Quasicrystal in QCD

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Low Energy Dense QCD

Spontaneous Chiral Symmetry Breaking

$$U(1)_V \times SU(N_f)_L \times SU(N_f)_R \to U(1)_V \times SU(N_f)_V$$

Nambu Goldstone (NG) boson

$$\Sigma = \exp\left(iT^a \pi^a(x)/f_\pi\right) \in SU(N_f)$$

Chiral Perturbation Theory (ChPT):

$$\mathcal{L}_{\text{chiral}} = \frac{f_\pi^2}{4}\text{Tr}\left(\partial^\mu \Sigma \partial_\mu \Sigma\right) - \frac{b}{2}\text{Tr}\left[M(\Sigma - 1) + \text{h.c.}\right]$$

$N_f = 2$ case with approximately $m_u \approx m_d$

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \quad m_u \approx m_d \equiv m$$

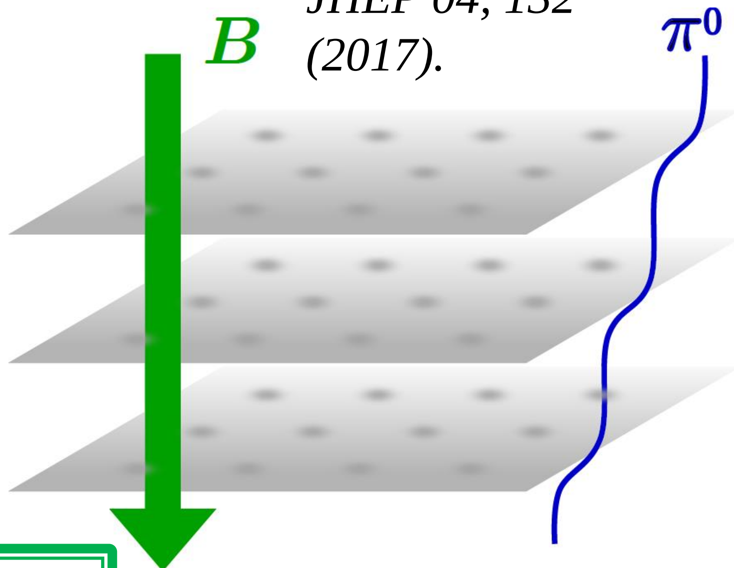
Under Magnetic Field

Pions $\xrightarrow{\boldsymbol{B}}$ Chiral Soliton Lattice (CSL) / π^0 domain wall (DW)

Idea by T. Brauner and N. Yamamoto, JHEP 04, 132 (2017).

LLL: $n = 0$, Pion: Spin $s = 0$

$$\varepsilon^2 = p_z^2 + 2|eB|(n + 1/2) + m^2 - 2seB$$



$\pi^\pm \xrightarrow{\boldsymbol{B}\uparrow}$ "massive", $\Sigma \to \exp(i\tau^3\pi^3)$

Transverse homogeneity $\partial_{x,y} = 0$ minimizes the Hamiltonian

$$\frac{f_\pi^2}{2}(\nabla\pi^0)^2 + m_\pi^2 f_\pi^2 (1 - \cos\pi^0) - \frac{\mu_B B}{4\pi^2}\partial_z \pi^0$$

WZW term → CSL

Kinetic ∂_z + mass term → SG soliton

Winding $\pi_1(U(1)) = \mathbb{Z}$

WZW Term by Triangle Anomaly

Coupling of two U(1) via triangle anomaly to NG boson ϕ_i

$$\mathcal{L}_B = \frac{1}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}\sum_i C_i \partial_\mu \phi_i A_\nu^B F_{\alpha\beta}$$

D. T. Son, M. A. Stephanov, and A. R. Zhitnitsky, Phys. Rev. Lett. 86, 3955 (2001);

Setup of chemical potential μ and magnetic field $\boldsymbol{B} = B\hat{z}$

$$U(1)_B : A_\nu^B = (\mu, \mathbf{0}) . \quad U(1)_{\mathrm{EM}} : A_\mu = (0, yB/2, -xB/2, 0)$$

