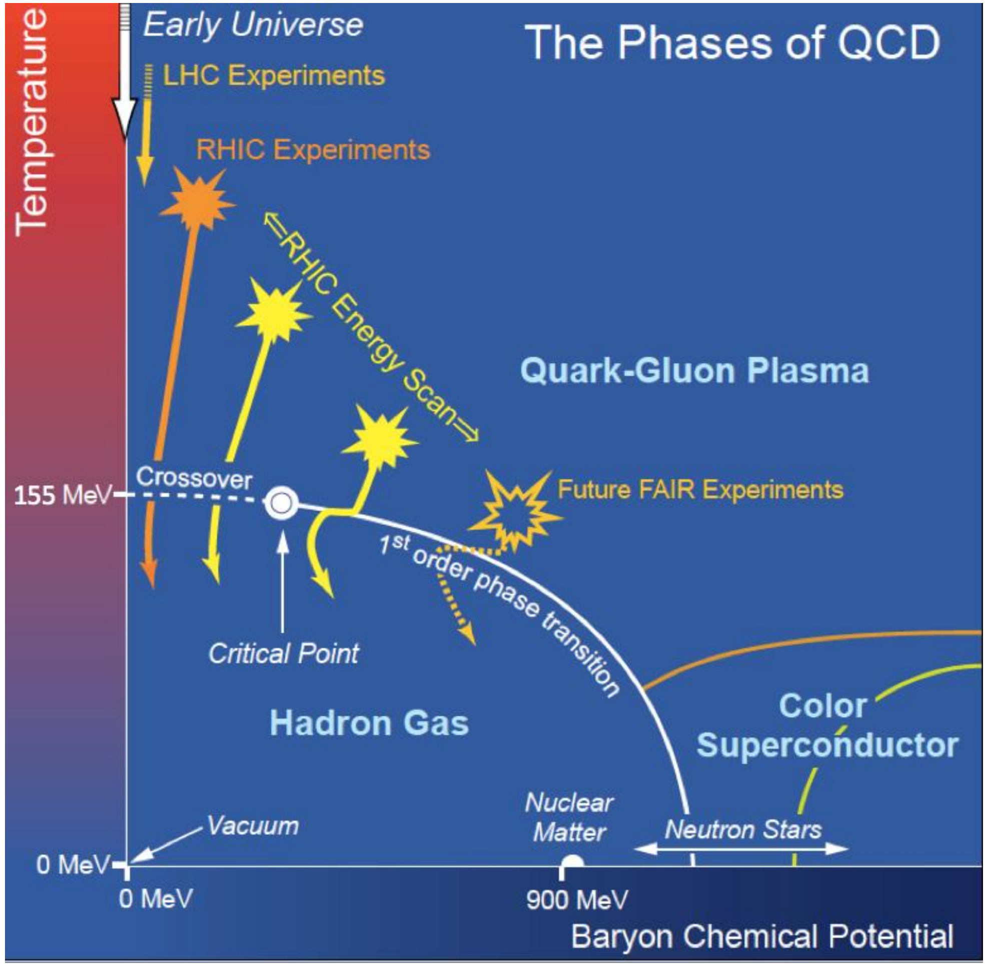
Chiral condensate for accelerated and rotated observer

Speaker: Zhibin Zhu

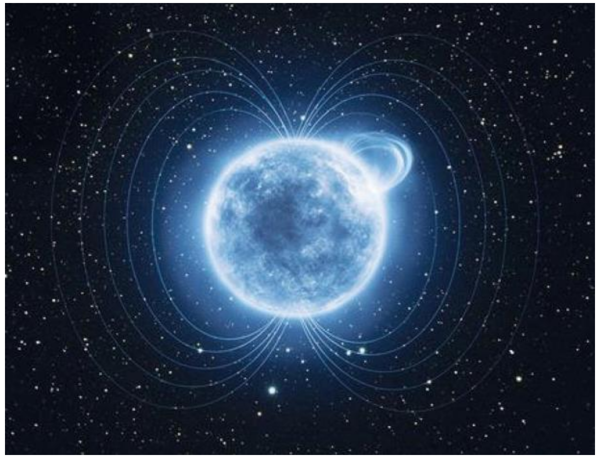
Outline

- Introduction：QCD phase transition under rotation
- Functional renormalization group study: Quark-meson model under rotation (arXiv:2306.08362)
- NJL model study: Chiral condensate under rotation and acceleration
- Summary

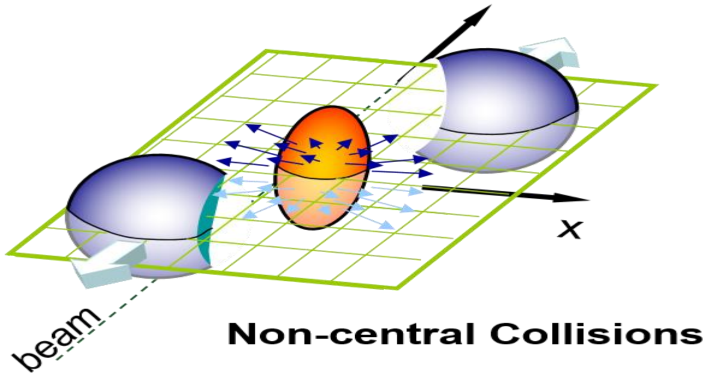
QCD phase transition



The Hot QCD White Paper (2015)



neutron star



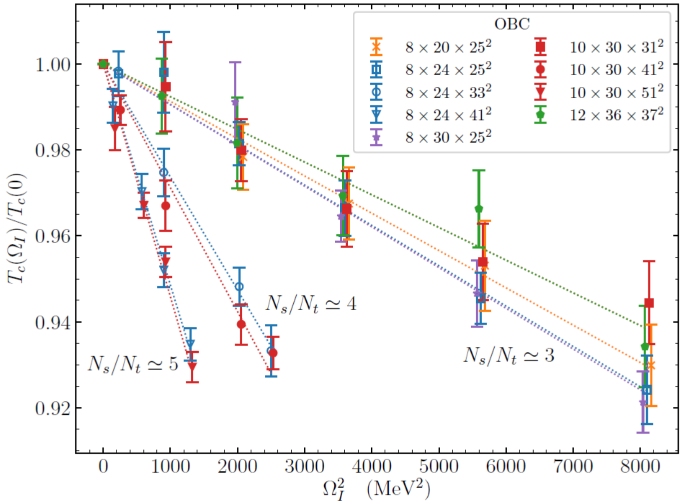
$\Omega \sim 0.01 - 0.1\ GeV$

Chiral condensate $<\bar{\psi}\psi>$

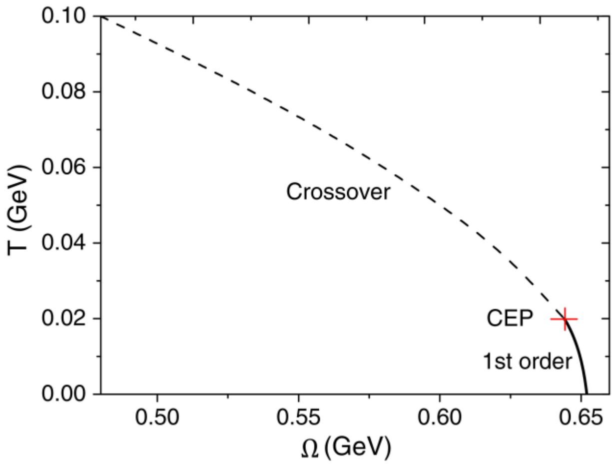
QGP phase $<\bar{\psi}\psi>=0$

Hadronic Phase $<\bar{\psi}\psi>=$finite

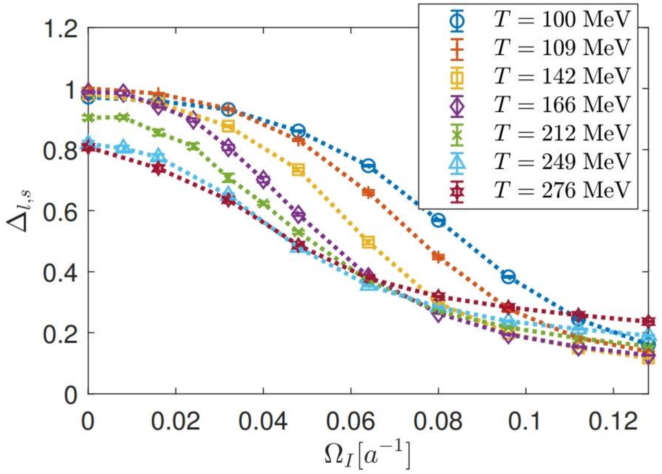
QCD under rotation



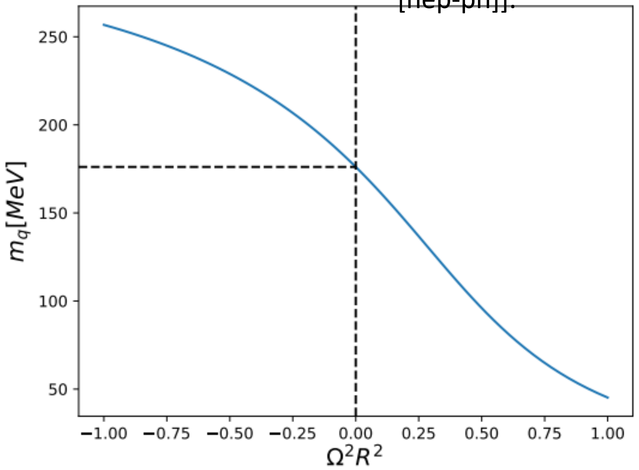
Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.



Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016) [arXiv:1606.03808 [hep-ph]].



