Chiral Magnetovortical Instability

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Outline

Outline

Alfvén Wave

Considering ideal magnetohydrodynamics(MHD), one obtains the known MHD waves - Alfvén wave in incompressible fluid by

$$\partial_t \boldsymbol{b}_1 = (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{v}_1, \qquad \partial_t \boldsymbol{v}_1 = \frac{1}{\mu_0 \rho_0}(\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{b}_1 \tag{1}$$

where $\boldsymbol{B}_0$-background field, $\boldsymbol{b}_1, \boldsymbol{v}_1$-perturbations, $\boldsymbol{B}_0 \perp \boldsymbol{b}_1(\boldsymbol{v}_1)$, ρ_0-mass density, μ_0-vacuum magnetic permeability . For plane waves

$$\boldsymbol{v}_1, \boldsymbol{b}_1 \sim exp[i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)] \tag{2}$$

the dispersion relation is

$$\omega^2 - \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\mu_0 \rho_0} = 0 \Rightarrow \omega = \frac{\boldsymbol{B}_0 \cdot \boldsymbol{k}}{\sqrt{\mu_0 \rho_0}} \tag{3}$$

Chiral Alfvén Wave-CAW

The Chiral hydrodynamics eqs. for plasmas of single right-handed chiral fermions in external electromagnetic fields are [1]

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \qquad \partial_\mu j^\mu = -C E^\mu B_\mu \tag{4}$$

$E^\mu = F^{\mu\nu} u_\nu, B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$ are defined in the fluid rest frame, C is anomaly coefficient. In the Landau-Lifshitz frame [1, 2], taking $|\boldsymbol{v}| \ll 1$, assuming plasmas with homogeneous and static ϵ, P, n, high temperature $T \gg \mu$(and so n)[2]

$$(\epsilon + P)\partial_t \boldsymbol{v} = \xi_\omega \boldsymbol{\omega} \times \boldsymbol{B}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 \tag{5}$$

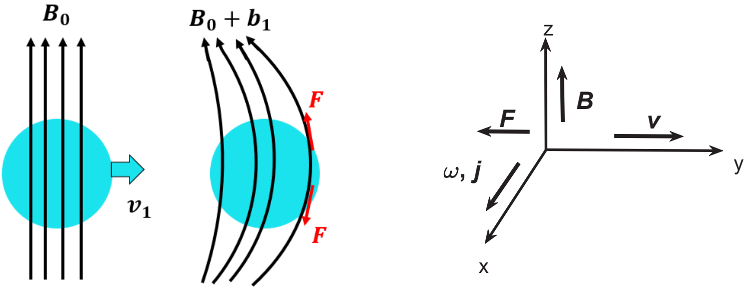
here $\xi_\omega \boldsymbol{\omega}$ is the chiral vortical effet(CVE).By setting $\boldsymbol{B} \cdot \boldsymbol{v} = 0$, $\boldsymbol{B} = B\hat{z}$. The plane wave solution of Right-CAW

$$\omega = -\frac{\xi_\omega B}{\epsilon + P} k_z \qquad (\xi_\omega \approx \frac{DT^2}{2}, D = \frac{1}{12}) \tag{6}$$

[1] D.T.Son and P.Surowka,PRL.103,191601(2009)

[2] N.Yamamoto,PRL.115,141601(2015)

Alfvén Wave and CAW



(a) Alfvén wave

(b) chiral Alfvén wave

Figure 1: the intuitive pictures

(a):the magnetic tension(restoring force)balances inertia,so disturbances are communicated by Alfvén waves.(b):the CVE induces a Lorentz force, where $\boldsymbol{v} = v(z)\hat{\boldsymbol{y}}$ with $\partial_z v(z) < 0$.

Outline

CVE and dynamical electromagnetic field

A natural extension is the CAW with dynamical EM field . [3, 4]

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\lambda} j_\lambda \\ \partial_\mu j_A^\mu &= C_A E^\mu B_\mu, \qquad \partial_\mu j^\mu = 0 \\ \partial_\mu F^{\mu\nu} &= j^\nu, \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0 \end{aligned} \tag{7}$$

where $\tilde{F}^{\mu\nu}$ is the dual of $F^{\mu\nu}$, j^μ-vector current,j_A^μ -axial current. Then

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \tau^{\mu\nu} \\ j^\mu &= n u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu + \nu^\mu \\ j_A^\mu &= n_A u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu \end{aligned} \tag{8}$$

$\xi_\omega(\kappa_\omega)$ -vector(axial) CVE, $\xi_B(\kappa_B)$ -vector(axial) CME.

