QCD phase transitions on rotating lattice

Ji-Chong Yang

2023-11-11 @fudan

Outline

- Lattice approach
- Results on phase transition
- Results on angular momentum

Two different approaches

- 1 – Simulate in a rotating frame
- 2 – Use a ‘rotwisted’ boundary condition

Rotating frame

- [Yamamoto et al. Phys.Rev.Lett. 111 (2013) 081601 • e-Print: 1303.6292]

$$g_{\mu\nu} = \begin{pmatrix} 1-r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$S_G = \int d^4x \sqrt{\det[g_{\mu\nu}]} g^{\mu\nu} g^{\alpha\beta} \frac{1}{2g_{YM}^2} \mathrm{tr}\left[F_{\mu\nu} F^{\alpha\beta}\right]$$

$$S_F = -\int d^4x \sqrt{\det[g_{\mu\nu}]} \bar{\psi} \left(i\gamma^\mu (D_\mu + \Gamma_\mu) - m\right) \psi$$

- There is sign problem in the case of real rotation.

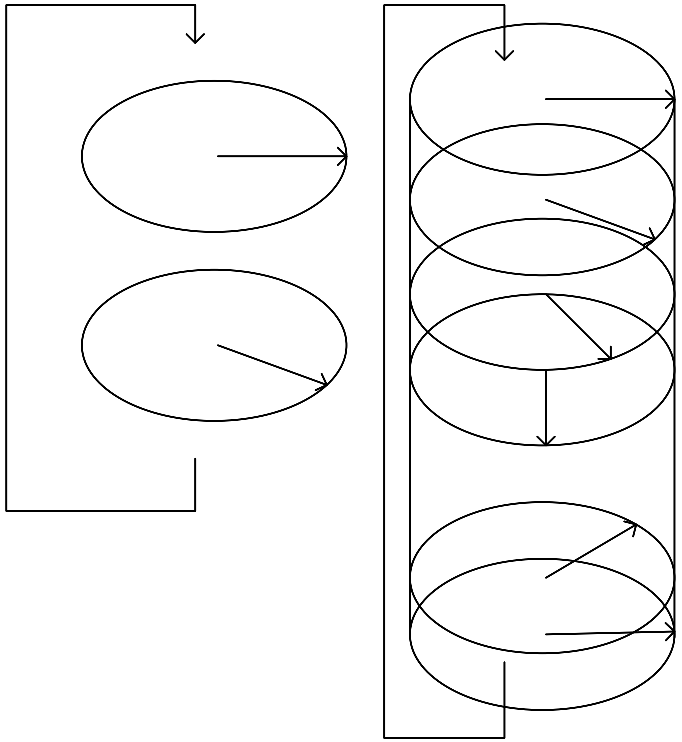
Rotwisted boundary condition

- [M. N. Chernodub et al. Phys.Rev.D 107 (2023) 11, 114502 • e-Print: 2209.15534]
- Change the periodic boundary condition in tau direction
- Redefine: $\phi = e^{-i\Omega_I \tau J_z}\psi$

$$\phi(\rho, \phi, z, \tau) = \Lambda\phi(\rho, \phi - \beta\Omega_I, z, \tau + \beta)$$

- In the case of square lattice

$$\Omega_I = n\frac{\pi T}{2}$$

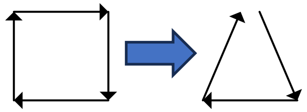


Different boundary conditions in the x-y plane

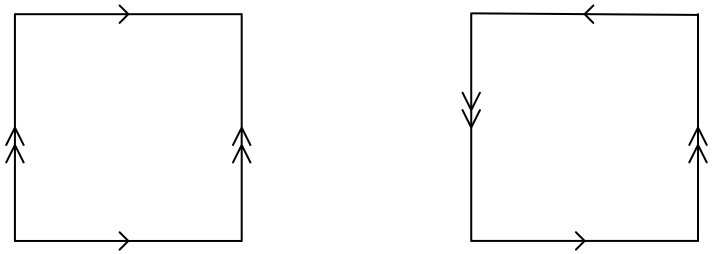
- Dirichlet & open(Neumann) boundary condition

$$A_{x,y}(x, y = \pm N_s/2) = 0 \qquad F_{x\mu}(x = \pm N_s/2) = F_{y\mu}(y = \pm N_s/2) = 0$$

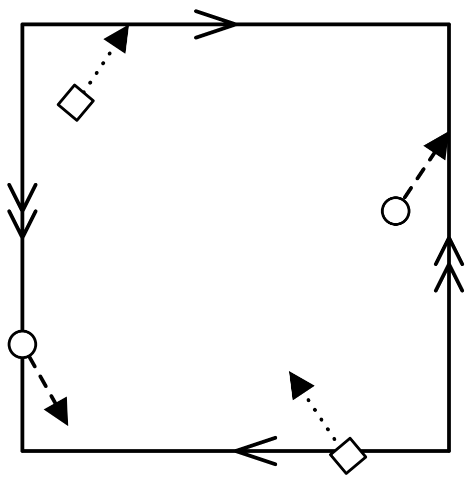
Respect Z3 symmetry

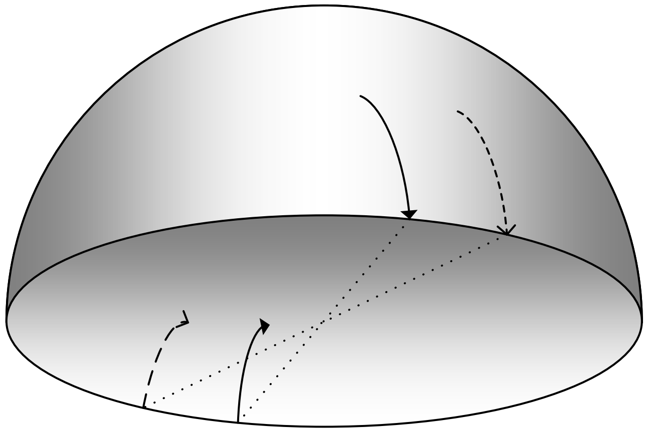


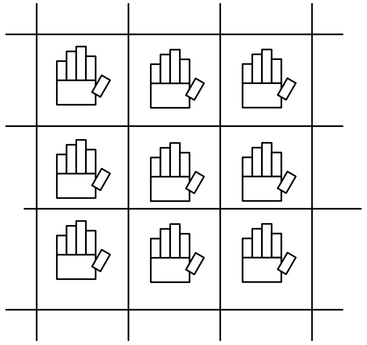
- Torus & projective plane periodic boundary condition

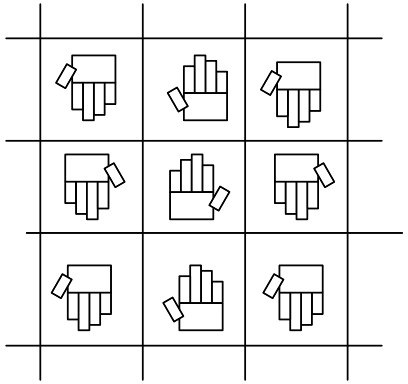


Projective plane boundary condition



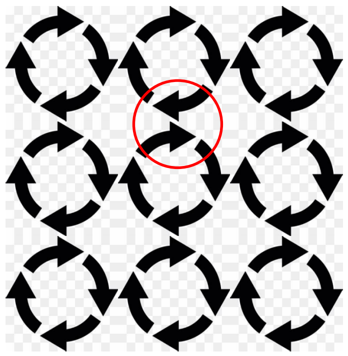


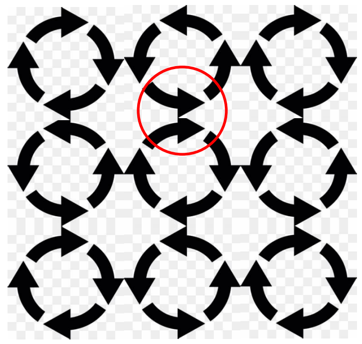




- gauge action is smooth

- Spinor eigenstates can be compatible in cylinder coordinate





Boundary condition is not so important

- Boundary conditions do not change the main properties of phase transitions. (at least at small angular velocity)

- spatial thermal correlation lengths << R (spatial extent) [Braguta et al., 2303.03147]

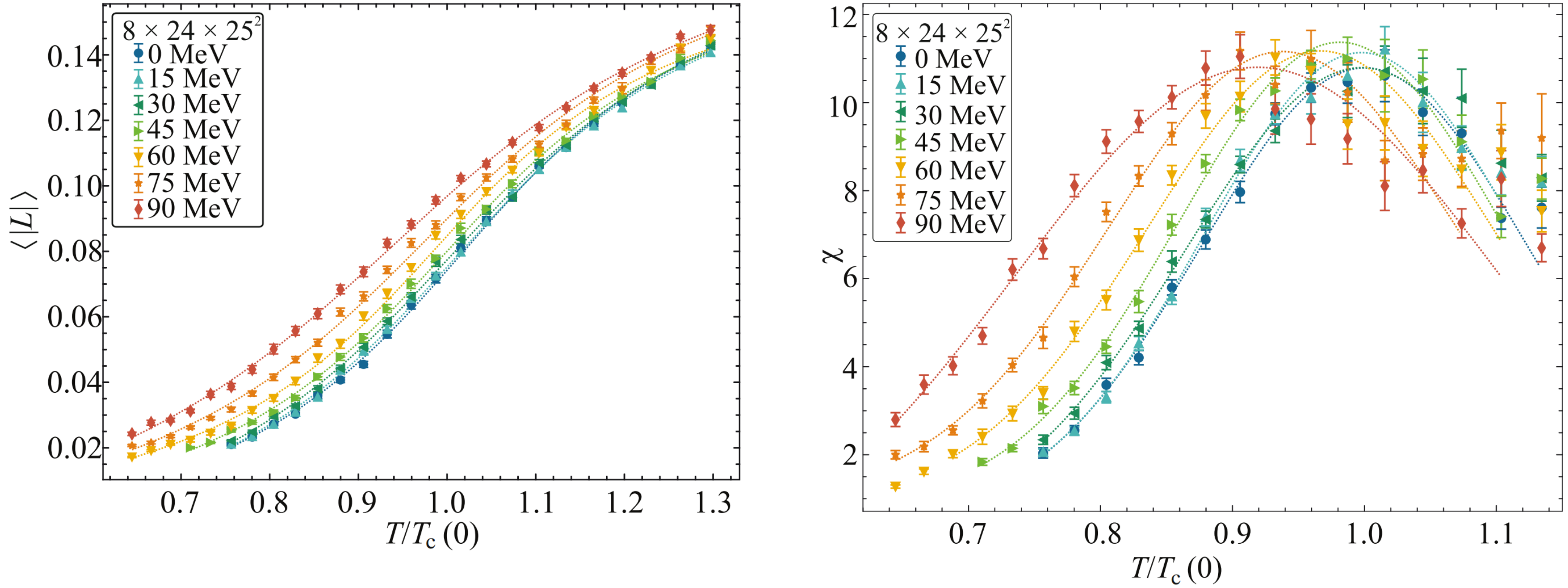
Results on phase transition:

- Chiral condensation & Polyakov loop
- chiral condensation: staggered fermion.
- analytical extension is assumed to be correct.
- the case of real rotation is also studied using Taylor expansion.

Quenched approximation

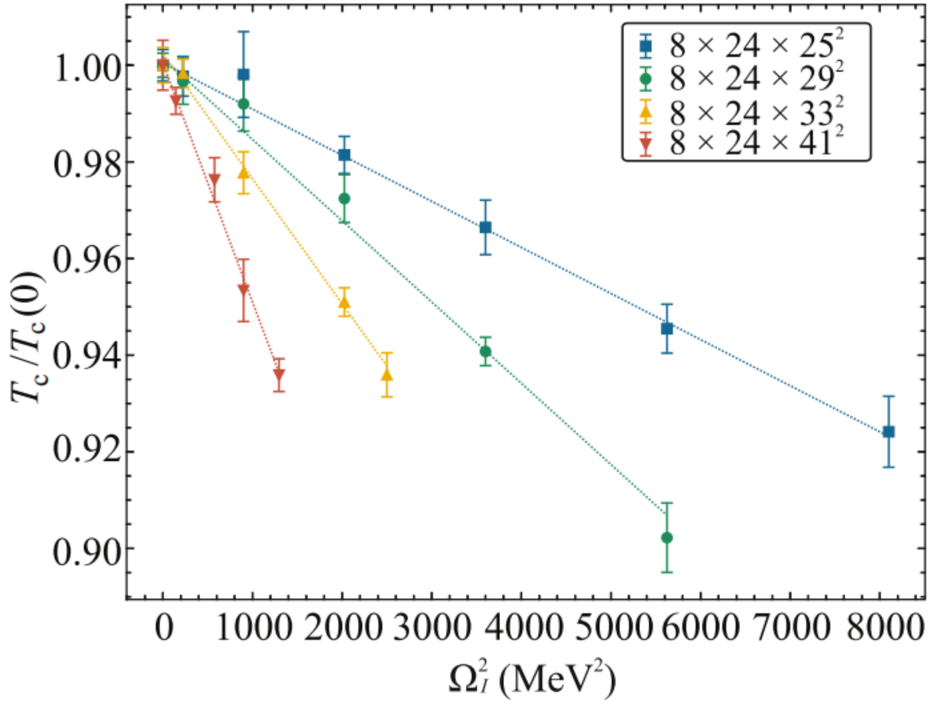
- [Braguta et al. JETP Letters, 2020, Vol. 112, No. 1, pp. 6–12,   Phys. Rev. D 103, 094515 (2021), arXiv:2102.05084]
- Open boundary condition, L increase with imaginary rotation

