Production of Doubly Heavy Hadrons at the Z pole

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Outline

1. **Background（Bc, doubly heavy baryons）**

2. **Production at e^+e^- colliders**

3. **Fragmentation functions**

4. **Conclusions**

1. Background for Bc ($c\bar{b}$)

➢ Only meson state with two different heavy flavors

- Only weak decay is possible => weak interaction

➢ Its production can be described by NRQCD factorization

- A lot of the dynamics can be calculated perturbatively
- The production mechanism of Bc is simpler than that of heavy quarkonium

➢ It was first observed by CDF collaboration in 1998

(u,d,s-1963; c-1974; b-1977; t-1995)

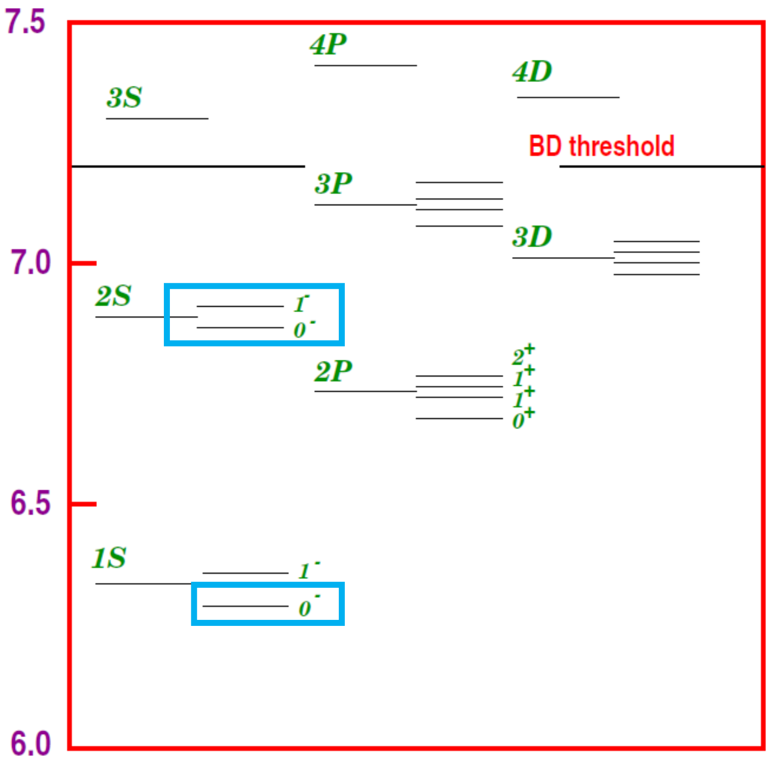
1. Background for Bc

➢ Many Bc excited states have not been observed experimentally

Bc(2S) and Bc*(2S) were observed in 2019

PRL122,132001(2019,CMS); PRL122,232001(2019,LHCb)

Excited Bc states



The mass spectrum of $c\bar{b}$ states

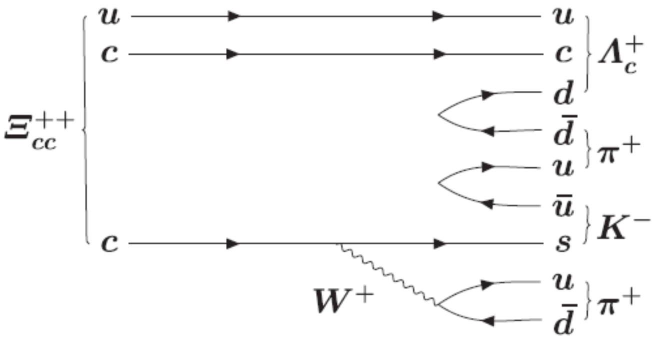
1. Background for doubly heavy baryons

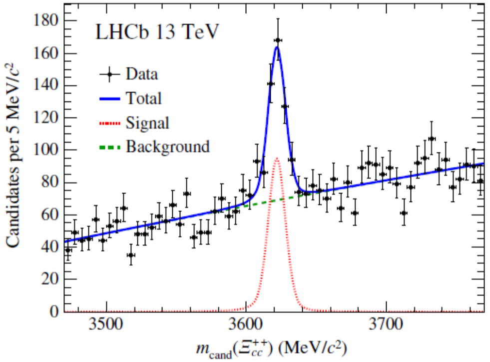
- **They provide a good platform for studying strong and weak interactions**

  Decay => weak interaction

  Production=> strong interaction=>pQCD, NRQCD

- **Ξ_{cc} was first observed by LHCb collaboration in 2017**





LHCb Collabration,PRL119, 112001 (2017)

2. Production at e^+e^- colliders

➢ Advantages of the production at e^+e^- colliders

- **The center-of-mass system of the process is known**

  Angle distributions and forward-backward asymmetry of doubly heavy hadrons have proper meaning in understanding the production.

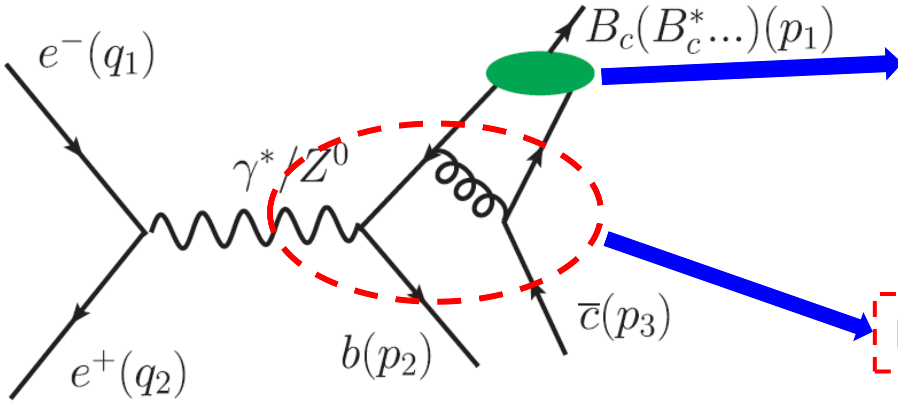
- **There are less backgrounds at an e^+e^- collider**

  A good platform for precision measurements.

➢ Running at the Z pole

- **Z-resonance effect**
- **CEPC**

➢ LO calculation for Bc



Universal nonperturbative factor

Perturbatively calculable

$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c})\langle O^{Bc}(n)\rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

B-factories cannot produce the Bc meson because the beam energy is not enough for the Bc production.

Numerical results

Phys. Rev. D 93, 034019, (2016),
X.-C. Zheng, C.-H. Chang et al.

