CEPC Flavor Physics/New Physics/
Detector Technology Workshop

Lepton Flavour Violation & Lepton Flavour Universality

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In the SM, EW interactions are *lepton flavour universal* and (with massless neutrinos) *lepton flavour conserving*

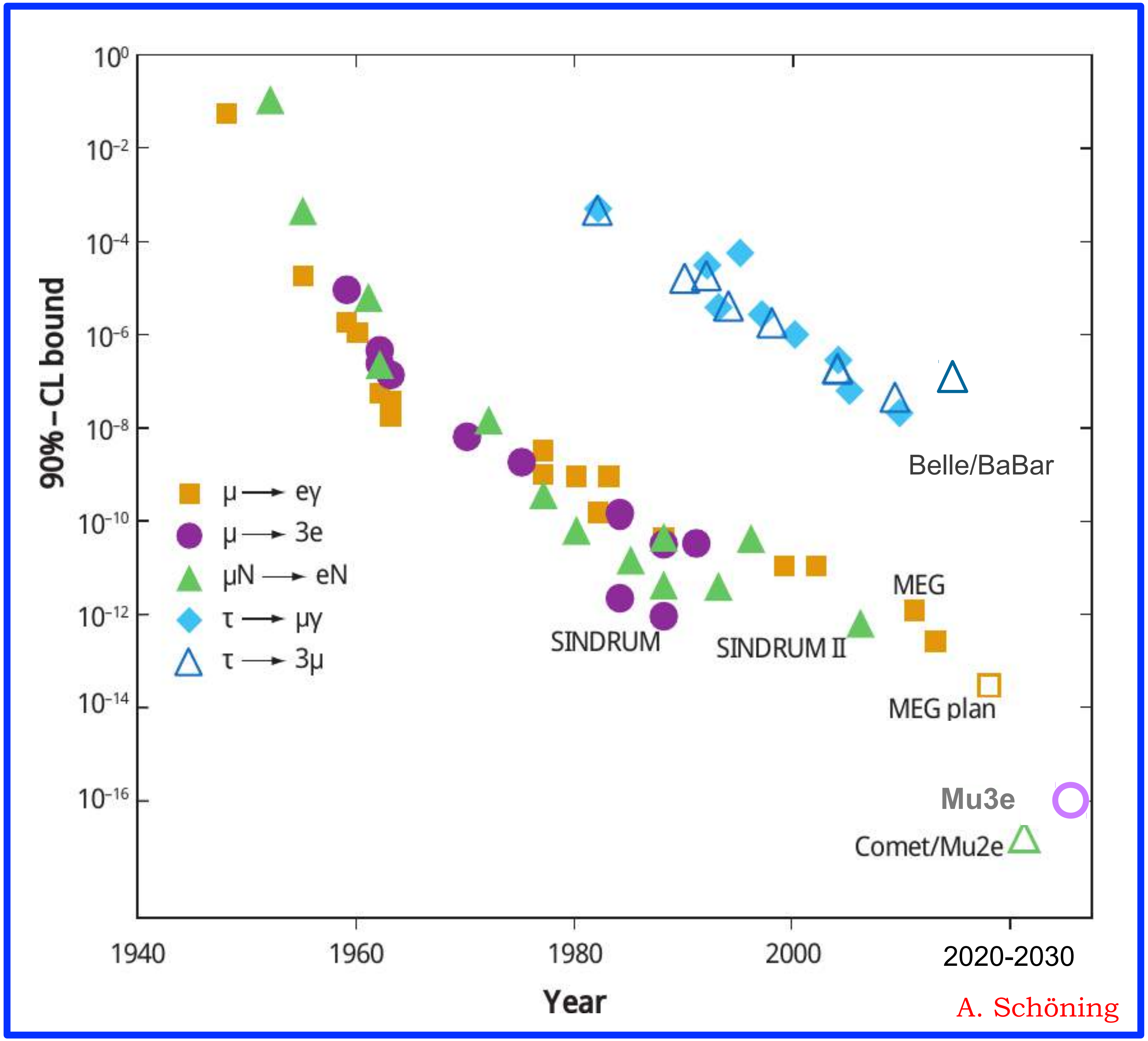
Neutrino masses/oscillations $\Longleftrightarrow \not{L}_e, \not{L}_\mu, \not{L}_\tau$

Lepton family numbers are not conserved

Why not *charged* lepton flavour violation (CLFV):

$$\mu \to e\gamma, \quad \tau \to \mu\gamma, \quad \mu \to eee, \; \text{etc.}\,?$$

CLFV has been sought for more than 70 years...



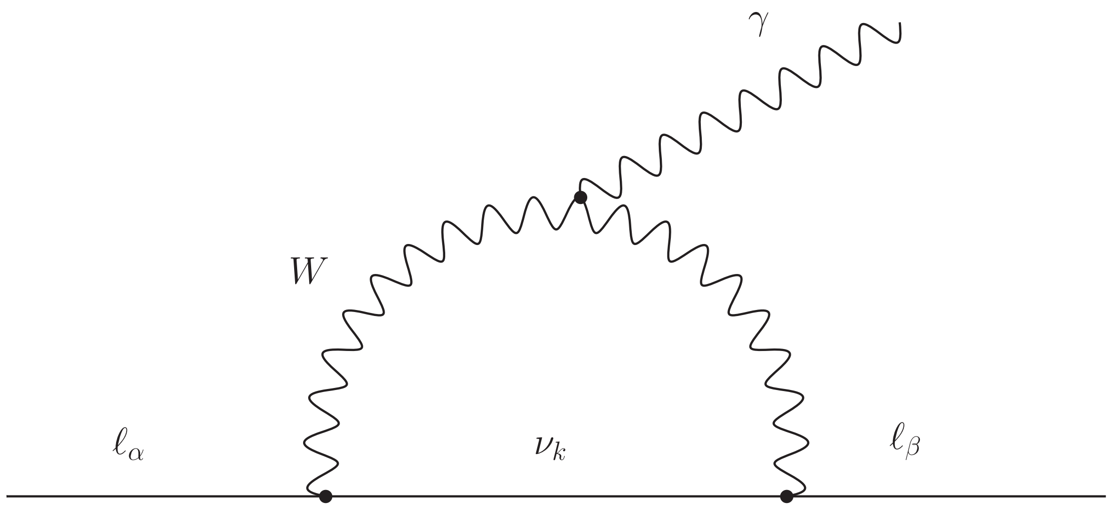
Why are we interested in CLFV?

- Neutrinos oscillate → Lepton family numbers are not conserved!
(while they would be exact global symmetries, if neutrinos were massless)

- Neutrino mass eigenstates couple to charged leptons of different flavours through the PMNS

- In the SM + massive neutrinos:

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W} \right|^2$$



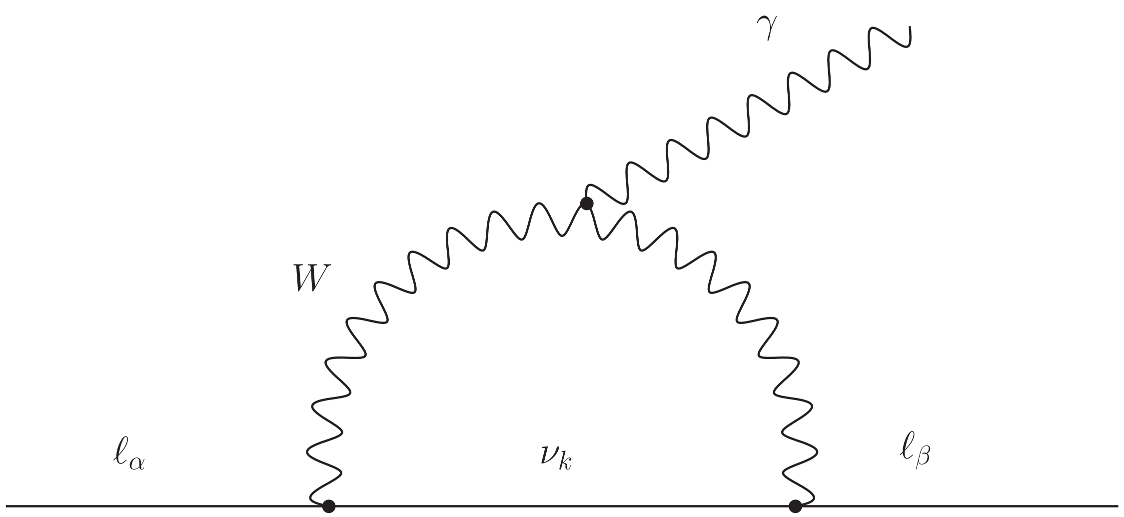
Cheng Li '77, '80; Petcov '77

⇨ $\mathrm{BR}(\mu \to e\gamma) \approx \mathrm{BR}(\tau \to e\gamma) \approx \mathrm{BR}(\tau \to \mu\gamma) = 10^{-55} \div 10^{-54}$

Large mixing, but huge suppression due to small neutrino masses

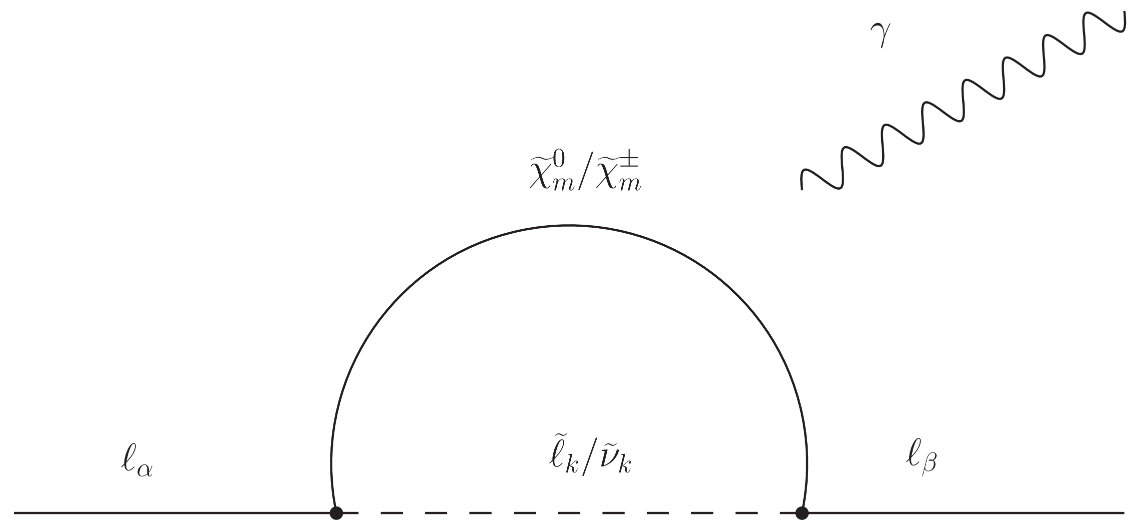
⇨ In presence of NP at the TeV we can expect large effects

Why are we interested in CLFV?



