DiPion light-cone distribution amplitudes and the semileptonic decays of $D_s^{(*)}$

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Overview

DiPion

- Light-cone distribution amplitudes
- Double expansion and the coefficient $B_{nl}^{(I)}(s)$

DiPion(DiKaon) LCDAs in $D_s^{(*)}$ weak decay

- $D_s \to (f_0 \to) [\pi^+\pi^-]_{\mathrm{S}} \, e^+\nu_e$
- D_s^* weak decay

Conclusion

DiPion

Why DiPion ?

- CKM matrix is a crucial criterion of the Standard Model[PDG 2022]

- long standing $|V_{ub}|$ tension

† $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$, mainly extracted from $B \to X_u l\nu$ and $B \to \pi l\nu$

† $|V_{ub}|_{\rm incl} = (4.13 \pm 0.25) \times 10^{-3}, |V_{ub}|_{\rm excl} = (3.70 \pm 0.16) \times 10^{-3}$, $\sim 2.5\sigma$

† enlarge the set of exclusive processes to determine $|V_{ub}|$, a candidate is $B \to \rho l\nu$

△ ρ is reconstructed by $\pi\pi$ invariant mass spectral, width effect/nonresonant contribution ?

△ the underlying consideration is $B \to \pi\pi l\bar{\nu}_l$ (B_{l4})[Faller 2014]

- V_{cs} issue

† $|V_{cs}| = 0.975 \pm 0.006$, mainly extracted from the (semi)leptonic $D_{(s)}$ decays

† $|V_{cs}| = 0.972 \pm 0.007$, $|V_{cs}| = 0.984 \pm 0.012$, $\sim 1.5\sigma$ derivation

† $\sim 3\sigma$ tension two years ago, 0.939 ± 0.038 and 0.992 ± 0.012

† new channels like semileptonic $D_s^{(*)}$ decays are highly anticipated

△ problems encountered, $D_s \to f_0 l\nu$ has large uncertainty due to the width and complicate structure

△ $D_s^* \to \phi l\nu$

- R_{D^*} anomaly, $B \to D^*$ form factors, $D\pi$ system $\cdots$

DiPion LCDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$[Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

△ $n^2 = 0$, △ index f, f' respects the (anti-)quark flavor, △ a, b indicates the electric charge

△ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, △ $k = k_1 + k_2$ is the invariant mass of dipion state

△ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs,

△ higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

† Three independent kinematic variables

△ momentum fraction z carried by anti-quark with respecting to the total momentum of DiPion state,

△ longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+/k^+$, $2q\cdot\bar{k}\,(\propto 2\zeta - 1)$ △ k^2

† Normalization conditions

$$\int_0^1 \Phi_{\parallel}^{I=1(0)}(u,\zeta,k^2) = (2\zeta-1)F_\pi(k^2)$$

$$\int_0^1 dz\,(2z-1)\Phi_{\parallel}^{I=0}(z,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\mathrm{EMT}}(k^2)$$

△ $F_\pi^{em}(0) = 1$, △ $F_\pi^{\mathrm{EMT}}(0) = 1$,

△ $M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

DiPion LCDAs

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π,ρ,f_0 mesons

$$B_{n\ell}(k^2,\mu) = B_{n\ell}(k^2,\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{[\gamma_n^{(0)}-\gamma_0^{(0)}]/[2\beta_0]}$$

$$\gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k}-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right)$$

- Watson theorem of π-π scattering amplitudes

△ implies an intuitive way to express the imaginary part of 2πDAs

△ leads to the Omnés solution of $N-$subtracted dispersion relation for the coefficients

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0)+\frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty}ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ 2πDAs in a wide range energies is given by δ_ℓ^I and a few subtraction constants

DiPion LCDAs

- Soft pion theorem relates the chirally even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel, I=1}(0) = a_n^\pi\,, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel, I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

△ express the moments of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as

$$M^\pi_{N=\mathrm{odd}} = \frac{3}{2}\frac{N+1}{2N+1} B^{I=1}_{N-1,N}(0), \quad M^\pi_{N=\mathrm{even}} = 3\frac{N+1}{2N+1} B^{I=0}_{N-1,N}(0)$$

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

△ relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} c_m^{n1} m_\rho^{2m}\right], \quad c_m^{(n1)} = \frac{1}{m!}\frac{d^m}{dk^{2m}}\left[\ln B_{n1}(0) - \ln B_{01}(0)\right]$$

