



# PERTURBATIVE STUDY OF YANG-MILLS FIELD UNDER (IMAGINARY) ROTATION

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2023年11月13日

**QCD**  
**Under Rotation**

November 11 - 13, 2023  
Room C101, Jiangwan Physics Building,  
Fudan University, Shanghai

Organizers: Defu Hou (CCNU), Mei Huang (UCAS),  
Xu-Guang Huang (Fudan U.), Guo-Liang Ma (Fudan U.)

The poster features a blue background with a glowing sphere and colorful particles, along with logos for Fudan University and the organizing institutions.

# OUTLINE

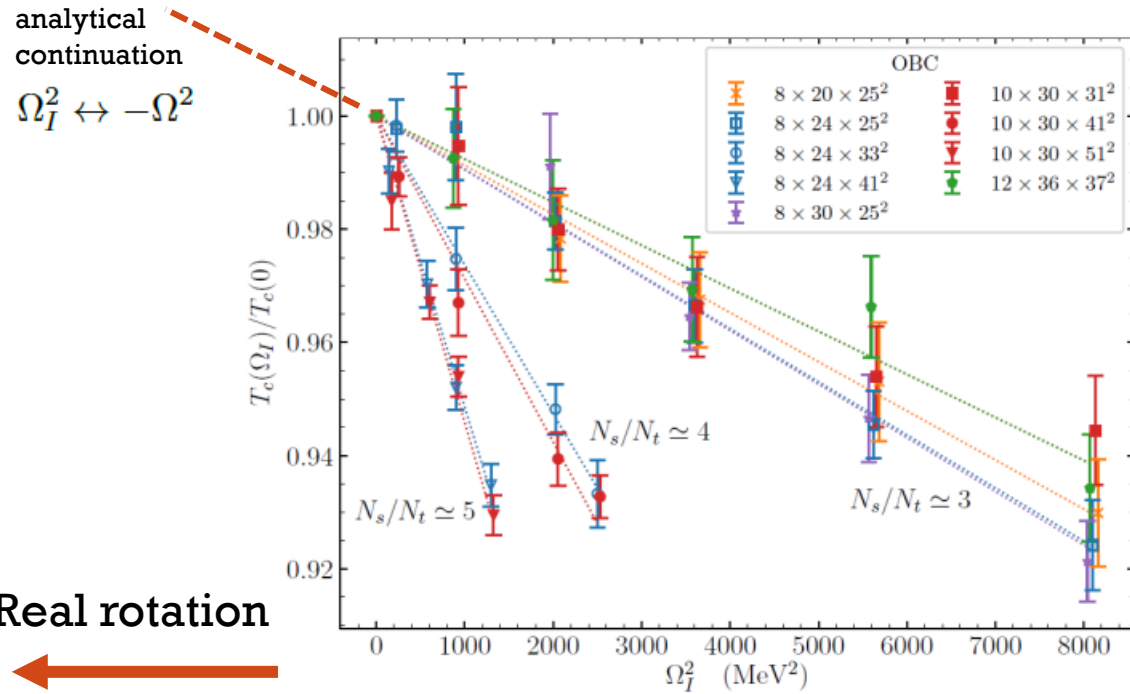
- **Introduction**
- **1-loop with background field:**
  - $A_0$ -condensate**
  - colormagnetic condensate**
- **2-loop**
  - Feynman rule**
- **Summary**

# CURRENT STAGE

- Model studies  
(P)NJL, QM, Bag model, Holographic...
- Rotation + other backgrounds (magnetic field, chemical potential, etc)
- Lattice  
V. V. Braguta, A. Y. Kotov, D. D. Kuznedev, and A. A. Roenko, Phys. Rev. D 103, 094515 (2021)  
J.-C. Yang and X.-G. Huang, arXiv:2307.05755 [hep-lat].
- Rotational effect on quark has been widely studied
- We don't know much about how rotation affects gluon sector  
S. Chen, K. Fukushima, and Y. Shimada, Phys. Rev. Lett. 129, 242002 (2022)  
M. N. Chernodub, Phys. Rev. D 103, 054027 (2021), Phys. Rev. D 107, 114502 (2023)  
Y-Q Zhao, S He, D Hou, L Li, and Z Li, JHEP04(2023)115

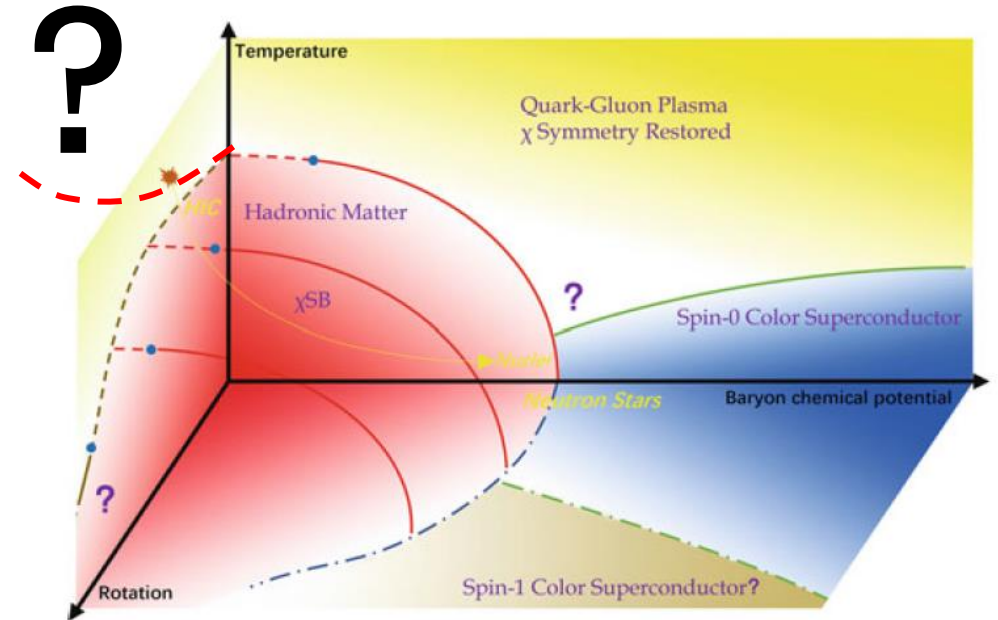
# DISCREPANCY

- Lattice result with imaginary rotation



Braguta V V, Kotov A Y, Kuznedev D D, et al. arXiv:2110.12302, 2021.