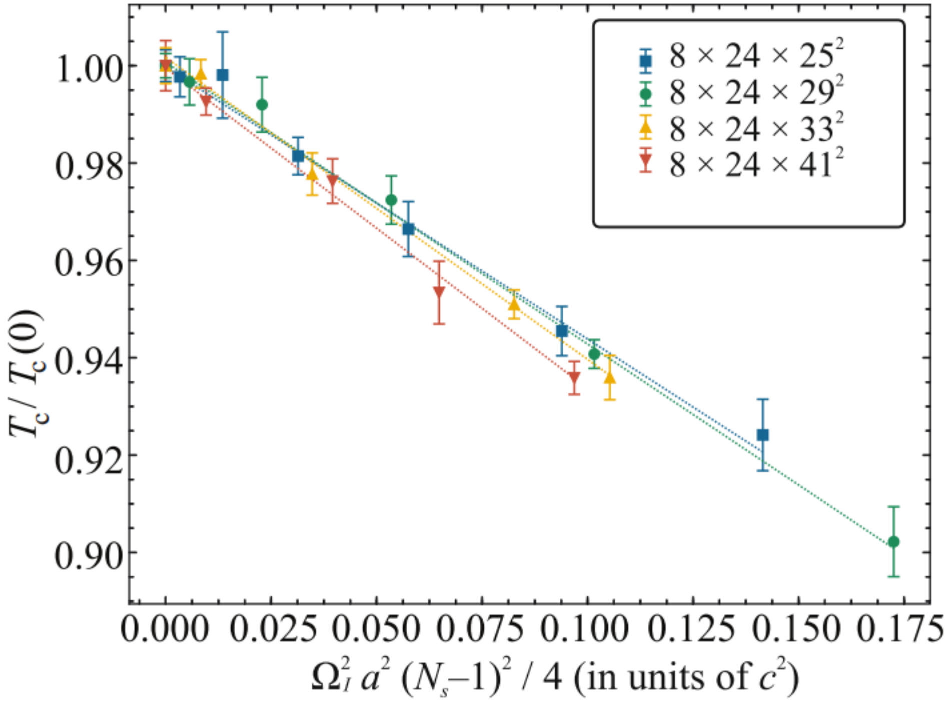
STUDY OF THE CONFINEMENT/DECONFINEMENT PHASE TRANSITION 9



Quenched approximation

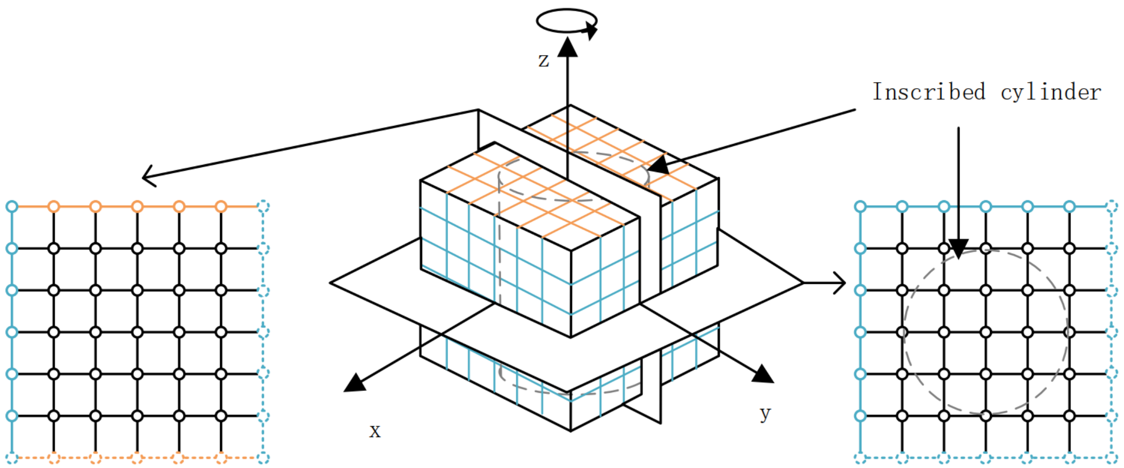
- The critical temperature is a function of $\Omega^2 R^2$

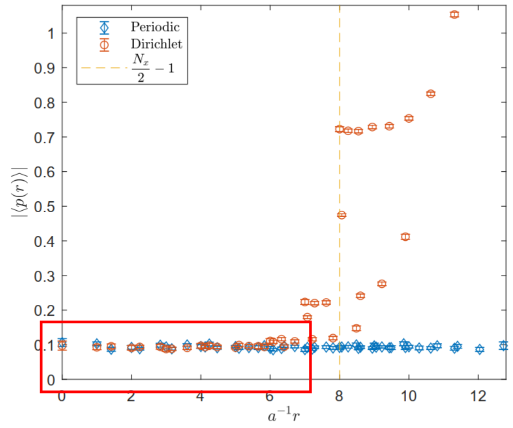


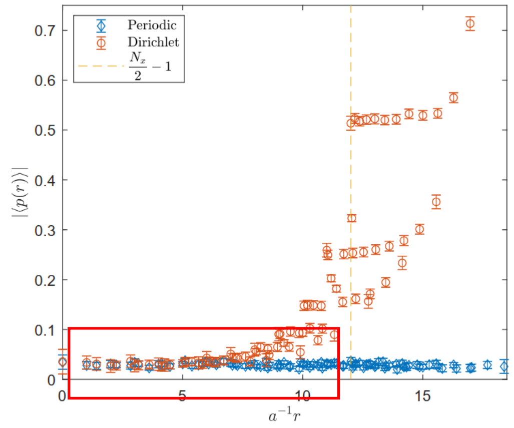


Wilson Dirac fermion

- [to be uploaded]
- Dirichlet boundary condition, Nf=2



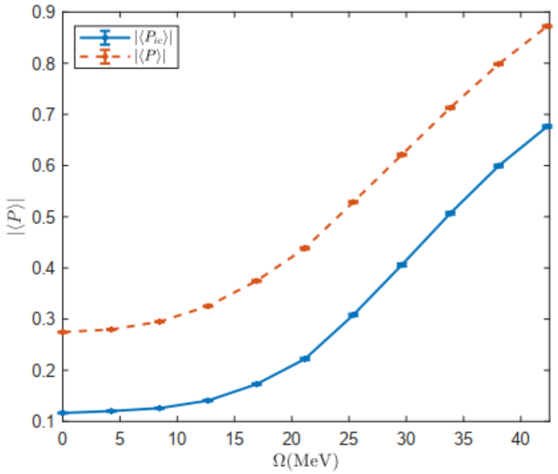




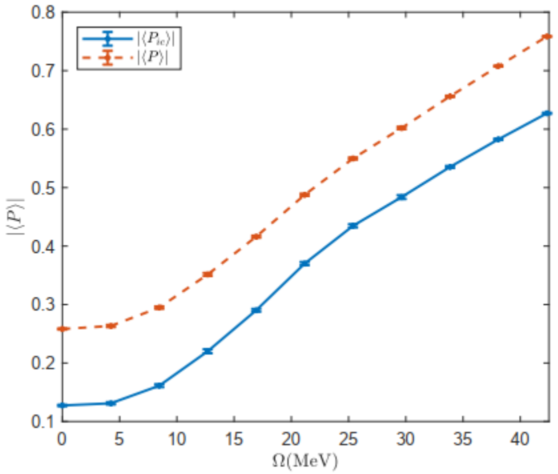
It only fits well when the fermion is turned on (implying, that the breaking of the Z3 symmetry at the boundary leads to some subtleties.)

Polyakov loop

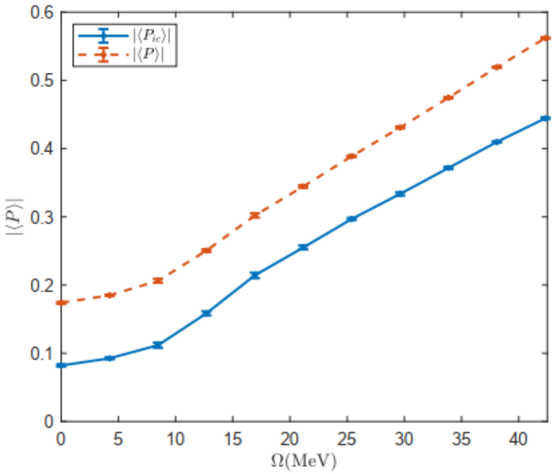
- Polyakov loop increase with imaginary rotation



(b) $N_x = 18$



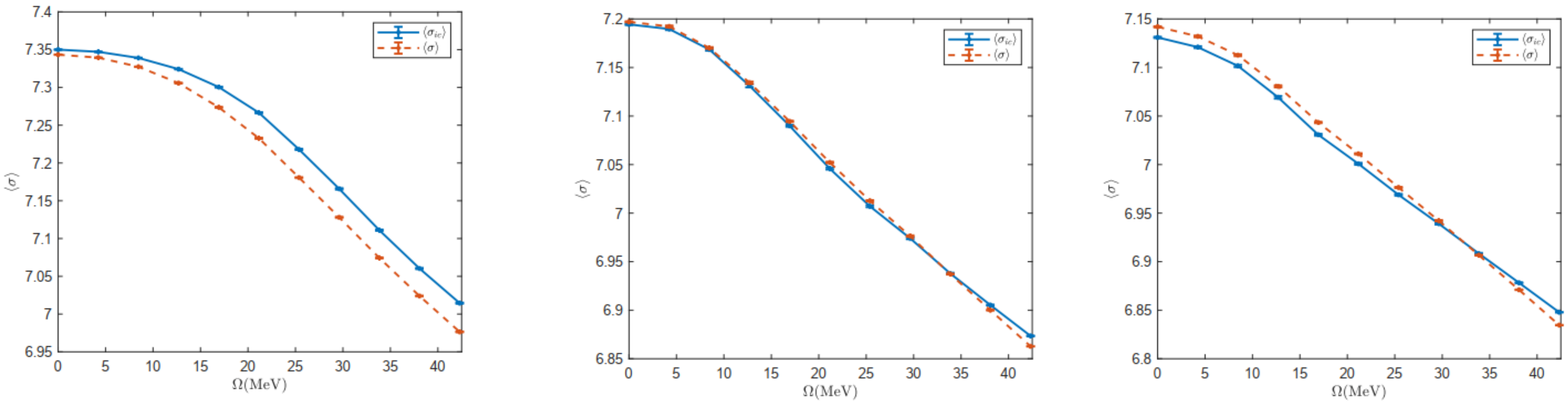
(c) $N_x = 22$



(d) $N_x = 26$

Chiral condensation

- Note that, Wilson Dirac fermion does not respect chiral symmetry
- Chiral condensation decrease with imaginary rotation