States	σ(**pb**)	Events/year
$B_c\ (^1S_0\ )$	2.73	2.7×10^6
$B_c^*(\,^3S_1\ )$	3.82	3.8×10^6
$B_c^{**}(\,^1P_1\ )$	0.27	2.7×10^5
$B_c^{**}(\,^3P_1\ )$	0.16	1.6×10^5
$B_c^{**}(\,^3P_2\ )$	0.34	3.4×10^5
$B_c^{**}(\,^3P_2\ )$	0.37	3.7×10^5

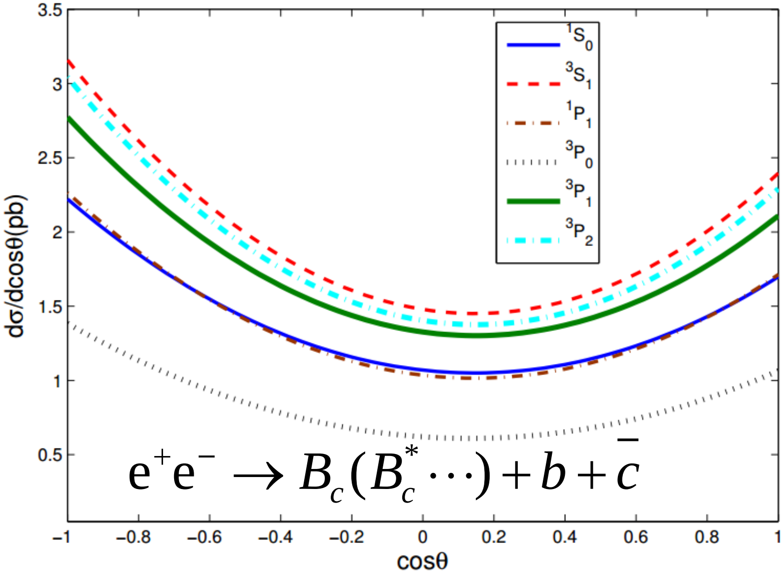
Cross sections at **the Z pole**
with $\mathrm{L}=10^{35}cm^{-2}s^{-1}$

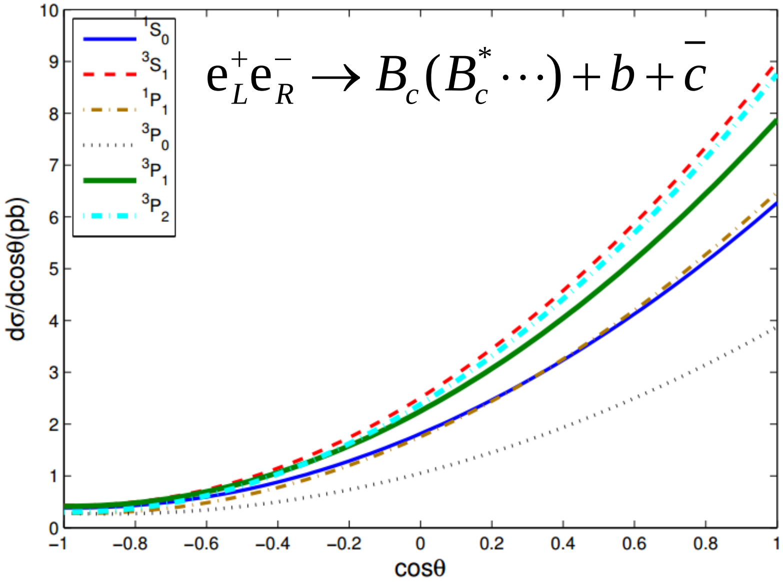
States	σ(**fb**)	Events/year
$B_c\ (^1S_0\ )$	0.47	4.7×10^2
$B_c^*(\,^3S_1\ )$	0.72	7.2×10^2
$B_c^{**}(\,^1P_1\ )$	0.05	50
$B_c^{**}(\,^3P_1\ )$	0.03	30
$B_c^{**}(\,^3P_2\ )$	0.07	70
$B_c^{**}(\,^3P_2\ )$	0.07	70

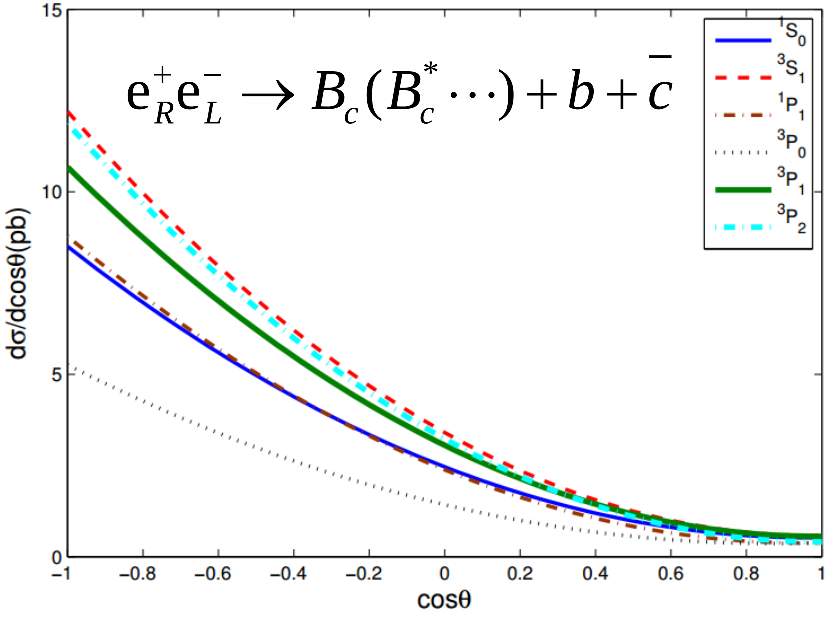
Cross sections at $\sqrt{s}=250GeV$
with $\mathrm{L}=10^{35}cm^{-2}s^{-1}$

- **The Z-resonance effect is important for studying Bc and its excited states**
- **The luminosity of the e⁺e⁻ collider should be $\mathbf{10^{35-36}cm^{-2}s^{-1}}$**

Production at e^+e^- colliders



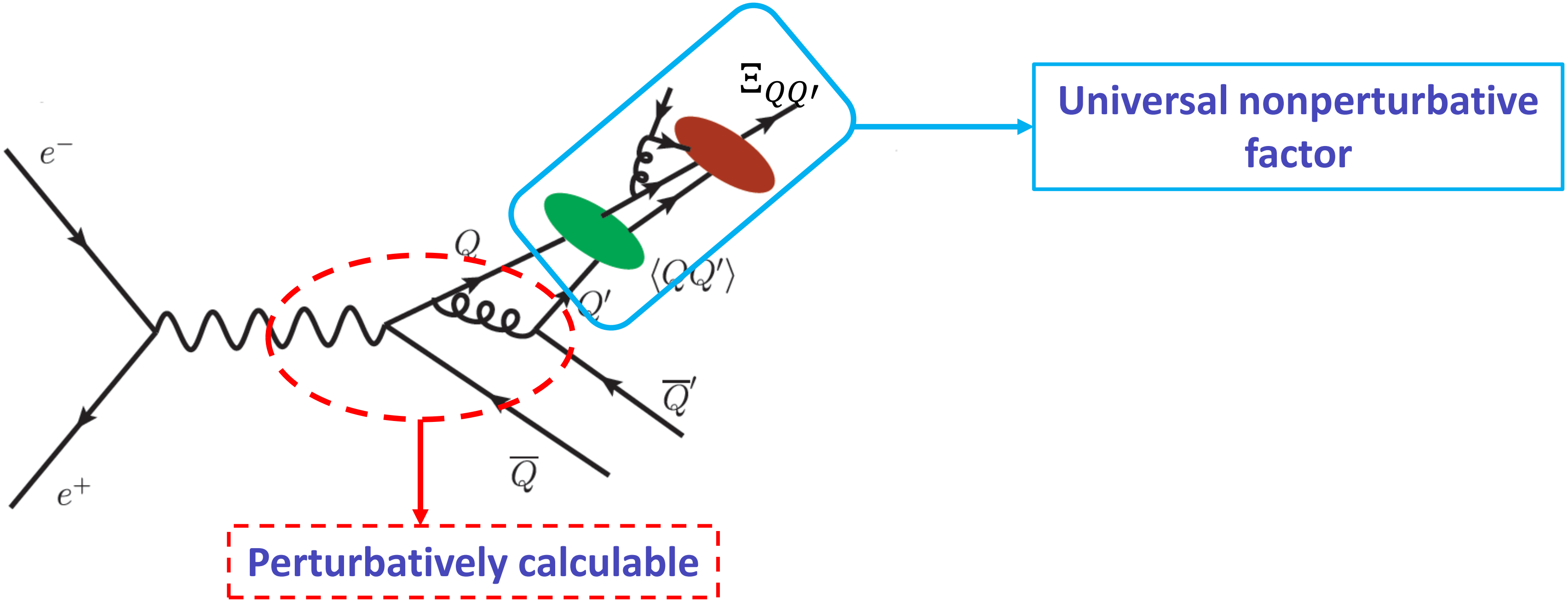




The angle distributions are forward-backward asymmetric.

