QCD under rotation workshop, Nov 2023

Vortical catalysis or …

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Rotation and magnetic field

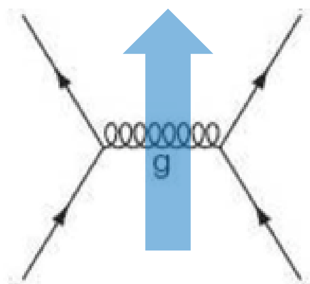
Magnetic field	Rotation
Pseduo-vector	Pseduo-vector
Polarize J	Polarize J
Chiral transportation	Chiral transportation
Anomaly effects	Anomaly effects
Chiral catalysis	Chiral inhibition
Inverse chiral catalysis	?

Quark fluctuation

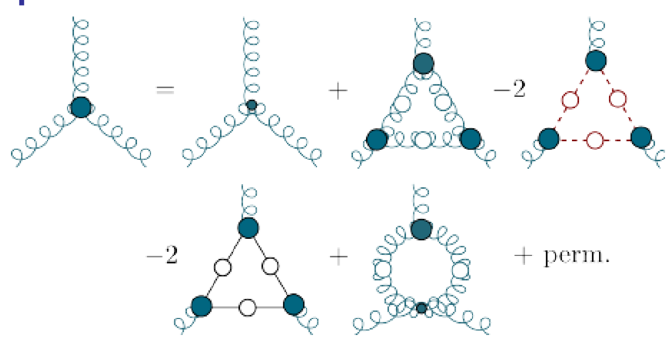
Chiral condensation

- With effective model, such as NJL, chiral condensation, which is typically non-perturbative, is studied intuitively by the 4-fermion interaction and mean-filed approx.

- All the gluon contributions is included in the coupling constant G.



- We should deal with analytical integration of 6 to 8 Bessel function multiplications



I am too lazy to go ahead in this direction.

$$D_{\mu\nu}(x,y) = \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)} e^{in(\phi-\theta)}$$
$$\times e^{-i(k_0-n\omega)(t-s)}\frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{k_0^2 - E_k^2 + i\eta}$$
$$= \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)} e^{in(\phi-\theta)}$$
$$\times e^{-ik_0(t-s)}\frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{(k_0-n\omega)^2 - E_k^2 + i\eta} \qquad (34)$$

$$D^n_{\mu\nu}(k_t,k_z;\rho,r) = \sum_\lambda A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)$$
$$= \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n^{++} & -iM_n^{+-} & 0 \\ 0 & iM_n^{-+} & M_n^{--} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad (35)$$
$$+\frac{E_{kt}^2}{4m^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n^{--} & -iM_n^{-+} & -2i\frac{k_tk_z}{E_{kt}^2}N_n^{1-} \\ 0 & iM_n^{+-} & M_n^{++} & 2\frac{k_tk_z}{E_{kt}^2}N_n^{1+} \\ 0 & 2i\frac{k_tk_z}{E_{kt}^2}N_n^{2-} & 2\frac{k_tk_z}{E_{kt}^2}N_n^{2+} & 4\frac{E_z^2}{E_{kt}^2}\Pi_n \end{pmatrix}$$
$$+\frac{k_tE_k}{2m^2}\begin{pmatrix} \frac{2k^2}{k_tE_k}\Pi_n & -iN_n^{2-} & -N_n^{2+} & -2\frac{k_z}{k_t}\Pi_n \\ iN_n^{1-} & 0 & 0 & 0 \\ -N_n^{1+} & 0 & 0 & 0 \\ -2\frac{k_z}{k_t}\Pi_n & 0 & 0 & 0 \end{pmatrix}$$

$$M_n^{++} = Z_n^+(\rho,\phi)Z_n^{+*}(r,\theta)$$
$$M_n^{+-} = Z_n^+(\rho,\phi)Z_n^{-*}(r,\theta)$$
$$M_n^{-+} = Z_n^-(\rho,\phi)Z_n^{+*}(r,\theta)$$
$$M_n^{--} = Z_n^-(\rho,\phi)Z_n^{-*}(r,\theta),$$

$$N_n^{1+} = Z_n^+(\rho,\phi)J_n(r)$$
$$N_n^{1-} = Z_n^-(\rho,\phi)J_n(r)$$
$$N_n^{2+} = J_n(\rho)Z_n^{+*}(r,\theta)$$
$$N_n^{2-} = J_n(\rho)Z_n^{-*}(r,\theta)$$

$$\Pi_n = J_n(\rho)J_n(r)$$

$$Z_n^+(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} + J_{n+1}(\rho)e^{i\phi}$$

$$Z_n^-(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} - J_{n+1}(\rho)e^{i\phi}$$

A novel method

- It is NOT a systematic method.
- In order to compute the coupling running of QED, consider the system the polarization energy in a background magnetic field.