ϕ_i be not only $\pi^{\pm,0}$ but also η mesons (for larger density)

$$\phi_3 = \frac{\pi_3}{f_\pi} : \quad \mathcal{L}_{\mathrm{WZW}} = \frac{\mu B}{4\pi^2}\partial_z \phi_3 \Rightarrow \mathrm{CSL}$$

D. T. Son and A. R. Zhitnitsky , Phys. Rev. D 70, 074018 (2004).

$$\phi_0 \equiv \frac{\eta}{f_\eta} : \quad \mathcal{L}_\eta = \frac{\mu B}{12\pi^2}\partial_z \phi_0 \Rightarrow \eta\text{-CSL?}$$

Separately equivalent.
But mix η and π^0 …
new ground state?

U(2) ChPT in Magnetic Field

$\pi^{\pm}$ decoupled, η and π^0 remained: $U = \exp\phi_0 \exp\left(i\tau_3\phi_3\right)$

Kinetic Anomaly

$$H = \frac{1}{2}\left[\alpha\left(\partial_z\phi_3\right)^2 + \left(\partial_z\phi_0\right)^2\right] - \frac{\gamma}{2\pi}\left(\partial_z\phi_3 + \frac{1}{3}\partial_z\phi_0\right)$$

$$+ \sin\beta\left(1 - \cos 2\phi_0\right) + \cos\beta\left(1 - \cos\phi_0\cos\phi_3\right)$$ Mass

Redefine parameters to have dimensionless quantities

$$\alpha \equiv \frac{f_\pi^2}{f_\eta^2}, \quad \beta \equiv \arctan\frac{a}{2mb}, \quad \gamma \equiv \frac{\mu B}{2\pi}\frac{\left[a^2 + (4mb)^2\right]^{1/4}}{f_\eta}$$

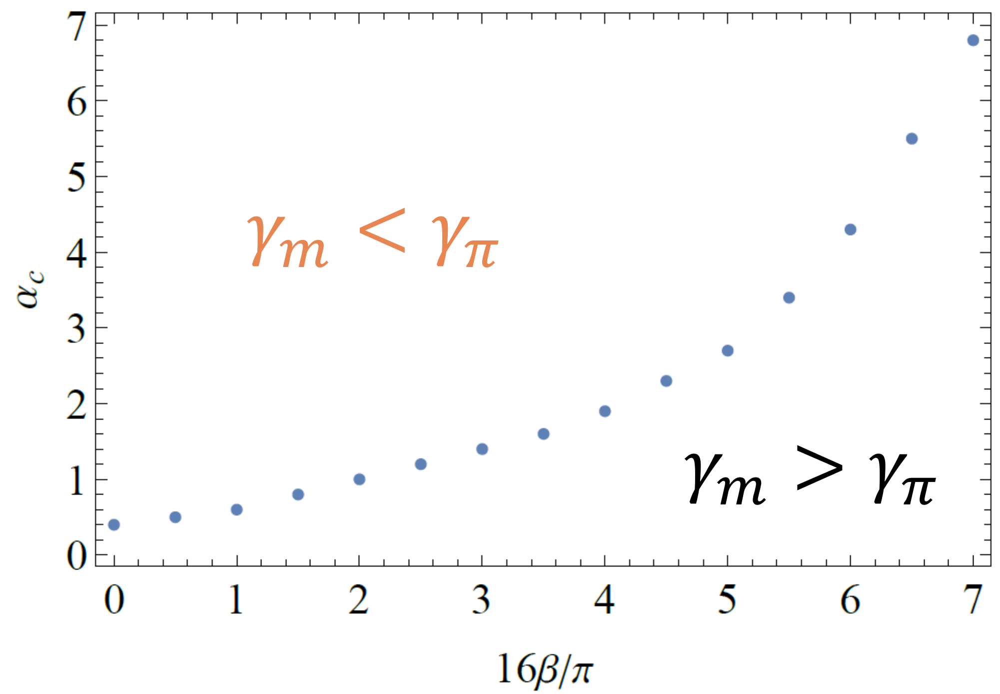
Boundary conditions to form Mixed Soliton Lattice

$$\left(\phi_3, \phi_0\right)\Big|_{z=0} = (0, 0), \quad \left(\phi_3, \phi_0\right)\Big|_{z=d} = (p\pi, q\pi), \quad \frac{p \pm q}{2} \in \mathbb{Z}$$