Production at e^+e^- colliders

Production of doubly heavy baryons



Universal nonperturbative factor

Perturbatively calculable

1) Production of diquark (in color $\overline{3}$ state)

The calculation is similar to the Bc case

2) The diquark fragments into the doubly heavy baryon

$\delta(1-z)$, Peterson model (a diquark to a doubly heavy baryon **is similar to a heavy quark to a heavy meson**)

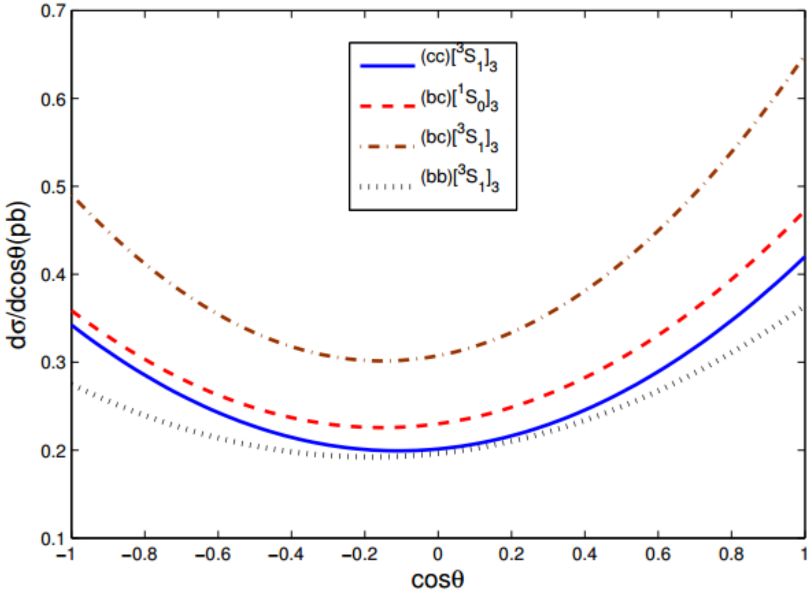
Production at e^+e^- colliders

➢ Production of doubly heavy baryons

Phys. Rev. D 93, 034019, (2016), X.-C. Zheng, C.-H. Chang et al.

States	σ(**pb**)	Events/year
Ξ_{cc}	0.52	5.2×10^5
Ξ_{bc}	1.37	1.4×10^6
Ξ_{bb}	0.05	5.0×10^4

Cross sections at **the Z pole**
with $\mathrm{L}=10^{35}cm^{-2}s^{-1}$



Differential angle distribution

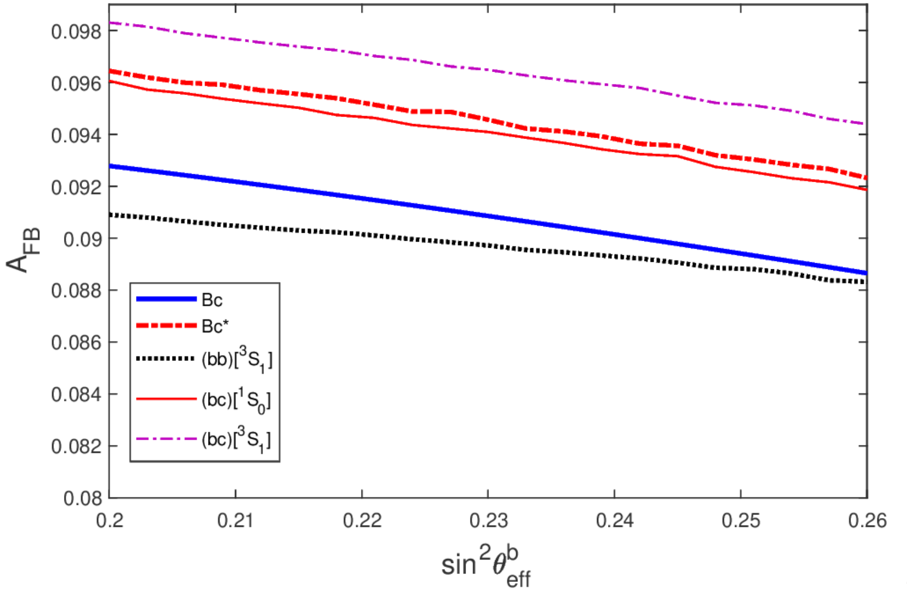
The angle distributions are also forward-backward asymmetric.

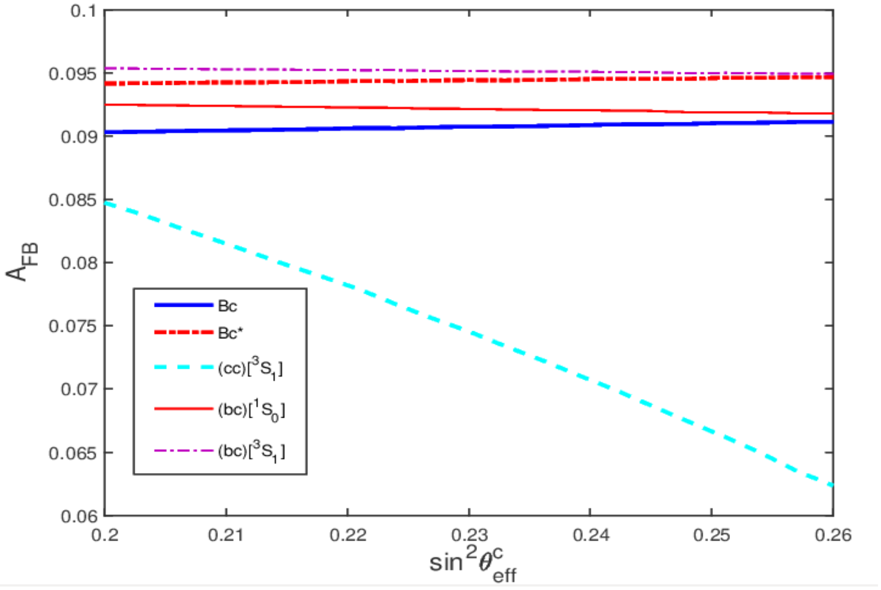
Production at e^+e^- colliders

Forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}.$$

Sci. China-Phys.Mech. Astron. 63, 281011,(2020), X.-C. Zheng, C.-H. Chang et al.





