Ripples of the QCD critical point

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“QCD under rotation”, Fudan University
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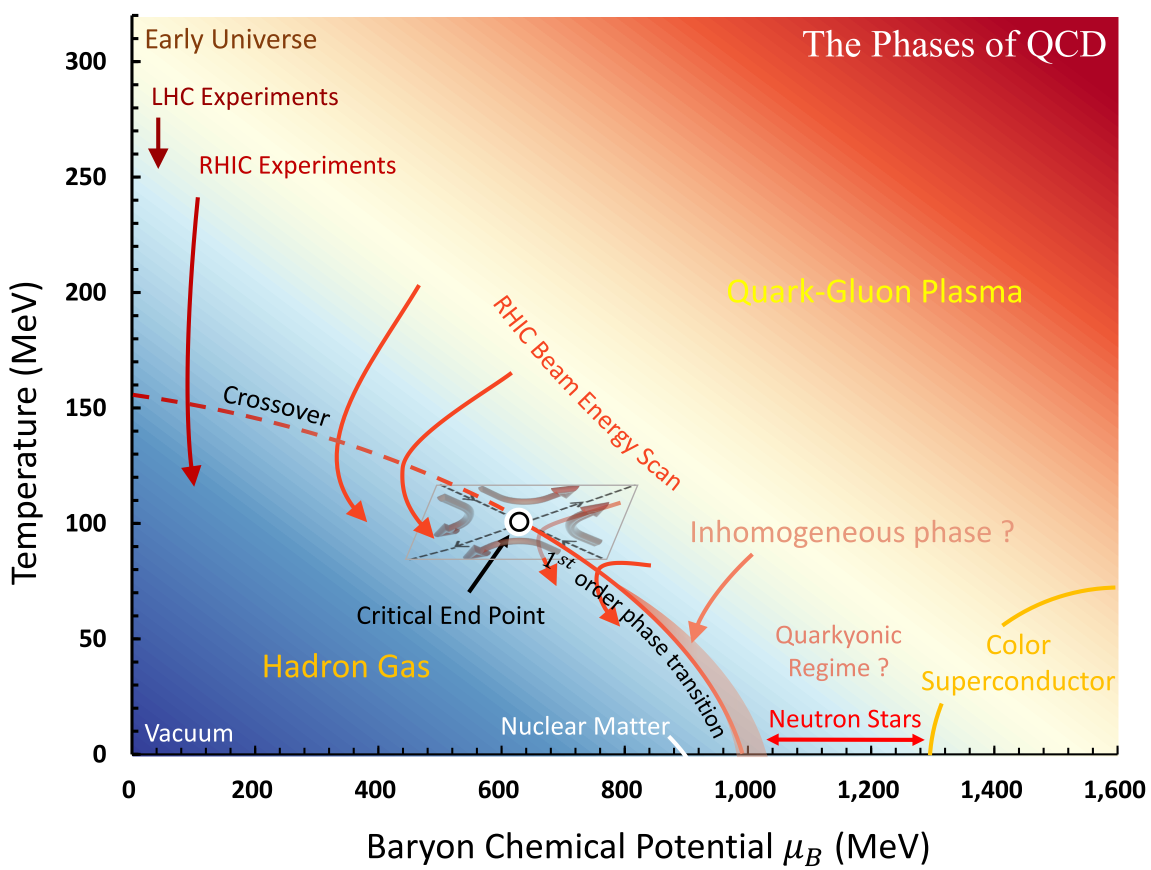
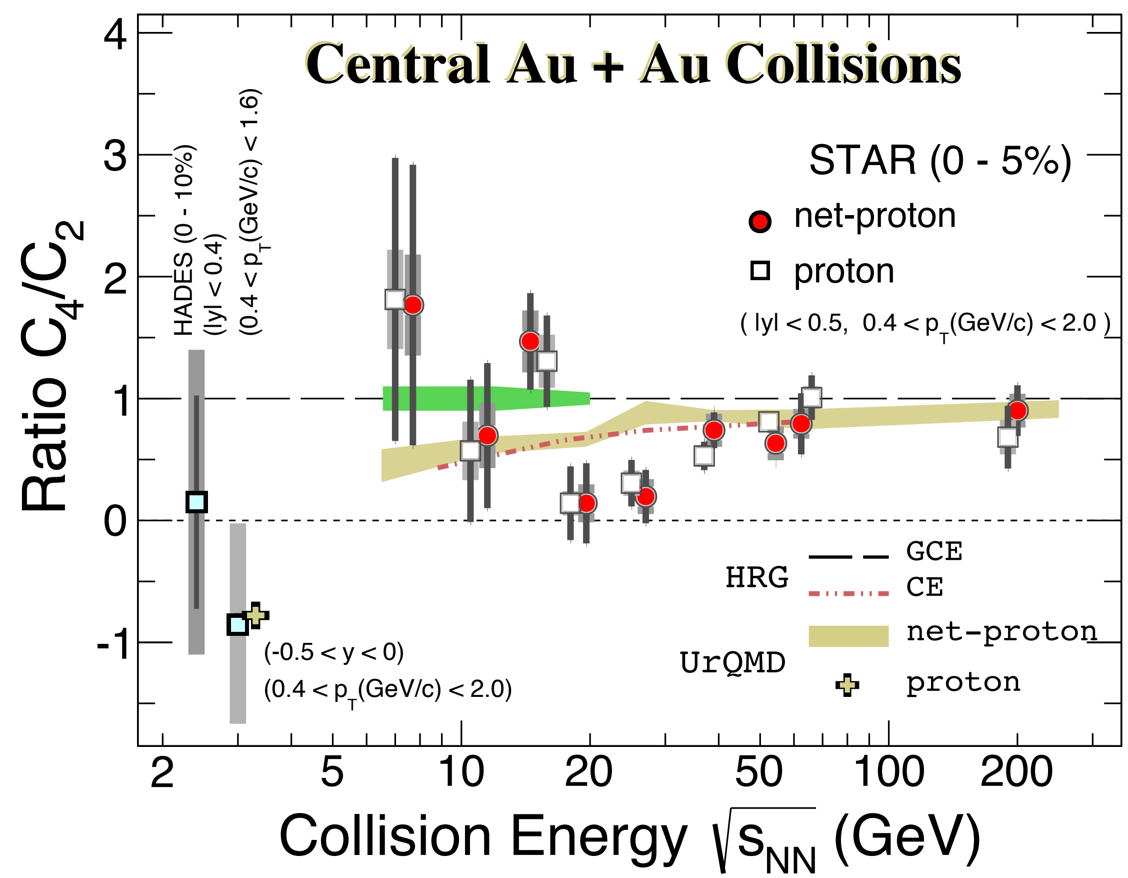
Based on :
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Shi Yin, *Ripples of the QCD Critical Point*, arXiv: 2308.15508;
Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, *Soft modes in hot QCD matter*, arXiv:2310.19853.

QCD phase structure

QCD phase diagram



Fluctuations measured by STAR



J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- Is there a "peak" structure in the regime of low colliding energy?

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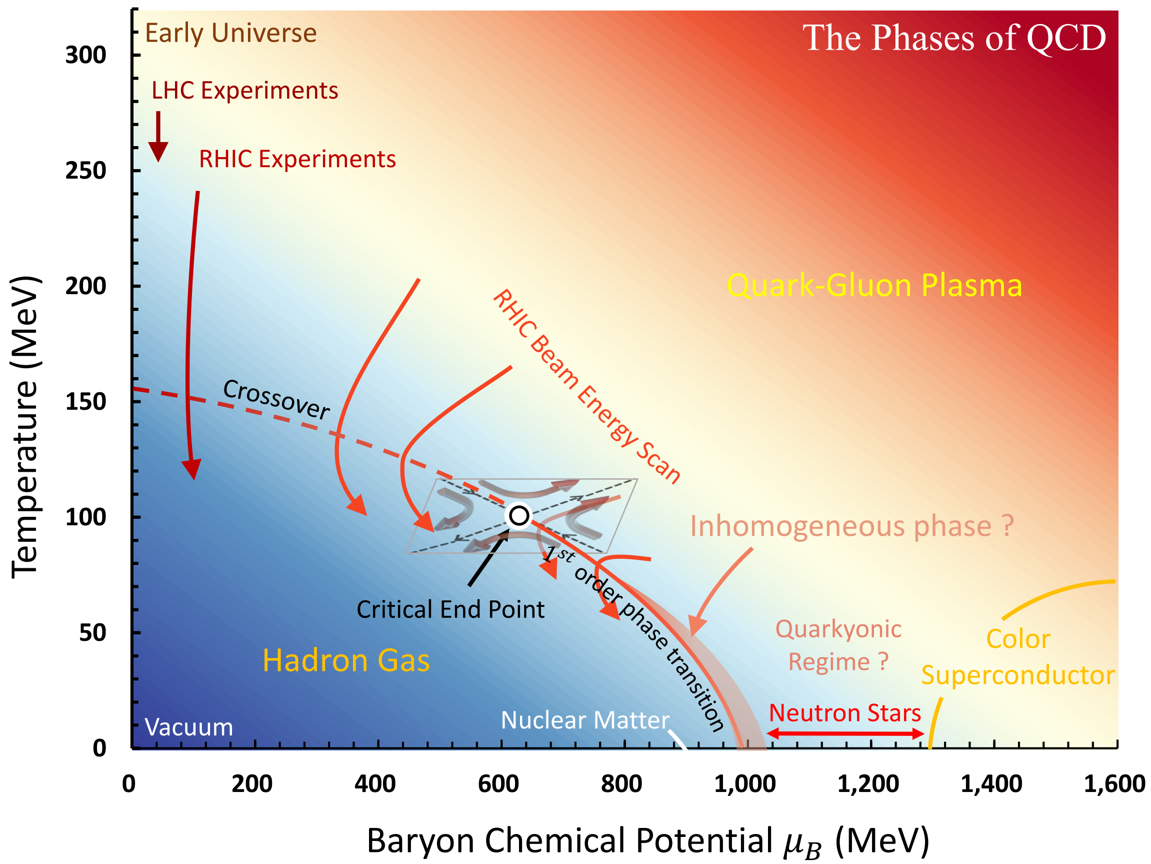
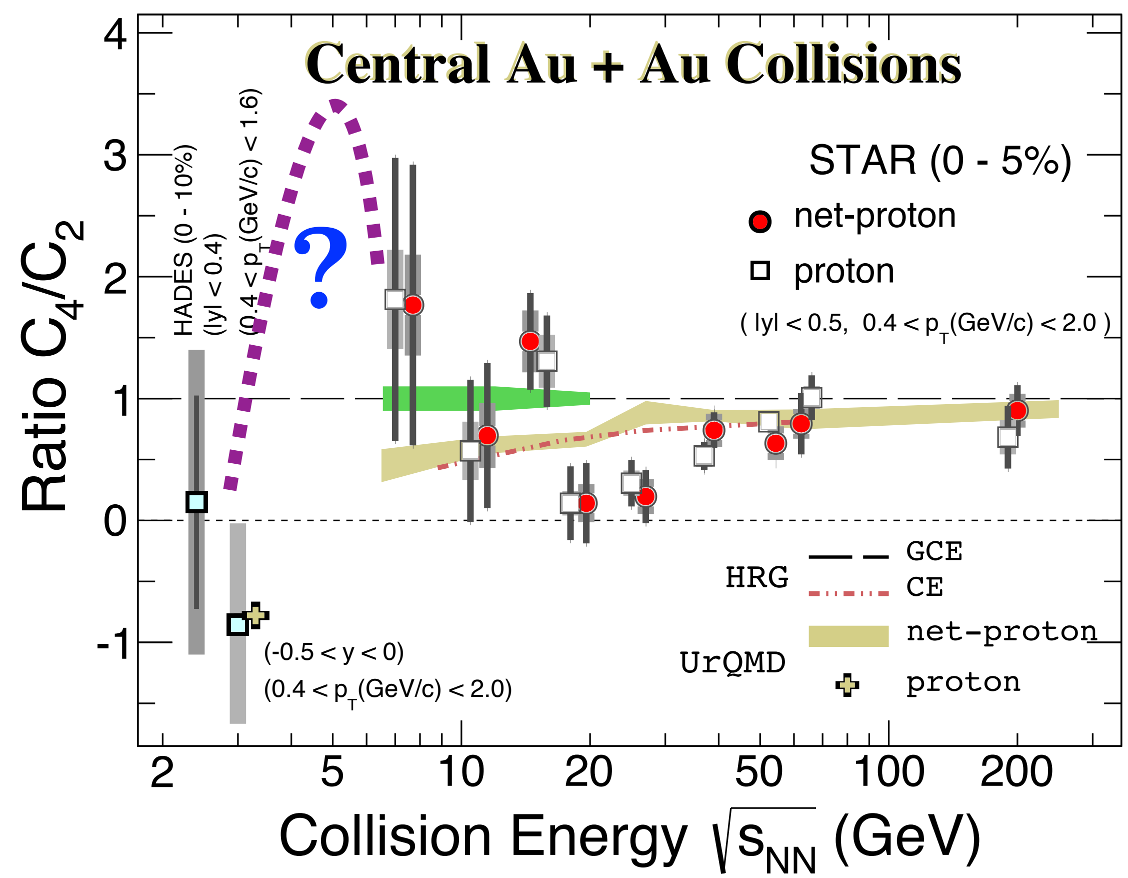


Fluctuations measured by STAR



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- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
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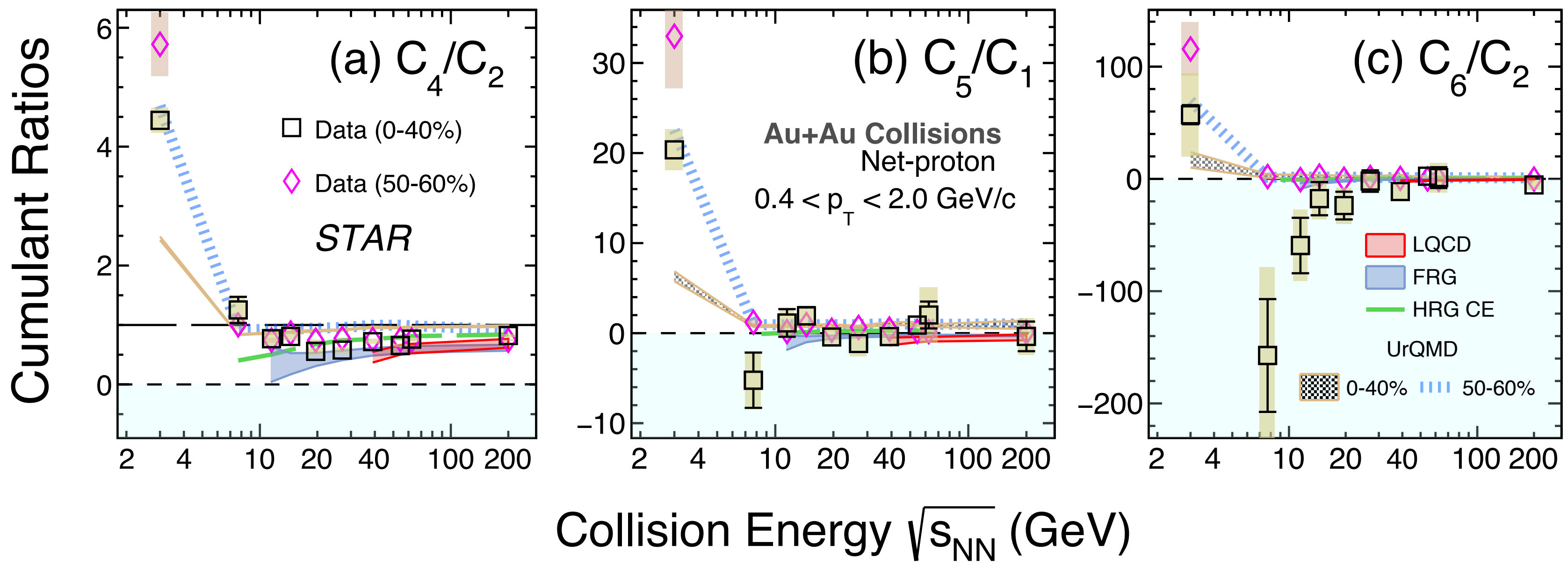
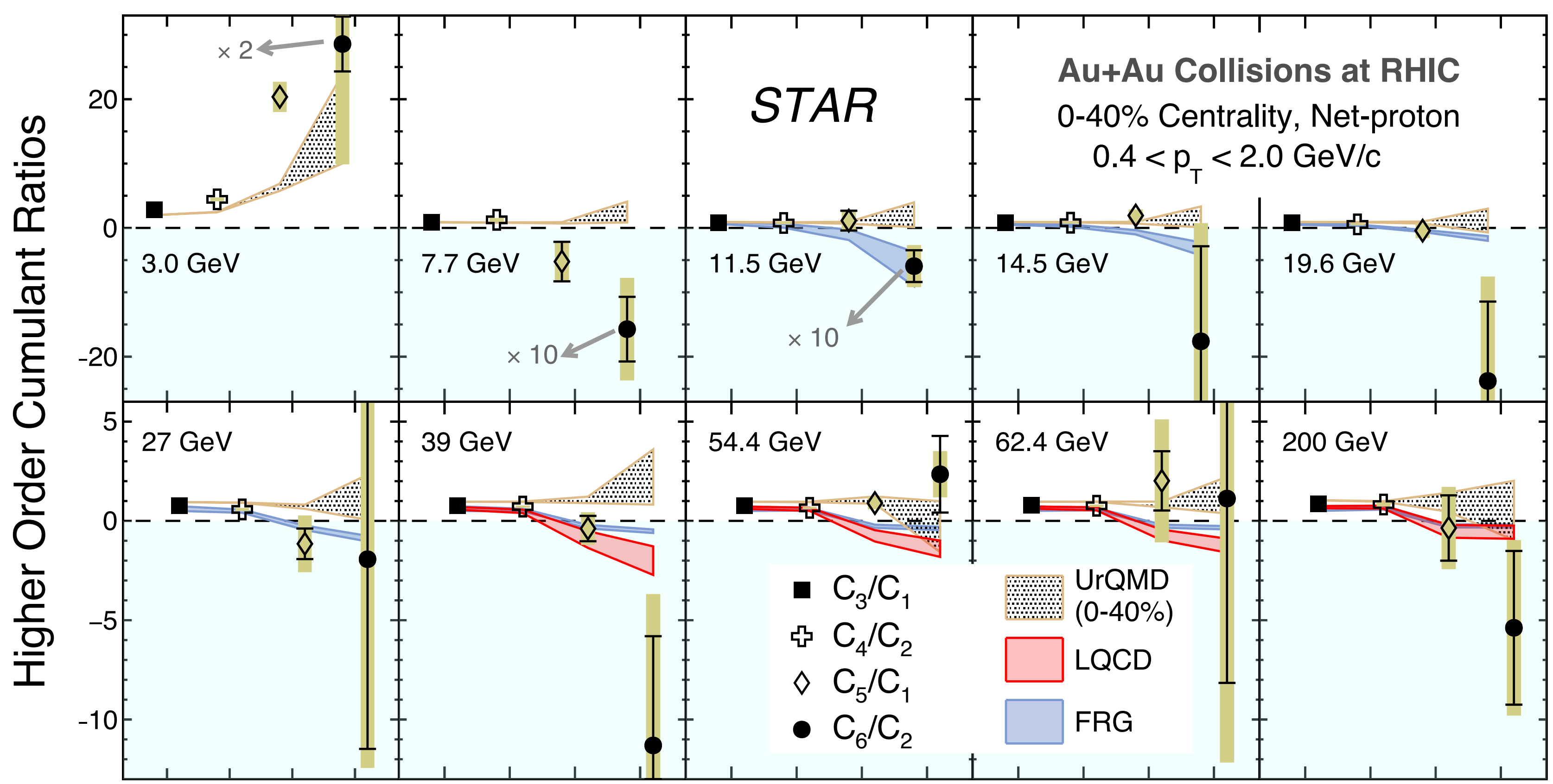
Hyper-order fluctuations





STAR: B. Aboona *et al.*, *PRL* 130 (2023), 082301, arXiv: 2207.09837

fRG: WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

Outline

- Introduction
- Brief review about fRG
- Baryon number fluctuations at high density
- Ripples of the QCD critical point
- Critical region and its size in QCD
- Summary

