QCD under Rotation

Effects of imaginary and real rotations on QCD matter

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GC, arXiv: 2310.03310.

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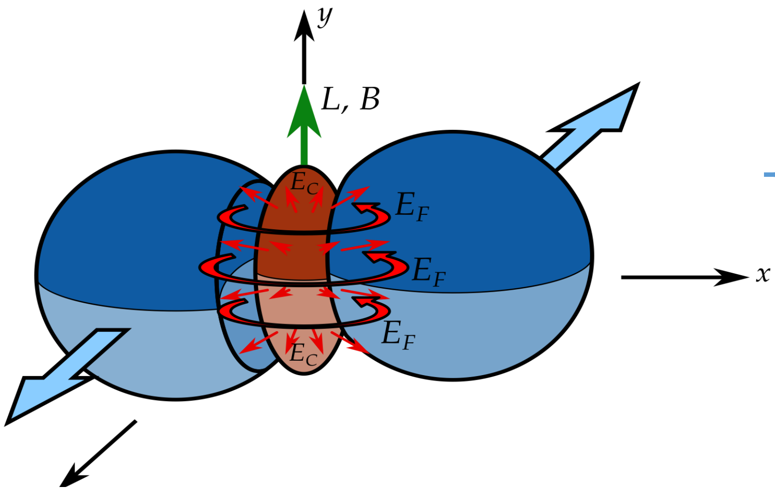
outline

- HIC and fast rotation
- Phase transitions in effective models
- Phase transitions in Lattice QCD
- Perturbative study
- Modified Polyakov loop potential
- Polyakov — Nambu—Jona-Lasinio model
- Summary and perspective

HIC and fast rotation

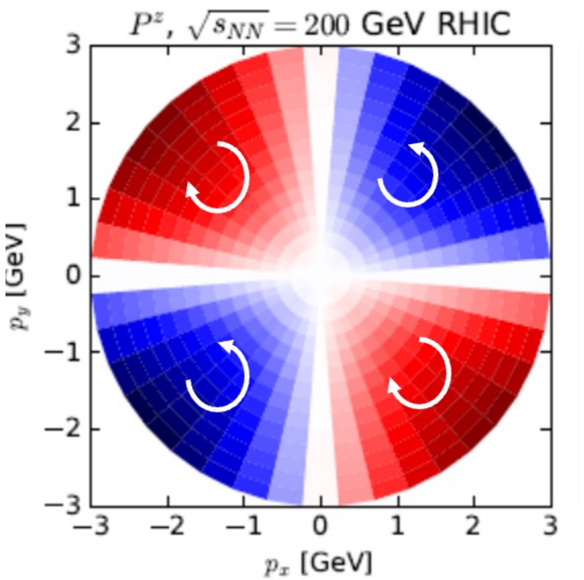


$\vec{\Omega} \parallel \vec{B}$ **in heavy ion collisions**

$$\boxed{\Omega \sim 10^{22} s^{-1} \sim 6\ MeV}$$

$$\Delta\epsilon \equiv \epsilon' - \epsilon = -S \cdot \omega$$

Ω plays a role of effective chemical potential



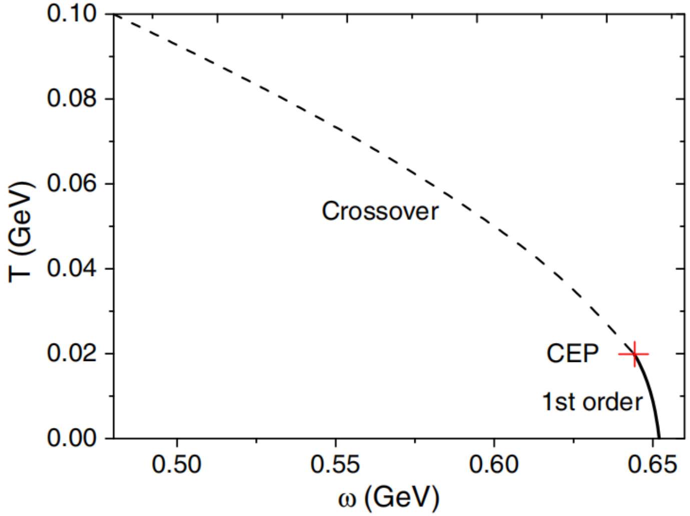
HIC is a good source of rotation

anomalous transport

- **magnetovorticity** K. Hattori & Y. Yin (PRL2016)
- **Chiral electric vortical effect** GC (PRD2021)

