Quantum computing for non-perturbative high-energy nuclear physics

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QuNu Collaboration

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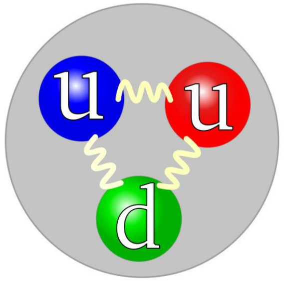
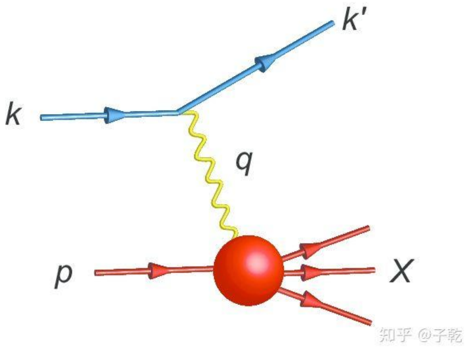
Quantum Computing and Machine Learning Workshop

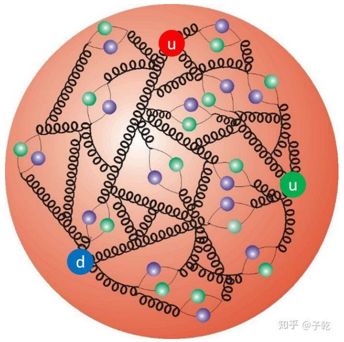
Non-perturbative aspects



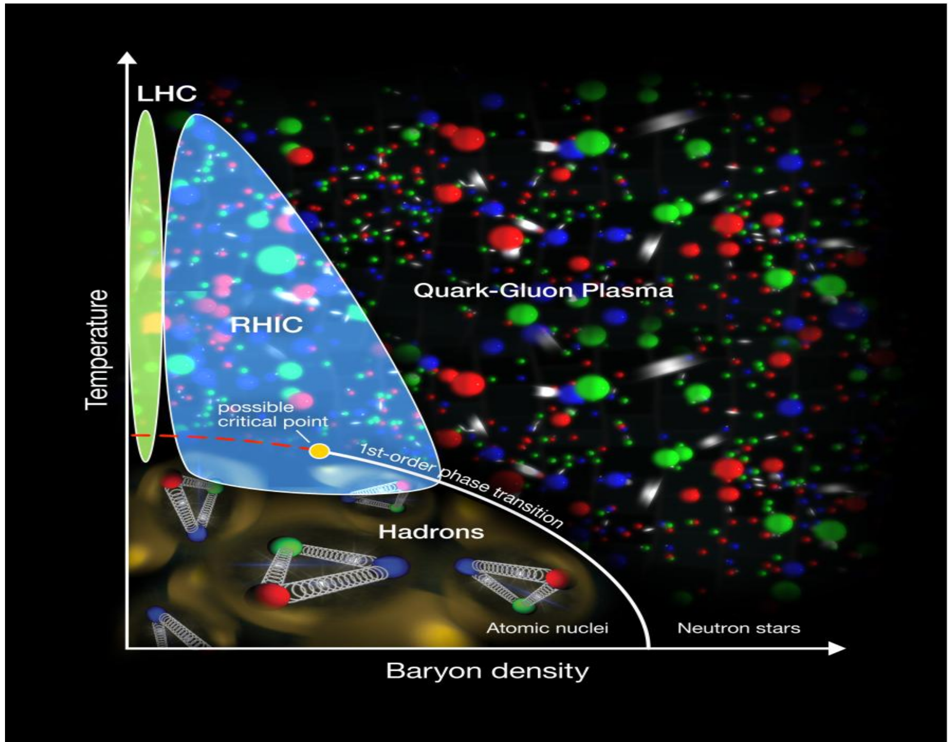
confinement
of quarks

Deep inelastic scattering
(parton distribution function)

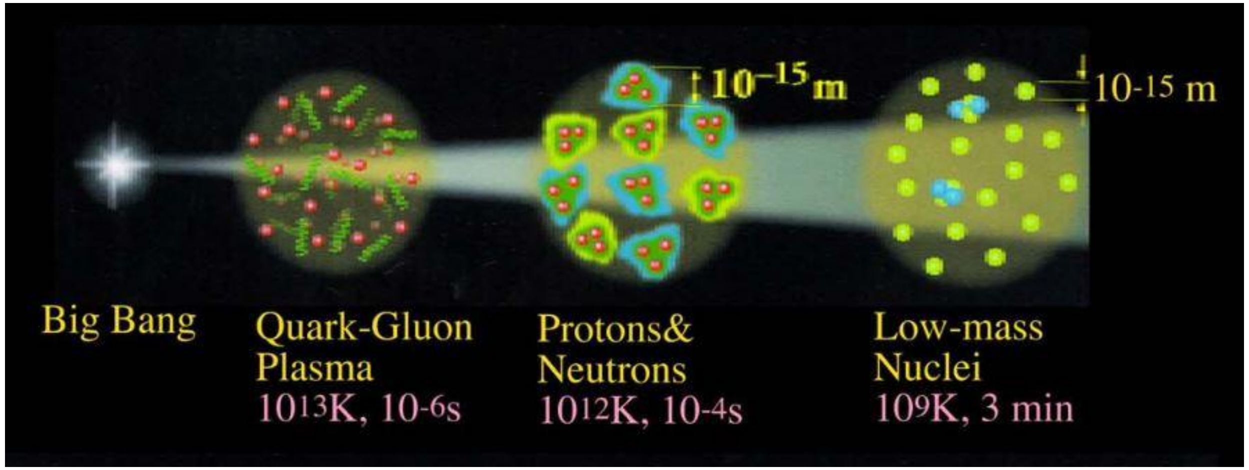




QCD at finite density &
finite temperature



Real-time evolution of the universe



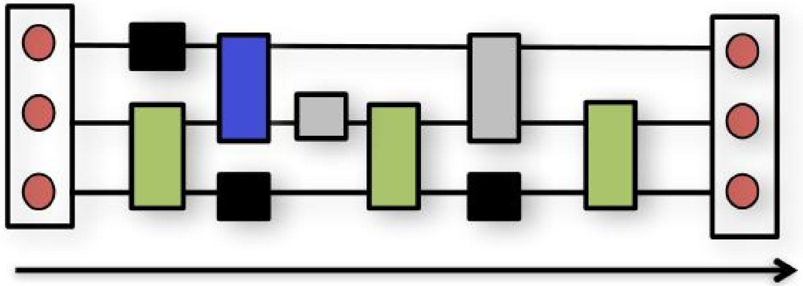
quantum computing

- in a word, directly perform calculation of quantum mechanics on quantum hardware of qubits

$$\langle \mathbf{M} \rangle = \langle \psi(t) | \mathbf{M} | \psi(t) \rangle \qquad |\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$$

- quantum simulation: framework of Hamiltonian & quantum states

- basically, prepare quantum states and do some measurements
- technically, controllable or programmable & quantum algorithm