Mixed Soliton Lattice

Irrelevant $\gamma_\eta \gg \gamma_{\pi,m}$. Duel is between γ_π and γ_m.

(p,q)	**Soliton Type**	**Critical magnetic field**
(2,0)	π^0	γ_π
(0,2)	η	γ_η
(1,1)	Mixed	γ_m



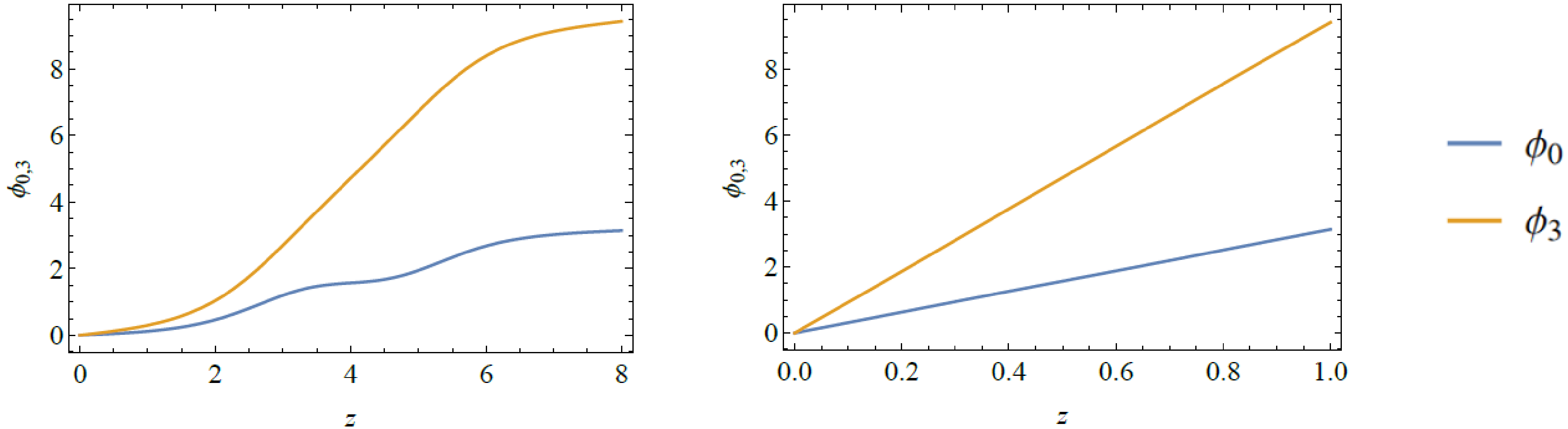
γ_m can be lowest! for $\alpha > \alpha_c(\beta)$

Strong Magnetic Field Limit

In $\gamma \to \infty$ limit, the ground state tends to take

$$r \equiv \frac{p}{q} = \frac{3}{\alpha}$$

Profiles: $\gamma \uparrow$, $d \downarrow$, $\phi_{3(0)} \to p(q)\pi z/d$; (almost linear)



Indicate: derivative is large, mass term is subleading.