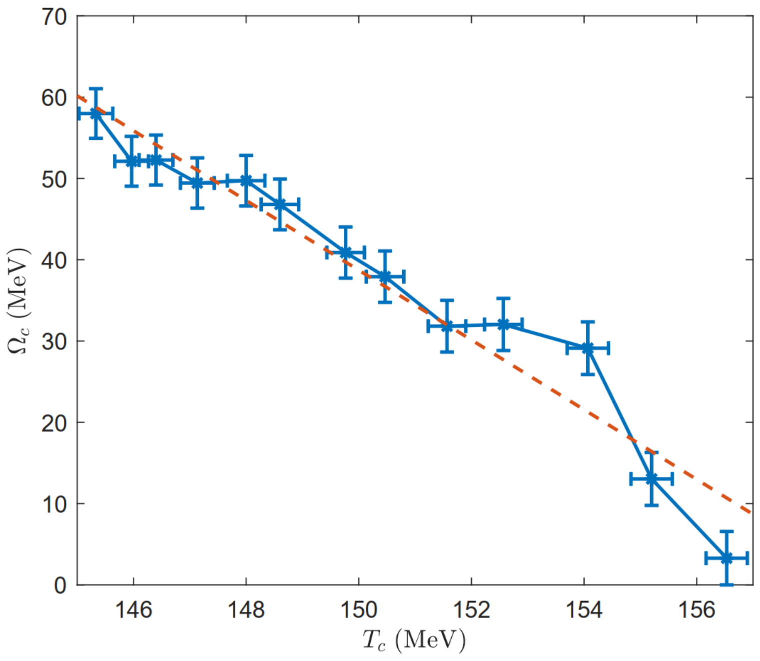


(b) $N_x = 18$ (c) $N_x = 22$ (d) $N_x = 26$

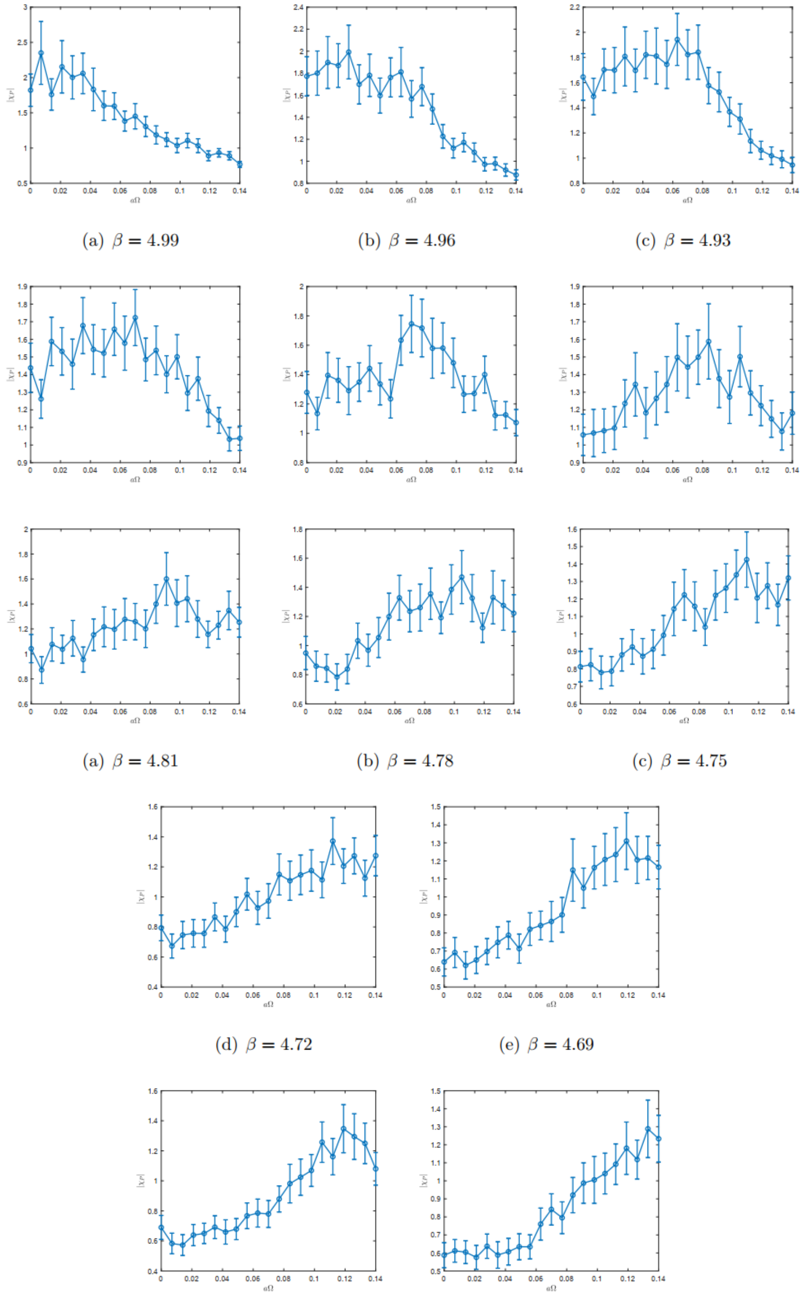
FIG. 10. $\langle\sigma\rangle$ and $\langle\sigma_{ic}\rangle$ as functions of Ω.

Critical temperature

- Susp of Polyakov loop
- Y-axis is imaginary rotation



(a) $\Omega - T$



(a) $\beta = 4.99$ (b) $\beta = 4.96$ (c) $\beta = 4.93$

(a) $\beta = 4.81$ (b) $\beta = 4.78$ (c) $\beta = 4.75$

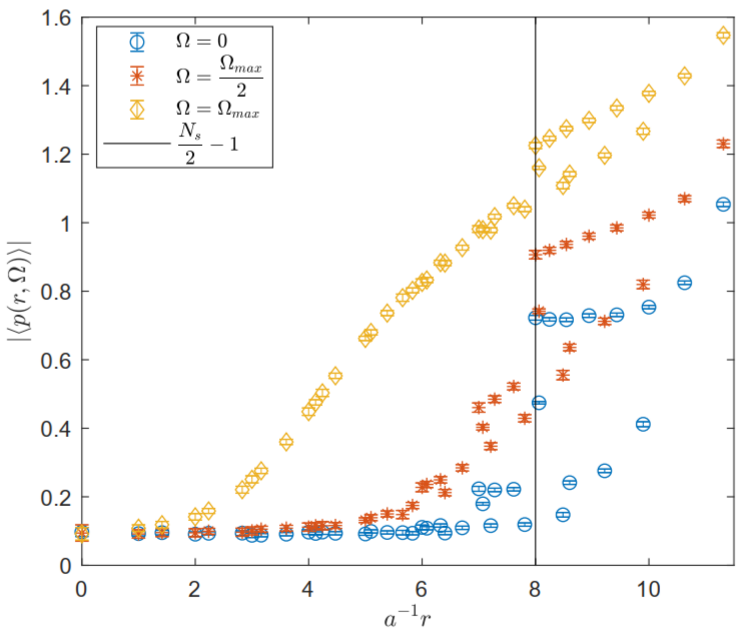
(d) $\beta = 4.72$ (e) $\beta = 4.69$

(f) $\beta = 4.66$ (g) $\beta = 4.63$

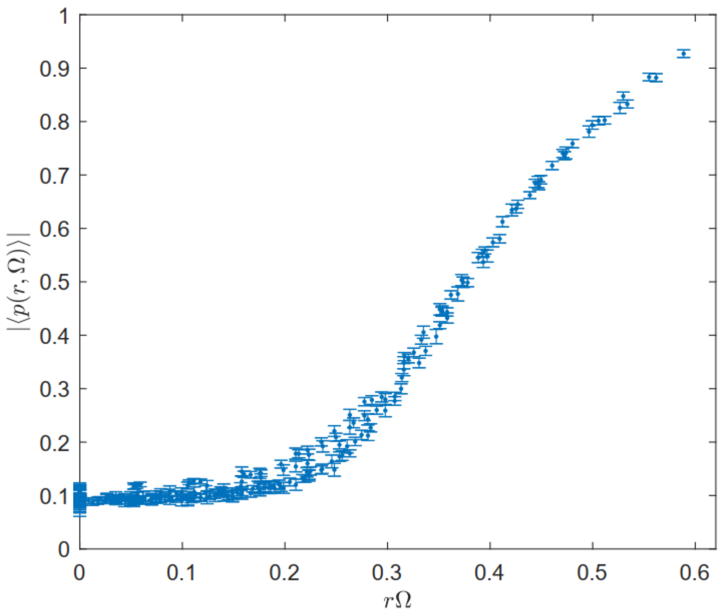
FIG. 23. $|\langle\chi_{ic}\rangle|$ as functions of Ω. $\beta = 4.63 \sim 4.81$.

Inhomogeneity

- Left, radius distribution of P, right, P as a function of v



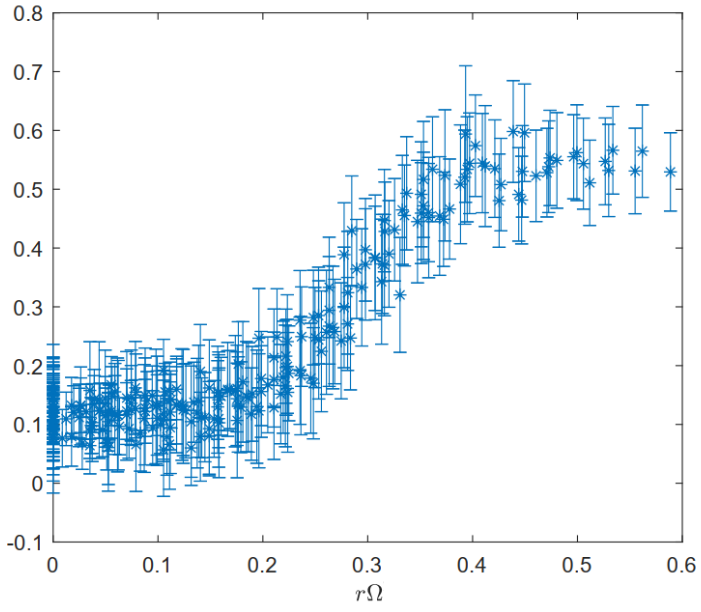
(a) $N_x = 18$



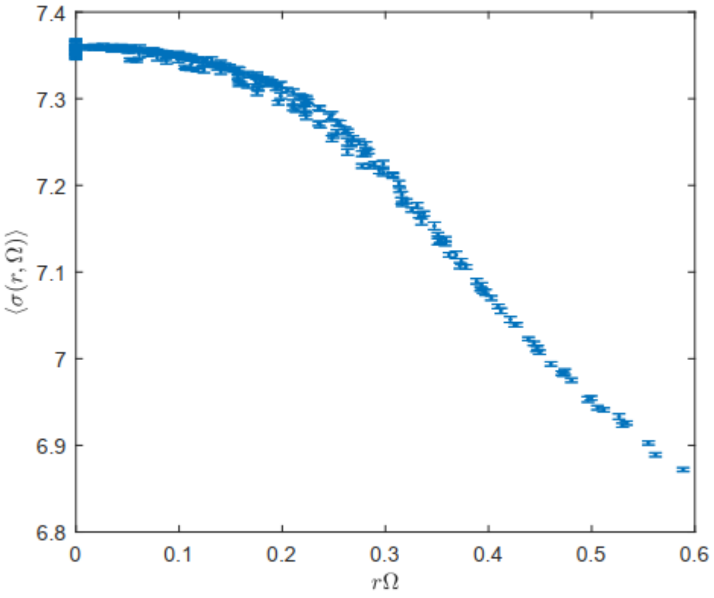
(a) $N_x = 18$

Inhomogeneity

- Left: susp of Polyakov loop as function of v, right: chiral condensation as function of v