T<sub>c</sub> decreases with increasing  $\Omega_I$



T<sub>c</sub> increases with increasing real rotation

# WHAT'S WRONG?

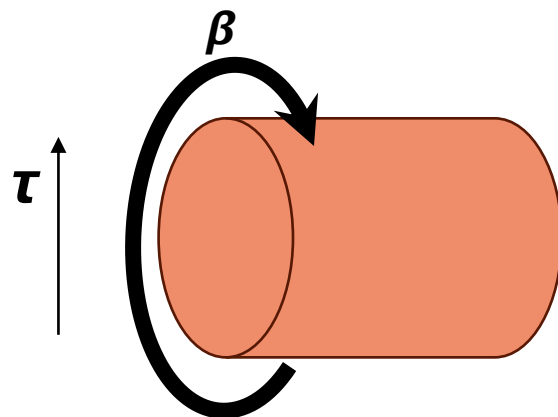
- Gauge sector contribution?
- Analytical continuation is not applicable?
- Something missed on lattice?
- Or something else?

# DIFFICULTIES OF REAL ROTATION

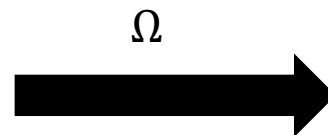
- Boundary condition to preserve causality
- Inhomogeneity
- Sign problem
- Subtlety of a Euclidean field theory

But anyway it works

Dmitri V. Fursaev, hep-th/0107089

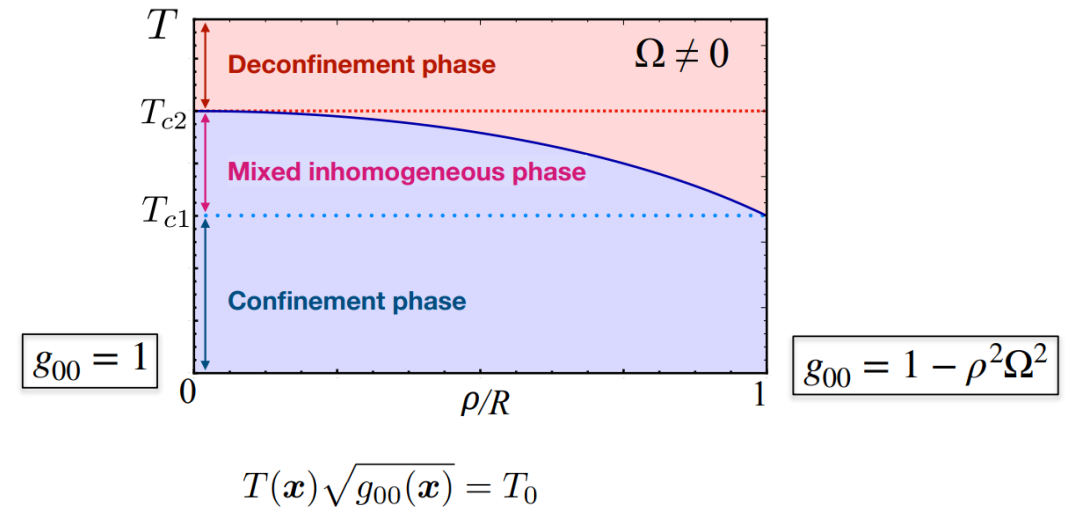


Winding number  $\longrightarrow$  Matsubara frequency



In Complex Spacetime?

Chernodub PRD (2021), (2022)



# MAYBE RELATED COMMENT

- E. Witten, A Note On Complex Spacetime Metrics, 2111.06514 [hep-th]

This is a little subtle. The thermodynamics of a Schwarzschild black hole is related to a standard thermal ensemble  $\text{Tr} \exp(-\beta H)$ . The thermodynamics of a rotating (Kerr) black hole is related to a more general ensemble  $\text{Tr} \exp(-\beta(H - \Omega J))$  where  $J$  is a conserved angular momentum and  $\Omega$  is called the angular velocity. However, in asymptotically flat spacetime, this ensemble is unstable because a particle far from the black hole can have a negative value of  $H - \Omega J$ . Hence the quantum corrections to this ensemble are not well-defined, and it turns out that this is reflected in the fact that the quasi-Euclidean metric is not allowable.

# IMAGINARY ROTATION

$$g_{\mu\nu} = \begin{pmatrix} -1 - \Omega_I^2 r^2 & y\Omega_I & -x\Omega_I & 0 \\ y\Omega_I & -1 & 0 & 0 \\ -x\Omega_I & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

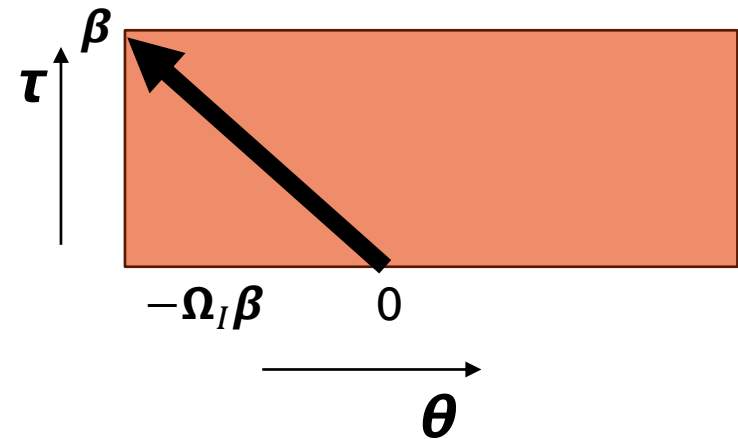
- Boundary condition for space is not necessary
- No sign problem
- Well defined Euclidean field theory
- twisted boundary condition

$$(\tau, \theta, r, z) \sim (\tau + \beta, \theta - \tilde{\Omega}_I, r, z),$$

- Analytical continuation back to real rotation

G.W. Gibbons and S.W. Hawking, Phys. Rev. D15 (1977) 2752.