△ f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ by $\quad \langle \pi(k_1)\pi(k_2)|\rho\rangle = g_{\rho\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha$

$$f_\rho^\parallel = \frac{\sqrt{2}\,\Gamma_\rho\,\mathrm{Im}B_{01}^\parallel(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^\perp = \frac{\sqrt{2}\,\Gamma_\rho\, m_\rho\,\mathrm{Im}B_{01}^\perp(m_\rho^2)}{g_{\rho\pi\pi}\, f_{2\pi}^\perp}$$

DiPion LCDAs

- The subtraction constants of $B_{n\ell}(s)$[Polyakov 1999, SC 2019, SC 2023]

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556 $\to$ -0.300	-	0.413 $\to$ 0.375	-	-	-
(12)	0.556 $\to$ 0.300	-	0.413 $\to$ 0.375	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum

- Above discussions are all at leading twist level
- Subleading twist LCDAs are still in lack
- but non-negligible even important in the decays of heavy flavor hadrons

DiPion LCDAs in D_s weak decay

DiKaon LCDAs in D_s^* weak decay

$D_s \to (f_0 \to) [\pi\pi]_{\rm S}\, e^+\nu_e$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons

  △ $D_{(s)} \to a_0 e^+\nu$[BESIII 18, 21], $D^+ \to f_0/\sigma e^+\nu$[BESIII 19], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[CLEO 09]

  △ $\mathcal{B}$ of $D_s \to f_0(\to \pi^0\pi^0, K_s K_s)e^+\nu$[BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$ form factor[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

  △ isospin symmetry expectation $\mathcal{B}(f_0 \to \pi^+\pi^-)/\mathcal{B}(f_0 \to \pi^0\pi^0) = 2$, possible ρ^0 pollution

  △ $f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$

- Theoretical consideration $\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2$

- Observed in the $\pi\pi$ invariant mass spectral, improvement with the width effect

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+\nu)}{ds dq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2\beta_\pi(s)}{|m_{\rm S}^2 - s + i\left(g_1^2\beta_\pi(s)) + g_2^2\beta_K(s)\right)|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\rm S}\, l^+\nu)}{dk^2 dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}\frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{16\pi}\sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

- $D_s \to f_0$ ffs to $D_s \to [\pi\pi]_{\rm S}$ ffs

  [Hambrock 2015, SC 2017,19,20, Descotes-Genon 2019,23] in $B_{(s)}$ cases

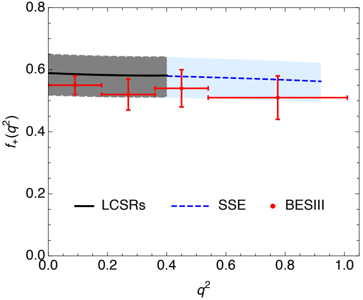
$D_s \to (f_0 \to) [\pi\pi]_S \, e^+ \nu_e$

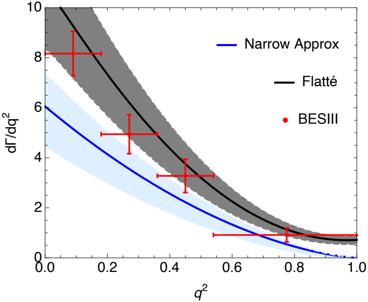
- Definitions of $D_s \to f_0$ form factors

$$\langle f_0(p_1)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -i\left[f_+(q^2)\,(p+p_1)_\mu + f_-(q^2) q_\mu\right]$$

- Form factor and the differential decay width [SC 2023]

$\triangle$ $M^2 = 5.0 \pm 0.5$ GeV^2 and $s_0 = 6.0 \pm 0.5$ GeV^2 , $\triangle$ $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV used in the previous LCSRs, $\triangle$ $a_1^{s/\sigma}$ term contributions are considered for the first time , $\triangle$ f_0 is not a pure $\bar{s}s$ state, the mixing angle is chosen at $20^\circ \pm 10^\circ$



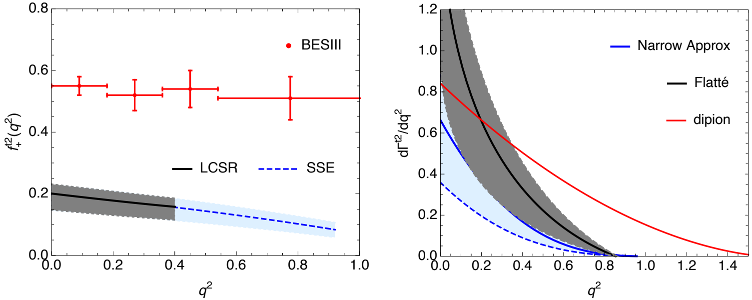


$D_s \to [\pi\pi]_{\rm S}\, e^+\nu_e$

- Definitions of $D_s \to [\pi\pi]_{\rm S}$ form factors

$$\langle[\pi(k_1)\pi(k_2)]_{\rm S}\,|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s^+(p)\rangle = -iF_t(q^2,s,\zeta)k_\mu^t - iF_0(q^2,s,\zeta)k_\mu^0 - iF_\parallel(q^2,s,\zeta)k_\mu^\parallel$$