Functional renormalization group

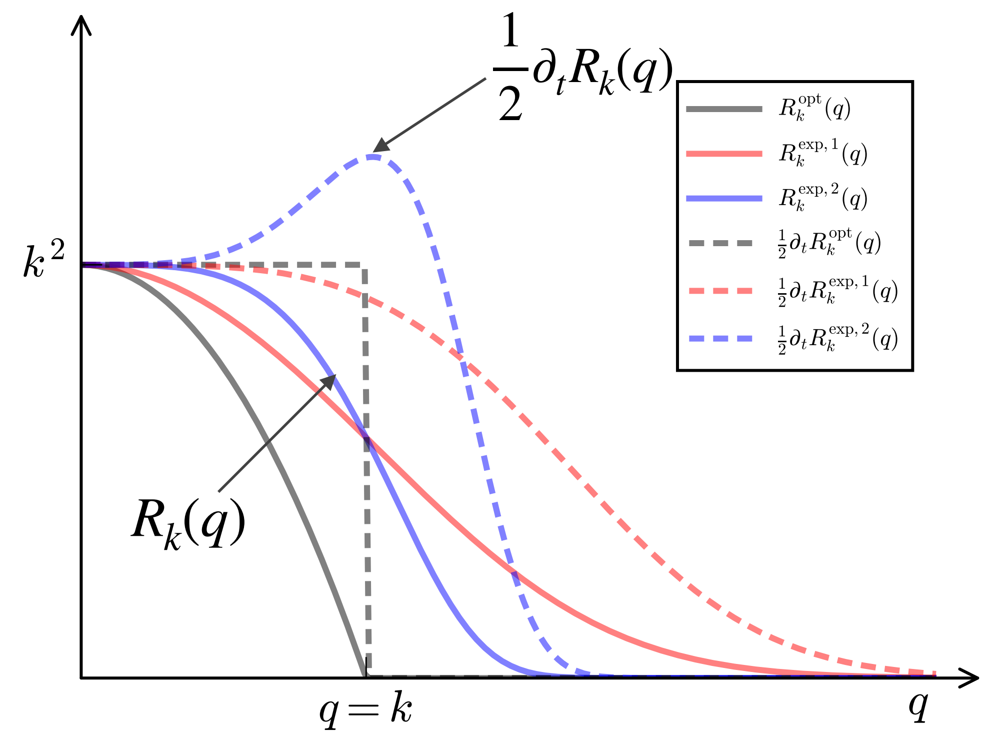
Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\left\{ -S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \right\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$



flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

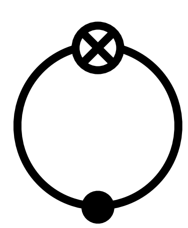
$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \Big)^{-1}_{cb},$$

Legendre transformation:

$$\Gamma_k[\Phi] = -W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$

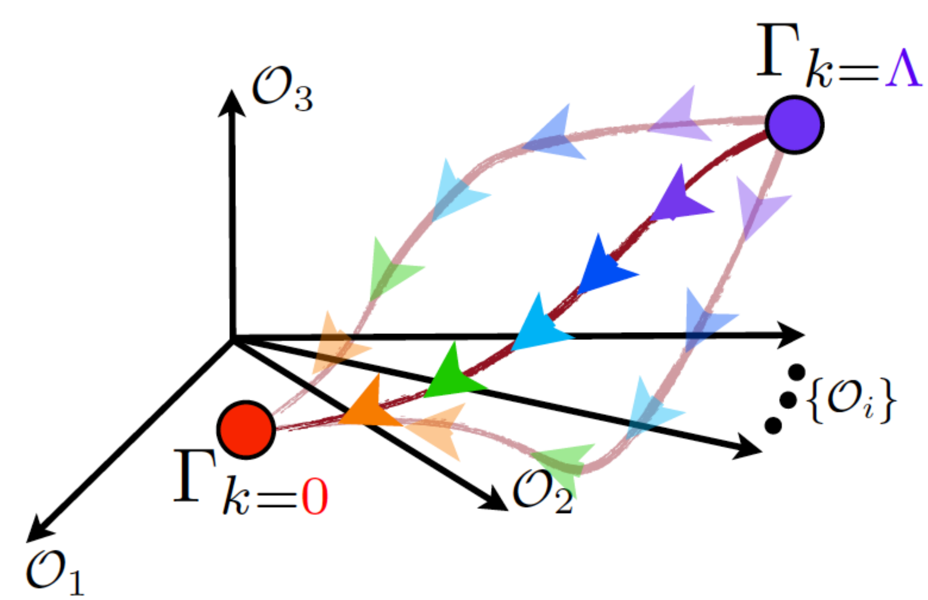
flow of the effective action:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] = \frac{1}{2}$$



Wetterich equation

C. Wetterich, *PLB*, 301 (1993) 90

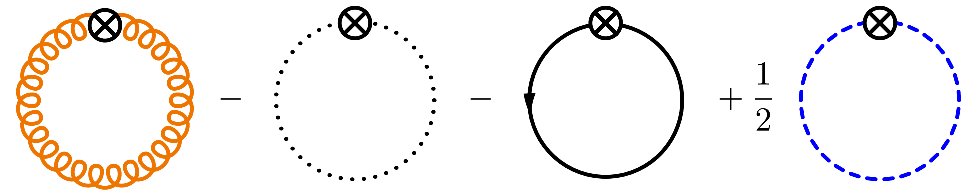


Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

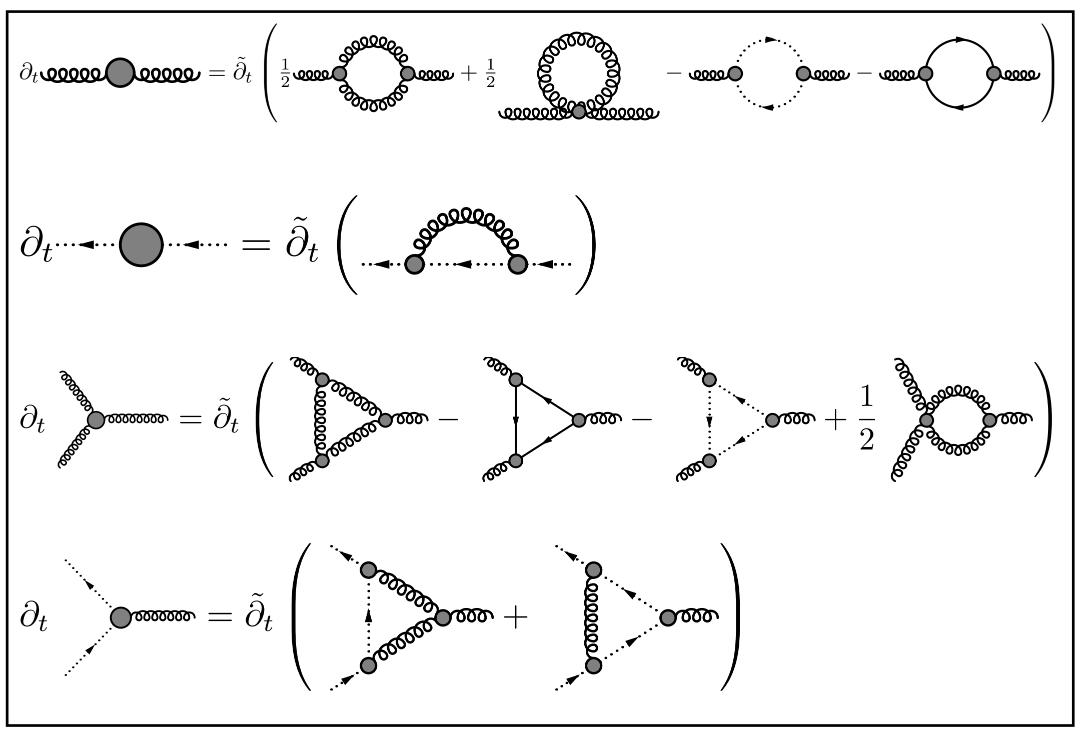
First-principles QCD within fRG

QCD flow equation:

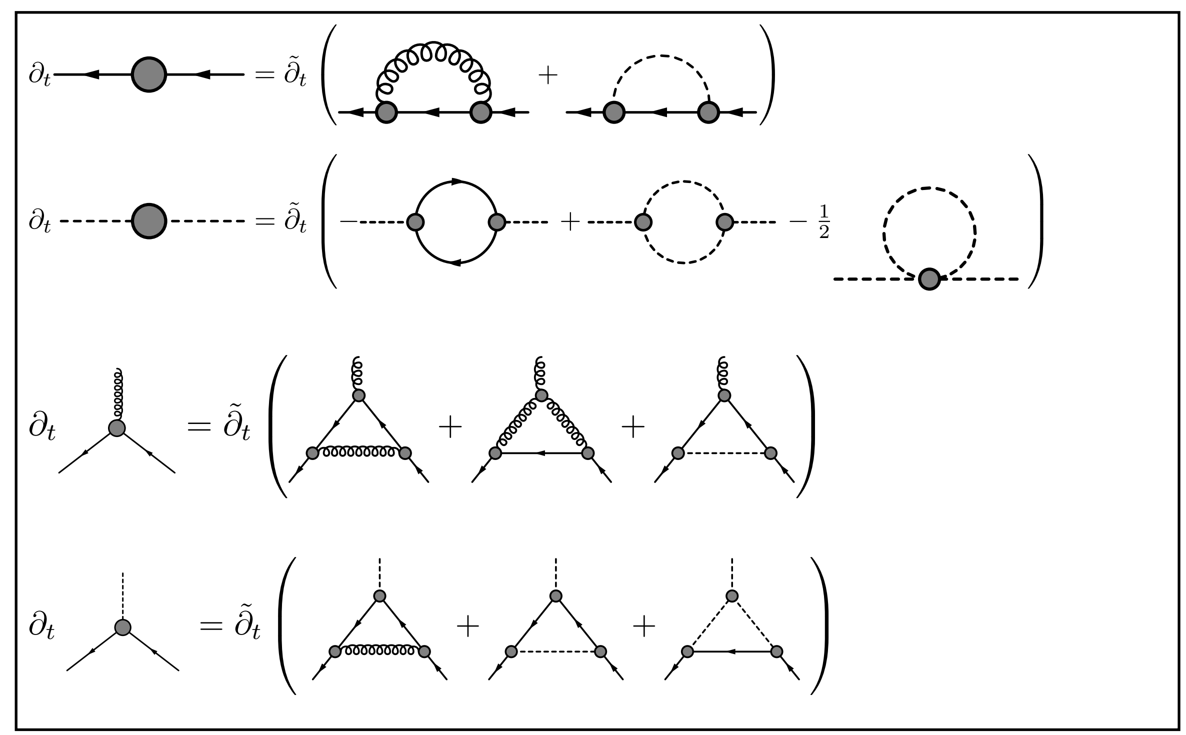
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$



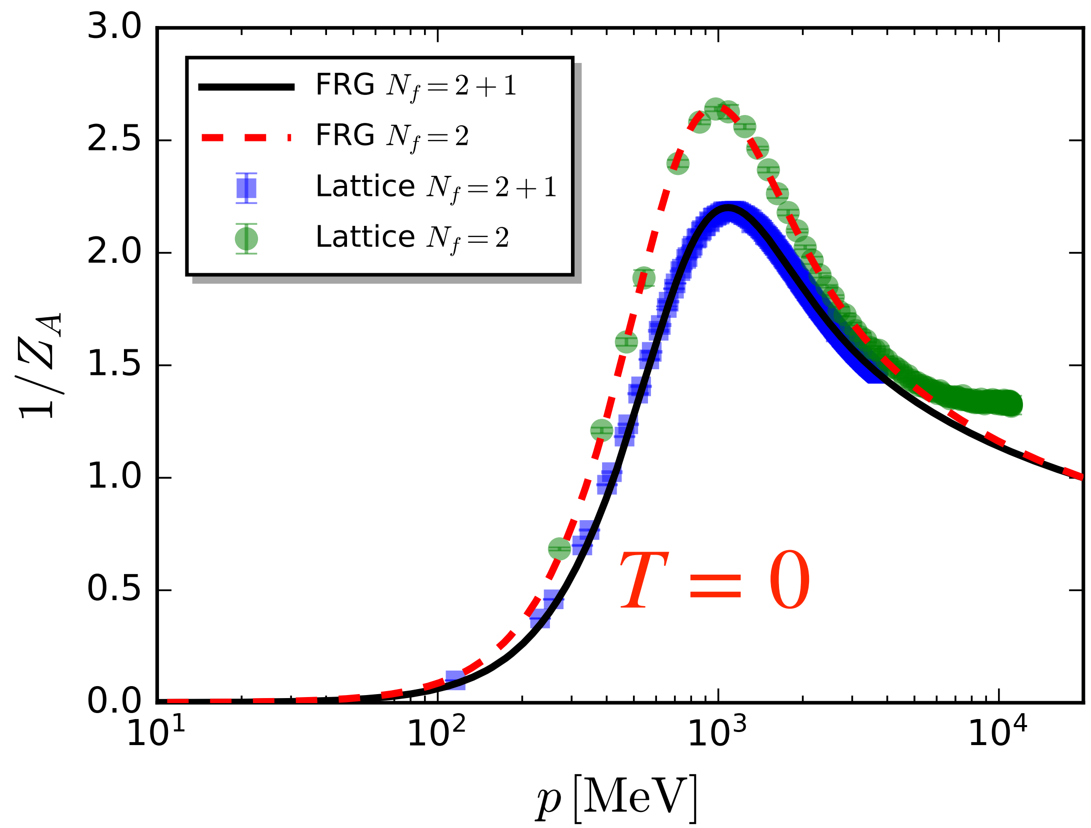
Glue sector:

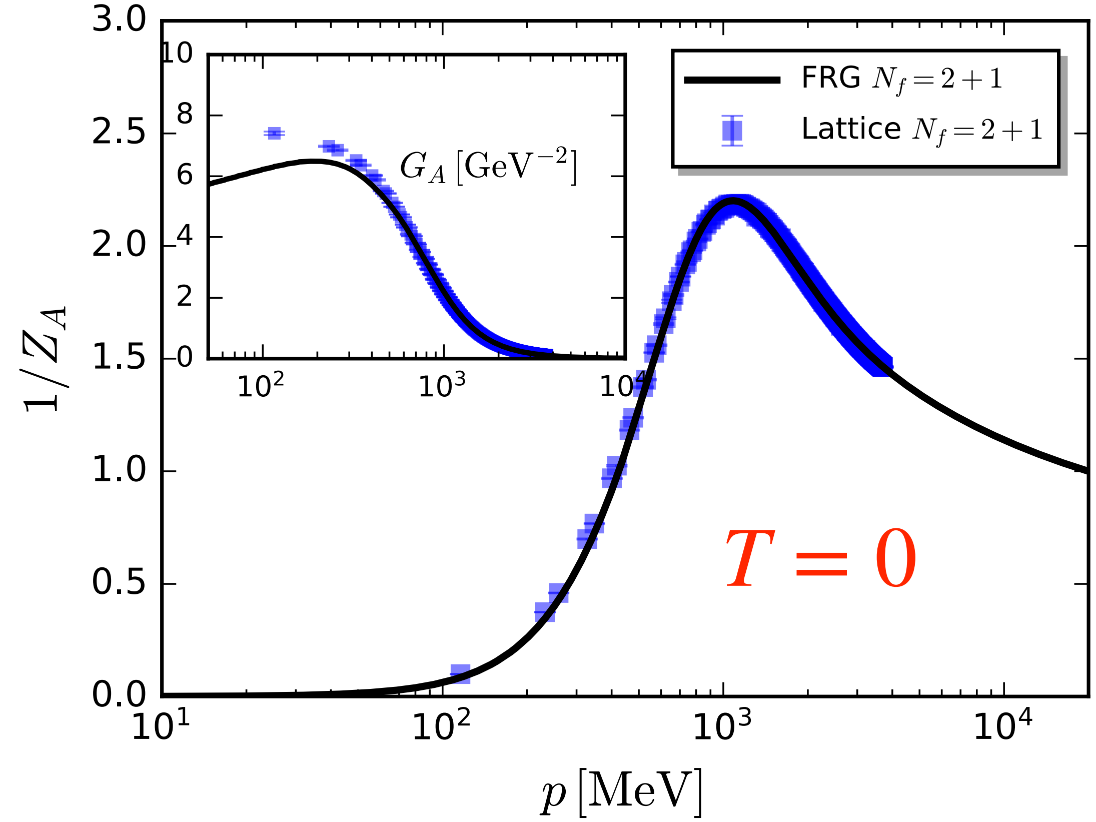


Matter sector:



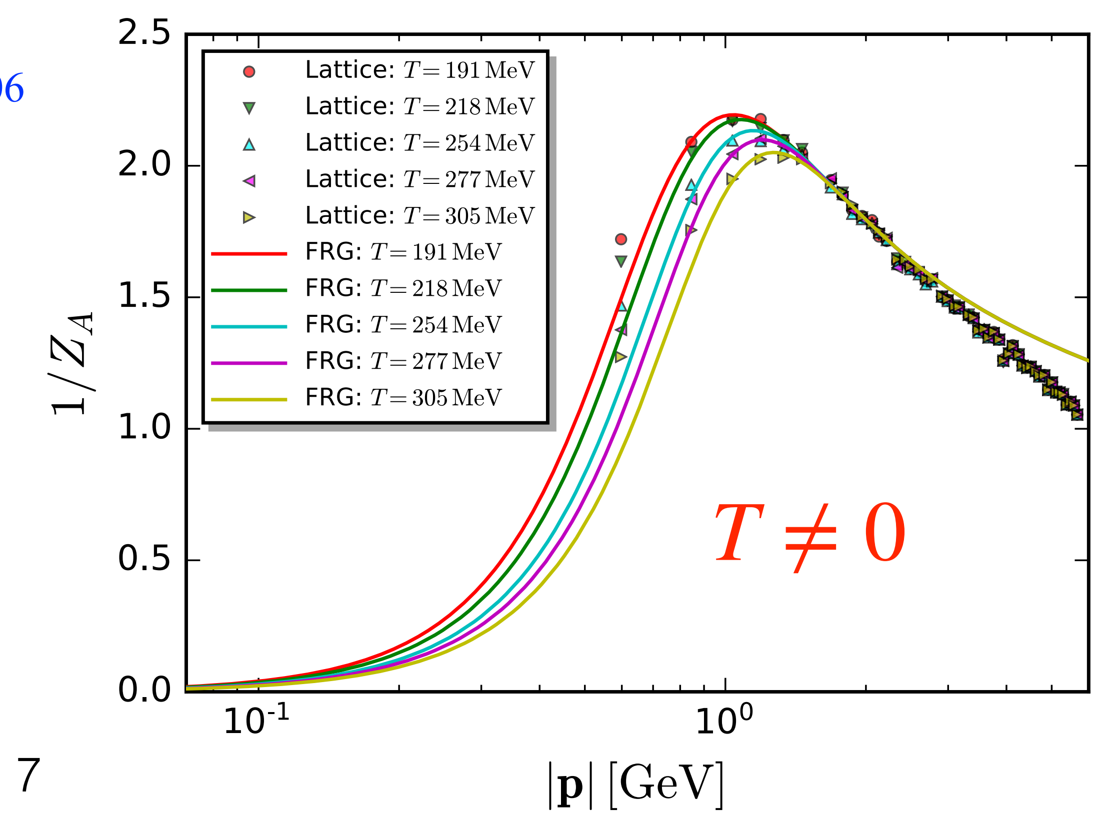
Gluon dressing functions



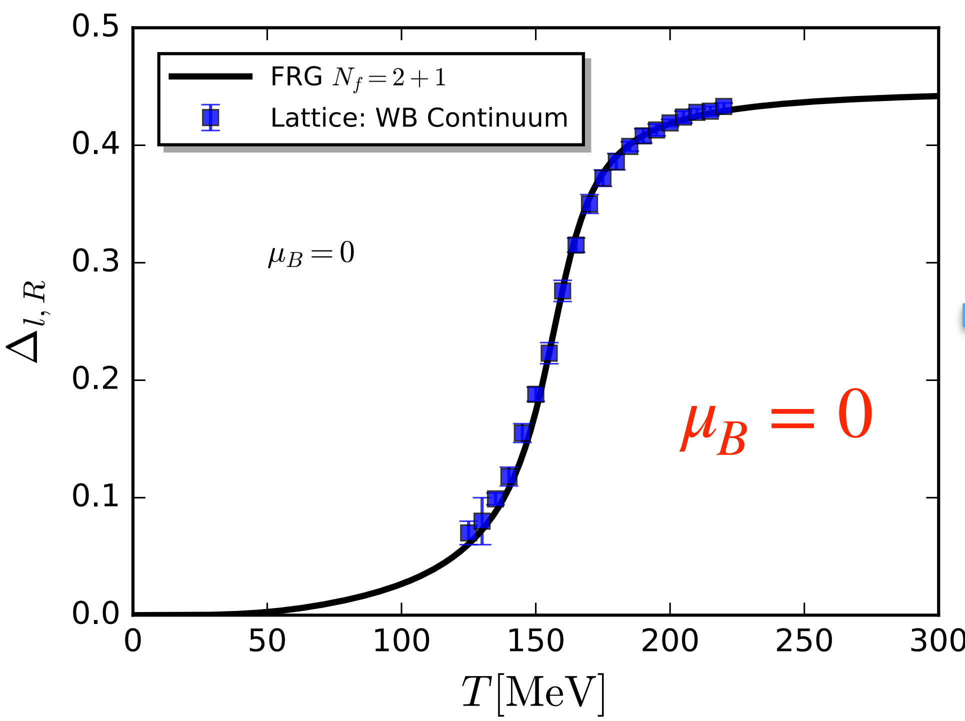


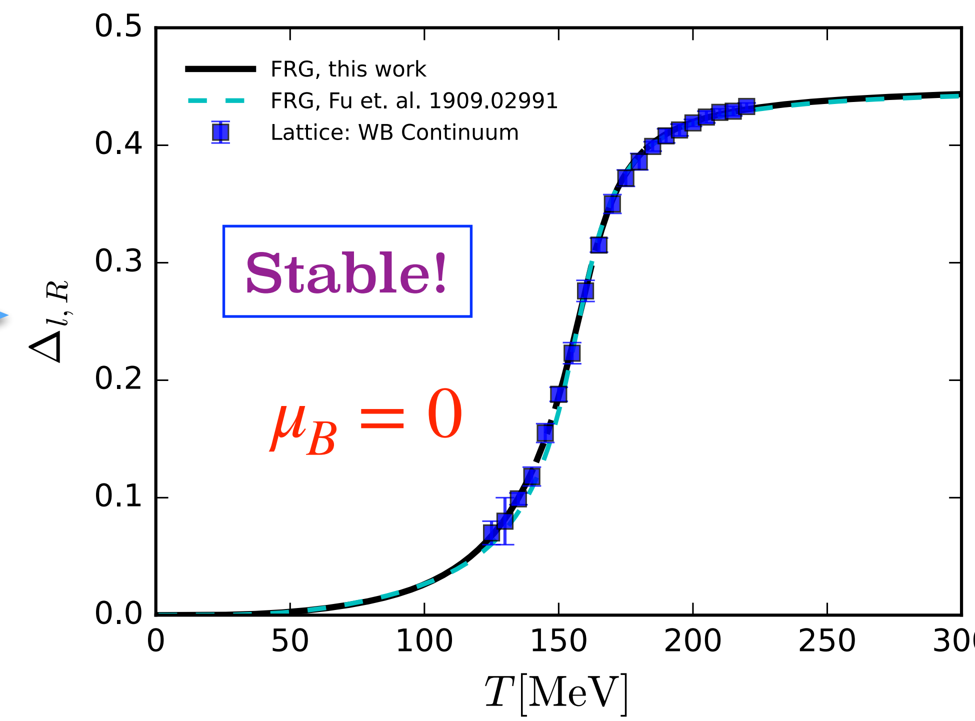
fRG $N_f = 2$: Cyrol, Mitter, Pawlowski,Strodthoff, *PRD* 97 (2018) 054006
Lattice $N_f = 2$: Sternbeck *et al.*, *PoS* (2012) LATTICE2012, 243
Lattice $N_f = 2+1$: Boucaud *et al.*, *PRD* 98 (2018) 114515

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032



Renormalized light quark condensate



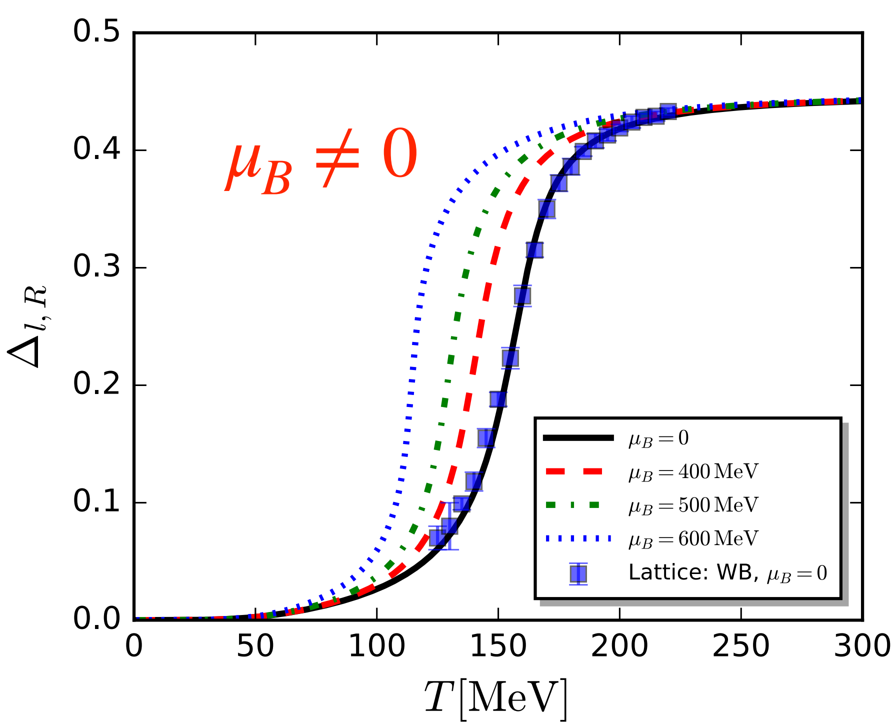


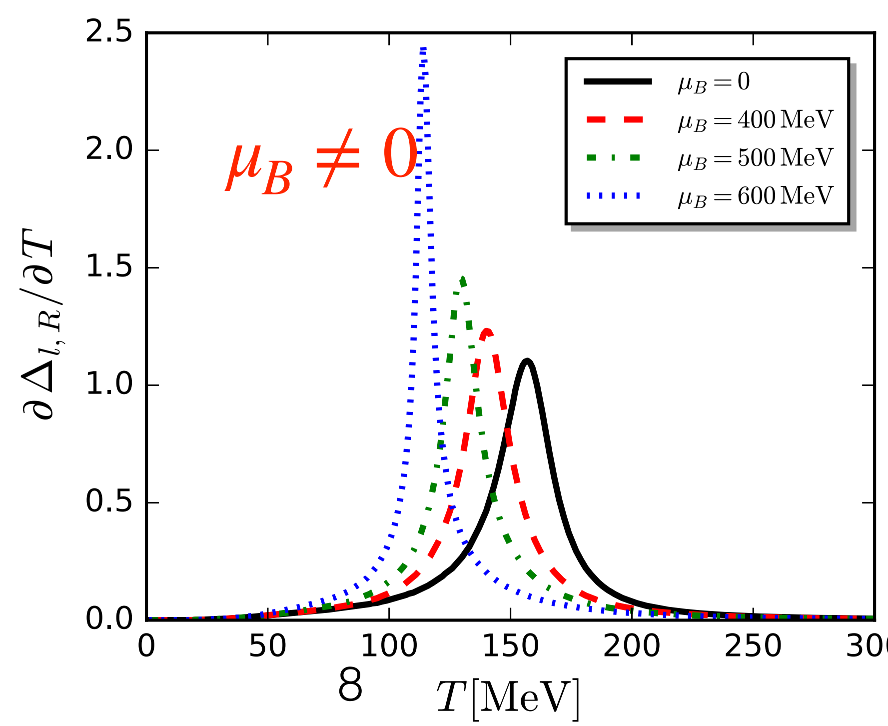
improved truncations for the sector of s quark and the full mesonic potential of $N_f = 2+1$.

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

fRG: WF, Pawlowski, Rennecke, Wen, Yin, (2023) in preparation



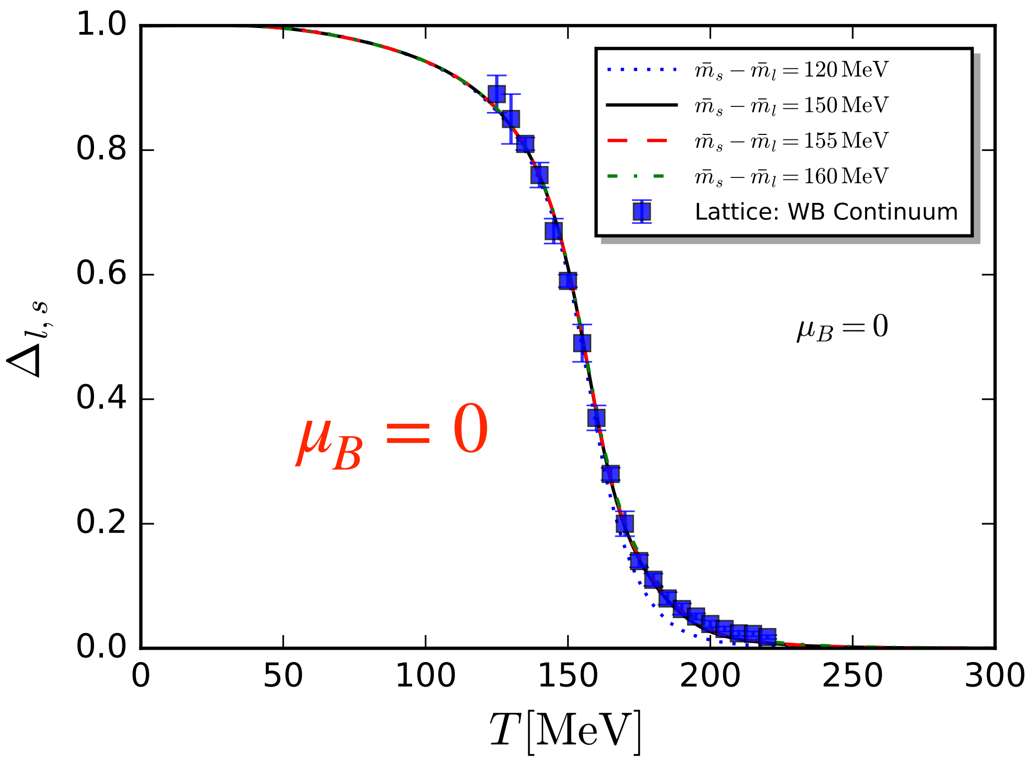


quark condensate:

$$\Delta_{q_i} \simeq -m_{q_i}^0 T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,,$$

$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R}\Big[\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)\Big]\,.$$

Other fermionic observables



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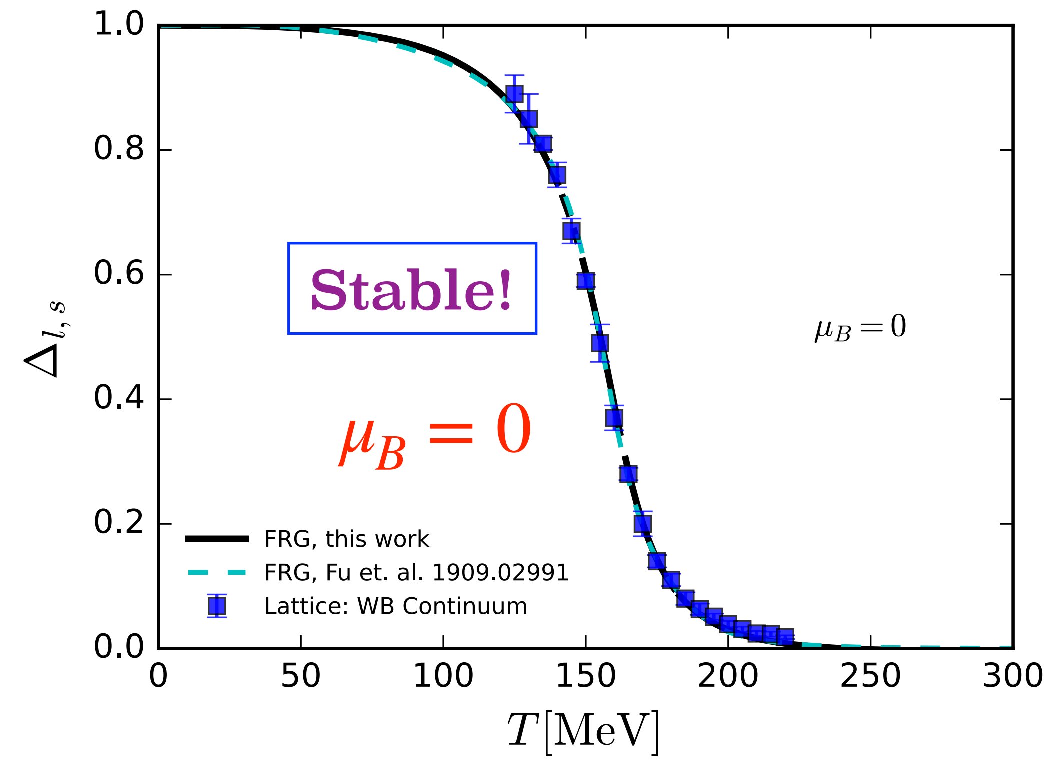


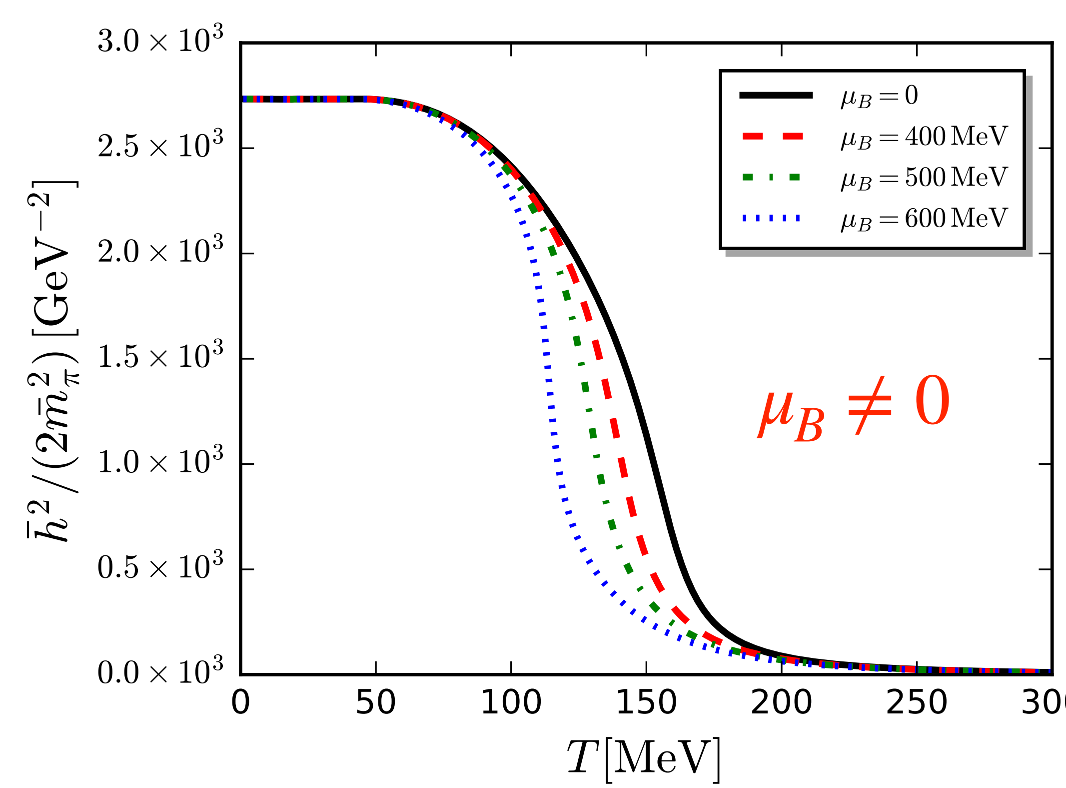
fRG: WF, Pawlowski, Rennecke, Wen, Yin, (2023) in preparation

improved truncations for the sector of s quark and the full mesonic potential of $N_f = 2+1$.

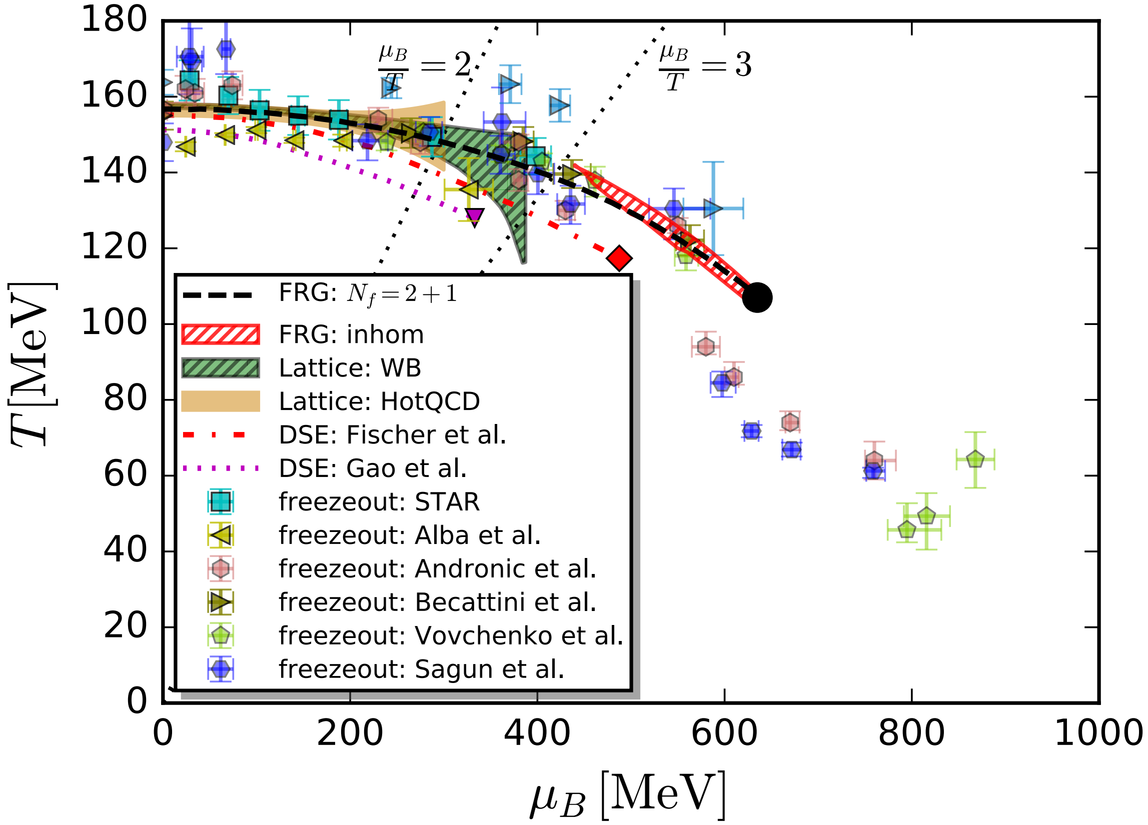
Reduced condensate:

$$\Delta_{l,s}(T,\mu_q) = \frac{\Delta_l(T,\mu_q) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_q)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