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[3] N.Yamamoto,PRD.93,065017(2016).

[4] S.Pu,J.Gao,Q.Wang, arXiv:1008.2418v2[nucl-th]; M.Isachenkov and A.Sadofyev,PLB 697,404(2011)

CVE and dynamical electromagnetic field

Considering the nonrelativistic limit $|\boldsymbol{v}| \ll 1$. The power counting : $\partial_t \sim \mathcal{O}(\epsilon_t), \boldsymbol{\nabla} \sim \mathcal{O}(\epsilon_s)$, $\boldsymbol{v}, \boldsymbol{b} \sim \mathcal{O}(\delta)$, they are three independent expansion parameters $\epsilon_s, \epsilon_t \ll 1$, $\delta \ll 1$, so the equations of motion

$$u_\nu \partial_\mu T^{\mu\nu} = u_\nu F^{\nu\lambda} j_\lambda, \quad (g^\rho_\nu - u^\rho u_\nu)\partial_\mu T^{\mu\nu} = (g^\rho_\nu - u^\rho u_\nu) F^{\nu\lambda} j_\lambda \tag{9}$$

keeping terms to $\mathcal{O}(\epsilon_t\delta, \epsilon_s\delta^2, \epsilon_s^2\delta)$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\epsilon + (\epsilon + P)\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0 \tag{10}$$

$$(\boldsymbol{v}\cdot\boldsymbol{\nabla})P = 0 \tag{11}$$

$$(\epsilon + p)(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla}P - \boldsymbol{v}(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})P + \boldsymbol{j}\times\boldsymbol{B} + n\boldsymbol{E} + \nu\boldsymbol{\nabla}^2\boldsymbol{v} \tag{12}$$

also the vector current conservation and axial current equation

$$\partial_t n + \boldsymbol{\nabla}\cdot\boldsymbol{j} = 0, \quad \boldsymbol{j} = n\boldsymbol{v} + \xi_\omega\boldsymbol{\omega} + \xi_B\boldsymbol{B} + \sigma(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) \tag{13}$$

$$\partial_t n_A + \boldsymbol{\nabla}\cdot\boldsymbol{j}_A = C_A\boldsymbol{E}\cdot\boldsymbol{B}, \quad \boldsymbol{j}_A = n_A\boldsymbol{v} + \kappa_\omega\boldsymbol{\omega} + \kappa_B\boldsymbol{B} \tag{14}$$

CVE and dynamical electromagnetic field

Taking the conductivity σ is large enough, $n\boldsymbol{v} = (\boldsymbol{\nabla} \cdot \boldsymbol{E})\boldsymbol{v} \ll \sigma \boldsymbol{E}$, also $n\boldsymbol{E}$, assuming P, ϵ are constants, $\nu = 0$, then keeping $\mathcal{O}(\delta)$

$$\begin{aligned}
(\epsilon + P)\partial_t \boldsymbol{v} &\approx \boldsymbol{j} \times \boldsymbol{B} \\
\boldsymbol{j} &\approx \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega} \\
\nabla \cdot \boldsymbol{v} &= 0 \\
\nabla \times \boldsymbol{B} &\approx \boldsymbol{j} \\
\nabla \times \boldsymbol{E} &= -\partial_t \boldsymbol{B} \\
\nabla \cdot \boldsymbol{B} &= 0
\end{aligned} \tag{15}$$

where $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$. In order to describe $\partial_t \boldsymbol{B}$, we rewrite $\boldsymbol{E}$ by $\boldsymbol{B}, \boldsymbol{v}$

$$\begin{aligned}
\boldsymbol{E} &= \eta(\boldsymbol{j} - \xi_B \boldsymbol{B} - \xi_\omega \boldsymbol{\omega}) - \boldsymbol{v} \times \boldsymbol{B} \\
&= \eta(\nabla \times \boldsymbol{B} - \xi_B \boldsymbol{B} - \xi_\omega \boldsymbol{\omega}) - \boldsymbol{v} \times \boldsymbol{B}
\end{aligned} \tag{16}$$

here $\eta = 1/\sigma$ is resistivity.

