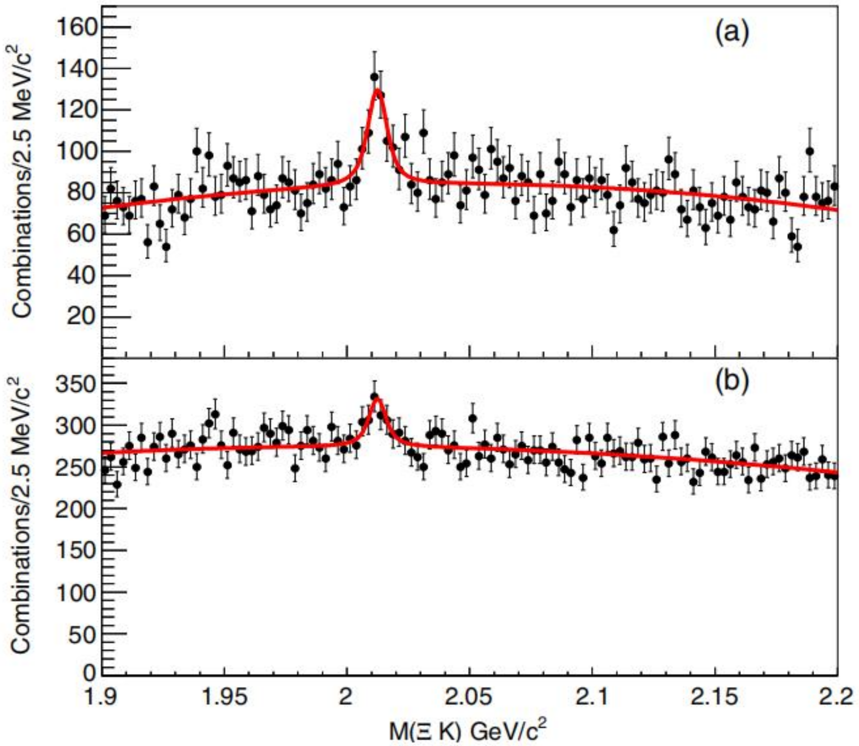


FIG. 2. The (a) $\Xi^0 K^-$ and (b) $\Xi^- K_S^0$ invariant mass distributions in data taken at the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonance energies. The curves show a simultaneous fit to the two distributions with a common mass and width.

A structure Ω(2012) was obsevred by Belle Collaboration in 2018

Experimental data of Ω(2012)

$$M[\Omega(2012)] = 2012.5 \pm 0.7 \pm 0.5 \text{ MeV}, \qquad (1)$$

$$\Gamma[\Omega(2012)] = 6.4^{+2.5}_{-2.0}(\text{stat}) \pm 1.6(\text{syst}) \text{ MeV}. \qquad (2)$$

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} \equiv \frac{\mathcal{B}[\Omega(2012) \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K}]}{\mathcal{B}[\Omega(2012) \to \Xi\bar{K}]}, \qquad (3)$$

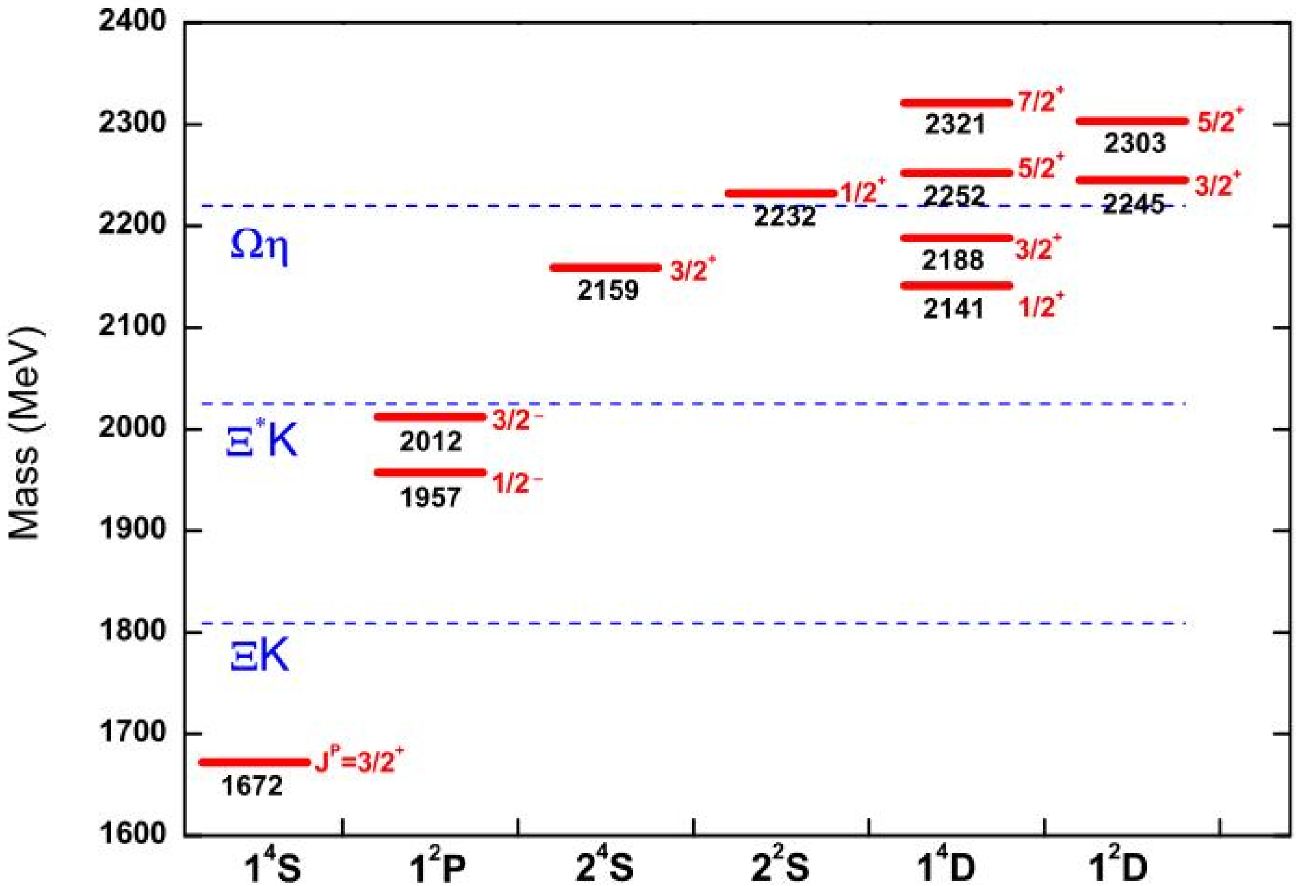
Two measurements in 2019 and 2022 and they are quite different.