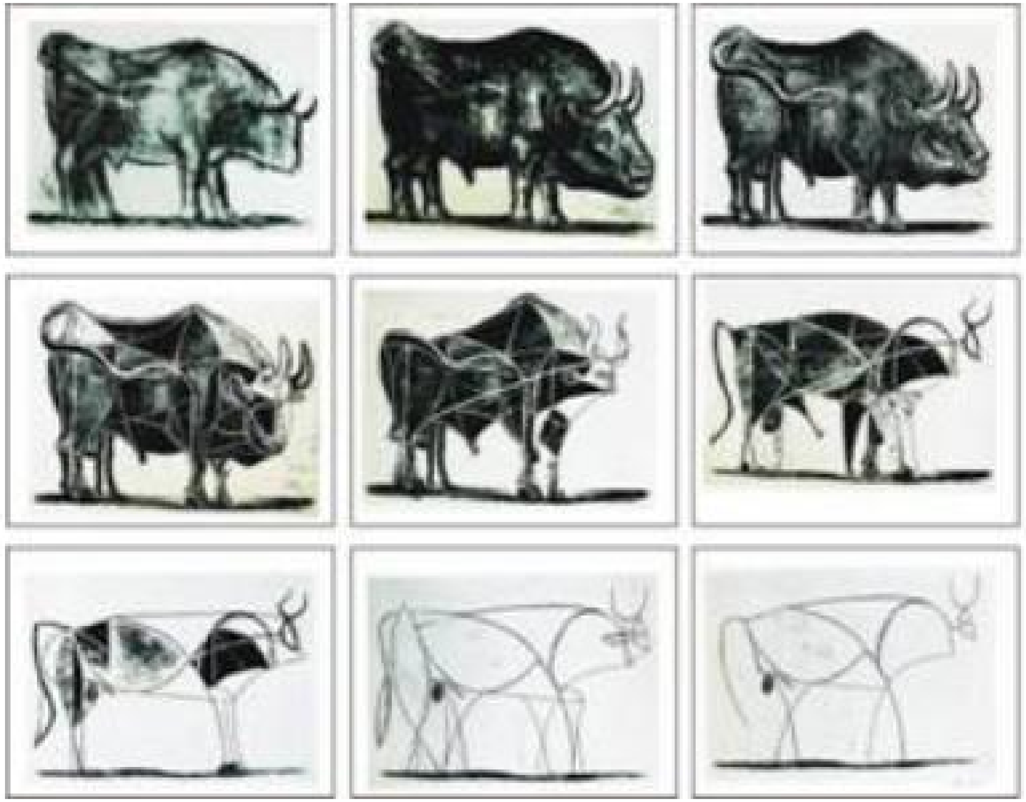


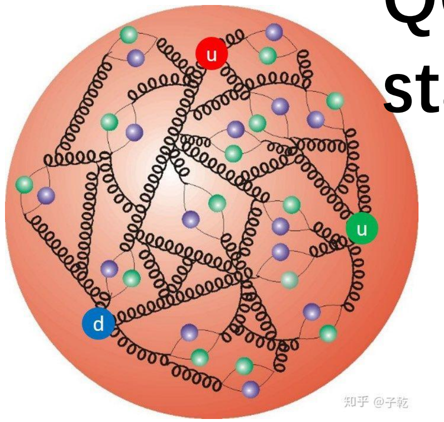
What make QCD special on a quantum computer?

QCD	mapping	Quantum computer
• fermion with flavor, spin, color	JW transf →	• qubit
• gluon (continuous-variable)	Digitalization →	• qubit
• local gauge-invariance	Local constraint → (Gauss law)	• physical Hilbert space
• continue limit		• renormalizaiton

The bottleneck is mapping

Nuclear matter in the eye of an artist





QCD at current stage is too hard

losing details but not the key features

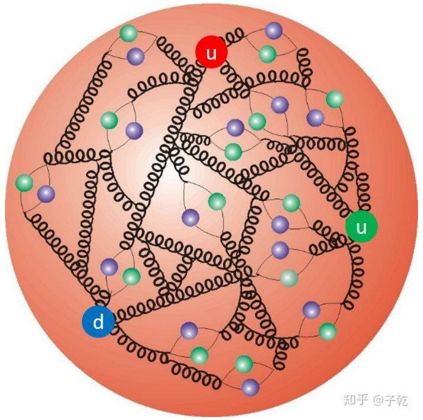
?

Starting with some “toy models”, play with quantum computers 沿途下蛋

QC for dynamical properties for nuclear structure

parton distribution functions (PDFs)

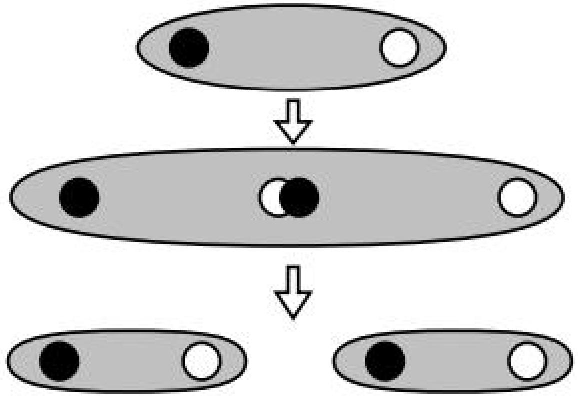
Parton in hadron



PhysRevD.105.L111502(2022)

Fragmentation function (FFs)

Hadron in parton



in preparation

PDFs and FFs

$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Data from experiments for global fitting FF