CVE and dynamical electromagnetic field

The evolution eqs of hydro and magnetic field are [5, 6, 7]

$$(\epsilon + P)\partial_t \boldsymbol{v} = (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = (\boldsymbol{B} \cdot \nabla)\boldsymbol{B} - \frac{1}{2}\nabla(\boldsymbol{B}^2) \quad (17)$$

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta(\nabla^2 \boldsymbol{B} + \xi_B \nabla \times \boldsymbol{B} + \xi_\omega \nabla \times \boldsymbol{\omega}) \quad (18)$$

Considering small perturbation $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b} = B_0 \hat{\boldsymbol{z}} + \boldsymbol{b}$ ($\boldsymbol{b} \sim \mathcal{O}(\delta)$)

$$(\epsilon + P)\partial_t \boldsymbol{v} = (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{b} - \nabla(\boldsymbol{B}_0 \cdot \boldsymbol{b}) \quad (19)$$

$$\partial_t \boldsymbol{b} = (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{v} + \eta\nabla^2 \boldsymbol{b} + \eta\xi_B \nabla \times \boldsymbol{b} - \eta\xi_\omega \nabla^2 \boldsymbol{v} \quad (20)$$

(19) implies $\nabla^2(\boldsymbol{B}_0 \cdot \boldsymbol{b}) = 0$, so we set $\boldsymbol{B}_0 \cdot \boldsymbol{b} = 0$, also $\boldsymbol{B}_0 \cdot \boldsymbol{v} = 0$(explicitly $v_z =$const), here we take $\boldsymbol{b} = (b_1, b_2, 0)$, $\boldsymbol{v} = (v_1, v_2, 0)$.

[5] N.Yamamoto,PRD.93,065017(2016).

[6] M.Giovannini,PRD.93,103518(2016)

[7] M.Giovannini,PRD.88,063536(2013)

Dispersion Relation and Instability

For plane-wave solution, (19),(20) lead to

$$(\epsilon + P)(-\omega)\boldsymbol{v} = (\boldsymbol{B}_0 \cdot \boldsymbol{k})\boldsymbol{b} \tag{21}$$

$$(\eta \boldsymbol{k}^2 - i\omega)\boldsymbol{b} - i\eta\xi_B \boldsymbol{k} \times \boldsymbol{b} = [i(\boldsymbol{B}_0 \cdot \boldsymbol{k}) + \eta\xi_\omega \boldsymbol{k}^2]\boldsymbol{v} \tag{22}$$

we just focus on the CVE (setting $\xi_B = 0$)

$$\omega^2 + i\eta\boldsymbol{k}^2\omega + i\eta\xi_\omega\boldsymbol{k}^2\frac{\boldsymbol{B}_0 \cdot \boldsymbol{k}}{\epsilon + P} - \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} = 0 \tag{23}$$

whose solutions are

$$\omega = \omega_\pm \equiv \frac{1}{2}[-i\eta\boldsymbol{k}^2 \pm \sqrt{\frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} - \eta^2\boldsymbol{k}^4 - i \cdot \eta\xi_\omega\boldsymbol{k}^2\frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\epsilon + P}}] \tag{24}$$

from (24), we will find a new instability which is related to the CVE and magentic field.

Dispersion Relation and Instability

We use geometric method to explain (24). First, setting

$$\omega = \omega_1 + i \cdot \omega_2 \tag{25}$$

here $\omega_1, \omega_2 \in \mathbb{R}$, then (23) leads to

$$C_1 : \quad \omega_1^2 - (\omega_2 + \eta \boldsymbol{k}^2/2)^2 = \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} - \frac{\eta^2 \boldsymbol{k}^4}{4} \tag{26}$$

$$C_2 : \quad \omega_2 = -\frac{\eta \boldsymbol{k}^2}{2} \frac{\xi_\omega}{\sqrt{\epsilon + P}} \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\sqrt{\epsilon + P}} \cdot \frac{1}{\omega_1} - \frac{\eta \boldsymbol{k}^2}{2} \tag{27}$$

C_1, C_2 are two hyperbola curves, the cross points of them are solutions. One can draw the figures of two curves. For example, considering a case: $(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2/(\epsilon + P) > \eta^2 \boldsymbol{k}^4/4$ and $\boldsymbol{B}_0 \cdot \boldsymbol{k} > 0$, we show a schematic diagram