- First things first, directly comparison to lattice results

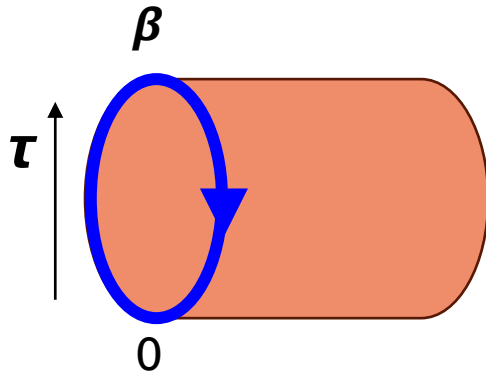




# POLYAKOV LOOP

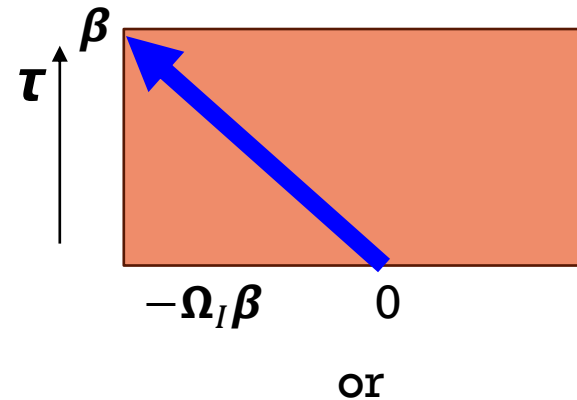
$$\Phi(x) = \frac{1}{N} \text{tr} \mathcal{P} \exp \left[ \int_0^\beta dx_4 A_4(x, x_f) \right],$$

- In flat spacetime



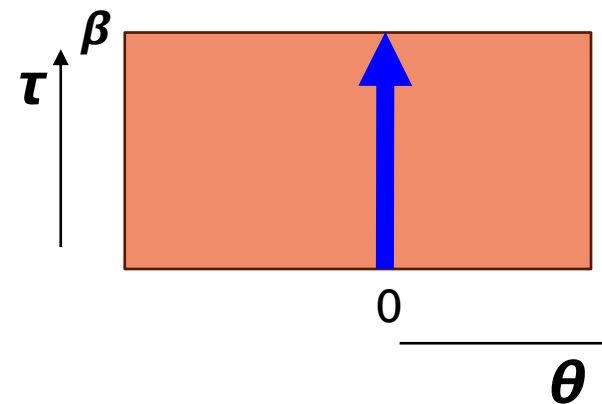
- Dependence on the boundary condition of imaginary time direction?

- In (imaginary) rotating frame, which one?



Killing vector  
 $n^\mu = (1, 0, -\Omega_I, 0)$   
 $n^2 = 1$

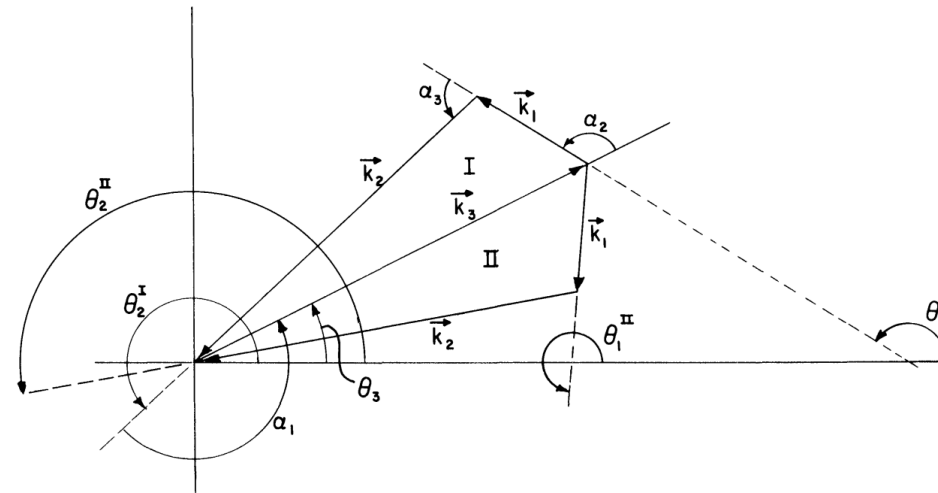
or



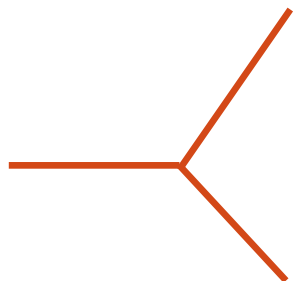
Killing vector  
 $n^\mu = (1, 0, 0, 0)$   
 $n^2 = -1 - \Omega_I^2 r^2$

# REMAINING DIFFICULTIES

- All fields feel rotation
- The interaction vertex is hard to deal with



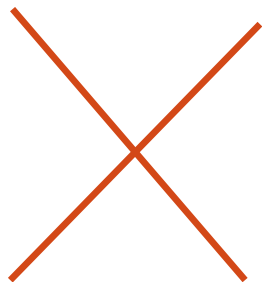
Jackson, SIAM J. MATH. ANAL.  
Vol. 3, No. 3, August 1972



$$\int_0^\infty J_{n_1}(k_1\rho)J_{n_2}(k_2\rho)J_{n_3}(k_3\rho)\rho d\rho$$

$$= \frac{\Delta}{6\pi A} [\cos(n_1\alpha_2 - n_2\alpha_1) + \cos(n_2\alpha_3 - n_3\alpha_2) + \cos(n_3\alpha_1 - n_1\alpha_3)],$$

where  $n_1 + n_2 + n_3 = 0$ .



$$\int r dr J_{l_1}(p_1 r) J_{l_2}(p_2 r) J_{l_3}(p_3 r) J_{l_4}(p_4 r) \delta(l_1 + l_2 + l_3 + l_4)$$

**= ?**

Fine at 2-loop level

# FAILED ATTEMPTS



- Kaluza-Klein method      Dmitri V. Fursaev, hep-th/0107089  
treat the curved background as an Abelian gauge vector field  
**But too hard for me...**

- Heat kernel expansion (Schwinger-DeWitt technique)

Feynman Propagator in Curved Space-Time: A Momentum Space Representation,  
T.S. Bunch and L. Parker, Phys.Rev.D 20 (1979), 2499-2510

Riemann normal coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3}R_{\mu\alpha\nu\beta} y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma} y^\alpha y^\beta y^\gamma + \left(-\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta} + \frac{2}{45}R_{\alpha\mu\beta\lambda}R^\lambda_{\gamma\nu\delta}\right) y^\alpha y^\beta y^\gamma y^\delta + \dots,$$

**We don't have curvature...**

- Stay in coordinate space?  $S_F(x, x') = (i\gamma^\mu \nabla_\mu - m)G(x, x')$

$$G = \frac{-i}{16\pi} \int_0^\infty \frac{ds}{s^2} e^{-i[\sigma^2/s + sm^2 - is\epsilon] + i\frac{\Omega\Sigma_3}{2}\Delta t}$$