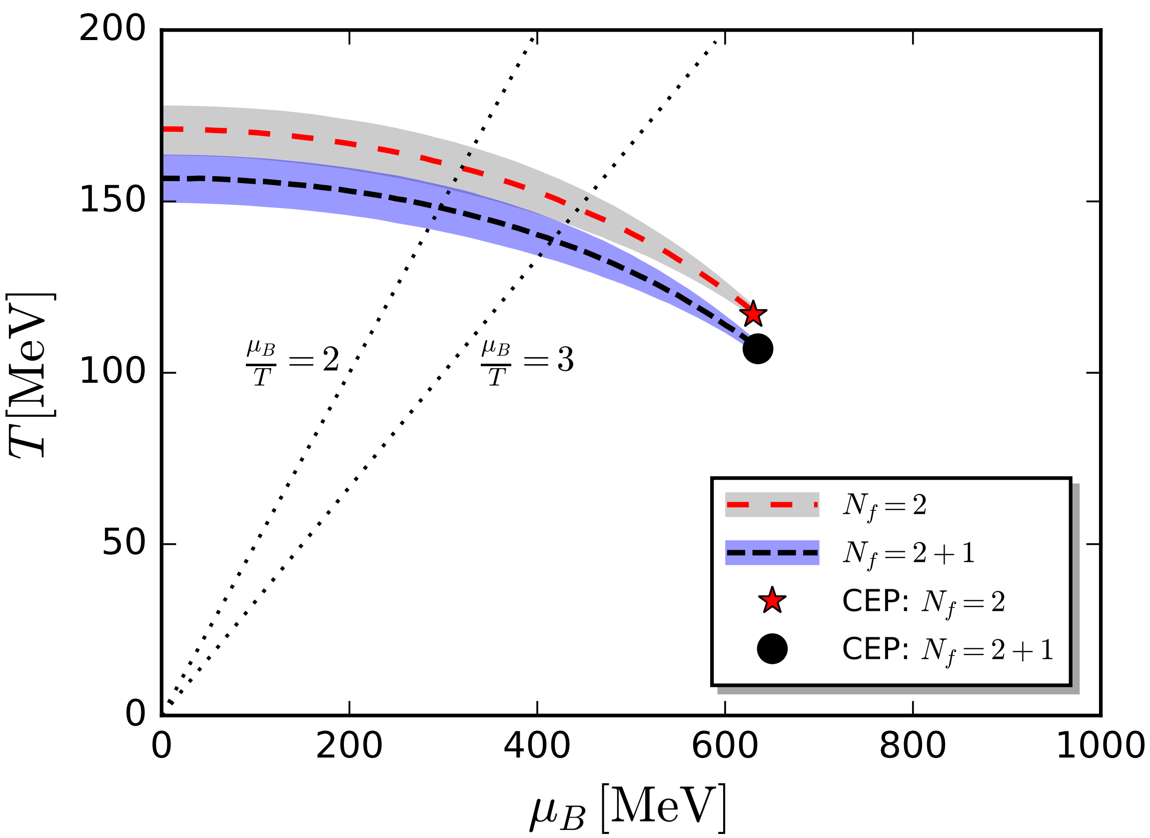
Effective four-quark coupling:



Phase boundary and curvature



fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032



CEP:

$$(T_{\rm CEP}, {\mu_B}_{\rm CEP})_{N_f=2+1} = (107\,\text{MeV}, 635\,\text{MeV})\,,$$

$$(T_{\rm CEP}, {\mu_B}_{\rm CEP})_{N_f=2} = (117\,\text{MeV}, 630\,\text{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

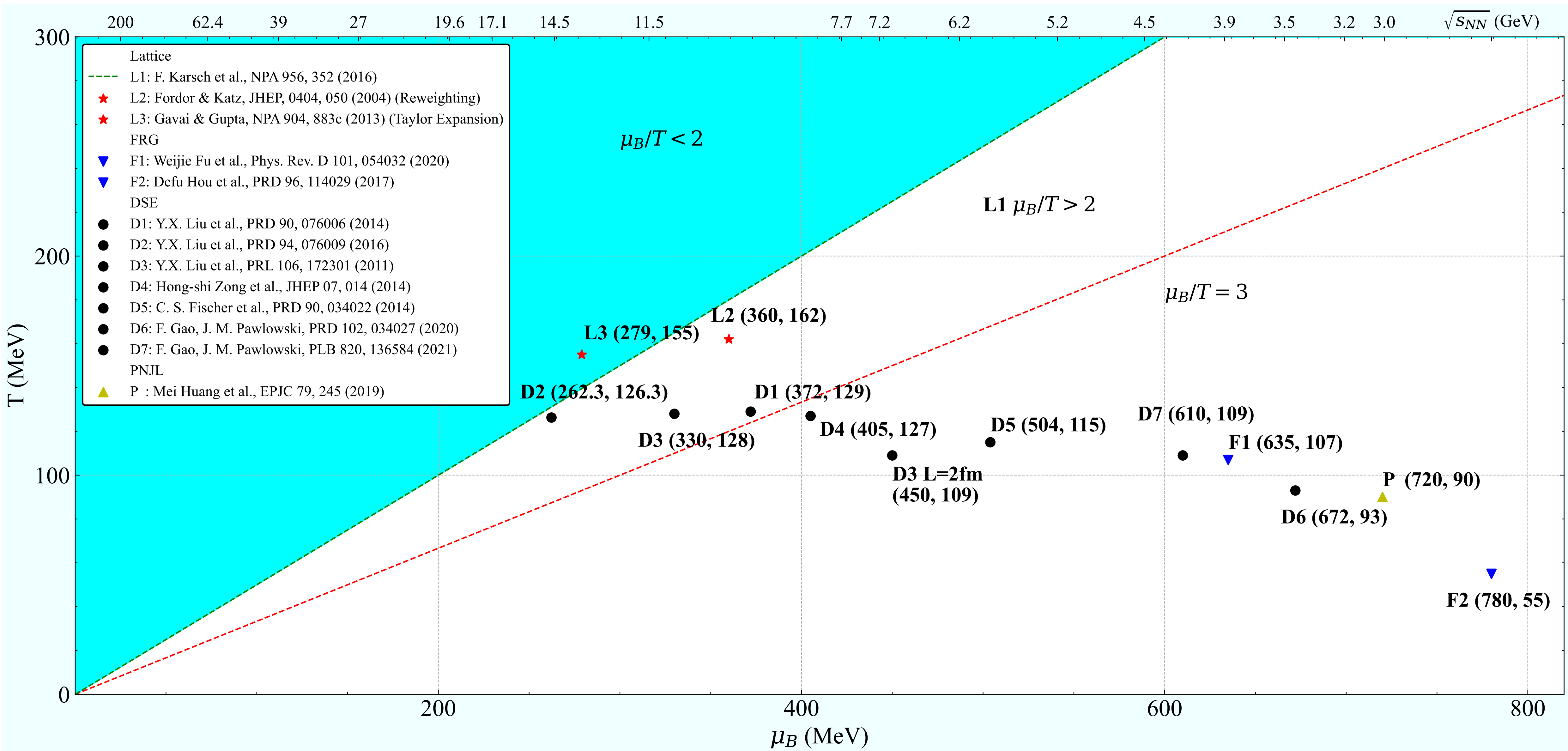
$$\kappa = 0.0149 \pm 0.0021$$

Lattice: Bellwied *et al.* (WB), *PLB* 751 (2015) 559

$$\kappa = 0.015 \pm 0.004$$

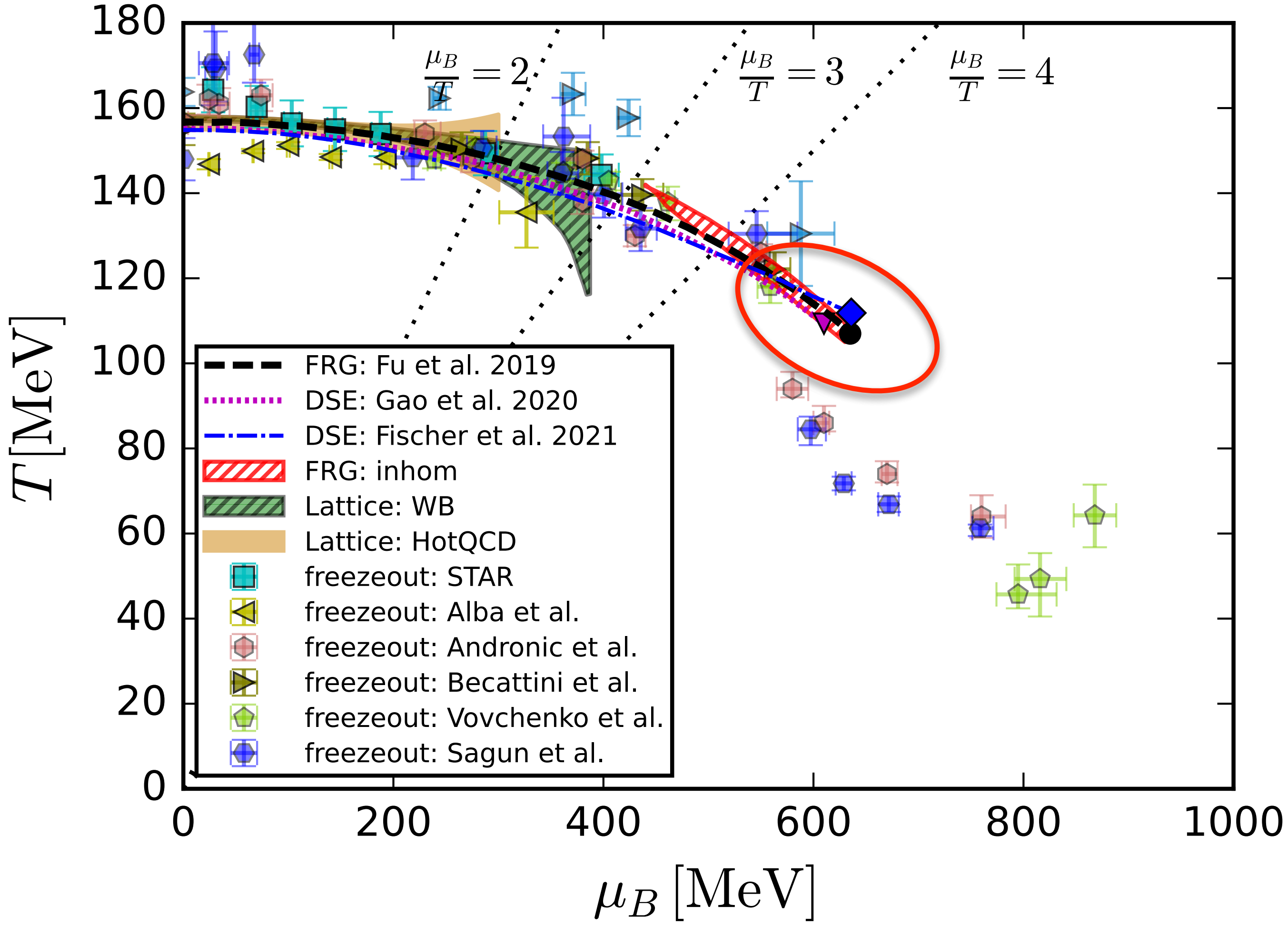
Lattice: Bazavov *et al.* (HotQCD), *PLB* 795 (2019) 15

CEP from different theoretical calculations



By courtesy of Xiaofeng Luo

CEP from first-principles functional QCD



Estimates of the location of CEP from first-principles functional QCD:

fRG:

● $(T, \mu_B)_{\mathrm{CEP}} = (107, 635)\mathbf{MeV}$

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

▼ $(T, \mu_B)_{\mathrm{CEP}} = (109, 610)\mathbf{MeV}$

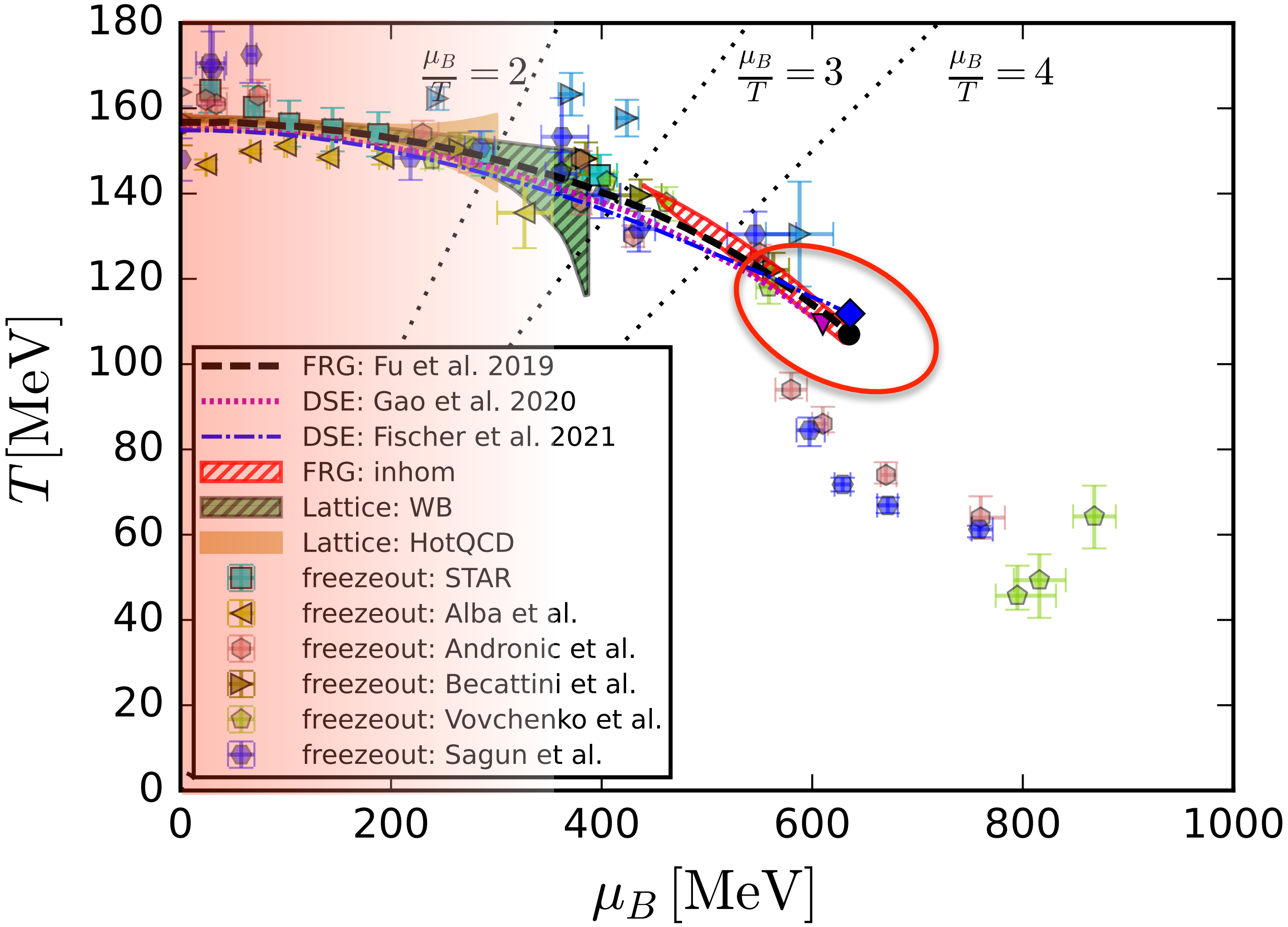
DSE (fRG): Gao, Pawlowski, *PLB* 820 (2021) 136584

◆ $(T, \mu_B)_{\mathrm{CEP}} = (112, 636)\mathbf{MeV}$

DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS* CORFU2018 (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: $600\ \mathrm{MeV} \lesssim \mu_{B_{\mathrm{CEP}}} \lesssim 650\ \mathrm{MeV}$.

CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Estimates of the location of CEP from first-principles functional QCD:

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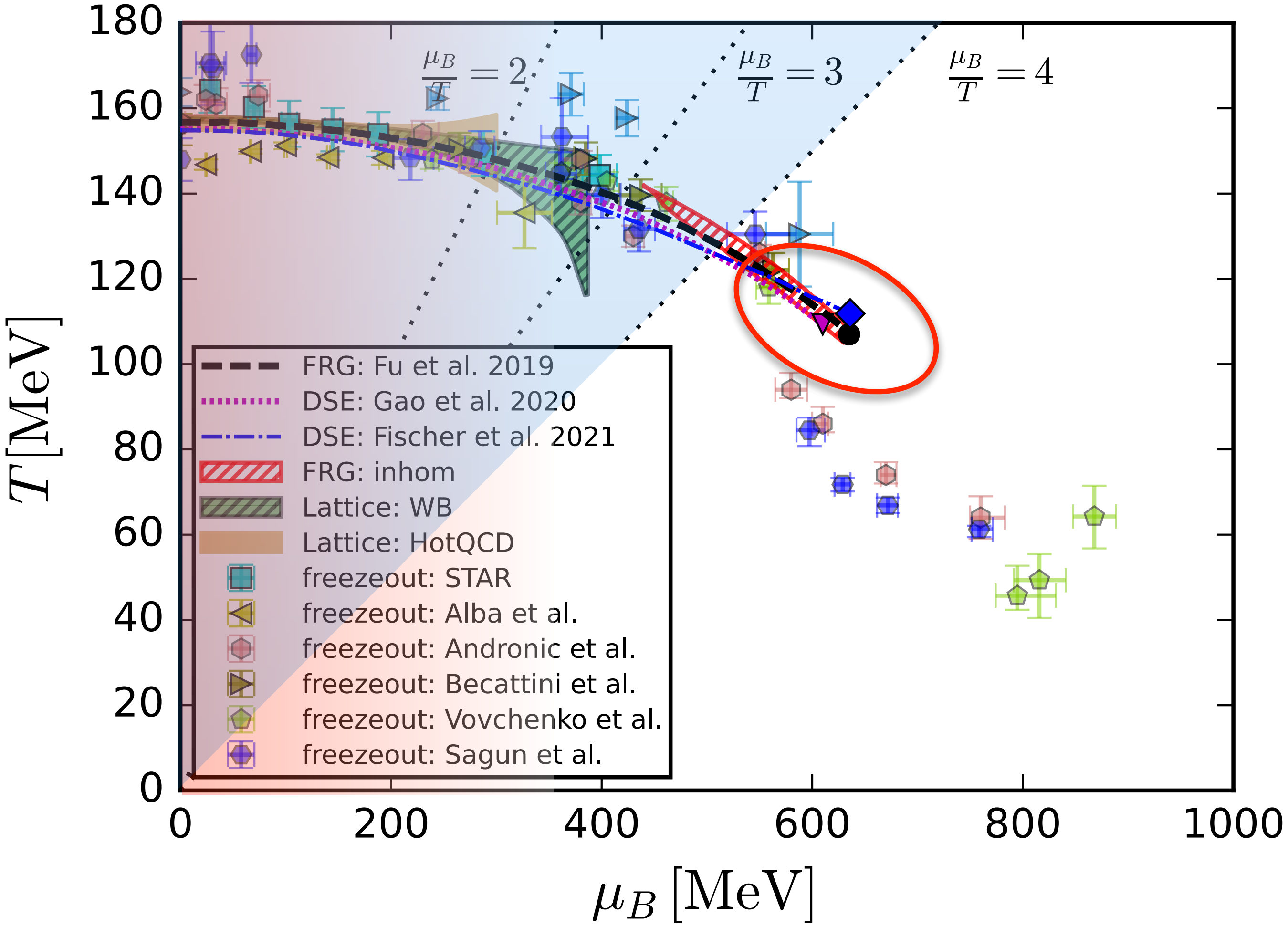
CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

Estimates of the location of CEP from first-principles functional QCD:

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$$\bullet \quad (T, \mu_B)_{\mathrm{CEP}} = (107, 635)\,\mathbf{MeV}$$