Semi-analytical Proof

$\gamma \to \infty$, mass term becomes irrelevant:

$$H\left(\gamma \to \infty\right) \simeq \frac{1}{2}\left(\alpha\phi_3'^2 + \phi_0'^2\right) - \frac{\gamma}{2\pi}\left(\phi_3' + \frac{1}{3}\phi_0'\right) \equiv H_\infty.$$

Simple trick of total square finds us the minimum:

$$H_\infty = \frac{1}{2}\left[\left(\sqrt{\alpha}\phi_3' - \frac{\gamma}{2\sqrt{\alpha}\pi}\right)^2 + \left(\phi_0' - \frac{\gamma}{6\pi}\right)^2\right] - \frac{\gamma^2}{8\pi^2}\left(\frac{1}{\alpha} + \frac{1}{9}\right)$$

$$\geq -\frac{\gamma^2}{8\pi^2}\left(\frac{1}{\alpha} + \frac{1}{9}\right) \equiv E_{\min}.$$

Lattice period d_L and ratio p/q that minimize the energy

$$d_L = \frac{2\pi^2}{\gamma} \cdot p\alpha = \frac{2\pi^2}{\gamma} \cdot 3q \quad \Rightarrow \quad \frac{p}{q} = \frac{3}{\alpha},$$

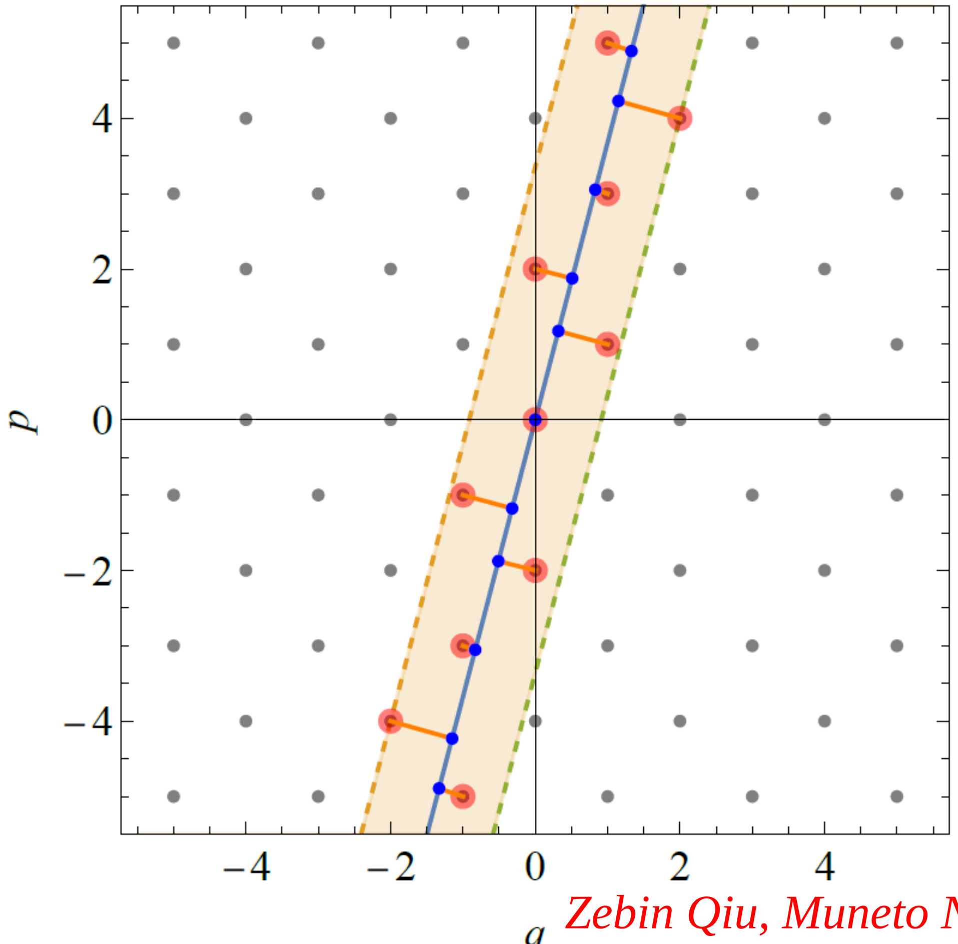
Quasicrystal in QCD

Grey dots: mixed soliton lattice of different *p*, *q*.

Blue solid line (exemplar) $p/q = 3f_\eta^2/f_\pi^2$ (irrational)

Orange dots: *(p,q)* that adjacent to the blue line.