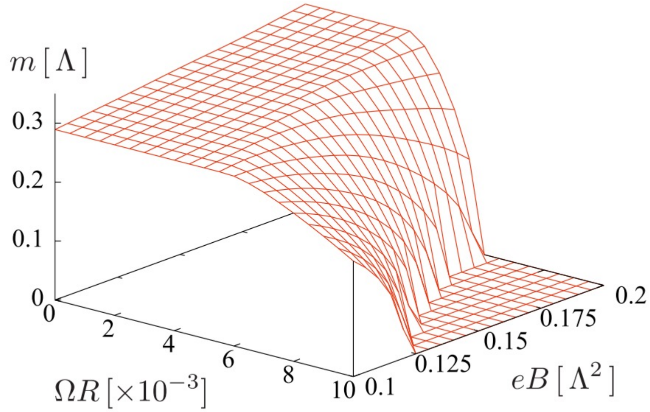
Phase transitions in effective models

NJL model



Y. Jiang & J. Liao (PRL2016)

1. Chiral symmetry restores with rotation
2. A critical end point

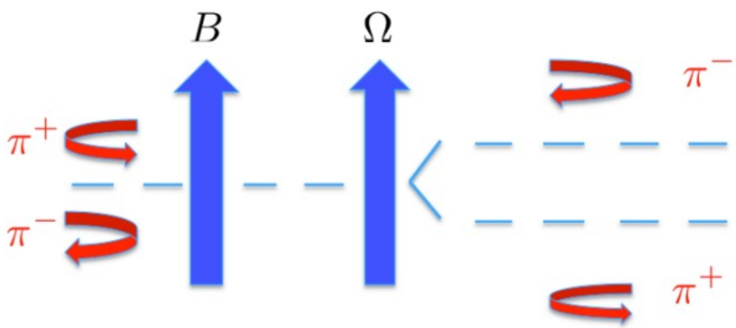


H. Chen et.al. (PRD2016)

Chiral symmetry restores with magnetic field at large rotation

-- de Haas-van Alphen effect

The case $\vec{\Omega} \parallel \vec{B}$

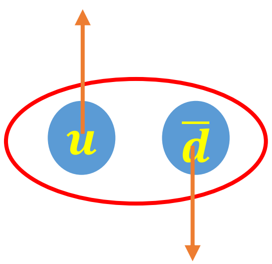


Y. Liu & I. Zahed (PRL2017)

1. **Energy splitting** between π^- and π^+

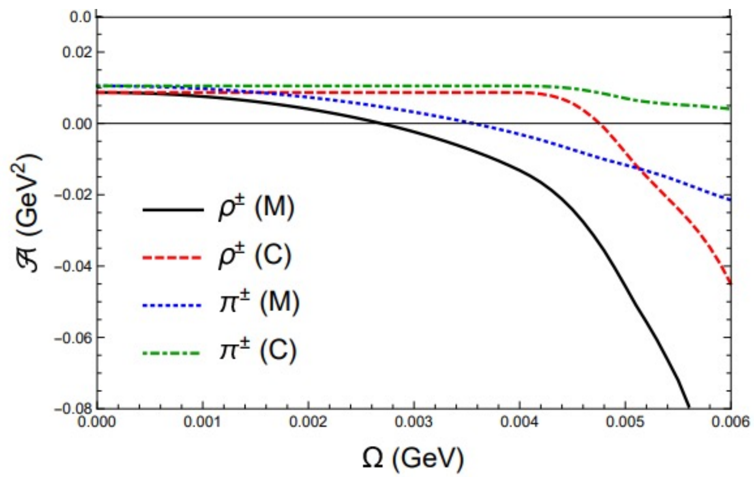
2. **Charge conservation**: π^- and π^+ **condensation** in different region

Internal spin structure of π^+



Unstable with both Ω and B

GC & L. He (PRD2019, 2020)



NJL model

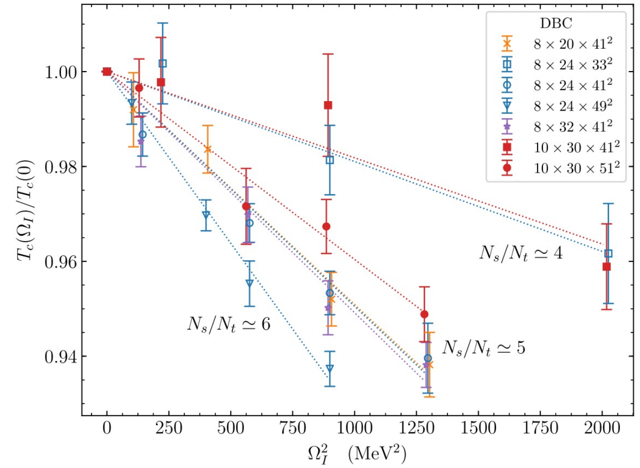
pion superfluid,
rho superconductor

increasing isospin density

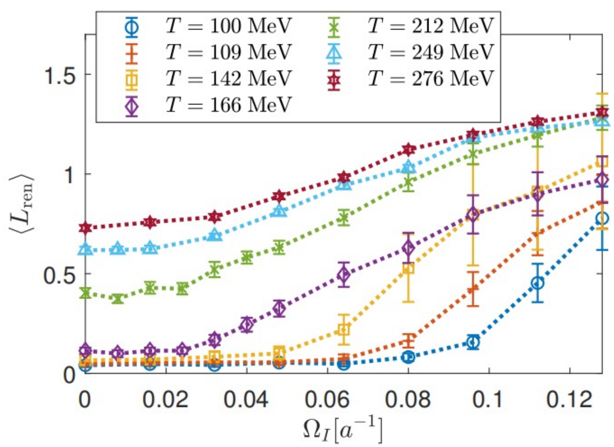
Phase transitions in Lattice QCD

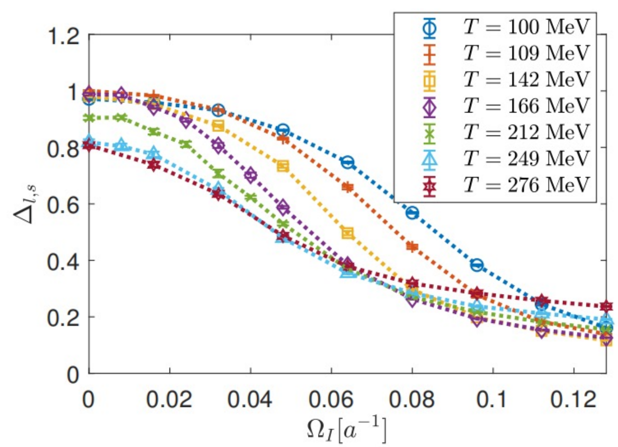
V. V. Braguta et.al. (PRD2021)

$$S_G = \frac{1}{2g^2}\int d^4x\,[(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz} + (1-x^2\Omega^2)F^a_{yz}F^a_{yz} + F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} - \underline{2iy\Omega}(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) + \underline{2ix\Omega}(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F^a_{xz}F^a_{zy}].$$



Imaginary rotation $\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2$ ⇒ **Real rotation** $\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$





J. Yang & X.G. Huang (2023)

Chiral symmetry restored by Ω_I

Perturbative study

Effective models
- Holographic QCD: D. Hou, M. Huang et.al. (JHEP, PRD).
- QM model: H.L. Chen et.al. (PRD2023).
- NJL model: Y. Jiang (EPJC2022).
- Bag model: K. Mameda & K. Takizawa (2023).