Ji-Chong Yang and Xu-Guang Huang arxiv:2307.05755



HL Chen, ZB Zhu, XG Huang arXiv:2306.08362

Functional renormalization group approach

Partition function with an IR regulator

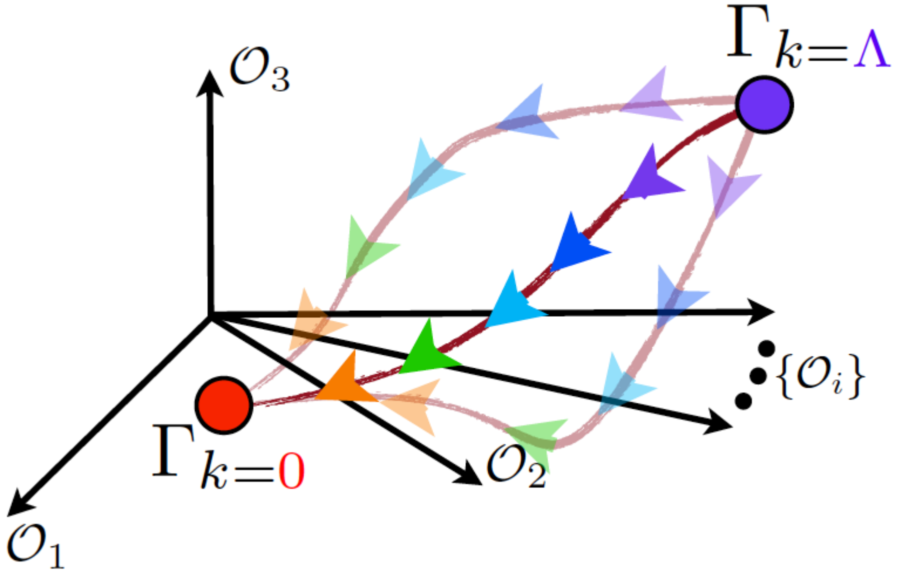
$$Z_k[J] = \int D\chi e^{-S[\chi]+\int_x \chi(x)J(x)-\Delta S_k[\chi]}$$

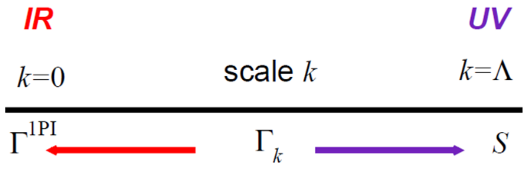
regulator

$$\Delta S_k[\chi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\chi^*(q)R_k(q)\chi(q)$$

Legendre transformation:

$$\Gamma_k[\phi] = -W_k[J] + \int_x \phi(x)J(x) + \Delta S_k[\phi]$$





flow equation

$$\partial_k\hat{\Gamma}_k[\Phi,\bar{\psi},\psi] = \frac{1}{2}\mathrm{STr}\left[\left(\hat{\Gamma}_k^{(2)}[\Phi,\bar{\psi},\psi]+\hat{R}_k\right)^{-1}\partial_k\hat{R}_k\right]$$

$$\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2,0)}+R_k^B}\partial_k R_k^B\right] - \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(0,2)}+R_k^F}\partial_k R_k^F\right]$$

QUARK-MESON MODEL AND FRG FLOW EQUATION

The QM model Lagrangian in Euclidean spacetime with rotation:

$$L = \phi\left[-\left(-\partial_\tau + \Omega\hat{L}_z\right)^2 - \nabla^2\right]\phi + U(\phi)$$
$$+\bar{q}\left[\gamma^0\left(\partial_\tau - \Omega\hat{J}_z\right) - i\gamma^i\partial_i + g\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma^5\right)\right]q$$

$$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 - c\sigma$$

$$\hat{L}_z = (-i\vec{r}\times\nabla)_z$$
$$\hat{J}_z = \hat{L}_z + \hat{S}_z$$

$$g_{\mu\nu} = \begin{pmatrix} 1-r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Effective action for mesons:

$$\Gamma_k^B = \frac{1}{2}\,l\,n\,d\,e\,t\left[-\left(-\partial_\tau + \Omega\hat{L}_z\right)^2 - \nabla^2 + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}\right]$$
$$= \frac{1}{2}\int d^4x_E T\sum_n\int\frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{N_{l,i}^2}\,t\,r\,l\,n\left[-(i\omega_n + \Omega ll)^2 + p_{l,i}^2 + p_z^2 + R_{\phi,k} + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}\right]J_l\left(p_{l,i}r\right)^2$$

effective action for fermion:

$$\Gamma_k^F = -\frac{1}{2}\,l\,n\,d\,e\,t\left(D_k\gamma^5 D_k^\dagger\gamma^5\right)$$
$$= -\frac{1}{2}\,t\,r\,l\,n\left[-\left(\partial_\tau - \Omega\hat{J}_z\right)^2 - \nabla^2 + \hat{R}_{q,k}^2 + gMM^\dagger\right]$$
$$= -\frac{1}{2}\int d^4x_E T\sum_n\int\frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{N_{l,i}^2}2N_cN_f\,l\,n\left[(\nu_n + i\Omega j)^2 + \tilde{p}^2 + R_{\phi,k} + g^2\phi^2\right]\left[J_l\left(\tilde{p}_{l,i}r\right)^2 + J_{l+1}\left(\tilde{p}_{l,i}r\right)^2\right]$$

QUARK-MESON MODEL AND FRG FLOW EQUATION

The QM model Lagrangian in Euclidean spacetime with rotation:

$$\Gamma_k = \int d^4x \frac{1}{2}\phi[-(-\partial_\tau + \Omega L_z)^2 - \nabla^2]\phi + \overline{q}[\gamma^0(\partial_\tau - i\Omega \hat{J_z}) - i\gamma^i\partial_i + g(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]q + U_k(\rho)$$

Effective action for mesons:

$$\Gamma_k^B = \frac{1}{2} ln\,det\left[-\left(-\partial_\tau + \Omega\hat{L}_z\right)^2 - \nabla^2 + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}\right]$$

$$= \frac{1}{2}\int d^4 x_E T\sum_n \int \frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{N_{l,i}^2} tr\,ln\left[-(i\omega_n + \Omega ll)^2 + p_{l,i}^2 + p_z^2 + R_{\phi,k} + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}\right] J_l\left(p_{l,i}r\right)^2$$

Effective action for fermion:

$$\Gamma_k^F = -\frac{1}{2} ln\,det\left(D_k\gamma^5 D_k^\dagger\gamma^5\right)$$

$$= -\frac{1}{2} tr\,ln\left[-\left(\partial_\tau - \Omega\hat{J}_z\right)^2 - \nabla^2 + \hat{R}_{q,k}^2 + gMM^\dagger\right]$$

$$= -\frac{1}{2}\int d^4 x_E T\sum_n \int \frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{N_{l,i}^2} 2N_c N_f\, ln\left[(\nu_n + i\Omega j)^2 + \tilde{p}^2 + R_{\phi,k} + g^2\phi^2\right]\left[J_l\left(\tilde{p}_{l,i}r\right)^2 + J_{l+1}\left(\tilde{p}_{l,i}r\right)^2\right]$$

QUARK-MESON MODEL AND FRG FLOW EQUATION

Regulator for mesons:

$$R_{\phi,k} = (k^2 - p^2)\theta(k^2 - p^2)$$

Flow equation for mesons:

$$\partial_k \Gamma_k^B = \frac{1}{2}\int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{N_{l,i}^2}\mathrm{tr}\frac{2k\theta(k^2-p^2)}{-(i\omega_n + \Omega l)^2 + k^2 + \frac{\partial^2 U}{\partial\phi_i\partial\phi_j}} J_l(p_{l,i}r)^2$$

Regulator for fermions:

$$\hat{R}_{\psi,k} = -i\gamma^i\partial_i\left(\frac{k}{\sqrt{-\nabla^2}} - 1\right)\theta(k^2 + \nabla^2)$$

Flow equation for fermions:

$$\partial_k \Gamma_k^F = -\frac{1}{2}\int d^4 x_E T \sum_n \int \frac{dp_z}{2\pi}\frac{1}{2\pi}\sum_{l,i}\frac{1}{\tilde{N}_{l,i}^2} 2N_c N_f \frac{\partial_k R_{\phi,k}}{(\nu_n + i\Omega j)^2 + \tilde{p}^2 + R_{\phi,k} + g^2\phi^2}\left[J_l(\tilde{p}_{l,i}r)^2 + J_{l+1}(\tilde{p}_{l,i}r)^2\right]$$

Flow equation:

$$\partial_k U_k = \frac{1}{\beta V}\left(\partial_k \Gamma_k^B + \partial_k \Gamma_k^F\right)$$

$$= \frac{1}{\beta V}\int d^4 x_E \frac{1}{(2\pi)^2}\left\{\sum_{l,i}\frac{1}{N_{l,i}^2}\mathrm{tr}\frac{k\sqrt{k^2 - p_{l,i}^2}}{\varepsilon_\phi}\frac{1}{2}\left[\coth\frac{\beta(\varepsilon_\phi + \Omega l)}{2} + \coth\frac{\beta(\varepsilon_\phi - \Omega l)}{2}\right] J_l(p_{l,i}r)^2\right.$$
$$\left. - \sum_{l,i}\frac{1}{\tilde{N}_{l,i}^2} 2N_c N_f \frac{k\sqrt{k^2 - \tilde{p}_{l,i}^2}}{\varepsilon_q}\frac{1}{2}\left[\tanh\frac{\beta(\varepsilon_q + \Omega j)}{2} + \tanh\frac{\beta(\varepsilon_q - \Omega j)}{2}\right]\left[J_l(\tilde{p}_{l,i}r)^2 + J_{l+1}(\tilde{p}_{l,i}r)^2\right]\right\}$$

$$\varepsilon_\sigma = \sqrt{k^2 + 2\bar{U}' + 4\rho\bar{U}''} = \sqrt{k^2 + \partial_\sigma^2 \bar{U}},$$

$$\varepsilon_\pi = \sqrt{k^2 + 2\bar{U}'} = \sqrt{k^2 + \partial_\sigma \bar{U}/\sigma},$$

$$\varepsilon_q = \sqrt{k^2 + g^2\rho},$$

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

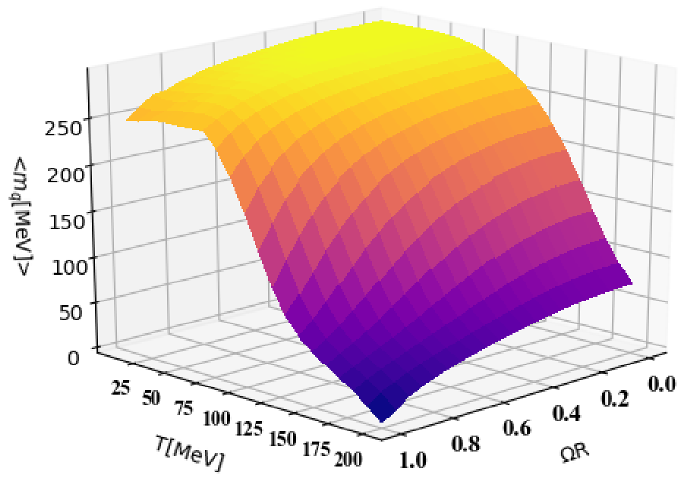


FIG. 1. The quark mass m_q as a function of Ω and T at r = 0.9R in QM model.