- Form factor and the differential decay width at leading twist [SC 2023]



- subleading twist LCDAs give dominate contribution in $D_s \to [\pi\pi]_{\rm S}$ transition
- shows relatively moderate evolution with larger allowed momentum transfer
- further measurements would help us to understand the dipion system, ρ, f_0
- different in B/Z case where the leading twist is dominate/overwhelming

DiPion LCDAs in D_s weak decay

DiKaon LCDAs in D_s^* weak decay

D_s^* weak decay

- $\alpha_s : \alpha : G_F \sim \mathcal{O}(1) : \mathcal{O}(1/137) : \mathcal{O}(10^{-5})$
- very hard to measure weak decay from strong and EM interactions
- the total widths of heavy-light vector mesons are still in lack[PDG 2022]
  - △ $\Gamma_{D^{*+}} = 84.3 \pm 1.8\,\mathrm{keV}$ ($\to D^0\pi^+, D^+\pi^0, D^+\gamma$)
  - △ $\Gamma_{D^{*0}} < 2.1\,\mathrm{MeV}$ ($\to D^0\pi^0, D^0\gamma$), $\Gamma_{D_s^{*+}} < 1.9\,\mathrm{MeV}$ ($\to D_s^+\gamma, D_s^+\pi^0, D_s^+e^+e^-$)
  - △ $\Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement
- but important to properties and $g_{D_s^* D_s \gamma}$ $\to$ non-perturbative approaches

	$g_{D^{*+}D^+\gamma}$ (GeV^{-1})	$g_{D^{*0}D^0\gamma}$ (GeV^{-1})	$g_{D_s^{*+}D_s^+\gamma}$ (GeV^{-1})
this work	$-0.15^{+0.11}_{-0.10}$	$1.48^{+0.29}_{-0.27}$	$-0.079^{+0.086}_{-0.078}$
HHχPT [24]	-0.27 ± 0.05	2.19 ± 0.11	0.041 ± 0.056
HQET+VMD [35]	$-0.29^{+0.19}_{-0.11}$	$1.60^{+0.35}_{-0.45}$	$-0.19^{+0.19}_{-0.08}$
HQET+CQM [71]	$-0.38^{+0.05}_{-0.06}$	1.91 ± 0.09	–
Lattice QCD [32]	-0.2 ± 0.3	2.0 ± 0.6	–
LCSR [21]	-0.50 ± 0.12	1.52 ± 0.25	–
QCDSR [20]	$-0.19^{+0.03}_{-0.02}$	0.62 ± 0.03	-0.20 ± 0.03
RQM [72]	-0.44 ± 0.06	2.15 ± 0.11	-0.19 ± 0.03
experiment [16–18]	-0.47 ± 0.06	1.77 ± 0.03	–

LCSRs, hadronic photon NLO[Li 2020]

LCSRs, LP NLO corrections[Pullin 2021]

$$g_{D_s^* D_s \gamma} = 0.60^{+0.19}_{-0.18}$$

very sensitive to different contributions (radiative corrections, power corrections)
a benchmark to probe the involved dynamics

- impressive lattice QCD evaluation[HPQCD 2013] $\Gamma^{\mathrm{HPQCD}}_{D_s^{*+}} = 0.070(28)\,\mathrm{keV}$
  - △ the longest-lived charged vector meson △ encourage us to study the exclusive D_s^* weak decay

D_s^* weak decay

- D_s^* weak decay are highly anticipated to determine $|V_{cs}|$
- leptonic decays, helicity enhanced $D_s^* \to l\nu$, $|V_{cs}| f_{D_s^*}$

$$\Gamma_{D_s^* \to l\nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 \left(1 - \frac{m_l^2}{m_{D_{s*}^2}}\right)\left(1 + \frac{m_l^2}{m_{D_{s*}^2}}\right) = 2.44 \times 10^{-12}\,\mathrm{GeV}$$