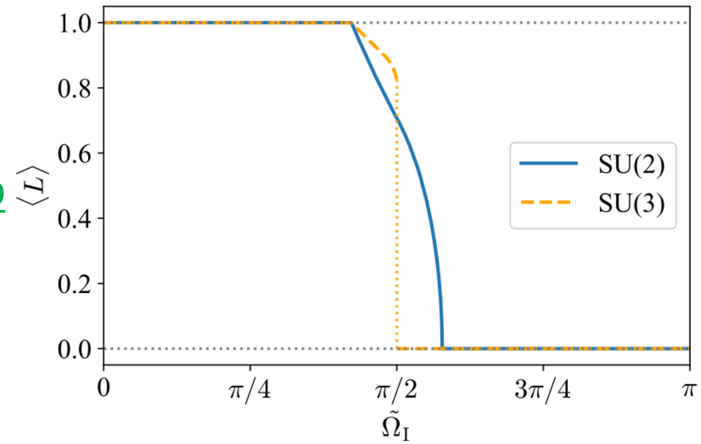
K. Fukushima et.al. (PRL2023).

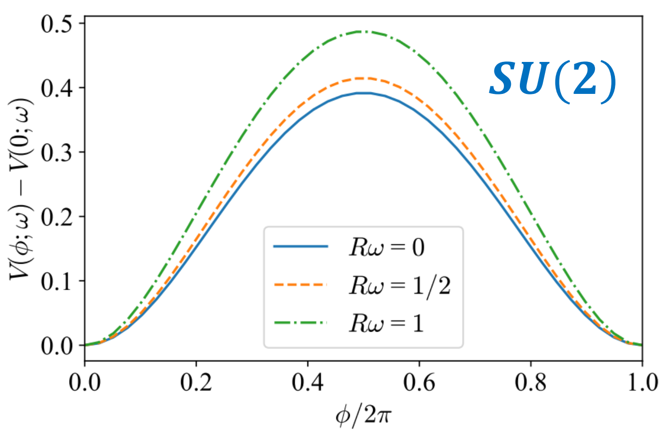
$$V = \frac{T}{4\pi^2}\sum_{\boldsymbol{\alpha}}\sum_{m\in\mathbb{Z}}\int_0^\infty k_\perp dk_\perp \int_{-\infty}^{\infty} dk_z \Big[\underline{J^2_{m-1}(k_\perp r)} + \underline{J^2_{m+1}(k_\perp r)}\Big]\mathrm{Re}\ln\Big[1 - e^{-(|\vec{k}|-i\Omega_{\mathrm{I}} m)/T + i\boldsymbol{\phi}\cdot\boldsymbol{\alpha}}\Big].$$

nonphysical gluons

cancelled by ghosts

Consistent with M. N. Chernodub et.al. (PRD2023)

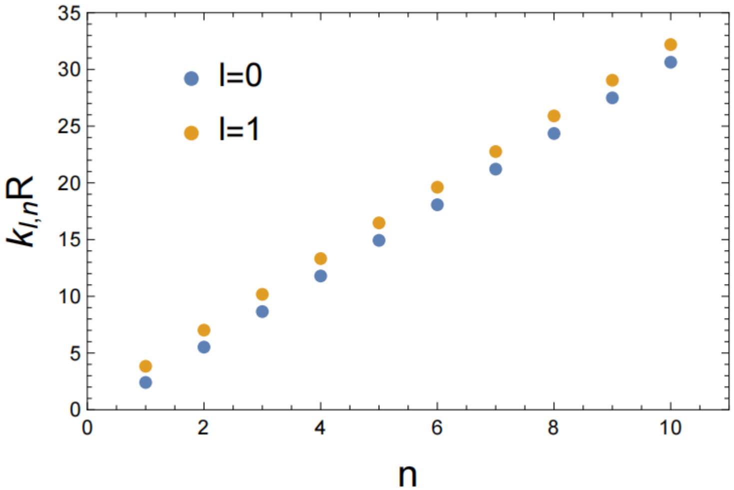




Modified Polyakov loop potential

Simple form: $$V(\phi;\tilde{\Omega}_{\rm I})|_{\tilde{r}=0} = \frac{\pi^2 T^4}{3}\sum_{\boldsymbol{\alpha}}\sum_{s=\pm 1} B_4\left(\left(\frac{\phi\cdot\boldsymbol{\alpha} + \boxed{s\tilde{\Omega}_{\rm I}}}{2\pi}\right)_{\rm mod\ 1}\right)$$

Rotation can affect ghosts significantly in confined phase



$k_{l,n}$ the transversal eigenenergy

$$J_l(k_{l,n}R) = 0$$

$$k_{0,n} < k_{1,n} < k_{0,n+1} < k_{1,n+1}$$

Leading-order nontrivial approximation

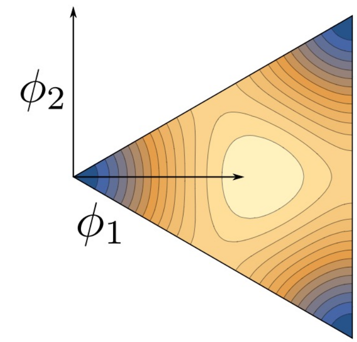
$$\begin{cases} \dfrac{V_1(L,L^*)}{T^4} = -\dfrac{a(\tilde{T})}{2}|L|^2 - \dfrac{1.75}{\tilde{T}^3}\ln H(L,L^*) \\ \dfrac{V_2(L,L^*)}{T^4} = -\dfrac{a(\tilde{T})}{2}|L|^2 - \dfrac{0.75}{6}\left(L^3 + L^{*3}\right) + \dfrac{7.5}{4}|L|^4 \end{cases} \Rightarrow \begin{cases} \bar{V}_1(q_{\rm ij},\Omega_{\rm I}) = \dfrac{1}{2}\sum_{s=\pm} V_1(q_{\rm ij} + s\,\Omega_{\rm I}), \\ \bar{V}_2(q_{\rm ij},\Omega_{\rm I}) = \dfrac{1}{2}\sum_{s=\pm} V_2(q_{\rm ij} + s\,\Omega_{\rm I}). \end{cases}$$