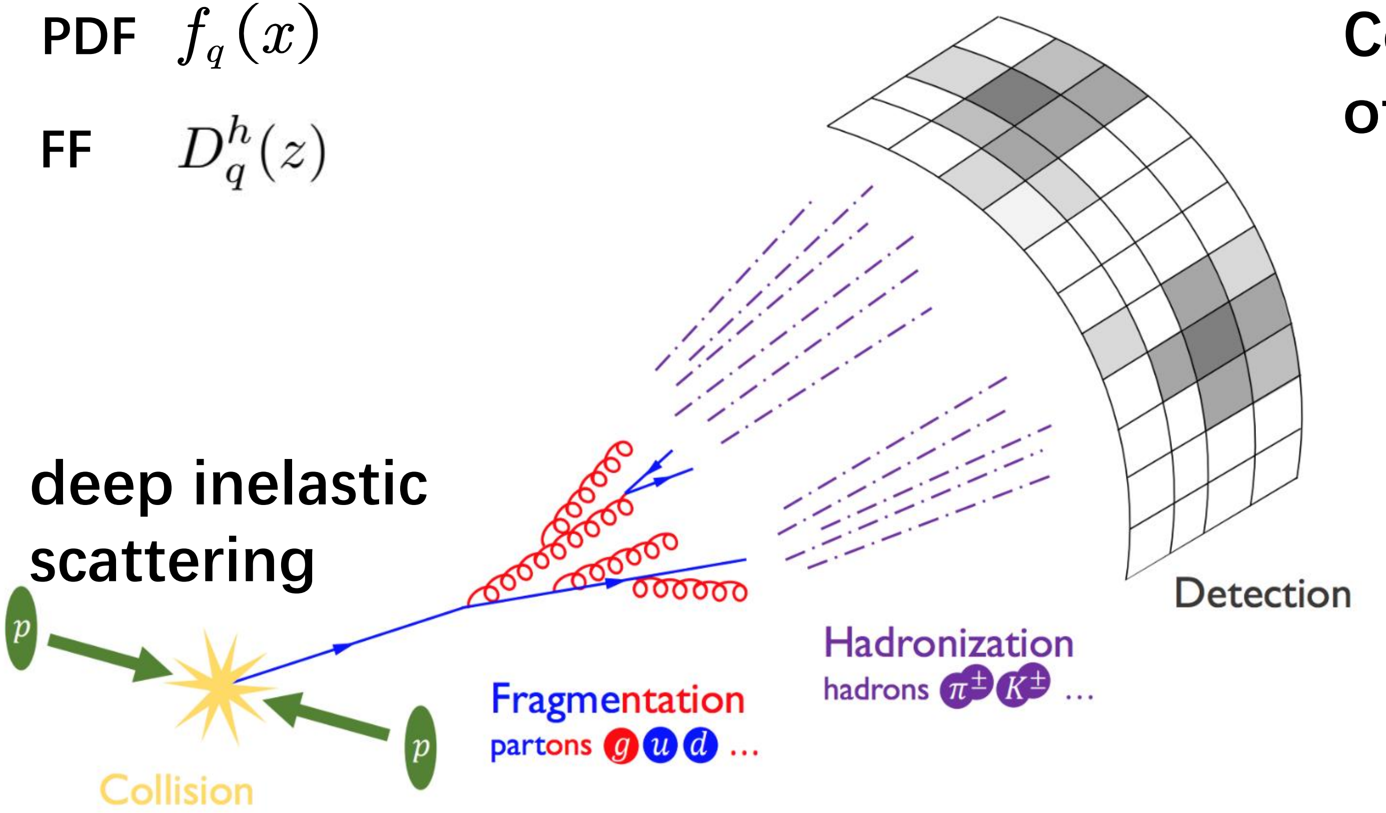
PDF $f_q(x)$

FF $D_q^h(z)$

Concern one kind of particle

h + X

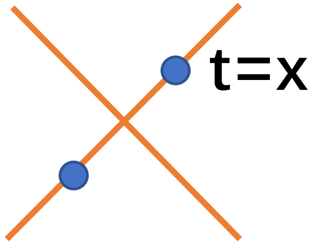
deep inelastic scattering



Operator definition of PDF and FF

PDF $$f_{q/h}(x) = \int \frac{dz}{4\pi} e^{-ixM_h z} \times \langle h| e^{iHt} \bar{\psi}(0,-z) e^{-iHt} \gamma^+ \psi(0,0) |h\rangle$$

Light-cone correlator for the hadron |h>



Lattice QCD:
Large momentum theory,
Xiaodong Ji

FF $$D_q^h(z) = z \int \frac{dy}{4\pi} e^{-iyM_h/z} \mathrm{Tr}\{\langle \Omega | \psi(yn^\mu) \times \underline{\sum_X |h, X\rangle \langle h, X|} \bar{\psi}(0,0) |\Omega\rangle \gamma^+\}$$

Light-cone correlator on the vacuum,
inserted with a projector of hadron

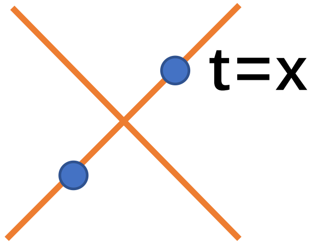
$$P_h = \sum_X |h, X\rangle \langle h, X|$$

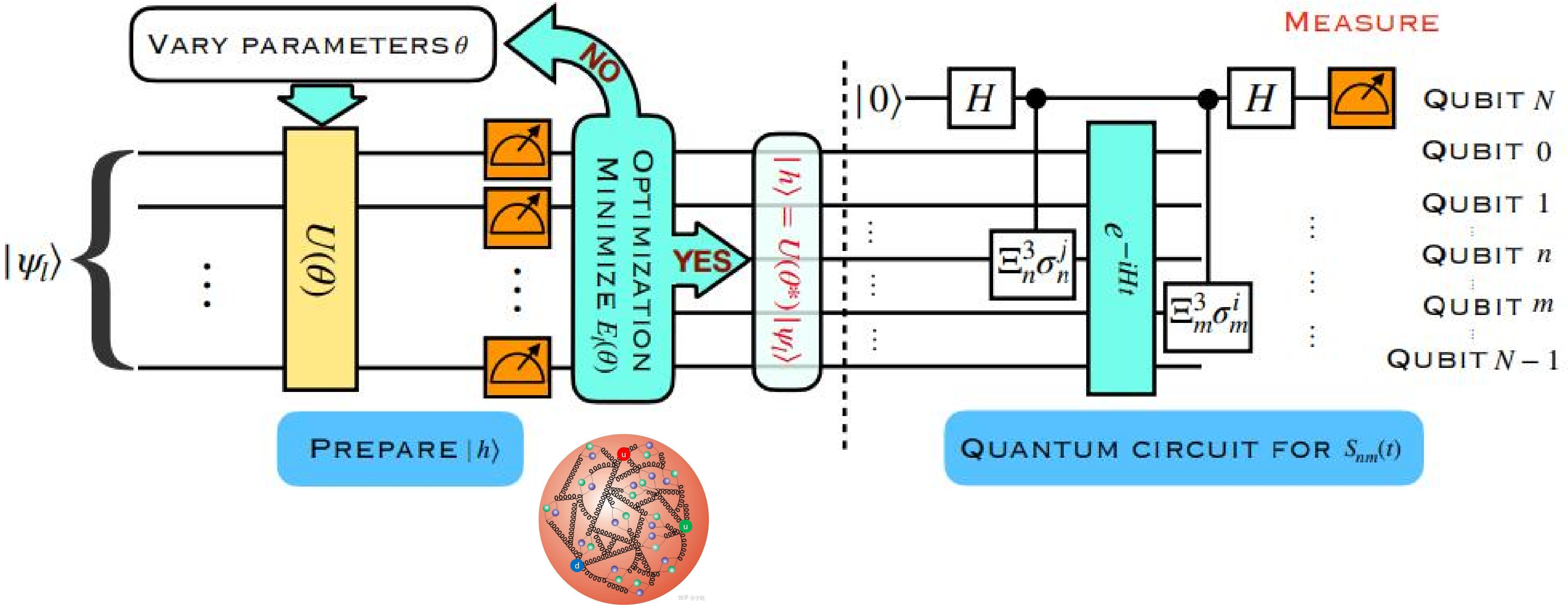
Still no Lattice QCD calculation:
How to sum up X?

Quantum algorithm for PDF

Prepare the hadronic state with VQE(nontrivial)

Probe the dynamical correlator (standard)





Demo with 1+1D NJL model

$$\mathcal{L} = \bar{\psi}_\alpha(i\gamma^\mu\partial_\mu - m_\alpha)\psi_\alpha + g(\bar{\psi}_\alpha\psi_\alpha)^2$$

using the staggered fermion approach

$$\psi_\alpha(x) = \begin{pmatrix}\psi_{\alpha,1}\\ \psi_{\alpha,2}\end{pmatrix} = \begin{pmatrix}\phi_{\alpha,\frac{(\alpha-1)N}{2}+2n}\\ \phi_{\alpha,\frac{(\alpha-1)N}{2}+2n+1}\end{pmatrix} \equiv \begin{pmatrix}\phi_{\alpha,2n}\\ \phi_{\alpha,2n+1}\end{pmatrix}$$

Bare vacuum (large m limit)

$$|010101\ldots01\rangle$$

10

Quark-antiquark pair

Lattice Hamiltonian

$$H = \sum_{\alpha,n}\left[-\frac{i}{2}\left(\psi^\dagger_{\alpha,n}\psi_{\alpha,n+1} - \psi^\dagger_{\alpha,n+1}\psi_{\alpha,n}\right) + (-1)^n m_\alpha \psi^\dagger_{\alpha,n}\psi_{\alpha,n}\right]$$
$$-g\sum_{\alpha,n=even}\left[\psi^\dagger_{\alpha,n}\psi_{\alpha,n} + \psi^\dagger_{\alpha,n+1}\psi_{\alpha,n+1} - 2\psi^\dagger_{\alpha,n}\psi_{\alpha,n}\psi^\dagger_{\alpha,n+1}\psi_{\alpha,n+1}\right]$$

