Isospin-$\frac{1}{2}$, $\frac{3}{2}$ $D\pi$ scattering and the D_0^* resonance from lattice QCD

The 8th XYZ Particle Symposium

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Jul 28, 2023

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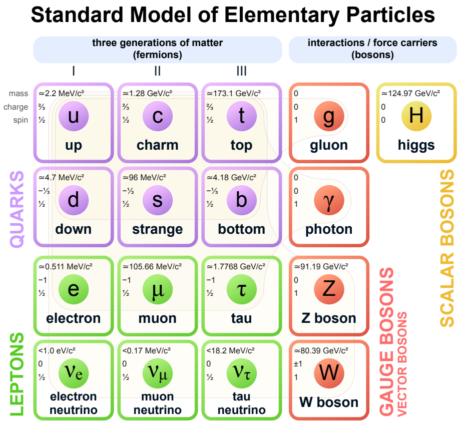
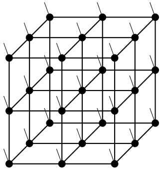
Lattice Quantum Chromodynamics (LQCD)



- QCD is the QFT that describes the strong interaction between quarks and gluons
- Lattice QCD is QCD formulated on a $4D$ discrete **Euclidean** spacetime grid

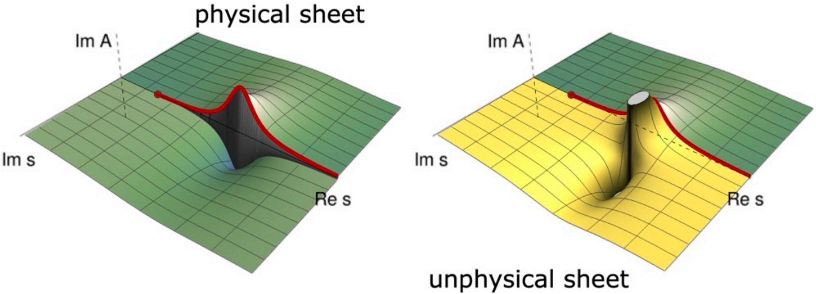


- Lattice QCD adopts path integral formalism

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i \left(\gamma^{\mu} D_{\mu} \right)_{ij} - m\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \tag{1}$$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] \mathrm{e}^{-S_F[\psi,\bar{\psi},U] - S_G[U]} O[\psi, \bar{\psi}, U] \tag{2}$$

- Most particles are hadronic resonances – scattering experiments

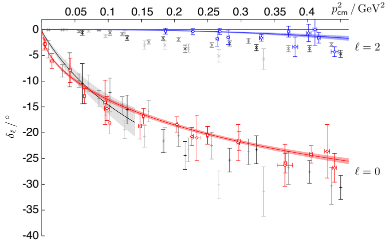


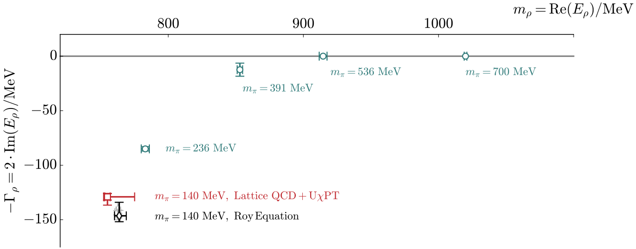
- Turn off the weak and electromagnetic interactions
- Since the invention of lattice field theory[1], the calculation of hadron spectroscopy in the non-perturbative regime has been pursued to understand the structure of particles from the first principle

[1]Wilson, PRD 10 (1974) 2445

A quick review of lattice hadron spectroscopy

- In 1991, Lüscher derived the pioneering formula[2]
- In 1992, Sharpe *et al.* calculated the first ever $\pi\pi$ scattering length[3]
- In the next 20 years, people vastly generalized the formula until Briceño wrote down the most general form[4]
- In 2012, JLab calculated the energy-dependent phase shift[5] (non-resonant)