The results of V_1

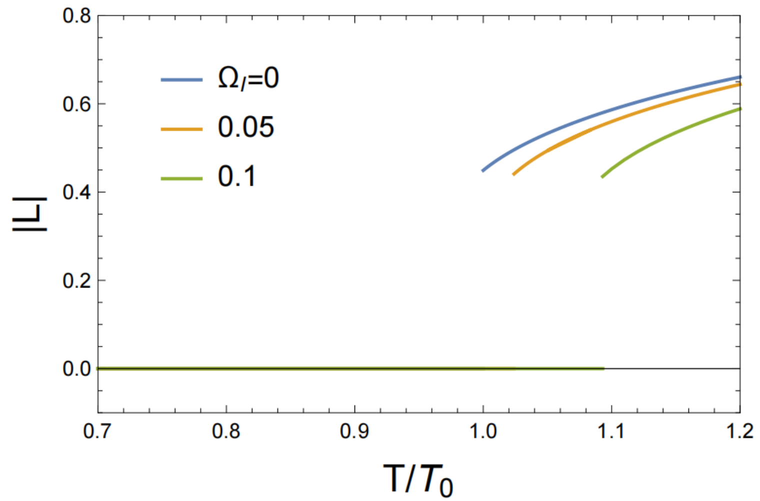
$$L(q_1,q_2,q_3)=\frac{1}{N_c}(e^{i\,q_1}+e^{i\,q_2}+e^{i\,q_3}) \qquad q_{ij}=q_i-q_j$$

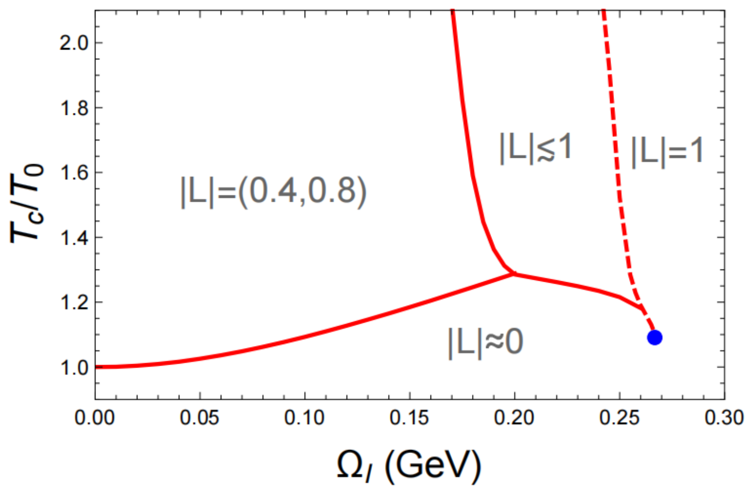
$$q_{31}=\phi_1,\; q_{21}=\frac{\phi_1}{2}+\frac{\sqrt{3}}{2}\phi_2,\; q_{32}=\frac{\phi_1}{2}-\frac{\sqrt{3}}{2}\phi_2$$

$$\phi_1 \in [0,2\pi] \quad \phi_2 \in \left[-\frac{\phi_1}{\sqrt{3}},\frac{\phi_1}{\sqrt{3}}\right]$$