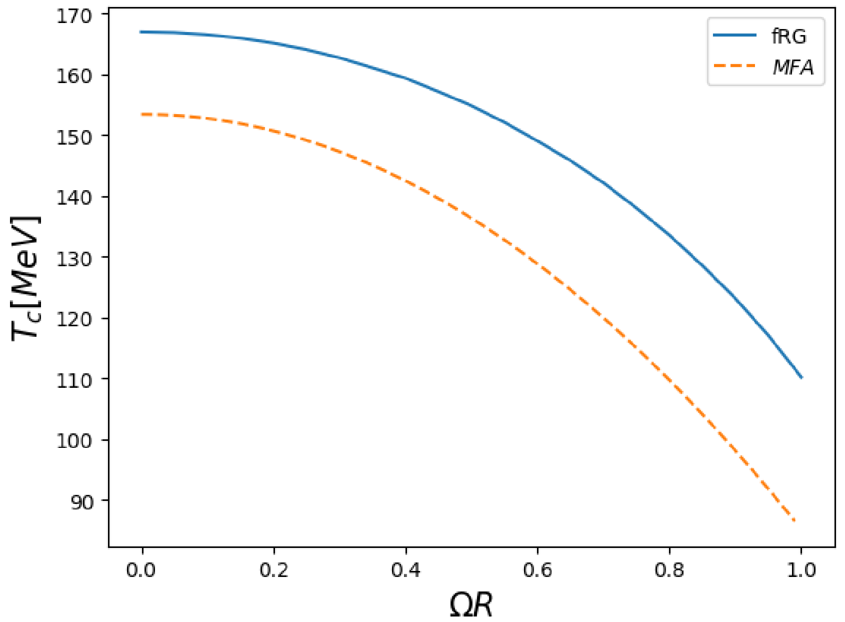
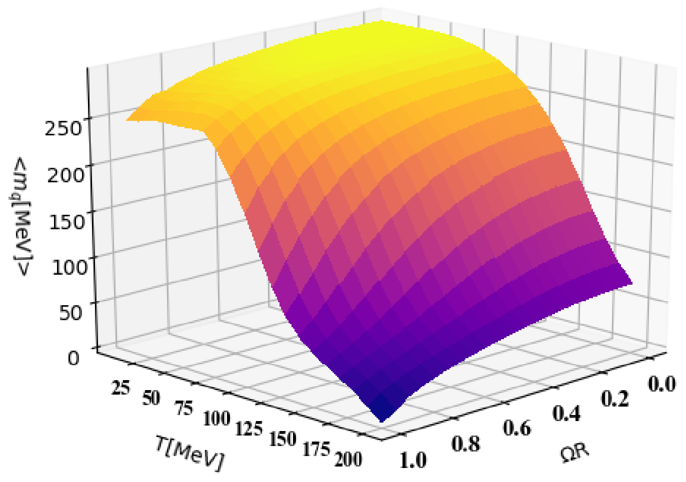


FIG. 2. The pesudo-critical temperature Tc as a function of Ω at r = 0.9R from fRG and MFA in QM model.

- **initial condition:** $$U_\Lambda = \frac{m_\Lambda^2}{2}\phi^2 + \frac{\lambda_\Lambda}{4}\phi^4 - c\sigma$$
- **Parameters chosen as:** $$m_\Lambda = 0.794\Lambda$$ $$\lambda_\Lambda = 2$$ $$c = 0.00175\Lambda^{-3}$$

- **Using the grid method**
- **Rotation effect almost invisible in low temperature**
- **Rotation effect suppress the chiral condensate**

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION



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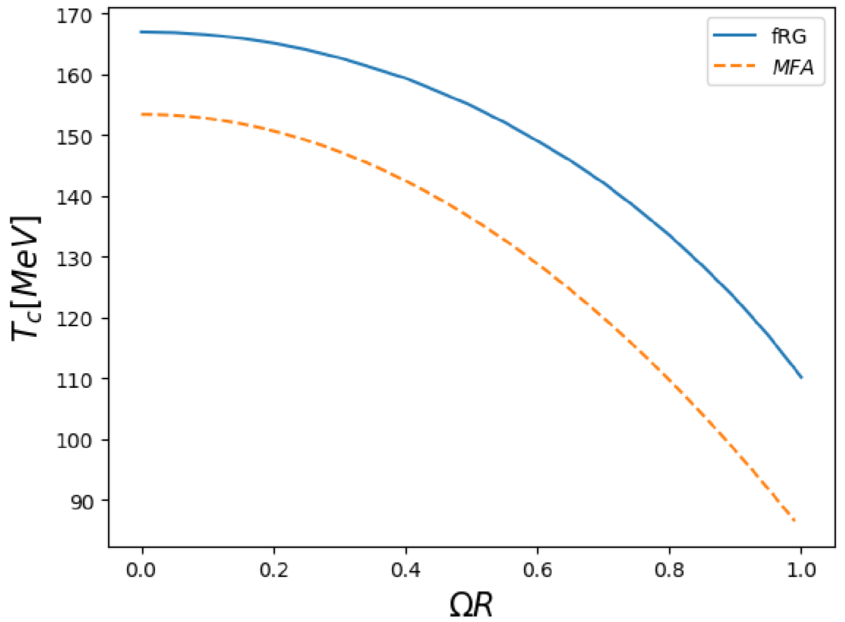


FIG. 2. The pesudo-critical temperature Tc as a function of Ω at r = 0.9R from fRG and MFA in QM model.

- Using the grid method
- Rotation effect almost invisible in low temperature
- Rotation effect suppress the chiral condensate

- T_c defined in where chiral susceptibility reach maximum
- T_c decreases with increasing rotation Ω

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

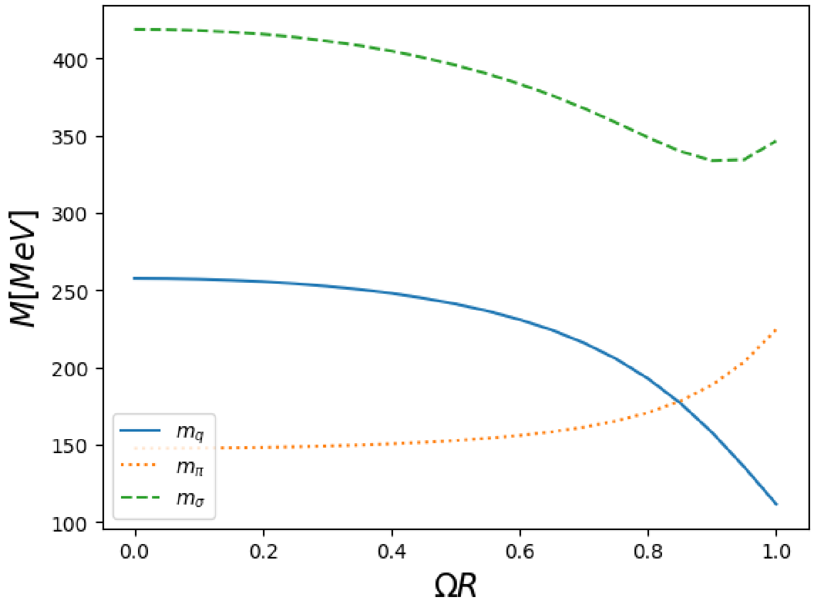


FIG. 3. Meson masses and quark mass as functions of Ω at T = 120 MeV from QM model FRG.

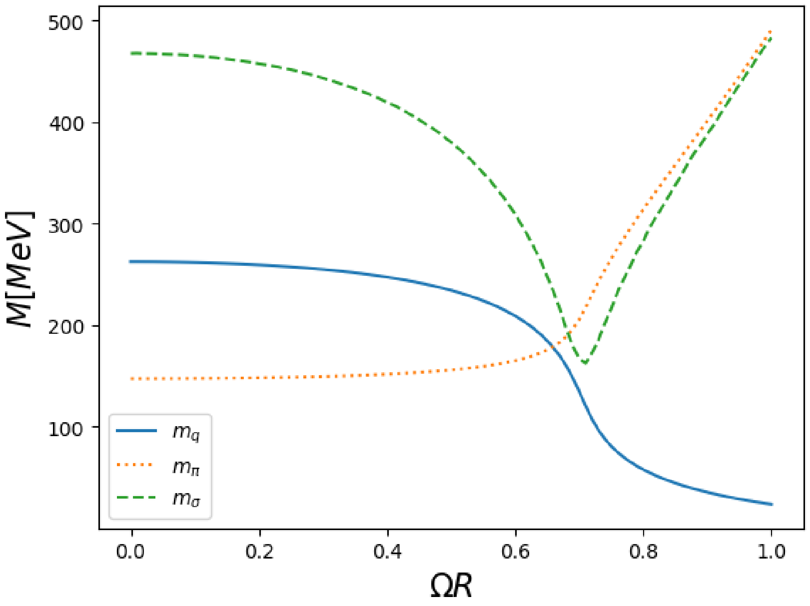


FIG. 4. Meson masses and quark mass as functions of Ω at T = 120 MeV from QM model MFA.

- m_π and m_σ have a tendency become degenerate
- rotational effect is milder in our FRG calculation compared to the mean-field approximation

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION

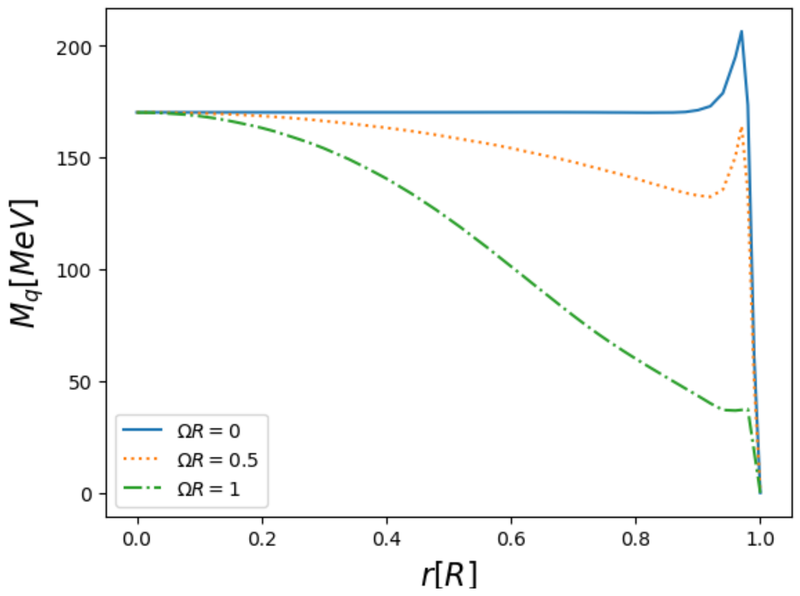


FIG. 5. The quark mass as a function of the radius r at different Ω at T = 160 MeV.