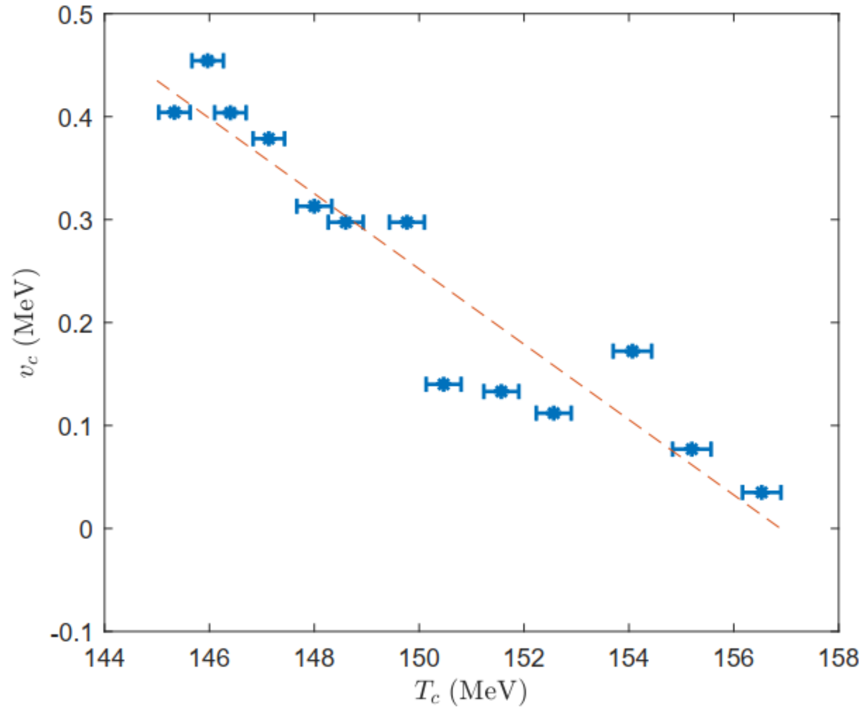


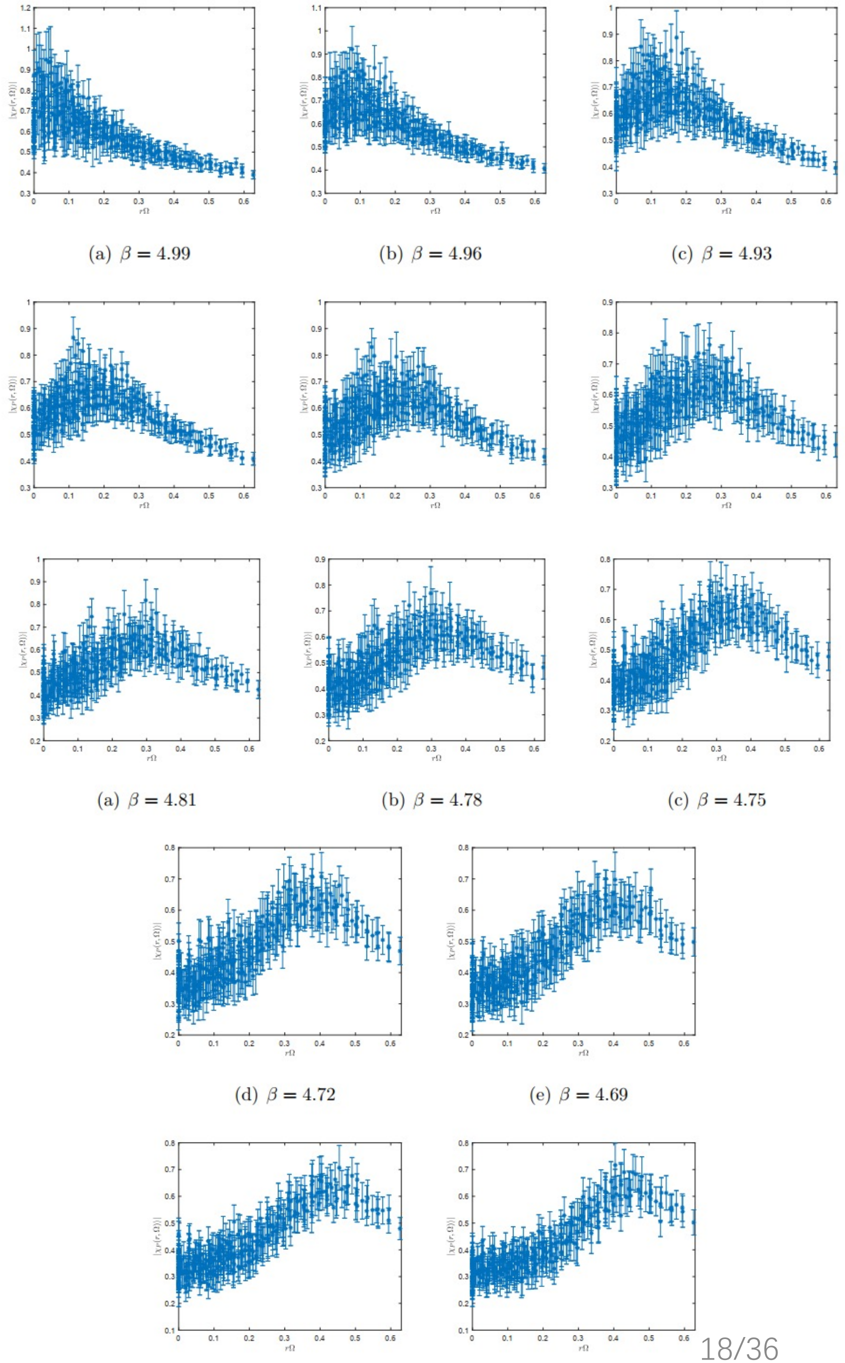
(a) $N_x = 18$



(a) $N_x = 18$

Critical temperature





(a) $\beta = 4.99$ (b) $\beta = 4.96$ (c) $\beta = 4.93$

(a) $\beta = 4.81$ (b) $\beta = 4.78$ (c) $\beta = 4.75$

(d) $\beta = 4.72$ (e) $\beta = 4.69$

(f) $\beta = 4.66$ (g) $\beta = 4.63$

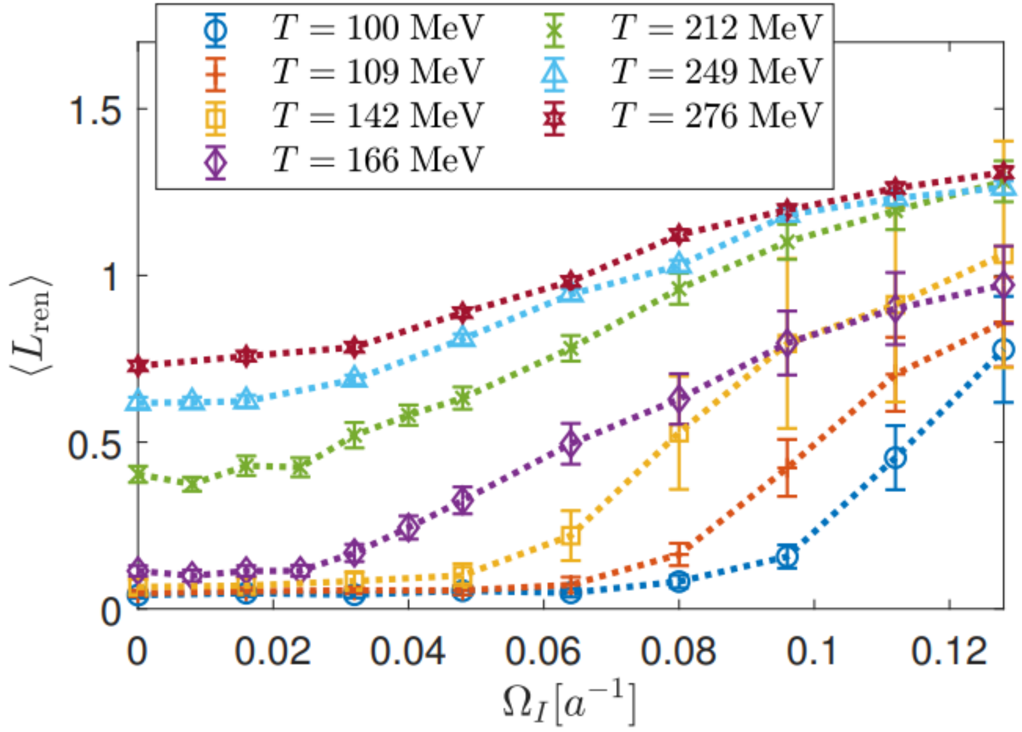
Staggered fermion with Nf=2+1

- [JCY, Xu-Guang Huang, 2307.05755]
- Projective periodic boundary condition is used.

$$L_{\rm ren} = e^{-N_\tau c(\beta) a/2} L_{\rm bare}$$

- Matching V at zero temperature

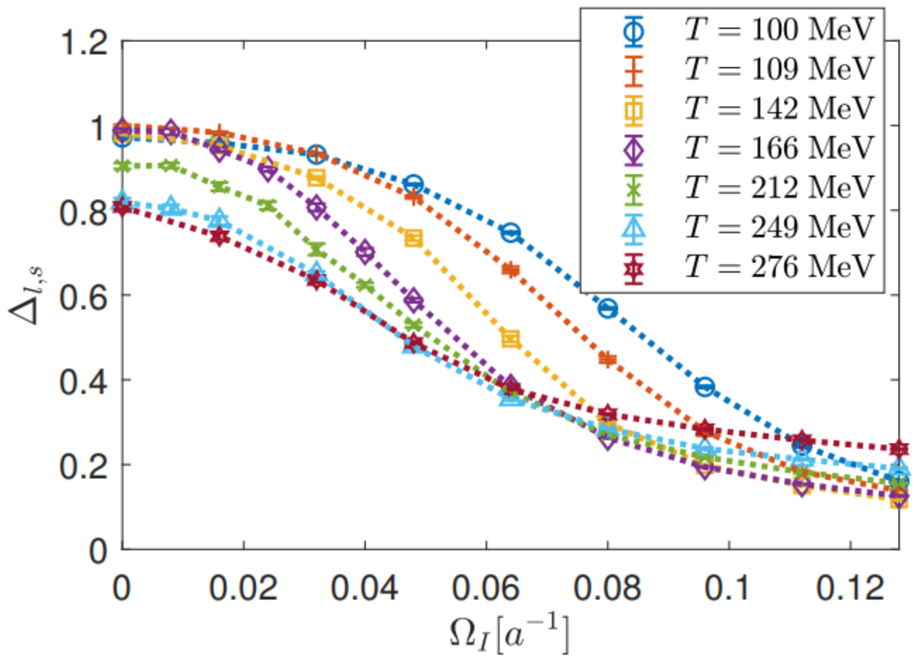
$$\langle W \rangle = C(r)\exp\left(-V_{\rm string}(r)\tau\right) = C(r)\exp\left(-(V_{qq}(r)+c)\tau\right)$$
$$V_{\rm string}(r) = -\frac{\pi}{12r} + \sigma r$$
$$V_{qq}(r) = -\frac{a}{r} + \sigma r + b$$



Staggered fermion

- Chiral condensation

$$\Delta_{l,s} = \frac{m_s \langle \bar{\psi}_l \psi_l \rangle_{T,\Omega} - m_l \langle \bar{\psi}_s \psi_s \rangle_{T,0}}{m_s \langle \bar{\psi}_l \psi_l \rangle_{0,0} - m_l \langle \bar{\psi}_s \psi_s \rangle_{0,0}}$$

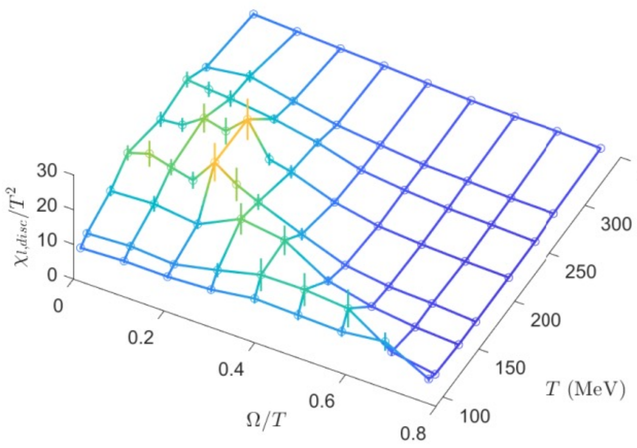


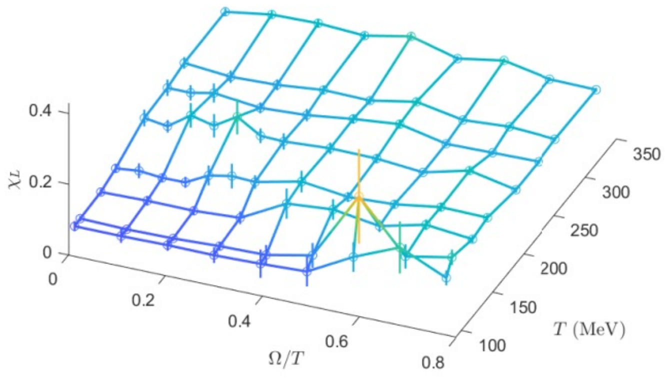
- P increase with i-rotation
- c decrease with i-rotation
- At large i-rotation, c increase with temperature

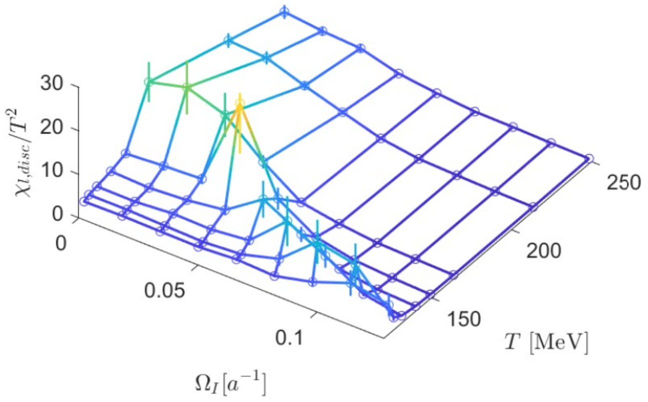
Staggered fermion

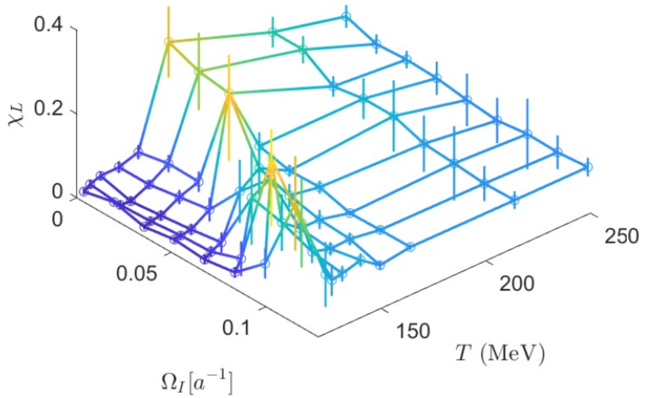
- Susp.