Imaginary rotation

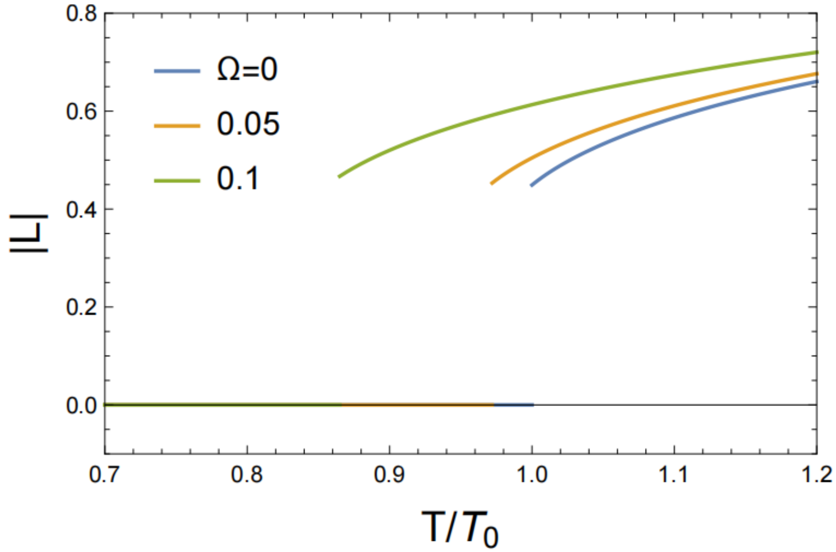


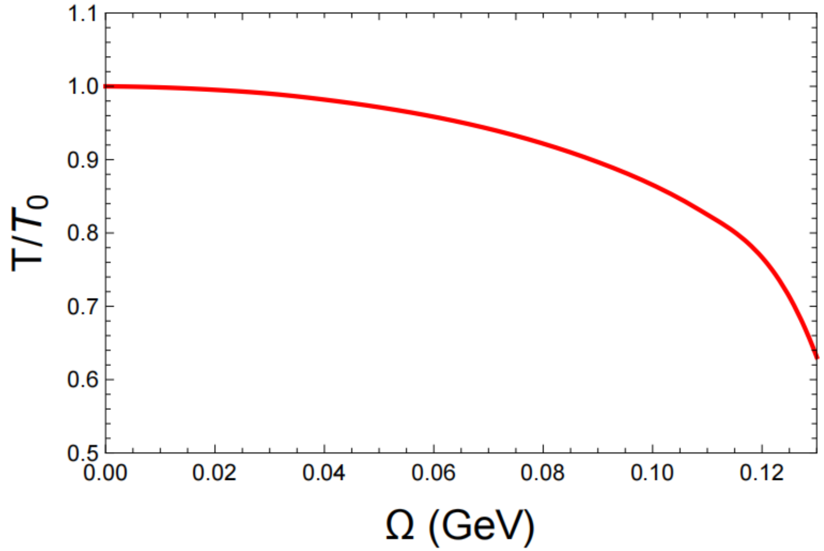


Several local minima → **Several transition branches**

The results of V_1

Real rotation



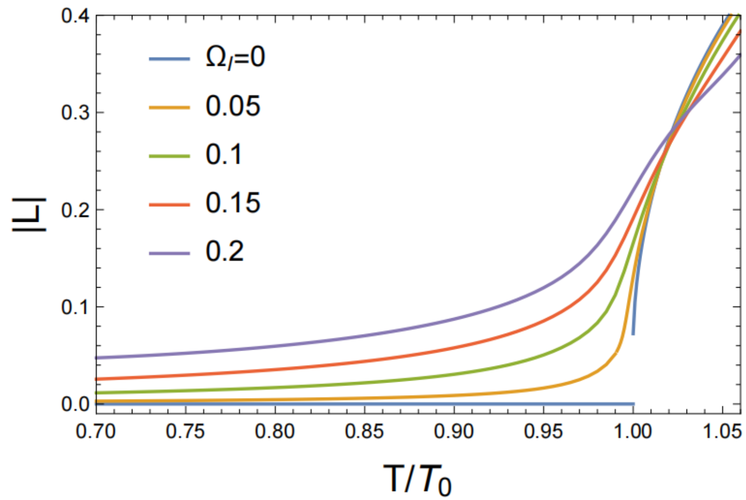


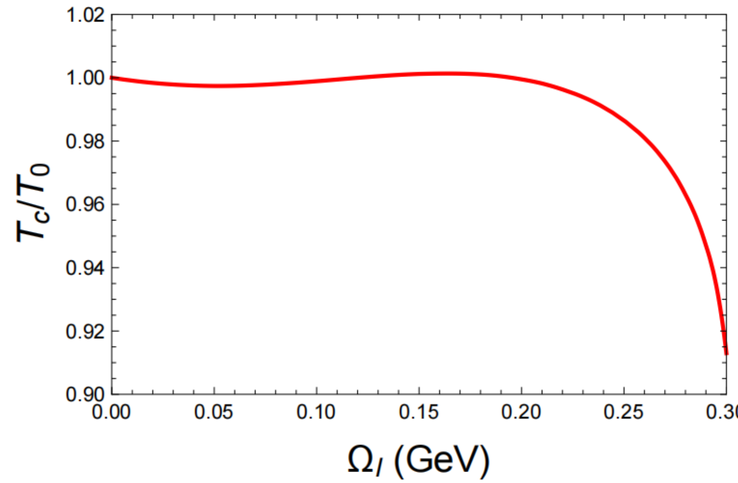
1. **First order transition at any Ω**

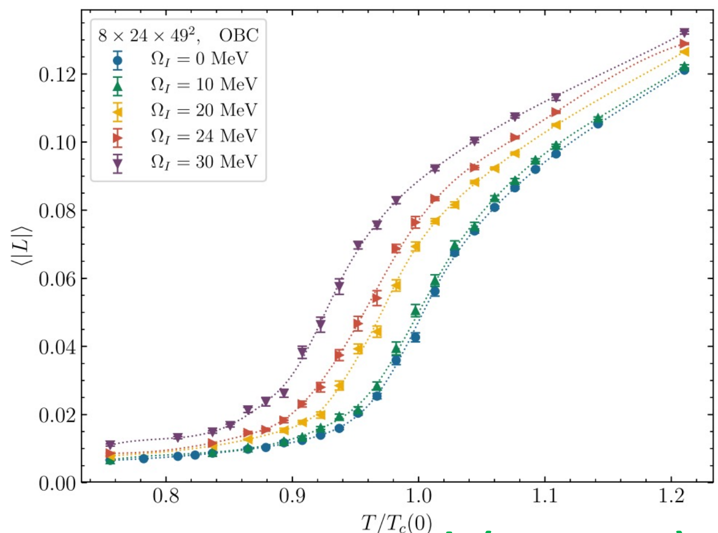
2. **T_C decreases with Ω – opposite to small Ω_I**