Dispersion Relation and Instability

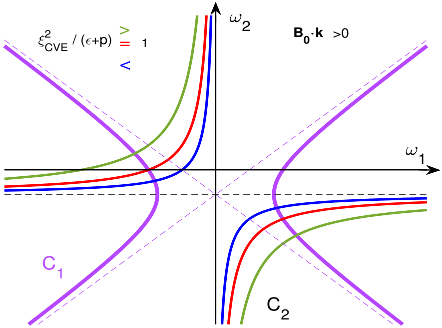


Figure 2: $\omega_2 - \omega_1$ of C_1, C_2

Green-$\xi_\omega > \sqrt{\epsilon + P}$, **Red**-$\xi_\omega = \sqrt{\epsilon + P}$, **Blue**-$\xi_\omega < \sqrt{\epsilon + P}$

Dispersion Relation and Instability

We summarize the real(Re) part and imaginary(Im) part of $\omega_\pm$ in (24).

$\boldsymbol{B}_0 \cdot \boldsymbol{k}$	CVE	Re(ω_+)	Im(ω_+)
$+$	$0 < \xi_\omega$	$+$	$-$
$-$	$\sqrt{\epsilon + P} < \xi_\omega$	$+$	$+$
	$\sqrt{\epsilon + P} = \xi_\omega$	$+$	0
	$\sqrt{\epsilon + P} > \xi_\omega$	$+$	$-$
$\boldsymbol{B}_0 \cdot \boldsymbol{k}$	CVE	Re(ω_-)	Im(ω_-)
$-$	$0 < \xi_\omega$	$-$	$-$
$+$	$\sqrt{\epsilon + P} < \xi_\omega$	$-$	$+$
	$\sqrt{\epsilon + P} = \xi_\omega$	$-$	0
	$\sqrt{\epsilon + P} > \xi_\omega$	$-$	$-$

$+(-)$ means a positive(negative) value. Because $\boldsymbol{v}, \boldsymbol{b} \sim exp[i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)]$, Im($\omega_\pm$) > 0 means an instability.

Dispersion Relation and Instability

Let's go back to (24) again

$$\omega = \omega_{\pm} \equiv \frac{1}{2}[-i\eta \boldsymbol{k}^2 \pm \sqrt{\frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} - \eta^2 \boldsymbol{k}^4 - i \cdot \eta \xi_\omega \boldsymbol{k}^2 \frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\epsilon + P}}] \tag{24}$$

the instability occurs once $\xi_\omega > \sqrt{\epsilon + P}$ and applicable to any $|\boldsymbol{k}|$.
Because the CVE and magnetic field are both included, we call this instability Chiral Magnetovortical Instability(CMVI).
Considering the small and large $|\boldsymbol{k}|$ expansions

$$\omega \approx \pm \underbrace{\frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\sqrt{\epsilon + P}}}_{Alfven\ wave} - i\frac{\eta}{2}(1 \pm \frac{\xi_\omega}{\sqrt{\epsilon + P}})\boldsymbol{k}^2 \tag{28}$$

$$\omega \approx -i\frac{\eta}{2}\boldsymbol{k}^2 \pm i\frac{\eta}{2}\boldsymbol{k}^2 \pm \underbrace{\frac{-\xi_\omega(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{(\epsilon + P)}}_{CAW} \pm i \cdot [\frac{\xi_\omega^2}{(\epsilon + P)} - 1] \cdot \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{(\epsilon + P)\eta \boldsymbol{k}^2} \tag{29}$$

Dispersion Relation and Instability

An intuitive picture about the CMVI.

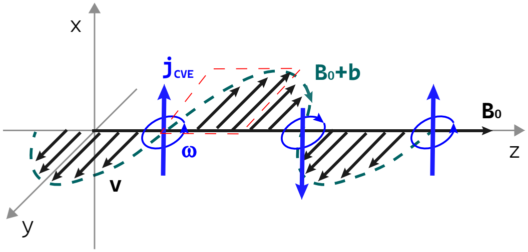


Figure 3: CMVI

The CVE currents induce new magnetic fileds around themselves, the new magnetic fields will add the perturbed magnetic fields $\boldsymbol{b}$. If the CVE is strong enough to overcome the dissipative effects ($\xi_\omega > \sqrt{\epsilon + P}$), then $\boldsymbol{b}$ increases in red region.