△ $\mathcal{B}(D_s^* \to \mu\nu) = \frac{\Gamma_{D_s^* \to \mu\nu}}{\Gamma_{D_s^*}} \sim \frac{\Gamma_{D_s \to \mu\nu}}{\Gamma_{D_s^*}} \frac{2m_{D_s^*}^2}{3m_\mu^2} \sim 2 \times 10^{-5}$, close to the LQCD[HPQCD 2013]

△ the most favored modes, $\left(2.1^{+1.3}_{-0.9}\right) \times 10^{-5}$[2304.12159 BESIII]

△ confirms the total width of D_s^* but need more precise lattice evaluation

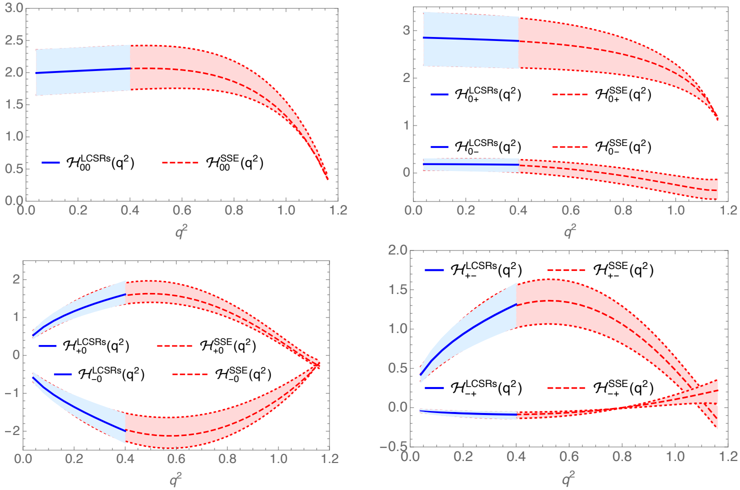
- semileptonic decays, $D_s^* \to \phi l\nu$, $|V_{cs}|$ and helicity form factors

△ heavy quark symmetry (HQS) has been examined in $\bar{B} \to D^*(D) l\bar{\nu}$, also in $D_s^*(D_s) \to \phi l^+\nu$?

△ lepton flavour university (LFU) in vector charm sector

- hadronic decays $D_s^* \to \phi\rho, \phi\pi$, factorisation theo. or topological analysis
- inclusive decays, $D_s^* \to X_s l\nu$, HQET and reliability of power expansion

D_s^* weak decay



- LCSRs parameters $s_0 = 6.8 \pm 1.0$ GeV^2, $M^2 = 4.50 \pm 1.0$ GeV^2
- Wigner-Eckart theorem: the helicity information at endpoint is only governed by the Clebsch-Gordan coefficients [Hiller 2014, Grattrex 2016, Hiller 2021]

D_s^* weak decay

- semileptonic decays $D_s^* \to \phi l \nu_l$

$$\frac{d\Gamma_{ij}(q^2)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s^*}^3} \lambda^{1/2}(m_{D_s^*}^2, m_\phi^2, q^2)\, q^2\, |H_{ij}(q^2)|^2$$

$$\Gamma_{D_s^* \to \phi l \nu_l} = \frac{1}{3} \int_0^{q_0^2} dq^2 \sum_{i,j=0,\pm} \frac{d\Gamma_{ij}(q^2)}{dq^2} = \left(3.28^{+0.82}_{-0.71}\right) \times 10^{-14}\,\mathrm{GeV}$$

△ DiKaon LCDAs and the width effect in $D_s^* \to \phi$ transition

- hadronic decays (naive factorisation)

$$\mathcal{A}(D_s^{*+} \to \phi\pi^+) = (-i)\frac{G_F}{\sqrt{2}} V_{cs}\, a_1\, m_\pi f_\pi \sum_{i=0,\pm} H_{0j}(m_\pi^2)$$

$$\mathcal{A}(D_s^{*+} \to \phi\rho^+) = \frac{G_F}{\sqrt{2}} V_{cs}\, a_1\, m_\rho f_\rho^{\parallel(\perp)} \sum_{i,j} H_{ij}(m_\rho^2)$$

△ $a_1(\mu) = 0.999$, $f_\pi = 0.130$ GeV, $f_\rho^{\parallel} = 0.210$ GeV

△ $\Gamma_{D_s^{*+} \to \phi\pi^+} = \left(3.81^{+1.52}_{-1.33}\right) \times 10^{-14}\,\mathrm{GeV}$, $\quad \Gamma_{D_s^{*+} \to \phi\rho^+} = \left(1.16^{+0.42}_{-0.39}\right) \times 10^{-13}\,\mathrm{GeV}$