- In the previous 10 years, ρ has been found on the lattice undisputedly[6]

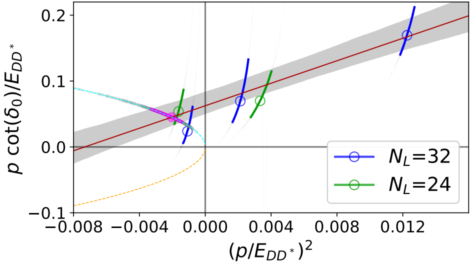
[2] Lüscher, NPB 354 (1991) 531
[3] Sharpe *et al.*, NPB 383 (1992) 309
[4] Briceño, PRD 89 (2014) 074507
[5] Dudek *et al.*, PRD 86 (2012) 034031
[6] Briceño *et al.*, RMP 90 (2018) 025001

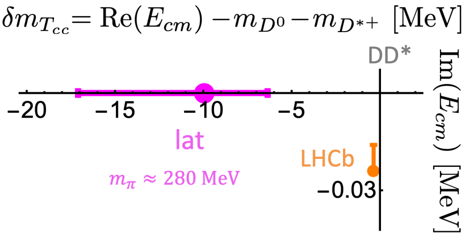
For example, $Z_c(3900)$:

- Chen *et al.*, PRD 89 (2014) 094506
- HALQCD, 117 (2016) 242001
- Chen *et al.*, 10 (2019) 103103

$T_{cc}^{+}(3875)$:

- Chen *et al.*, PRD 833 (2022) 137391
- Padmanath *et al.*, PRL 129 (2022) 032002





$X(6900)$

- Meng *et al.*, arxiv:230x.xxxxx (only the scattering length)

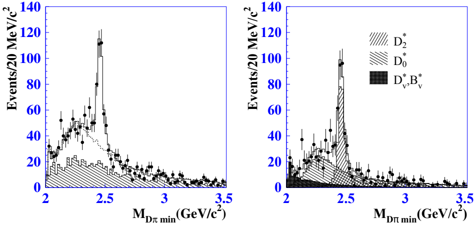
Make hadrons more charming

- Spectroscopy:
  - Liu *et al.*, JHEP 07 (2012) 126: charmonium excited and exotic spectroscopy
  - Moir *et al.*, JHEP 05 (2013) 021: D excited spectroscopy
  - Cheung *et al.*, JHEP 12 (2016) 089: D_s excited spectroscopy
  - ... ...
- Scattering:
  - Mohler *et al.*, PRD 87 (2013) 034501: $D\pi$ $I = \frac{1}{2}$ scattering
  - Moir *et al.*, JHEP 10 (2016) 011: Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ scattering
  - Gayer *et al.*, JHEP 07 (2021) 123: $D\pi$ $I = \frac{1}{2}$ scattering
  - Cheung *et al.*, JHEP 02 (2021) 100: DK $I = 0$, $D\bar{K}$ $I = 0, 1$ scattering
  - ... ...

We try to conduct a systematic study on the $D\pi$ scattering

The D_0^* resonance

- The D_0^* was found in 2004 by Belle collboration[7]



- The mass of $D_0^*(2300)$ is almost identical to $D_{s0}^*(2317)$, which is **not** consistent with the traditional quark model predictions[8]. This can be explained by the strong coupling to DK[9]
- UChPT: $D_0^*(2100)$ should be the lightest charmed scalar meson[10]
- The possible two-pole structure mentioned by many people
- Towards the understanding of $\psi_0(4360) \to D^* \bar{D}_1$ (0^{--})[11]

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[7] Satpathy *et al.*, PRB 159 (2003) 553.
[8] Du *et al.*, PRD 98 (2018) 094018.
[9] Chen *et al.*, Rep. Prog. Phys. 80 (2017) 076201
[10] Albaladejo *et al.*, PLB 767 (2017) 465.
[11] Ji *et al.*, PRL 129 (2022) 102002.