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

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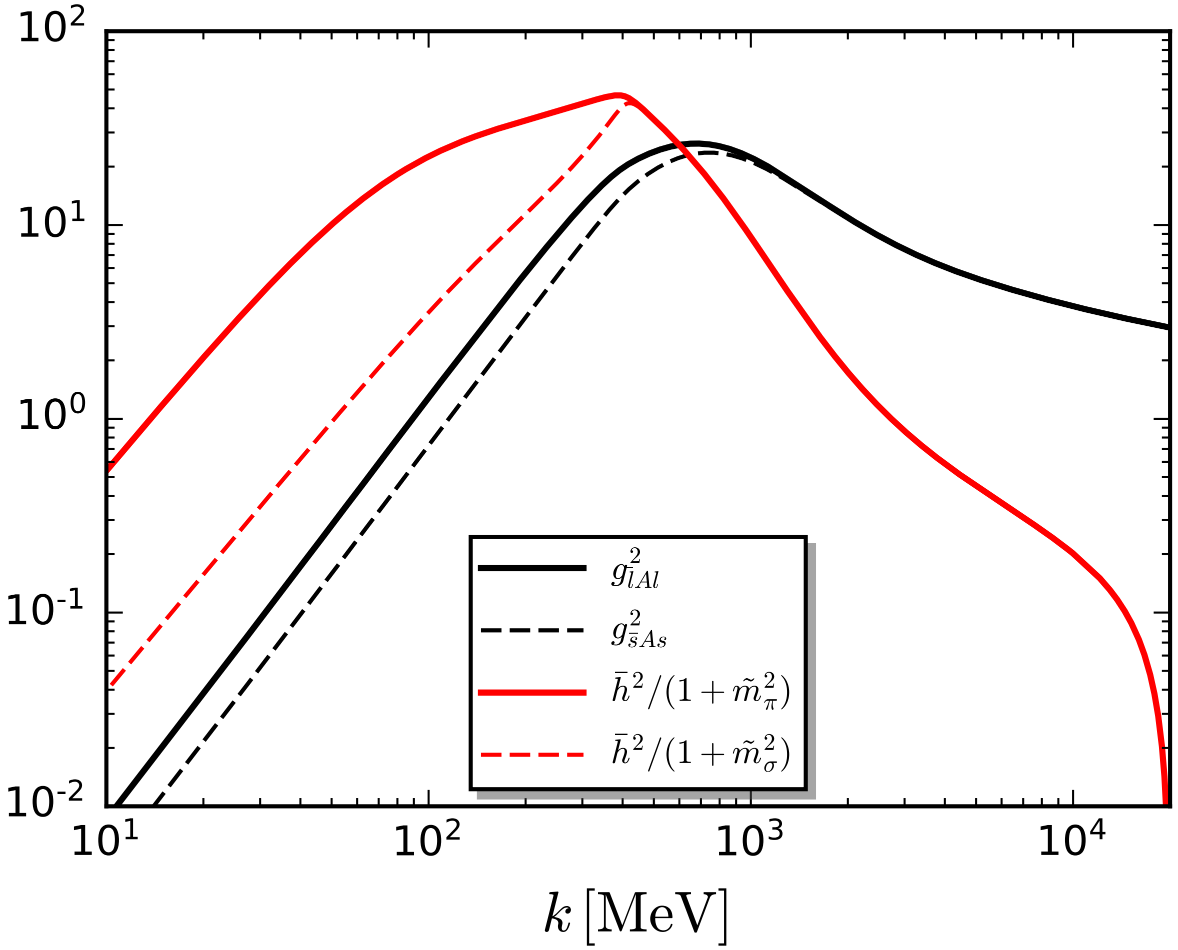
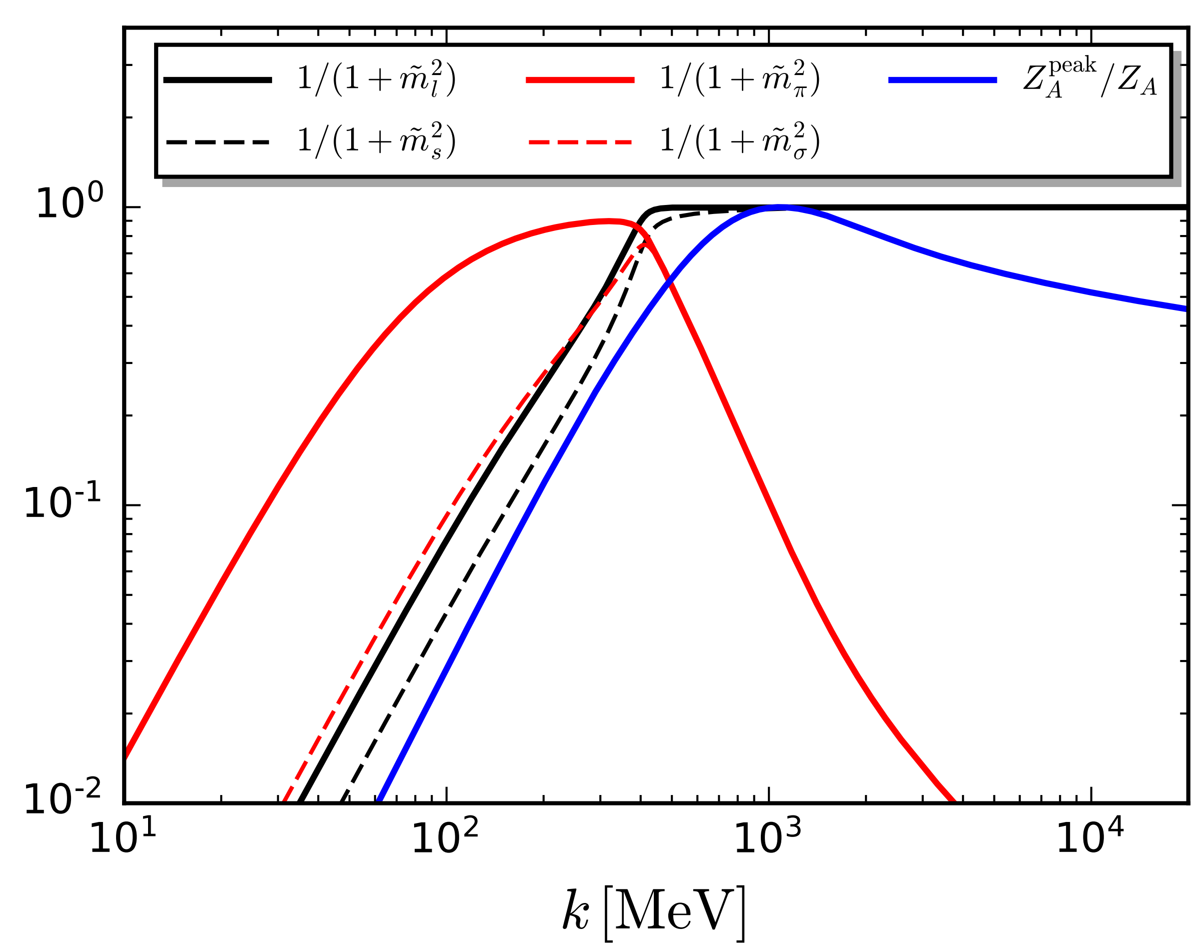
Natural emergence of LEFTs from QCD

- **Exchange couplings**
- **Propagator gapping**



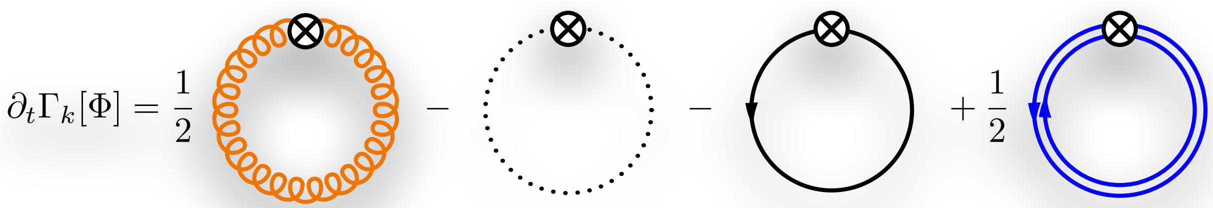


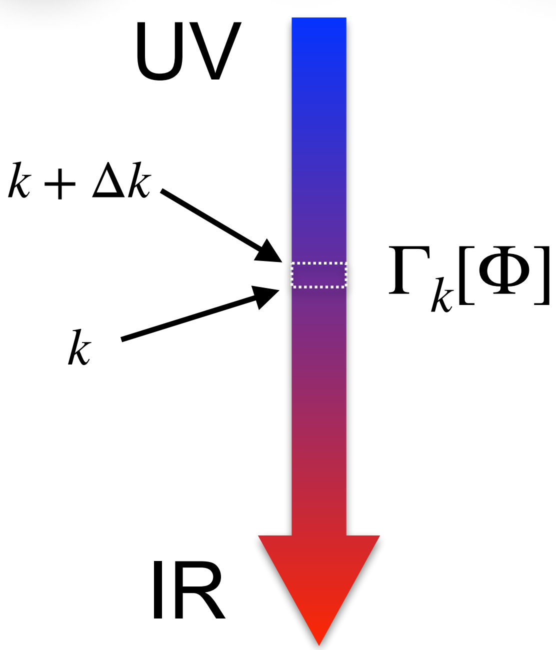
- Composite (mesonic) degrees of freedom take over active dynamics from partonic ones when the RG scale is lowered down $k \lesssim 600 \sim 800$ MeV.
- LEFTs emerge naturally from fundamental theory in the regime of low energy, in agreement with the viewpoint of RG.

WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

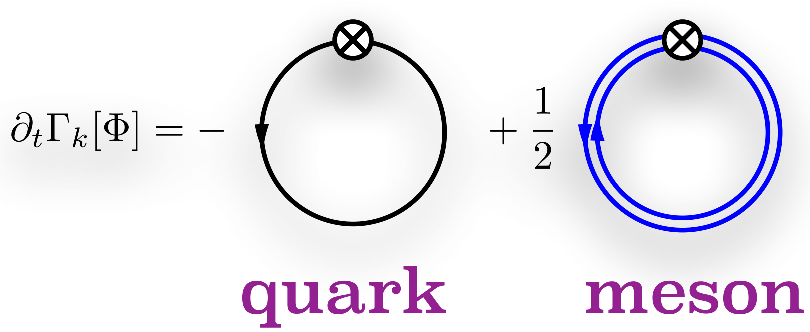
QCD-assisted LEFT

QCD flow equation:

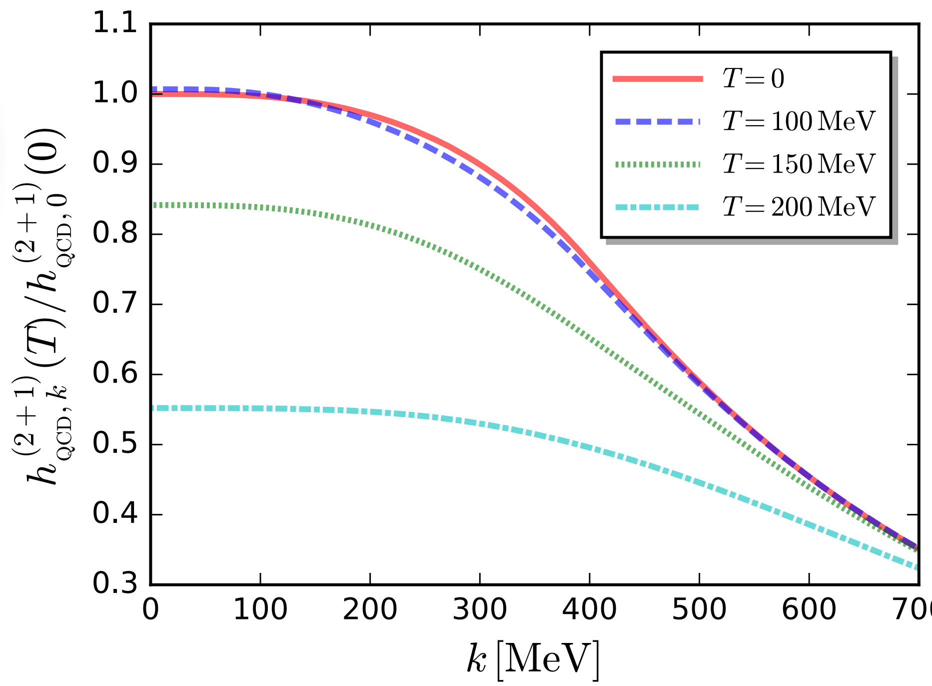




LEFT flow equation:

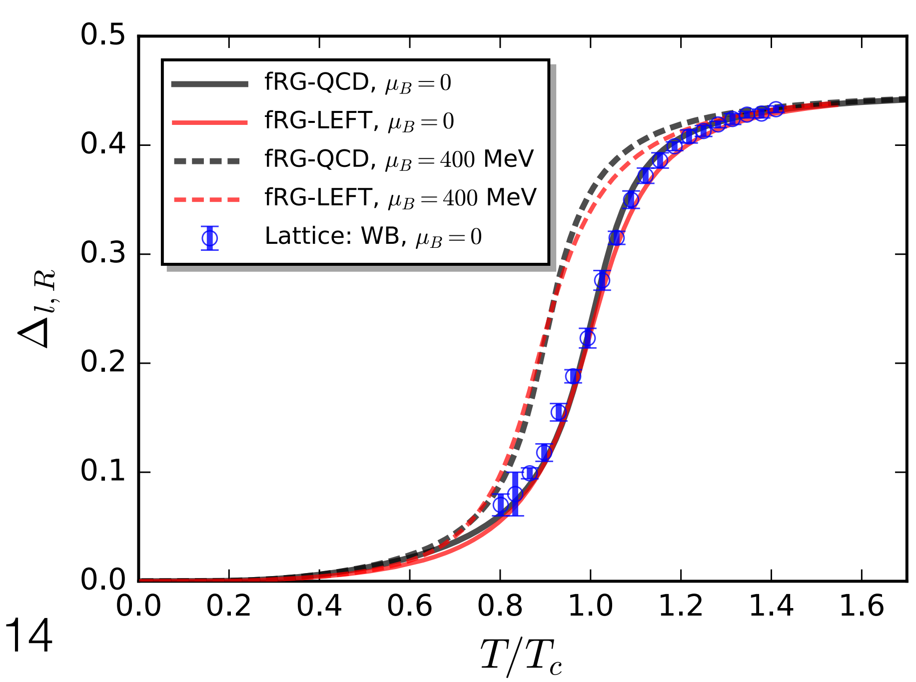


- Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT



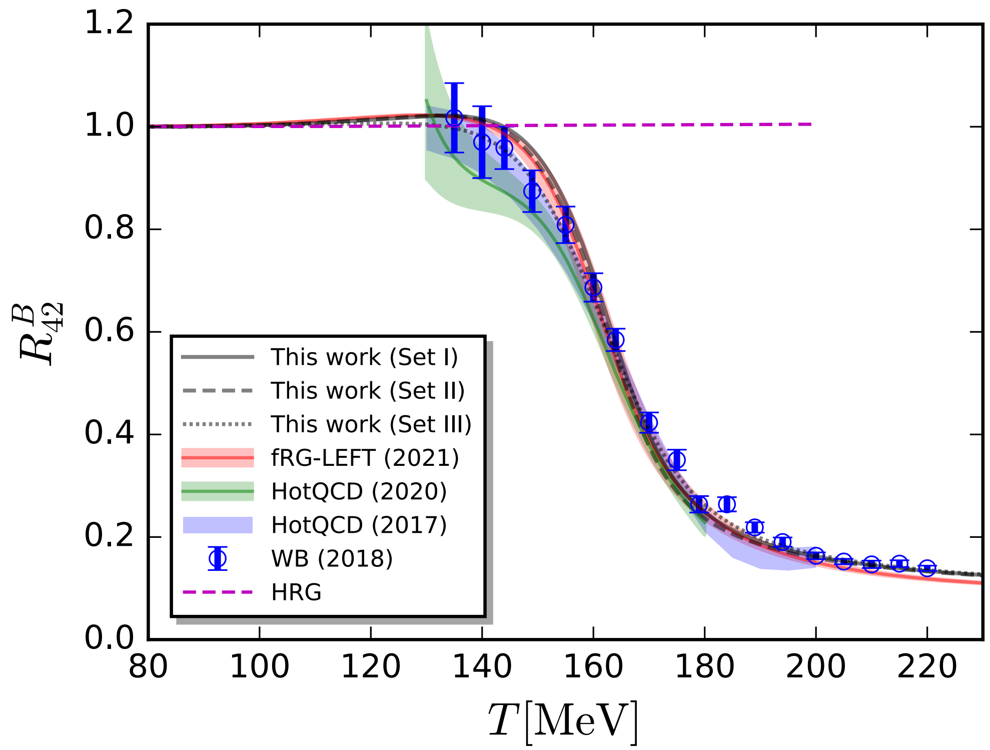
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

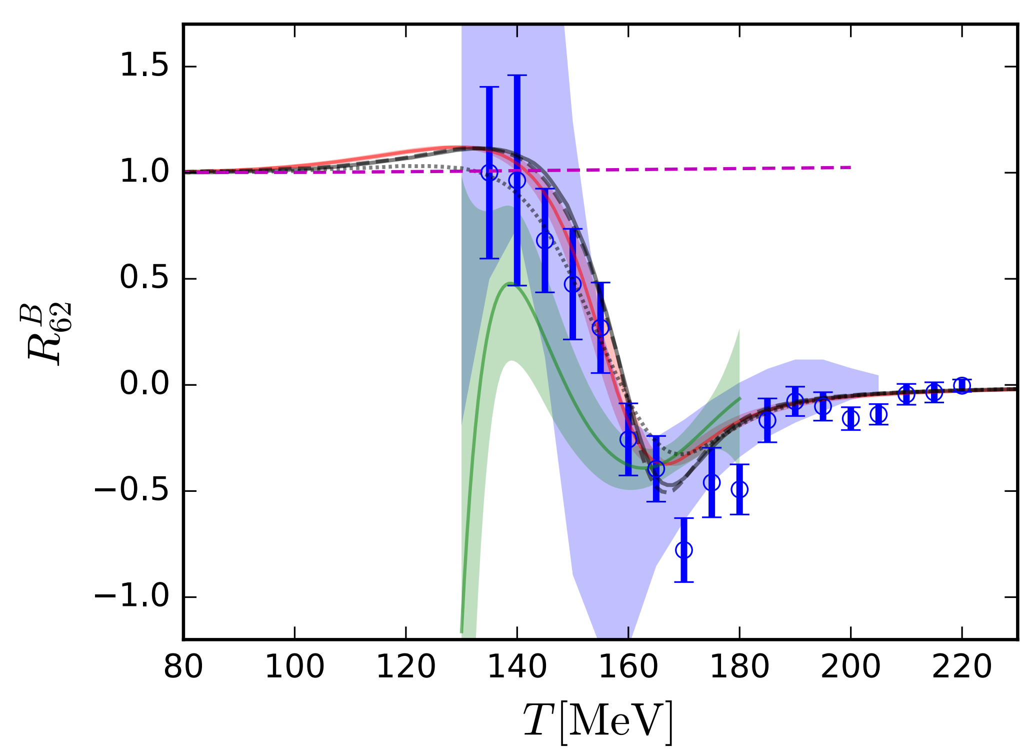
- Chiral condensates in QCD and QCD-assisted LEFT in agreement



WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Baryon number fluctuations





$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508;
WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