See N.K.Nielsen, Am.J. Phys.49, 1171(1981);
R.A.Schneider, Phys. Rev. D 66, 036003(2002);
R.A.Schneider, Phys.Rev.D 67,057901(2003)

$$E = -\frac{1}{2}[4\pi\chi]VB^2 = \sum_{n,i=\{fermions\}} E_n^i$$

$$E = \sqrt{2gB(l+1/2\pm 1)+k_z^2}$$

- Eigen energy can be obtained by solving the Dirac equation in a background magnetic field.
- The B-dependent coupling constant is extracted by noticing

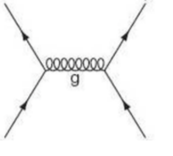
$$\mu(B) = 1+4\pi\chi(B) \quad \mu\epsilon = 1. \qquad \alpha_{eff} = \alpha/\epsilon$$

Leading order result

- Include both gluon and quark contributions in QCD, replace the background color B field with A_8^μ, we get the familiar leading order result

$$-g^2 \frac{11N_c - 3N_f}{48\pi^2} ln(\frac{2gB}{\Lambda^2})$$

- Gives reasonable running coupling at finite temperature, B and chemical potential.
- Notice $\partial A \sim kA \sim B$ and $\partial \sim gA$ in equation of motion we can translate the background 2gB into the energy scale k^2.
- Why it works?
- If we neglect the back action to the virtual gluons, the full quantum corrections to the physical particles are all encoded in their equations of motion.



Rotating system

- Consider a pure gluon system. Introduce both background color magnetic field and rotation into the eigen equation. Calculate the ω-dependent QCD coupling constant.
- At given n, the l should be chosen to make the boundary locates at one of the zeros of Laguerre function

$$L^{l}_{n+\frac{1}{2}\pm 1}\left(\frac{gBR^2}{2}\right) = 0$$

- The eigen energies corresponding to the two spin modes are

$$E = \sqrt{2gB(l(n,R)+1/2\pm 1)+k_z^2} - n\omega$$

Coupling constant running

- Summation over all the energy levels of gluon. Extract the QCD coupling constant as a function of the rotation speed.

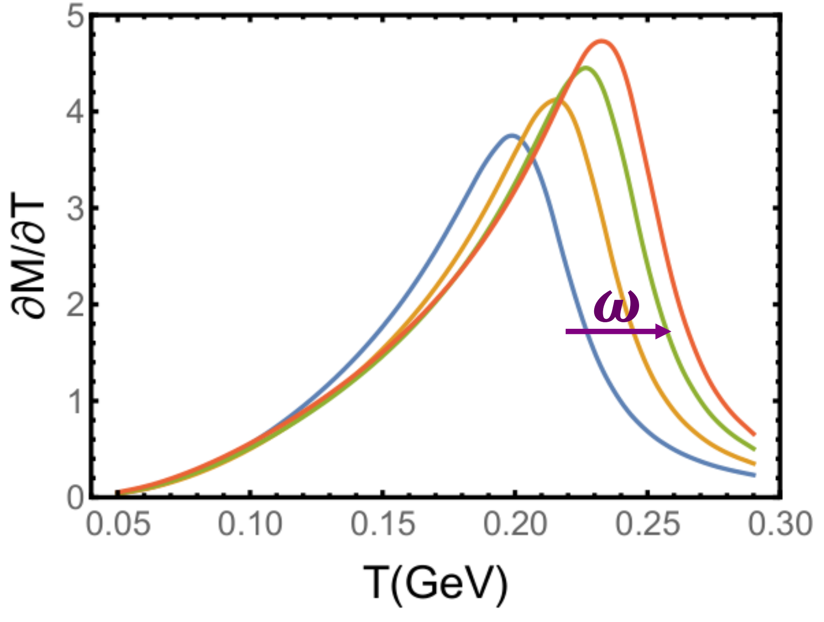
$$\alpha_{eff} = \alpha[1 + \frac{\alpha}{\pi}(ln(R\Lambda) + \frac{1.08}{6\langle k^2\rangle R^2}\frac{\omega}{\Lambda})]$$

- Here we again replace the $2gB$ with k^2. We concern the running behavior of rotation. Coupling is larger when rotation is faster.