- Up:Nt=6
- Down:Nt=4
- Left: chiral condensation
- Right: Polyakov





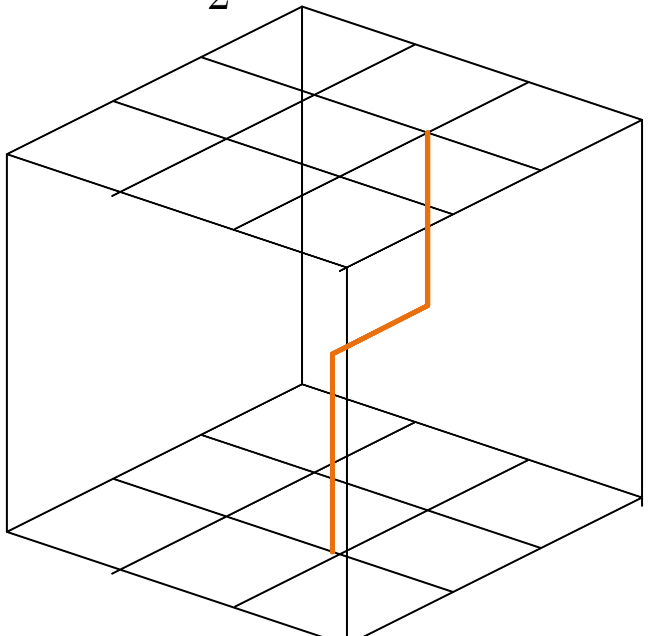


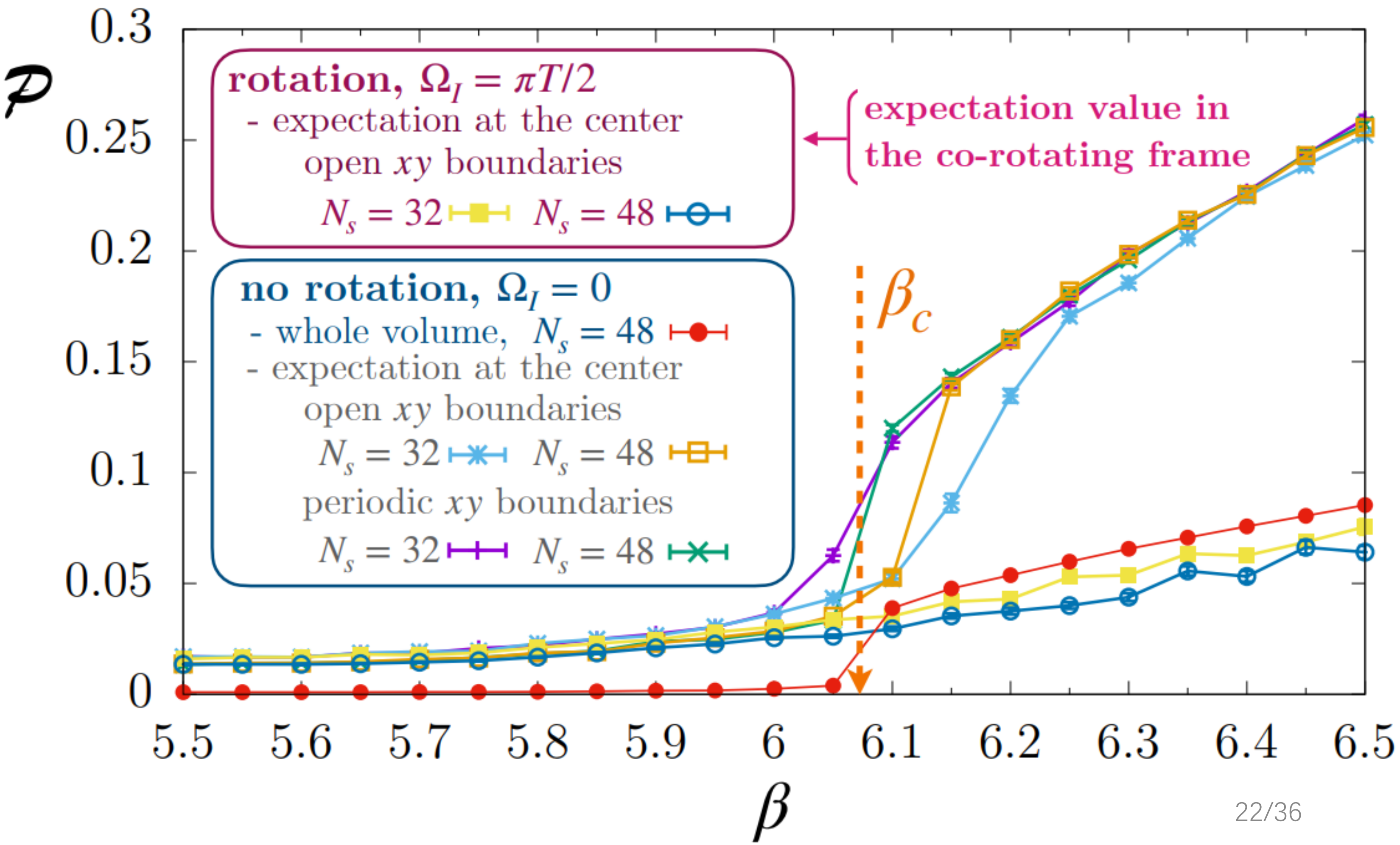


Rotwist boundary condition

- [M. N. Chernodub et al. Phys.Rev.D 107 (2023) 11, 114502 • e-Print: 2209.15534]
- Quenched
- Open boundary
- Polyakov loop

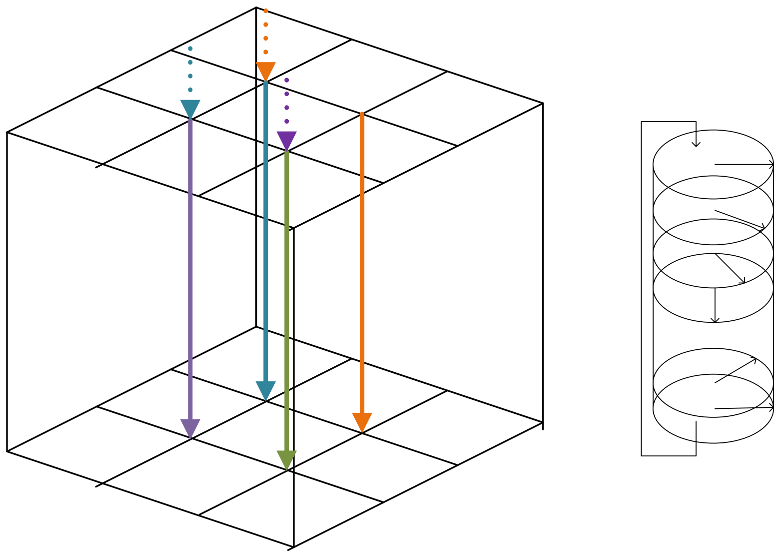
$$\Omega_I = \frac{\pi T}{2}$$

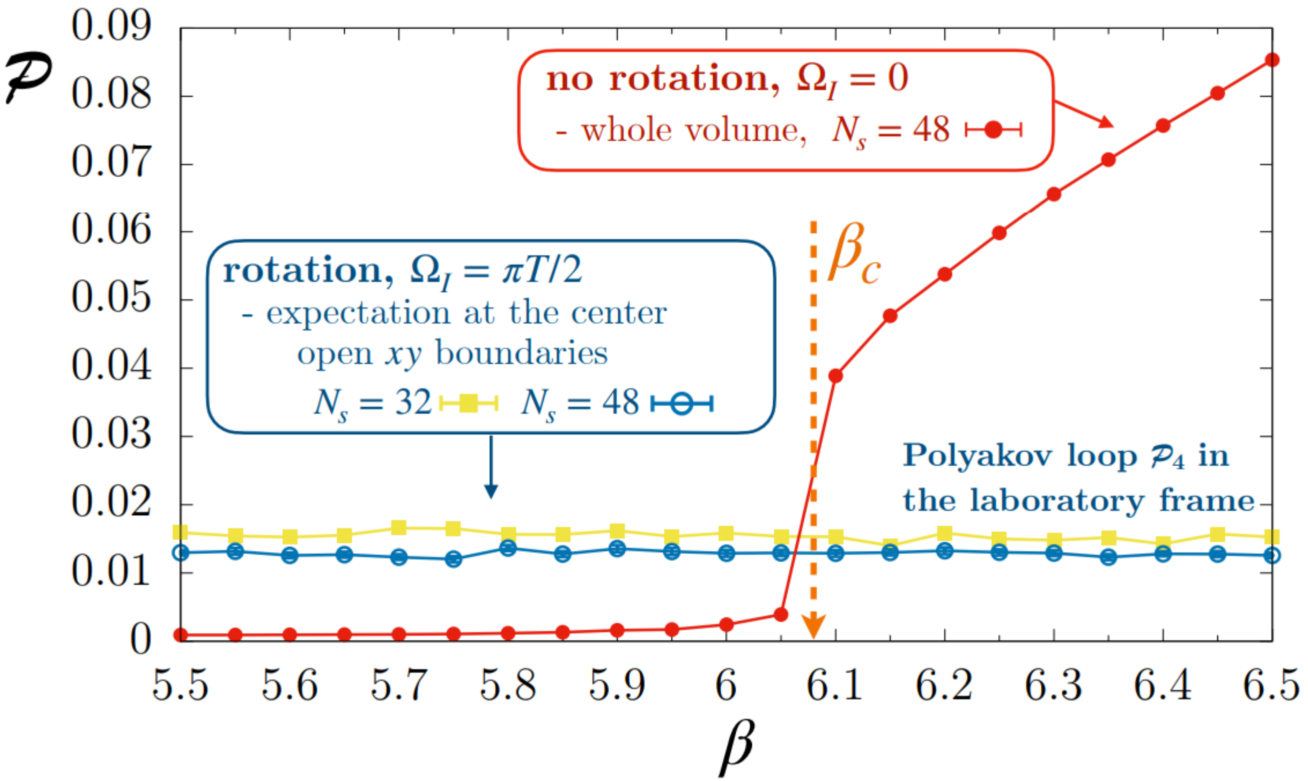




Rotwist boundary condition

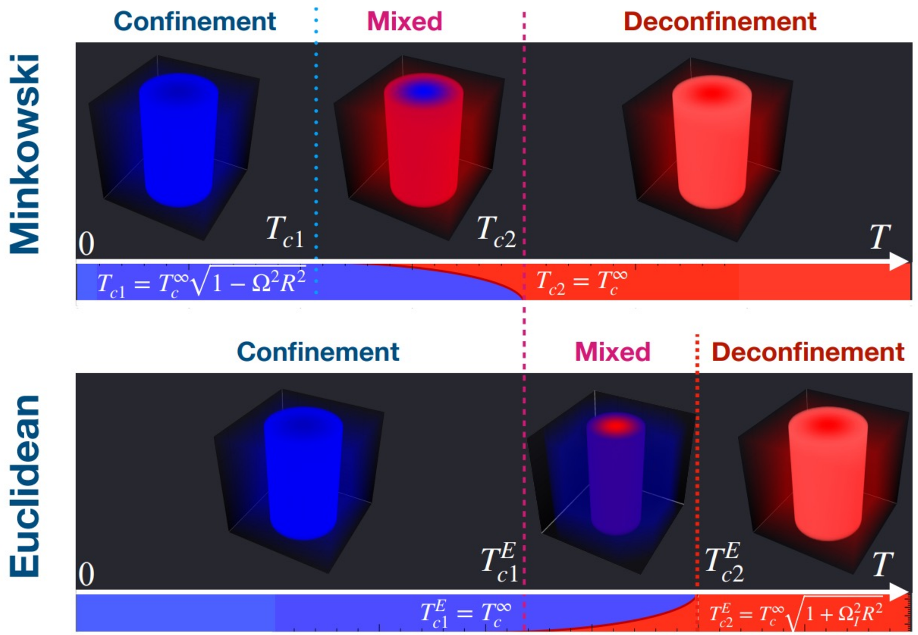
- Four fold Polyakov loop
- P decrease with i-rotation

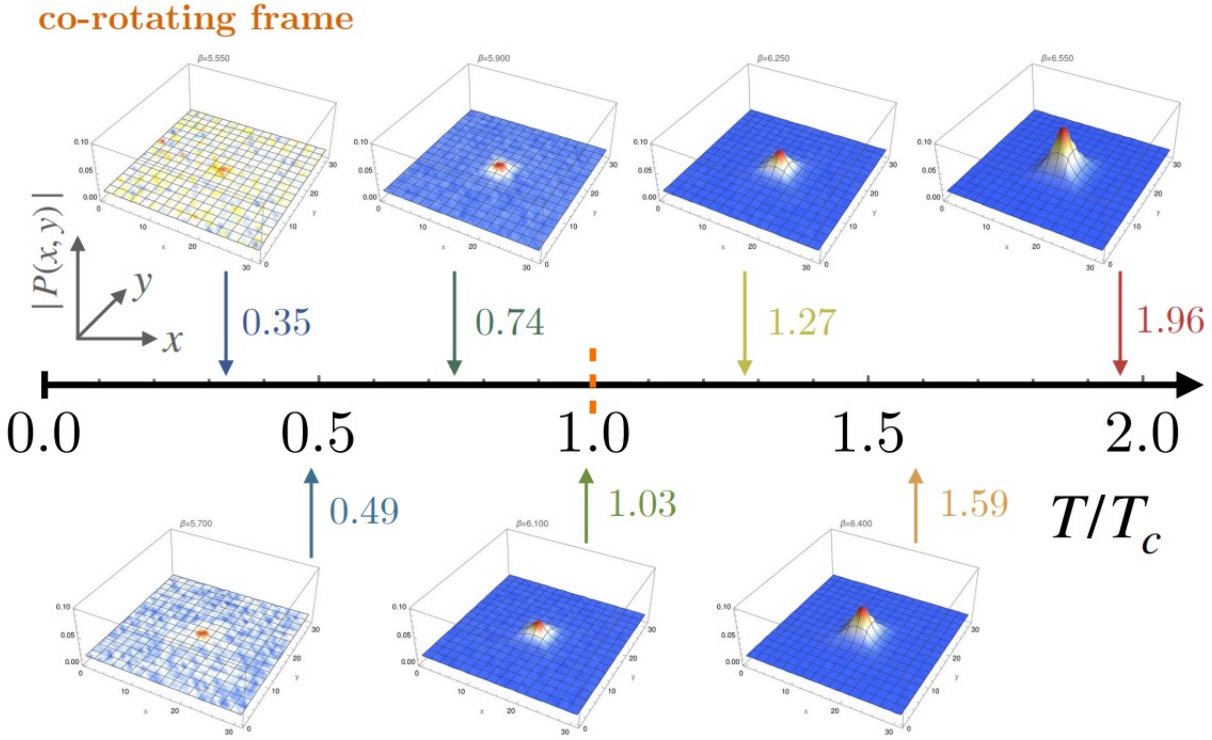




Rotwist boundary condition

- Inhomogeneity





Analytical extension

- In general, the rotating frame approach indicates:
- P increase with imaginary rotation
- Chiral condensation decrease with imaginary rotation
- The rotwist approach indicates:
- P at $\Omega_I = \frac{\pi T}{2}$ is smaller than $\Omega_I = 0$

Analytical extension

- At small angular velocity, the observables as functions of angular velocity change sign:

$$f(\Omega_I^2) = f(0) + \Omega_I^2 f'(0) + \mathcal{O}(\Omega_I^4)$$
$$\to f(-i\Omega_I^2) = f(0) - \Omega^2 f'(0) + \mathcal{O}(\Omega^4)$$

- At small angular velocity rotating frame approach indicates:
- P **decrease** with **real** rotation
- Chiral condensation **increase** with **real** rotation
- Which is contradict with most model predictions.