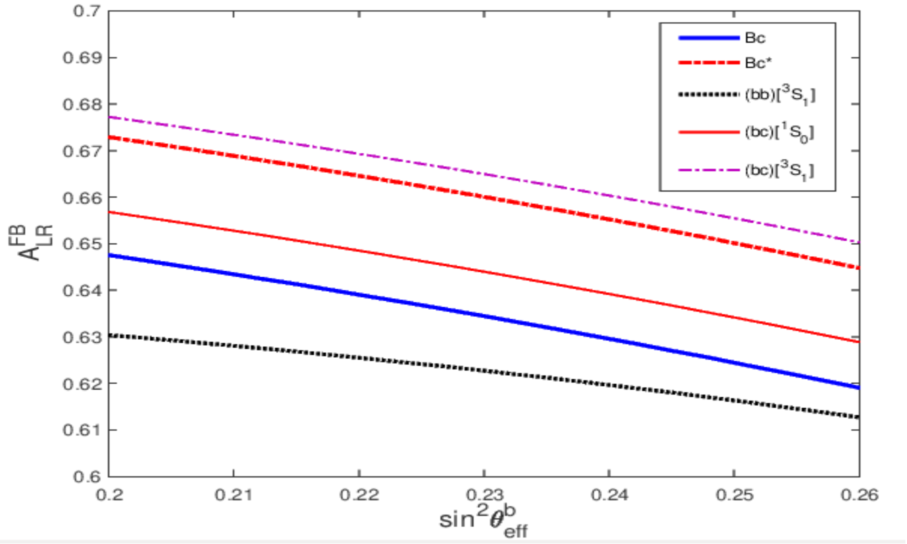
$\sin^2\theta^f_{eff}$ can be determined through measuring the forward-backward asymmetry of the doubly heavy-flavored hadrons.

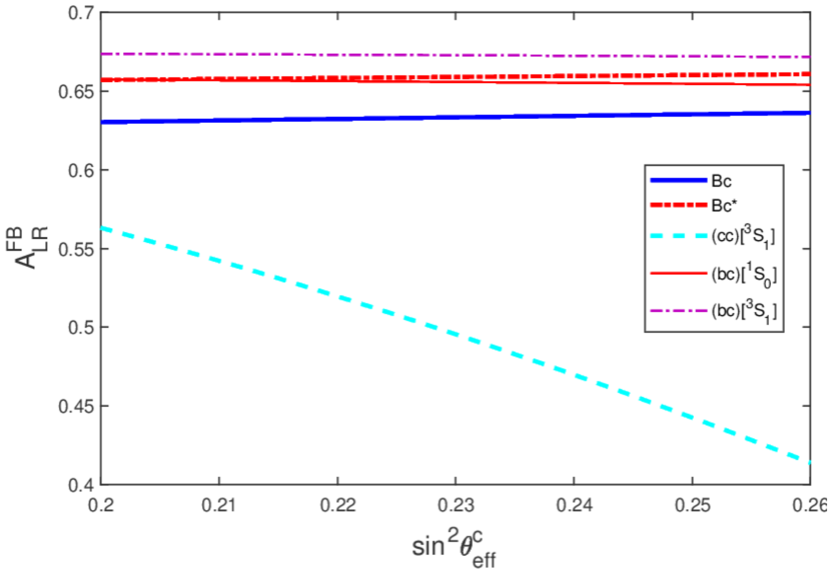
Production at e^+e^- colliders

Left-right forward-backward asymmetry:

$$A_{LR}^{FB} = \frac{\sigma_{LF} - \sigma_{LB} - \sigma_{RF} + \sigma_{RB}}{\sigma_{LF} + \sigma_{LB} + \sigma_{RF} + \sigma_{RB}}.$$

Sci. China-Phys.Mech. Astron. 63, 281011,(2020),
X.-C. Zheng, C.-H. Chang et al.



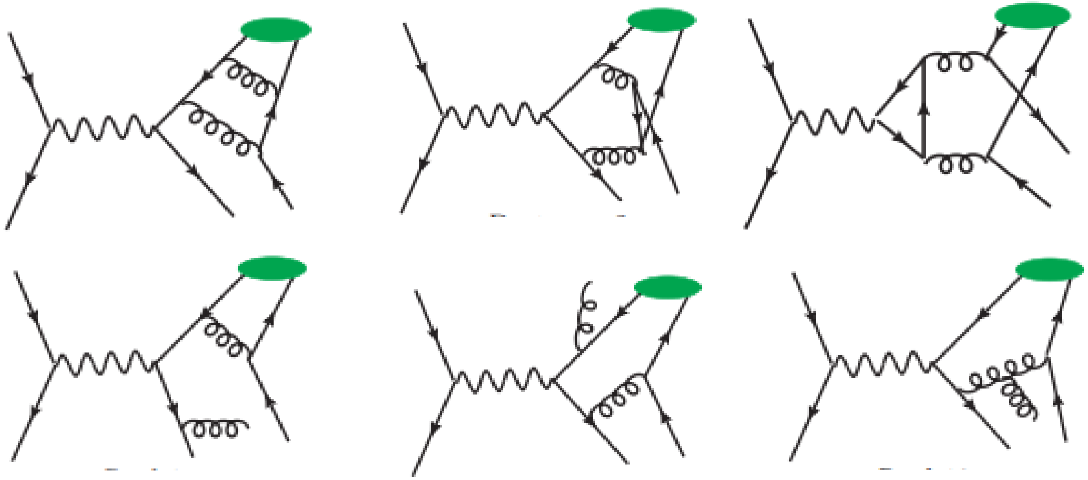


$\sin^2\theta_{eff}^{f}$ can be determined through measuring the left-right-forward-backward asymmetry of the doubly heavy-flavored hadrons.

Production at e^+e^- colliders

➢ NLO calculations for Bc and Bc*

- To see the **changes** of the physical observables from the **LO** calculations to the **NLO** calculations.
- To see how the dependence on the **renormalization scale** changes after including the NLO QCD corrections.



84 Feynman diagrams for the virtual correction, 24 Feynman diagrams for the real correction.

Numerical results

Sci. China-Phys.Mech. Astron. 61, 031012,(2018), X.-C. Zheng, C.-H. Chang et al.

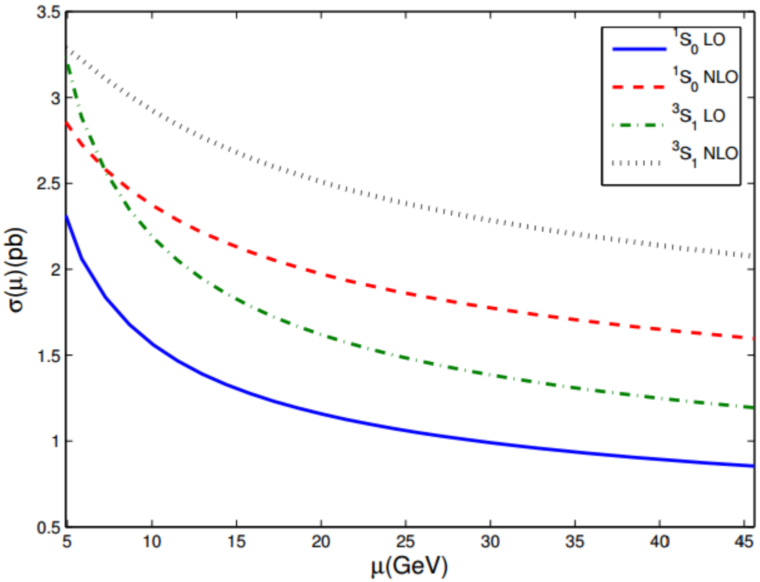
μ	$\alpha_s(\mu)$	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	0.180	1.58	2.38	1.51
$m_Z/2$	0.132	0.85	1.58	1.86

Cross section of Bc

μ	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	2.20	2.93	1.33
$m_Z/2$	1.18	2.06	1.74

Cross section of Bc*

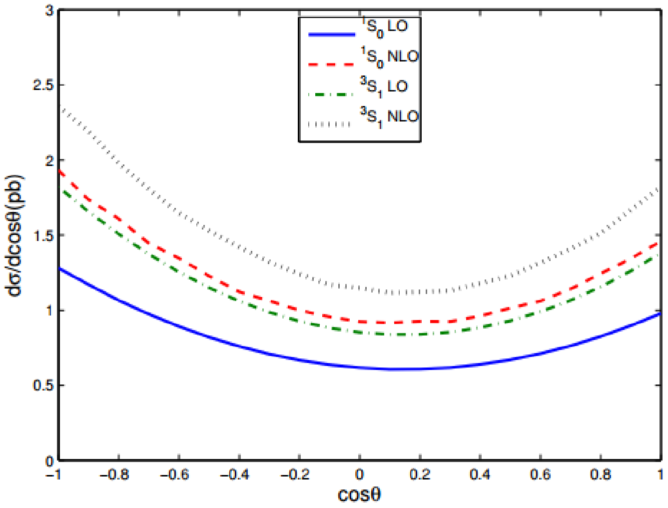
The NLO corrections are significant!