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} < 11.9\%$$

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$$

J. Yelton et al. (Belle Collaboration), Phys. Rev. Lett. 121, 052003 (2018).
S. Jia et al. (Belle Collaboration), Phys. Rev. D 100, 032006 (2019).
Y. Li et al. (Belle Collaboration), Phys. Rev. D 104, 052005 (2021).
(Belle Collaboration), arXiv:2207.03090 (2022).

Canonical interpretation



Mass spectrum of Ω baryons

M. S. Liu, K. L. Wang, Q. F. Lü and X. H. Zhong, Phys. Rev. D 101, 016002 (2020)

Canonical interpretation

$n^{2S+1}L_{J^P}$	$\|N_6,^{2S+1}N_3,N,L,J^P\rangle$	Ours	Exp.	Ref. [3]	Ref. [5]	Ref. [6]	Ref. [7]	Ref. [8]	Ref. [12]	Ref. [13]
$1^4S_{\frac{3}{2}^+}$	$\|56,^4 10,0,0,\frac{3}{2}^+\rangle$	1672	1672.45 [2]	1694	1635	1678	1675	1673	1656	1642(17)
$1^2P_{\frac{1}{2}^-}$	$\|70,^2 10,1,1,\frac{1}{2}^-\rangle$	1957	$\cdots$	1837	1950	1941	2020	2015	1923	1944(56)
$1^2P_{\frac{3}{2}^-}$	$\|70,^2 10,1,1,\frac{3}{2}^-\rangle$	2012	2012.4 [15]	1978	2000	2038	2020	2015	1953	2049(32)
$2^2S_{\frac{1}{2}^+}$	$\|70,^2 10,2,0,\frac{1}{2}^+\rangle$	2232	$\cdots$	2140	2220	2301	2190	2182	2191	2350(63)
$2^4S_{\frac{3}{2}^+}$	$\|56,^4 10,2,0,\frac{3}{2}^+\rangle$	2159	$\cdots$	$\cdots$	2165	2173	2065	2078	2170	$\cdots$
$1^2D_{\frac{3}{2}^+}$	$\|70,^2 10,2,2,\frac{3}{2}^+\rangle$	2245	$\cdots$	2282	2345	2304	2265	2263	2194	2470(49)
$1^2D_{\frac{5}{2}^+}$	$\|70,^2 10,2,2,\frac{5}{2}^+\rangle$	2303	$\cdots$	$\cdots$	2345	2401	2265	2260	2210	$\cdots$
$1^4D_{\frac{1}{2}^+}$	$\|56,^4 10,2,2,\frac{1}{2}^+\rangle$	2141	$\cdots$	2140	2255	2301	2210	2202	2175	2481(51)
$1^4D_{\frac{3}{2}^+}$	$\|56,^4 10,2,2,\frac{3}{2}^+\rangle$	2188	$\cdots$	2282	2280	2304	2215	2208	2182	2470(49)
$1^4D_{\frac{5}{2}^+}$	$\|56,^4 10,2,2,\frac{5}{2}^+\rangle$	2252	2252 [2]	$\cdots$	2280	2401	2225	2224	2178	$\cdots$
$1^4D_{\frac{7}{2}^+}$	$\|56,^4 10,2,2,\frac{7}{2}^+\rangle$	2321	$\cdots$	$\cdots$	2295	2332	2210	2205	2183	$\cdots$

Y. Oh, Phys. Rev. D 75, 074002 (2007).
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R. N. Faustov and V. O. Galkin, Phys. Rev. D 92, 054005 (2015).
K. T. Chao, N. Isgur and G. Karl, Phys. Rev. D 23, 155(1981).
Y. Chen and B. Q. Ma, Nucl. Phys. A 831, 1 (2009).
M. Pervin and W. Roberts, Phys. Rev. C 77, 025202 (2008).
G. P. Engel et al. [BGR Collaboration], Phys. Rev. D 87, 074504 (2013).
J. Yelton et al. [Belle Collaboration], Phys. Rev. Lett. 121, 052003 (2018).

Canonical interpretation

The strong decay widths (MeV) of Ω baryons up to $N=2$ shell:

$n^{2S+1}L_{J^P}$	Mass	α(MeV)	$\Gamma[\Xi K]$		$\Gamma[\Xi(1530)K]$		$\Gamma[\Omega(1672)\eta]$		Γ^{th}_{total}		$\mathcal{B}$	
			Ours	Ref. [16]	Ours	Ref. [16]	Ours	Ref. [16]	Ours	Ref. [16]	Ours	Ref. [16]
$1^2P_{\frac{1}{2}^-}$	1957	428	12.43	12.64	⋯	⋯	⋯	⋯	12.43	12.64	⋯	⋯
$1^2P_{\frac{3}{2}^-}$	2012	411	5.69	5.81	⋯	⋯	⋯	⋯	5.69	5.81	⋯	⋯
$2^2S_{\frac{1}{2}^+}$	2232	387	0.04	0.27	5.09	8.32	0.006	0.08	5.14	8.67	0.008	0.03
$2^4S_{\frac{3}{2}^+}$	2159	381	0.99	4.72	5.12	8.96	⋯	⋯	6.11	13.68	0.19	0.53
$1^2D_{\frac{3}{2}^+}$	2245	394	2.49	2.52	4.27	4.24	0.055	0.06	6.82	6.82	0.58	0.59
$1^2D_{\frac{5}{2}^+}$	2303	380	3.07	3.04	14.30	14.51	1.65	1.81	19.02	19.36	0.21	0.21
$1^4D_{\frac{1}{2}^+}$	2141	413	39.52	39.34	2.17	2.21	⋯	⋯	41.69	41.55	18.21	17.80
$1^4D_{\frac{3}{2}^+}$	2188	399	20.25	20.26	10.93	10.92	⋯	⋯	31.18	31.18	1.85	1.86
$1^4D_{\frac{5}{2}^+}$	2252	383	5.28	5.21	21.37	21.48	0.79	0.90	27.44	27.59	0.25	0.24
$1^4D_{\frac{7}{2}^+}$	2321	367	34.38	34.36	7.17	7.00	0.066	0.13	41.62	41.49	4.79	4.91

All of them are narrow states. Present calculations are consistent with the results by using simple harmonic oscillator wavefunctions.

Status of canonical interpretation

- ✓ Can describe the mass and total decay width well.
- ✓ The calculated ratio of the branching fractions $\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}}$ is small and consistent with measurement in 2019.
- ✓ Describe the weak production from Ω_c reasonably.
- ✗ The ratio $\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}}$ contradicts with the new measurement in 2022.

Molecular interpretation

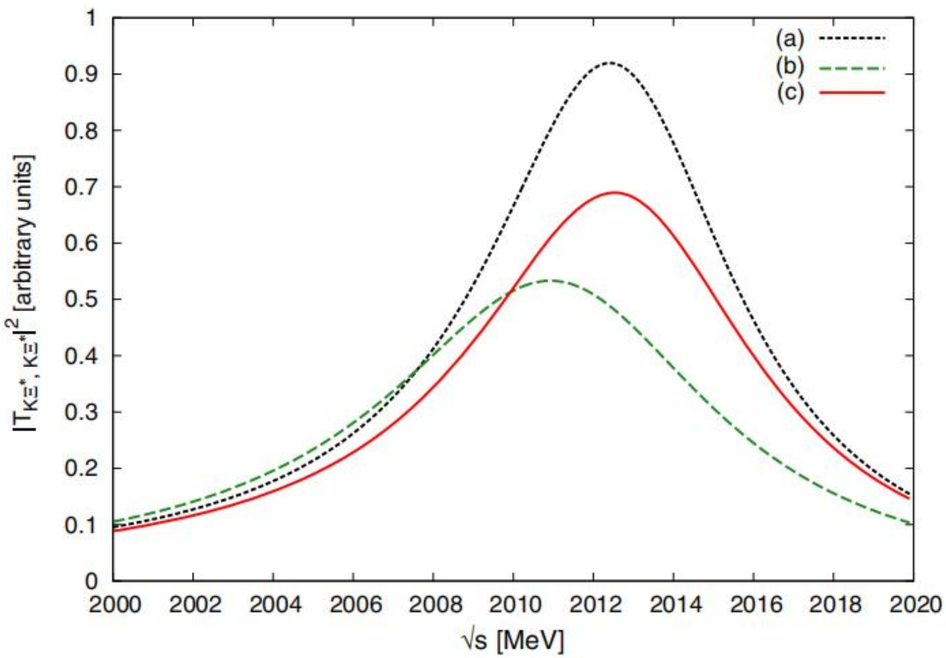

FIG. 3. $|T|^2$ for the diagonal $\bar{K}\Xi^*$ channel with three options: (a) $G_{\bar{K}\Xi^*}$ without convolution; (b) $G_{\bar{K}\Xi^*}$ with convolution and Γ_{Ξ^*} fixed; (c) $G_{\bar{K}\Xi^*}$ with convolution and Γ_{Ξ^*} energy dependent.

$\Xi^*\overline{K}$ and $\Omega\eta$ molecular state

N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020)

Molecular interpretation

R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018).
M. P. Valderrama, Phys. Rev. D 98, 054009 (2018).
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Y. H. Lin, F. Wang, and B. S. Zou, Phys. Rev. D 102, 074025 (2020).
N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).
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X. Liu, H. Huang, J. Ping, and D. Chen, Phys. Rev. C 103, 025202 (2021).
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X. Hu and J. Ping, Phys. Rev. D 106, 054028 (2022).

Status of molecular interpretation

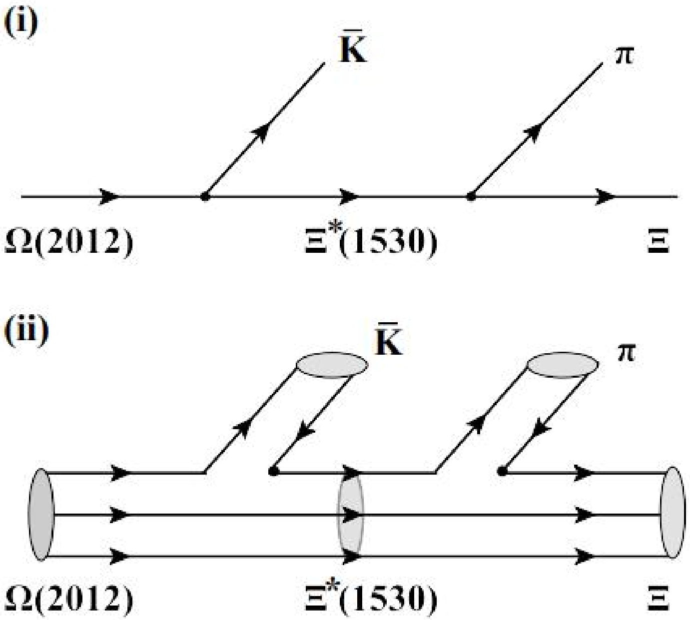
- ✓ Can describe the mass and total decay width well.
- ✓ Can give significant three-body decay decay width.
- ✓ Describe the weak production from Ω_c reasonably.
- ✗ Can not provide dynamic mechanism for two-body decay.