CME-Chiral Plasma Instability

Checking the CME term ξ_B, thus (19),(20) lead to

$$\omega = \frac{1}{2}[-i\eta k_3(k_3 - \lambda\xi_B) \pm |k_3|\sqrt{-\eta^2(k_3 - \lambda\xi_B)^2 + \frac{4B_0^2}{\epsilon + P}}] \quad (\lambda = \pm 1) \tag{30}$$

The instability appears in $|k_3| \in (0, \xi_B)$-Chiral Plasma Instability(CPI) [8].

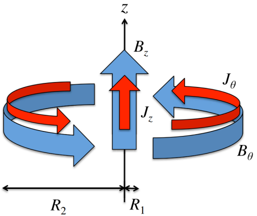


Figure 4: Intuitive picture of the CPI[9].

[8] Y.Akamatsu and N.Yamamoto,PRL.111,052002(2013)

[9] Y.Akamatsu and N.Yamamoto, Phys.Rev.D 90,125031(2014)

Outline

The conservation of helicities

The chiral anomaly equation in nonrelativistic limit-(14)

$$\partial_\mu j_A^\mu = \partial_0 j_A^0 + \boldsymbol{\nabla} \cdot \boldsymbol{j}_A = C_A \boldsymbol{E} \cdot \boldsymbol{B} \tag{31}$$

one can integrate it and ignore the surface term

$$\partial_t \Big(\int d^3\boldsymbol{x} j_A^0 + \frac{C_A}{2} \mathcal{H}_B \Big) = 0 \tag{32}$$

where $j_A^0 = n_A + \kappa_B \boldsymbol{v} \cdot \boldsymbol{B} + \kappa_\omega \boldsymbol{v} \cdot \boldsymbol{\omega}$. Next we introduce different helicities

$$\mathcal{H}_B \equiv \int d^3\boldsymbol{x} \boldsymbol{A} \cdot \boldsymbol{B} \qquad \mathcal{H}_A \equiv \int d^3\boldsymbol{x}\, n_A \tag{33}$$

$$\mathcal{H}_c \equiv \int d^3\boldsymbol{x} \boldsymbol{v} \cdot \boldsymbol{B} \qquad \mathcal{H}_v \equiv \int d^3\boldsymbol{x} \boldsymbol{v} \cdot \boldsymbol{\omega} \tag{34}$$

here $\mathcal{H}_{B/A/c/v}$ are magnetic,fermion,cross,fluid helicities. Therefore

$$\partial_t [\mathcal{H}_A + \kappa_B \mathcal{H}_c + \kappa_\omega \mathcal{H}_v + \frac{C_A}{2} \mathcal{H}_B] = 0 \tag{35}$$

The conservation of helicities

However, the fermion helicity is not conserved due to the mass of chiral fermions. Generally, near-equilibrium decay law tells us

$$\partial_t n_A \propto -\Gamma(m) n_A \tag{36}$$

the notation $\Gamma(m)$ is the chirality relaxation rate in order to account for the chirality-flipping process due to mass, with $\Gamma(m=0)=0$. Thus

$$\chi_A \partial_t \mu_A = -\frac{C_A}{2}\partial_t \bar{\mathcal{H}}_b - \kappa_B \partial_t \bar{\mathcal{H}}_c - \kappa_\omega \partial_t \bar{\mathcal{H}}_v - \Gamma \chi_A \mu_A \tag{37}$$

where we have assumed a homogeneity of the system, $n_A = \chi_A \mu_A(t)$, $\chi_A \propto T^2$, χ_A is the chiral susceptibility. And $\kappa_\omega \propto T^2$, $\kappa_B \propto \mu$, the chermical potential μ and temperature T are taken as constants. Here, we define the quantum statistical ensemble average helicities $< ... > \equiv V^{-1} \int d^3\boldsymbol{x}(...)$, so one get $\bar{\mathcal{H}}_{A/c/v/B} \equiv V^{-1}\mathcal{H}_{A/c/v/B}$.