e.g. SUSY

Borzumati Masiero ‘86;
Hisano et al. ‘95



$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W} \right|^2$$

$$\frac{\Gamma(\ell_\alpha \to \ell_\beta \gamma)}{\Gamma(\ell_\alpha \to \ell_\beta \nu \bar{\nu})} \sim \frac{|\delta_{\alpha\beta}|^2}{G_F^2\, m^4_{\rm SUSY}}$$

- Unambiguous signal of New Physics (beyond neutrino masses)
- Stringent test of any NP coupling to leptons
- Probe of scales far beyond the LHC reach

⇒ For a pedagogical introduction (exp + th) cf. LC and Signorelli ‘17

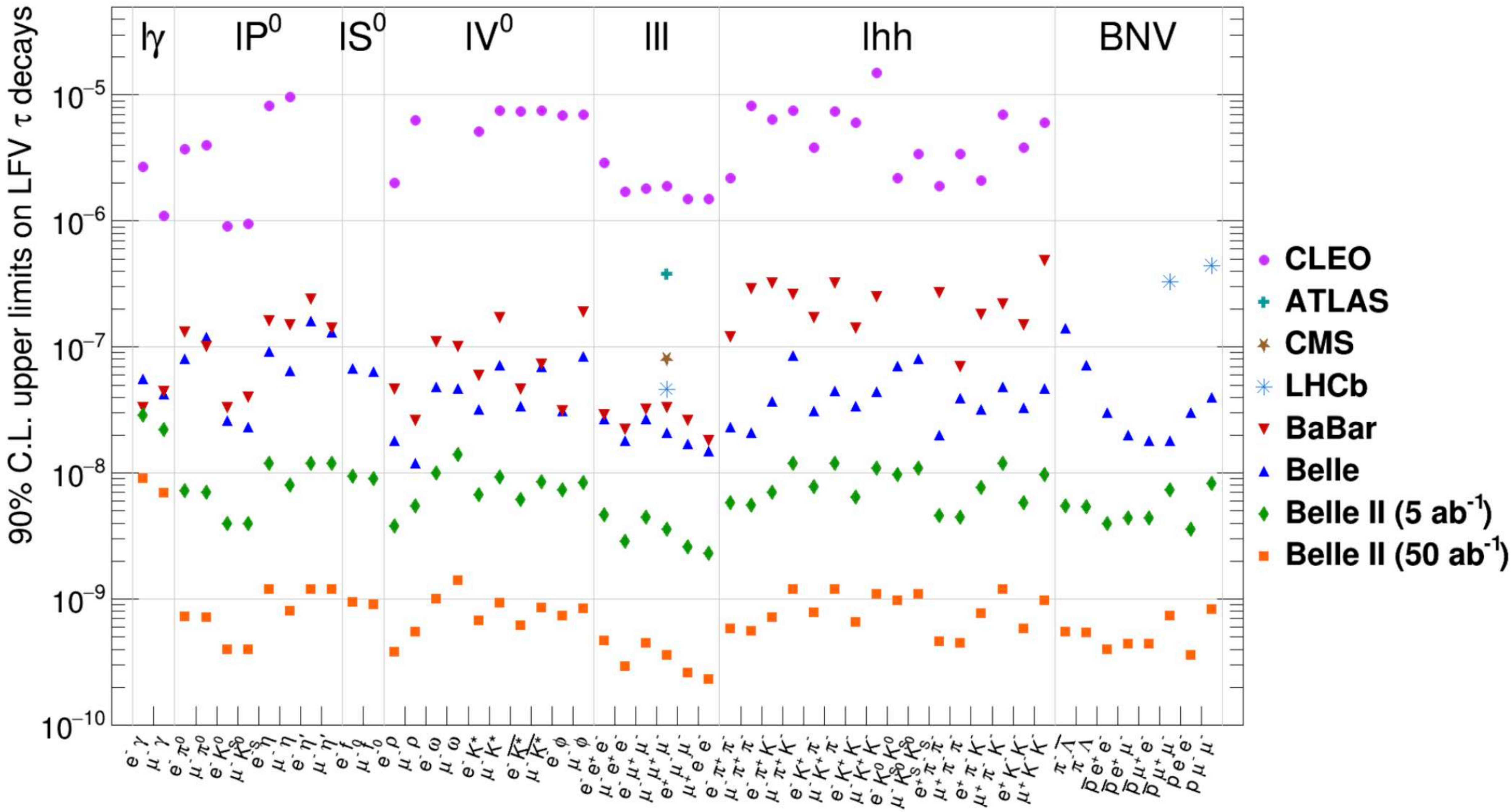
… and we have experiments!

LFV observable	Present bounds		Expected future limits	
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13}	MEG (2016) [28]	6×10^{-14}	MEG II [29]
$\mathrm{BR}(\mu \to eee)$	1.0×10^{-12}	SINDRUM (1988) [30]	10^{-16}	Mu3e [31]
$\mathrm{CR}(\mu \to e, \mathrm{Au})$	7.0×10^{-13}	SINDRUM II (2006) [32]		–
$\mathrm{CR}(\mu \to e, \mathrm{Al})$		–	6×10^{-17}	COMET/Mu2e [33, 34]
$\mathrm{BR}(Z \to e\mu)$	2.62×10^{-7}	ATLAS (2022) [35]	$10^{-8} - 10^{-10}$	FCC-ee/CEPC [36]
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	BaBar (2010) [37]	9×10^{-9}	Belle II [25, 38]
$\mathrm{BR}(\tau \to eee)$	2.7×10^{-8}	Belle (2010) [39]	4.7×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to e\mu\mu)$	2.7×10^{-8}	Belle (2010) [39]	4.5×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \pi e)$	8.0×10^{-8}	Belle (2007) [40]	7.3×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \rho e)$	1.8×10^{-8}	Belle (2011) [41]	3.8×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(Z \to e\tau)$	5.0×10^{-6}	ATLAS (2021) [42]	10^{-9}	FCC-ee/CEPC [36]
$\mathrm{BR}(\tau \to \mu\gamma)$	4.2×10^{-8}	Belle (2021) [43]	6.9×10^{-9}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.1×10^{-8}	Belle (2010) [39]	3.6×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \mu ee)$	1.8×10^{-8}	Belle (2010) [39]	2.9×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \pi\mu)$	1.1×10^{-7}	Babar (2006) [44]	7.1×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(\tau \to \rho\mu)$	1.2×10^{-8}	Belle (2011) [41]	5.5×10^{-10}	Belle II [25, 38]
$\mathrm{BR}(Z \to \mu\tau)$	6.5×10^{-6}	ATLAS (2021) [42]	10^{-9}	FCC-ee/CEPC [36]

Table 2: Present 90% CL upper limits (95% CL for the Z decays) and future expected sensitivities for the set of LFV transitions relevant for our analysis.

→ searches for muon LFV will soon test new physics up to scales of the order of 10^7–10^8 GeV

LFV tau decays:



Belle II Snowmass, arXiv:2207.06307

CEPC can play a role in the search for LFV tau decays:

$$\mathrm{BR}(Z \to \tau^+\tau^-) \simeq 3.4\%, \quad 4 \times 10^{12}\ Z \quad \Rightarrow \quad 10^{11}\ \tau^+\tau^-$$

Measurement	Current [129]	FCC [112]	CEPC prelim. [113]	Comments
Lifetime [sec]	$\pm 5 \times 10^{-16}$	$\pm 1 \times 10^{-18}$		from 3-prong decays, stat. limited
$\mathrm{BR}(\tau \to \ell\nu\bar{\nu})$	$\pm 4 \times 10^{-4}$	$\pm 3 \times 10^{-5}$		$0.1\times$ the ALEPH systematics
$\mathrm{m}(\tau)$ [MeV]	± 0.12	$\pm 0.004 \pm 0.1$		$\sigma(p_{\mathrm{track}})$ limited
$\mathrm{BR}(\tau \to \mu\mu\mu)$	$< 2.1 \times 10^{-8}$	$\mathcal{O}(10^{-10})$	same	bkg free
$\mathrm{BR}(\tau \to eee)$	$< 2.7 \times 10^{-8}$	$\mathcal{O}(10^{-10})$		
$\mathrm{BR}(\tau \to e\mu\mu)$	$< 2.7 \times 10^{-8}$	$\mathcal{O}(10^{-10})$		
$\mathrm{BR}(\tau \to \mu ee)$	$< 1.8 \times 10^{-8}$	$\mathcal{O}(10^{-10})$		
$\mathrm{BR}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$	$\sim 2 \times 10^{-9}$	$\mathcal{O}(10^{-10})$	$Z \to \tau\tau\gamma$ bkg , $\sigma(p_\gamma)$ limited
$\mathrm{BR}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$	$\sim 2 \times 10^{-9}$		

Table 7: Projected sensitivities for some τ physics measurements at the Z-factory run of FCC-*ee* [112] and CEPC [113]. Absolute instead of relative uncertainties are quoted. For the LFV modes $\tau \to eee$, $\tau \to \mu ee$, and $\tau \to e\mu\mu$, we assume that the sensitivity is similar to that of $\tau \to \mu\mu\mu$.

CEPC flavour WP, to appear

Also colliders: LFV Higgs decays

In the SM Higgs couplings and masses aligned → flavour conserving

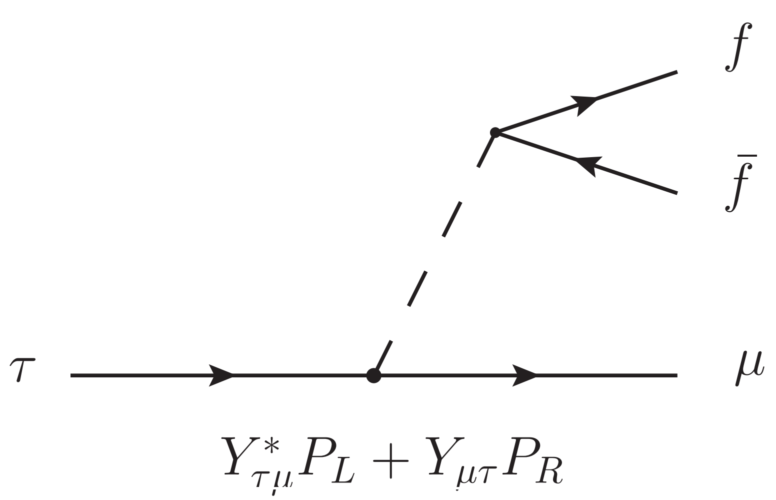
$$(m_f)_{ij} = \frac{v}{\sqrt{2}}(Y_f)_{ij}, \qquad -\mathcal{L}_{h\bar{f}f} = \frac{m_f}{v}\,\bar{f}_L f_R\, h + \text{h.c.}$$