Jordan Wigner transformation (Fermion-> qubit)

Variational quantum eigensolver for preparing hadronic state

- hadron: excited state with given quantum numbers
- quantum-number-resolving VQE

$$|\psi_{lk}(\theta)\rangle = U(\theta)\,|\psi_{lk}\rangle$$

Input state has the given quantum number, and U is symmetry-preserving

- ansatz: quantum-number-resolving, efficient and easy for optimization

$$H = H_1 + H_2 + \cdots + H_n \qquad U(\theta) \equiv \prod_{i=1}^{p}\prod_{j=1}^{n} \exp(i\,\theta_{ij} H_j)$$

- subspace-search VQE: get a set of excited states at one time

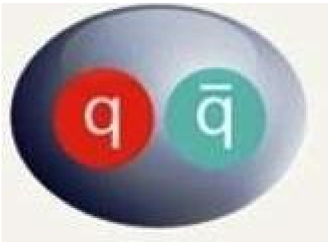
PRR **1**, 033062 (2019)

Min $E_l(\theta) = \sum_i^k w_{li}\,\langle\psi_{li}(\theta)|\,H\,|\psi_{li}(\theta)\rangle \qquad w_{l1} > w_{l2} > \cdots > w_{lk}$

1-flavor meson state

Large m limit as input states:

$$|\psi_{\Omega,1}\rangle = |010101\ldots01\rangle\ ,$$ bare vacuum



$$|\psi_{\Omega,2}\rangle = \frac{1}{\sqrt{N/2}}\left(|\underline{10}01,\ldots,01\rangle + |01\underline{10},\ldots,01\rangle + \cdots + |0101,\ldots,\underline{10}\rangle\right)\ ,$$

Zero momentum:
superposition of a Quark-antiquark pair

Trial states:

$$|\psi_{lk}(\theta)\rangle = U(\theta)\,|\psi_{lk}\rangle$$

$$U(\theta) \equiv \prod_{i=1}^{p}\prod_{j=1}^{n}\exp(i\,\theta_{ij}H_j)$$

U: quantum fluctuations

Both vacuum and the meson should be superposition of different number of quark-antiquark pairs

Light-cone correlator and PDF (N=18)

- Real part consistent with 0.

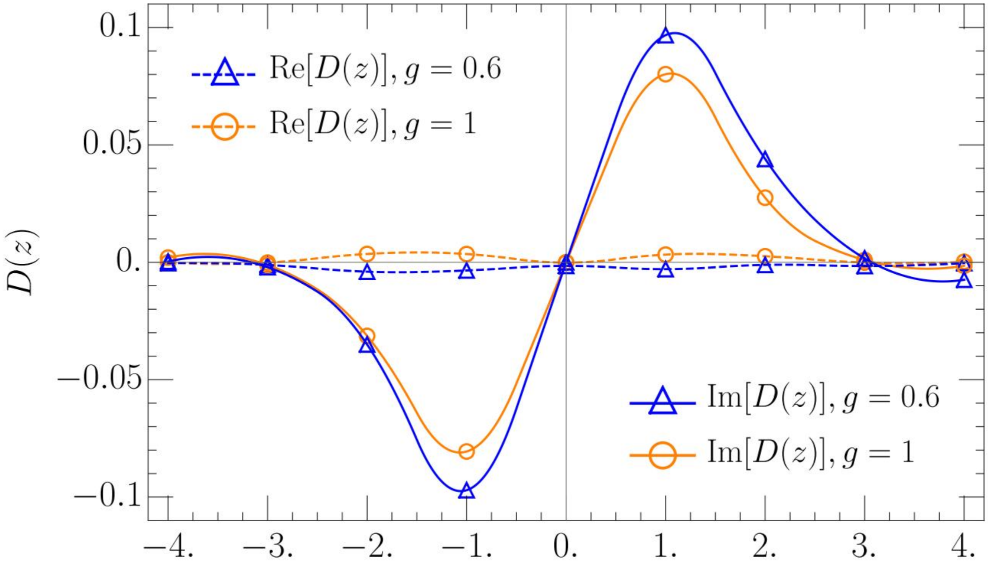
f is odd, so D is imaginary

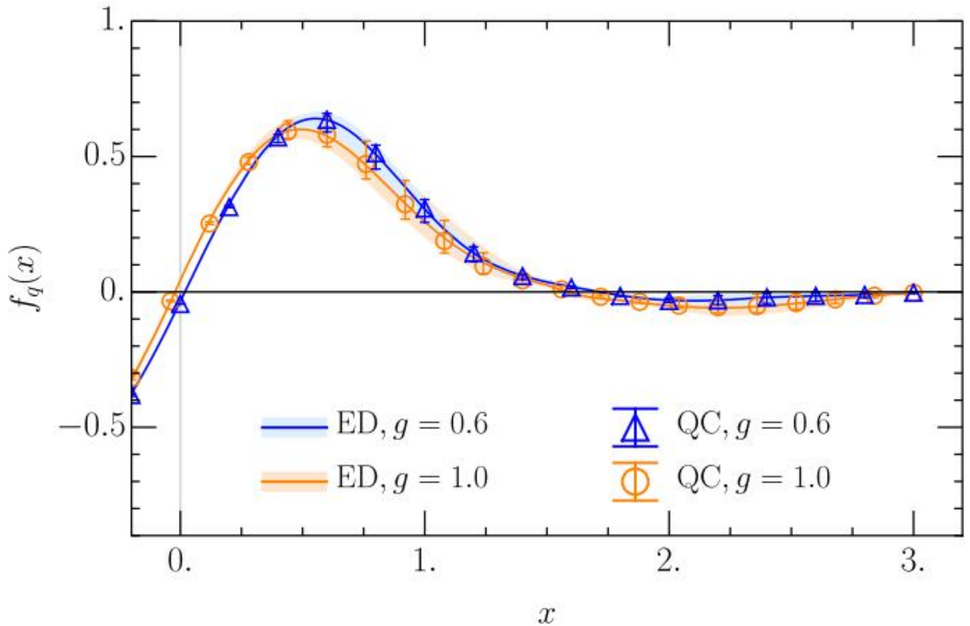
$$f_q(x) = -f_{\bar{q}}(-x)$$

- Bounded state behavior.

$$D(z) \equiv \sum_{ij} T_{ij}$$

$$f_{q_\alpha/h} = \sum_{i,j=0}^{1} \sum_{z} \frac{1}{4\pi} e^{-ixM_h z} T_{ij,\alpha}(z)$$





Hadron projector in FF

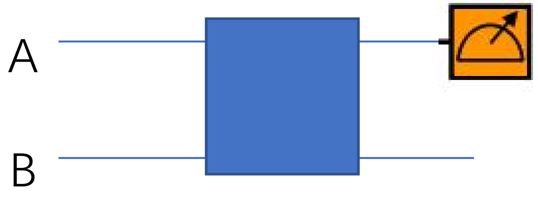
- X is unknown, how can one sum unknow?