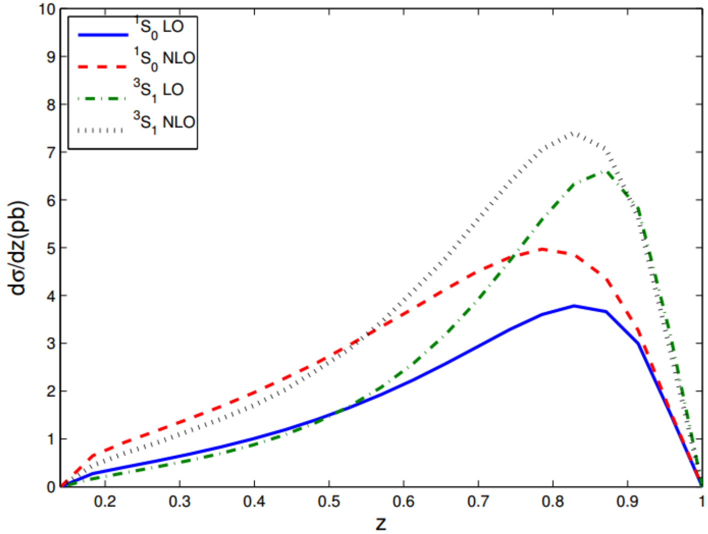
The dependence on μ is weaken significantly due to NLO corrections.

Production at e^+e^- colliders

Sci. China-Phys.Mech. Astron. 61, 031012,(2018),
X.-C. Zheng, C.-H. Chang et al.



Differential angle distribution



Differential energy distribution

- The K-factor changes very little with different θ;
- The NLO corrections change the energy distribution significantly.

3. Fragmentation functions

- NRQCD factorization

$$
\begin{aligned}
&d\sigma(e^+ + e^- \to Bc + b + \bar{c}) \\
&= \sum\nolimits_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c}) \left\langle O^{\mathrm{Bc}}(n) \right\rangle
\end{aligned}
$$

- Fragmentation approach

$$
\begin{aligned}
&d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c}) \\
&= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i \to Bc}(z, \mu_F) + \mathrm{O}(m_Q^2/s)
\end{aligned}
$$

Fragmentation function

Partonic production cross section

- The production of the Bc meson is dominated by the fragmentation mechanism when $s \gg m_Q^2$

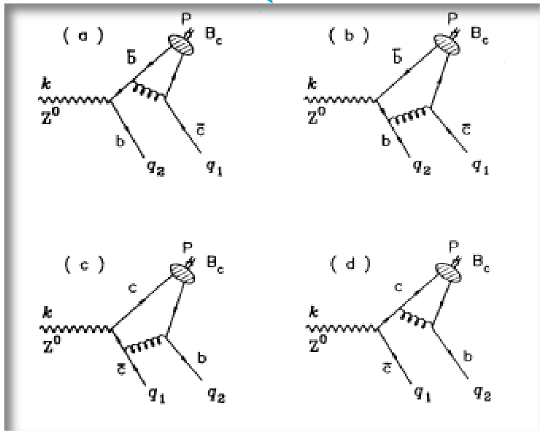
Fragmentation functions

- **LO fragmentation functions** for the Bc production
  - Extracting from the LO calculation of process $Z^0 \to Bc + b + \overline{c}$

    C.-H. Chang, Y.-Q. Chen, Phys. Rev. D 46, 3845, (1992);

  - Calculating from the definition:

    J.-P. Ma, Phys. Lett. B 332, 398, (1994);



◆ There were no NLO results for $D_{i\to Bc}(z,\mu_F)$ before our calculation.

◆ In order to obtain the theoretical predictions under the fragmentation approach up to NLO QCD accuracy, the NLO results for $D_{i\to Bc}(z,\mu_F)$ are needed.

Fragmentation functions

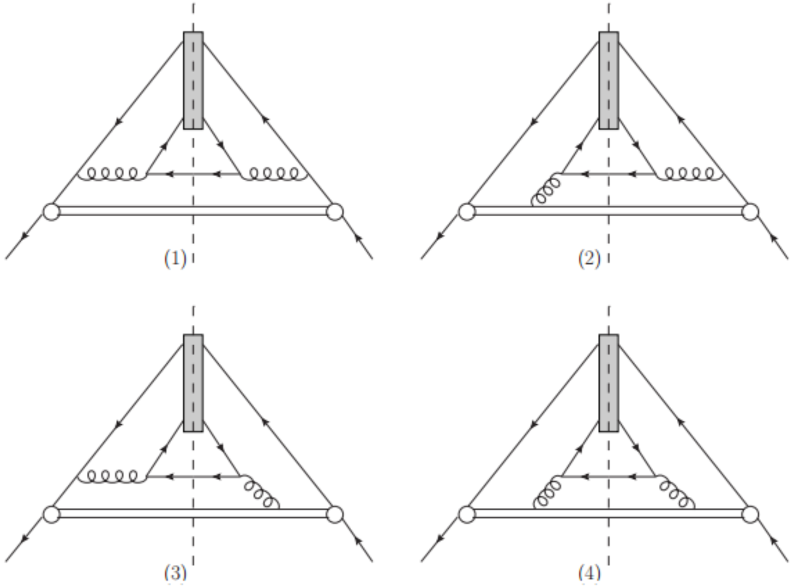
Fragmentation function calculation

Based on the definition of FFs by **Collins and Soper.**

Nucl. Phys. B 194, 445, (1982).