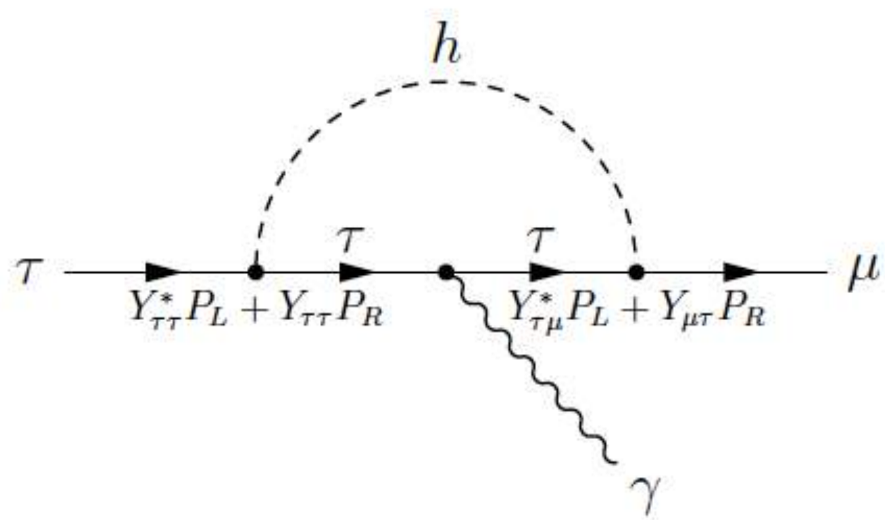
This is not the case if there is 2nd Higgs doublet or ops such as $\overline{L}_L e_R \Phi(\Phi^\dagger\Phi)$

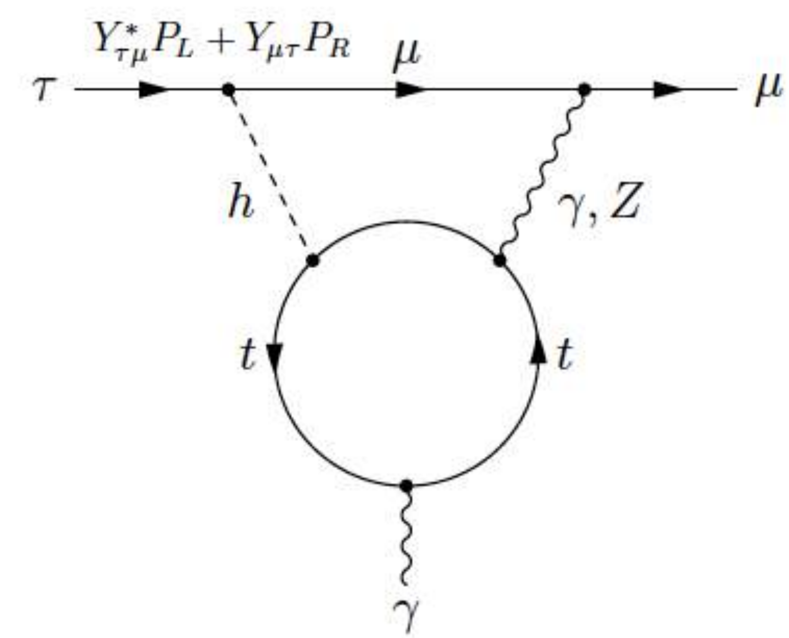
Useful parameterisation: $-\mathcal{L} \supset (m_e)_i \bar{e}_{L\,i}\, e_{R\,i} + (Y_e^h)_{ij}\,\bar{e}_{L\,i}\, e_{R\,j}\, h + \text{h.c.}$

Harnik Kopp Zupan '12

These couplings induce both LFV Higgs decays and low-energy processes:







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Harnik Kopp Zupan '12

Limits: $\text{BR}(h \to e\mu) < 4.4 \times 10^{-5}$, $\text{BR}(h \to e\tau) < 2.0 \times 10^{-3}$, $\text{BR}(h \to \mu\tau) < 1.8 \times 10^{-3}$

ATLAS, CMS '23

Process	Coupling	Bound
$h \to \mu e$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.9 \times 10^{-4}$
$\mu \to e\gamma$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 2.1 \times 10^{-6}$
$\mu \to eee$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$\lesssim 3.1 \times 10^{-5}$
$\mu\,\text{Ti} \to e\,\text{Ti}$	$\sqrt{\lvert Y^h_{\mu e}\rvert^2 + \lvert Y^h_{e\mu}\rvert^2}$	$< 1.2 \times 10^{-5}$
$h \to \tau e$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	$< 1.3 \times 10^{-3}$
$\tau \to e\gamma$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	< 0.014
$\tau \to eee$	$\sqrt{\lvert Y^h_{\tau e}\rvert^2 + \lvert Y^h_{e\tau}\rvert^2}$	$\lesssim 0.12$
$h \to \tau\mu$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	$< 1.2 \times 10^{-3}$
$\tau \to \mu\gamma$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	< 0.016
$\tau \to \mu\mu\mu$	$\sqrt{\lvert Y^h_{\tau\mu}\rvert^2 + \lvert Y^h_{\mu\tau}\rvert^2}$	$\lesssim 0.25$

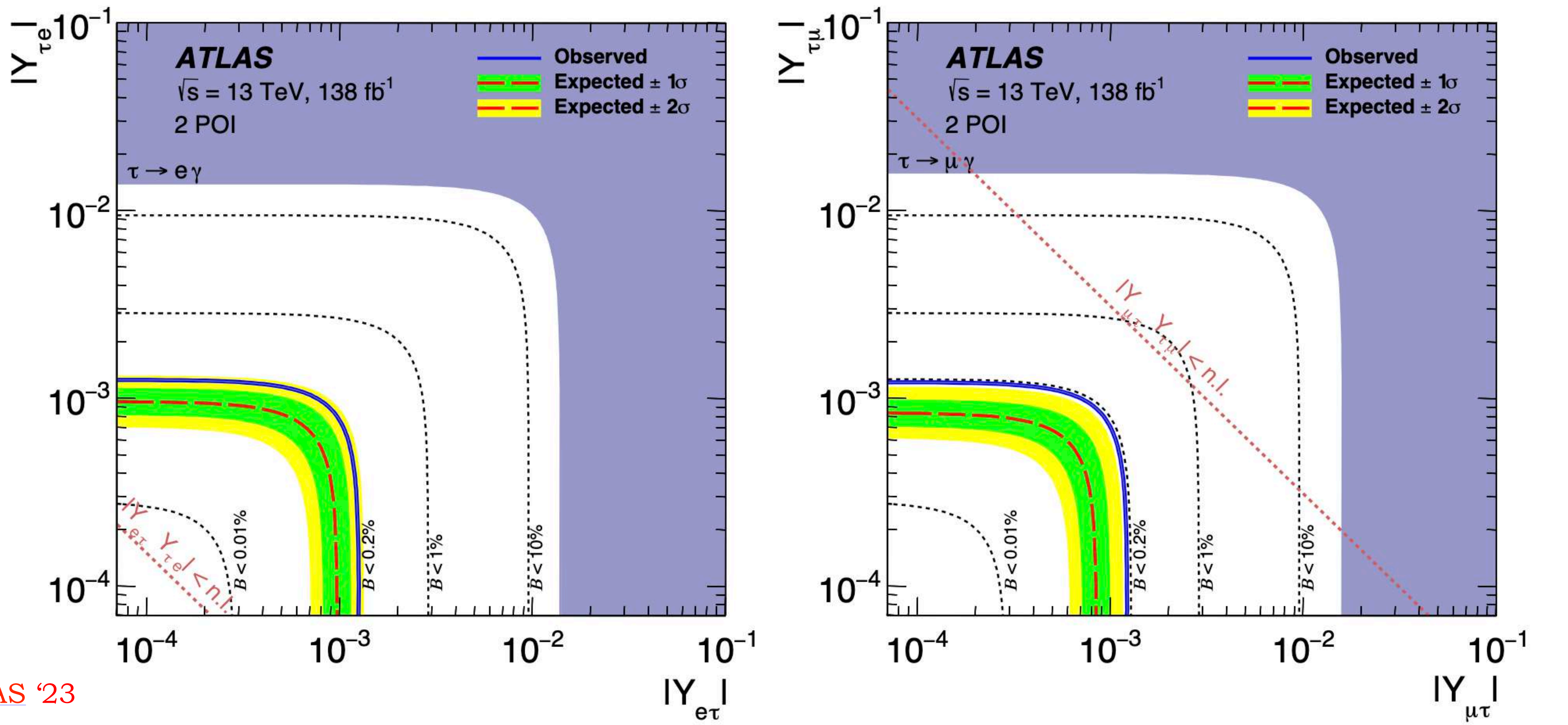
Also colliders: LFV Higgs decays

In the SM Higgs couplings and masses aligned → flavour conserving

$$(m_f)_{ij} = \frac{v}{\sqrt{2}}(Y_f)_{ij}, \qquad -\mathcal{L}_{h\bar{f}f} = \frac{m_f}{v}\,\bar{f}_L f_R\, h + \text{h.c.}$$

This is not the case if there is 2nd Higgs doublet or ops such as $\overline{L}_L e_R \Phi(\Phi^\dagger\Phi)$

Useful parameterisation: $-\mathcal{L} \supset (m_e)_i \bar{e}_{L\,i}\, e_{R\,i} + (Y_e^h)_{ij}\, \bar{e}_{L\,i}\, e_{R\,j}\, h + \text{h.c.}$



ATLAS '23

Also colliders: LFV Z decays at future circular e+e-

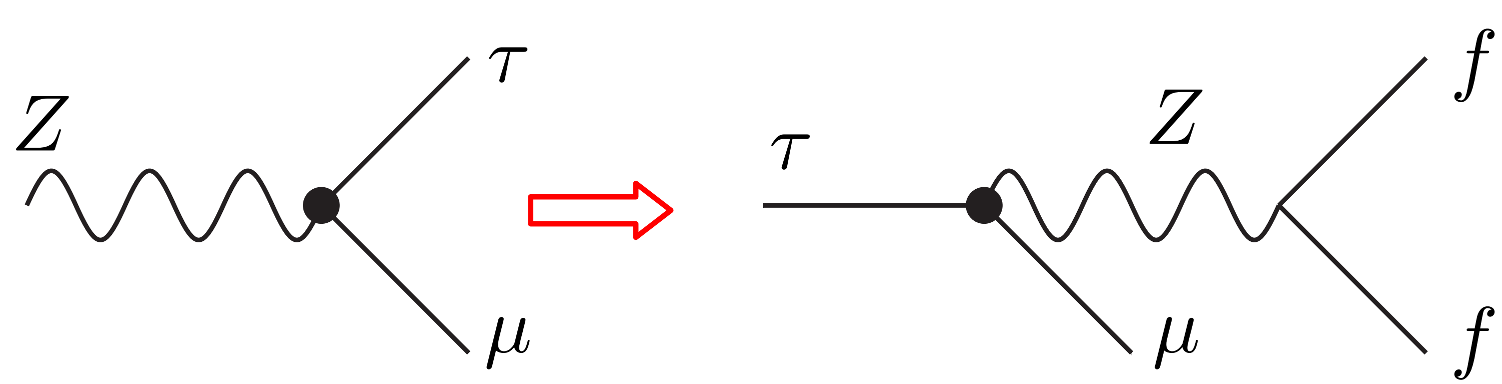
CEPC/FCC-ee Z-pole run: $O(10^{12})$ Z

M. Dam '18

Mode	LEP bound (95% CL)	LHC bound (95% CL)	CEPC/FCC-ee exp.
$\mathrm{BR}(Z \to \mu e)$	1.7×10^{-6} [2]	7.5×10^{-7} [3]	$10^{-8} - 10^{-10}$
$\mathrm{BR}(Z \to \tau e)$	9.8×10^{-6} [2]	5.0×10^{-6} [4, 5]	10^{-9}
$\mathrm{BR}(Z \to \tau \mu)$	1.2×10^{-5} [6]	6.5×10^{-6} [4, 5]	10^{-9}

- LHC searches limited by backgrounds (in particular $Z \to \tau\tau$):
  max ~10 improvement can be expected at HL-LHC (3000/fb)
- A Tera Z factory can improve the present (future) bounds by 4 (3) orders of magnitude
- The question is: can we find new physics searching for these modes?
  Low-energy LFV decays are unavoidably induced, giving *indirect* bounds

e.g. :



$\tau \to \mu \ell \ell$

$\tau \to \mu \pi$

$\tau \to \mu \rho$

...