The numerical solution

First, we define

$$\boldsymbol{B}_0' \equiv \frac{\boldsymbol{B}_0}{\sqrt{\epsilon + P}} \qquad \boldsymbol{b}' \equiv \frac{\boldsymbol{b}}{\sqrt{\epsilon + P}} \qquad \xi_\omega' \equiv \frac{\xi_\omega}{\sqrt{\epsilon + P}} \tag{38}$$

thus the (19),(20) are rewrited as

$$\partial_t \boldsymbol{v} = \boldsymbol{B}_0' \cdot \boldsymbol{\nabla} \boldsymbol{b}' \tag{39}$$

$$\partial_t \boldsymbol{b}' = \boldsymbol{B}_0' \cdot \boldsymbol{\nabla} \boldsymbol{v} + \eta \nabla^2 \boldsymbol{b}' - \eta \xi_\omega' \nabla^2 \boldsymbol{v} \tag{40}$$

we expand the fields in their Fourier modes

$$\boldsymbol{v}(t, \boldsymbol{x}) = \int_{\boldsymbol{k}} \boldsymbol{v}(t, \boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \qquad \boldsymbol{b}'(t, \boldsymbol{x}) = \int_{\boldsymbol{k}} \boldsymbol{b}'(t, \boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \tag{41}$$

where $\int_{\boldsymbol{k}} \equiv \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}$.

The numerical solution

For each Fourier mode, we further expand it in helicity basis with $\boldsymbol{e}_3(\boldsymbol{k}) = \hat{\boldsymbol{k}} \equiv \boldsymbol{k}/|\boldsymbol{k}|$, $\boldsymbol{e}_\pm(\boldsymbol{k})$ as the right-hand and left-hand helicity basis vectors, they satisfy the relations $\hat{\boldsymbol{k}} \times \boldsymbol{e}_\pm(\boldsymbol{k}) = \mp i \boldsymbol{e}_\pm(\boldsymbol{k})$,$\hat{\boldsymbol{k}} \cdot \boldsymbol{e}_\pm(\boldsymbol{k}) = 0$, $\boldsymbol{e}_\pm(\boldsymbol{k}) \cdot \boldsymbol{e}^*_\pm(\boldsymbol{k}) = 1$,$\boldsymbol{e}_\pm(\boldsymbol{k}) \cdot \boldsymbol{e}^*_\mp(\boldsymbol{k}) = 0$. The solenoidal conditions for $\boldsymbol{v}$ and $\boldsymbol{B}$ imply that

$$\boldsymbol{v}(t,\boldsymbol{k}) = v_+(t,\boldsymbol{k})\boldsymbol{e}_+(\boldsymbol{k}) + v_-(t,\boldsymbol{k})\boldsymbol{e}_-(\boldsymbol{k}) \tag{42}$$

$$\boldsymbol{b}'(t,\boldsymbol{k}) = b'_+(t,\boldsymbol{k})\boldsymbol{e}_+(\boldsymbol{k}) + v'_-(t,\boldsymbol{k})\boldsymbol{e}_-(\boldsymbol{k}) \tag{43}$$

Thus

$$\partial_t v_\pm = i(\boldsymbol{B}'_0 \cdot \boldsymbol{k}) b'_\pm \tag{44}$$

$$\partial_t b'_\pm = i(\boldsymbol{B}'_0 \cdot \boldsymbol{k}) v_\pm + \eta \xi'_\omega \boldsymbol{k}^2 v_\pm - \eta \boldsymbol{k}^2 b'_\pm \tag{45}$$

we can see that both helicities satisfy the same equations.

The numerical solution

so the average kinetic energy,magnetic energy and various helicities are

$$\mathcal{E}_{\boldsymbol{v}}(t) = \frac{1}{2} < \boldsymbol{v}^2 >= \frac{1}{2V}\int_{\boldsymbol{k}} (|v_+|^2 + |v_-|^2) \tag{46}$$

$$\mathcal{E}_{\boldsymbol{b}'}(t) = \frac{1}{2} < \boldsymbol{b}' >= \frac{1}{2V}\int_{\boldsymbol{k}} (|b'_+|^2 + |b'_-|^2) \tag{47}$$

$$\bar{\mathcal{H}}_c(t) =< \boldsymbol{v} \cdot \boldsymbol{B} >= \frac{1}{V}\int_{\boldsymbol{k}} (v_+ b_+^* + v_- b_-^*) \tag{48}$$

$$\bar{\mathcal{H}}_{\boldsymbol{v}}(t) =< \boldsymbol{v} \cdot \boldsymbol{\omega} >= \frac{1}{V}\int_{\boldsymbol{k}} |\boldsymbol{k}| \cdot (|v_+|^2 - |v_-|^2) \tag{49}$$

$$\bar{\mathcal{H}}_b(t) =< \boldsymbol{A} \cdot \boldsymbol{B} >= \frac{1}{V}\int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|} \cdot (|b_+|^2 - |b_-|^2) \tag{50}$$

when a finite Γ is present,to highlight the effect of CMVI, we have chosen an initial condition such that $\mathcal{H}_b(t=0) = \mathcal{H}_v(t=0) = 0$, which implies that they remain zero throughout the time evolution.