Taylor expansion

- It is possible that, the imaginary rotation does not work correctly.
- One way to go back to real rotation at small angular velocity.

$$\Omega_I = -i\Omega \qquad S^i = S_0 + \Omega_I S_1 + \Omega_I^2 S_2 = S^r = S_0 - i\Omega S_1 - \Omega^2 S_2$$

$$\langle O \rangle = \frac{f(\Omega)}{g(\Omega)} = \frac{\sum O e^{-S^r}}{\sum e^{-S^r}} \qquad \frac{d^2}{d\Omega^2}\langle O \rangle\Big|_{\Omega=0} = \frac{-2g'(0)}{g(0)}\left(\frac{d}{d\Omega}\langle O \rangle\Big|_{\Omega=0}\right) + \frac{f''(0)}{g(0)} - \frac{f(0)}{g(0)}\frac{g''(0)}{g(0)}$$

$$\frac{d^2\langle O \rangle}{d\Omega^2}\Big|_{\Omega=0} = \langle O(2S_2 - S_1^2)\rangle_0 - \langle O \rangle_0 \langle (2S_2 - S_1^2)\rangle_0$$

- 12^3 x 4, quenched, torus boundary:[JCY Xu-Guang Huang, 2303.03147]

$$\langle |L_{\text{bare}}| \rangle = 0.10498(3) - (3.0 \pm 1.1) \times 10^2 (a\Omega)^2 + \mathcal{O}(a^4\Omega^4)$$

$$\langle O \rangle_{i,r} = \frac{\sum O e^{-S^{i,r}}}{\sum e^{-S^{i,r}}} \rightarrow \frac{d^2\langle O \rangle_r}{d\Omega^2}\Big|_{\Omega=0} = -\frac{d^2\langle O \rangle_i}{d\Omega_I^2}\Big|_{\Omega_I=0}$$

Results on angular momentum

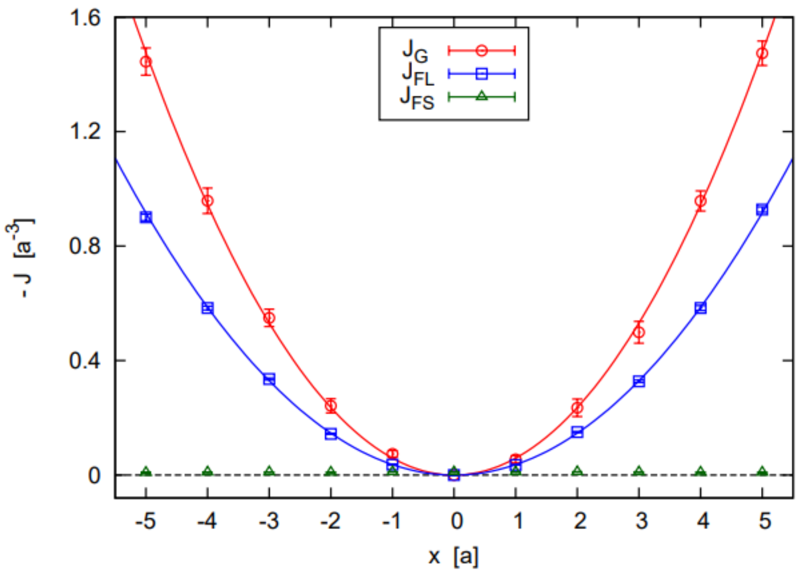
- The spin of fermion is related to the CVE.

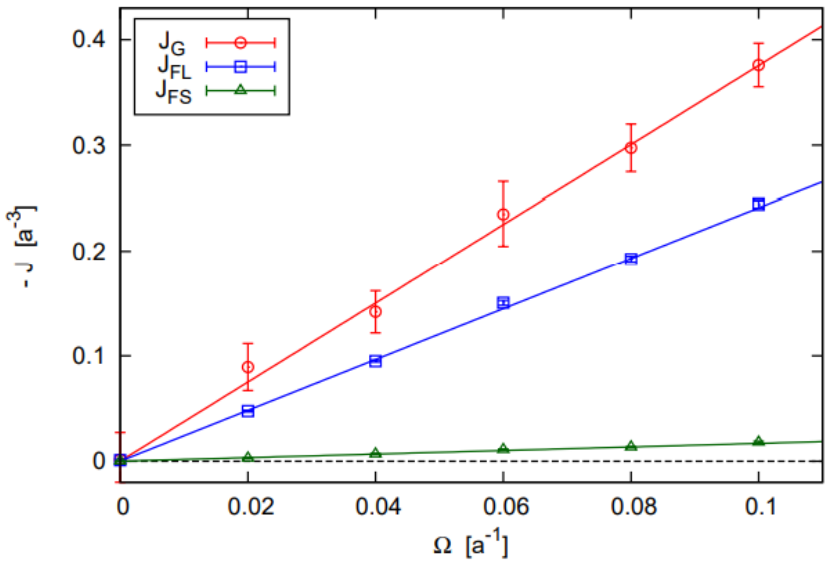
- Reveals the importance of the surface.

Quenched approximation

- [Yamamoto et al. Phys.Rev.Lett. 111 (2013) 081601 • e-Print: 1303.6292]
- Quenched approximation, Dirichlet boundary
- Negative angular momentum

$$J_G, J_F \propto \Omega r^2$$
$$J_S \propto \Omega$$



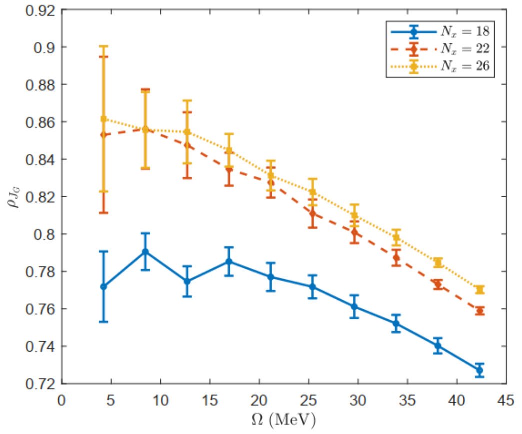


Wilson Dirac fermion

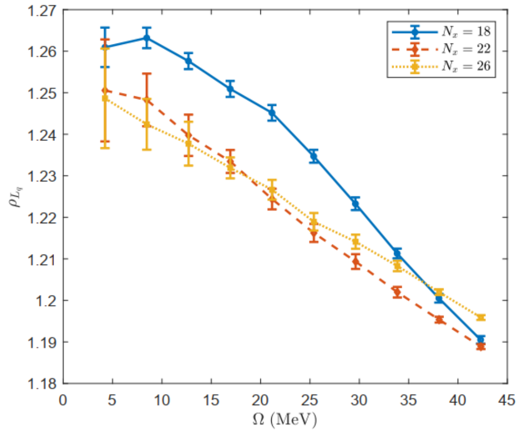
- Similar results are obtained (same temperature with different lattice spacing)

$$\rho = -\frac{1}{V}\frac{J_{G,F}}{\Omega r^2}$$

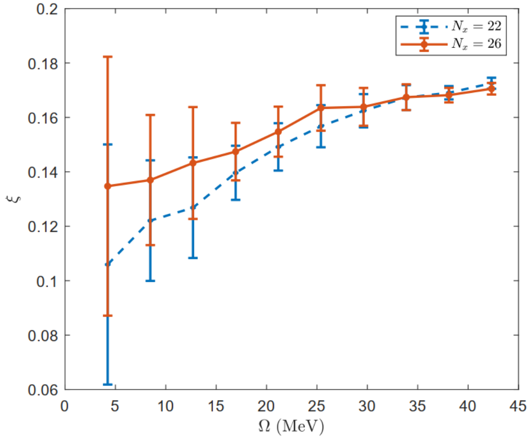
$$\xi = -\frac{1}{V}\frac{J_S}{\Omega}$$



(c) $N_x = 18, 22, 26,\ \rho_{J_G}$



(d) $N_x = 18, 22, 26,\ \rho_{L_q}$



- The order of magnitude is also as same as [Yamamoto et al. Phys.Rev.Lett. 111 (2013) 081601 • e-Print: 1303.6292]

$$J_G = -(0.94 \pm 0.01)a^{-4} \times r^2\Omega, \quad (25)$$

$$J_{FL} = -(0.60 \pm 0.01)a^{-4} \times r^2\Omega, \quad (26)$$

$$J_{FS} = -(0.17 \pm 0.01)a^{-2} \times \Omega. \quad (27)$$

Staggered fermion

gamma matrix in Euclidian space

$J_S \propto \Omega T^2$

$\langle \bar{\psi}\gamma_3\gamma_5\psi \rangle = 2J_S = -0.472(5)\Omega T^2$

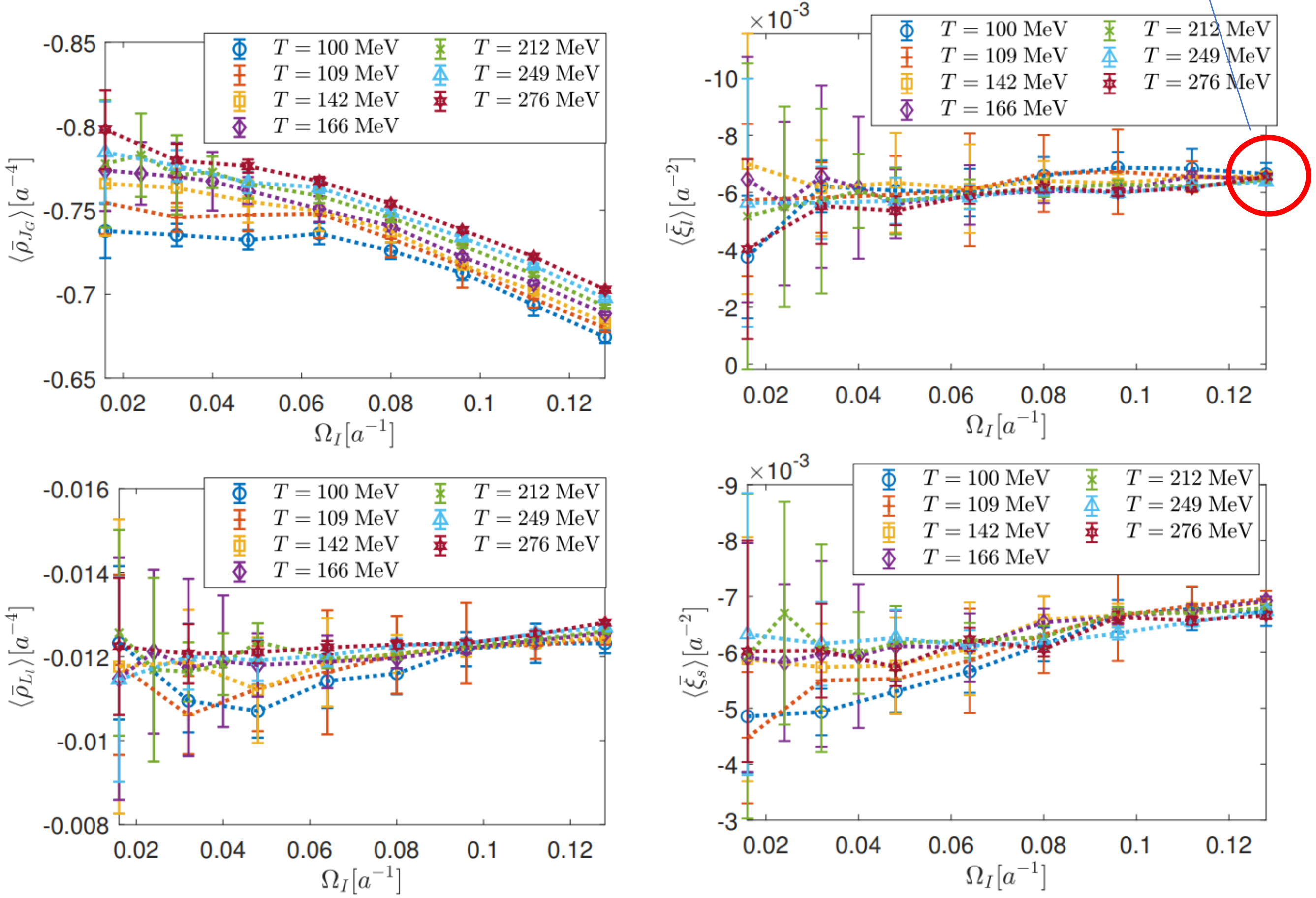
- [JCY Xu-Guang Huang, 2307.05755]
- Similar results

$$\rho = \frac{1}{V}\frac{J_{G,F}}{\Omega r^2}$$

$$\xi = \frac{1}{V}\frac{J_S}{\Omega}$$

$$\rho = \frac{1}{V}\frac{J_{G,F}}{\Omega r^2}$$

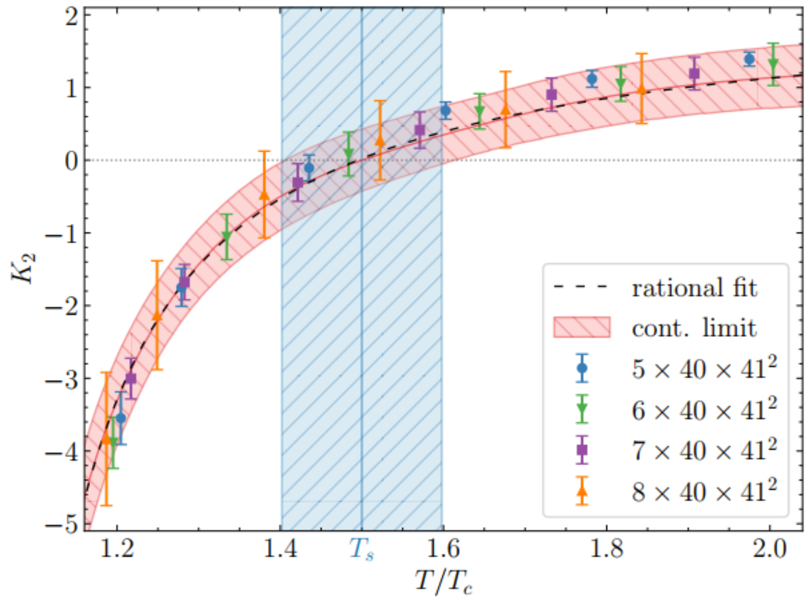
$$\xi = \frac{1}{V}\frac{J_S}{\Omega}$$



- Note: the spin is introduced in a linear form, typically, there is additive divergence renormalization is needed