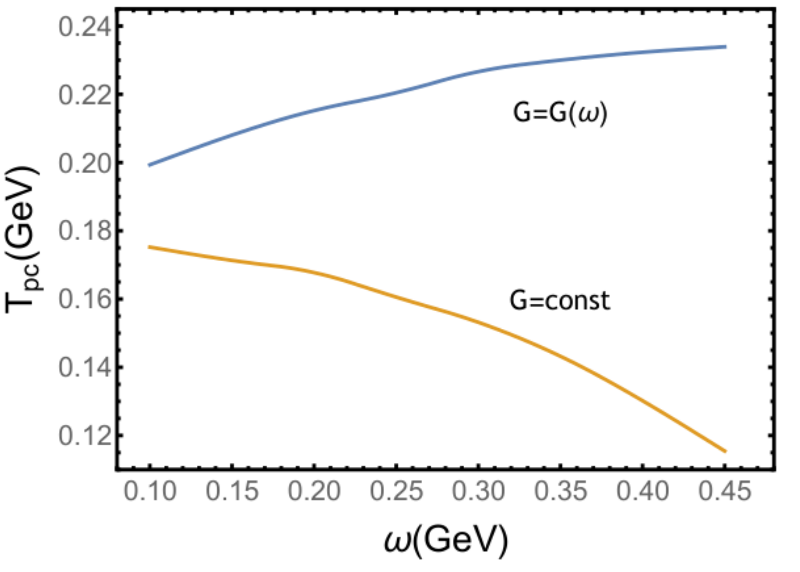
$$G(\omega) = G_0(1 + 0.32\frac{\omega}{\Lambda_{NJL}})$$

Chiral condensate

- Replace the NJL coupling G with $G(\omega)$, in mean field approximation



approaches saturation eventually because of the model cutoff



The same behavior has also been observed in recent lattice QCD simulation for deconfinement transition. (V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev and A. A. Roenko, Phys. Rev. D 103, no.9, 094515 (2021))

What's more

- It is better to check the running coupling as function of angular velocity with traditional perturbative computation.
- The increasing trend appears not so significant. It is not so convincible that the chiral symmetry will not restore at very large angular velocity even at very high temperature.
- Go beyond NJL model. The QCD vacuum is controlled by some non-trivial gluon configurations. The vacuum structure may be modified by the rotation.
- Gluon fluctuations around the non-trivial gluon profile will be changed by the finite-size and rotation polarization effects as well.
- These motivate us to investigate the gauge field seriously in a globally rotating system and switch to deconfinement transition.

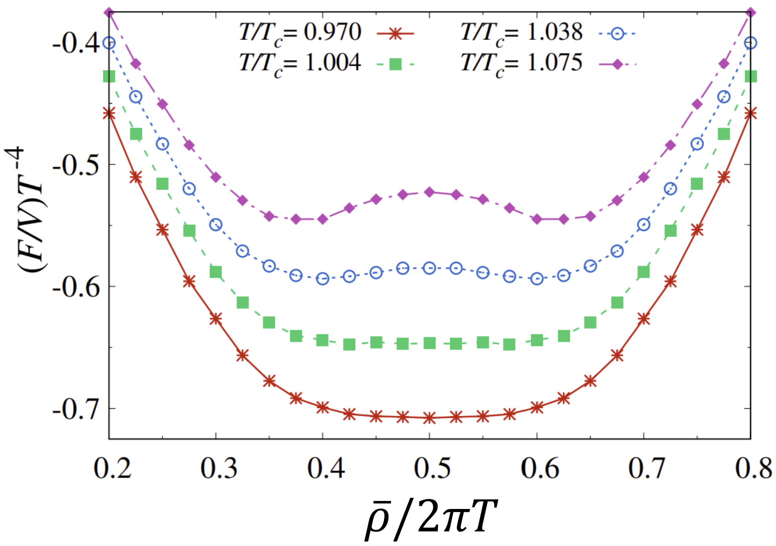
Understand confinement with KvBLL CALORON

- No matter what kind of non-trivial gluon configuration is, the fluctuation integration gives a thermodynamic potential which can not confine the color charge.
- Focus on the SU(2) gauge group case from now on. A potential non-trivial gluon field which may be responsible for the confinement is Caloron

$$A_4^{calron}(r \to +\infty) = \bar{\rho}\frac{\tau_3}{2}$$

$$F_p(T,\omega) = \frac{1}{3(2\pi)^2 T}\bar{\rho}^2(2\pi T - \bar{\rho})^2.$$

$$F_{np}(T) = -c[|\bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|\bar{\rho}|}{6\pi T}} + |2\pi T - \bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|2\pi T - \bar{\rho})|}{6\pi T}}]$$



$$L = \mathcal{P}e^{i\int_0^\beta dx_4 A_4} : \frac{\bar{\rho}}{2\pi T} = 0.5, Tr(L) = 0$$ confinement