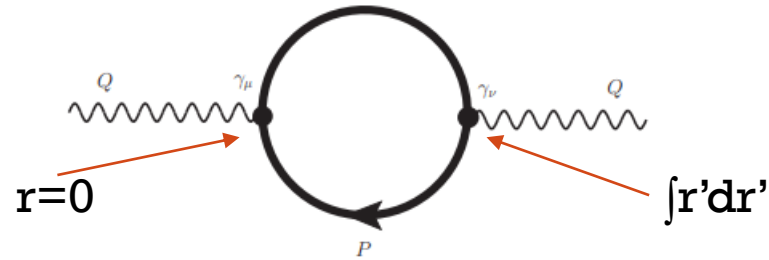
NJL gap Eq.  $\rightarrow \frac{m}{g} = \frac{m}{4\pi} \sum_n \int_{1/\Lambda_{UV}^2}^\infty \frac{ds}{s^2} e^{\frac{-(n\beta)^2 - 2r^2 + 2r^2 \cosh(\Omega n\beta)}{s} - sm^2} \cosh\left(\frac{\Omega}{2}n\beta\right) (-1)^n$

**Is this really useful?**

# ONE STRATEGY

- Focus on local quantities at the center

M. Wei, et al,  
Chin.Phys.C 46 (2022) 2, 024102



- Assumption:  $\theta + \Omega t = 0$

A. Ayala et al., Phys. Rev. D 103, no.7, 076021 (2021),  
[erratum: Phys. Rev. D 104, no.3, 039901 (2021)],

$$S(p) = \frac{(p_0 + \Omega/2 - p_z + ip_\perp)(\gamma_0 + \gamma_3) + m(1 + \gamma_5)}{(p_0 + \Omega/2)^2 - \vec{p}^2 - m^2 + i\epsilon} \mathcal{O}^+ + \frac{(p_0 - \Omega/2 + p_z - ip_\perp)(\gamma_0 - \gamma_3) + m(1 + \gamma_5)}{(p_0 - \Omega/2)^2 - \vec{p}^2 - m^2 + i\epsilon} \mathcal{O}^-.$$

equivalent

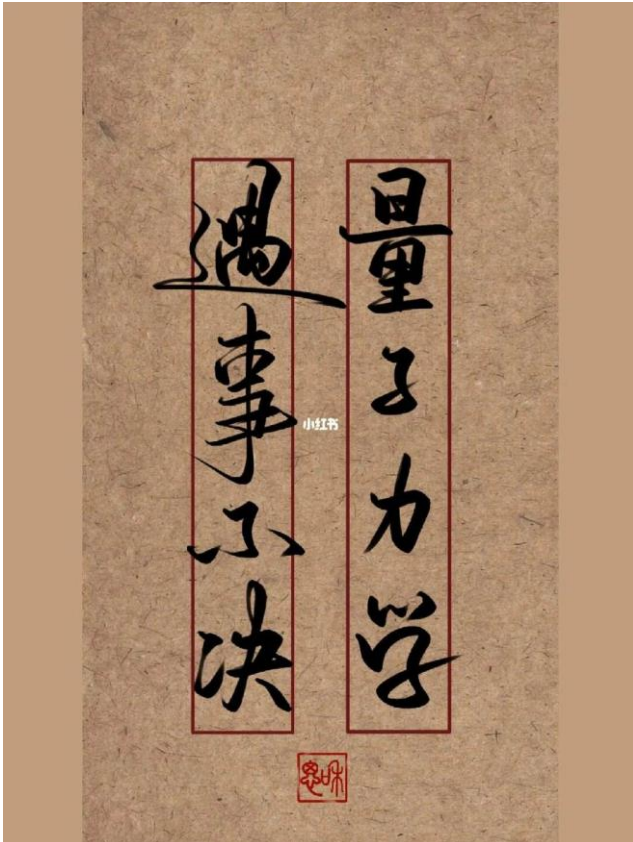
- Momentum space representation
- Account for contribution from spin
- No r-dependent part (orbit contribution)  $\longrightarrow$  Maybe important when  $\Omega$  is large

# WHAT ELSE?

- I'm not smart enough to solve this exactly, so

# WHAT ELSE?

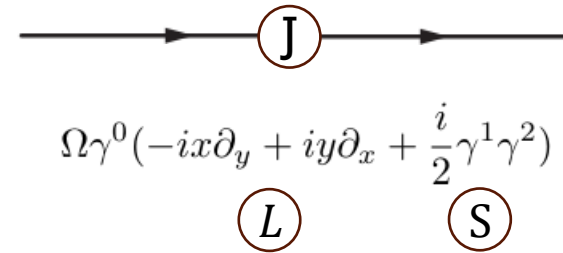
- I'm not smart enough to solve this exactly, so



Just do  
perturbation

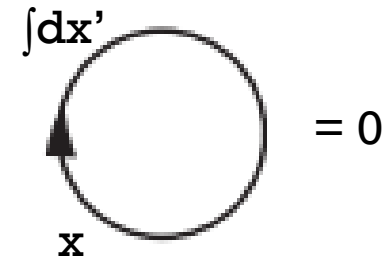
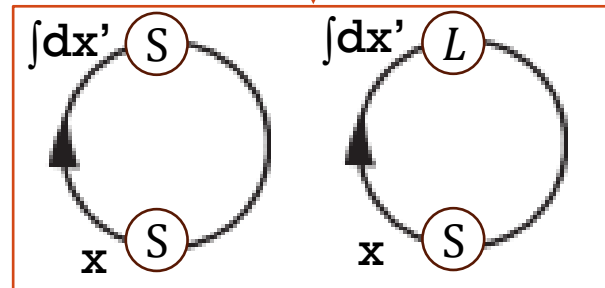
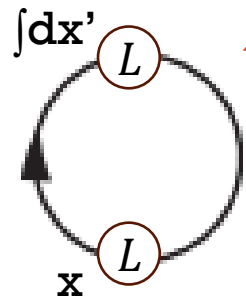
# PERTURBATIVE EXPANSION OF $\Omega$

Feynman Rule



▪ Exercise 1: Fermion 1-loop effective action

$$\begin{aligned} \ln Z &= \int d^3x \int \frac{p_\perp dp_\perp dp_z}{(2\pi)^2} \sum_l \{ \beta\varepsilon + \ln[1 + e^{-\beta(\varepsilon - \Omega j)}] + \ln[1 + e^{-\beta(\varepsilon + \Omega j)}] \} [J_l(p_\perp r)^2 + J_{l+1}(p_\perp r)^2] \\ &= \int d^3x \int \frac{p_\perp dp_\perp dp_z}{(2\pi)^2} \sum_l \{ \beta\varepsilon + 2 \ln[1 + e^{-\beta\varepsilon}] + \frac{e^{\beta\varepsilon}}{(1 + e^{\beta\varepsilon})^2} \beta^2 \Omega^2 j^2 + O(\Omega^4) \} [J_l(p_\perp r)^2 + J_{l+1}(p_\perp r)^2] \\ \ln Z_2 &= 2 \int d^3x \int d^3p \frac{e^{\beta\varepsilon}}{(1 + e^{\beta\varepsilon})^2} \beta^2 \Omega^2 \left( \frac{1}{2} r^2 p_\perp^2 + \frac{1}{4} \right) \end{aligned}$$