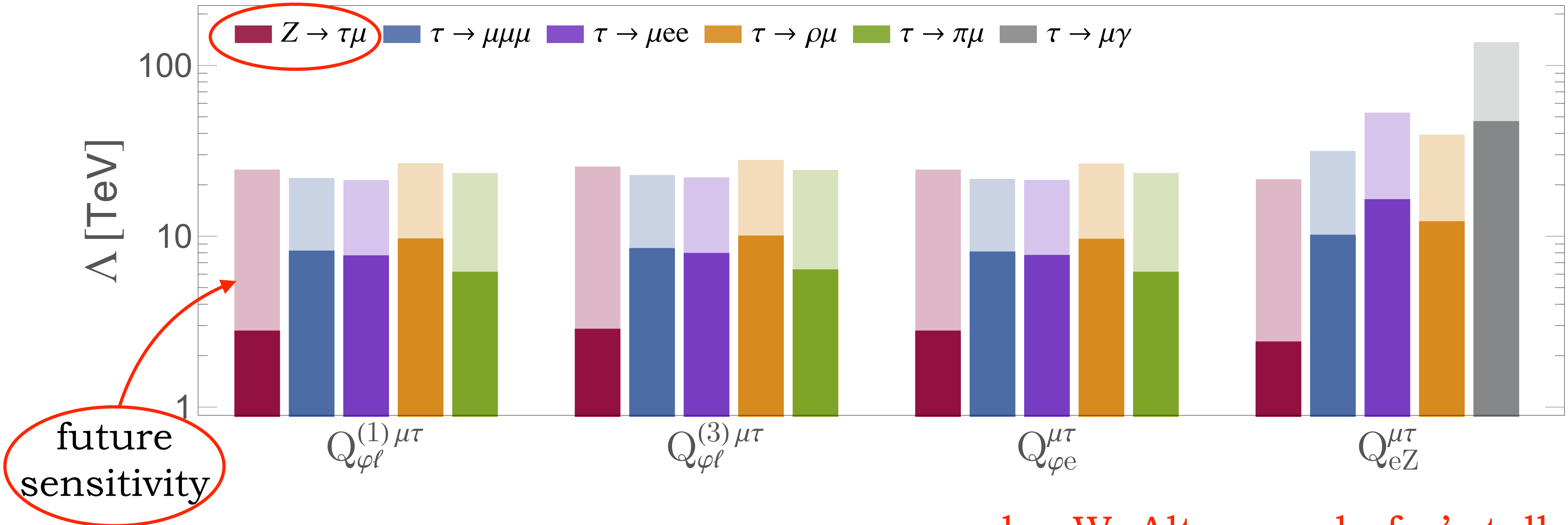
Model-independent indirect limits on Z LFV decays

Observable	Operator	Indirect Limit on LFVZD	Strongest constraint
lepton-Higgs ops	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{e\mu}$	3.7×10^{-13}	$\mu \to e$, Au
$\mathrm{BR}(Z \to \mu e)$	$Q_{\varphi e}^{e\mu}$	9.4×10^{-15}	$\mu \to e$, Au
dipole ops	$Q_{eB}^{e\mu}$	1.4×10^{-23}	$\mu \to e\gamma$
	$Q_{eW}^{e\mu}$	1.6×10^{-22}	$\mu \to e\gamma$
$\mathrm{BR}(Z \to \tau e)$	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{e\tau}$	6.3×10^{-8}	$\tau \to \rho\, e$
	$Q_{\varphi e}^{e\tau}$	6.3×10^{-8}	$\tau \to \rho\, e$
	$Q_{eB}^{e\tau}$	1.2×10^{-15}	$\tau \to e\gamma$
	$Q_{eW}^{e\tau}$	1.3×10^{-14}	$\tau \to e\gamma$
$\mathrm{BR}(Z \to \tau\mu)$	$(Q_{\varphi\ell}^{(1)} + Q_{\varphi\ell}^{(3)})^{\mu\tau}$	4.3×10^{-8}	$\tau \to \rho\, \mu$
	$Q_{\varphi e}^{\mu\tau}$	4.3×10^{-8}	$\tau \to \rho\, \mu$
	$Q_{eB}^{\mu\tau}$	1.5×10^{-15}	$\tau \to \mu\gamma$
	$Q_{eW}^{\mu\tau}$	1.7×10^{-14}	$\tau \to \mu\gamma$

LC Marcano Roy '21

Observable	Operator	Indirect Limit on LFVZD	Strongest constraint

- A Tera Z can test LFV new physics scales searching for $Z \to \tau\,\ell$ at the level of what Belle II will do through LFV tau decays (or better)



→ see also W. Altmannshofer's talk

LC Marcano Roy '21

Assume there is a *light*, *invisible*, new particle "*a*"
with *flavour-violating couplings* to leptons

That's natural, if it is the (pseudo) Nambu-Goldstone boson (PNGB)
of a broken global U(1), *aka* an axion-like particle (ALP)

Global symmetry:	PNGB:
• Lepton Number	Majoron
• Peccei-Quinn	Axion
• Flavour symmetry	Familon
...	

Wilczek '82

Pilaftsis '93

Feng et al. '97

LC Goertz Redigolo Ziegler Zupan '16

Di Luzio et al. '17, '19

...

CLFV modes would then be $\mu \to e\,a, \;\; \tau \to \mu\,a, \;\; \mu \to e\,\gamma\,a, \;\; \text{etc}\,.$

Interesting interplay with cosmo/astro:

- DM candidate? (if long-lived enough)
- Bounds from star cooling/supernovae (if light and feeble enough)

Generic couplings to leptons:

$$\mathcal{L}_{a\ell\ell} = \frac{\partial^\mu a}{2 f_a} \left(C_{ij}^V \, \bar{\ell}_i \gamma_\mu \ell_j + C_{ij}^A \, \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j \right)$$

This generic Lagrangian induces 2-body LFV decays such as:

$$\Gamma(\ell_i \to \ell_j \, a) = \frac{1}{16\pi} \frac{m_{\ell_i}^3}{F_{ij}^2} \left(1 - \frac{m_a^2}{m_{\ell_i}^2} \right)^2 \qquad F_{ij} \equiv \frac{2 f_a}{\sqrt{|C_{ij}^V|^2 + |C_{ij}^A|^2}}$$

Goal: constrain the effective LFV scales F_{ij} using experimental data

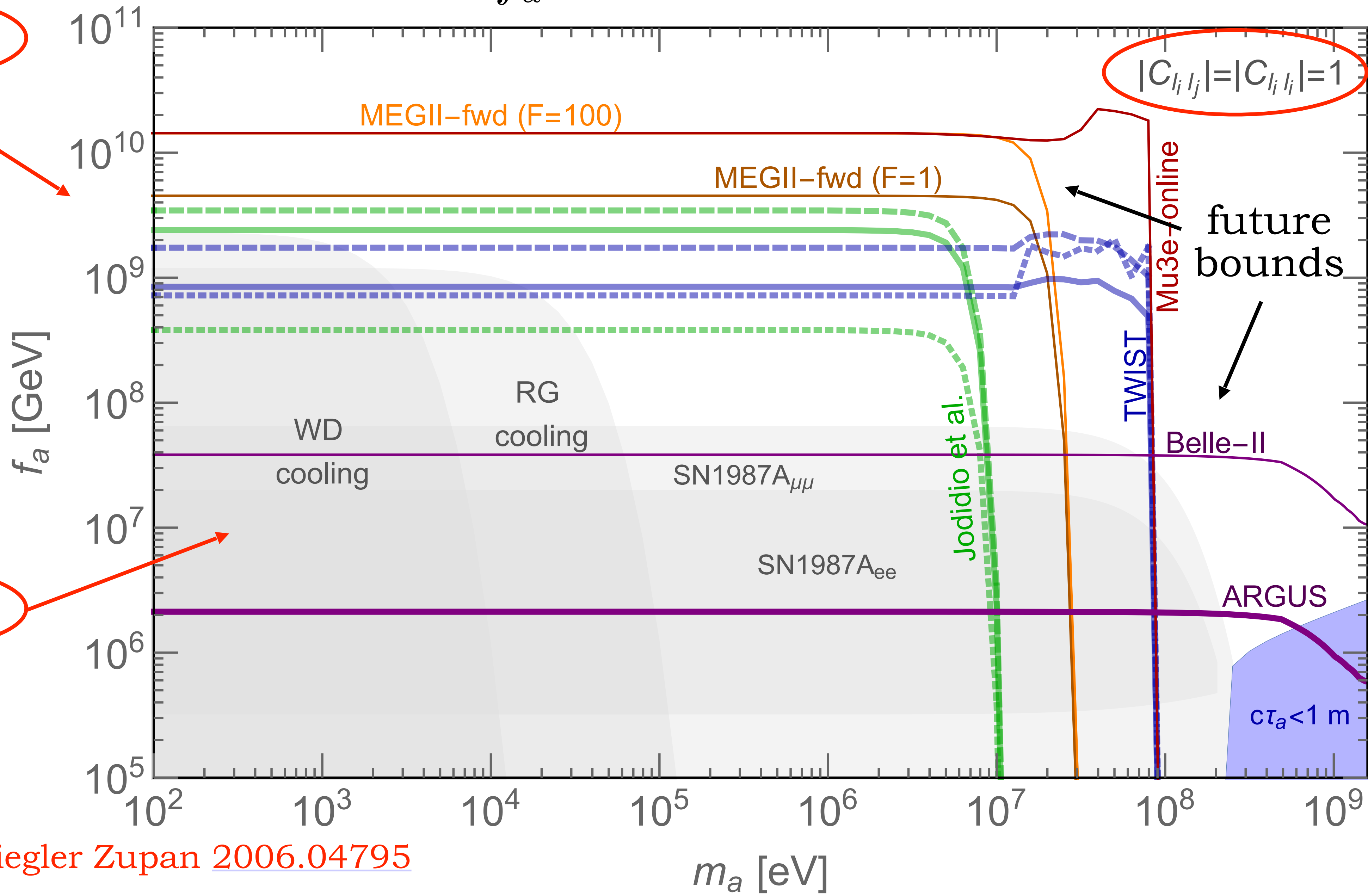
Where does *lepton flavour violation* come from?