Configurations generated by the CLQCD collaboration (中国格点合作组)

By 24:00, Jul. 27, 2023

configuration	volume	a/fm	β	m_π/MeV	m_{η_s}/MeV	$m_\pi L$	N_{cfgs}
C24P34	$24^3 \times 64$	0.1053	6.20	340	748	4.38	301
C24P29	$24^3 \times 72$	0.1053	6.20	292	658	3.75	879
C32P29	$32^3 \times 64$	0.1053	6.20	292	658	5.01	984
C32P23	$32^3 \times 64$	0.1053	6.20	228	643	3.91	451
C48P23	$48^3 \times 96$	0.1053	6.20	225	643	5.79	278
C48P14	$48^3 \times 96$	0.1053	6.20	135	706	3.56	203
F32P30	$32^3 \times 96$	0.0775	6.41	303	681	3.81	568
F48P30	$48^3 \times 96$	0.0775	6.41	303	679	5.72	278
F32P21	$32^3 \times 64$	0.0775	6.41	210	665	2.67	459
F48P21	$48^3 \times 96$	0.0775	6.41	207	667	3.91	270
H48P32	$48^3 \times 144$	0.0519	6.72	321	709	4.06	274
H64P32	$64^3 \times 128$	0.0519	6.72	321	709	5.41	preparing

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$D\pi$ scattering

- Create them from the vacuum! (in a world where $m_\pi \approx 300$ MeV)
- Severe partial wave mixing – need many many operators
- Project the operators into specific quantum numbers – irrep. and group[12]

$$O_{|p|,\Gamma,r,n} = \sum_{\tilde{R}\in G} T^{\Gamma}_{r,r}(\tilde{R})\tilde{R}D(p_1)\pi(p_2)\tilde{R}^{-1}$$

$$Dic_4(A_1)\begin{cases} \mathcal{O}_{D_0^*} & = D_0^{*+}(e_z), \\ \mathcal{O}_{D^*} & = D_z^{*+}(e_z), \\ \mathcal{O}_{D^*_\nabla} & = \sum_i \bar{d}(\gamma_x\vec{\nabla}_y - \gamma_y\vec{\nabla}_x)c, \\ \mathcal{O}_{D(0)\pi(1),|\vec{p}^2_{\text{rel}}|=|\frac{1}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [0], \\ \mathcal{O}_{D(1)\pi(0),|\vec{p}^2_{\text{rel}}|=|\frac{1}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_z], \\ \mathcal{O}_{D(1)\pi(2),|\vec{p}^2_{\text{rel}}|=|\frac{5}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_{-x}, e_x, e_{-y}, e_y], \\ \mathcal{O}_{D(2)\pi(1),|\vec{p}^2_{\text{rel}}|=|\frac{5}{4}|} & = \sum_\alpha D(\vec{p}_\alpha)\pi(\vec{P}_{\text{tot}} - \vec{p}_\alpha), \alpha \in [e_{xz}, e_{-x,z}, e_{yz}, e_{-y,z}], \\ \ldots \end{cases}$$

- These operators are constructed to map out the scattering phase shift

[12] Prelovsek *et al.*, JHEP 2017 (2017) 1.

Correlation functions

- Create $D\pi$ from a spacetime point, and annihilate them later

$$\langle \mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]}_{D^{(*)}\pi,\Gamma,p}(t')\mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]\dagger}_{D^{(*)}\pi,\Gamma,p}(t)\rangle = \sum_{\beta\alpha ji}(6\mathbb{E}+9\mathbb{F}-3\mathbb{G})^{[\beta,P-\beta;-\alpha,-(P-\alpha)]}_{[\gamma_j,\gamma_5;\gamma_i,\gamma_5]}$$

$$\mathbb{F} = \langle \bar{u}\Box e^{-ip_\delta\cdot x}\Omega\Box c(t')\cdot \bar{d}\Box e^{-ip_\gamma\cdot x}\Xi\Box u(t')\cdot \bar{c}\Box e^{-ip_\beta\cdot x}\Lambda\Box u(t)\cdot \bar{u}\Box e^{-ip_\alpha\cdot x}\Gamma\Box d(t)\rangle$$