# PERTURBATIVE EXPANSION OF $\Omega$

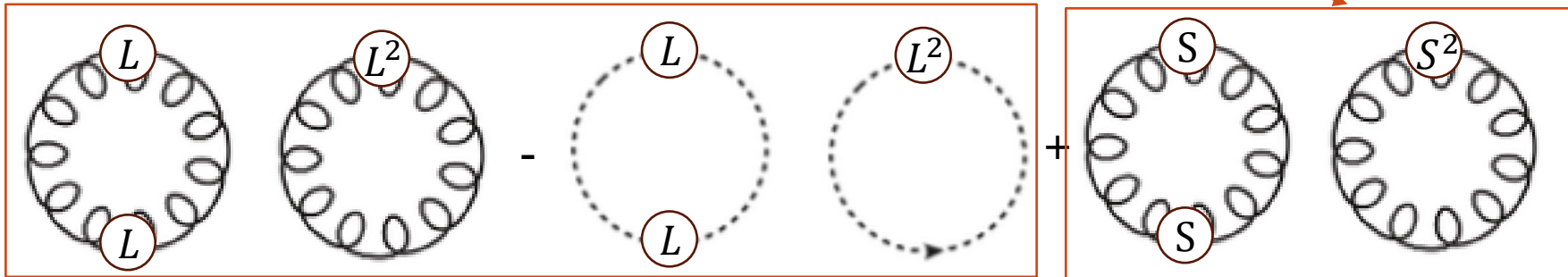
Feynman Rule

$$\text{wavy line} \text{---} \text{circle } J \text{---} \text{wavy line} \quad 2\Omega \hat{J}_z \partial_\tau$$

$$\text{wavy line} \text{---} \text{circle } J^2 \text{---} \text{wavy line} \quad -\Omega^2 \hat{J}_z^2$$

- Exercise 2: massless Gauge boson  
No S-L contribution, since Sz is off diagonal

$$\ln Z_2 = 2N_g \int d^3x \int d^3p \frac{e^{\beta\varepsilon}}{(1 - e^{\beta\varepsilon})^2} \beta^2 \Omega^2 \left( \frac{1}{2} r^2 p_\perp^2 + 1 \right)$$



- Despite tedious calculations, it seems no difficulties to go to higher loop



# ADD SOME BACKGROUND FIELDS TO SU(2)

Because its simple

- Metric

$$g_{\mu\nu} = \begin{pmatrix} -1 - \Omega_I^2 r^2 & y\Omega_I & -x\Omega_I & 0 \\ y\Omega_I & -1 & 0 & 0 \\ -x\Omega_I & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Background field formalism

$$\mathcal{A}_\mu^a = \bar{A}_\mu^a + A_\mu^a$$

$$\bar{A}_\mu^a = \delta_3^a \bar{A}_\mu^3 + \delta_8^a \bar{A}_\mu^8$$

in tangent space, we choose  $\bar{A}_\mu^3 = (\phi, \frac{1}{2}Hy, -\frac{1}{2}Hx, 0)$  for SU(2)

A0 condensate,  
related to Polyakov loop

chromo-magnetic condensate

**Assumption:  
homogeneous condensate**

- Lagrangian

$$\mathcal{L}_E = \frac{1}{4} \bar{F}_{\mu\nu}^a \bar{F}^{a\mu\nu} + \frac{1}{2} \bar{F}_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

$$\mathcal{L}_{gf} = \frac{1}{2} (\nabla_\mu^B A^{a\mu})^2 \quad \text{Feynman gauge}$$

$$\mathcal{L}_{gh} = \bar{c} \nabla_\mu^B D^\mu c$$

## Some lattice results

V.I. Demchik, N.V. Kolomojets, and V.V. Skalozub. Spatial structure of the polyakov loop in external chromomagnetic field in lattice su(2) gluodynamics. *Visnyk Dnipropetrovskogo universytetu. Fizyka. Radioelectronika*, 21(2):13–17, 2013.

V. Demchik and V. Skalozub. Spontaneous creation of chromomagnetic field and A(0)-condensate at high temperature on a lattice. *J. Phys.*, A41:164051, 2008.

# GAUGE FIELD GO TO TANGENT SPACE

- With the help of vierbein

$$A_{\hat{\mu}} = e_{\hat{\mu}}^{\mu} A_{\mu}$$

- Simple replacement in Lagrangian

$$\begin{aligned} \partial_{\tau} &\rightarrow \partial_{\tau} - i\Omega_I \hat{J}_z \\ \hat{J}_z A_{\hat{\mu}} &= (\hat{L}_z \delta_{\mu}^{\nu} + \hat{S}_{z\mu}^{\nu}) A_{\hat{\nu}} \\ \hat{J}_z c &= \hat{L}_z c \end{aligned} \quad \hat{S}_z = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- At one loop level, the Lagrangian can be diagonalized with
- 1-loop effective potential

$$\begin{aligned} A_{\pm}^a &= \frac{1}{\sqrt{2}}(A_{\hat{x}}^a + A_{\hat{y}}^a), & A_{\pm}^a &= \frac{1}{\sqrt{2}}(A_{\hat{x}}^a - A_{\hat{y}}^a) \\ A_{\pm}^{\pm} &= \frac{1}{\sqrt{2}}(A_{\pm}^1 + A_{\pm}^2), & A_{\pm}^{\pm} &= \frac{1}{\sqrt{2}}(A_{\pm}^1 - A_{\pm}^2) \end{aligned}$$

$$V(r) = \frac{1}{2}H^2 + \frac{1}{S} \sum_n \sum_{l=-\lambda}^{N-\lambda} \sum_{\lambda \in \mathbb{N}} \sum_{\pm} \int \frac{dk_z}{2\pi} \ln[(\omega_n - \Omega_I(l \mp 1) + g\phi)^2 + gH(2\lambda + 1 \pm 2) + k_z^2] \Phi_l^2(\lambda, \frac{1}{2}gHr^2)$$