- If lepton U(1) charges are flavour non-universal ⇒ naturally flavour-violating couplings
- Alternatively, loop-induced flavour-violating couplings (several explicit examples in arXiv:2006.04795)

Lepton-flavour-violating invisible ALPs

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu (C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5) f_j$$

$\mu \to ea$



$\tau \to \ell a$

star cooling

LC Redigolo Ziegler Zupan 2006.04795

Decays mediated by dim-5 operators: much larger NP scales can be reached
Essential interplay among μ decays, τ decays, and astrophysical bounds

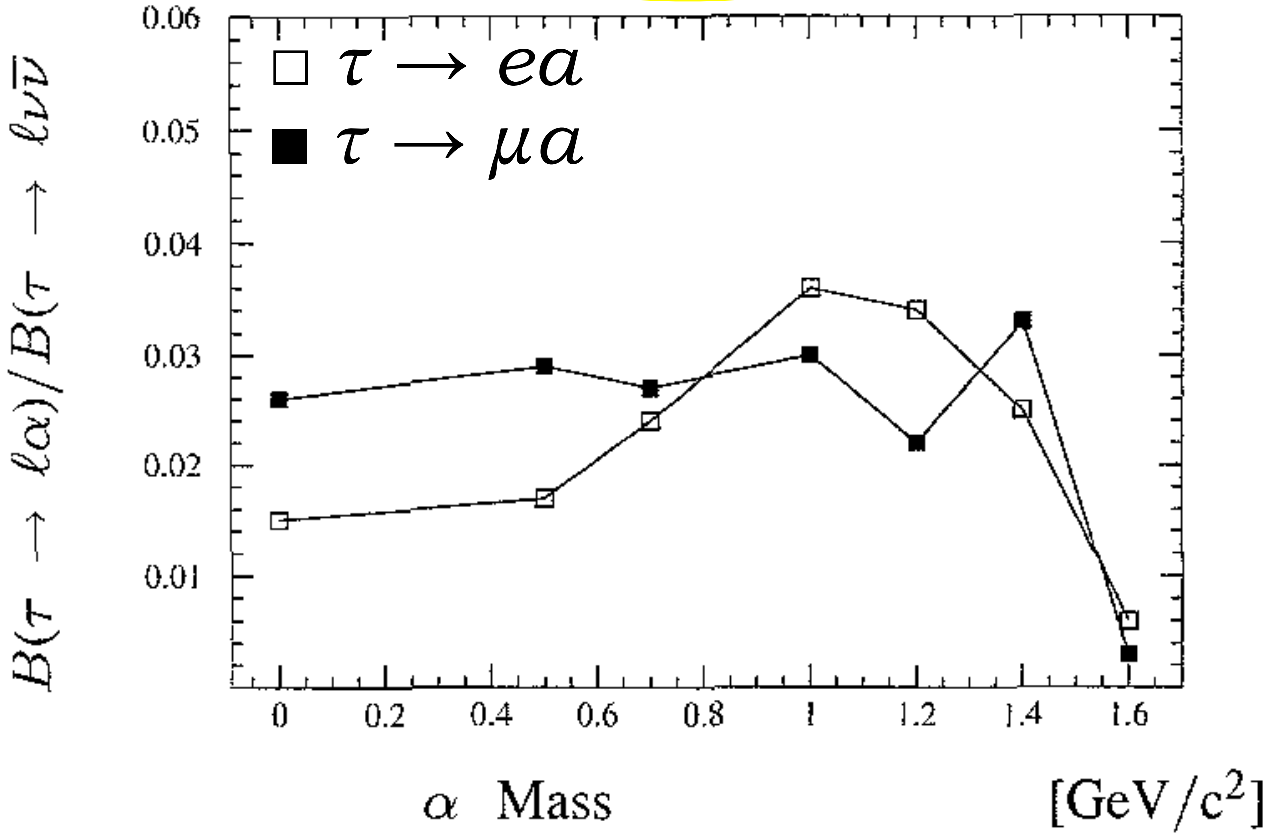
- ARGUS 1995

A search for the lepton-flavour violating decays $\tau \longrightarrow e\alpha$, $\tau \longrightarrow \mu\alpha$

Z. Phys. C 68, 25–28 (1995)

ARGUS Collaboration

With 472 pb^{-1}:

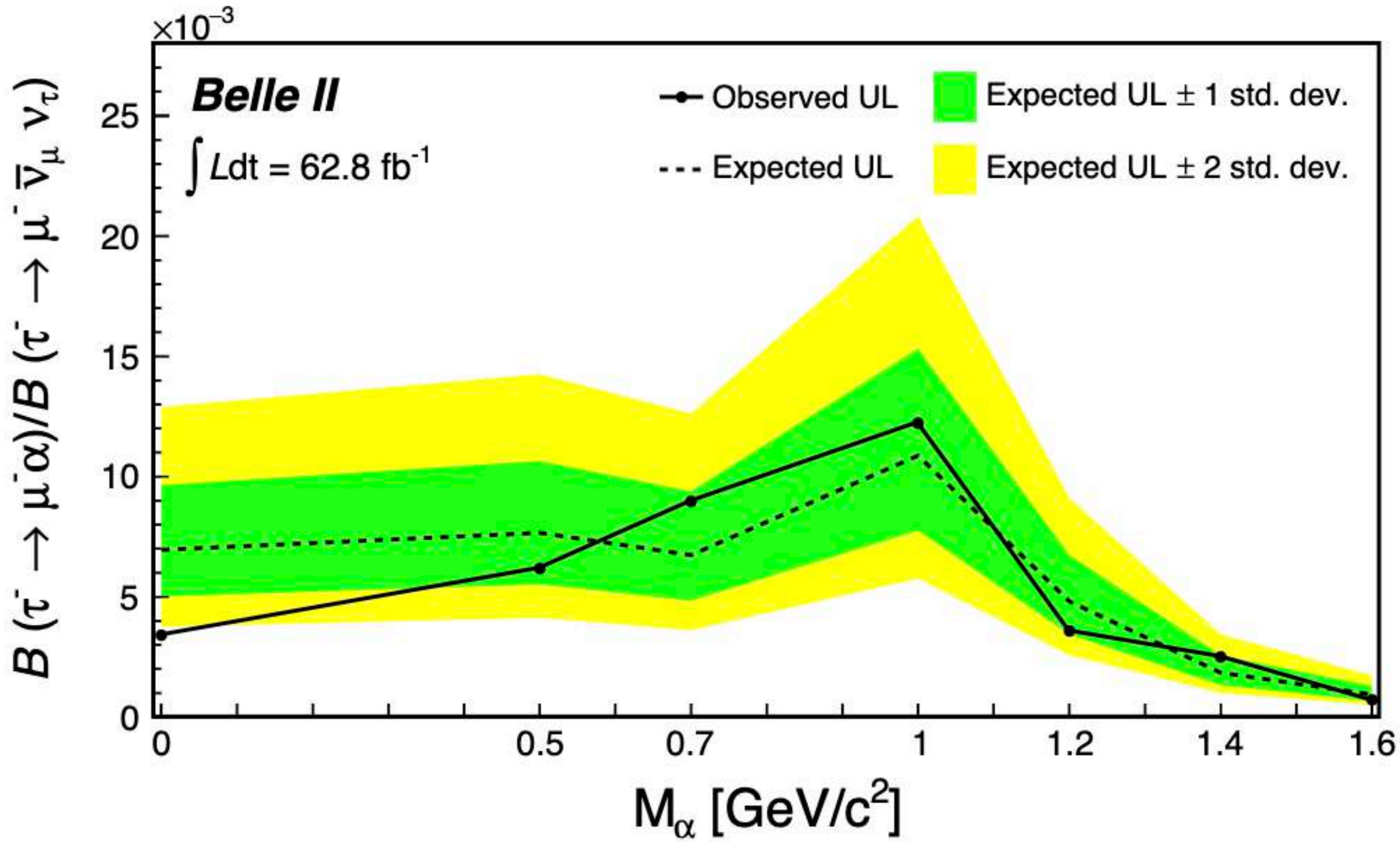


$m_a \approx 0$:

$$\mathrm{BR}(\tau \to e\, a) < 2.7 \times 10^{-3} \quad (95\%\ \mathrm{CL}) \quad \Rightarrow \quad F_{\tau e} \gtrsim 4.3 \times 10^{6}\ \mathrm{GeV}\,,$$
$$\mathrm{BR}(\tau \to \mu\, a) < 4.5 \times 10^{-3} \quad (95\%\ \mathrm{CL}) \quad \Rightarrow \quad F_{\tau \mu} \gtrsim 3.3 \times 10^{6}\ \mathrm{GeV}\,.$$

Present bounds: $\tau \to e\,a\,, \tau \to \mu\,a$

- NEW! Belle II, Phys.Rev.Lett. 130 (2023)