- The Wick contractions contain the following diagrams

(a) $\mathbb{E}$ (b) $\mathbb{F}$ (c) $\mathbb{J}$ (d) $\mathbb{G}$

We apply the distillation method[13] to make the calculation possible

$$\Box(t) = V(t)\,V^\dagger(t) \longrightarrow \Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t)v_y^{(k)\dagger}(t)$$

[13] Peardon *et al.*, PRD 80 (2009) 054506.

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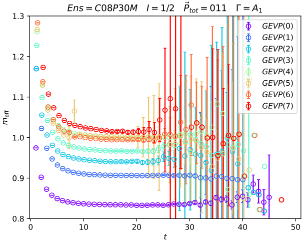
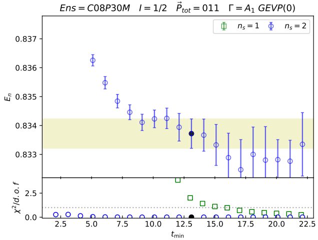
[14] Peardon *et al.*, PRD 80 (2009) 054506.

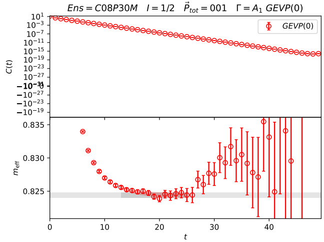
- Inserting a complete basis, we know

$$\langle \mathcal{O}^{[J=L=S=0]}_{D\pi,A_1^+,p}(t')\mathcal{O}^{[J=L=S=0]\dagger}_{D\pi,A_1^+,p}(t)\rangle = \sum_n |\langle n|\mathcal{O}^{[J=L=S=0]}_{D\pi,A_1^+,p}|0\rangle|^2\, \mathrm{e}^{-E_n t}$$

- For more than one operator, we use the GEVP method to diagonalize them
- The effective mass $m_{\text{eff}}(t)$ would go asymptotic to a plateau of energy levels







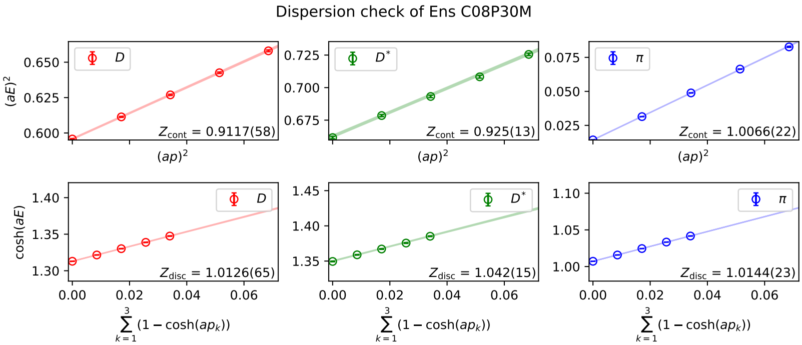
- The high precision enables precise phase shift determination