Negative inertia

- [Braguta et al. 2303.03147]
- Quenched approximation



The problem of negative angular momentum

$$\Omega = i\Omega_I$$

$$S^i = S_0 + \Omega_I S_1 + \Omega_I^2 S_2 = S^r = S_0 - i\Omega S_1 - \Omega^2 S_2$$

- What we have measured?
- It is the imaginary part of angular momentum under i-rotation

$$J_I = \langle \frac{\partial S}{\partial \Omega_I}\Big|_{\Omega_I=0} \rangle = -i\langle - \frac{\partial S}{\partial \Omega}\Big|_{\Omega=0} \rangle$$

$$\frac{J_I}{\Omega_I} = \frac{1}{\Omega}\langle - \frac{\partial S}{\partial \Omega}\Big|_{\Omega=0} \rangle = \frac{J}{\Omega}$$

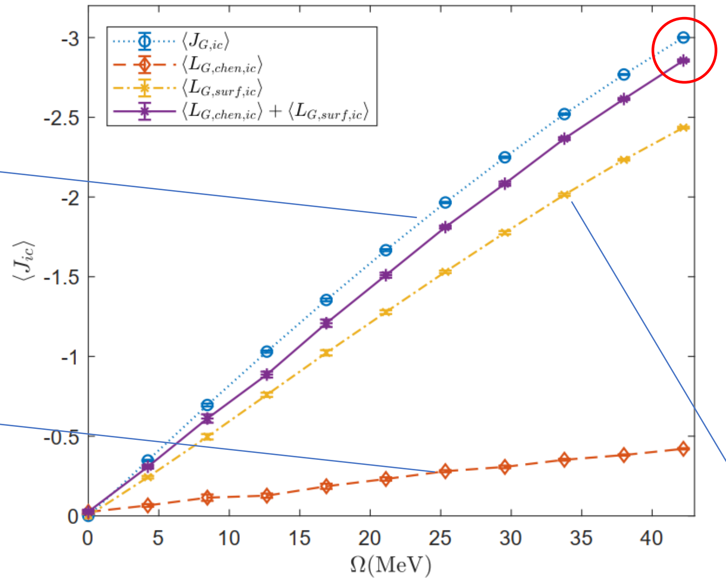
The angular momentum on the surface

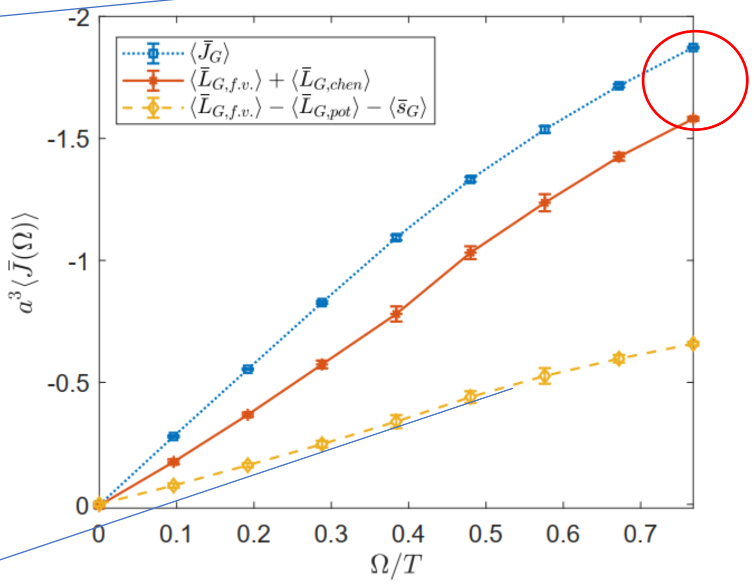
- Chen decomposition vs Wakamatsu decomposition
- Left: Wilson Dirac, right: staggered fermion

$$s_G + L_{G,pot} - L_{G,f.v} = (\nabla\cdot\mathbf{E})(\mathbf{r}\times\mathbf{A}) + \sum_i E^i\epsilon_{jkl}\left(\delta_{ki}A^l + r^k(\partial_i A^l)\right) = \partial_\mu\left(\sum_i E^i\epsilon_{jkl}r^kA^l\right)$$

Different decomposition with surface included

Different decomposition without surface included





surface

Summary

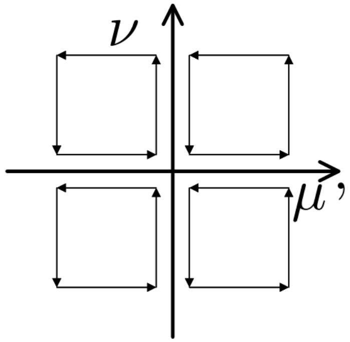
- Lattice simulations verify the phase transition caused by rotation, however,
- At small angular velocity, real rotation drives the system towards deconfinement & chiral symmetry broken.
- There is problem with the sign of angular momentum.

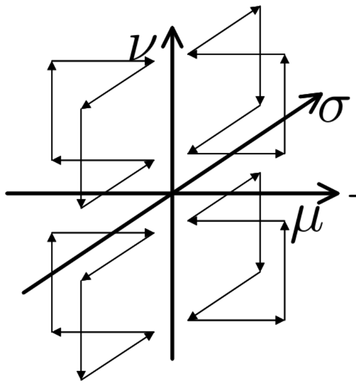
- Thank you very much!

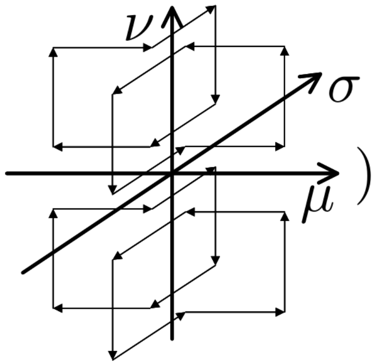
Back up

$$
S_G^E = \frac{2}{g_{YM}^2} \times \sum_n \left\{ \sum_{\mu<\nu} \mathrm{Retr}[1 - U_{\mu,\nu}(n)] - \Omega \left[x \left(\mathrm{Retr}[V_{412}] + \mathrm{Retr}[V_{432}] \right) - y \left(\mathrm{Retr}[V_{421}] + \mathrm{Retr}[V_{431}] \right) \right] \right.
$$

$$
\left. + \Omega^2 \left[r^2 \mathrm{Retr}[1 - \bar{U}_{1,2}(n)] + x^2 \mathrm{Retr}[1 - \bar{U}_{2,3}(n)] + y^2 \mathrm{Retr}[1 - \bar{U}_{1,3}(n)] - xy \mathrm{Retr}[V_{231}] \right] \right\}
$$

$\bar{U}_{\mu\nu} = \frac{1}{4}$ 

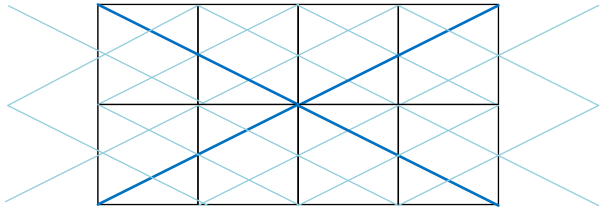
, $V_{\mu\nu\sigma} = \frac{1}{8}\Big($ 

$-$ 

$\Big)$

Staggered fermions

$$S_F^E = \int d^4x\bar{q}\left[\sum_{i=1}^{4}\gamma_i^E D_i + y\Omega\gamma_0^E D_x - x\Omega\gamma_0^E D_y + \frac{i}{2}\gamma_E^0\Omega\sigma_E^{12} + m\right] q$$



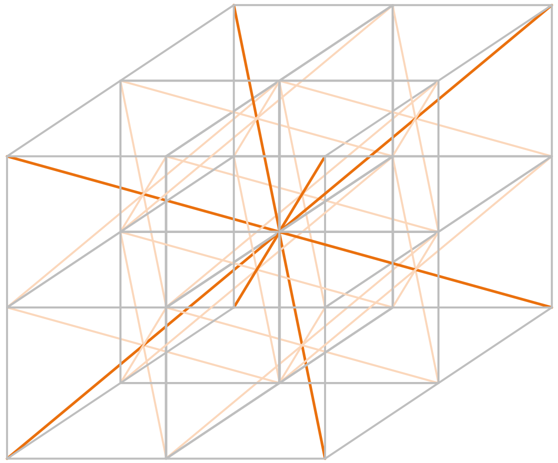
$$(2a)^4\sum_h \bar{q}(2h)\left(\sum_\mu \gamma_\mu\partial_\mu + y\Omega\gamma_4\partial_x - x\Omega\gamma_4\partial_y + \frac{i}{2}\Omega\gamma_4\sigma_E^{12} + m\right) q(2h) + \mathcal{O}(a)$$

$$= \frac{1}{2a}a^4\sum_n\left\{\sum_\mu \eta_\mu(n)\left(\bar{\chi}(n)\chi(n+\mu) - \bar{\chi}(n)\chi(n-\mu)\right) + 2am\bar{\chi}(n)\chi(n)\right.$$

$$+y\Omega\frac{1}{4}\eta_\tau(n)\bar{\chi}(n)\left[\chi(n-\tau+2x) + \chi(n+\tau+2x) - \chi(n-\tau-2x) - \chi(n+\tau-2x)\right]$$

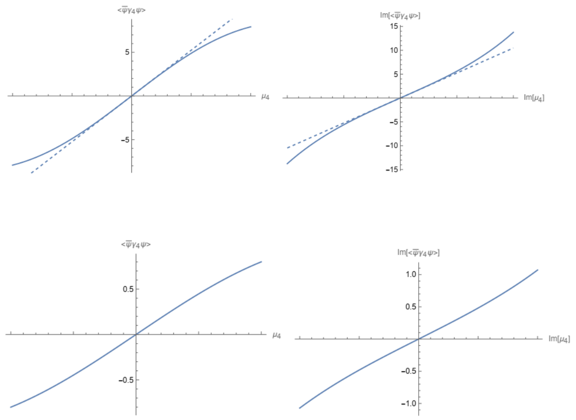
$$-x\Omega\frac{1}{4}\eta_\tau(n)\bar{\chi}(n)\left[\chi(n-\tau+2y) + \chi(n+\tau+2y) - \chi(n-\tau-2y) - \chi(n+\tau-2y)\right]$$

$$\left.+\frac{1}{8}a\Omega\sum_{\delta_{x,y,\tau}=\pm1}\left(\eta_{124}(n)\bar{\chi}(n)\chi(n+\delta)\right)\right\}$$



It seems that it is not due to analytical extension

The case of chemical potential



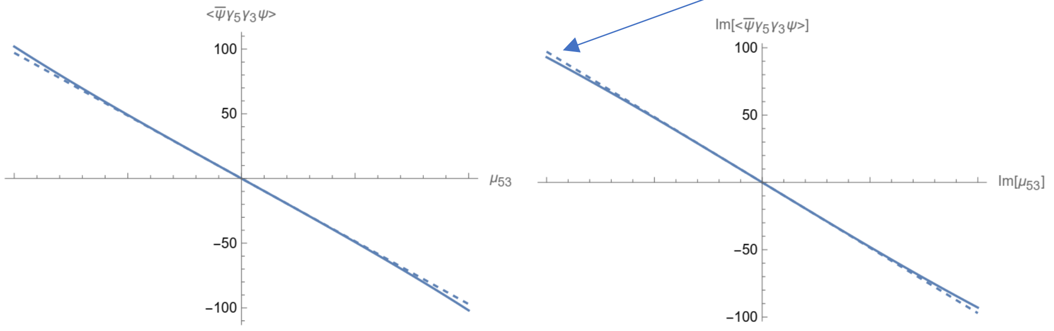
- Assuming free fermion, with only the spin term