The results of V_2

Imaginary rotation





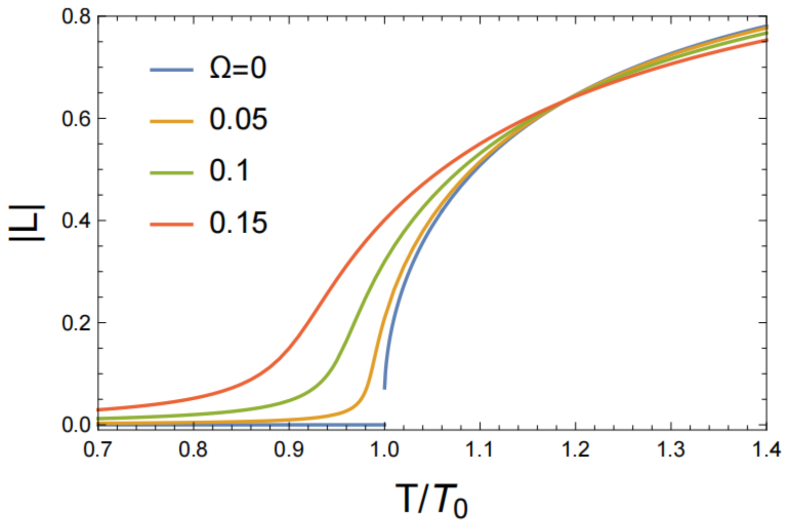


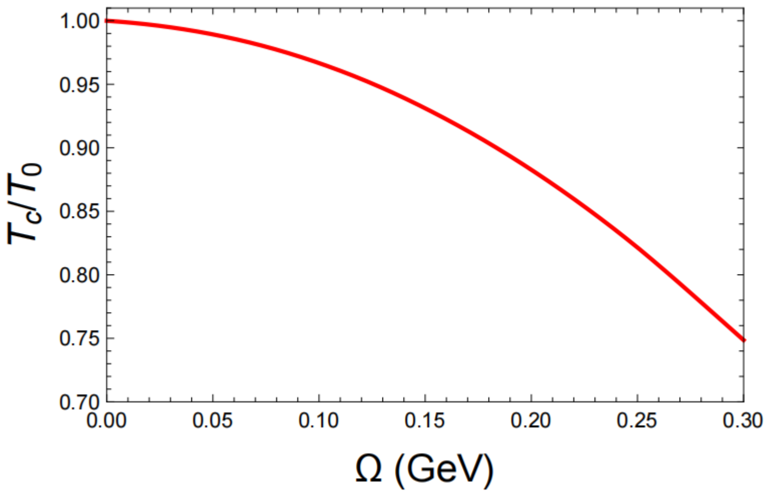
V. V. Braguta et.al. (PRD2021)

1. Crossover transition at larger Ω_I
2. T_C decreases with Ω_I basically
3. Consistent with LQCD results

The results of V_2

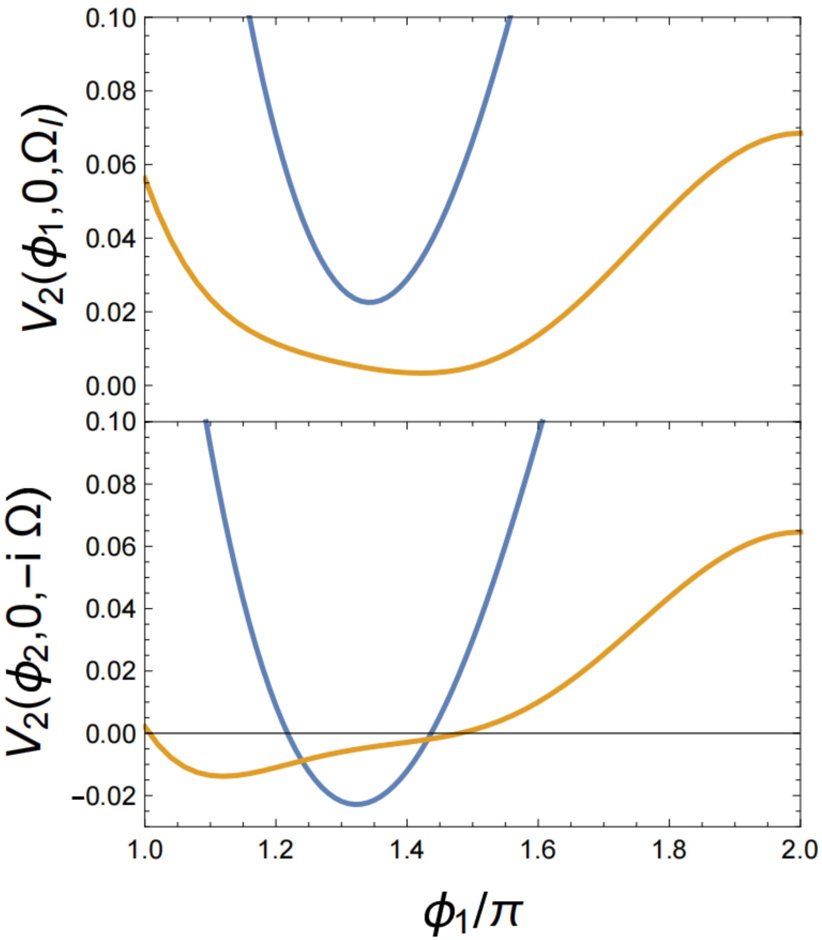
Real rotation





1. **Crossover transition at larger Ω**
2. **T_C decreases with Ω**
3. **Analytic continuation breaks down for the phase diagram**

Blue: $0.9\,T_0$, Yellow: T_0



$\phi_1 > \frac{4\pi}{3}$ for Ω_I

$\Downarrow$

$Re\ L < 0$

$\phi_2 < \frac{4\pi}{3}$ for Ω

$\Downarrow$

$Re\ L > 0$

Polyakov — Nambu—Jona-Lasinio model

Three-flavor Lagrangian

$$\begin{cases} \mathcal{L}_{\mathrm{PNJL}} = \bar{\psi}\left[i\partial\!\!\!/ - i\gamma^4\left(ig\mathcal{A}_4 + i\,\Omega_{\mathrm{I}}(\hat{L}_z + \hat{S}_z)\right) - m_0\right]\psi \\ \qquad + G\sum_{a=0}^{8}\left[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2\right] + \mathcal{L}_{\mathrm{tH}} \\ \qquad - \boxed{V_2(\phi_1, \phi_2, \Omega_{\mathrm{I}})}, \\ \mathcal{L}_{\mathrm{tH}} = -\frac{K}{2}\sum_{t=\pm}\epsilon_{ijk}\epsilon_{imn}(\bar{\psi}^i\Gamma^t\psi^i)(\bar{\psi}^j\Gamma^t\psi^m)(\bar{\psi}^k\Gamma^t\psi^n) \end{cases}$$