$$P_h = \sum_X |h, X\rangle \langle h, X|$$

- sum=trace on a quantum computer = do nothing

$$\rho_A = \mathrm{Tr}_B |\psi\rangle \langle\psi|$$

$$Tr[O_A \psi] = Tr[O_A \rho_A]$$



- **sum_X = Trace = Do nothing**
- example: counting domain walls in a quantum state

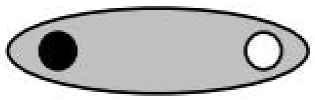
  |0001111>, |1110001>,

  then, the projector of domain wall is

$$N_F = \sum_i \frac{1}{2}(1 - Z_i Z_{i+1}) \qquad Z|n\rangle = (-1)^n |n\rangle$$

Hadron projector: bare meson

- meson in a digital word **(large m limit)**

01	11	00	10	
No particle	q quark	$\bar{q}$ antiquark	$q\bar{q}$ Quark-antiquark	

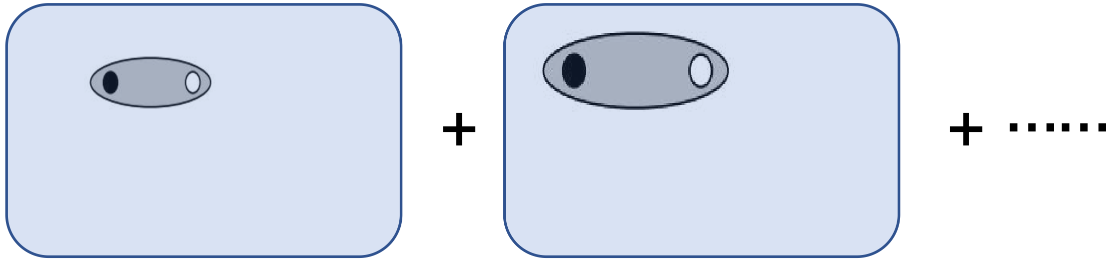
- at site i: $|\boldsymbol{I}_i^0\rangle = |01\rangle, |\boldsymbol{I}_i^1\rangle = |00\rangle, |\boldsymbol{I}_i^2\rangle = |11\rangle, |\boldsymbol{I}_i^3\rangle = |10\rangle$

- the hadron projector (T for translation symmetry)

$$P_h^{(0)} = \frac{1}{\sqrt{M}} \sum_{ij} |\boldsymbol{I}_i^3\rangle \langle \boldsymbol{I}_i^3| T^j \qquad |\boldsymbol{I}_i^3\rangle \langle \boldsymbol{I}_i^3| = 1 - \sigma_{2i}^z + \sigma_{2i+1}^z - \sigma_{2i}^z \sigma_{2i+1}^z$$

Meson in a quantum digital world

- superposition of $q\bar{q}$



- a dictionary of digital → quantum digital: vacuum, quark, antiquark, meson ….

$$|\Omega\rangle = U\,|\boldsymbol{I}_0^0, \boldsymbol{I}_1^0, ..., \boldsymbol{I}_0^{M-1}\rangle \equiv U\,|\tilde{\Omega}\rangle$$

$$|h_\alpha\rangle = \frac{1}{\sqrt{M}}\sum_{i=0}^{m-1} U\,|\boldsymbol{I}_0^0, \boldsymbol{I}_1^0, ..., \boldsymbol{I}_i^\alpha, \boldsymbol{I}_{i+1}^0, ..., \boldsymbol{I}_{M-1}^0\rangle$$

$$\equiv \frac{1}{\sqrt{M}}\sum_{i=0}^{m-1} U\,|\tilde{h}_\alpha(i)\rangle\,.$$

Eigenstates with specified quantum numbers!

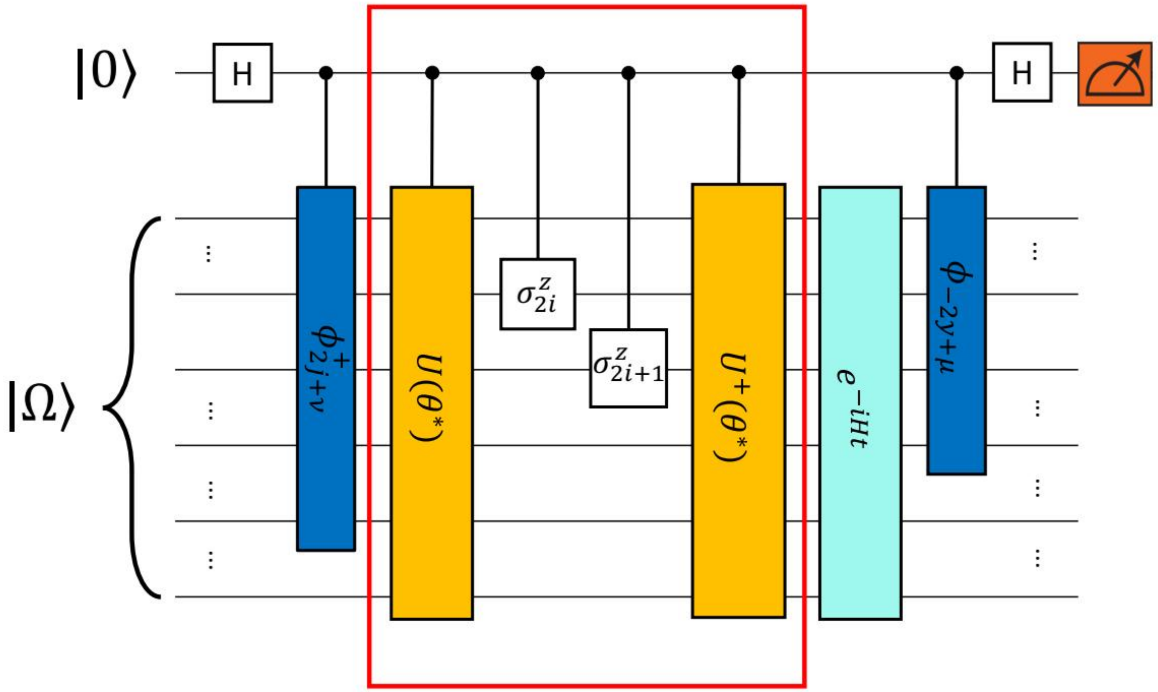
The unitary U can be found with quantum-number preserving VQE (see PRD 105, L111502 (2022))

Quantum circuit for correlator of FF

- variational constructing the hadron projector:

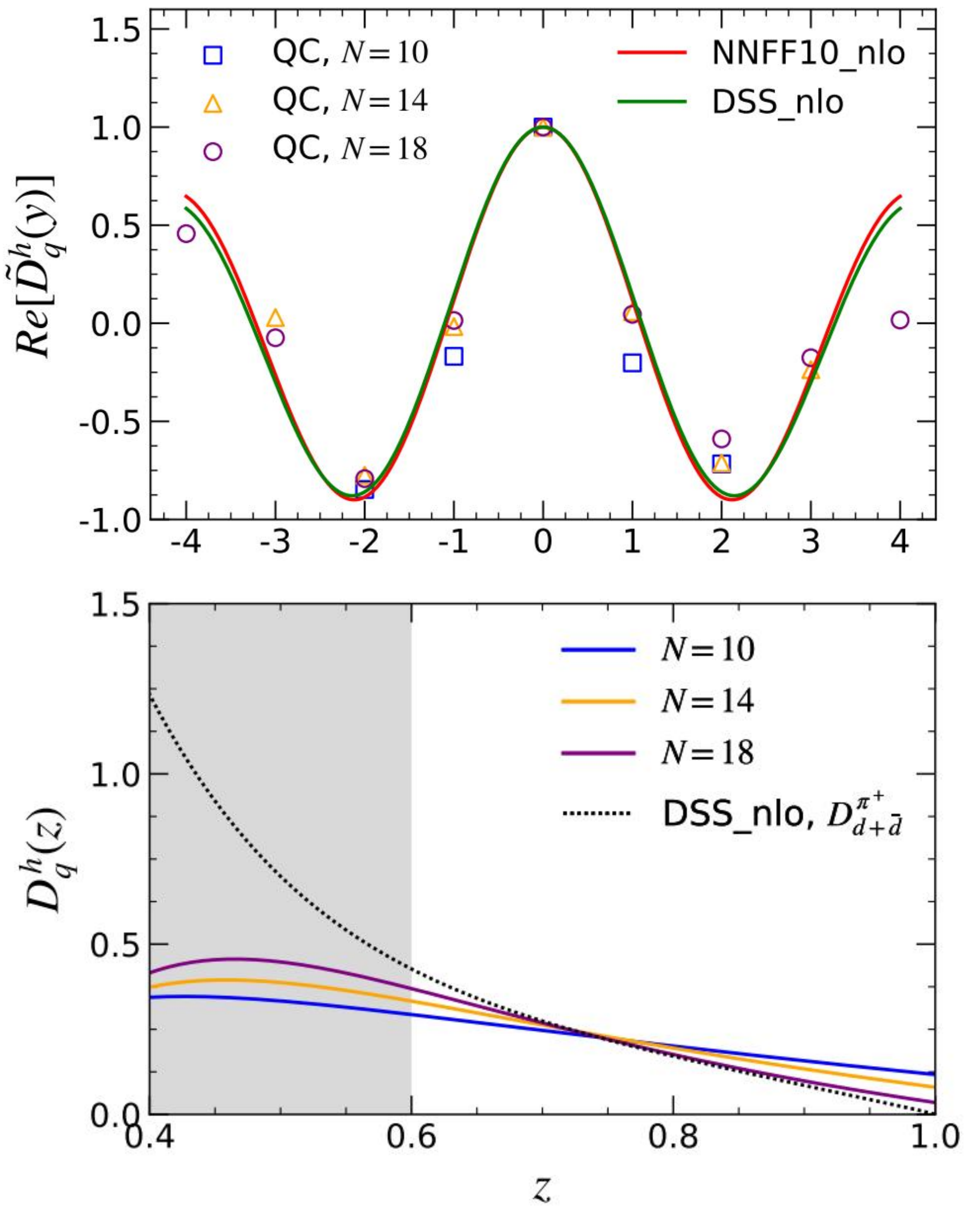
$$P_h = \sum_{ij} \frac{1}{4\sqrt{M}} U(\theta^*) \tilde{P}_h(i) U^\dagger(\theta^*) T^j$$

$$\tilde{P}_h(i) = (I - \sigma^z_{2i})(I + \sigma^z_{2i+1})$$



- Evaluate the light-cone correlator inserted with a projector

Demo of FF: numeral simulation



As the lattice size is limited, a comparison with global-fitting is obtained in short-range:

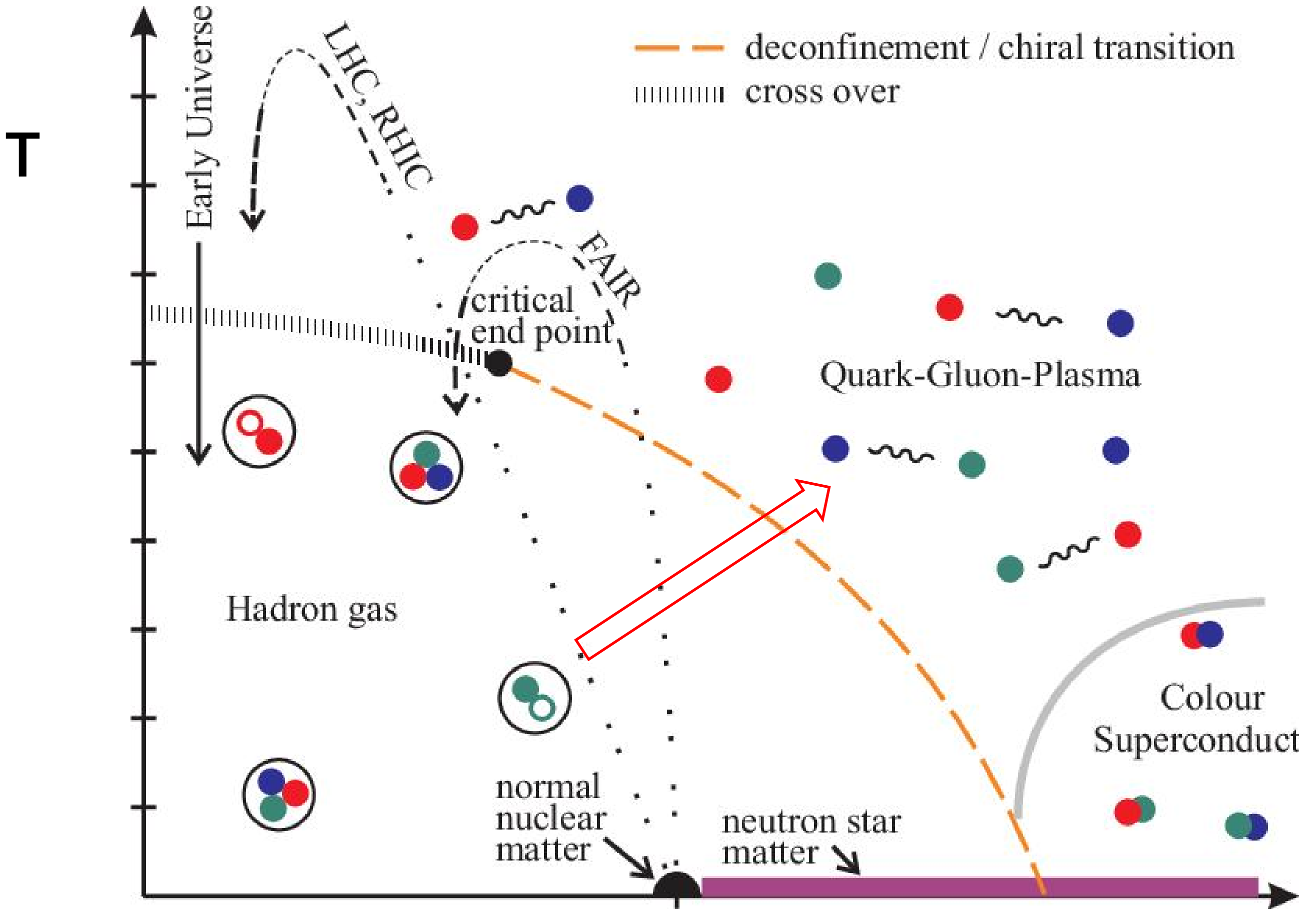
$$\tilde{D}^{\pi^+}_{d+\bar{d}}(y) = \int_{z_{min}}^{1} \frac{M_h}{z^3} e^{iyM_h/z} D^{\pi^+}_{d+\bar{d}}(z)$$

$$m_q a = 0.4, \ m_h a = 0.6$$

$$z_{min} = 0.35$$

QC for confinement-deconfinement transition

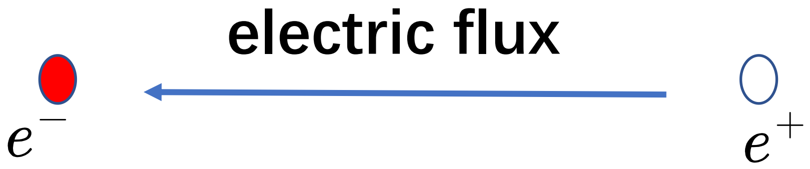
PhysRevD.106.054509(2022)



Model: 1+1D Schwinger model

- 1+1D quantum electrodynamics (U(1) gauge field)

$$\mathcal{L} = \bar{\psi}\left(\gamma^{\mu}(i\partial_{\mu} + gA_{\mu}) - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



linear potential

- 麻雀虽小，五脏俱全。 confinement, dynamical generated mass(gapped at m=0), chiral symmetry breaking…

- familiar to quantum computing community as easy to simulate.