Spin the system

- Solve the configurations again. Compute the fluctuations in a hard bounded system with radius R.

$$F_p^\omega(T,\omega) = -\sum_{\substack{s,m=1\\ n=-\infty}}^{+\infty} \frac{e^{\frac{sn\omega}{T}}}{\pi^2 s R^3} \frac{4\xi_n^{(m)} cos(s\frac{\bar{\rho}}{T})}{J_{n+1}(\xi_n^{(m)})^2} K_1(s\frac{\xi_n^{(m)}}{TR})$$

$$\begin{aligned}
F_{np}(T,\omega) &= -\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}+i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)}{6\pi T}}] \\
&-\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}-i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)}{6\pi T}}]
\end{aligned}$$

$$(\frac{\Lambda}{\pi T})^{22/3} = e^{-8\pi^2/g^2}$$

$$\bar{\rho}_c = 2\pi T - \bar{\rho}.$$

- Real angular velocity. Caloron and anti-caloron contribution are different because of CP violation by rotation. Finite size effect in the perturbative part.

Angular velocity changes

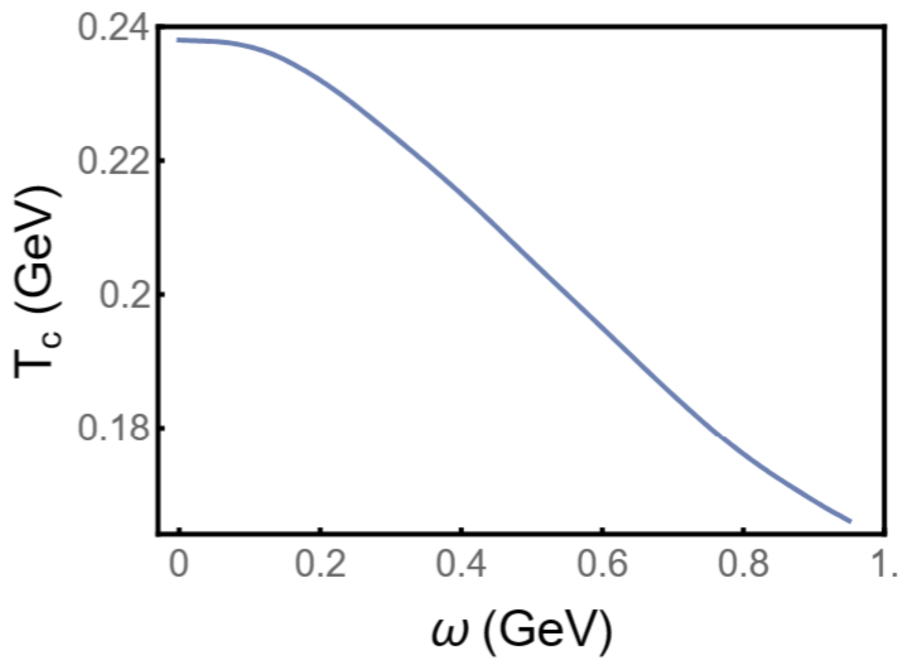
- The gluon non-trivial configuration. Calorons actions are modified by rotation.
- Perturbative part of the thermodynamic potential.
- Mystical running coupling.
- Which of them gives vortical catalysis?
- Calorons actions are necessary to confine the color charge.

1. Choose the perturbative part as the static one and combine with the Calorons actions. (finite size and rotation polarization effects)
2. Choose the static perturbative potential and the Calorons actions. And replace the coupling with a running one.