HotQCD: A. Bazavov *et al.*, arXiv: *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: *JHEP* 10 (2018) 205

baryon number fluctuations

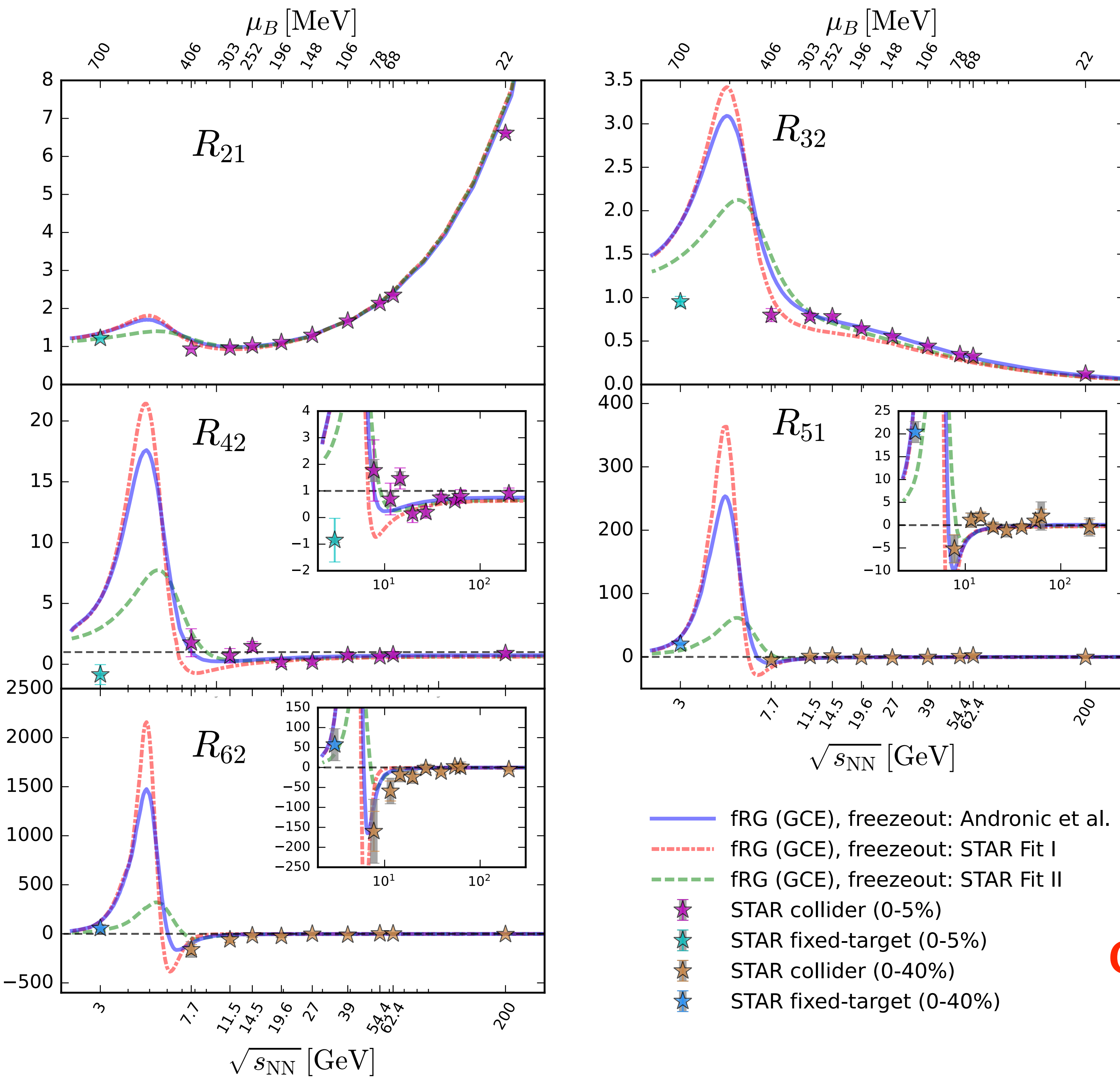
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,, \frac{\sigma^2}{VT^3} = \chi_2^B\,, S = \frac{\chi_3^B}{\chi_2^B\sigma}\,, \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

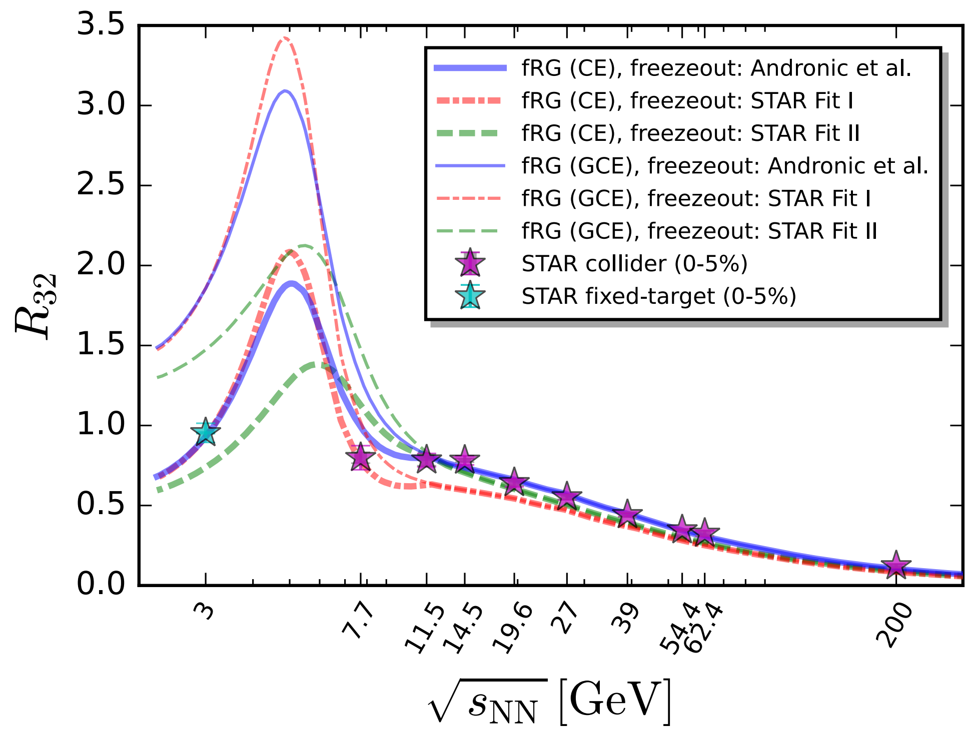
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (98{,}643)$ MeV.**
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Agreement between the theory and experiment is worsening with $\sqrt{s_{\rm NN}} \lesssim$ 11.5 GeV.**
- **Effects of global baryon number conservation in the regime of low collision energy should be taken into account.**

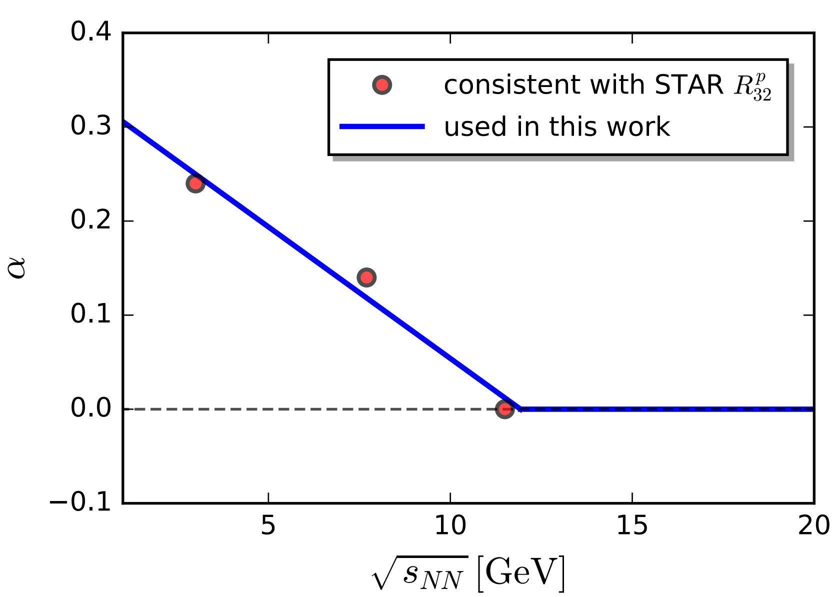
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- **Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{\rm NN}} \lesssim$ 11.5 GeV.**
- **We choose the simplest linear dependence**



$$\alpha(\bar{s}) = a\left(1 - \sqrt{\bar{s}}\right)\theta\left(1 - \bar{s}\right)$$

$$a = 0.33\,, \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,\text{GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1: the subensemble volume measured in the acceptance window, V: the volume of the whole system.

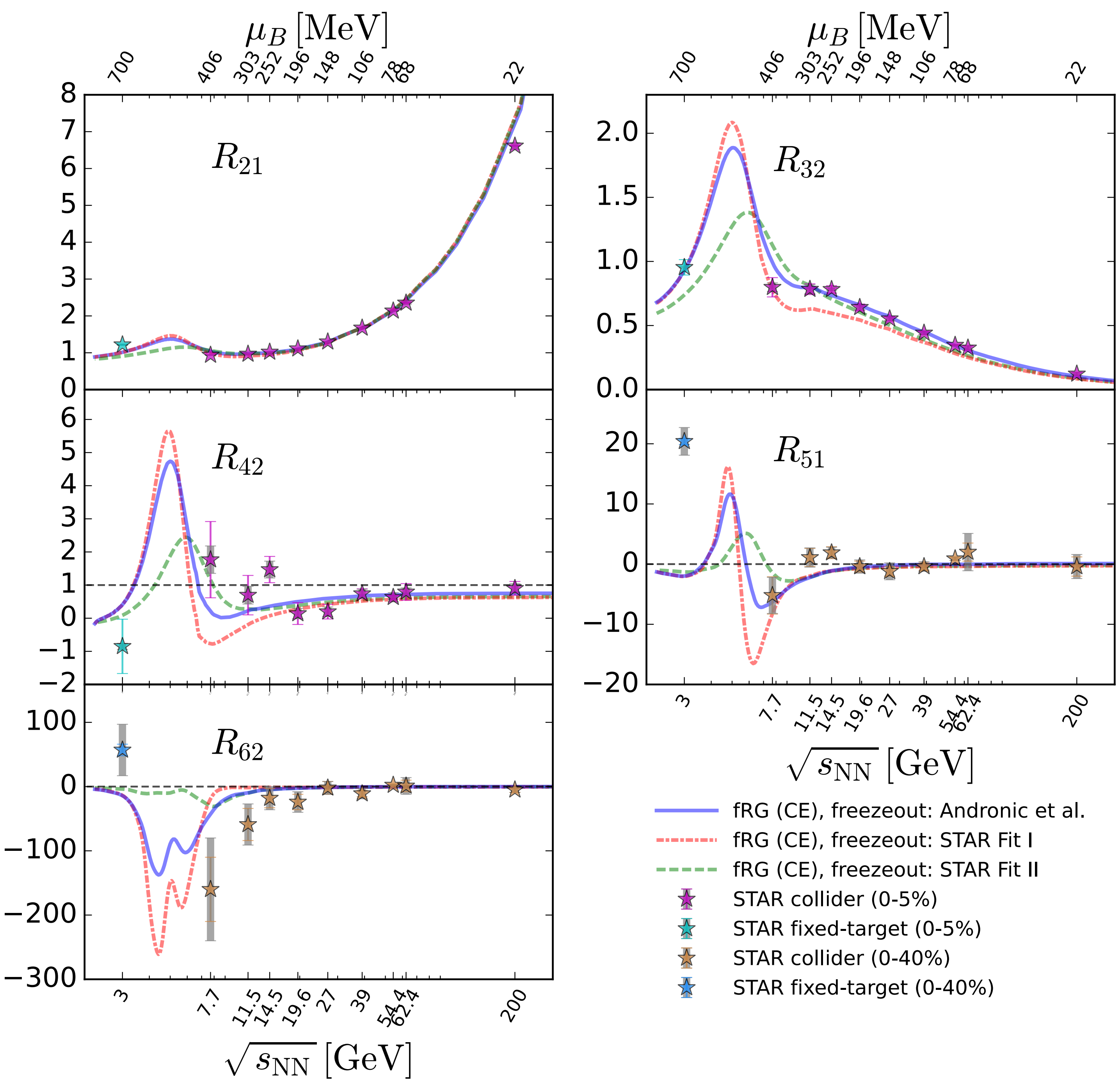
- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

$$\bar{R}^B_{21} = \beta R^B_{21}, \qquad \bar{R}^B_{32} = (1-2\alpha)R^B_{32},$$

$$\bar{R}^B_{42} = (1-3\alpha\beta)R^B_{42} - 3\alpha\beta(R^B_{32})^2$$

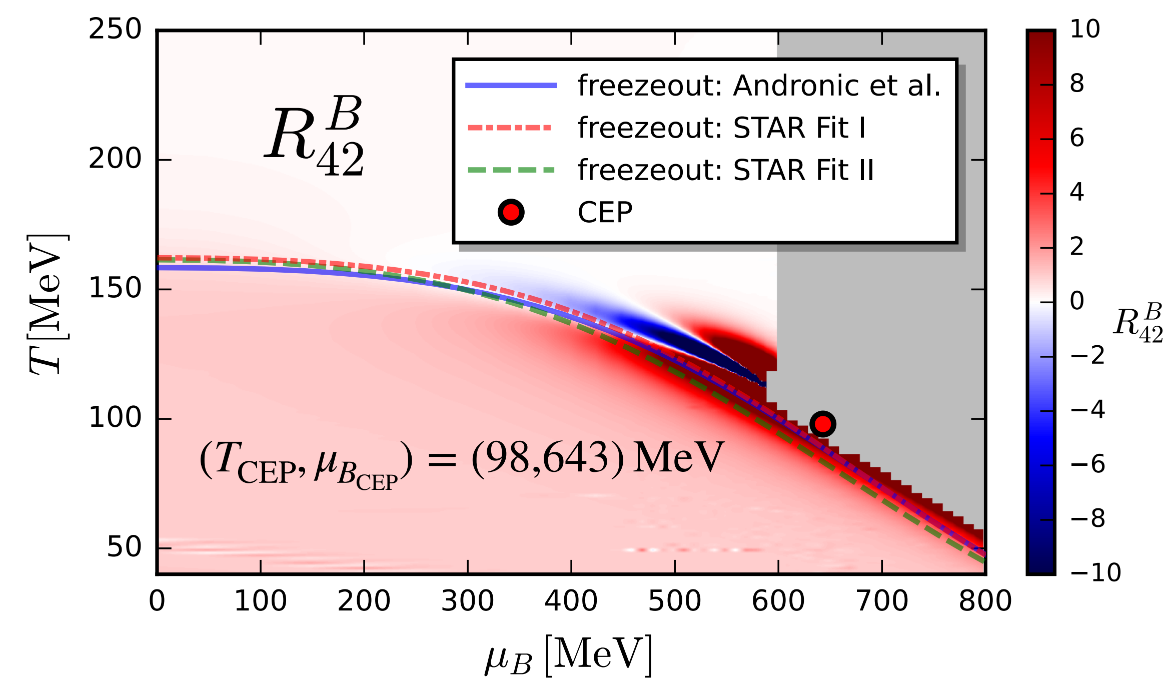
SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , *PLB* 811 (2020) 135868

Canonical fluctuations at the freeze-out



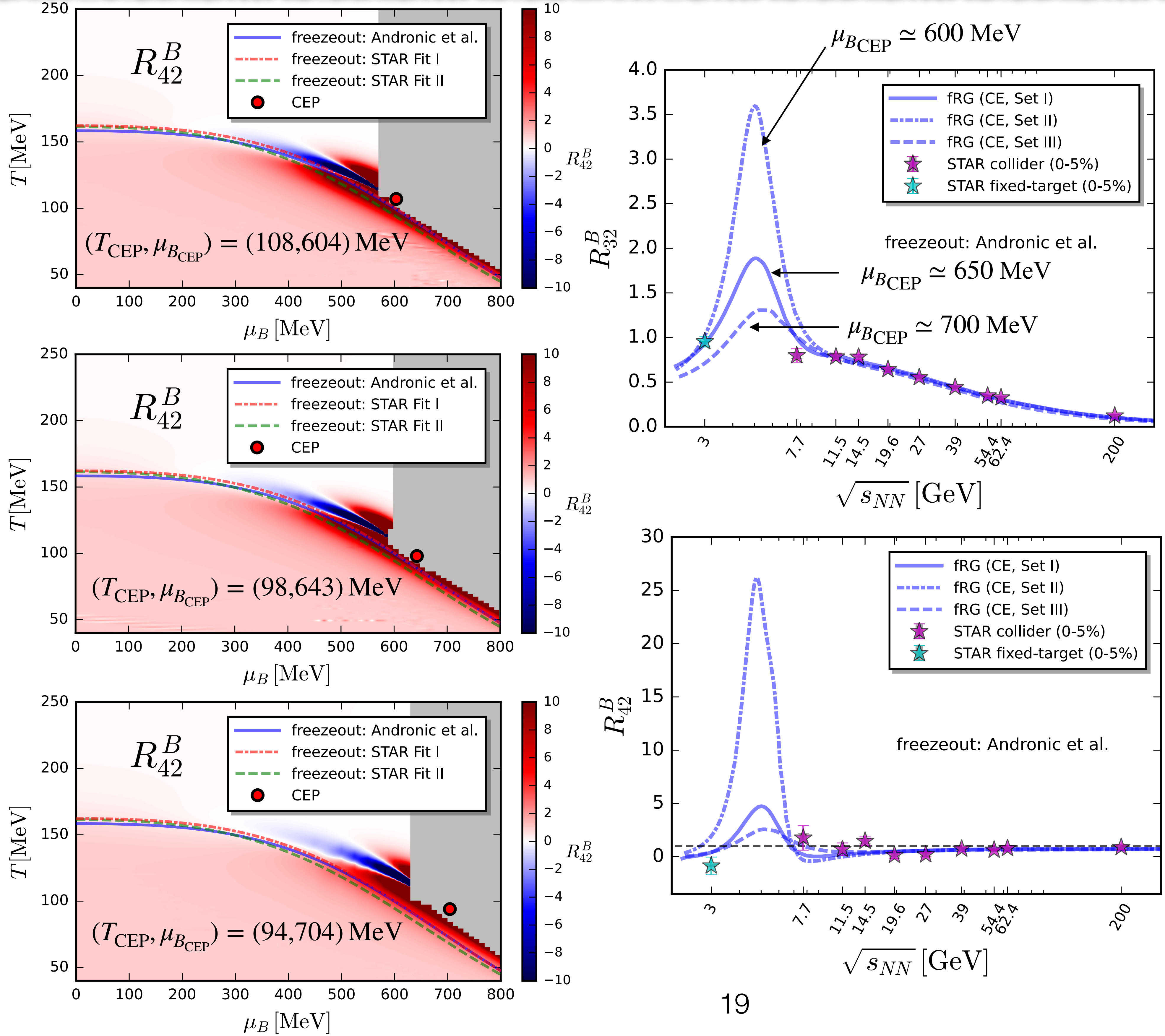
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301;
Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303;
Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508



- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{\rm peak}}$ = 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\rm CEP}}$ = 643 MeV.**

Dependence on the location of the CEP



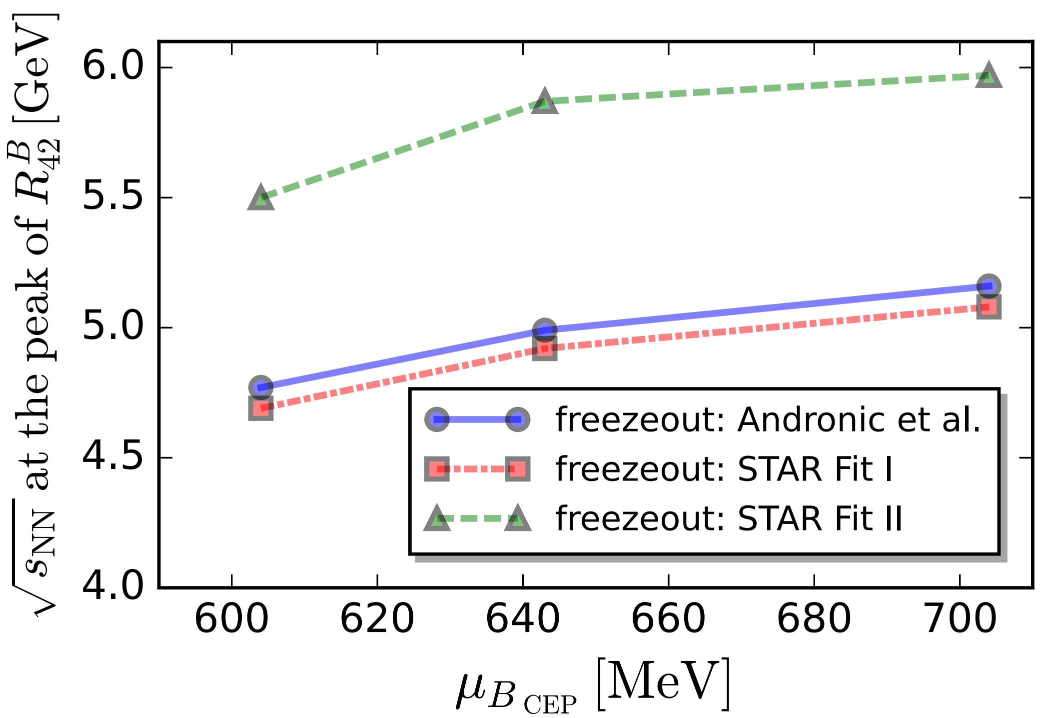
STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