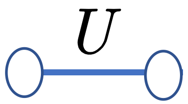
- deconfinement at infinite temperature
  PRD 19.1188(1979)

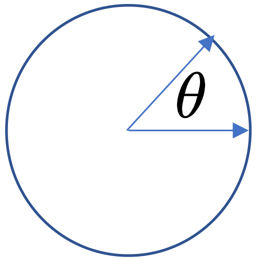
Lattice Hamiltonian + Gauss law

U(1) gauge theory

- lattice Hamiltonian (is not enough)

$$H = \frac{1}{2a}\sum_{j=1}^{N-1}[\hat{\Phi}_j^\dagger \hat{U}_{j,j+1}\hat{\Phi}_{j+1} + h.c.]$$
$$+m\sum_{j=1}^{N}(-1)^j\hat{\Phi}_j^\dagger\hat{\Phi}_j + \frac{g^2a}{2}\sum_{j=1}^{N-1}\hat{L}_{j,j+1}^2$$





$$\hat{U}_{j,j+1} = e^{i\theta_{j,j+1}}$$

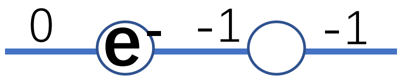
$$[\theta_{j,j+1}, L_{j',j'+1}] = -i\delta_{j,j}$$

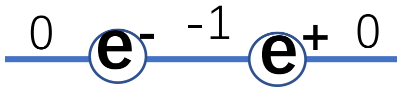
$$\hat{U}|l\rangle = |l+1\rangle$$

- Gauss law: physical Hilbert space with local constraints

stagger fermion

$$\hat{L}_{j,j+1} - \hat{L}_{j-1,j} = \hat{\Phi}_j^\dagger\hat{\Phi} - \frac{1-(-1)^j}{2}$$





Odd: 0 (e$^-$), 1 (empty)

even: 1 (e$^+$), 0 (empty)

- Eliminate gauge field by Gauss law $\hat{L}_{j,j+1} = \varepsilon + \sum_{l=1}^{j} [\hat{\Phi}_j^\dagger \hat{\Phi}_j - \frac{1-(-1)^l}{2}]$

- Qubit Hamiltonian: gauge field eliminated by Gauss law

$$H_\varepsilon = \varpi \sum_{j=1}^{N-1} [\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + h.c.] + \frac{m}{2} \sum_{j=1}^{N} (-1)^j \hat{\sigma}_j^z$$
$$\underline{+ \frac{g^2 a}{2} \sum_{j=1}^{N-1} \left\{ \varepsilon + \sum_{l=1}^{j} [\hat{\sigma}_l^z + (-1)^l] \right\}^2,}$$

non-local interaction

- Finite density

$$H_\varepsilon \quad \Rightarrow \quad G_\varepsilon(\mu) = H_\varepsilon - \frac{\mu}{2} \sum_{j=1}^{N} \hat{\sigma}_j^z$$

String tension: indicator of confinement/deconfinement

String tension as the difference of free energies with/without a pair of charges at boundaries

H_0

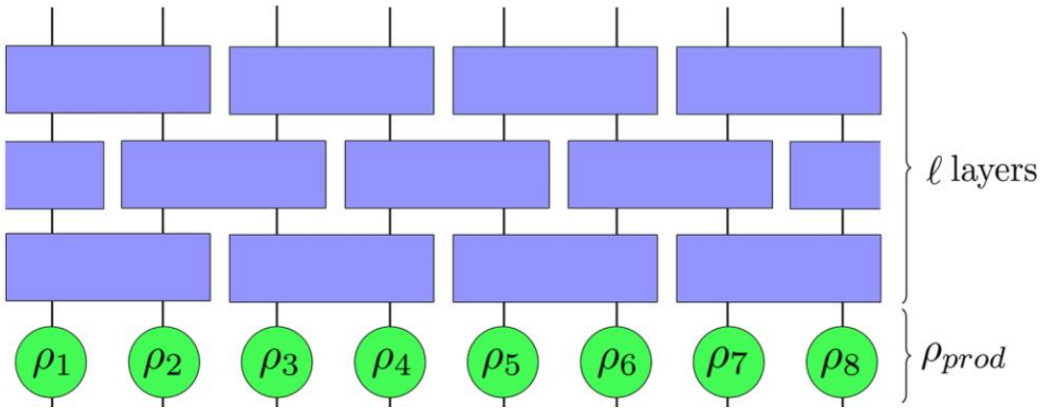
H_ε

- **Zero T. A linear potential between two charges(qqbar)**
- **Increasing T: the string is weaken by thermal fluctuations of qqbar.**