There are many references about Ω(2012), They can be found in

https://inspirehep.net/literature?sort=mostrecent&size=25&page=1&q=refersto%3Arecid%3A1674698

Problem

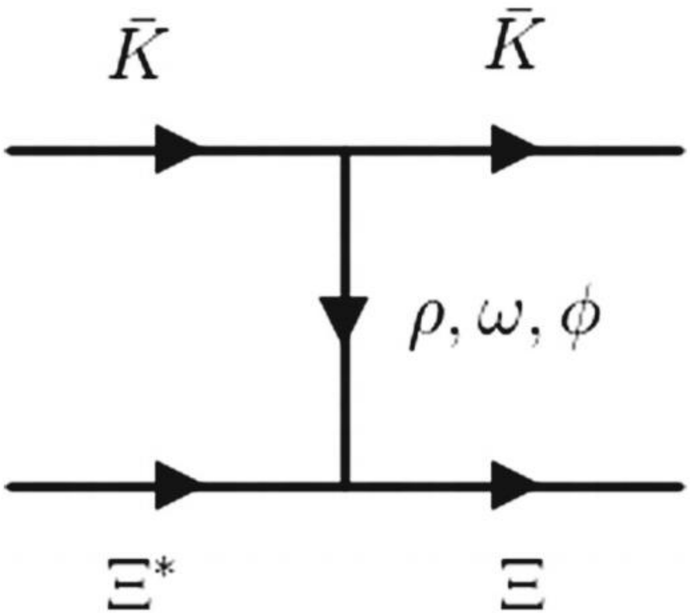
- The canonical interpretation gives small three-body decay width compared with new experimental data. Because it needs quark pair creation twice.

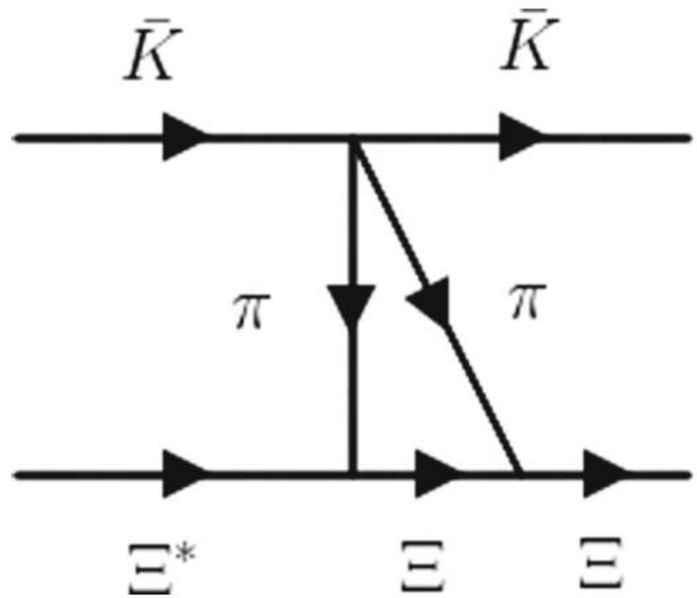


H. H. Zhong, R. H. Ni, M. Y. Chen, X. H. Zhong, and J. J. Xie, Chin.Phys.C 47, 063104 (2023)

Problem

- The molecular interpretation can not give dynamic mechanism for two-body decay. It needs assuming free higher order term.





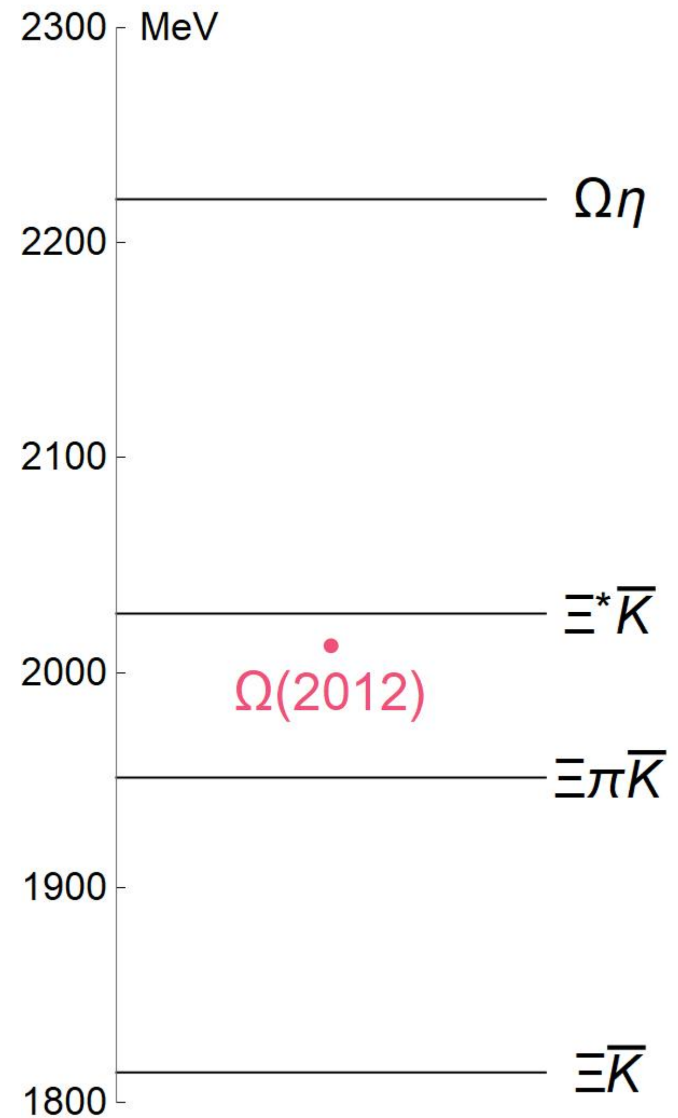
$$\langle M \rangle \propto \sum_{\epsilon\ \mathrm{pol.}} (\vec{\epsilon} \times \vec{q}) \cdot \vec{S}\ \vec{\epsilon} \cdot (\vec{p}_K + \vec{p}_K{}') \\ = \sum_{\epsilon\ \mathrm{pol.}} (\vec{\epsilon} \times \vec{q}) \cdot \vec{S}\ \vec{\epsilon} \cdot \vec{q} = (\vec{q} \times \vec{q}) \cdot \vec{S} = 0,$$