# A0 CONDENSATE

S. Chen, K. Fukushima, and Y. Shimada  
 Phys.Rev.Lett. 129 (2022) 24, 242002

- $gH=0$
- Weiss potential

$$V(\phi; \tilde{\Omega}_I)|_{\tilde{r}=0} = \frac{\pi^2 T^4}{3} \sum_{\alpha} \sum_{s=\pm 1} B_4 \left( \left( \frac{\phi \cdot \alpha + s \tilde{\Omega}_I}{2\pi} \right)_{\text{mod } 1} \right).$$

- Simple replacement

$$\phi \cdot \alpha \rightarrow \phi \cdot \alpha \pm \beta \Omega_I$$

- Still true at higher loop?  
 Rotation contribution to 3-gluon interaction  
 and gluon-ghost vertex differently

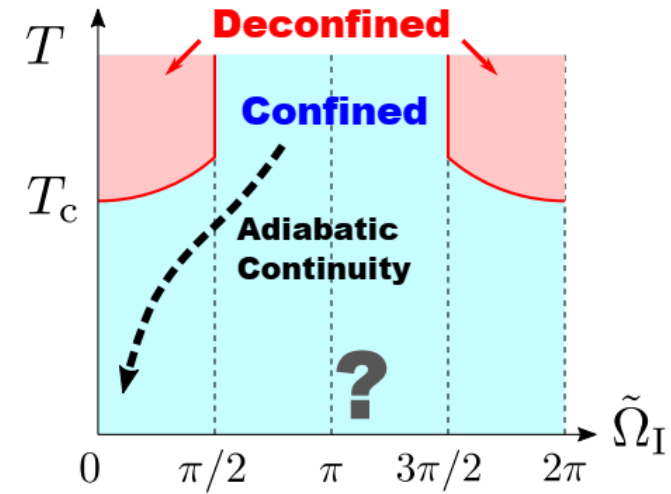
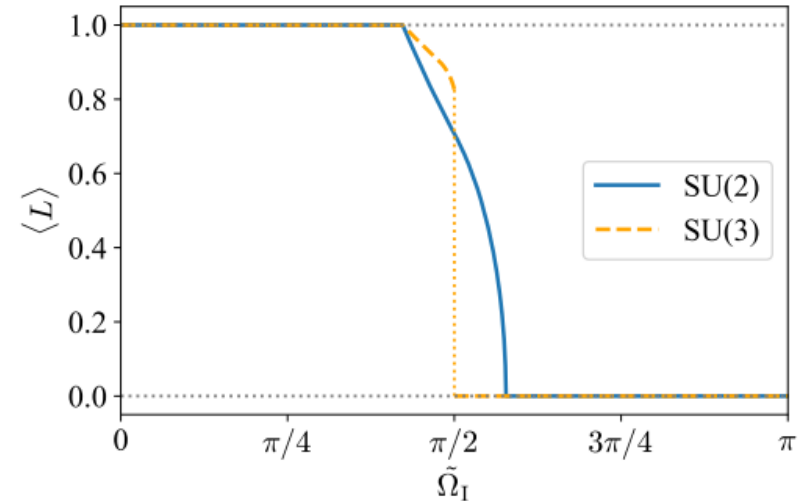


FIG. 4. Conjectured phase diagram on the  $\tilde{\Omega}_I$ - $T$  plane around the rotation axis,  $\tilde{r} = 0$ , for the SU(3) case. Solid curves represent the phase transition.

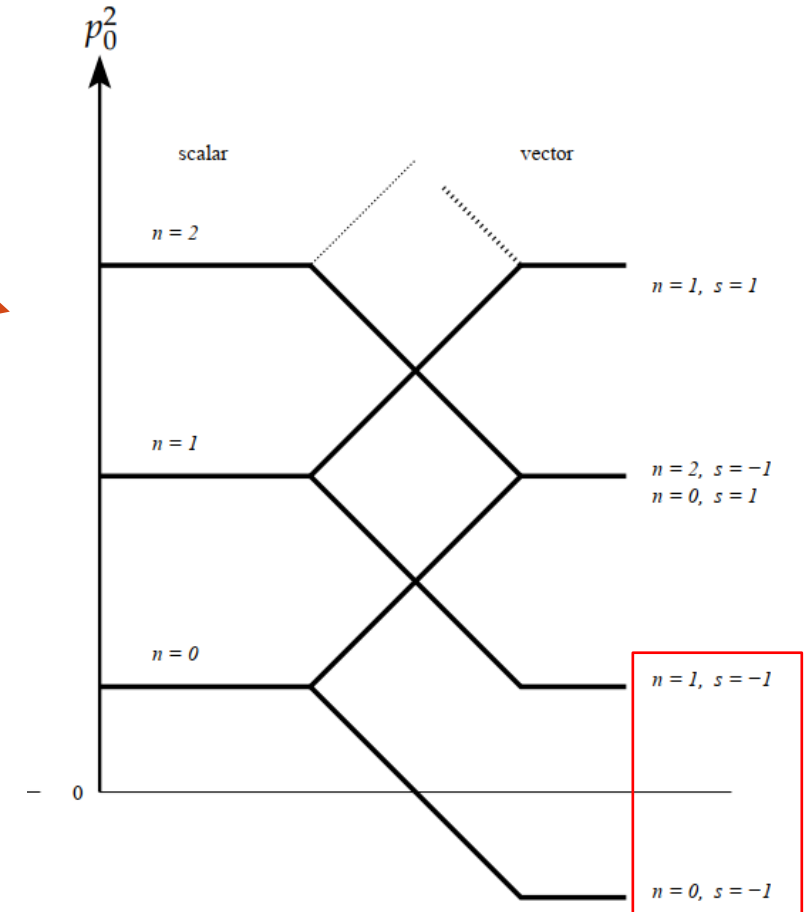


# CHROMO-MAGNETIC CONDENSATE

- At  $r=0$

$$V(0) = \frac{1}{2}H^2 + \frac{gH}{2\pi\beta} \sum_n \sum_{\lambda \in \mathbb{N}} \sum_{s=\pm 1} \int \frac{dk_z}{2\pi} \ln[(\omega_n + s\Omega_I + g\phi)^2 + \boxed{gH(2\lambda + 1 + 2s)} + k_z^2]$$

- $\Omega=0, T=0$ : Savvidy Vacuum G.I. Savvidy, Phys. Lett. B, 71:133, 1977
- Nielsen-Olesen instability N.K Nielsen and P. Olesen., Nucl. Phys. B, 144(2-3):376-396, 1978.
- When  $\Omega=0$ , A0 condensate can help