Process independent approach

LO cut diagrams:



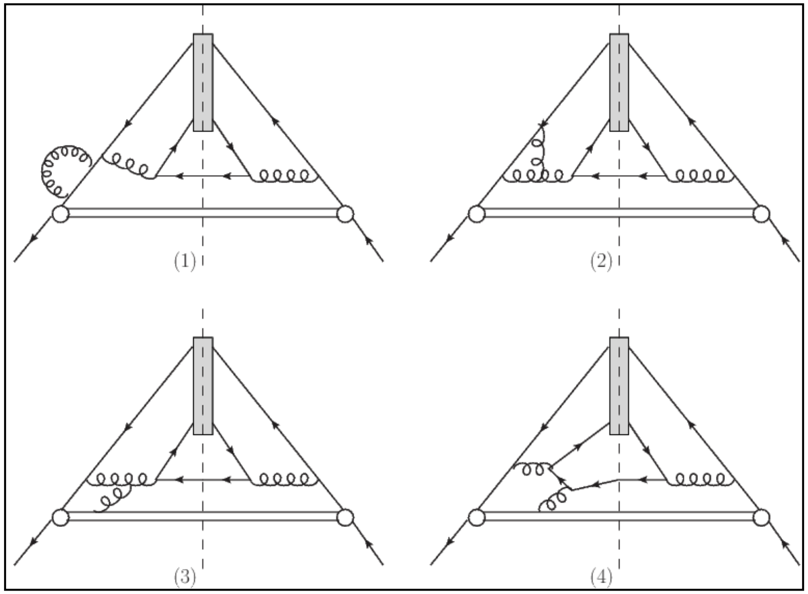
LO fragmentation functions:

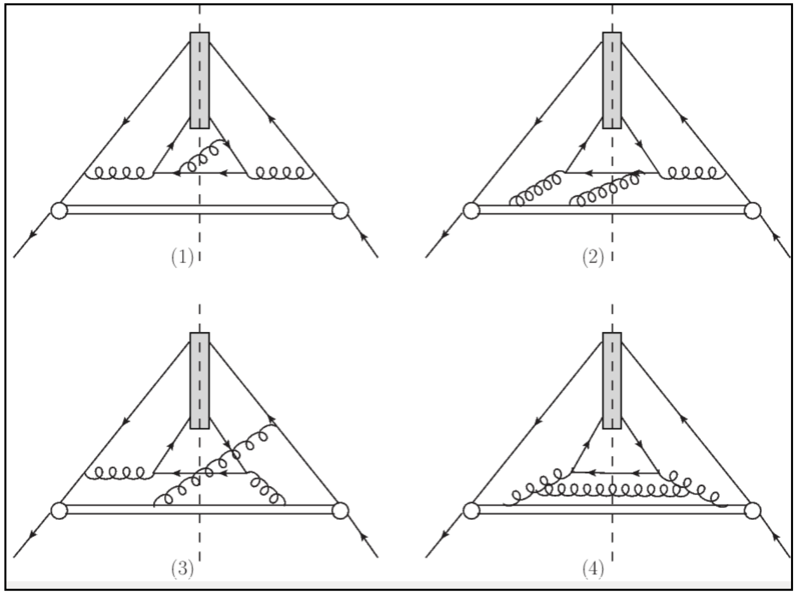
$$
\begin{aligned}
&D^{\rm LO}_{\bar{b}\to B_c}(z)\\
&=\frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{81\pi r_c^2(1-r_b z)^6 M^3}[6-18(1-2r_c)z+(21-74r_c+68r_c^2)z^2\\
&\quad -2r_b(6-19r_c+18r_c^2)z^3+3r_b^2(1-2r_c+2r_c^2)z^4],
\end{aligned}
$$

$$
\begin{aligned}
&D^{\rm LO}_{\bar{b}\to B_c^*}(z)\\
&=\frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{27\pi r_c^2(1-r_b z)^6 M^3}[2-2(3-2r_c)z+3(3-2r_c+4r_c^2)z^2\\
&\quad -2r_b(4-r_c+2r_c^2)z^3+r_b^2(3-2r_c+2r_c^2)z^4].
\end{aligned}
$$

NLO corrections

Sample NLO cut diagrams





54 virtual cut diagrams， 72 real cut diagrams.

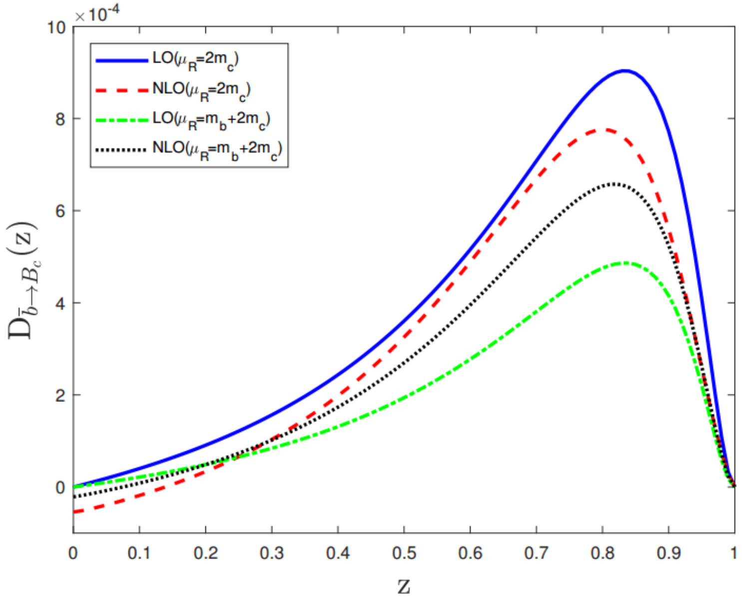
最困难的是计算实修正，实修正既有紫外发散，又有红外发散。需要解析计算这些发散的积分，由于末态3个粒子，导致相空间积分异常复杂！

Fragmentation functions

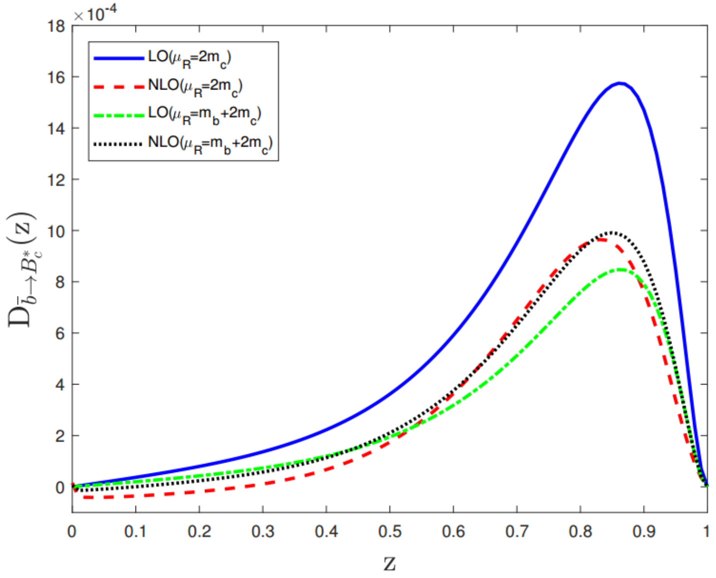
NLO results

Phys. Rev. D 100, 034004, (2019), X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$



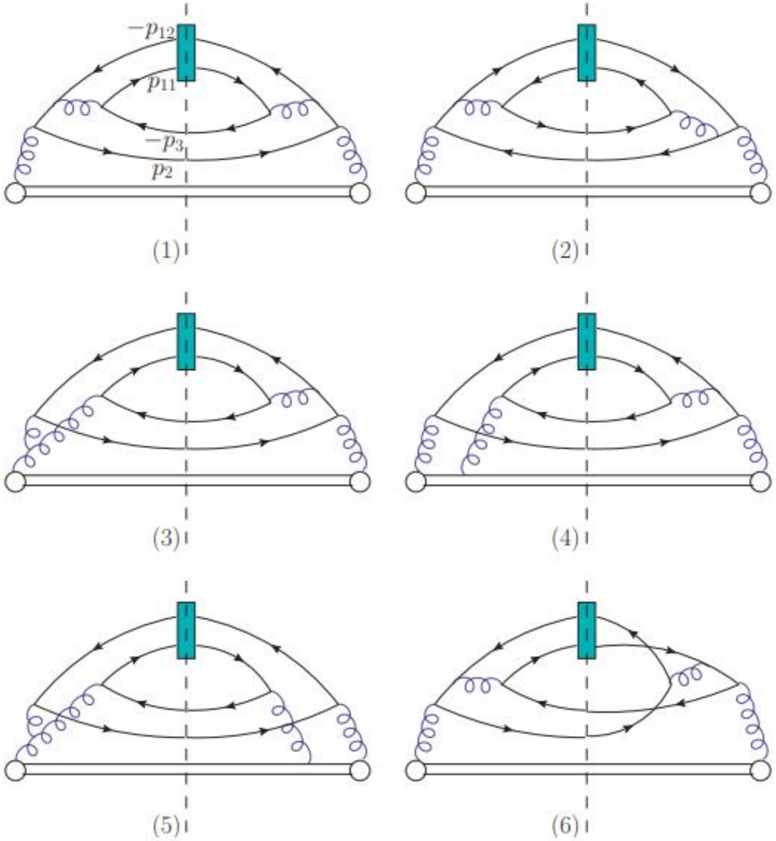
$$D_{\bar{b}\to Bc}(z,\mu_F = m_b + 2m_c)$$