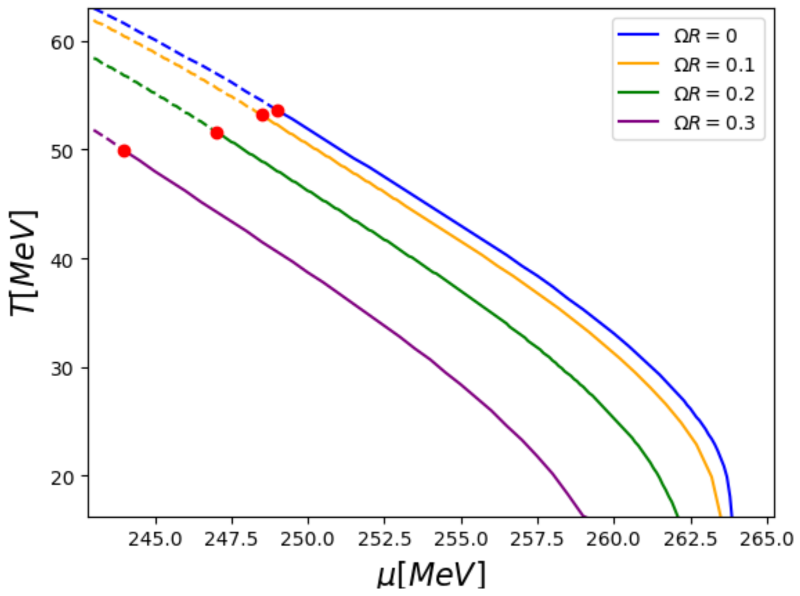


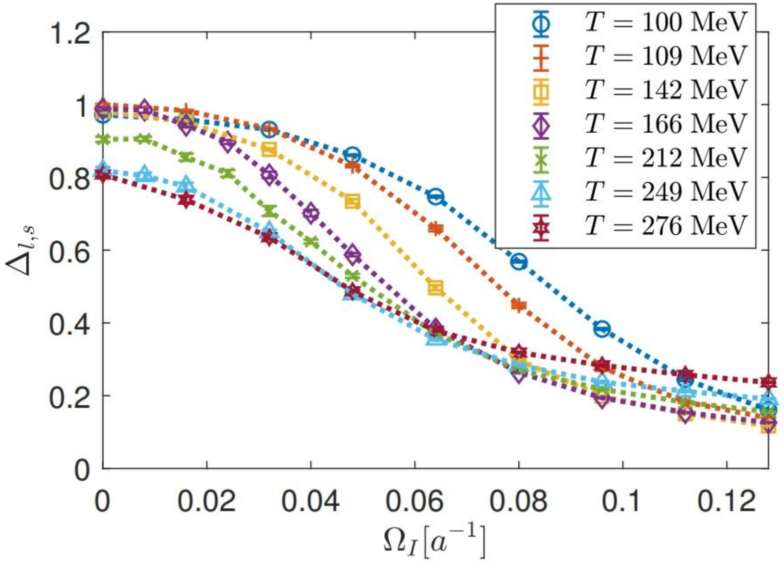
FIG. 6. T − μ phase diagram near the critical end point at different angular velocity.

- **Fig 6 choose the parameters:**

$$\Lambda = 500MeV,\quad m_\Lambda = 0,\quad \lambda_\Lambda = 10,\quad c = 0.$$

- **Rotational suppression is strong near the boundary**
- **The fermions feel a effective chemical potential of $\Omega/2$**
- **The CEP shifting into low T and μ with increasing angular velocity**

NUMERICAL RESULTS FROM THE FRG FLOW EQUATION



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arxiv:2307.05755

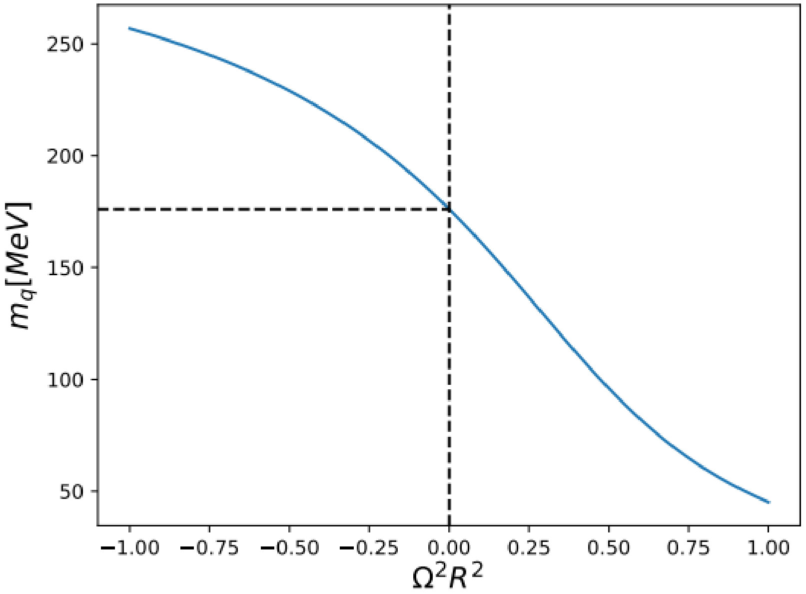


FIG. 7. The quark mass as a function of the square of the rotating angular velocity at T = 160 MeV and r = 0.9R from fRG calculation

- **Quark mass is a smooth function of Ω^2 which provide a condition for analytic continuation**
- **The behavior with increasing imaginary angular velocity is contrary**

Summary

- **Calculate the chiral condensate using FRG approach under rotating QM model**
- **Rotation will suppress chiral condensate at non-zero temperature which is agree with the model calculation**

Unruh effect in heavy ion collisions

The Hawking–Unruh effect predicts that the accelerated observer sees Minkowski vacuum state as a thermal bath of particles with temperature $T = a/2\pi$.

According to **Dmitri** work[1], the Unruh effect under strong color fields in color glass condensate picture should be observable.

The strength of the color-electric field $E \sim Q_S^2/g$, where $\boldsymbol{Q_s}$ is the saturation scale and g is the strong coupling and the typical acceleration is $a \sim Q_S \sim 1GeV$ $(T = \frac{a}{2\pi} \sim 200MeV)$

[1] Kharzeev D, Tuchin K. From color glass condensate to quark–gluon plasma through the event horizon[J]. Nuclear Physics A, 2005, 753(3-4): 316-334.

Related papers for acceleration

Ohsaku T. Dynamical chiral symmetry breaking and its restoration for an accelerated observer[J]. Physics Letters B, 2004, 599(1-2): 102-110

Castorina P, Finocchiaro M. Symmetry restoration by acceleration[J]. arXiv preprint arXiv:1207.3677, 2012.

Basu P, Haridev S R, Samantray P. Aspects of spontaneous symmetry breaking in Rindler and anti–de Sitter spacetimes for the O (N) linear sigma model[J]. Physical Review D, 2023, 107(10): 105004.

Casado-Turrión A, Dobado A. Triggering the QCD phase transition through the Unruh effect: Chiral symmetry restoration for uniformly accelerated observers[J]. Physical Review D, 2019, 99(12): 125018.

Chiral symmetry have been discuss by applied low energy effective model calculation in Rindler space

Prokhorov G Y, Teryaev O V, Zakharov V I. Effects of rotation and acceleration in the axial current: density operator vs Wigner function[J]. Journal of High Energy Physics, 2019, 2019(2): 1-17.

Prokhorov G Y, Teryaev O V, Zakharov V I. Unruh effect for fermions from the Zubarev density operator[J]. Physical Review D, 2019, 99(7): 071901.

Becattini F, Buzzegoli M, Palermo A. Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: scalar field[J]. Journal of High Energy Physics, 2021, 2021(2): 1-51.

Palermo A, Buzzegoli M, Becattini F. Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: Dirac field[J]. Journal of High Energy Physics, 2021, 2021(10): 1-49.

Acceleration (and rotation) studied by equilibrium quantum statistical density operator or Wigner function. The phase transition under both acceleration and rotation have not been discuss.

Acceleration in relativity case

Minkowski coordinates (T,X,Y,Z)

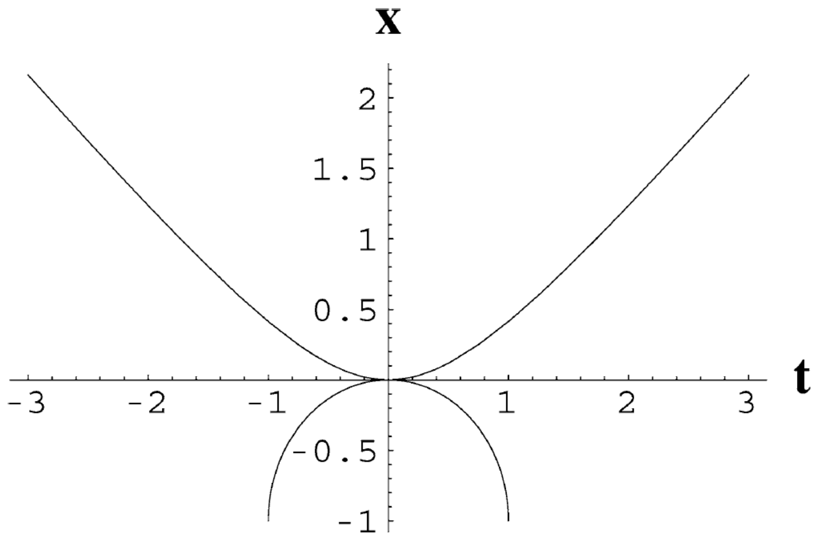
The world line for accelerated particle is $\boldsymbol{T = \frac{1}{a}\sinh(a\tau)\ , X = \frac{1}{a}cosh(a\tau)}$

The trajectory is hyperbola in Minkowski coordinates $\boldsymbol{X^2 - T^2 = \frac{1}{a^2}}$

$\boldsymbol{where\ \tau\ is\ the\ proper\ time\ , a\ is\ the\ proper\ acceleration}$

$$v(t) = \frac{at}{\sqrt{1 + a^2t^2}},$$
$$x(t) = a^{-1}\left(\sqrt{1 + a^2t^2} - 1\right)$$

$$a = \frac{d}{dt}\frac{v}{\sqrt{1 - v^2}}$$



Taken from Kharzeev D, Tuchin K. Nuclear Physics A, 2005, 753(3-4): 316-334.