- The dispersion relation

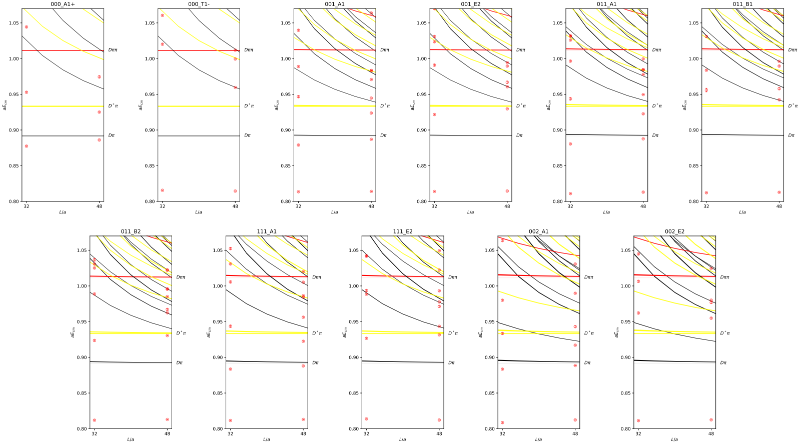
$$E(\vec{p}) = \sqrt{m_H^2 + \vec{p}^2}(1 + \mathcal{O}(ap)) \tag{3}$$

should be replaced by the discretized version

$$\cosh(aE(\vec{p})) = \cosh(am_H) + \sum_{k=1}^{3}(1 - \cos(ap_k)) \tag{4}$$



- The extracted finite volume scattering spectra



- The emergence of D^*
- Strong attraction in S-wave and small δ_1

Scattering analysis (preliminary)

- The Lüscher's equation[15]

$$\det\left[e^{2i\delta} - U(\Gamma)\right] = 0$$

  relates the spectrum to the scattering phase shifts in infinite volume
- Underconstrained problem
- Parametrize the phase shifts by the effective range expansion

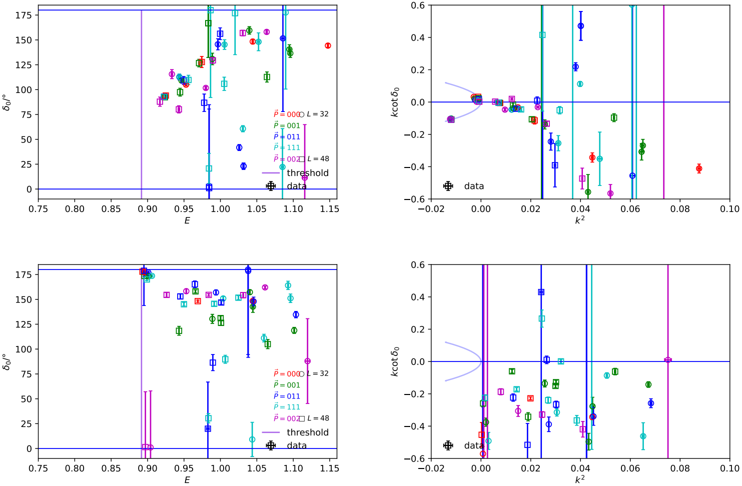
$$k^{2l+1}\cot\delta_l = \frac{1}{a_l} + \frac{1}{2}r_l k^2 + P_2 k^4 + \mathcal{O}\left(k^6\right),$$

- Coupling to $D^*\pi$ is to be considered

[15] Lüscher, NPB 354 (1991) 531.

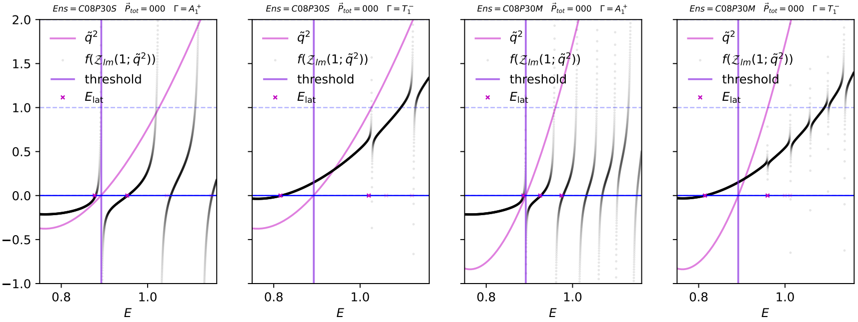
Scattering analysis (preliminary)