$$D_{\bar{b}\to B_c^*}(z,\mu_F = m_b + 2m_c)$$

Fragmentation functions

g->Bc(Bc*) FFs



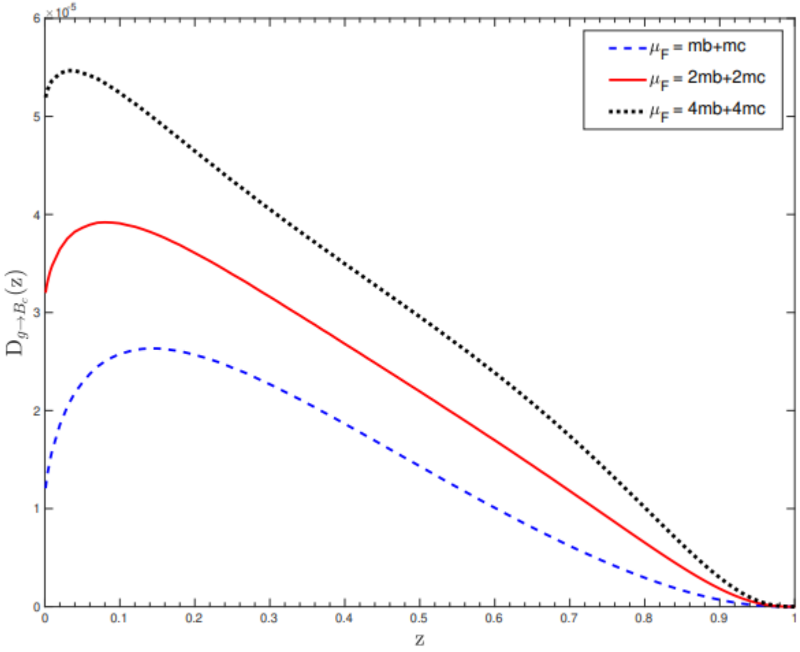
末态有三个粒子，
且相空间积分存
在紫外发散。

Six of the 49 cut diagrams

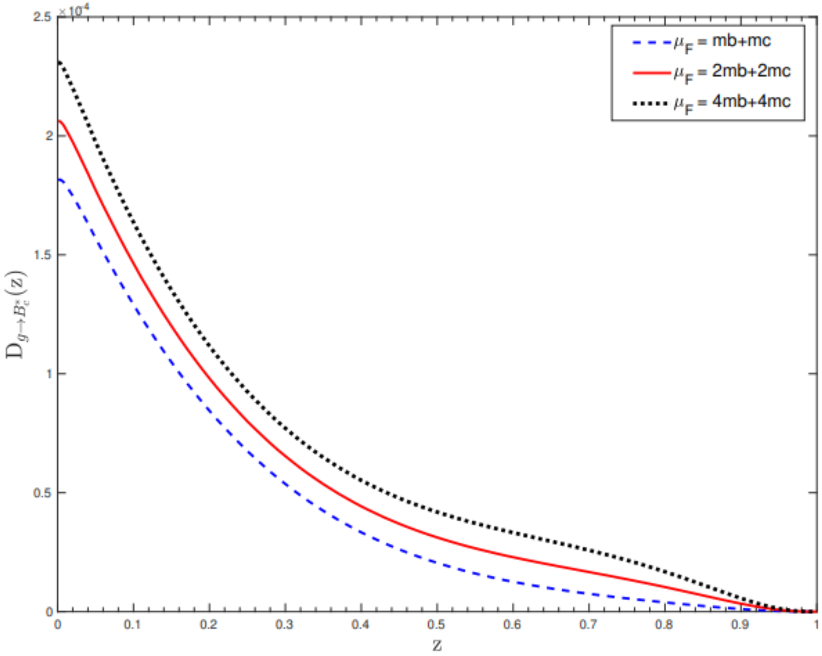
Fragmentation functions

g->Bc(Bc*) FFs

JHEP 05, 036, (2022),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.



g->Bc FFs



g->Bc* FFs

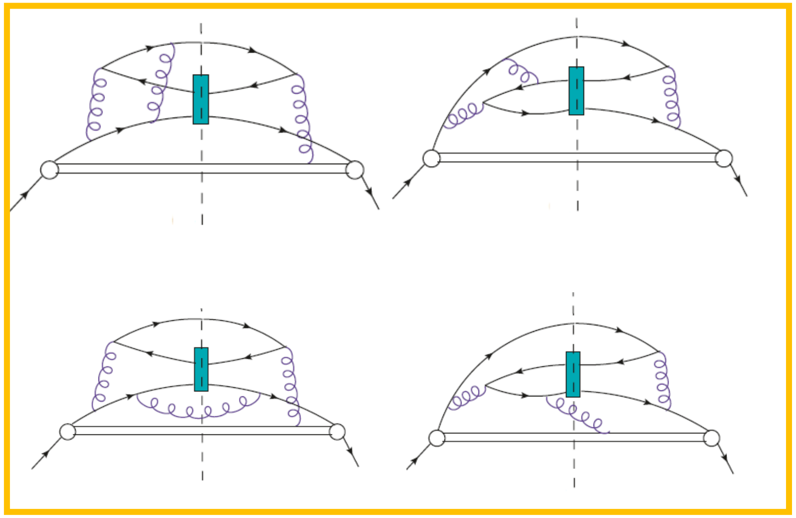
Fragmentation functions

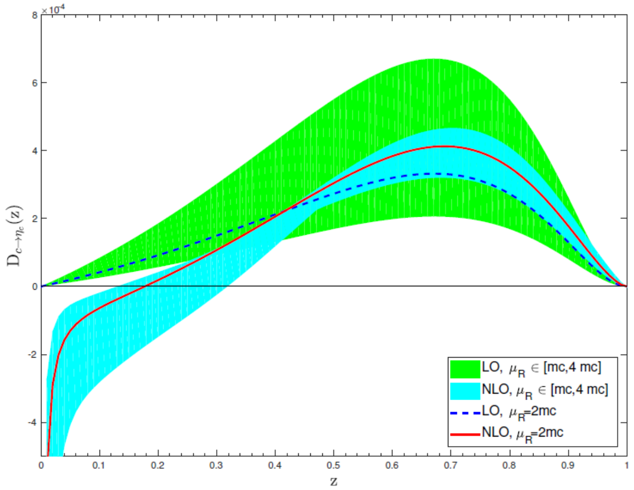
Heavy quarkonium FFs

$c(b) \to J/\psi(\Upsilon)$ NLO FFs, X.C. Zheng et al, PRD 100, 014005, (2019) .