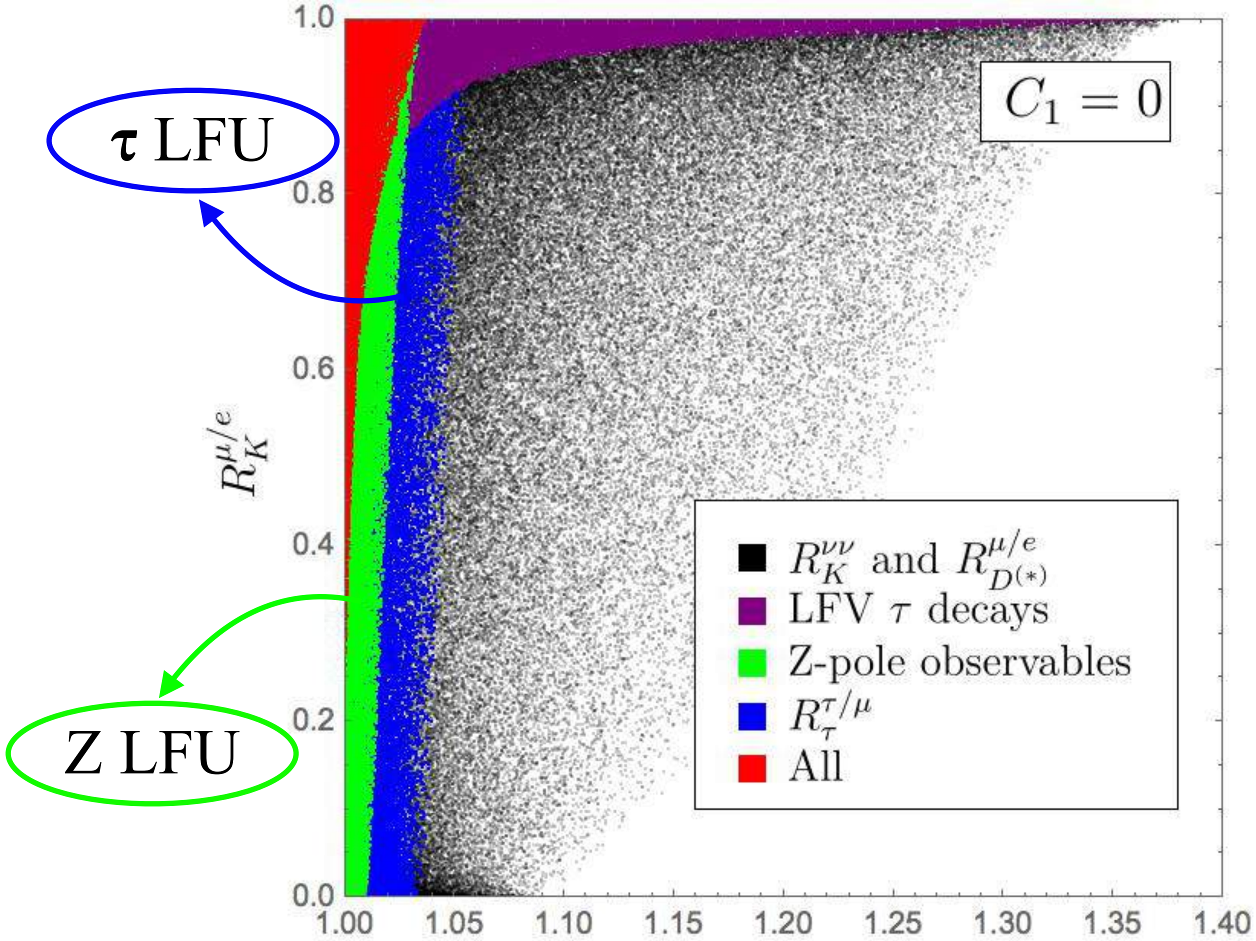


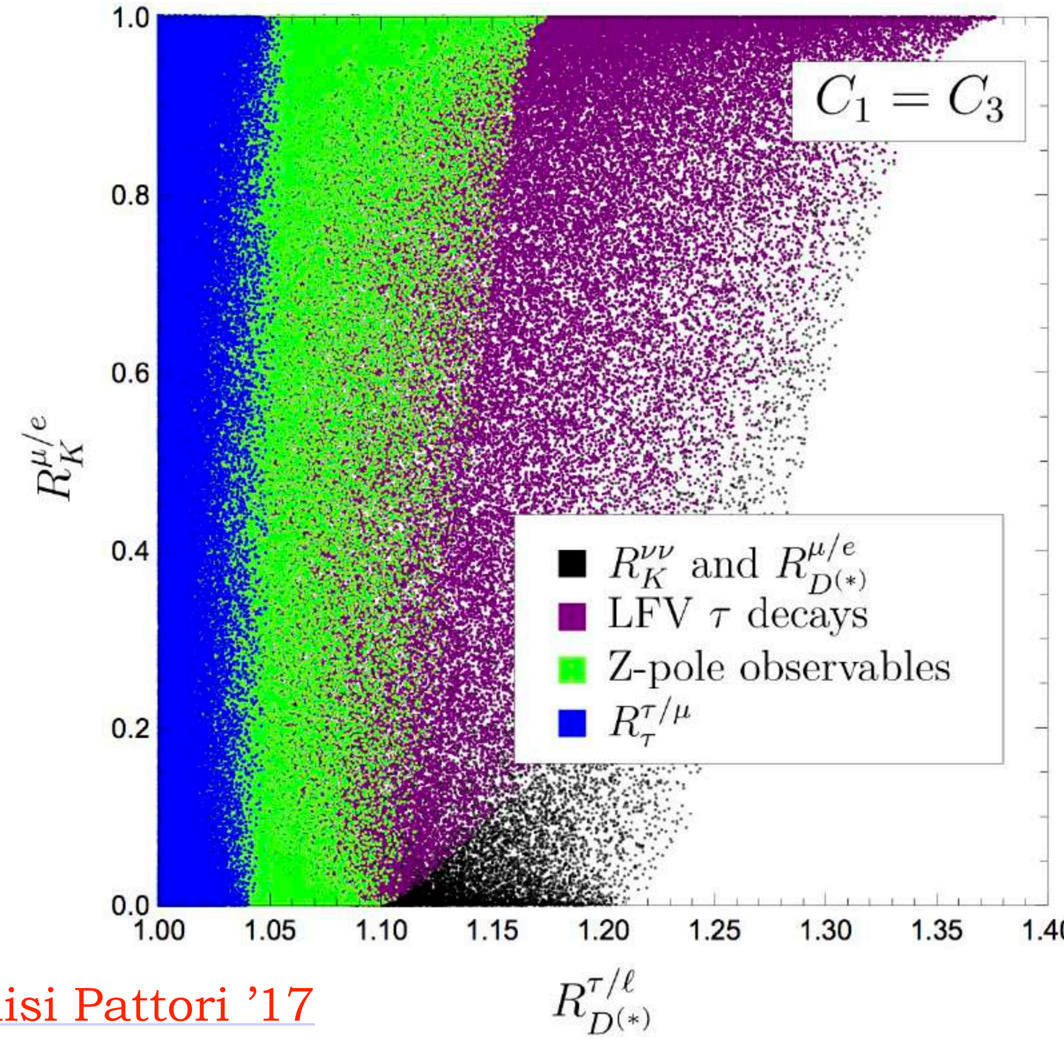
$$m_a \approx 0, \quad \mathrm{BR}(\tau \to \mu a) \simeq 7 \times 10^{-4}, \quad \mathrm{BR}(\tau \to e a) \simeq 9 \times 10^{-4}$$

→ see Li Lingfeng's talk on Friday for the CEPC prospect

Example: new physics inducing operators involving mainly 3rd family fermions

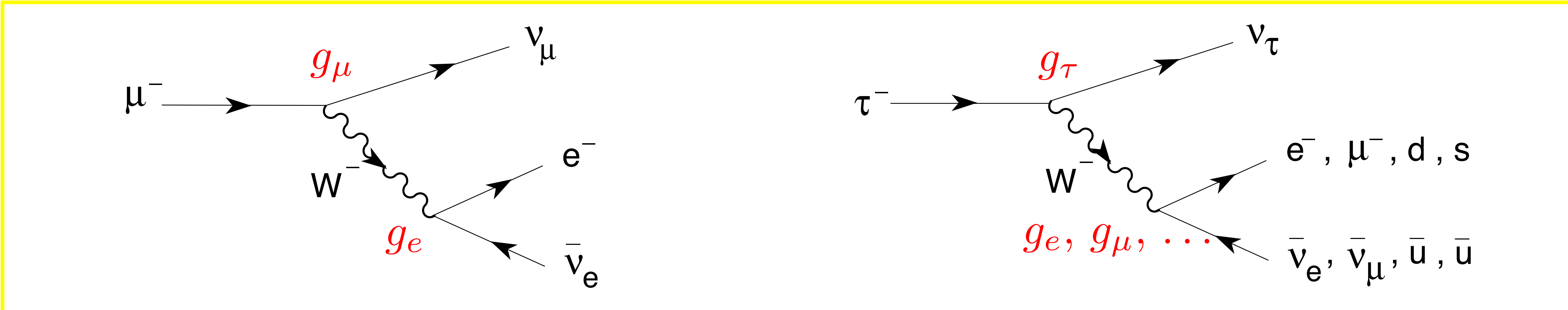
$$Q^{(1)}_{\ell q} = (\bar{L}_3 \gamma^\mu L_3)(\bar{Q}_3 \gamma_\mu Q_3) \ , \quad Q^{(3)}_{\ell q} = (\bar{L}_3 \gamma^\mu \tau_I L_3)(\bar{Q}_3 \gamma_\mu \tau^I Q_3)$$





Feruglio Paradisi Pattori '17

Z and Tau LFU (and LFV) observables are a limiting factor
⇒ crucial test of the B anomalies!
(true also for more general flavour structures)



phase-space factors

radiative corrections

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{\mathrm{BR}(\tau\to\mu\nu\bar\nu)}{\mathrm{BR}(\tau\to e\nu\bar\nu)}\,\frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}\,\frac{R_W^{\tau e}}{R_W^{\tau\mu}}\,,$$

$$\left(\frac{g_\tau}{g_\ell}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{\mathrm{BR}(\tau\to\ell\nu\bar\nu)}{\mathrm{BR}(\mu\to e\nu\bar\nu)}\,\frac{f(m_e^2/m_\mu^2)}{f(m_\ell^2/m_\tau^2)}\,\frac{R_W^{\mu e}R_\gamma^{\mu}}{R_W^{\tau\ell}R_\gamma^{\tau}}\,,\qquad(\ell=e,\mu)$$

Currently LFU tested with per mil level precision:

HFLAV '22: $\left(\frac{g_\mu}{g_e}\right) = 1.0009\pm0.0014\,,\quad \left(\frac{g_\tau}{g_e}\right) = 1.0027\pm0.0014\,,\quad \left(\frac{g_\tau}{g_\mu}\right) = 1.0019\pm0.0014$

→ see A. Lusiani's talk on Friday for the prospect at Tera-Z factories

LFU tests in Z decays

Universality presently tested at the per-mil level

LEP exps/SLD combination: hep-ex:0509008

$$\frac{\mathrm{BR}(Z \to \mu^+\mu^-)}{\mathrm{BR}(Z \to e^+e^-)} = 1.0009 \pm 0.0028\,, \qquad \frac{\mathrm{BR}(Z \to \tau^+\tau^-)}{\mathrm{BR}(Z \to e^+e^-)} = 1.0019 \pm 0.0032$$

(1.7×10^7 Z decays at LEP + 6×10^5 Z decays with polarised beams at SLC)

- Very important test in view of the LFU anomalies in B decays
- At LEP statistical and systematic uncertainties of the same order
- With 10^{12} Z, CEPC has no problem of statistics
- Can systematics be controlled e.g. at the 10^{-4} level?
- This would test new physics coupling preferably to tau up to scales of the order of 10-20 TeV

Summary

CLFV observables among the cleanest and most stringent tests of physics beyond the Standard Model

Still plenty of room also to discover (tau) LFV in Higgs and Z decays (and complementarity with B-factory searches)

ALPs from non-universal global U(1)s (or due to loop effects) give rise to lepton-flavour-violating decays

We have huge room for improvement over old limits: next generation experiments may discover axions in lepton decays!

Leptonic Z and tau decays are crucial tests of LFU, complementary to semileptonic B decays

Thanks! 谢谢!

Additional slides

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*, stems from the Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\ \overline{Q}_{L\,i}\, u_{R\,j}\, \widetilde{\Phi} + (Y_d)_{ij}\ \overline{Q}_{L\,i}\, d_{R\,j}\, \Phi + (Y_e)_{ij}\ \overline{L}_{L\,i}\, e_{R\,j}\, \Phi + h.c.$$

Rotations to the fermion mass basis: $$Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$$

Unitary rotation matrices, couplings to photon and Z remain flavour-diagonal:

$$e\ \bar{f}\gamma_\mu f A^\mu \qquad\qquad (g_L\ \bar{f}_L\gamma_\mu f_L + g_R\ \bar{f}_R\gamma_\mu f_R)Z^\mu$$

Couplings to the Higgs are also flavour-conserving (aligned to the mass matrix):

$$\frac{m_f}{v}\ \bar{f}_L f_R\, h$$

No (tree-level) flavour-changing neutral currents

Why no CLFV in the Standard Model?