Rindler spacetime

Minkowski coordinates (T,X,Y,Z)

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$$

Rindler coordinates (t,x,y,z)

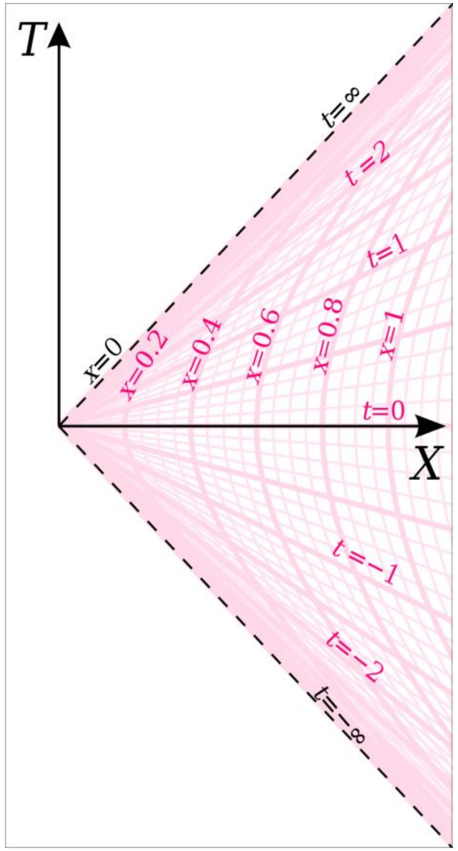
$$\boldsymbol{ds^2 = -(ax)^2 dt^2 + dx^2 + dy^2 + dz^2}$$

Coordinates transformation :

$$T = x\sinh(at)\ , X = x\cosh(at), Y = y, Z = z$$

The world line in Minkowski coordinates: $T = x\sinh(a\tau)\ , X = x\cosh(a\tau)$

The world line in Rindler coordinates: $x = \frac{1}{a}\ , t = \tau$



Rindler spacetime

Kottler-Moller coordinates:

$$T = \left(x + \frac{1}{\alpha}\right)\sinh(\alpha t)$$
$$X = \left(x + \frac{1}{\alpha}\right)\cosh(\alpha t) - \frac{1}{\alpha}$$
$$Y = y$$
$$Z = z$$

$$t = \frac{1}{\alpha}\,\mathrm{artanh}\left(\frac{T}{X + \frac{1}{\alpha}}\right)$$
$$x = \sqrt{\left(X + \frac{1}{\alpha}\right)^2 - T^2} - \frac{1}{\alpha}$$
$$y = Y$$
$$z = Z$$

$$ds^2 = -(1 + \alpha x)^2 dt^2 + dx^2 + dy^2 + dz^2$$

Radar coordinates:

$$T = \frac{1}{\alpha} e^{\alpha x} \sinh(\alpha t)$$
$$X = \frac{1}{\alpha} e^{\alpha x} \cosh(\alpha t)$$
$$Y = y$$
$$Z = z$$

$$t = \frac{1}{\alpha}\,\mathrm{artanh}\,\frac{T}{X}$$
$$x = \frac{1}{2\alpha}\ln\left[\alpha^2\left(X^2 - T^2\right)\right]$$
$$y = Y$$
$$z = Z$$

$$ds^2 = e^{2\alpha x}\left(-dt^2 + dx^2\right) + dy^2 + dz^2$$

NJL model in general spacetime

$$\mathcal{L}_{NJL} = \bar{\psi}[i\gamma^{\mu}\nabla_{\mu} - m_0]\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]$$

$$\{\gamma_{\mu}(x), \gamma_{\nu}(x)\} = 2g_{\mu\nu}(x), \{\gamma_{\hat{m}}, \gamma_{\hat{n}}\} = 2\eta_{\hat{m}\hat{n}}$$

$$g_{\mu\nu}g^{\nu\rho} = \delta^{\rho}_{\mu}, g^{\mu\nu}(x) = e^{\mu}_{\hat{m}}(x)e^{\nu\hat{m}}(x), \gamma_{\mu}(x) = e^{\hat{m}}_{\mu}(x)\gamma_{\hat{m}}.$$

Covariant derivative : $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}, \Gamma_{\mu} = -\frac{i}{4}\omega_{\mu\hat{i}\hat{j}}\sigma^{\hat{i}\hat{j}}, \sigma^{ij} = \frac{i}{2}[\gamma^{\hat{i}}, \gamma^{\hat{j}}], \omega_{\mu\hat{i}\hat{j}} = g_{ab}e^{a}_{\hat{i}}\nabla_{\mu}e^{b}_{\hat{j}}$

$$g_{\mu\nu} = \begin{pmatrix} (1+az)^2 - \omega^2 r^2 & \omega y & -\omega x & 0 \\ \omega y & -1 & 0 & 0 \\ -\omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^0(x) = \frac{1}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}}, \gamma^i(x) = \frac{(\boldsymbol{\omega}\times\boldsymbol{x})^i}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}} + \gamma^{\hat{i}}$$

$$\Gamma_0 = -\frac{i}{2}\boldsymbol{\omega}\cdot\sigma + \frac{1}{2}\boldsymbol{a}\cdot\boldsymbol{\alpha},$$

NJL model action in rotation and acceleration frame:

$$S = \int d^4x \left\{\bar{\psi}\left[i\gamma^{\hat{\mu}}\partial_{\mu} + i\boldsymbol{a}\cdot\boldsymbol{x}\gamma^{\hat{i}}\partial_i + \frac{i}{2}\boldsymbol{a}\cdot\gamma + \gamma^{\hat{0}}\boldsymbol{\omega}\cdot\boldsymbol{J} - m_0\phi\right]\psi + \frac{G}{2}\phi\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right]\right\}$$

Where $\phi = 1 + \boldsymbol{a}\cdot\boldsymbol{x}$

NJL model in general spacetime

$$\mathcal{L}_{NJL} = \bar{\psi}[i\gamma^{\mu}\nabla_{\mu} - m_0]\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]$$

$$Z = \int D[\bar{\psi},\psi]exp\left(i\int d^4x\sqrt{-g}\mathcal{L}_{\mathrm{NJL}}\right)$$
$$= \int D[\bar{\psi},\psi,\sigma,\pi]exp\left\{i\int d^4x\sqrt{-g}\left[\bar{\psi}(i\gamma^{\mu}\nabla_{\mu} - m - i\gamma^5\pi)\psi - \frac{\sigma^2+\pi^2}{2G}\right]\right\}$$

$$\Gamma = \frac{1}{i}\ln Z = -\int d^4x\sqrt{-g}\frac{\sigma^2+\pi^2}{2G} + \frac{1}{i}\ln\det(i\gamma^{\mu}\nabla_{\mu} - m - i\gamma^5\pi)$$

$$V_{eff} = -\frac{\Gamma}{\int d^4x\sqrt{g}} = \frac{\sigma^2+\pi^2}{2G} - \frac{1}{i}\mathrm{tr}\ln(i\gamma^{\nu}\nabla_{\nu} - m - i\gamma^5\pi)$$

$$\frac{\partial V_{\mathrm{eff}}}{\partial\sigma} = 0, \frac{\partial V_{\mathrm{eff}}}{\partial\pi} = 0$$

$$\frac{m - m_0}{G} = i\,\mathrm{tr}(S)$$

$$\frac{\pi}{G} = i\,\mathrm{tr}(i\gamma^5 S)$$

Where S the propagator

Dirac equation in Rindle spacetime

The Dirac equation:

$$\left(\frac{i\gamma^{\hat{\mu}}\partial_\mu + i\boldsymbol{a}\cdot\boldsymbol{x}\gamma^{\hat{\imath}}\partial_i + \frac{i}{2}\boldsymbol{a}\cdot\boldsymbol{\gamma} + \gamma^{\hat{0}}\boldsymbol{\omega}\cdot\boldsymbol{J}}{\phi} - m\right)\psi = 0$$

$$\left[\frac{i\gamma^{\hat{0}}}{\phi}\left(\partial_0 - \frac{i}{2}\omega\cdot\sigma_3 + \frac{1}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{\imath}}\partial_i + \frac{\gamma^{\hat{0}}}{\phi}\omega[x_1(-i\partial_2) - x_2(-i\partial_1)] - s\right]\psi = 0$$

$$\left[\frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0 + \omega\left(\frac{\sigma_3}{2} + \hat{L}_z\right) + \frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{\imath}}\partial_i - s\right]\psi = 0$$

$$\frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0 + \omega\left(\frac{\sigma_3}{2} + \hat{L}_z\right) + \frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{\imath}}\partial_i \equiv \hat{A}$$

$$(\hat{A} - m)\psi = 0$$

Green's functions

The Green's functions S(x,y,s) and G(x,y;s) :

$$\left(\hat{A} - s\right)S(x, y, s) = \frac{1}{\sqrt{-g}}\delta(x, y),$$
$$\left(\hat{A} + s^{\dagger}\right)G(x, y, s) = S(x, y, s),$$
$$\left(\hat{A}^2 - s^{\dagger}s\right)G(x, y, s) = \frac{1}{\sqrt{-g}}\delta(x, y).$$

$$\hat{A} \equiv \frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0 + \omega\left(\frac{\sigma_3}{2} + \hat{L}_z\right) + \frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{\imath}}\partial_i$$

$$A^2 = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\right] + \partial_1^2 + \partial_2^2$$

Dirac equation in z direction

$$A^2 = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\right] + \partial_1^2 + \partial_2^2$$

Projection operator:

$$\hat{P}^{\pm} = \frac{1}{2}\left(1 \pm \gamma^{\hat{0}}\gamma^{\hat{3}}\right), G = \hat{P}_+ G_+ + \hat{P}_- G_-$$

$$\hat{P}^+ + \hat{P}^- = 1, \hat{P}^{\pm}\hat{P}^{\pm} = \hat{P}^{\pm}, \hat{P}^+\hat{P}^- = 0$$

$$\hat{Q} = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \partial_1^2 + \partial_2^2 - s^2$$

$$\hat{R} = \frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\right]$$

$$\hat{A}^2 - s^2 = \left(\hat{Q} + \hat{R}\right)\hat{P}^+ + \left(\hat{Q} - \hat{R}\right)\hat{P}^-$$

$$\left(\hat{Q} \pm \hat{R}\right) = \left\{\frac{1}{\phi^2}\left[(E + j\omega)^2 - \frac{1}{4}a^2\right] - p_{l,k}^2 - s^2 + \partial_3^2 + \frac{1}{\phi}a\partial_3 \pm \frac{a}{\phi^2}i[E + j\omega]\right\}$$

Eigenfunction

$$\left(\hat{Q} \pm \hat{R}\right) = \left\{\frac{1}{\phi^2}\left[(E + j\omega)^2 - \frac{1}{4}a^2\right] - p_{l,k}^2 - s^2 + \partial_3^2 + \frac{1}{\phi}a\partial_3 \pm \frac{a}{\phi^2}i[E + j\omega]\right\}$$

$$\left\{a^2\frac{d^2}{d\phi^2} + \frac{a^2}{\phi}\frac{d}{d\phi} - \left(p_{l,k}^2 + s^2 - \frac{\varepsilon^2 - \frac{1}{4}a^2}{\phi^2} \mp \frac{a}{\phi^2}i\varepsilon\right)\right\}f^{\pm}(\phi) = 0$$

$$a^2\alpha^2\left\{\frac{d^2}{d\alpha^2\phi^2} + \frac{1}{\alpha\phi}\frac{d}{d\alpha\phi} - \left(1 + \frac{\left(-\frac{i\varepsilon}{a} \pm \frac{1}{2}\right)^2}{\alpha^2\phi^2}\right)\right\}f^{\pm}(\alpha\phi) = 0$$

$$,where\ \varepsilon = E + \omega j\ , \phi = 1 + az\ , \alpha = \left[\frac{p_{l,k}^2 + s^2}{a^2}\right]^{\frac{1}{2}}$$

The solution:

$$f_{\Omega}^{\pm}(\alpha\phi) = \frac{\sqrt{(\mp 2i\Omega/a - 1)\cosh(\pi\Omega/a)}}{\pi}K_{\frac{i\Omega}{a}\pm\frac{1}{2}}(\alpha\phi)$$