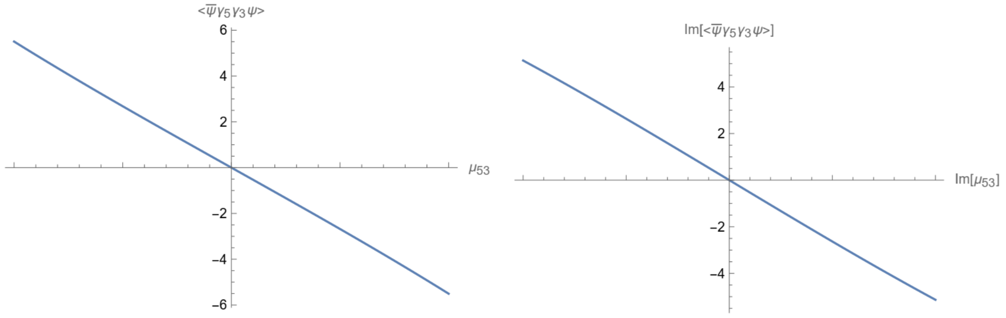
$$S_F = \bar{\psi} D \psi, \quad D = D_0 - \mu\Gamma, \quad D_0 = \sum \partial_\mu \gamma_\mu + m$$

$$c = \langle -\frac{\partial S_F}{\partial \mu} \rangle_F \qquad\qquad c = -\mathrm{tr}[\Gamma D_0^{-1}] - \mu \mathrm{tr}[\Gamma D_0^{-1} \Gamma D_0^{-1}] + \mathcal{O}(\mu^2)$$

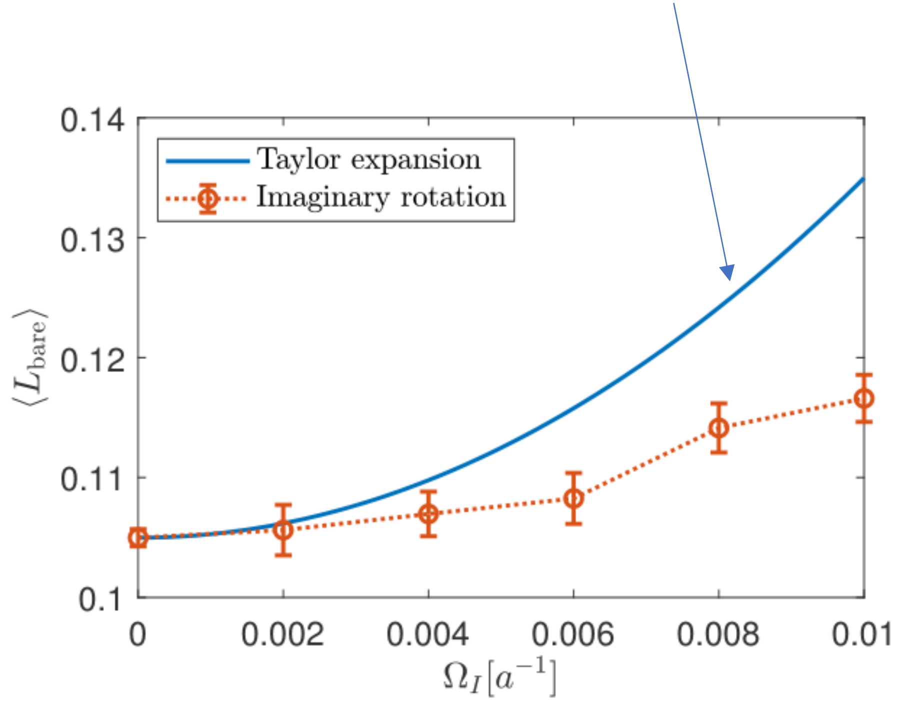
Left: naïve discretization,
Right: staggered fermion

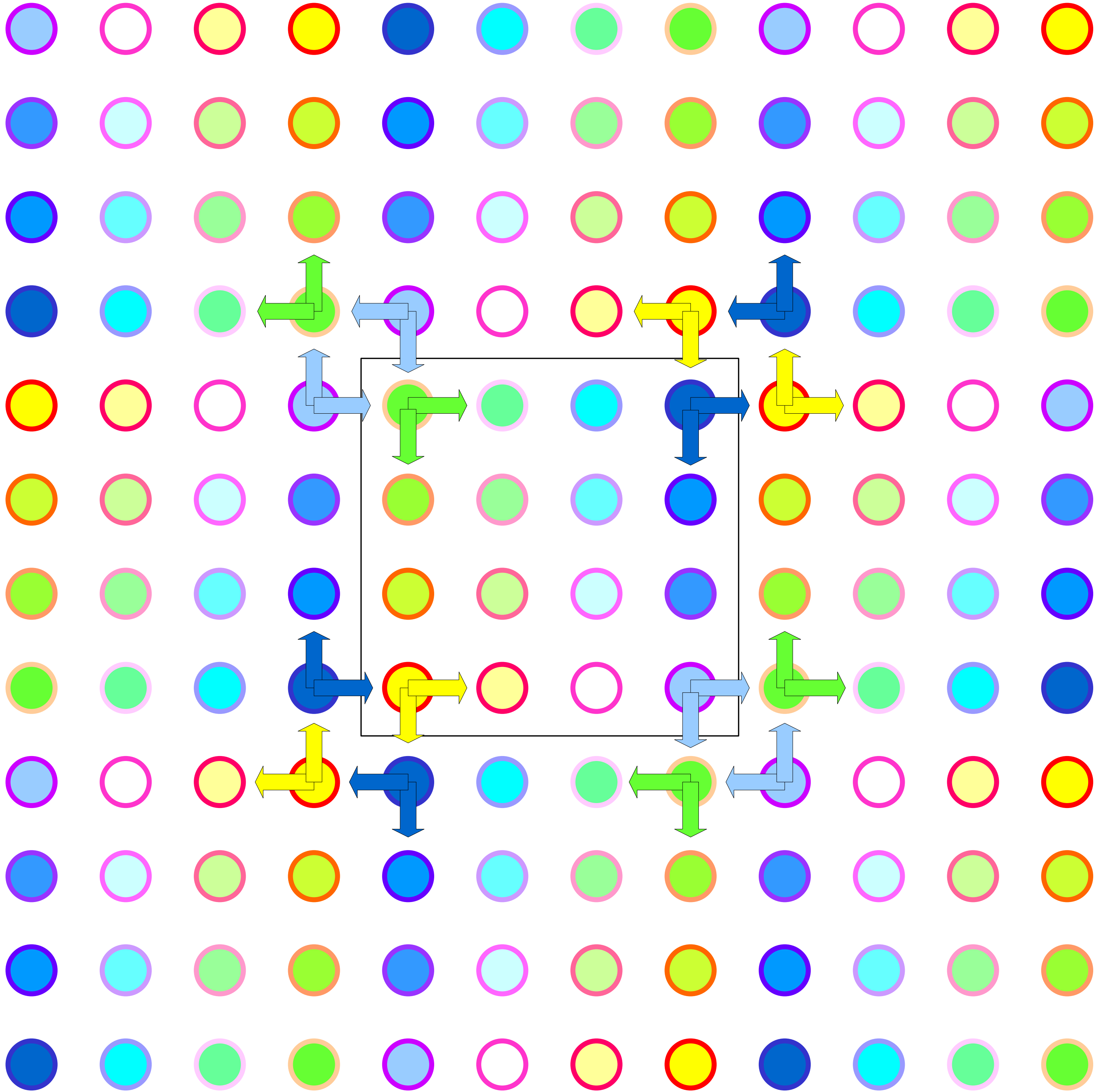
Dashed line is the first order Taylor expansion





$$0.10498(3) + (-3.0 \pm 1.1) \times 10^2 (ai\Omega_I)^2,$$





Eigen value

- 1606.03808
- Considering in a cylinder

$$z = re^{i\theta},\ z^{2n} = (-z)^{2n}.$$

$$u = \frac{e^{in\theta + ik_z z}\sqrt{E_k + m}}{\sqrt{4E_k}} \begin{pmatrix} J_n(k_t r) \\ s e^{i\theta} J_{n+1}(k_t r) \\ \frac{k_z - isk_t}{E_k + m} J_n(k_t r) \\ \frac{ik_t - sk_z}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \end{pmatrix},$$

$e^{2in\theta} f(r)$ satisfy the projective plane boundary condition.

$$v = \frac{e^{in\theta - ik_z z}\sqrt{E_k + m}}{\sqrt{4E_k}} \begin{pmatrix} \frac{k_z - isk_t}{E_k + m} J_n(k_t r) \\ \frac{ik_t - sk_z}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \\ J_n(k_t r) \\ -s e^{i\theta} J_{n+1}(k_t r) \end{pmatrix},$$

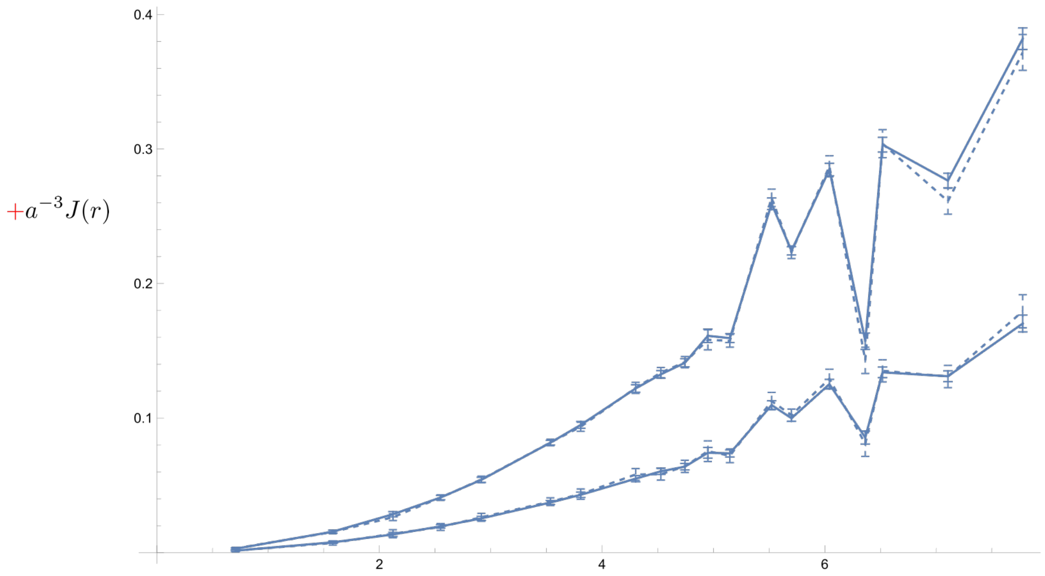
$$J_{2n+1}(k_t r) = 0.$$

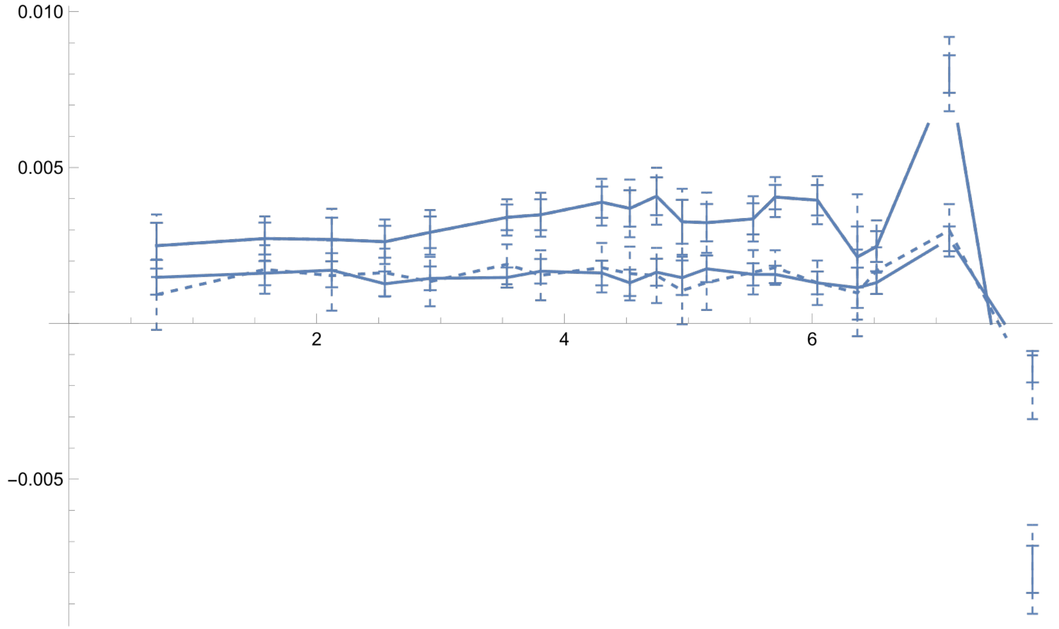
→ Satisfy projective plane boundary condition

The case of external magnetic field is similar

Lq and Sq

- Dashed line for ud-quark, solid line for s-quark
- aOmega = 0.064, 0.128





$$\langle \bar{\psi}_l \psi_l \rangle = \mathrm{tr}\left[D_l^{-1} \right] / 4 N_x^3 N_\tau$$

$$\Delta_{l,s}(T,\Omega) = \frac{\langle \bar{\psi}_l \psi_l \rangle_{T,\Omega} - \frac{m_l}{m_s} \langle \bar{\psi}_s \psi_s \rangle_{T,0}}{\langle \bar{\psi}_l \psi_l \rangle_{0,0} - \frac{m_l}{m_s} \langle \bar{\psi}_s \psi_s \rangle_{0,0}},$$

$$\langle \bar{\psi}_q \psi_q \rangle = c_q + m_q \times \infty$$

$$\langle \bar{\psi}_q \psi_q \rangle \propto a^{-3}$$

Analytical extension

- When higher order are considered, it could be this:
- Too early to conclude that, real rotation will drive to confinement and chiral symmetry broken.

Solid and dots:

Fit to

$$a + b\Omega^2 + c\Omega^4$$

dashed

$$a - b\omega^2 + c\omega^4$$