△ the result of $\phi\pi$ channel is marginally consistent with the PQCD[Yang 2022]

- with the lattice evaluation of $\Gamma_{D_s^*} = (0.70 \pm 0.28) \times 10^{-8}$ GeV[HPQCD 2013]

[**SC** 2022] $\mathcal{B}(D_s^* \to l\nu) = (3.49 \pm 1.40) \times 10^{-5}$, $\quad \mathcal{B}(D_s^* \to \phi l\nu) = (0.47\,^{+0.12}_{-0.10} \pm 0.19) \times 10^{-6}$

$\mathcal{B}(D_s^{*+} \to \phi\pi^+) = (0.54\,^{+0.22}_{-0.19} \pm 0.22) \times 10^{-6}$, $\quad \mathcal{B}(D_s^{*+} \to \phi\rho^+) = (1.65\,^{+0.61}_{-0.56} \pm 0.66) \times 10^{-6}$

D_s^* weak decay   without the background analysis

- Belle II   clear background

  △ 2022, $400\,\mathrm{fb}^{-1}$, reconstruct 2×10^5 data samples of $D_s^*(D_s)$ from $\phi\pi$ channel

  △ phase 3 running (2024-2026), $10\,\mathrm{ab}^{-1}$, $\mathcal{O}(1 \times 10^7)$ data sample of $D_s^*(D_s)$

  △ the number of D_s^* production is $\mathcal{O}(10^9)$ $\Leftarrow \mathcal{B}(D_s \to \phi\pi) = (4.5 \pm 0.4)\,\%$

  △ excellent potential to study the D_s^* weak decays, 50 ab^{-1} is hottest expected

- LHCb   excellent particle identification to distinguish K, π and μ

  △ the channel $D_s^* \to \phi(KK)\pi$ with the D_s^* producing by $B_s \to D_s^* \mu\nu$

- BESIII   low background

  △ directly produced from e^+e^- collision at the $D_s D_s^*$ threshold

  △ have collected $\sim 6 \times 10^6$ D_s^* mesons with the $3.2\,\mathrm{fb}^{-1}$ data at 4.178 GeV

  △ provides the good chance for the leptonic decay $D_s^* \to l\nu$, Statistical error

  △ first $\mathcal{B}$ measurement [2304.12159], determination of spin and parity [2305.14631]

$D_s^{(*)}$ weak decay

without the background analysis

CEPC CDR

Particle	Tera-Z	Belle II	LHCb
b hadrons			
B^+	6×10^{10}	3×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	3×10^{13}
B^0	6×10^{10}	3×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	3×10^{13}
B_s	2×10^{10}	3×10^{8} (5 ab^{-1} on $\Upsilon(5S)$)	8×10^{12}
b baryons	1×10^{10}		1×10^{13}
Λ_b	1×10^{10}		1×10^{13}
c hadrons			
D^0	2×10^{11}		
D^+	6×10^{10}		
D_s^+	3×10^{10}		
Λ_c^+	2×10^{10}		
τ^+	3×10^{10}	5×10^{10} (50 ab^{-1} on $\Upsilon(4S)$)	

Table 2.4: Collection of expected number of particles produced at a tera-Z factory from 10^{12} Z-boson decays. We have used the hadronization fractions (neglecting p_T dependencies) from Refs. [431, 432] (see also Ref. [433]). For the decays relevant to this study we also show the corresponding number of particles produced by the full 50 ab^{-1} on $\Upsilon(4S)$ and 5 ab^{-1} on $\Upsilon(5S)$ runs at Belle II [430], as well as the numbers of b hadrons at LHCb with 50 fb^{-1} (using the number of $b\bar{b}$ pairs within the LHCb detector acceptance from [435] and the hadronization fractions from [431]).

CEPC (4×Tera-Z)
8.8×10^{10}

BESIII $\mathcal{O}(10^6) D_s^+ / D_s^{*+}$ production
Belle II $\mathcal{O}(10^9) D_s^+ / D_s^{*+}$ production

Conclusion

- DiPion (DiKaon) structure in LCDAs

  △ width effect of ρ, ϕ, f_0 in CKM determinations and anomalies study

  △ could be studied in heavy flavors and Z decays at CEPC

- $D_s^{(*)}$ weak decay

  △ semileptonic D_s decay provides clean environment to study scale meson and (subleading twist) DiPion LCDAs △ D_s^* provides the opportunity of first measurement of weak decay of vector meson and further more physics

Thank you for your patience.