In the SM fermion masses, thus the *flavour sector*, stems from the Yukawa interactions:

$$-\mathcal{L}_Y = (Y_u)_{ij}\ \overline{Q}_{L\,i}\, u_{R\,j}\, \widetilde{\Phi} + (Y_d)_{ij}\ \overline{Q}_{L\,i}\, d_{R\,j}\, \Phi + (Y_e)_{ij}\ \overline{L}_{L\,i}\, e_{R\,j}\, \Phi + h.c.$$

Rotations to the fermion mass basis: $Y_f = V_f \hat{Y}_f W_f^\dagger, \qquad f = u, d, e$

Flavour violation occurs in charged currents only:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\overline{u}_L \gamma^\mu (V_u^\dagger V_d) d_L + \overline{\nu}_L \gamma^\mu (V_\nu^\dagger V_e) e_L \right) W_\mu^+ + h.c.$$

$$V_{\rm CKM} \equiv V_u^\dagger V_d \qquad\qquad U_{\rm PMNS} \equiv V_\nu^\dagger V_e$$

However, if neutrinos are massless, we can choose:

$$V_\nu = V_e$$

No LFV (Y_e only 'direction' in the leptonic flavour space)

If NP scale **Λ**≫m_W : $$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \ldots$$

Dimension-6 effective operators that can induce CLFV

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i \overleftrightarrow{D}^I_\mu \Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

Grzadkowski et al. ’10; Crivellin Najjari Rosiek ‘13

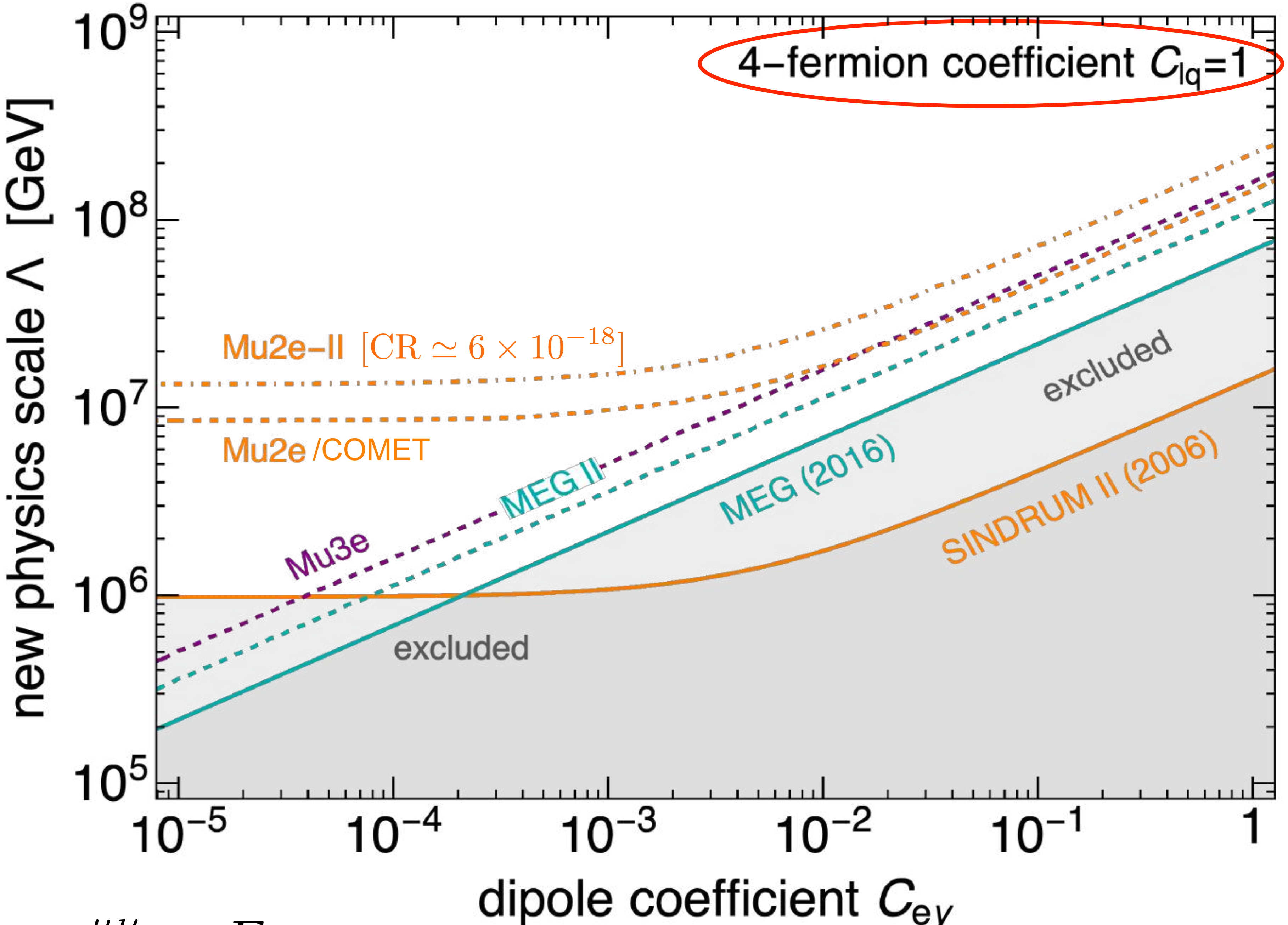
Probing very high-energy scales

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2}\sum_a C_a^{(6)} Q_a^{(6)} + \ldots$$

	$\lvert C_a\rvert$ [$\Lambda = 1$ TeV]	Λ (TeV) [$\lvert C_a\rvert = 1$]	CLFV Process
$C_{e\gamma}^{\mu e}$	2.1×10^{-10}	6.8×10^{4}	$\mu\to e\gamma$
$C_{\ell e}^{\mu\mu\mu e,e\mu\mu\mu}$	1.8×10^{-4}	75	$\mu\to e\gamma$ [1-loop]
$C_{\ell e}^{\mu\tau\tau e,e\tau\tau\mu}$	1.0×10^{-5}	312	$\mu\to e\gamma$ [1-loop]
$C_{e\gamma}^{\mu e}$	4.0×10^{-9}	1.6×10^{4}	$\mu\to eee$
$C_{\ell\ell,ee}^{\mu eee}$	2.3×10^{-5}	207	$\mu\to eee$
$C_{\ell e}^{\mu eee,ee\mu e}$	3.3×10^{-5}	174	$\mu\to eee$
$C_{e\gamma}^{\mu e}$	5.2×10^{-9}	1.4×10^{4}	$\mu^-{\rm Au}\to e^-{\rm Au}$
$C_{\ell q,\ell d,ed}^{e\mu}$	1.8×10^{-6}	745	$\mu^-{\rm Au}\to e^-{\rm Au}$
$C_{eq}^{e\mu}$	9.2×10^{-7}	1.0×10^{3}	$\mu^-{\rm Au}\to e^-{\rm Au}$
$C_{\ell u,eu}^{e\mu}$	2.0×10^{-6}	707	$\mu^-{\rm Au}\to e^-{\rm Au}$
$C_{e\gamma}^{\tau\mu}$	2.7×10^{-6}	610	$\tau\to\mu\gamma$
$C_{e\gamma}^{\tau e}$	2.4×10^{-6}	650	$\tau\to e\gamma$
$C_{\ell\ell,ee}^{\mu\tau\mu\mu}$	7.8×10^{-3}	11.3	$\tau\to\mu\mu\mu$
$C_{\ell e}^{\mu\tau\mu\mu,\mu\mu\mu\tau}$	1.1×10^{-2}	9.5	$\tau\to\mu\mu\mu$
$C_{\ell\ell,ee}^{e\tau ee}$	9.2×10^{-3}	10.4	$\tau\to eee$
$C_{\ell e}^{e\tau ee,eee\tau}$	1.3×10^{-2}	8.8	$\tau\to eee$

Example: dipole *and* 4-fermion operators

$$\frac{C_{\ell q}}{\Lambda^2}\,(\bar{e}_L\,\gamma^\mu\,\mu_L)\,(\overline{Q}\,\gamma_\mu Q)$$



$$\frac{C_{e\gamma}}{\Lambda^2}\,\langle H\rangle\,\bar{e}_L\,\sigma^{\mu\nu}\,\mu_R\,F_{\mu\nu}$$

Mu2e-II Snowmass, arXiv:2203.07569

Z LFV in the SMEFT

The couplings of Z to leptons are protected by the SM gauge symmetry
→ LFV effects must be proportional to the EW breaking:

$$\mathrm{BR}(Z \to \ell\ell') \sim \mathrm{BR}(Z \to \ell\ell) \times C_{\mathrm{NP}}^2 \left(\frac{v}{\Lambda_{\mathrm{NP}}}\right)^4$$

In the SM EFT, only 5 operators contribute at the tree level:

$$Q_{\Phi\ell}^{(1)} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar\ell_L \gamma^\mu \ell'_L), \qquad Q_{\Phi\ell}^{(3)} = (\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi)(\bar\ell_L \tau_I \gamma^\mu \ell'_L), \qquad Q_{\Phi e} = (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar\ell_R \gamma^\mu \ell'_R)$$

$$Q_{eW} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\tau_I \Phi\, W_{\mu\nu}^I, \qquad Q_{eB} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\Phi\, B_{\mu\nu}$$

$$\mathrm{BR}\,(Z \to \ell_i \ell_j) = \frac{m_Z}{12\pi\Gamma_Z}\left\{ \left|g_{VR}\delta_{ij} + \delta g_{VR}^{ij}\right|^2 + \left|g_{VL}\delta_{ij} + \delta g_{VL}^{ij}\right|^2 + \frac{m_Z^2}{2}\left(\left|\delta g_{TR}^{ij}\right|^2 + \left|\delta g_{TL}^{ij}\right|^2\right)\right\}$$

$$\mathcal{L}_{\mathrm{eff}}^Z = \Big[\left(g_{VR}\,\delta_{ij} + \delta g_{VR}^{ij}\right)\ \bar\ell_i \gamma^\mu P_R \ell_j \ + \ \left(g_{VL}\,\delta_{ij} + \delta g_{VL}^{ij}\right)\ \bar\ell_i \gamma^\mu P_L \ell_j\Big] Z_\mu \ +$$
$$\Big[\delta g_{TR}^{ij}\ \bar\ell_i \sigma^{\mu\nu} P_R \ell_j \ + \ g_{TL}^{ij}\ \bar\ell_i \sigma^{\mu\nu} P_L \ell_j\Big] Z_{\mu\nu} \ + \ h.c.\,,$$