R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018)

Why considering the mixture/superposition?

Why considering the mixture?

- Provide interpretations of both two-body and three-body decays.
- Three-quark state is near the meson-baryon threshold.
- The mixture between bare and hadron-hadron channels may also exist in other systems: X(3872), $D_{s0}(2317)$, and $\Lambda_c(2940)$...



Interactions

$$\mathcal{L}_{WT} = -\frac{i}{8f_p^2}\bar{q}\gamma_\mu(\phi^\mu\phi - \phi\phi^\mu)q,$$

$$\mathcal{M}[\Xi^*\bar{K} \to \Xi^*\bar{K}] = \mathcal{M}[\Omega\eta \to \Omega\eta] = 0,$$

$$\begin{aligned}\mathcal{M}[\Xi^*\bar{K} \to \Omega\eta] &= \mathcal{M}[\Omega\eta \to \Xi^*\bar{K}],\\ &= -\frac{3}{4f_K^2}(2\sqrt{s} - M_\Omega - M_{\Xi^*}),\end{aligned}$$

$$G_{\Omega\eta}(\sqrt{s}) = \frac{M_\Omega}{2\pi^2}\int_0^\Lambda d|\boldsymbol{q}|\frac{\boldsymbol{q}^2}{s-(E_\Omega+\omega_\eta)^2+i\epsilon} \times \frac{E_\Omega+\omega_\eta}{E_\Omega\omega_\eta}\left(1+\frac{2\boldsymbol{q}^2}{9M_\Omega}\right),$$

$$v_{\mathrm{com}} = \mathcal{M}[\Xi^*\bar{K} \to \Omega\eta]G_{\Omega\eta}(\sqrt{s})\mathcal{M}[\Omega\eta \to \Xi^*\bar{K}].$$

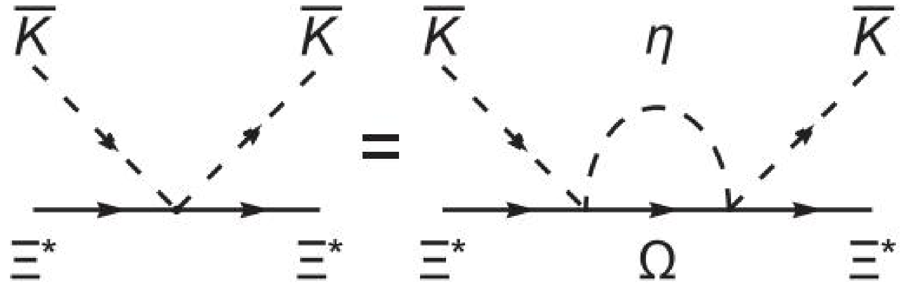


FIG. 1. The Feynman diagram for the effective $\Xi^*\bar{K}$ interaction via $\Omega\eta$ loop.

Interactions

$$\mathcal{L}_{M_p qq} = \frac{g_A^q}{2f_p}\bar{q}\gamma_\mu\gamma_5\phi^\mu q,$$

$$\mathcal{H}_{\rm NR} = \frac{g_A^q}{2f_p}\left[\boldsymbol{\sigma}\cdot\boldsymbol{q} + \frac{\omega}{2m}(\boldsymbol{\sigma}\cdot\boldsymbol{q} - 2\boldsymbol{\sigma}\cdot\boldsymbol{p}_i)\right].$$

$$\mathcal{M}[\Omega(1^2P_{3/2^-}) \to \Xi^*\bar{K}] = \frac{g_A^q\omega_{\bar{K}}\alpha}{f_K m}e^{-\frac{q^2}{9\alpha^2}},$$

$$v_{\rm bare} = \frac{2M_0 g_A^{q2}\alpha^2(\sqrt{s} - M_{\Xi^*})^2}{f_K^2 m^2(s - M_0^2)},$$

$$v_{\rm full} = v_{\rm com} + v_{\rm bare}.$$

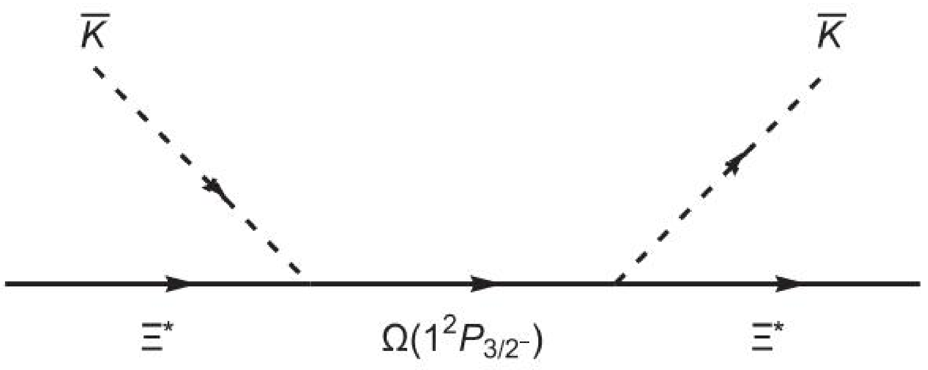


FIG. 2. The tree-level Feynman diagram for the interaction between the bare state and $\Xi^*\bar{K}$ channel.