$$g(\omega) = (1 + 0.1\omega/\Lambda)g$$

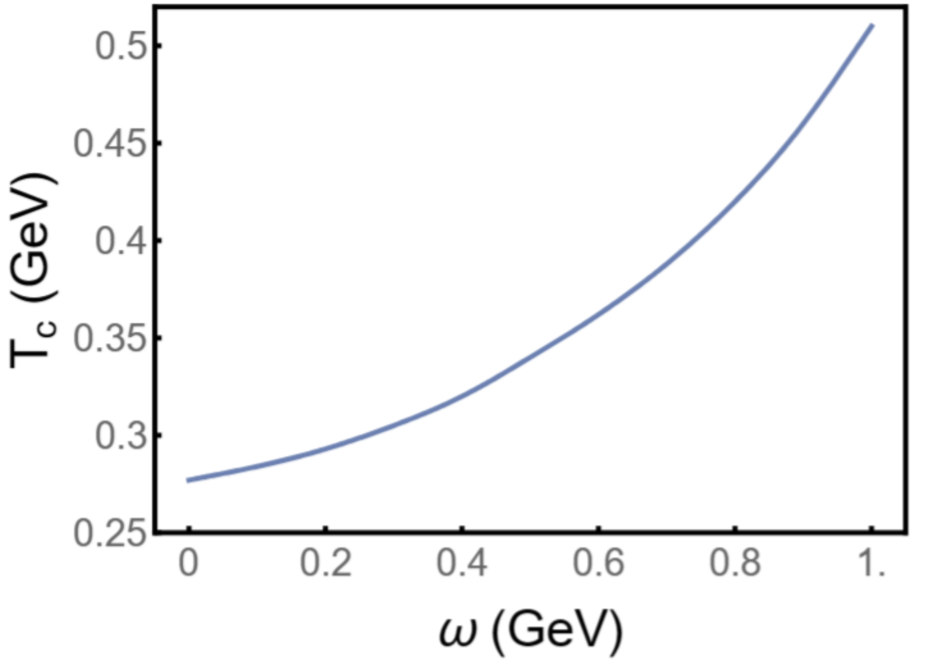
3. Choose the rotational perturbative potential and the Calorons actions. And replace the coupling with a running one.

Static perturbation and the Calorons actions



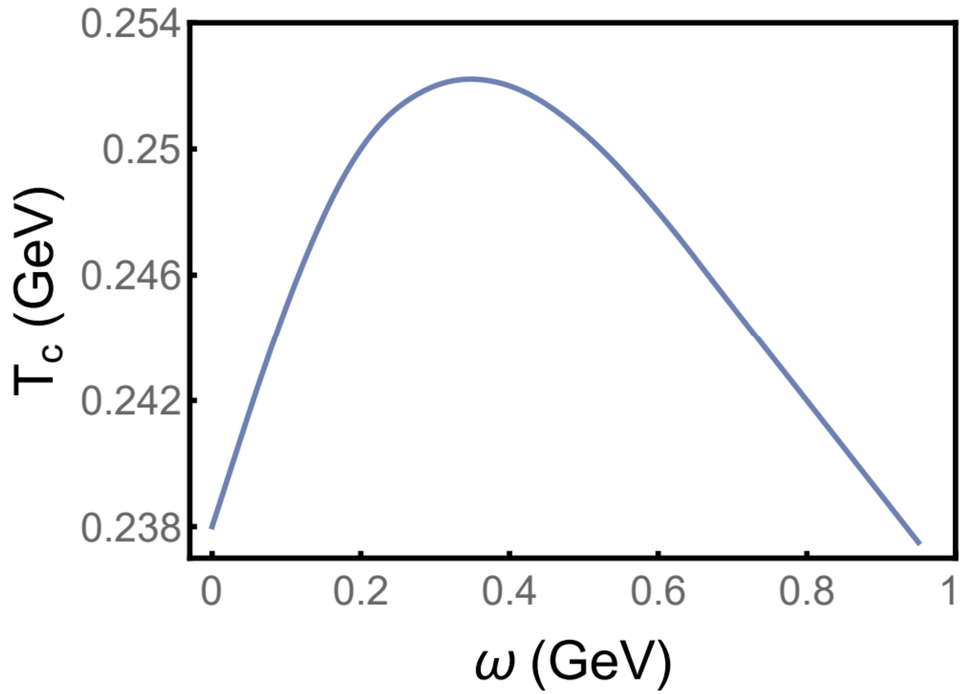
- Rotation helps to free color charge.
- The perturbation part will not be helpful to confine the color charge. Replacing the static part with rotational one gives the same qualitative result.
- Finite-size and polarization help to free color charge.

Static perturbation and Calorons(running g)



- Running coupling helps to confine color charge.
- The only ambiguity in this computation.

Rotational perturbation and Calorons(running g)



- Competition between running coupling and the other two contributions.
- The increasing range is short and unsignificant. It may disappear if the coupling dependence on rotation is weaker.

Outlook

- Achieved in this work
- With a novel method it is shown that the effective coupling will become larger when the rotation becomes faster.
- The pseudo critical temperature of the chiral restoration increases with rotation and approaches saturation eventually which may be induced by the model cutoff.
- Modified QCD vacuum and fluctuation contribution(finite size and polarization) are not powerful enough to enhance the critical temperature.
- The increase coupling constant may be the only reason to give us vortical catalysis.

- Double check the coupling running behavior.
- Consider dyon ensemble beyond dilute limit.
- Compute spatial dependent results to compare with lattice QCD.

Thank you for your attention!