- Ignore all $l > 0$ partial wave



Scattering analysis (preliminary)

- Using only data from $\vec{P} = 0$
- The Lüscher's equations

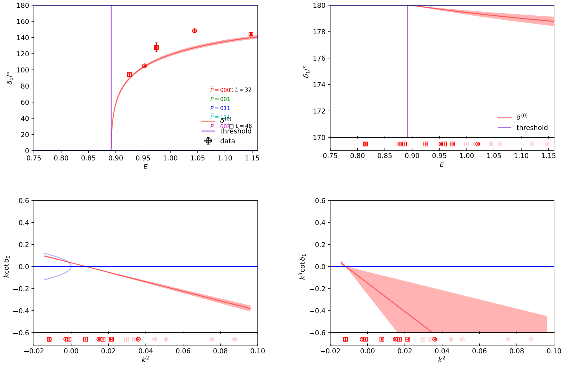


- The scattering length and the effective range

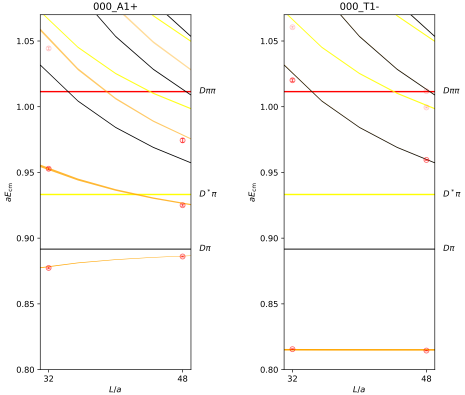
$$\begin{cases} a_0 = 2.26(19)\text{fm} \\ r_0 = -0.670(47)\text{fm} \end{cases} \quad \begin{cases} a_1 = -0.52(15)\text{fm} \\ r_1 = -2.0(1.3)\text{fm} \end{cases} \tag{5}$$

Scattering analysis (preliminary)

The $I = \frac{1}{2}$ phase shifts



The predicted spectrum



- With the phase shifts, the poles in the Riemann sheet can be found
- For now, D_0^* looks like a virtual state on our lattice

Conclusions

- There has been a renaissance in hadron spectroscopy
- People are still far from calculating the exact hadron spectrum (well, ρ is good enough)
- A large number of $D\pi$ operators are constructed
- Obtained many finite-volume energy levels in the $D\pi$ system
- Found the D_0^* virtual state on our configuration
- To-dos:
  - Analysis for non-inertial frames
  - Interpretation of the pole(s)
  - Chiral extrapolation
  - Continuum extrapolation

Thank you!

Appendix

6 Appendix A: Extract the energy spectra

Appendix A: Extract the energy spectra

For the correlated analysis, one fits the data in a range $n_{\min} \leq n_t, n_t' \leq n_{\max}$ by minimizing

$$\chi^2 = \sum_{n_t, n_t' = n_{\min}}^{n_{\max}} \left(C\left(n_t\right) - f\left(n_t\right) \right) w\left(n_t, n_t'\right) \left(C\left(n_t'\right) - f\left(n_t'\right) \right)$$

with regard to the overlap factor $A_{\alpha 0}$ and the ground state energy m_α

$$f\left(n_t\right) = A_{\alpha 0} \cosh\left(\left(n_t - N_T/2\right) m_\alpha \right)$$

The estimation of the weight $w\left(n_t, n_t'\right) = \mathrm{Cov}^{-1}\left(n_t, n_t'\right)$ is

$$\mathrm{Cov}_N\left(n_t, n_t'\right) = \frac{1}{N-1} \left\langle \left(C\left(n_t\right) - \left\langle C\left(n_t\right) \right\rangle_N \right) \left(C\left(n_t'\right) - \left\langle C\left(n_t'\right) \right\rangle_N \right) \right\rangle_N$$

In this study, all statistical errors are estimated by jackknife resampling