K_μ is the modified Bessel function

The orthonormal condition :

$$\int_0^\infty \frac{d\xi}{\xi}\Psi_{\Omega}^{\pm}(\xi)\Psi_{\Omega'}^{\pm}(\xi) = \int_0^\infty \frac{d\xi}{\xi}\frac{(\mp 2i\Omega - 1)\cosh\pi\Omega}{\pi^2}K_{i\Omega\pm1/2}(\alpha\xi)K_{i\Omega'\pm1/2}(\alpha\xi) = \delta(\Omega,\Omega')$$

Green's functions

Fudan University 复旦大学

$$G(x_1, x_2, s) = \sum_{l,k} \int \frac{dp_0}{2\pi} e^{-ip_0(t_1-t_2)} \frac{1}{2\pi} \frac{2}{J_{l+1}^2(p_{l,k}R)R^2} \mathcal{M}(l, k, x_1, x_2) \mathcal{G}(z_1, z_2, l, k, s)$$

$$\mathcal{M}(l, k, x_1, x_2) = diag\left(\eta_l \eta_l', \xi_l \xi_l', \eta_l \eta_l', \xi_l \xi_l'\right) \; and \; \eta_l = e^{il\theta} J_l\left(p_{l,k} r\right), \xi_l = e^{i(l+1)\theta} J_{l+1}\left(p_{l,k} r\right)$$

Projection operator for rotation part:

$$\widehat{P_1}^{\pm} = \frac{1}{2}\left(1 \pm i\gamma^{\hat{1}}\gamma^{\hat{2}}\right), \mathcal{M} = \widehat{P_1}^{+} \eta\eta' + \widehat{P_1}^{-} \xi_l \xi_l'$$

$$G = \sum_{s_1, s_2 = \pm 1} \frac{1}{2}(1 + s_1 \gamma^{\hat{0}} \gamma^{\hat{3}}) \frac{1}{2}(1 + s_2 i\gamma^{\hat{1}}\gamma^{\hat{2}}) \mathcal{G}^{s_1, s_2}$$

$$\mathcal{G}^{s_1,s_2} = \sum_{l,k} \int \frac{dp_0}{2\pi} - \frac{1}{\left(-i\varepsilon \pm \frac{1}{2}a\right)^2 - \left(i\Omega \pm \frac{1}{2}a\right)^2} \frac{(\mp 2i\Omega/a - 1)\cosh(\pi\Omega/a)}{\pi^2} K_{\frac{i\Omega}{a} \pm \frac{1}{2}}(\alpha\phi_1) K_{\frac{i\Omega}{a} \pm \frac{1}{2}}(\alpha\phi_2)$$

$$\times \frac{1}{2\pi} \frac{1}{N_{l,k}} e^{i(j - s_2/2)(\theta_1 - \theta_2)} J_{j - s_2/2}(p_{l,k} r_1) J_{j - s_2/2}(p_{l,k} r_2)$$

Gap equation

When $m_0 = 0$, we can set π condensate $\pi = 0$

$$\frac{m}{G} = i\,\mathrm{tr}(S)$$

When $T = \frac{a}{2\pi}$, , $m_0 = 0$, the gap equation become

$$\frac{1}{G} = \sum_{l,k,s_1} \int d\,\Omega \frac{1}{2\pi} \frac{1}{N_{l,k}} \frac{-is_1}{2a} \frac{cosh(\pi\,\Omega/a)}{\pi^2} \left\{ \tan\mathrm{h}\left(\frac{\Omega - \omega j}{2T}\right) + \tan\mathrm{h}\left(\frac{\Omega + \omega j}{2T}\right) \right\} \times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r) + J_{l+1}^2(p_{l,k}r)\right]$$

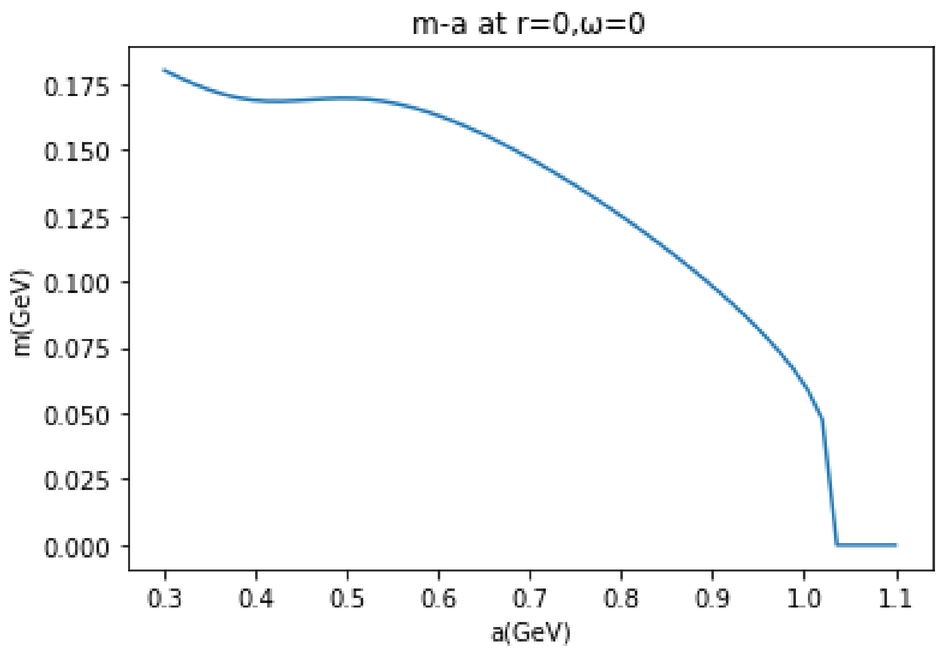
If we ignore the boundary and take the non rotation limit, we have

$$1 = G \int d\Omega \int \frac{d^2 p_t}{(2\pi)^2} \frac{-i}{a} \frac{sinh(\pi\Omega/a)}{\pi^2} \left\{ K^2_{\frac{i\Omega}{a}+\frac{1}{2}}(\alpha) - K^2_{\frac{i\Omega}{a}-\frac{1}{2}}(\alpha) \right\}$$

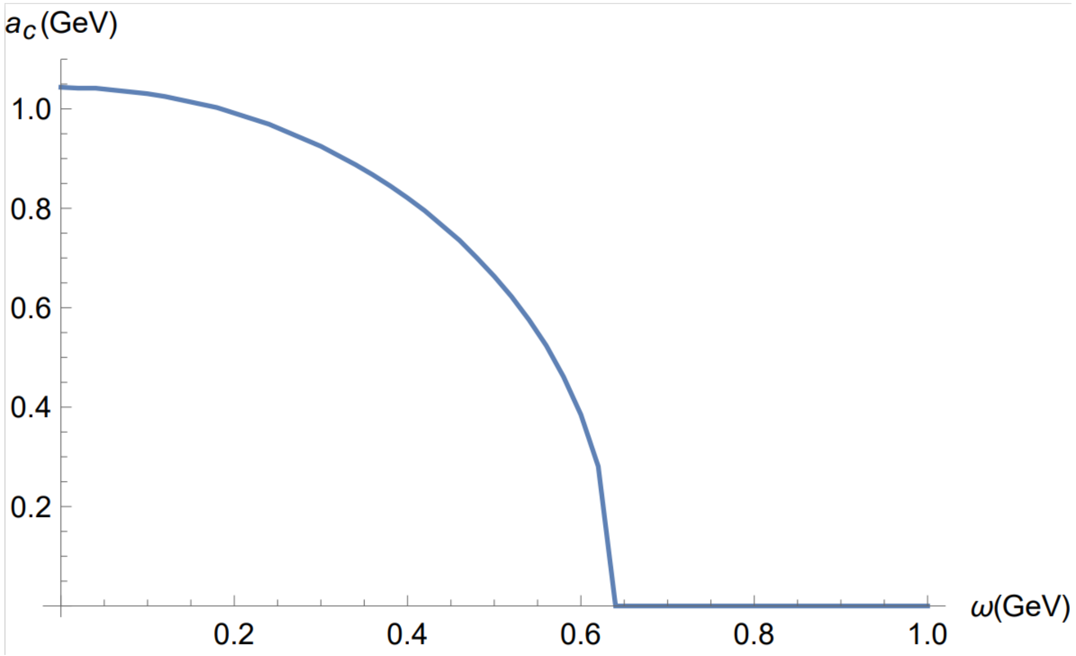
Where we have set $\phi = 1$. The result is same as Ohsaku work[1]

[1] Ohsaku T. Dynamical chiral symmetry breaking and its restoration for an accelerated observer[J]. Physics Letters B, 2004, 599(1-2): 102-110

Result from gap equation



chiral condensate as a function of acceleration obtained form gap equation



Critical acceleration a_c as a function of angular velocity ω

- **Acceleration suppress the chiral condensate at large value**
- **Critical acceleration suppressed with increasing angular velocity**

Gap equation

The case $m_0 \neq 0$

$$\frac{m-m_0}{G} = i\,\mathrm{tr}(S) \qquad \frac{\pi}{G} = i\,\mathrm{tr}(i\gamma^5 S)$$

$$i\,\mathrm{tr}(S) = i\,\mathrm{tr}\left[\left(i\gamma^\nu\nabla_\nu + m - i\gamma^5\pi\right)G\right] = i\sum_{s_1,s_2}\mathrm{tr}\left[\left(i\gamma^\nu\nabla_\nu + \underline{m} - \underline{i\gamma^5\pi}\right)\frac{1}{2}(\underline{1} + \underline{s_1\gamma^{\hat{0}}\gamma^{\hat{3}}})\frac{1}{2}(\underline{1} + \underline{s_2 i\gamma^{\hat{1}}\gamma^{\hat{2}}})\mathcal{G}^{s_1,s_2}\right]$$

$$i\,\mathrm{tr}(i\gamma^5 S) = i\,\mathrm{tr}\left[i\gamma^5\left(i\gamma^\nu\nabla_\nu + m - i\gamma^5\pi\right)G\right] = i\sum_{s_1,s_2}\mathrm{tr}\left[i\gamma^5\left(i\gamma^\nu\nabla_\nu + m - i\gamma^5\pi\right)\frac{1}{2}(1 + s_1\gamma^{\hat{0}}\gamma^{\hat{3}})\frac{1}{2}(1 + s_2 i\gamma^{\hat{1}}\gamma^{\hat{2}})\mathcal{G}^{s_1,s_2}\right]$$

$$\mathrm{tr}(\gamma^5) = \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0$$

$$\frac{m-m_0}{G} = i\,\mathrm{tr}(S) = i\sum_{s_1,s_2}\mathrm{tr}\left[\frac{m}{4}\mathcal{G}^{s_1,s_2} - \frac{i\pi}{4}s_1 s_2\mathcal{G}^{s_1,s_2}\right]$$

$$\frac{\pi}{G} = i\,\mathrm{tr}(i\gamma^5 S) = i\sum_{s_1,s_2}\mathrm{tr}\left[\frac{im}{4}s_1 s_2\mathcal{G}^{s_1,s_2} + \frac{\pi}{4}\mathcal{G}^{s_1,s_2}\right]$$