The two-level problem

$$t_{\text{com}} = \frac{v_{\text{com}}}{1 - v_{\text{com}} G_{\Xi^* \bar{K}}},$$

$$t_{\text{full}} = \frac{v_{\text{full}}}{1 - v_{\text{full}} G_{\Xi^* \bar{K}}} = \frac{v_{\text{com}} + v_{\text{bare}}}{1 - (v_{\text{com}} + v_{\text{bare}}) G_{\Xi^* \bar{K}}}.$$

$$G_{\Xi^* \bar{K}}(\sqrt{s}) = \frac{M_{\Xi^*}}{2\pi^2} \int_0^\Lambda d|\boldsymbol{q}| \frac{\boldsymbol{q}^2}{s - (E_{\Xi^*} + \omega_{\bar{K}})^2 + i\epsilon} \times \frac{E_{\Xi^*} + \omega_{\bar{K}}}{E_{\Xi^*} \omega_{\bar{K}}} \left(1 + \frac{2\boldsymbol{q}^2}{9M_{\Xi^*}^2}\right).$$

$$t_{\text{com}} \equiv g_R(s) \frac{1}{\sqrt{s} - \sqrt{s_p}} g_R(s),$$

$$v_{\text{bare}} \equiv g(s) \frac{1}{\sqrt{s} - M_0} g(s).$$

$$t_{\text{full}} = (g_R, g) \frac{1}{\hat{D}_0^{-1} - \hat{\Sigma}} \begin{pmatrix} g_R \\ g \end{pmatrix},$$

$$\hat{D}_0^{-1} = \begin{pmatrix} \sqrt{s} - \sqrt{s_p} & 0 \\ 0 & \sqrt{s} - M_0 \end{pmatrix},$$

$$\hat{\Sigma} = \begin{pmatrix} 0 & g_R G_{\Xi^* \bar{K}} g \\ g G_{\Xi^* \bar{K}} g_R & g G_{\Xi^* \bar{K}} g \end{pmatrix}.$$

$$\hat{D} = \frac{1}{\hat{D}_0^{-1} - \hat{\Sigma}}$$

$$z^{ii} = \frac{1}{2\pi i} \oint_\gamma D^{ii}(s) d\sqrt{s}, \quad (i = 1, 2),$$

$$\Omega(2012) = \sqrt{z^{11}} |1\rangle + \sqrt{z^{22}} |2\rangle,$$

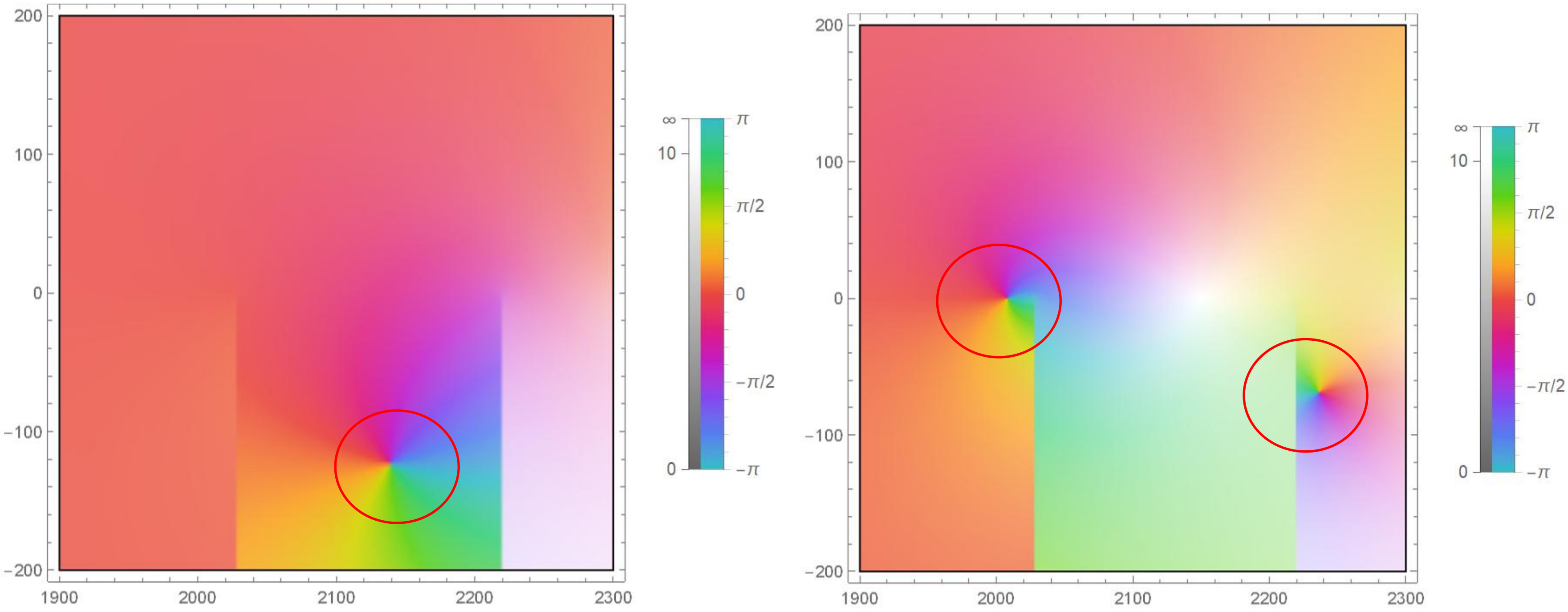
Solve Bethe-Salpeter equation to obtain the pole and coupling strength

Parameters

- Decay constant $f_{ps} = f_K = 111$ MeV
- Cut off in the loop integral $\Lambda = 500$ MeV
- Average quark mass (u/d/s) m = 450 MeV
- Harmonic oscillator parameter $\alpha = 331$ MeV
- Bare mass of $\Omega(1^2P_{3/2}{-})$ is supposed to be heavier than predictions (1953 ~ 2142 MeV) of physical mass, we choose $M_0 = 2150$ MeV.