Tachyonic mode

when  $\lambda = 0, s = -1$

# CHROMO-MAGNETIC CONDENSATE

- Effective potential

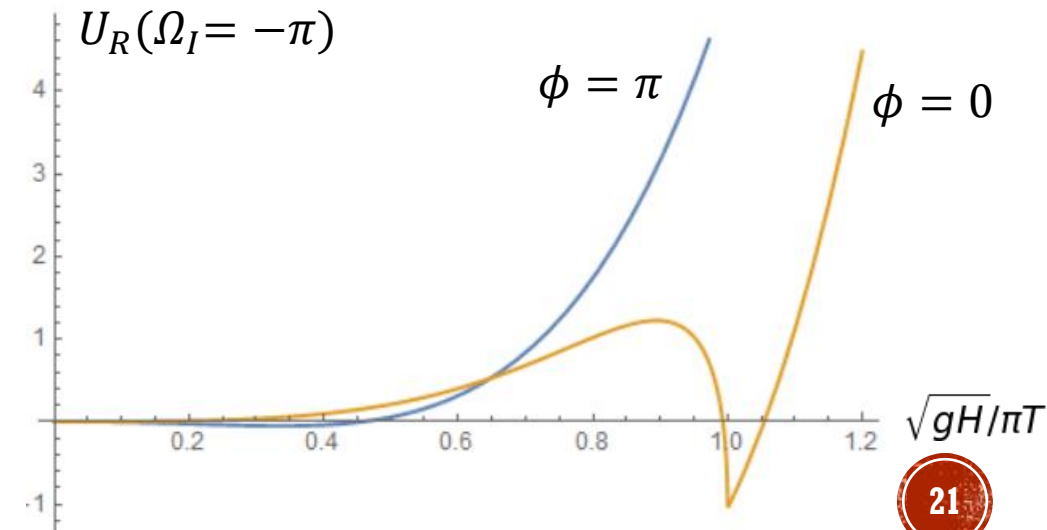
$$U_R = \frac{11g^2H^2}{48\pi^2} \ln\left(\frac{gH}{\mu_0^2}\right) - \frac{(gH)^{\frac{3}{2}}}{\pi^2\beta} \sum_{n=1}^{\infty} \frac{1}{n} [K_1(n\beta\sqrt{gH}) - \frac{\pi}{2}Y_1(n\beta\sqrt{gH})] \cos n\beta(g\phi - \Omega_I)$$

$$- 2\frac{(gH)^{\frac{3}{2}}}{\pi^2\beta} \sum_{n=1}^{\infty} \sum_{\lambda=0}^{\infty} \frac{1}{n} \sqrt{2\lambda+3} K_1(n\beta\sqrt{gH(2\lambda+3)}) \cos n\beta g\phi \cos n\beta\Omega_I$$

$$U_I = -\frac{(gH)^2}{8\pi} - \frac{(gH)^{\frac{3}{2}}}{2\pi^2\beta} \sum_{n=1}^{\infty} \frac{1}{n} J_1(n\beta\sqrt{gH}) \cos n\beta(g\phi - \Omega_I)$$

- No replacement  $\phi \cdot \alpha \rightarrow \phi \cdot \alpha \pm \beta\Omega_I$
- Imaginary part can be cured at 2-loop

M. Bordag, V. Skalozub, Eur.Phys.J.C 82 (2022) 5, 390



# GO TO HIGHER LOOP

- 2-loop correction

$$- \frac{1}{2} \text{ (loop with dashed outer boundary) } + \frac{1}{12} \text{ (loop with wavy outer boundary) } + \frac{1}{8} \text{ (two-loop figure-eight) }$$

- $g_H=0$ , again to check  $\phi \cdot \alpha \rightarrow \phi \cdot \alpha \pm \beta \Omega_I$
- So we need Feynman rule under rotation

# FEYNMAN RULE FOR SU(2)

- Gluon propagator  $a, \mu$    $b, \nu$

$$G_{\mu\nu}^{ab} = \frac{1}{2} \begin{pmatrix} G_{\mu\nu}(p_4 + g\phi) + G_{\mu\nu}(p_4 - g\phi) & -iG_{\mu\nu}(p_4 + g\phi) + iG_{\mu\nu}(p_4 - g\phi) & 0 \\ i(G_{\mu\nu}(p_4 + g\phi) - iG_{\mu\nu}(p_4 - g\phi)) & G_{\mu\nu}(p_4 + g\phi) + G_{\mu\nu}(p_4 - g\phi) & 0 \\ 0 & 0 & 2G_{\mu\nu}(p_4) \end{pmatrix} \xrightarrow{g\phi = 0} \delta^{ab} G_{\mu\nu}$$

- where

$$G_{\mu\nu}(x, x') = \frac{e^{ip_4\Delta\tau + ip_z\Delta z}}{(ip_4 - i\Omega_I l)^2 + \mathbf{p}^2} \begin{pmatrix} \phi_l \phi_l'^* & 0 & 0 & 0 \\ 0 & \frac{1}{2}(\phi_{l+1} \phi_{l+1}'^* + \phi_{l-1} \phi_{l-1}'^*) & \frac{i}{2}(\phi_{l+1} \phi_{l+1}'^* - \phi_{l-1} \phi_{l-1}'^*) & 0 \\ 0 & -\frac{i}{2}(\phi_{l+1} \phi_{l+1}'^* - \phi_{l-1} \phi_{l-1}'^*) & \frac{1}{2}(\phi_{l+1} \phi_{l+1}'^* + \phi_{l-1} \phi_{l-1}'^*) & 0 \\ 0 & 0 & 0 & \phi_l \phi_l'^* \end{pmatrix}$$

- Gauge dependence

$$G_{\mu\nu}^\xi(x, x') = G_{\mu\nu}(x, x') - (1 - \xi) \partial_\mu \partial'_\nu \frac{e^{ip_4\Delta\tau + ip_z\Delta z} \phi_l \phi_l'^*}{[(ip_4 - i\Omega_I l)^2 + \mathbf{p}^2]^2}$$

# GLUON PROPAGATOR

Euclidean thermal Green functions of photons in generalized Euclidean Rindler spaces for any Feynman - like gauge

V. Moretti, Int.J.Mod.Phys.A 12 (1997), 3787-3798, hep-th/9607178

- Eigenvalue equation

$$\nabla_\mu \nabla^\mu A_\nu - (1 - \frac{1}{\xi})(\nabla_\nu \nabla^\mu A_\mu) = \lambda A_\nu$$

- For Feynman gauge

$$\begin{aligned} A_\mu^{(1)} &= (1, 0, 0, 0)\phi_l & j &= l \\ A_\mu^{(2)} &= (0, 1, i, 0)\phi_l & j &= l - 1 \\ A_\mu^{(3)} &= (0, 1, -i, 0)\phi_l & j &= l + 1 \\ A_\mu^{(4)} &= (0, 0, 0, 1)\phi_l & j &= l \end{aligned} \quad \text{with}$$