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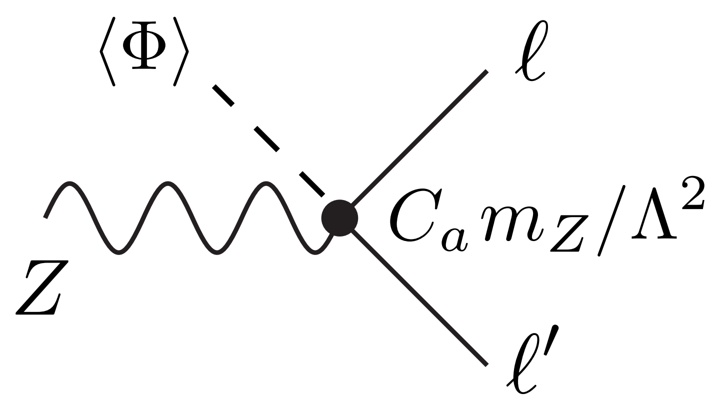
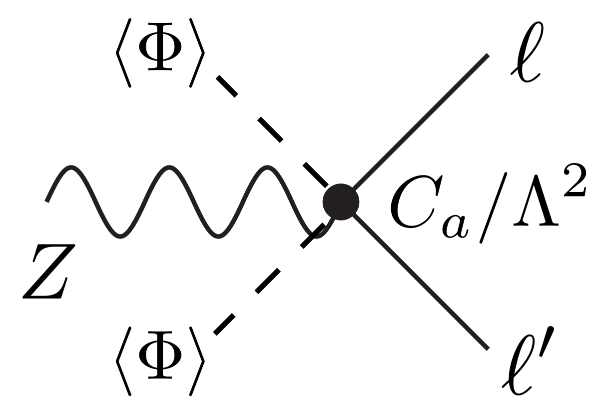
$$\delta g_{VR}^{ij} = -\frac{ev^2}{2s_{\mathrm{w}}c_{\mathrm{w}}\Lambda^2} C_{\varphi e}^{ij}, \quad \delta g_{VL}^{ij} = -\frac{ev^2}{2s_{\mathrm{w}}c_{\mathrm{w}}\Lambda^2}\left(C_{\varphi\ell}^{(1)\,ij} + C_{\varphi\ell}^{(3)\,ij}\right),$$

$$\delta g_{TR}^{ij} = \delta g_{TL}^{ji\,*} = -\frac{v}{\sqrt{2}\Lambda^2}\left(s_{\mathrm{w}} C_{eB}^{ij} + c_{\mathrm{w}} C_{eW}^{ij}\right),$$

Dipole operators:



Higgs-lepton operators:



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$$Q_{eW} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\tau_I \Phi\, W^I_{\mu\nu}, \qquad Q_{eB} = (\bar\ell_L \sigma^{\mu\nu} \ell'_R)\Phi\, B_{\mu\nu}$$

If a single operator dominates, $Z \to \ell\ell'$ constrain NP scales up to

$$C_a = 1: \quad \Lambda \gtrsim 5 \text{ TeV} \quad (\mathrm{Z} \to \mu \mathrm{e}), \quad \Lambda \gtrsim 3 \text{ TeV} \quad (\mathrm{Z} \to \tau\ell)$$

$$\delta g^{ij}_{VR} = -\frac{ev^2}{2s_{\rm w}c_{\rm w}\Lambda^2} C^{ij}_{\varphi e}, \quad \delta g^{ij}_{VL} = -\frac{ev^2}{2s_{\rm w}c_{\rm w}\Lambda^2}\left(C^{(1)\,ij}_{\varphi\ell} + C^{(3)\,ij}_{\varphi\ell}\right),$$

$$\delta g^{ij}_{TR} = \delta g^{ji\,*}_{TL} = -\frac{v}{\sqrt{2}\Lambda^2}\left(s_{\rm w} C^{ij}_{eB} + c_{\rm w} C^{ij}_{eW}\right),$$

SMEFT semileptonic operators

SU(2)-invariant operators ('SMEFT'):

Differ by SU(2) contractions:

$$(Q_{\ell q}^{(1)})_{\mu\mu bs} = (\bar{L}_{L\,2}^a \gamma^\mu L_{L\,2}^a)(\bar{Q}_{L\,2}^b \gamma_\mu Q_{L\,3}^b)$$

"singlet-singlet"

$$(Q_{\ell q}^{(3)})_{\mu\mu bs} = \sum_{I=1,3} (\bar{L}_{L\,2}^a \gamma^\mu (\tau_I)_{ab} L_{L\,2}^b)(\bar{Q}_{L\,2}^c \gamma_\mu (\tau_I)_{cd} Q_{L\,3}^d)$$

"triplet-triplet"

They both give C_9=-C_{10}

it gives also rise to charged-current, it can address also $R_{D(*)}$

One can attempt to explain class 1 and 2 anomalies simultaneously

Relevant constraints from $B \to K^{(*)} \nu\bar{\nu}$ which can be however relaxed if C_S=C_T

Alonso Grinstein Camalich '15
LC Crivellin Ota '15

Simultaneous explanation of the anomalies

Ops with only 3[rd] family: $Q_{\ell q}^{(1)} = (\bar{L}_3\gamma^\mu L_3)(\bar{Q}_3\gamma_\mu Q_3)$, $Q_{\ell q}^{(3)} = (\bar{L}_3\gamma^\mu \tau_I L_3)(\bar{Q}_3\gamma_\mu \tau^I Q_3)$

(in the interaction basis)

Flavour structure justified by:

- Theoretical considerations (SM hierarchies, MFV paradigm, ...)
- Observed anomalies (3rd generation affected more than 2nd generation, 2nd generation more than 1st generation)

Glashow Guadagnoli Lane '14, Bhattacharya et al. '14, LC Crivellin Ota '15, Feruglio Paradisi Pattori '16,'17 ...

Operators involving 2nd generations generated by rotations to the mass basis:

$$Y^f = V^{f\dagger}\hat{Y}^f W^f, \qquad f = u, d, e$$

Giving e.g. :

$$C_S(\bar{L}_3\gamma^\mu L_3)(\bar{Q}_3\gamma_\mu Q_3) \longrightarrow C_S V_{23}^d V_{33}^{d\,*}|V_{23}^e|^2\ (\bar{L}_2\gamma^\mu L_2)(\bar{Q}_2\gamma_\mu Q_3)$$

$\Rightarrow$ $b \to s\mu\mu$ $\qquad$ $V_{23}^d V_{33}^{d\,*} \sim V_{cb} \times V_{tb}$

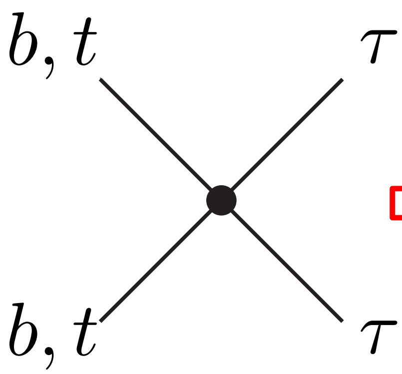
LFU in both NC and CC B decays are induced. However…

Radiatively generated LFV and LFUV effects

Ops with only 3rd family: $Q_{\ell q}^{(1)} = (\bar{L}_3 \gamma^\mu L_3)(\bar{Q}_3 \gamma_\mu Q_3)$, $Q_{\ell q}^{(3)} = (\bar{L}_3 \gamma^\mu \tau_I L_3)(\bar{Q}_3 \gamma_\mu \tau^I Q_3)$

Important radiative effects:

Feruglio Paradisi Pattori ’16 & ’17



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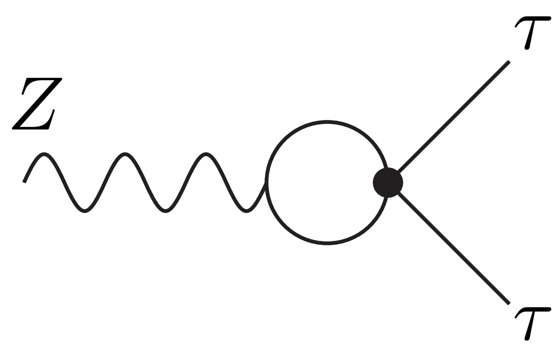
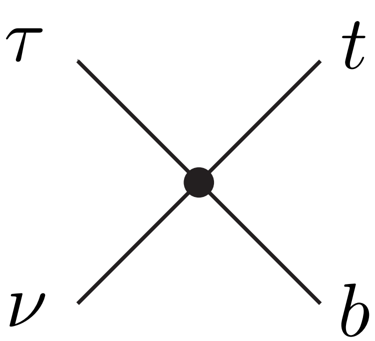


: $$\frac{\mathrm{BR}(Z \to \tau\tau)}{\mathrm{BR}(Z \to ee)}$$

(LFU in Z couplings tested at the permil level)



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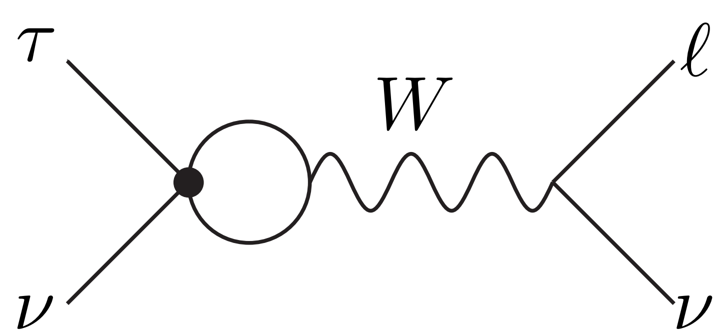
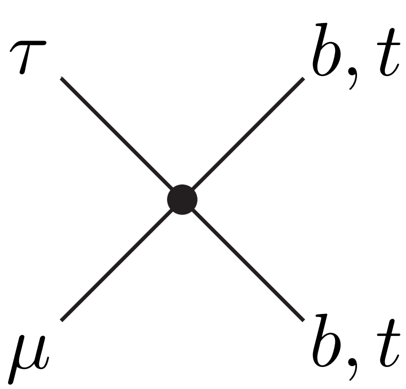


: $$\frac{\mathrm{BR}(\tau \to \ell\nu\bar{\nu})}{\mathrm{BR}(\mu \to e\nu\bar{\nu})}$$

(LFU in tau decays tested below the percent level)



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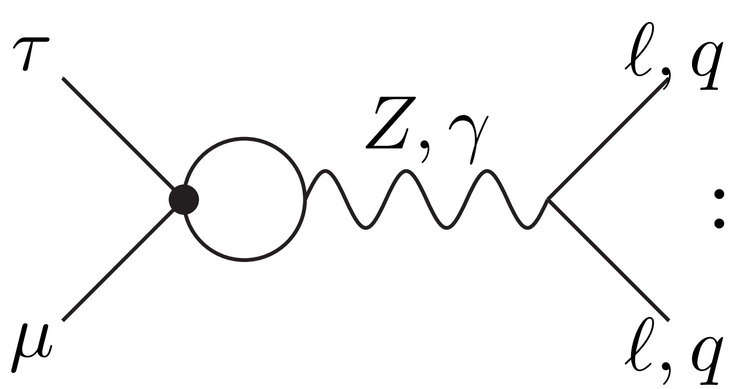

: $\tau \to \mu\ell\ell$ $\quad \tau \to \mu\pi$ $\quad \tau \to \mu\rho$

Tau CLFV!