Beige band: possibly degenerate groun states, forming Quasicrystal!



Zebin Qiu, Muneto Nitta, JHEP 05, 170 (2023)

*valid range: $\mu B \ll \Lambda^3, \quad \Lambda \simeq 4\pi f_{\pi,\eta}; \quad \gamma \ll 5.6\pi^2$

Rotational Counterpart

In parallel to WZW term, anomalous rotational ($\boldsymbol{\Omega} = \Omega\hat{z}$) Lagrangian is the coupling j^5_{CVE} to axial rotation.

$$\mathcal{L}^{\Omega}_{\eta} = \frac{\mu^2\Omega}{6\pi^2}\partial_z\phi_0, \quad \mathcal{L}^{\Omega}_{\pi} = \frac{\mu\mu_I\Omega}{2\pi^2}\partial_z\phi_3$$

X.-G. Huang, K. Nishimura and N. Yamamoto, JHEP 02 (2018) 069

We consider no isospin checmial potential $\mu_I = 0$. Recap:

	$\boldsymbol{\eta}$	$\boldsymbol{\pi^0}$
Rotation ***Ω***	$(\partial_z\phi_0)\mu^2\Omega/6\pi^2$	0
Magnetic field ***B***	$(\partial_z\phi_0)\mu B/12\pi^2$	$(\partial_z\phi_3)\mu B/4\pi^2$

In rotational case, the net winding number must be from η only. But π^0 can have local "up and down"

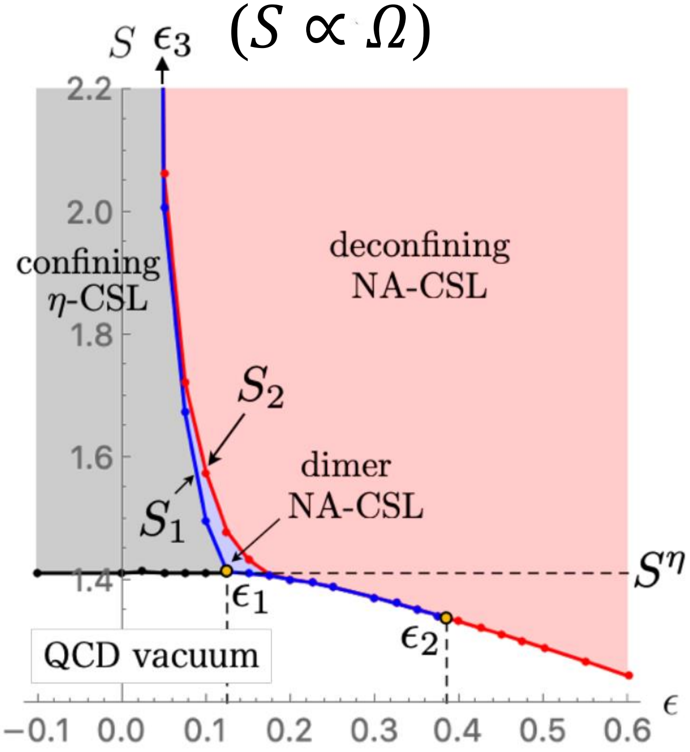
Non-Abelion (NA) CSL (Rotation)

η-CSL can split into "dimer", i.e., "Up" ⊕ "Down"

$$(\phi_3, \phi_0)\Big|_{z=0} = (0,0)\,;$$

$$(\phi_3, \phi_0)\Big|_{z=l} = \begin{cases} (0, 2\pi) & \eta\text{-CSL} \\ (\pi, \pi) & \text{"Up" branch} \\ (-\pi, \pi) & \text{"Down" branch} \end{cases}$$

$(S \propto \Omega)$



$(\epsilon = 1 - f_\pi^2/f_\eta^2 \sim 0.3)$

Each branch is a "mixed soliton"

The interaction between branches

1. attractive: η-CSL; 2. null: dimer; 3. repulsive: NA-CSL

$$\mu^2\Omega_c \sim 1.3 \cdot N_c 2\pi^2 f_\eta^2 m_\eta$$

M. Eto, K. Nishimura and M. Nitta, JHEP 08 (2022) 305

Conclusion

- Depending on decay constants and effective masses, the **mixed soliton lattice** of η and π^0 can have lower energy / critical magnetic field than separate ones.