$$\boxed{g\mathcal{A}_4 = T\,\mathrm{diag}(q_1, q_2, q_3)}$$

Thermodynamic potential with boundary condition

$$\Omega_{\mathrm{bl}} = -\sum_{\mathrm{f=u,d,s}}\boxed{\sum_{l=0}^{\infty}}\frac{1}{\pi^2R^2}\sum_{\mathrm{n}=1}^{\infty}\Bigg\{N_{\mathrm{c}}\underline{\sum_{j=0}^{3}(-1)^{j-1}C_3^j(\epsilon_{\mathrm{f0}}^2 + j\Lambda^2)\ln(\epsilon_{\mathrm{f0}}^2 + j\Lambda^2)}$$

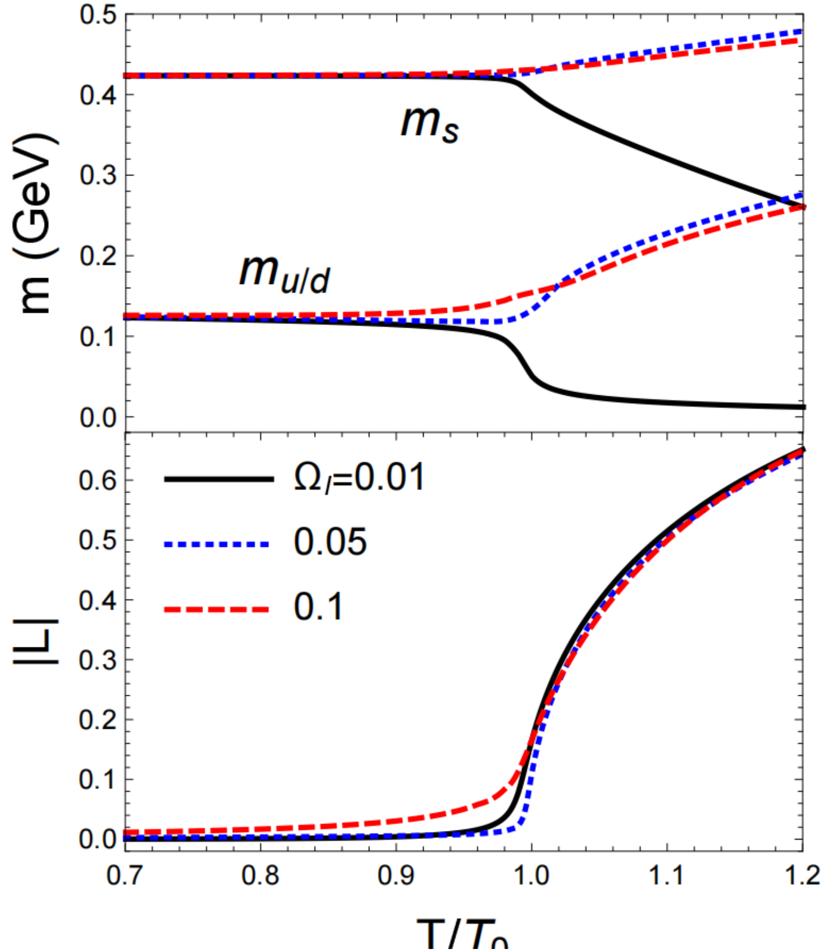
$$+2T\sum_{t=\pm}\int_0^{\infty}\mathrm{d}k_3\left[\ln\left(1 + 3Le^{-\tilde{\epsilon}_{\mathrm{f}}+i\,t(l+\frac{1}{2})\tilde{\Omega}_{\mathrm{I}}} + 3L^*e^{-2\tilde{\epsilon}_{\mathrm{f}}+2i\,t(l+\frac{1}{2})\tilde{\Omega}_{\mathrm{I}}} + e^{-3\tilde{\epsilon}_{\mathrm{f}}+3i\,t(l+\frac{1}{2})\tilde{\Omega}_{\mathrm{I}}}\right) + c.c.\right]\Bigg\}$$

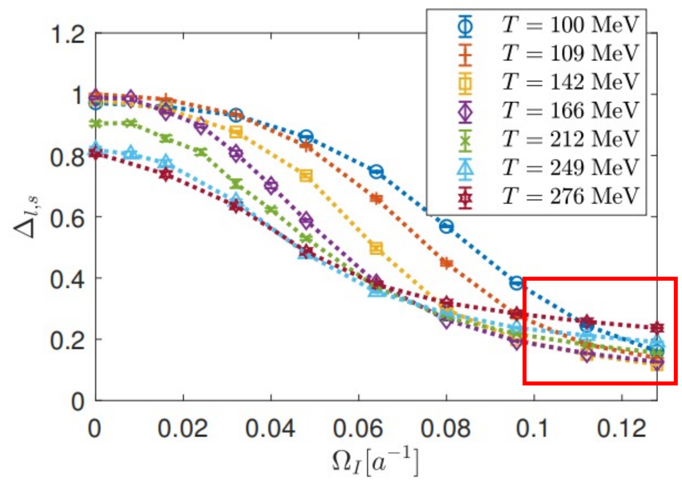
PV regularization

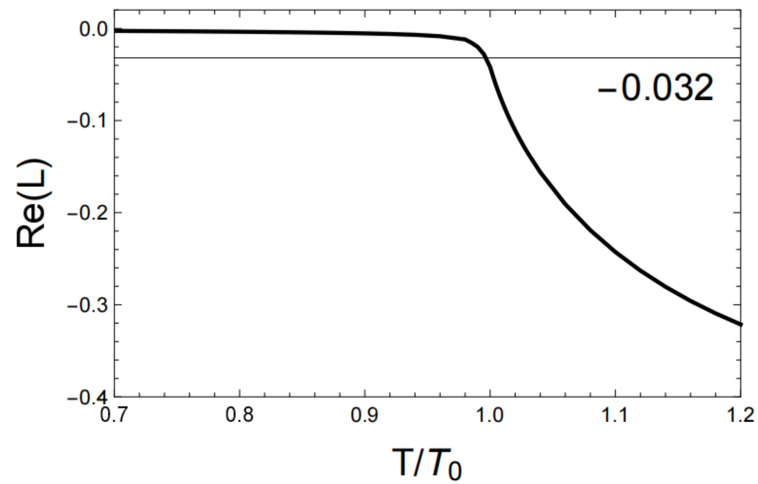
$$\epsilon_f = \sqrt{k_{l,n}^2 + k_3^2 + m_f^2}$$

Real !