The numerical solution

Initial values: $\xi'_\omega = 5$, $|\boldsymbol{B}'_0| = 5$, $v_+(0) = b'(0) = \frac{0.1}{exp(10\eta|k_z|-100)+1}$.(lines in red/blue/purple/orange correspond to $\eta\Gamma = 0, 0.01, 0.02, 0.03$.)

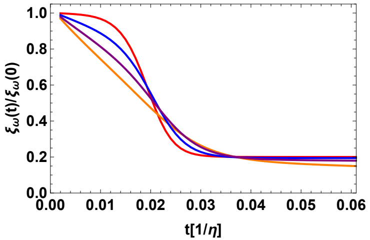
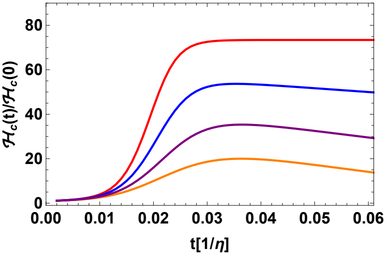


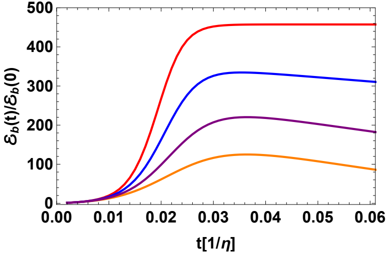
Figure 5: Evolution of ξ'_ω

- The CVE coefficient ξ'_ω decrease with time
- $\Gamma = 0$ means a pure Alfvén wave, $\frac{\xi'_\omega}{\xi'_\omega(0)} = 0.2$ in final time,so $\xi'_\omega = 1$
- $\Gamma \neq 0$, $\xi'_\omega(t) \to 0$ with time increases

The numerical solution



(a) Evolution of $\mathcal{H}_c$



(b) Evolution of $\mathcal{E}_b$

Figure 6: the cross helicity $\mathcal{H}_c$ and magnetic energy $\mathcal{E}_b$

- $\mathcal{H}_c$ and $\mathcal{E}_b$ increase with time
- $\Gamma = 0$ means a pure Alfvén wave, $\mathcal{H}_c/\mathcal{E}_b$ are stationary in final time
- $\Gamma \neq 0$, $\mathcal{H}_c/\mathcal{E}_b \uparrow$ first, then $\downarrow$. The turning point satisfies $\xi'_\omega(t) = 1$
- $\mathcal{E}_b \uparrow$ means a dynamo effect, the magnification is 10-20

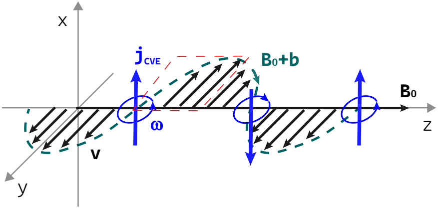
Outline

Summary and Outlook

1.A new instability-Chiral Magnetovortical Instability (CMVI).

$$\omega = \omega_{\pm} \equiv \frac{1}{2}[-i\eta \boldsymbol{k}^2 \pm \sqrt{\frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} - \eta^2 \boldsymbol{k}^4 - i \cdot \eta \xi_\omega \boldsymbol{k}^2 \frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\epsilon + P}}]$$

the CMVI occurs when $\xi_\omega > \sqrt{\epsilon + P}$ and for any value of $|\boldsymbol{k}|$.



2. The CMVI will supply a new dynamo action.

Summary and Outlook

3. A pure Alfvén wave appears even in dissipative chiral-MHD with $\xi_\omega = \sqrt{\epsilon + P}$. This is different from the conventional case, a pure Alfvén wave appears only in the ideal MHD.
4. The CMVI ceases when $\xi_\omega < \sqrt{\epsilon + P}$, because the CMVI is not enough to overcome the dissipations.
5. Considering the CMVI in an anisotropic chiral MHD due to the strong magnetic field.

Thank you for your attention!