- In strong magnetic field / density limit, the ground state ratio *p/q* approaches $3f_\eta^2/f_\pi^2$ which is generally irrational. The result is a **mesonic quasicrystal**.

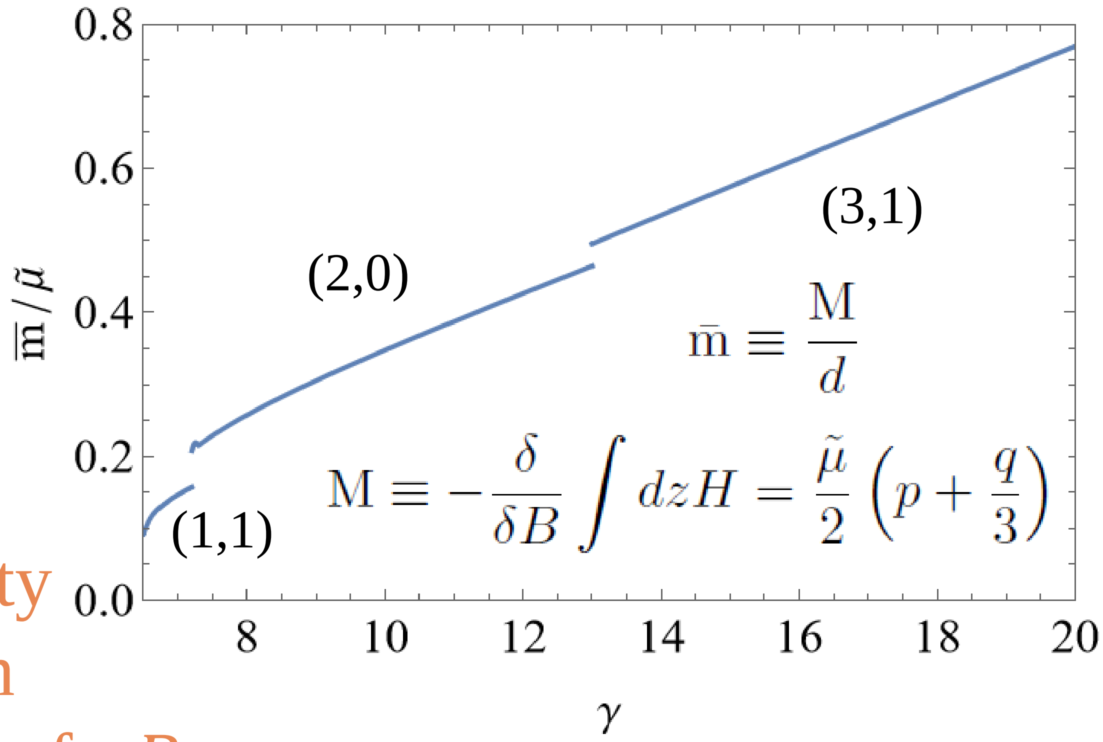
- In rotation case, the η-CSL can split into dimer and then **NA-CSL**, during increasing the angular velocity.

Outlook: Intermediate *B*

Example: up to $p+q=4$, under $\alpha = 0.7$ and $\beta = \pi/16$: competitive configurations are (1,1), (2,0), and (3,1) with critical $\gamma_m = 6.5$, $\gamma_\pi = 6.7$ and $\gamma_{31} = 7.3$.

Ground State

$$= \begin{cases} (1,1) & \gamma \in [6.4,\ 7.2) \\ (2,0) & \gamma \in [7.2,\ 13.0) \\ (3,1) & \gamma \in (13.0,\ \ldots) \end{cases}$$



Magnetic moment density quantifies the alternation as a piece-wise function of μB