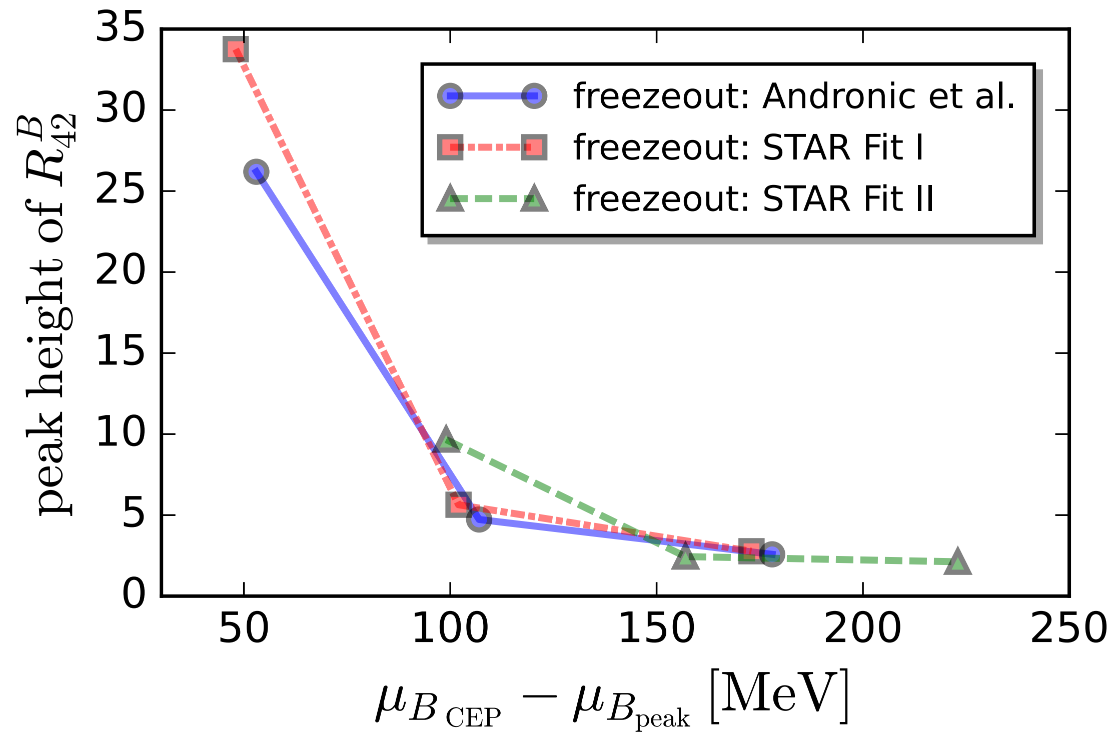
- **Position of the peak is insensitive to the location of CEP.**
- **Height of peak decreases as CEP moves towards larger μ_B.**

Ripples of the QCD critical point

Postion of peak:



Height of peak:



fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical exponents.
- But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.

Magnetic equation of state

- The magnetic equation of state (EoS) is obtained via the chiral condensate:

$$\Delta_q = m_q \frac{\partial \Omega\big(T; m_q(T)\big)}{\partial m_q} = m_q \frac{T}{V} \int_x \big\langle \bar{q}(x)\, q(x) \big\rangle$$

- The chiral properties of the magnetic EoS are encoded in the magnetic susceptibility:

$$\chi_M = -\frac{\partial \bar{\Delta}_l}{\partial m_l}\,, \qquad \text{with} \qquad \bar{\Delta}_l = \frac{\Delta_l}{m_l}$$

- In the critical region, the magnetic EoS can be expressed as a universal scaling function $f_G(z)$ through

$$\bar{\Delta}_l = m_l^{1/\delta} f_G(z)$$

with

$$z = t\, m_l^{-1/\beta\delta}\,, \qquad \text{and} \qquad t = (T - T_c)/T_c$$

z is the scaling variable and t is the reduced temperature.

- The pseudo-critical temperature T_{pc}, which is defined through the peak location of χ_M, is readily obtained from the scaling function as

$$T_{pc}(m_\pi) \approx T_c + c\, m_\pi^p\,, \qquad \text{with} \qquad p = 2/(\beta\delta)$$

Critical exponent in fRG for 3d-O(4):

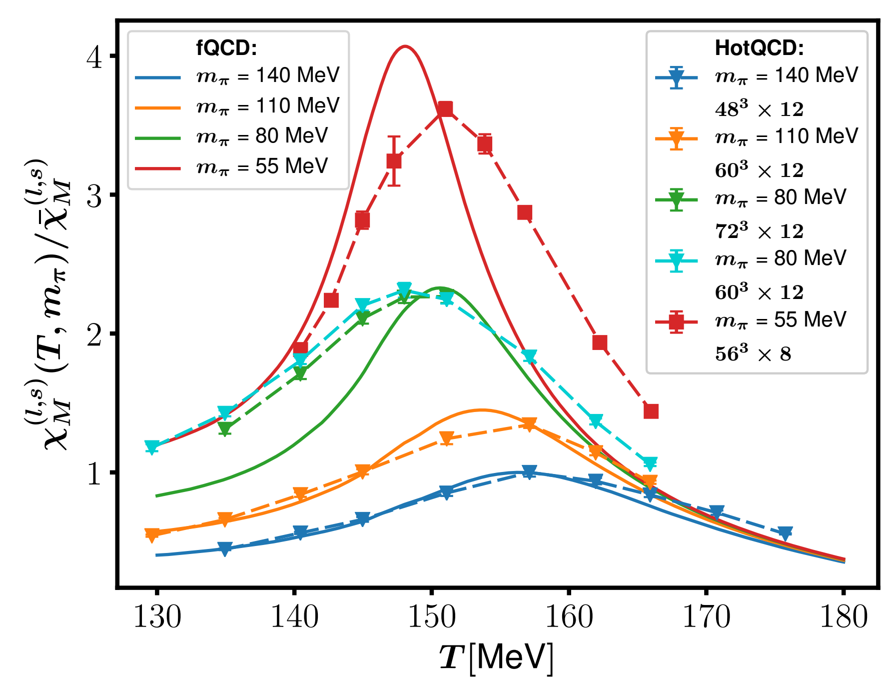
$$\beta = 0.405\,, \quad \delta = 4.784\,, \quad \theta_H = 0.272\,,$$

obtained from the fixed-point equation for the Wilson-Fisher fixed point, which leads us $p_{\rm fRG} = 1.03$

Critical exponent in mean field:

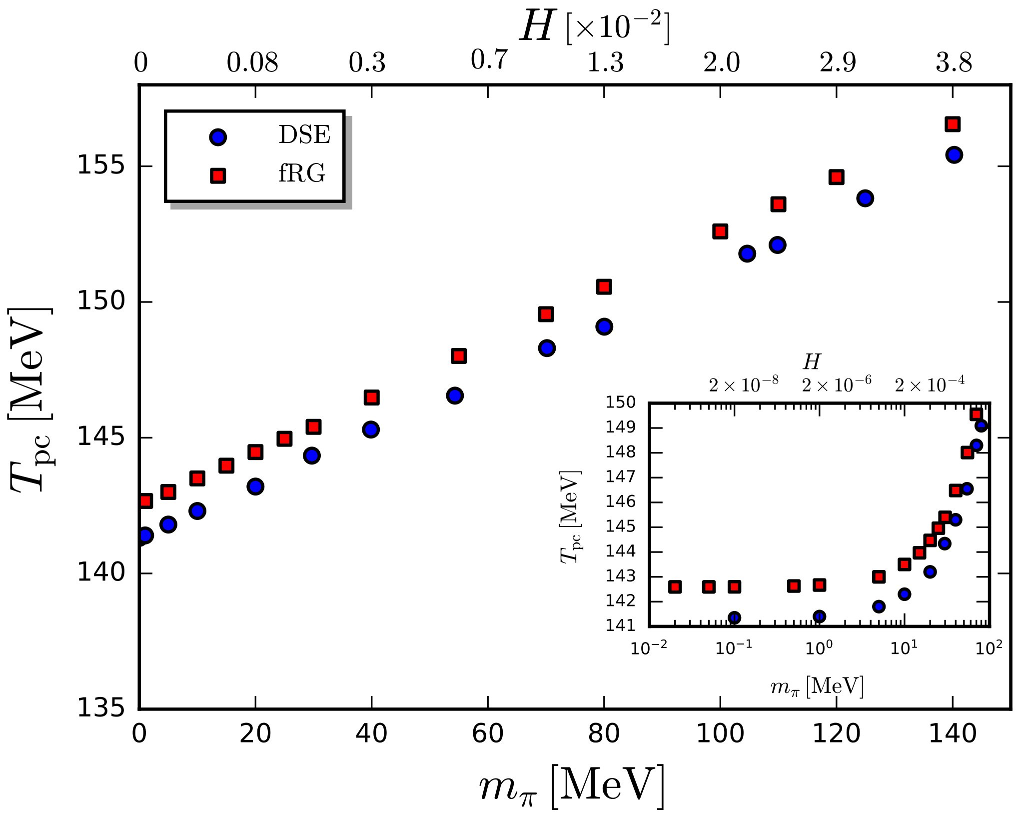
$$\beta_{\rm MF} = 1/2\,, \quad \delta_{\rm MF} = 3\,,$$

thus, one has $p_{\rm MF} = 4/3$



Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

Magnetic equation of state



$$T_{\rm pc}(m_\pi) \approx T_c + c\, m_\pi^p$$

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Lattice (HotQCD):

$$T_c^{\rm lattice} = 132^{+3}_{-6}\,\text{MeV},$$

Ding *et al.*, *PRL* 123 (2019) 062002.

fRG:

$$T_c^{\rm fRG} \approx 142\,\text{MeV}\,, \qquad p_{\rm fRG} = 1.024$$

Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020) 056010.

DSE:

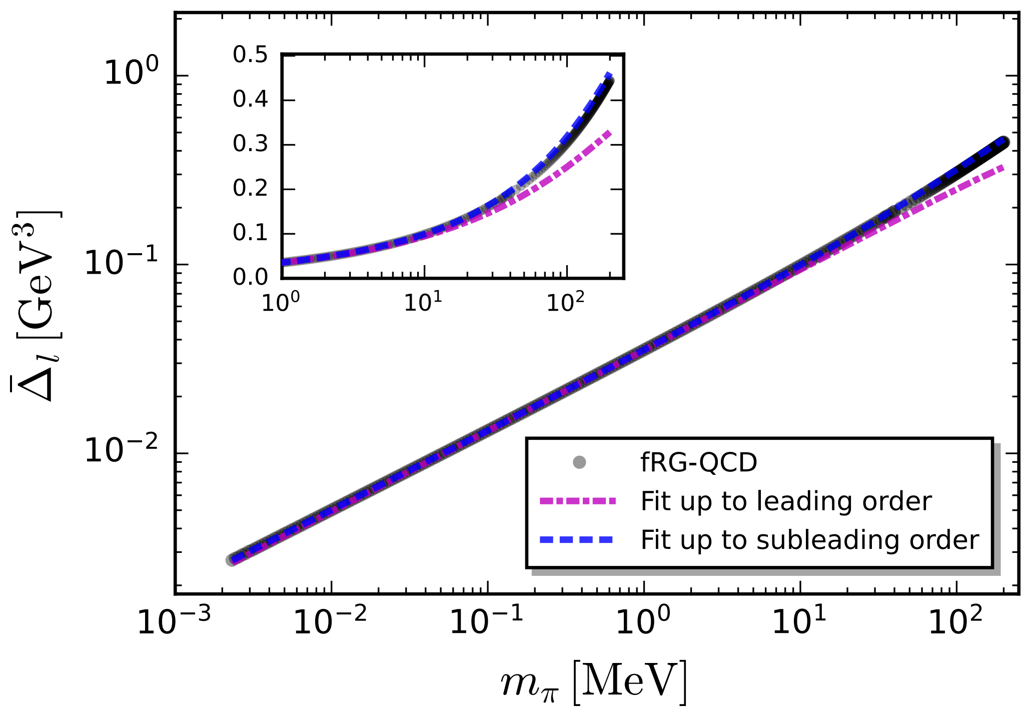
$$T_c^{\rm DSE} \approx 141\,\text{MeV}\,, \qquad p_{\rm DSE} = 0.9606$$

Gao, Pawlowski, *PRD* 105 (2022) 9, 094020, arXiv: 2112.01395.

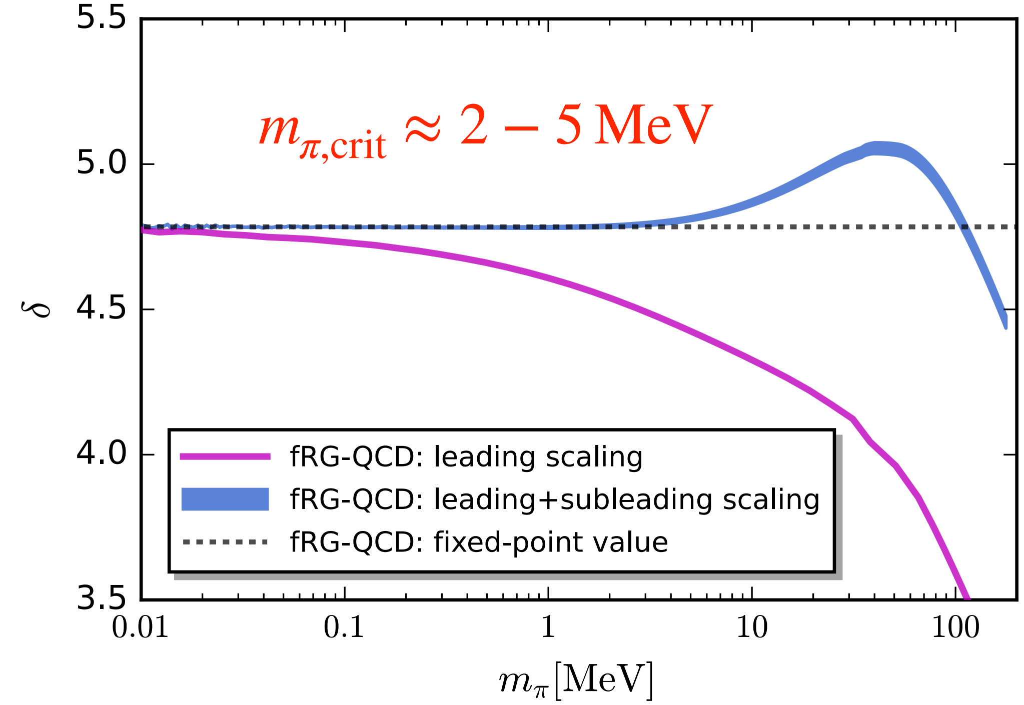
- **The almost linear dependence of the pseudo-critical temperature on the pion mass has nothing to do with the criticality.**
- **So what is the size of the critical region in QCD?**

Critical region in QCD

Scaling in the external field:

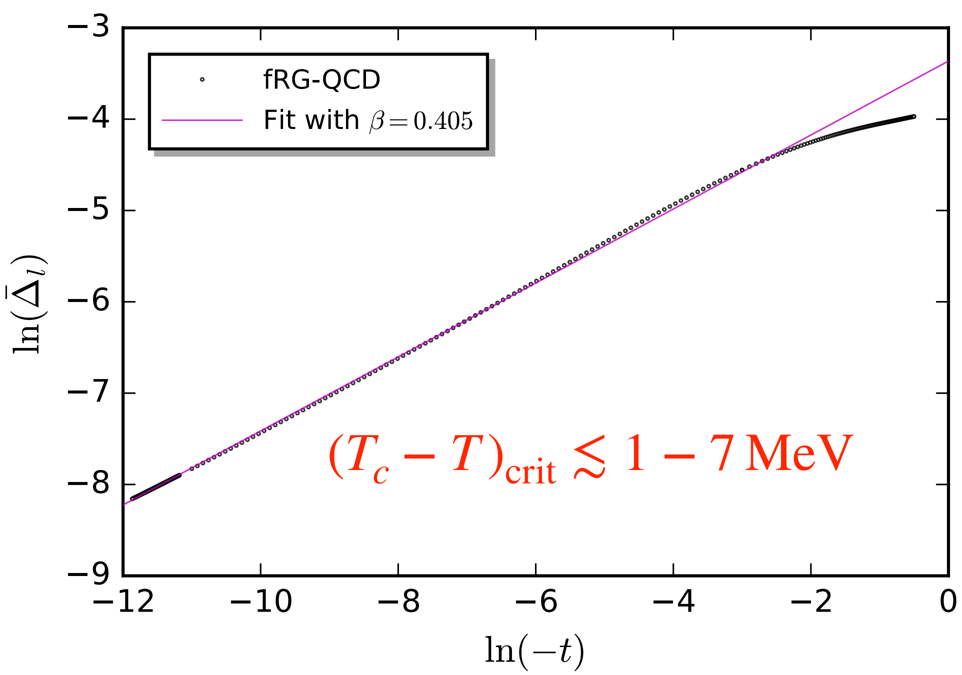


Critical exponent δ:



$$\bar{\Delta}_l^{(\mathrm{crit})}(m_\pi) = B_c\, m_\pi^{2/\delta}\left[1 + a_m m_\pi^{2\theta_H}\right]$$

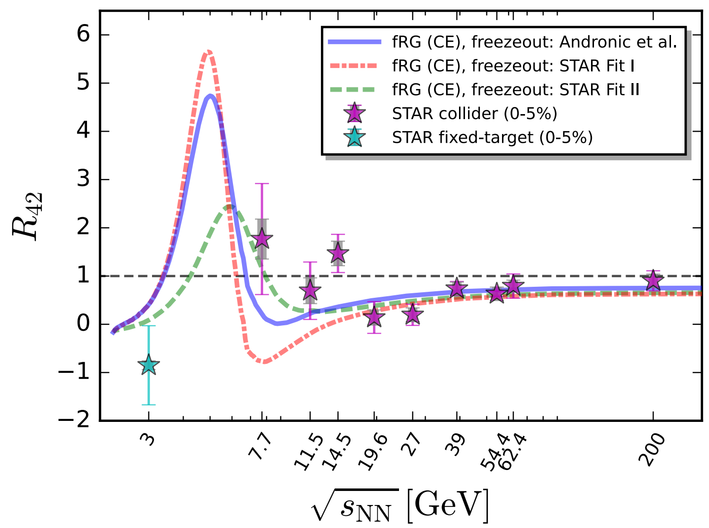
Scaling in the temperature:

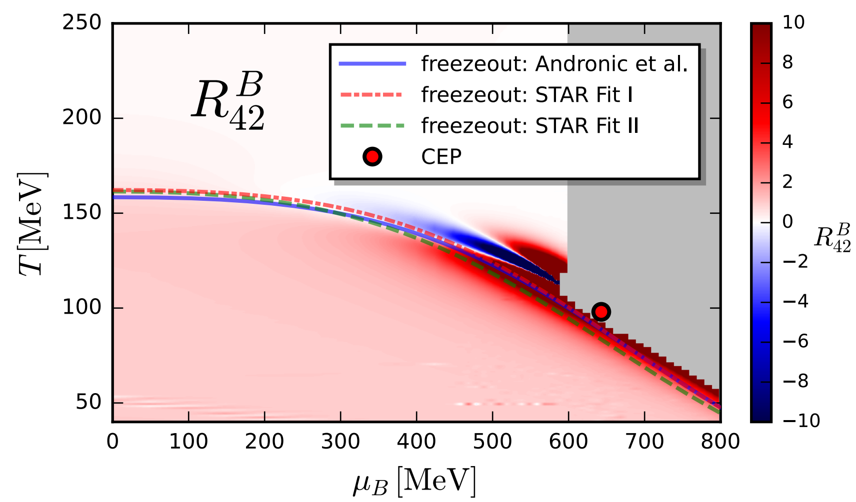


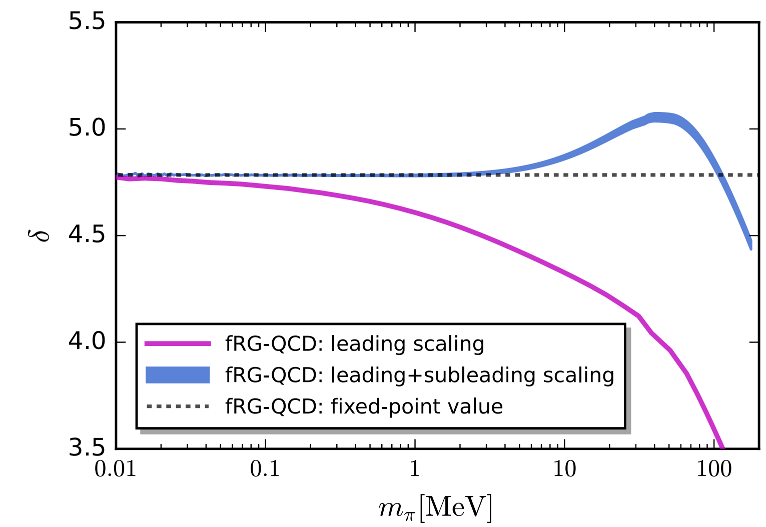
- **QCD at physical light quark mass is far away from the critical region.**
- **The scaling behavior is observed for the first time in the calculations of first-principles QCD.**

Braun, Chen, WF, Gao, Huang, Ihssen, Pawlowski, Rennecke, Sattler, Tan, Wen, and Yin, arXiv:2310.19853.