Results for imaginary rotation

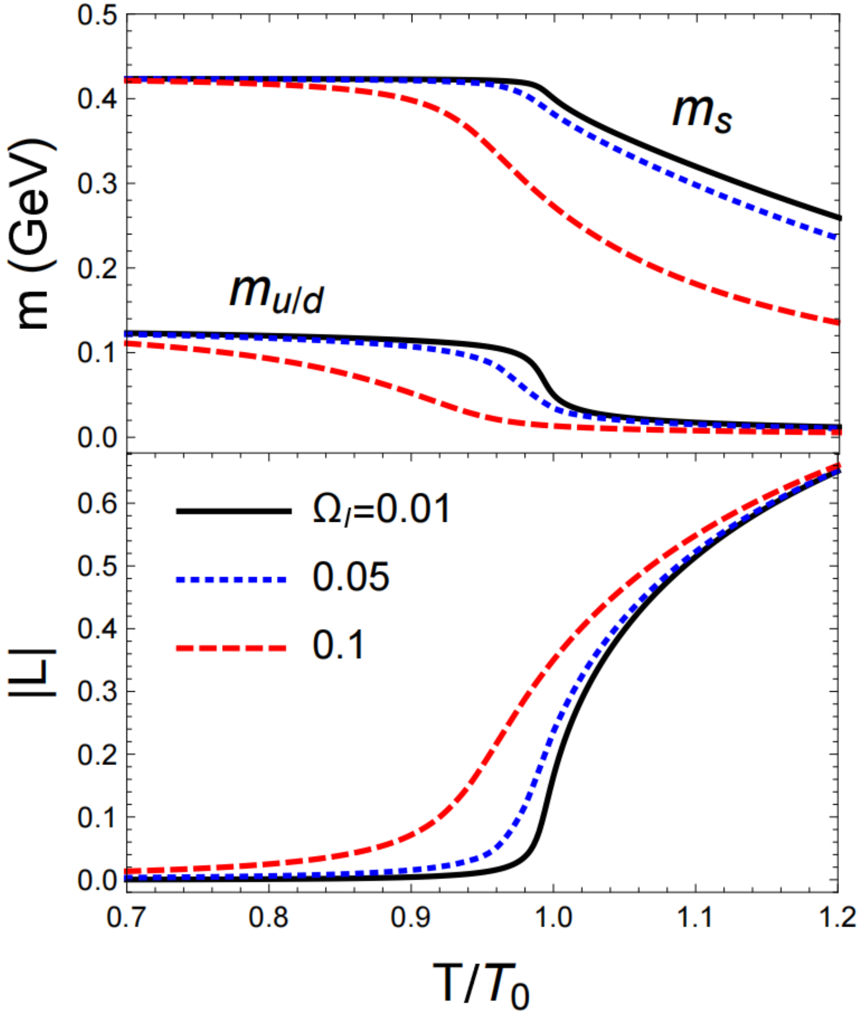


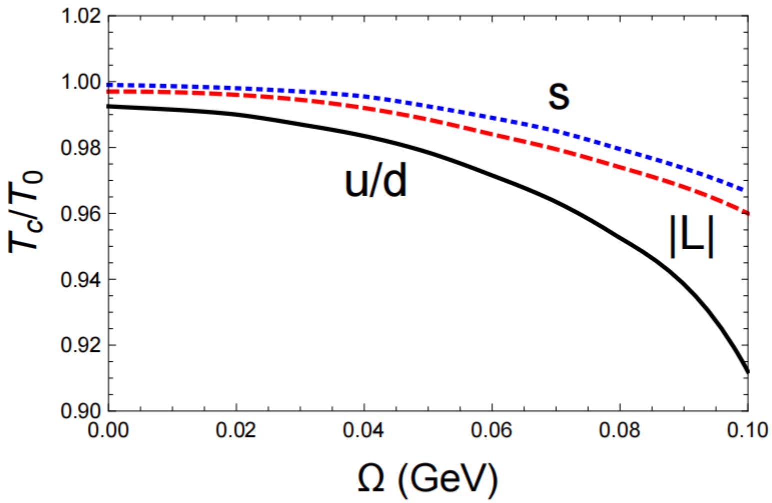




$$Re\left[3Le^{-\tilde{\epsilon}_{\rm f}+i\,\frac{t}{2}\tilde{\Omega}_{\rm I}}+3L^{*}e^{-2\tilde{\epsilon}_{\rm f}+2i\,\frac{t}{2}\tilde{\Omega}_{\rm I}}+e^{-3\tilde{\epsilon}_{\rm f}+3i\,\frac{t}{2}\tilde{\Omega}_{\rm I}}\right]<0. \Rightarrow Re(L) < -0.032.$$

Results for real rotation





1. crossover transition for all Ω
2. T_C decreases with Ω

consistent with most effective models

Summary

- Polyakov loop potential modified according to perturbative study

- Munich's potential works very well

- Analytic continuation breaks down for the phase diagram

- The effects of real rotation are consistent with the expections from effective models

Thank you very much！