- For arbitrary  $\xi$ , use

$$A_\mu^\xi = (\partial_\tau, \partial_x, \partial_y, \partial_z)\phi_l$$

- Propagator end at the center can be expanded as plane wave

$$\begin{aligned} \int p_t dp_t \phi_l(0) \phi_l'^*(r) &= \int p_t dp_t J_0(p) \\ &= \int p_t dp_t dp_\theta e^{ip_t \cos \theta_p} = \int dp_x dp_y e^{ip_x x + ip_y y} \end{aligned}$$

$$G_{\mu\nu} = -\eta_{\mu\nu}^\parallel G(ip_4) - \eta_{\mu\nu}^\perp [G(ip_4 + i\Omega_I) + G(ip_4 - i\Omega_I)] + S_{z\mu\nu} [G(ip_4 + i\Omega_I) - G(ip_4 - i\Omega_I)]$$



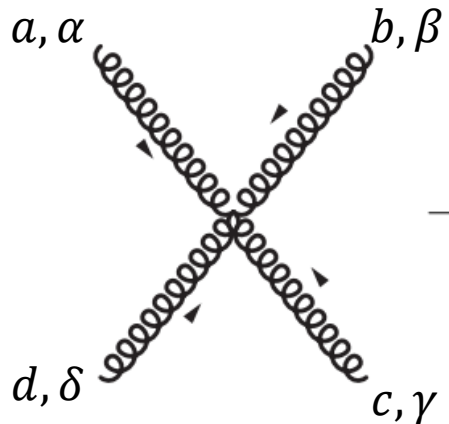
# FEYNMAN RULE FOR SU(2)

- Ghost propagator

$a$  .....  $\rightarrow$  .....  $b$

$$G^{ab} = \frac{1}{2} \begin{pmatrix} G(p_4 + g\phi) + G(p_4 - g\phi) & -iG(p_4 + g\phi) + iG(p_4 - g\phi) & 0 \\ i(G(p_4 + g\phi) - iG(p_4 - g\phi)) & G(p_4 + g\phi) + G(p_4 - g\phi) & 0 \\ 0 & 0 & 2G(p_4) \end{pmatrix} \xrightarrow{g\phi = 0} \delta^{ab}G$$

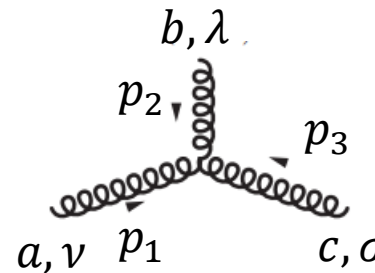
- Four-gluon interaction



$$-g^2 [f^{eab} f^{ecd} (\eta^{\alpha\gamma} \eta^{\beta\delta} - \eta^{\alpha\delta} \eta^{\beta\gamma}) + f^{eac} f^{edb} (\eta^{\alpha\delta} \eta^{\beta\gamma} - \eta^{\alpha\beta} \eta^{\gamma\delta}) + f^{ead} f^{ebc} (\eta^{\alpha\beta} \eta^{\gamma\delta} - \eta^{\alpha\gamma} \eta^{\beta\delta})]$$

# FEYNMAN RULE FOR SU(2)

- Three-gluon interaction

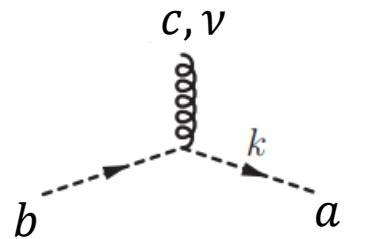


A Feynman diagram showing three gluons meeting at a central vertex. The top gluon has momentum  $p_2$  and indices  $b, \lambda$ . The bottom-left gluon has momentum  $p_1$  and indices  $a, \nu$ . The bottom-right gluon has momentum  $p_3$  and indices  $c, \sigma$ . The gluons are represented by wavy lines.

$$-g[(iP_1^{ia\langle\lambda\rangle\sigma\rangle\nu} f^{ibc} - (iP_2^{ib\langle\nu\rangle\sigma\rangle\lambda} f^{aic} + (iP_3^{ic\langle\nu\rangle\lambda\rangle\sigma} f^{abi}$$

where  $(iP^{ia\lambda})^{\sigma\nu} = ip^\lambda \eta^{\sigma\nu} \delta^{ia} - \eta^{\lambda 4} i\Omega_I \hat{J}_z^{\sigma\nu} \delta^{ia} - \eta^{\lambda 4} \eta^{\sigma\nu} g\phi f^{i3a}$

- Gluon-ghost interaction



A Feynman diagram showing a ghost line and a gluon line meeting at a central vertex. The ghost line (dashed) has momentum  $k$  and indices  $a, b$ . The gluon line (wavy) has momentum  $c, \nu$ . The ghost line is labeled with  $b$  at the bottom-left and  $a$  at the bottom-right.

$$-g(iP_\nu^{aa'}) f^{a'bc}$$

Orbit contribution only

# 2-LOOP CORRECTION AT THE CENTER

Rotation only result

- Tedious but possible

$$- \frac{1}{2} \text{ (diagram: a dashed circle with a wavy line inside)}$$

$$\frac{3}{2} g^2 \int_k \int_q [2(k_3 p_3 + k_4 p_4) G(ik_4) + (k_1 p_1 + k_2 p_2) (G(ik_4 + i\Omega_I) + G(ik_4 - i\Omega_I))] G(ip_4) G(-ik_4 + ip_4)$$

$$\frac{1}{8} \text{ (diagram: two overlapping circles with wavy lines inside)}$$

$$- \frac{3}{4} g^2 \int_k \int_q \{ 4G(ik_4)G(ip_4) + 4G[ik_4](G(ip_4 + i\Omega_I) + G(ip_4 - i\Omega_I)) + 4[G(ik_4 + i\Omega_I) + G(ik_4 - i\Omega_I)]G(ip_4) + [G(ik_4 + i\Omega_I) + G(ik_4 - i\Omega_I)][G(ip_4 + i\Omega_I) + G(ip_4 - i\Omega_I)] \}$$

$$\frac{1}{12} \text{ (diagram: a circle with a wavy line inside)}$$

Still ongoing...

Too long to be shown

# SUMMARY

- As less assumption as possible
- At least we can do some calculation of QCD under (imaginary) rotation
- Some other quantities to compare with lattice
- zzz

**THANKS !**