Variational quantum algorithm for finite-temperature system

- free energy F=E-TS: energy vs entropy

- prepare thermal state $\rho(\beta) = e^{-\beta H}/Z(\beta)$ with VQA



Product spectrum ansatz

$$\rho(\omega) = U(\phi)\rho_0(\theta)U^\dagger(\phi)$$

Entropy not change with U

- **minimize free energy:**

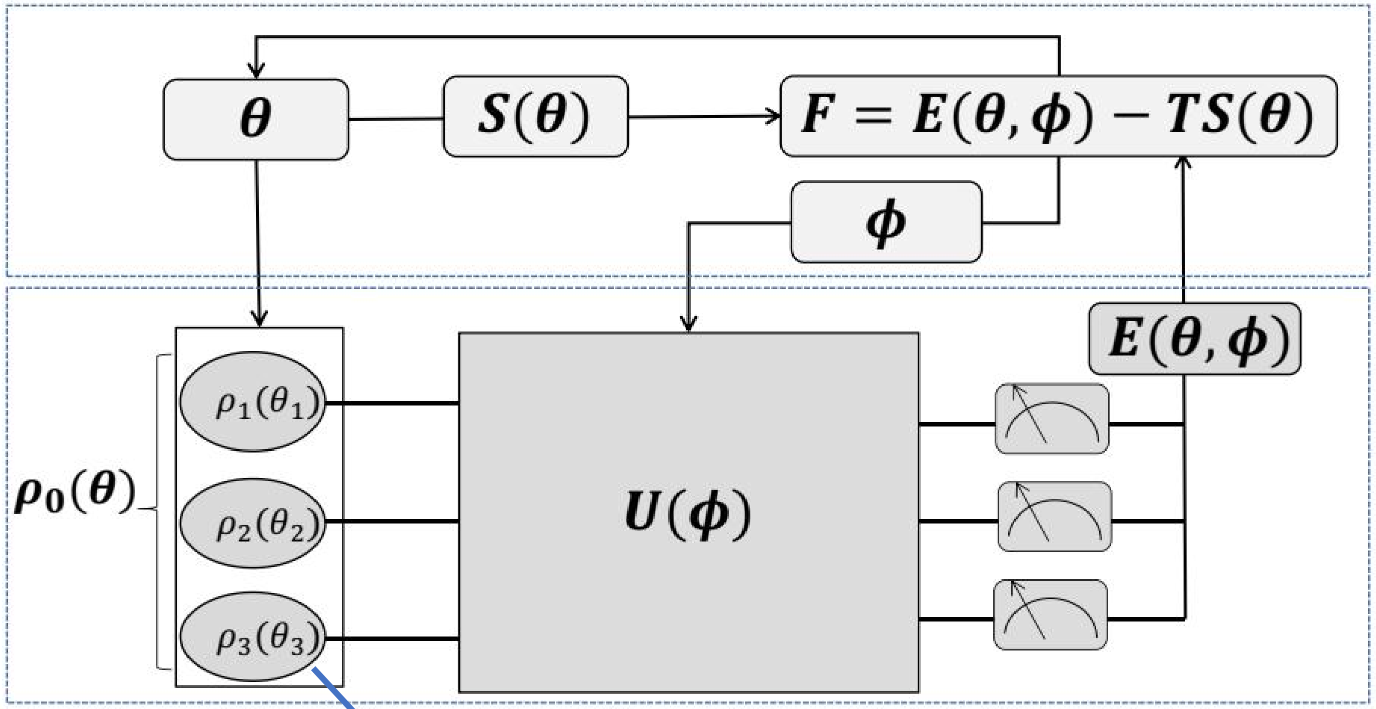
Classical computer

$$F(\beta) = E(\beta) - TS(\beta)$$

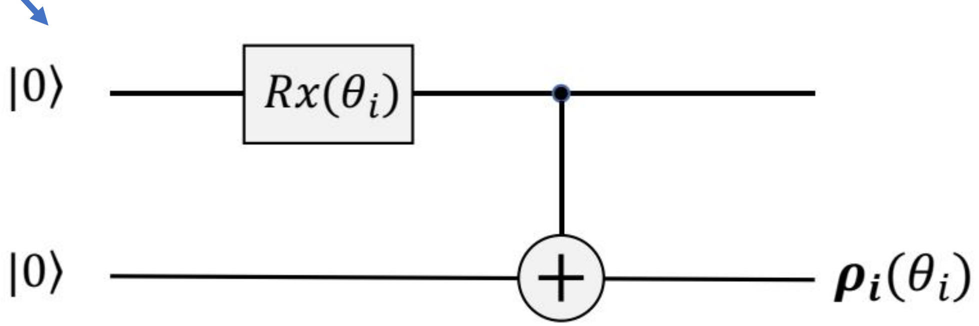
Quantum computer

Illustration of the algorithm

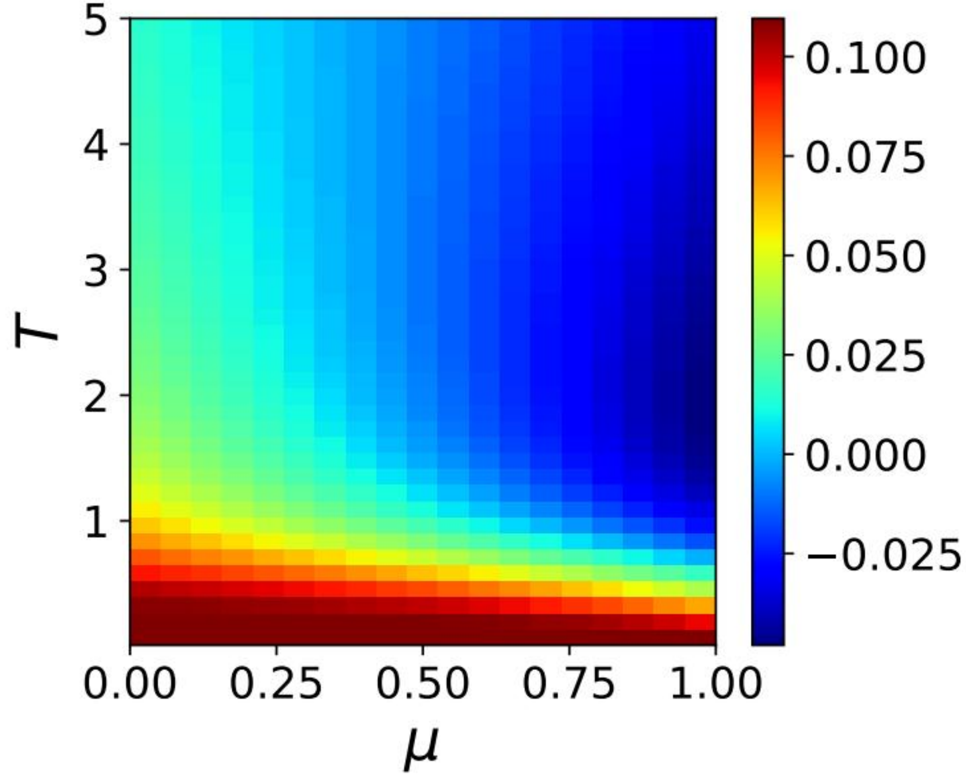
Hybrid quantum-classical optimization



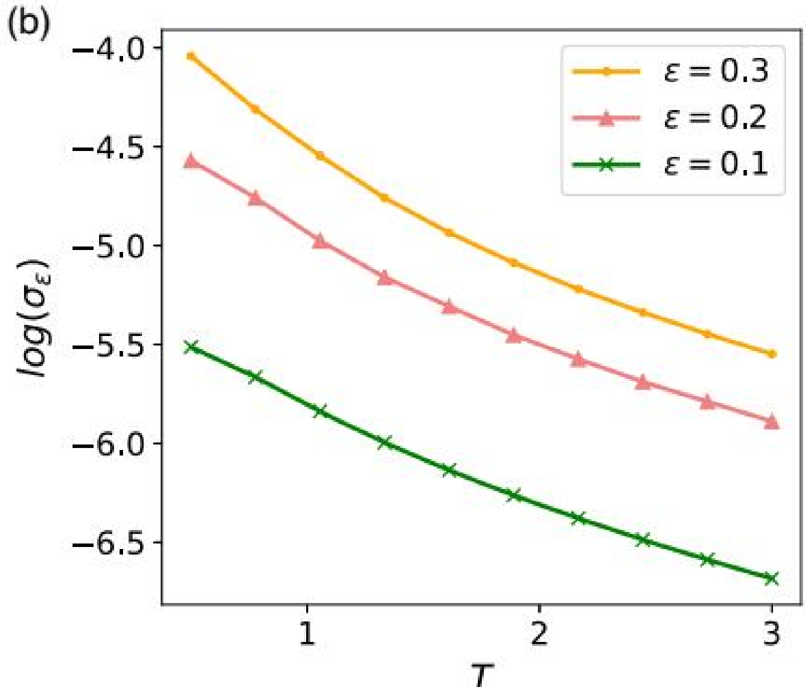
One-qubit mixed state
from a two-qubit entangled state



String tension at finite-T finite-density



Exponential decay of string tension with T



Conclusion

- QC for dynamic properties of hadron structure

- QC for finite-temperature & finite-density nuclear matter

Outlook:

- gluon PDFs
- phase diagram of QCD
- 2+1D systems