Summary





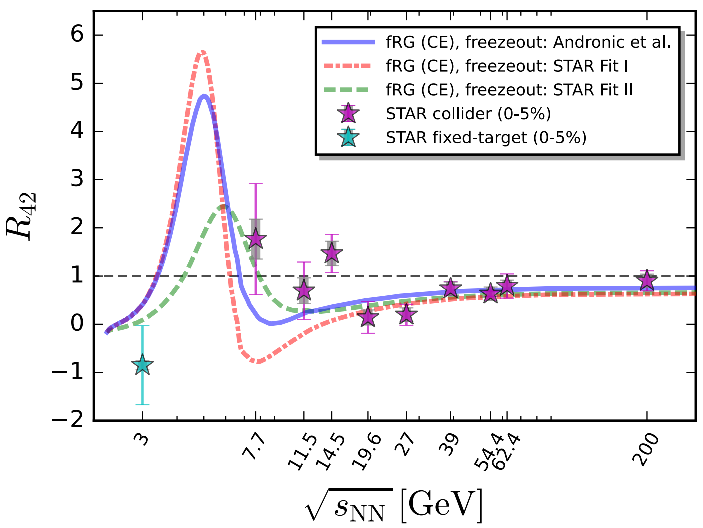


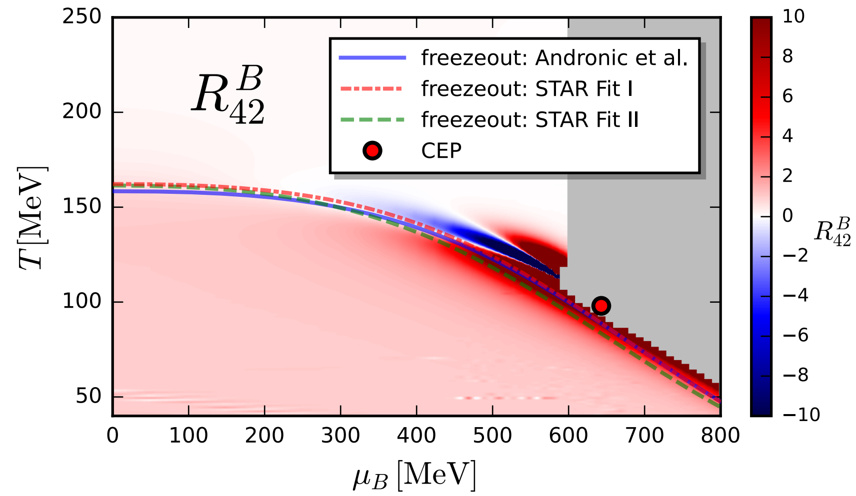
★ A prominent peak structure is found in baryon number fluctuations in the collision energy range of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.

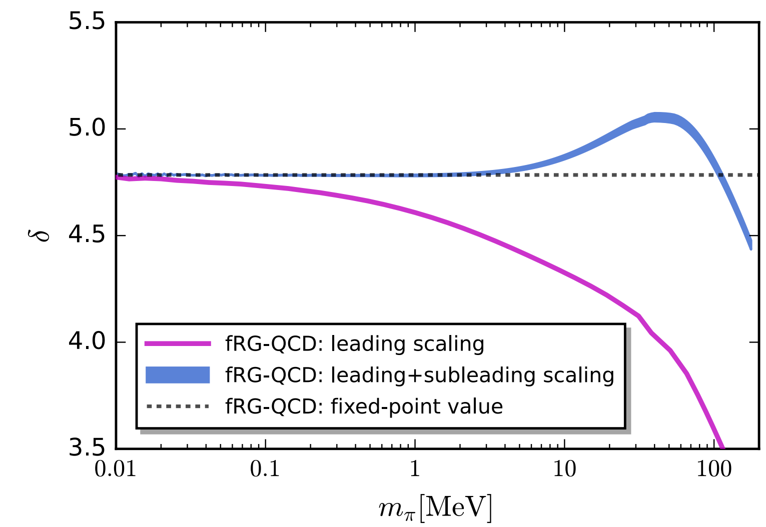
★ Information of the peak, i.e., the ripples of CEP can be used to reconstruct the location and properties of CEP.

★ The size of the critical region in QCD is determined for the first time.

Summary







★ A prominent peak structure is found in baryon number fluctuations in the collision energy range of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.

★ Information of the peak, i.e., the ripples of CEP can be used to reconstruct the location and properties of CEP.

★ The size of the critical region in QCD is determined for the first time.

Thank you very much for your attentions!

Backup

QCD with dynamical hadronization

Introducing a RG scale dependent composite field:

$$\hat{\phi}_k(\hat{\varphi}),\ \text{with}\ \hat{\varphi} = (\hat{A}, \hat{c}, \hat{\bar{c}}, \hat{q}, \hat{\bar{q}}),$$

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k\, \bar{q}\tau q + \dot{B}_k\, \phi + \dot{C}_k\, \hat{e}_\sigma\,,$$

Gies, Wetterich , *PRD* 65 (2002) 065001; 69 (2004) 025001
Pawlowski, *AP* 322 (2007) 2831
Flörchinger, Wetterich, *PLB* 680 (2009) 371

Wetterich equation is modified as

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\big(G_k[\Phi]\,\partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_a}[\Phi]\frac{\delta\langle\partial_t\hat{\phi}_k\rangle}{\delta\Phi_a}\,R_\phi\right)$$

$$-\int \langle \partial_t \hat{\phi}_{k,i}\rangle \left(\frac{\delta\Gamma_k[\Phi]}{\delta\phi_i} + c_\sigma \delta_{i\,\sigma}\right),$$

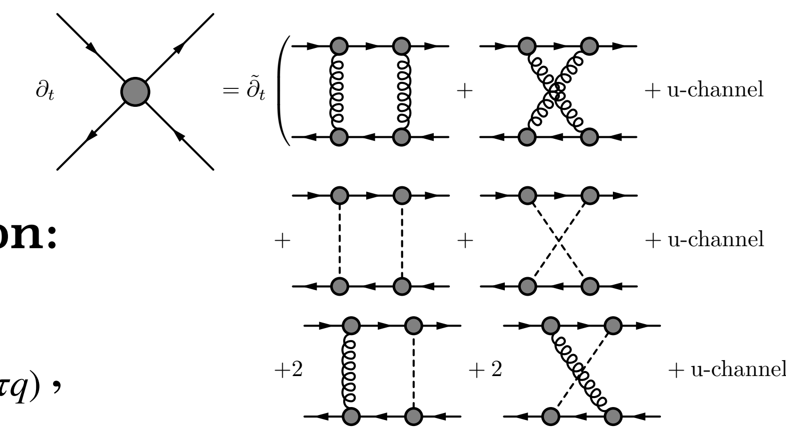
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Flow of four-quark couplings:

$$\partial_t \bar{\lambda}_q - 2\left(1+\eta_q\right)\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \overline{\mathbf{Flow}}^{(4)}_{(\bar{q}\tau q)(\bar{q}\tau q)}\,,$$

choosing

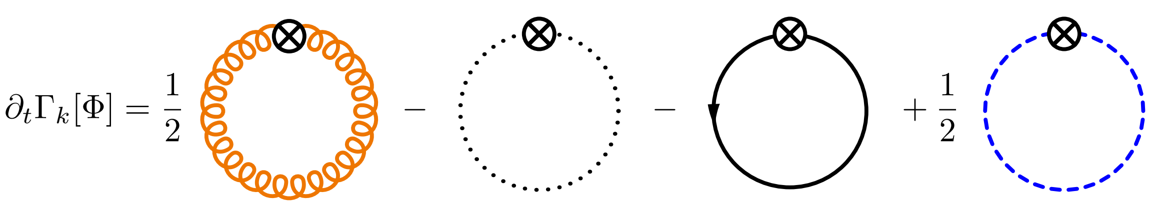
$$\bar{\lambda}_q \equiv 0\,, \qquad \forall k\,,$$

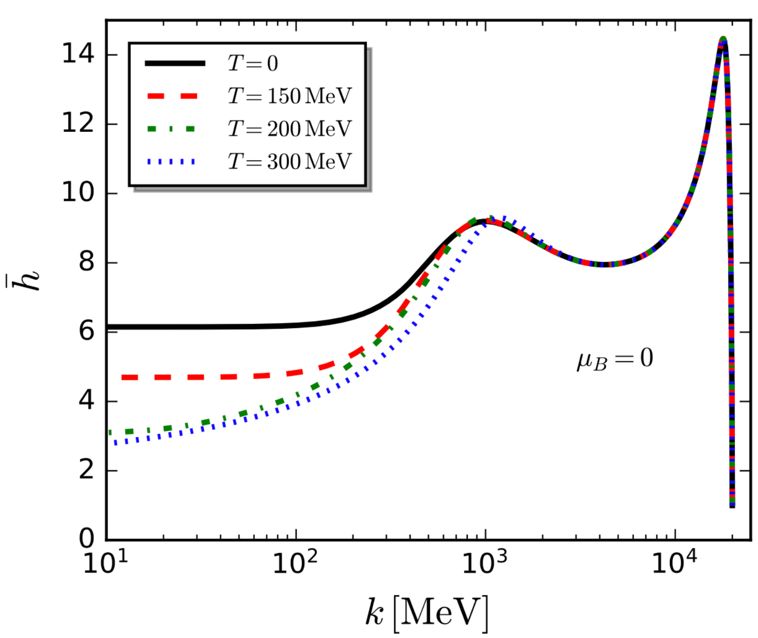


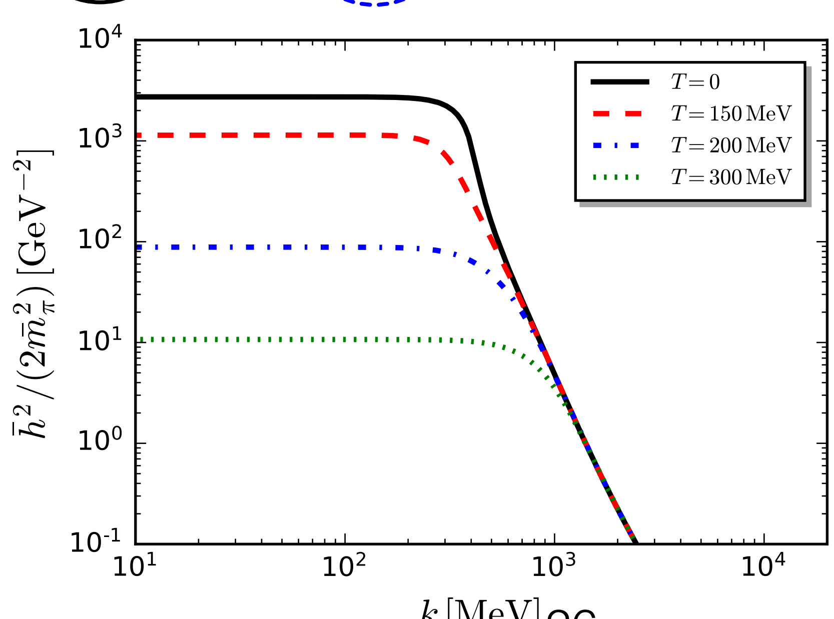
Hadronization function:

$$\dot{\bar{A}} = -\frac{1}{\bar{h}}\overline{\mathbf{Flow}}^{(4)}_{(\bar{q}\tau q)(\bar{q}\tau q)}\,,$$

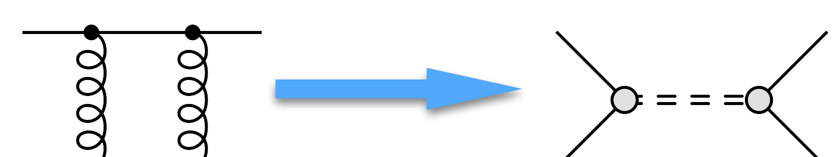
Flow equation:



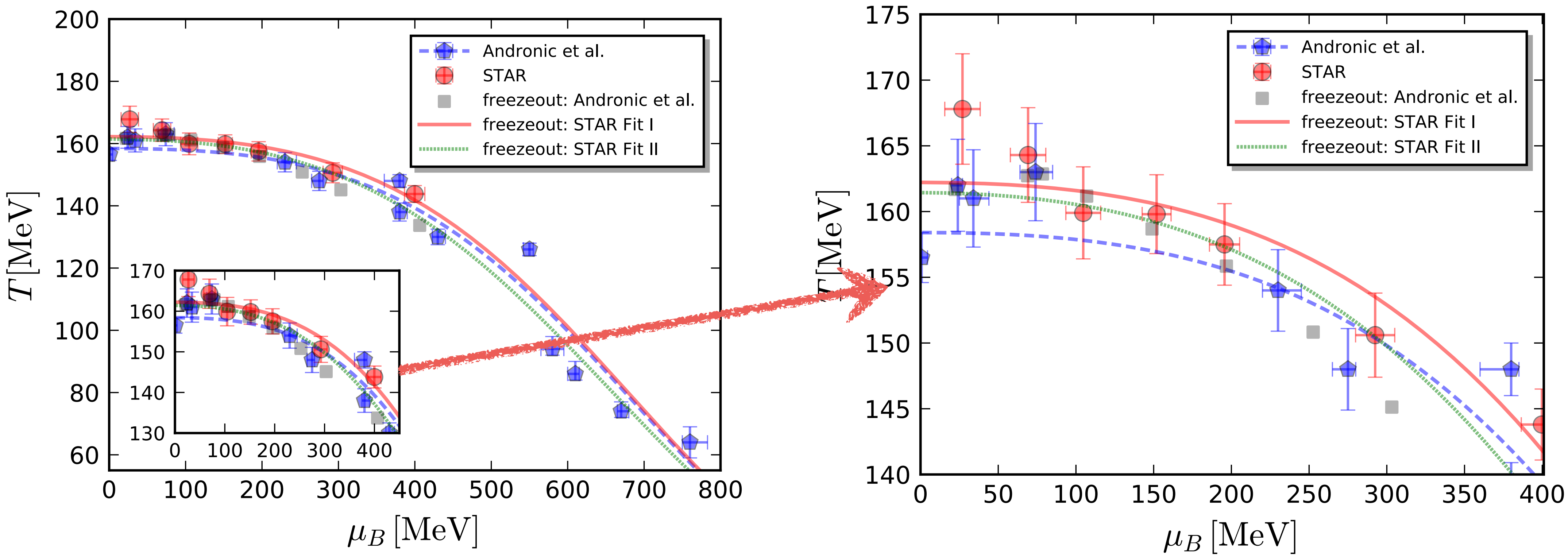




four-quark interaction encoded in Yukawa coupling:



Determination of the freeze-out curve



three freeze-out curves

1. **freeze-out: Andronic *et al.***

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. **freeze-out: STAR Fit I**

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

$$\mu_{B_{\mathrm{CF}}} = \frac{a}{1 + 0.288\sqrt{s_{\mathrm{NN}}}},$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

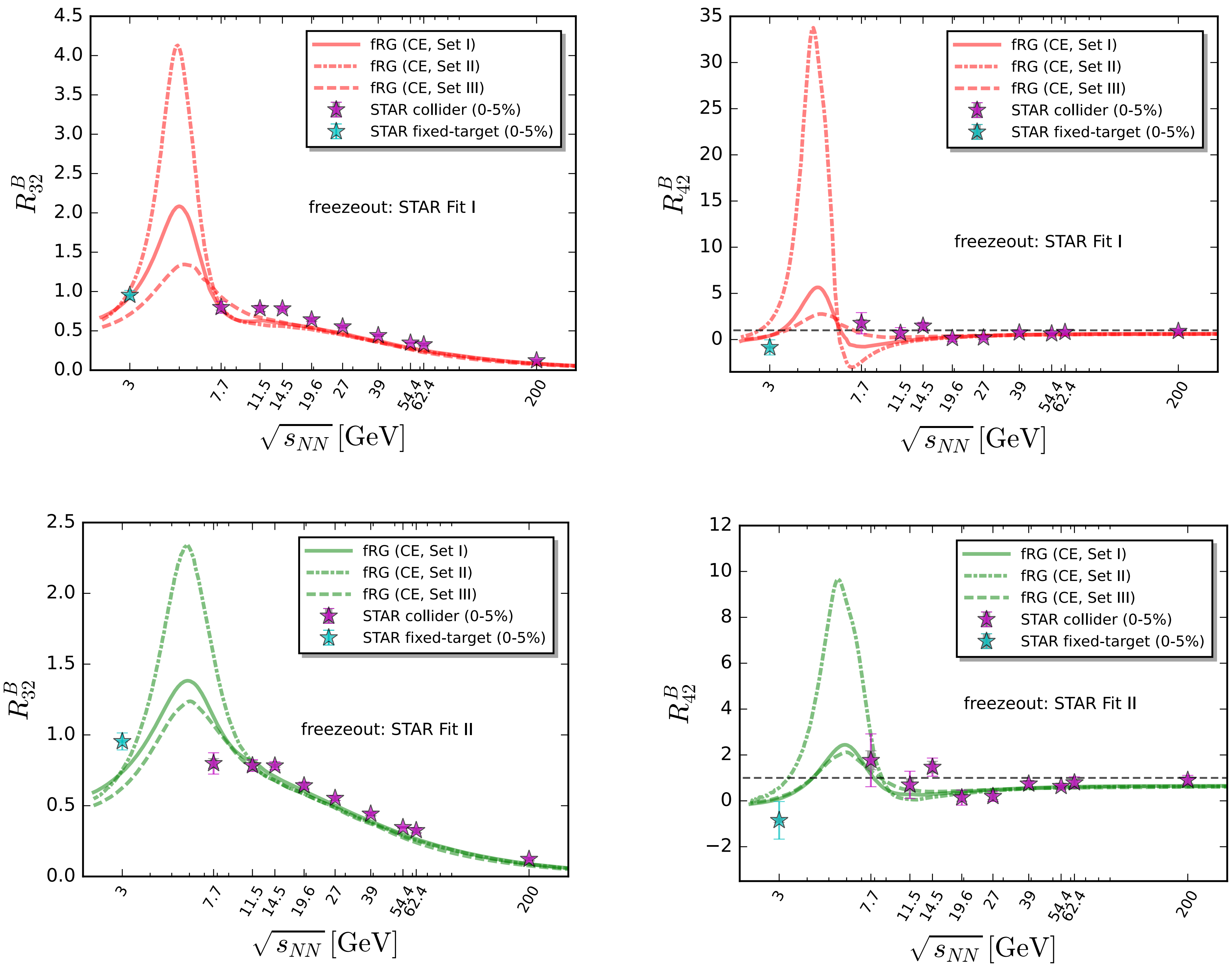
all data points

3. **freeze-out: STAR Fit II**

neglecting first two at low μ_B and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Dependence of the location of CEP



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