Gap equation

The case $m_0 \neq 0$

$$\frac{m-m_0}{G} = i\,\mathrm{tr}(S) = i\sum_{s_1,s_2}\mathrm{tr}\left[\frac{m}{4}\mathcal{G}^{s_1,s_2} - \frac{i\pi}{4}s_1 s_2 G^{s_1,s_2}\right]$$

$$\frac{\pi}{G} = i\,\mathrm{tr}(i\gamma^5 S) = i\sum_{s_1,s_2}\mathrm{tr}\left[\frac{im}{4}s_1 s_2\mathcal{G}^{s_1,s_2} + \frac{\pi}{4}\mathcal{G}^{s_1,s_2}\right]$$

$$tr[\frac{\mathcal{G}^{s_1,s_2}}{4}] = -i\sum_{l,k}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\,J^2_{j-s_2/2}(p_{l,k}r)$$

$$tr[\frac{s_1 s_2\mathcal{G}^{s_1,s_2}}{4}] = -is_1 s_2\sum_{l,k}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\,J^2_{j-s_2/2}(p_{l,k}r)$$

$$\frac{m-m_0}{G} = m\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] - i\pi\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

$$\frac{\pi}{G} = \pi\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] + im\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

Gap equation

The case $m_0 \neq 0$

$$\frac{m-m_0}{G} = m\sum_{l,k,s_1}\int d\Omega \frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] - i\pi\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

$$\frac{\pi}{G} = \pi\sum_{l,k,s_1}\int d\Omega \frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] + im\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

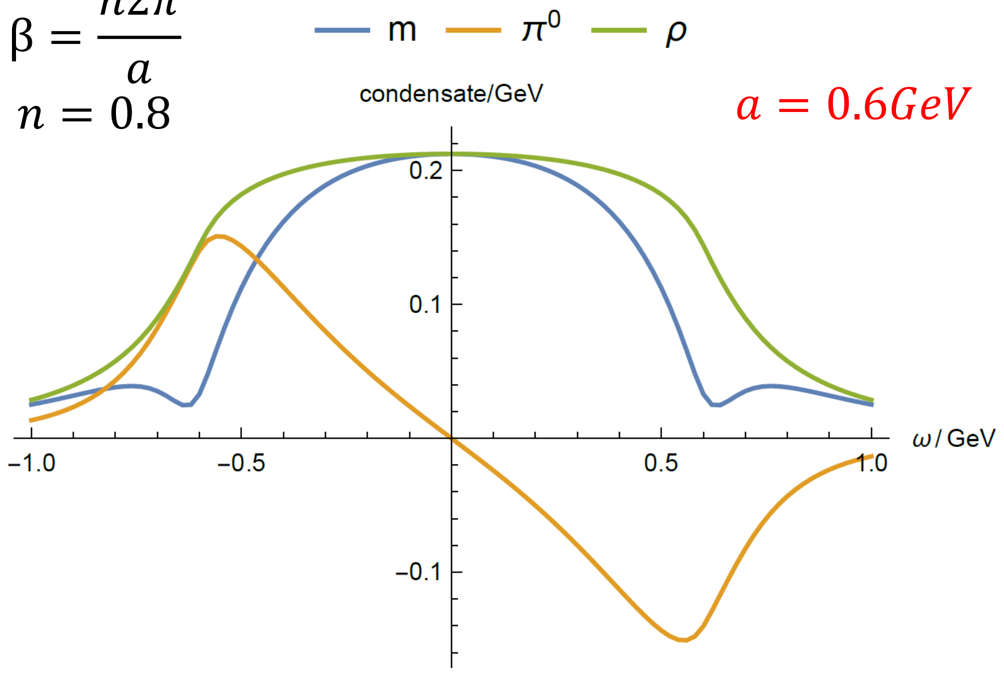
We consider the point $r=0$.When $T=\frac{a}{2\pi}$, the second term vanish such that no pion condensate

Result from gap equation

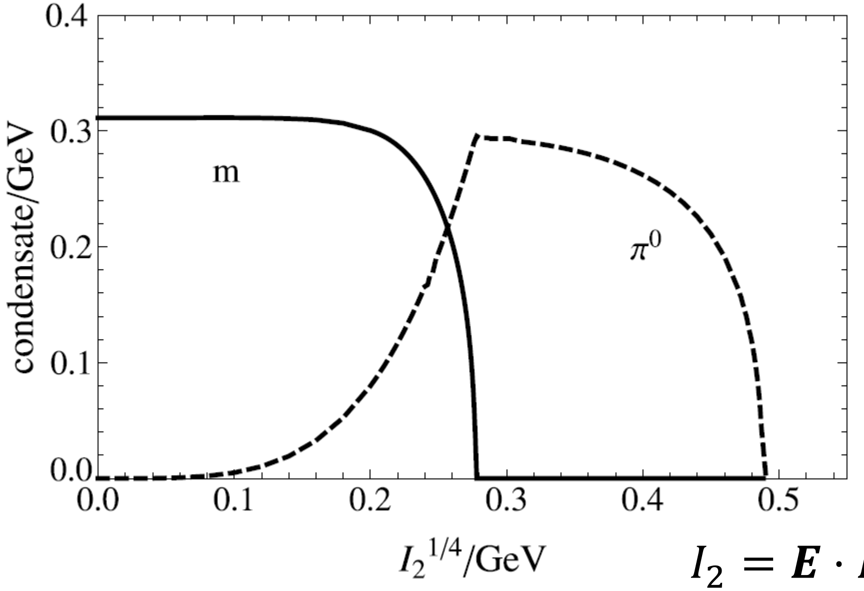
$\beta = \frac{n2\pi}{a}$

$n = 0.8$

$a = 0.6GeV$



Condensate as functions of rotation $\boldsymbol{\omega}$



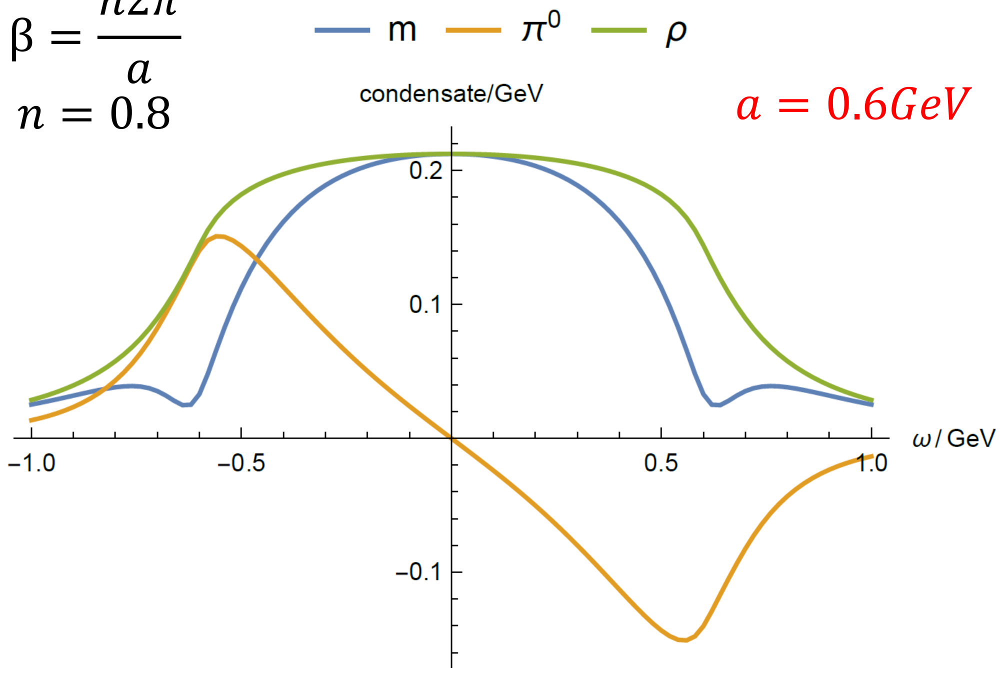
$I_2 = \boldsymbol{E} \cdot \boldsymbol{B}$

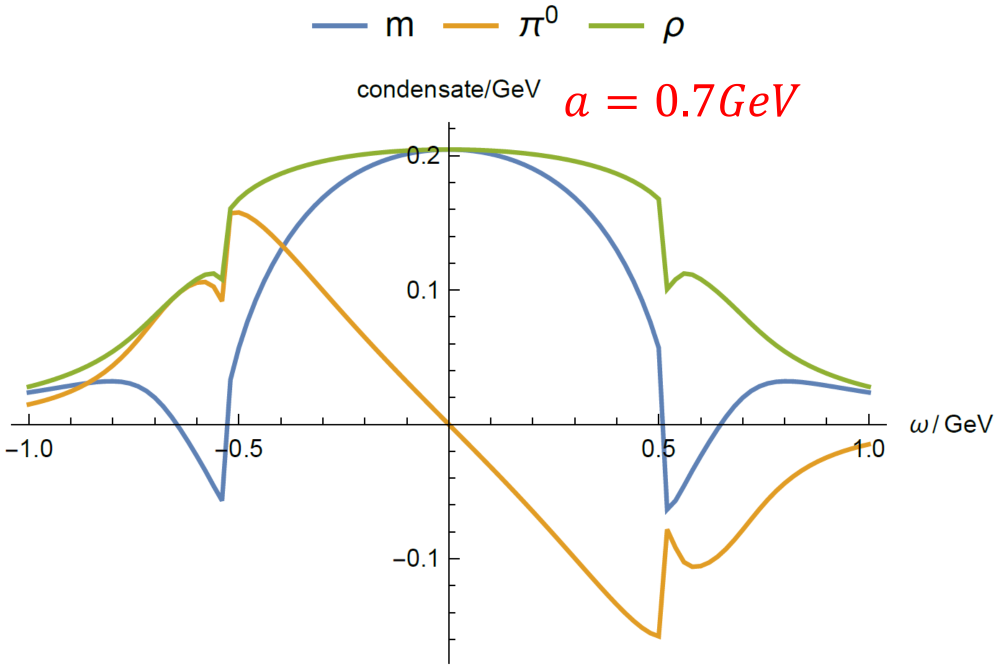
The constitute quark mass m and $\boldsymbol{\pi}^0$ condensate as functions of $I^{\frac{1}{4}}$ (Cao, G., & Huang, X. G. (2016). Physics Letters B, 757, 1-5. arXiv:1509.06222)

- Acceleration and rotation induced the pion ($\boldsymbol{\pi}^0$) condensate when $\boldsymbol{T} \neq \boldsymbol{T}_U$
- The presence of $\boldsymbol{a} \cdot \boldsymbol{\omega}$ tends to diminish $\boldsymbol{\sigma}$ condensate while drive $\boldsymbol{\pi}^0$ condensation which is similar to the presence of $\boldsymbol{E} \cdot \boldsymbol{B}$