$q \to \eta_Q \ (q \neq Q)$ FFs, X.C. Zheng et al, PRD 103, 074004, (2021) .

$c(b) \to \eta_c \ (\eta_b)$ NLO FFs, X.C. Zheng et al, JHEP 07, 014, (2021) .





These fragmentation functions can be studied at the CEPC.

Conclusions

- An e^+e^- collider running around the Z pole can provide a good platform for studying the properties of doubly heavy hadrons;
- The NLO fragmentation functions for a quark or gluon into a doubly heavy meson (Bc, J/ψ, Υ, η_c, η_b) have been obtained;
- These fragmentation functions can be studied at the CEPC running at the Z pole.

Thank you！

- **Virtual corrections**

Tensor reduction, IBP reduction

Many integrals containing an eikonal line, e.g,

$$\int \frac{d^D l}{[(l-p_1)^2-m_1^2+i\varepsilon][(l-p_2)^2-m_2^2+i\varepsilon][(l-p_3)^2-m_3^2+i\varepsilon](l\cdot n+i\varepsilon)}$$

- **Real corrections**

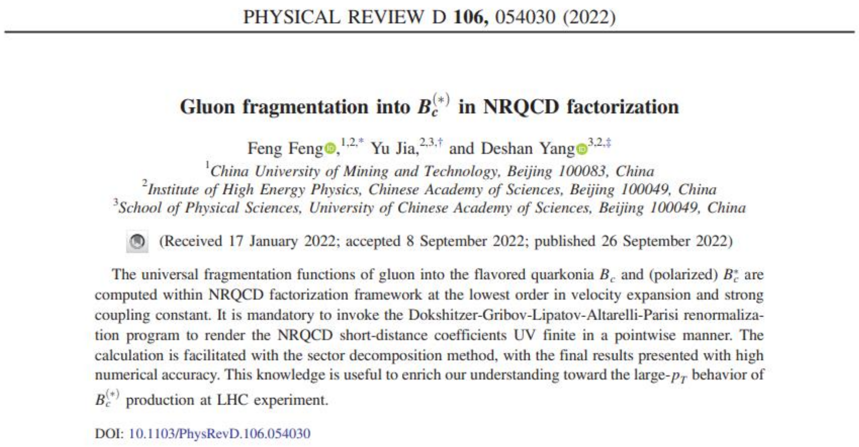
UV and IR divergences!

$$\mathrm{D}^{real}_{\bar{b}\to Bc}(z)=\int N_{CS}d\phi_{real}(A_{real}-A_S)+\int N_{CS}d\phi_{real}A_S$$

Calculated in
4 dimensions

Calculated in
d dimensions

Various types of subtraction terms need to be integrated!



PHYSICAL REVIEW D **106,** 054030 (2022)

Gluon fragmentation into $B_c^{(*)}$ in NRQCD factorization

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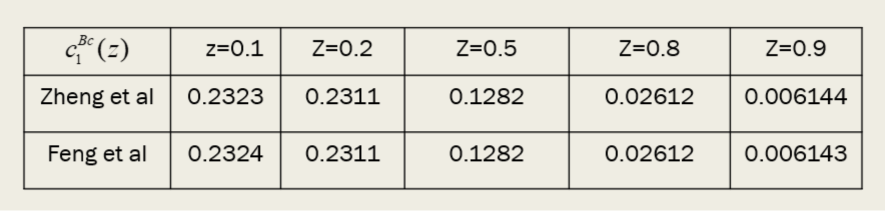
[3]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

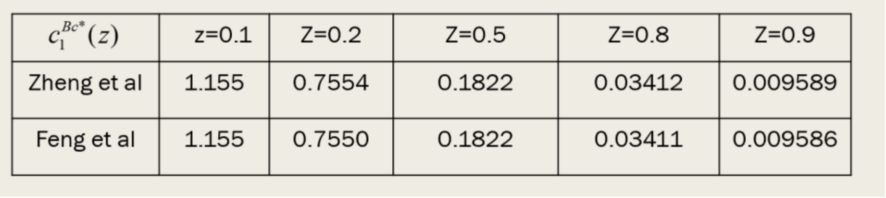
(Received 17 January 2022; accepted 8 September 2022; published 26 September 2022)

The universal fragmentation functions of gluon into the flavored quarkonia B_c and (polarized) B_c^* are computed within NRQCD factorization framework at the lowest order in velocity expansion and strong coupling constant. It is mandatory to invoke the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi renormalization program to render the NRQCD short-distance coefficients UV finite in a pointwise manner. The calculation is facilitated with the sector decomposition method, with the final results presented with high numerical accuracy. This knowledge is useful to enrich our understanding toward the large-p_T behavior of $B_c^{(*)}$ production at LHC experiment.

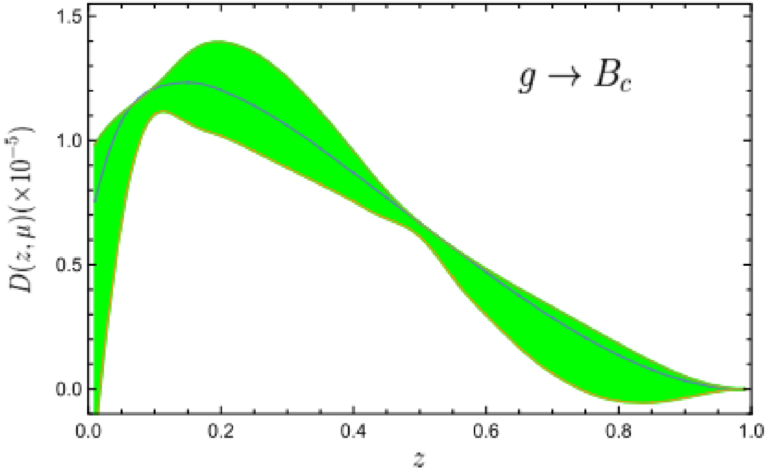
DOI: 10.1103/PhysRevD.106.054030

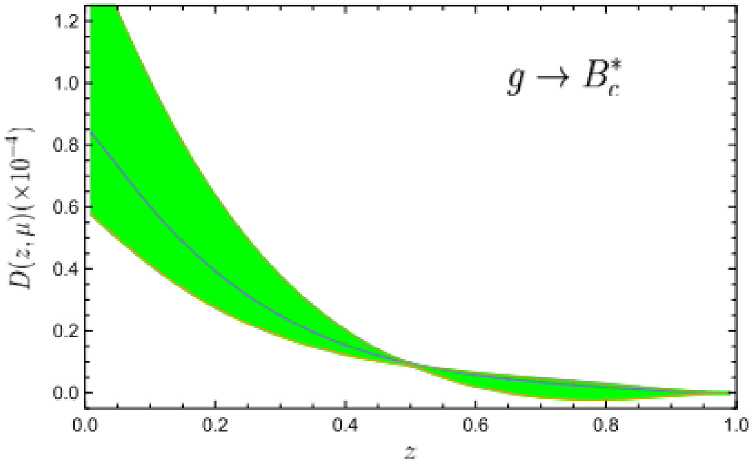


$c_1^{Bc}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	0.2323	0.2311	0.1282	0.02612	0.006144
Feng et al	0.2324	0.2311	0.1282	0.02612	0.006143



$c_1^{Bc^*}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	1.155	0.7554	0.1822	0.03412	0.009589
Feng et al	1.155	0.7550	0.1822	0.03411	0.009586





贾宇老师课题组使用了和我们完全不同的计算方法，他们将相空间积分转换为圈图积分，并用双圈计算程序进行计算。我们两个组的结果是完全一致的。