Qi-Fang Lü, Hideko Nagahiro, Atsushi Hosaka, Phys. Rev. D 107, 014025 (2023)

Pole positions



Left: Pole in meson-baryon channels only

Right: Poles in three-quark state and meson-baryon channels

Pole positions

- If only considering the meson-baryon channels, the pole is $2139.2 - 124.3i$ MeV.
- When the bare state, $\Xi^*\bar{K}$, and $\Omega\eta$ channels are considered, we obtain two poles: 2007.9 MeV and $2236.8 - 69.3i$ MeV.
- Three-quark component *sss* is about 28.7%.
- Since we do not consider the decay channels in Bethe-Salpeter equation, the physical pole 2007.9 MeV is a bound state.
- Strong decays are dealt with perturbatively.

Strong decays

$$\Gamma[\Omega(2012) \to \Xi\bar{K}] = z^{22}\frac{1}{4\pi}\frac{|\boldsymbol{q}|}{2M_i^2}\frac{1}{2J+1}\sum_h |\mathcal{A}_h|^2,$$

$$g^2_{\Omega(2012)\Xi^*\bar{K}} = \lim_{\sqrt{s}\to M^*}(\sqrt{s}-M^*)t_{\rm full} = \frac{1}{2\pi i}\oint_\gamma t_{\rm full}(s)d\sqrt{s}.$$

$$\Gamma_{\Xi^*} = \Gamma_{\Xi^*,{\rm on}}\frac{|\tilde{\boldsymbol{q}}_\pi|^3}{|\tilde{\boldsymbol{q}}_{\pi,{\rm on}}|^3}\theta(M_{\rm inv}(\pi\Xi)-m_\pi-M_\Xi),$$

$$\frac{d\Gamma[\Omega(2012)\to\Xi^*\bar{K}\to\Xi\pi\bar{K}]}{dM_{\rm inv}(\pi\Xi)} = \frac{1}{(2\pi)^2}\frac{M_{\Xi^*}}{M_{\Omega(2012)}}|\tilde{\boldsymbol{q}}_{\bar{K}}|g^2_{\Omega(2012)\Xi^*\bar{K}} \times \frac{\Gamma_{\Xi^*}}{|M_{\rm inv}(\pi\Xi)-M_{\Xi^*}+i\frac{\Gamma_{\Xi^*}}{2}|^2}.$$

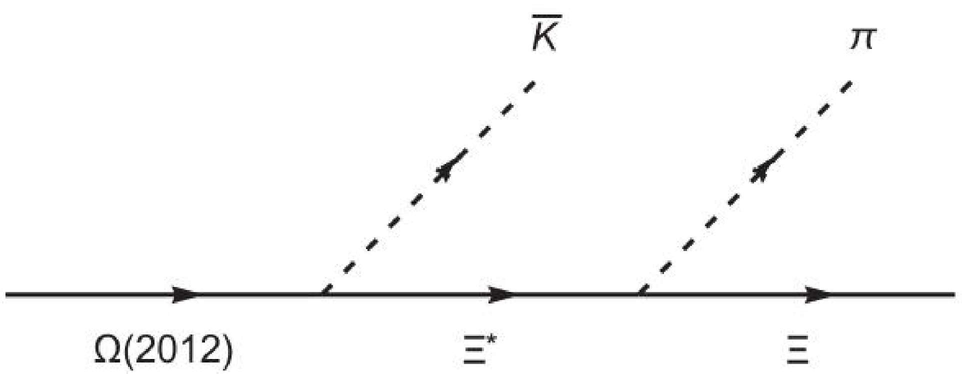
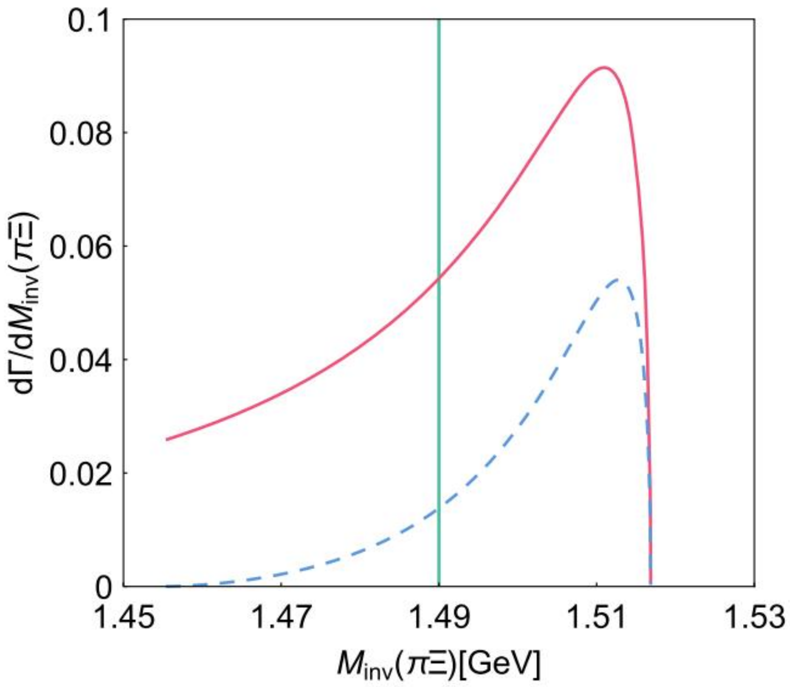


FIG. 3. The diagram of sequential decay chain $\Omega(2012) \to \Xi^*\bar{K} \to \Xi\pi\bar{K}$.