Result from gap equation

$$\beta = \frac{n2\pi}{a}$$
$$n = 0.8$$





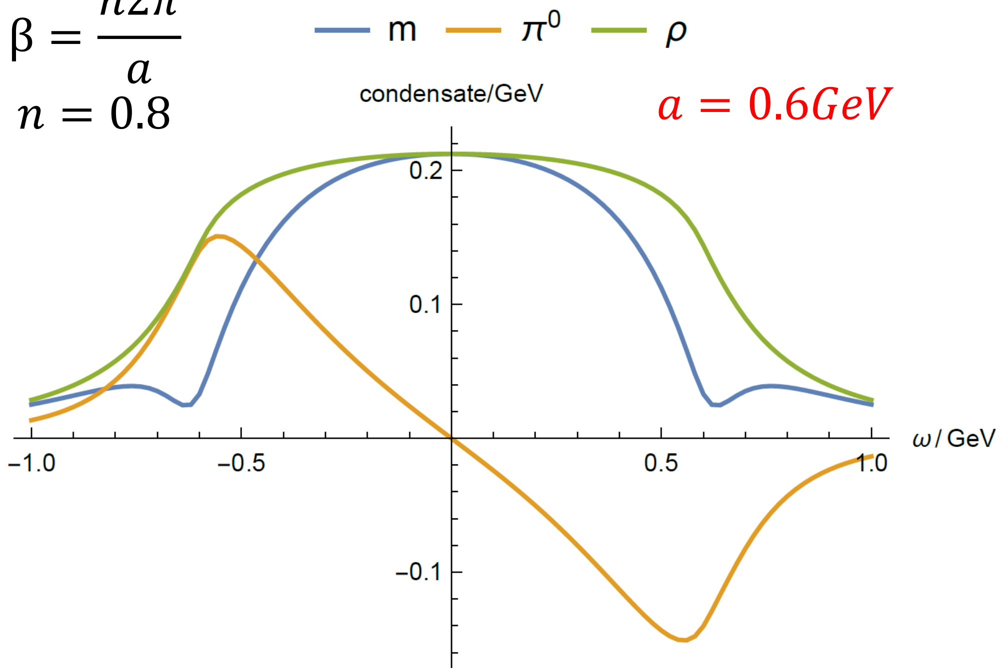
condensate as a function of rotation with different acceleration

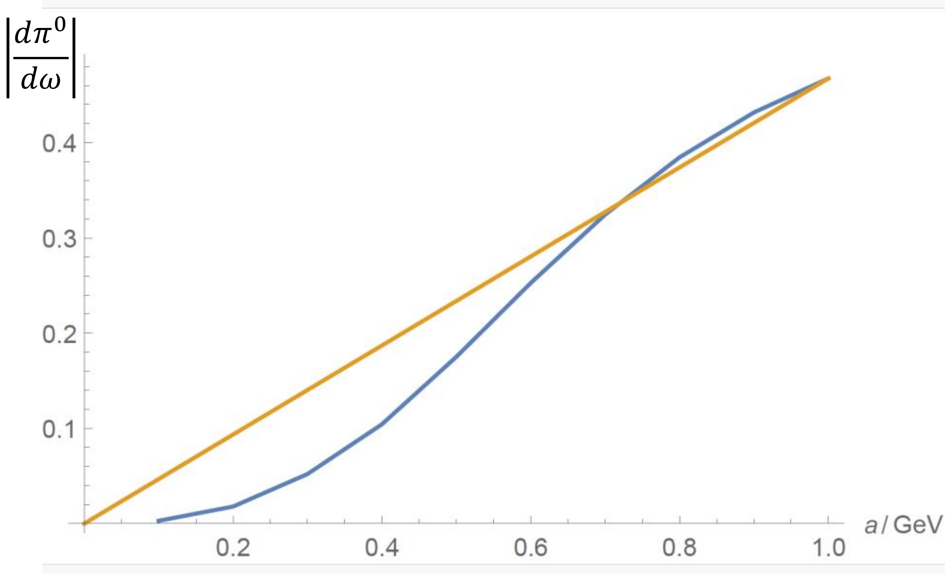
- **The pion (π^0) condensate is a odd function of ω and constitute quark mass m is even**
- **The discontinuous behavior showed near Inflection point showed when a is large enough**
- **The total condensate decrease with increasing ω**

Result from gap equation

$\beta = \frac{n2\pi}{a}$

$n = 0.8$

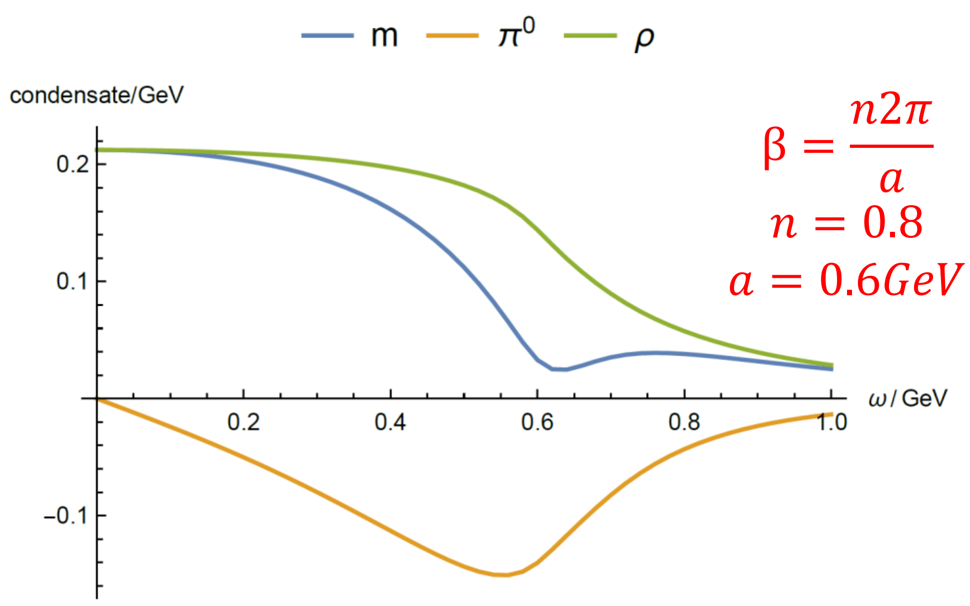


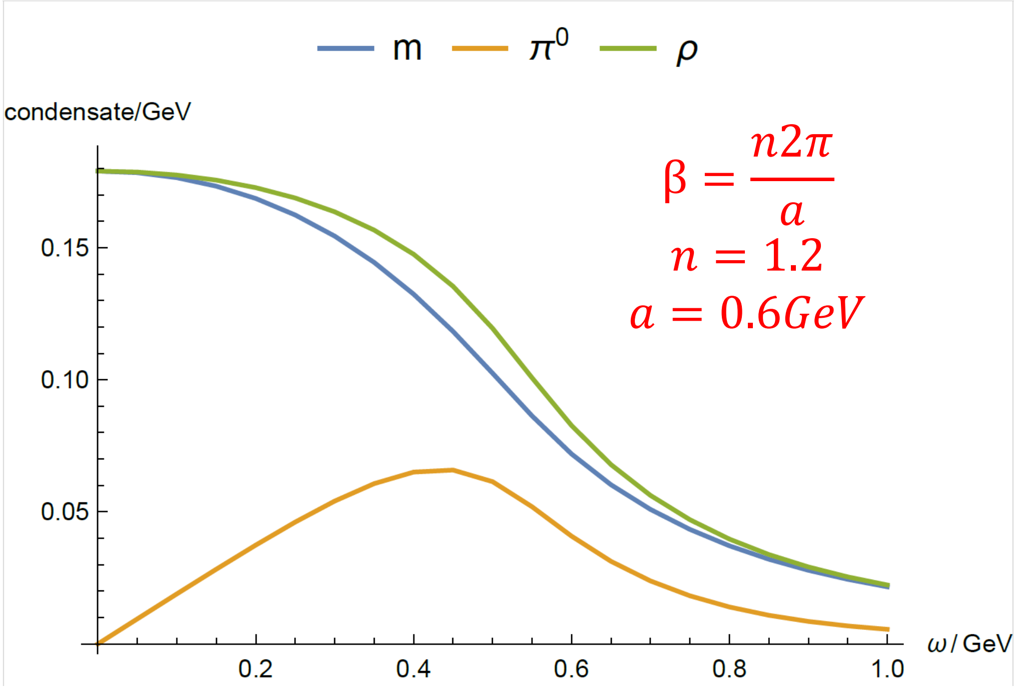


condensate as a function of rotation with different acceleration

- The pion (π^0) condensate is near a linear function in the region ω is small
- The slope $\left|\frac{d\pi^0}{d\omega}\right|$ increase with increasing a

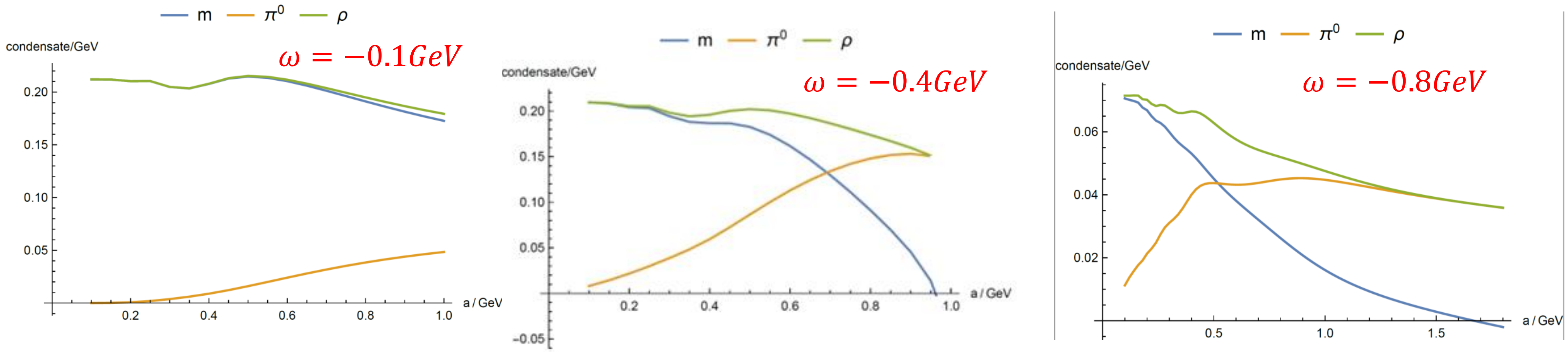
Result from gap equation





- **The sign of pion condensate change while temperature cross the Unruh temperature**

Result from gap equation



condensate as a function of acceleration with different rotation

- **The pion condensate vanish when $a \to 0$**
- **Acceleration suppress the total condensate at large value**

Summary

- **The acceleration and rotation restore the chiral symmetry**
- **The presence of $a \cdot \omega$ induce the pion condensate when $T \neq a/2\pi$**
- **Exist a chiral rotation from σ to π^0**

Thanks!

Thanks!