Strong decays

TABLE I. Theoretical results of the strong decays for the $\Omega(2012)$ resonance in MeV. The lower cut affects the three-body decays, ratio of branching fractions, and total width.

	No cut		With cut	
Decays	Independent	Dependent	Independent	Dependent
$\Xi\bar{K}$		3.02		
$\Xi\pi\bar{K}$	3.27	1.07	1.99	0.92
$\Gamma[\Omega(2012)]$	6.29	4.09	5.01	3.94
$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}}$	1.08	0.35	0.66	0.30
$\Gamma[\Omega(2012)]_{\text{exp}}$		$6.4^{+2.5}_{-2.0} \pm 1.6$ [2]		
$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}\ \text{exp}}$	$0.97 \pm 0.24 \pm 0.07$ [9]		<0.119 [7]	



The calculated strong decay behaviors are consistent with the new experimental data in 2022 well.

Advantages of coupled-channel interpretation

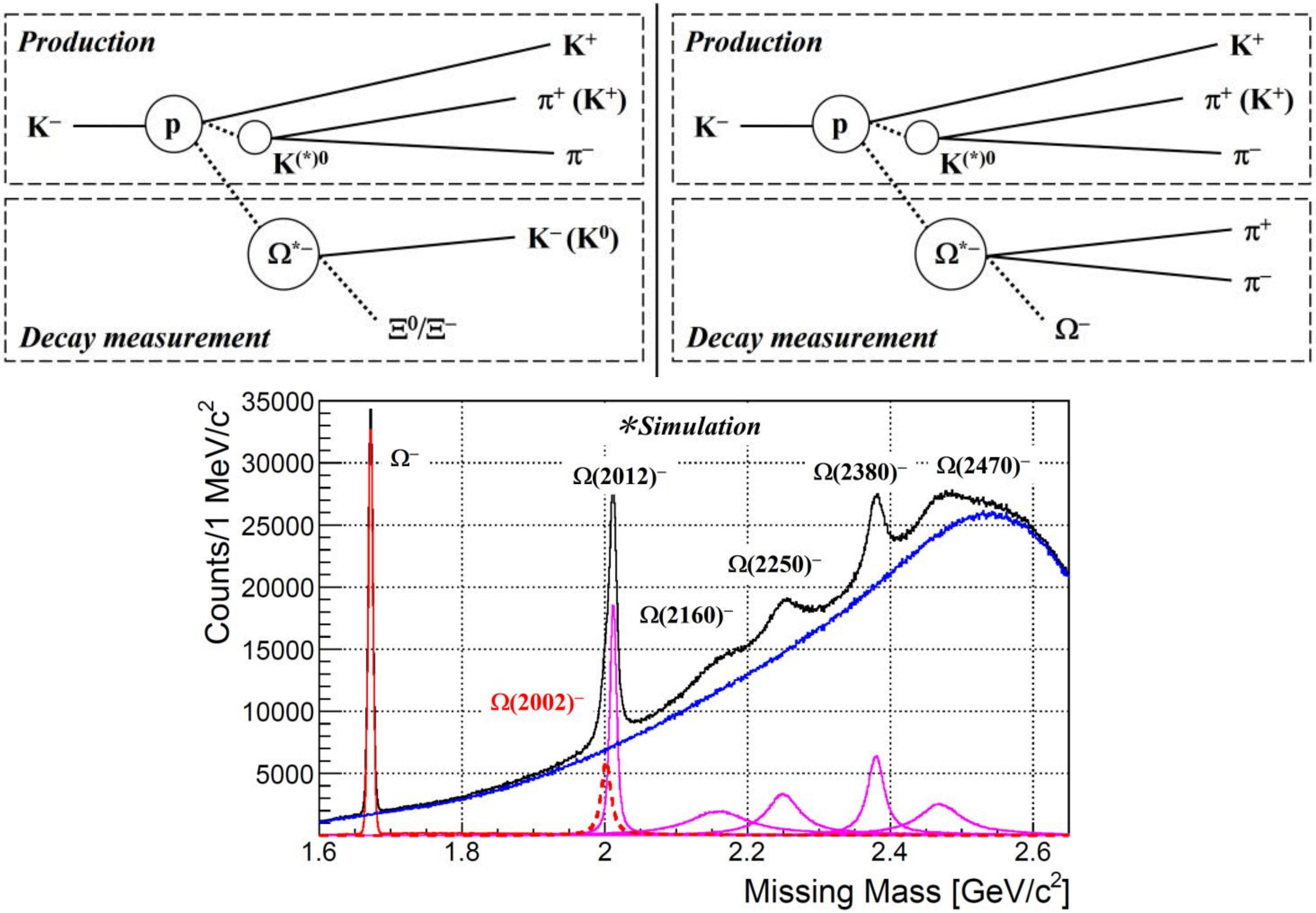
- ✓ Can describe the mass and total decay widths well.
- ✓ The calculated two-body and three-body strong decays agree the new experimental measurement well.
- ✓ Solve the difficulties of canonical and molecular interpretations.
- ✓ Several commonly used parameters.

Remaining Problems

- Three-body decay calculations are model sensitive.
- How about other properties, such as weak and electromagnetic?
- What about other analogous systems?
- All experimental data come from Belle Collaboration, no others.

The BESIII, BelleII, and J-PARC can also search for $\Omega(2012)$

Simulation of J-PARC



The third white paper of J-PARC, arXiv:2110.04462

Summary

- $\Omega(2012)$ is a mixture of three-quark core and meson-baryon channels.
- Describe the mass and strong decay behaviors well.
- Investigating other properties and systems.
- Need both future theoretical and experimental efforts.

More works about spectroscopy for mesons, baryons, tetraquarks, pentaquarks, and hexaquarks can be found in

My homepage **https://inspirehep.net/authors/1383269**

Thanks for your attentions！