CEPC味物理-新物理和相关探测技术研讨会

2023年8月16日-18日，复旦大学

Some theoretical aspects of hadronic τ decays

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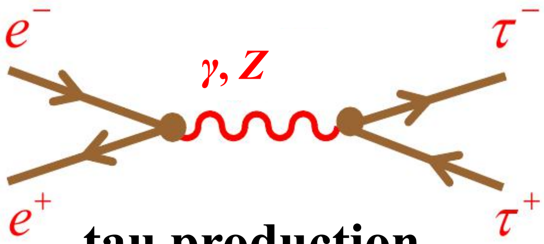
Overview



tau production

Z exchange dominates at CEPC.

Number of taus produced at e^+e^- colliders:

ALEPH: ~ 3×10^5

BaBar /Belle: ~ 1×10^9

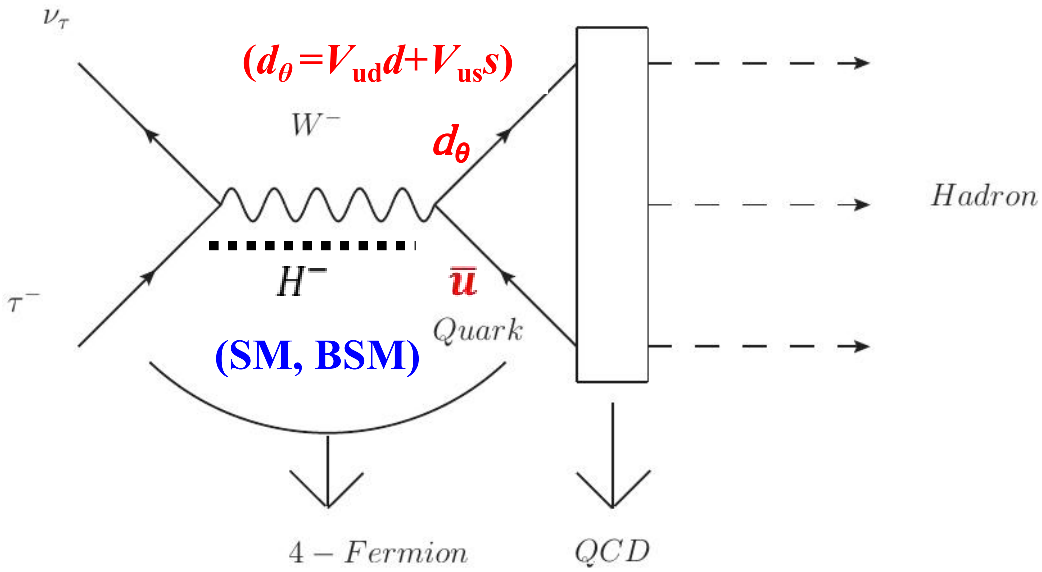
Belle-II: ~ 5×10^{10}

CEPC (Tera-*Z* factory): ~ 3×10^{10}

Tau provides broad interests for particle physics:

- ✓ Precision tests for electroweak sector: V_{CKM}, lepton universality, g-2, ...
- ✓ Stong interactions: α_s, hadron resonances, chiral symmetry, ...
- ✓ Possible discoveries for new physics: cLFV, CPV, ...

Sketch for hadronic tau decays (similar for leptonic decays by dropping QCD part)



Theoretical tools: SM EFT + Chiral EFT

- **SM EFT → LEFT** [Cirigliano et al, '10] ... .

$$\mathcal{L}_{\rm eff} = -\frac{G_\mu V_{uD}}{\sqrt{2}}\Big[\left(1+\epsilon_L^{D\ell}\right)\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1-\gamma_5)D+\epsilon_R^{D\ell}\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1+\gamma_5)D$$

$$+\bar{\ell}(1-\gamma_5)\nu_\ell\cdot\bar{u}\left[\epsilon_S^{D\ell}-\epsilon_P^{D\ell}\gamma_5\right]D+\frac{1}{4}\hat{\epsilon}_T^{D\ell}\bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Big]+\text{h.c.},$$

ε_X parameterize various new physics at high energy scale

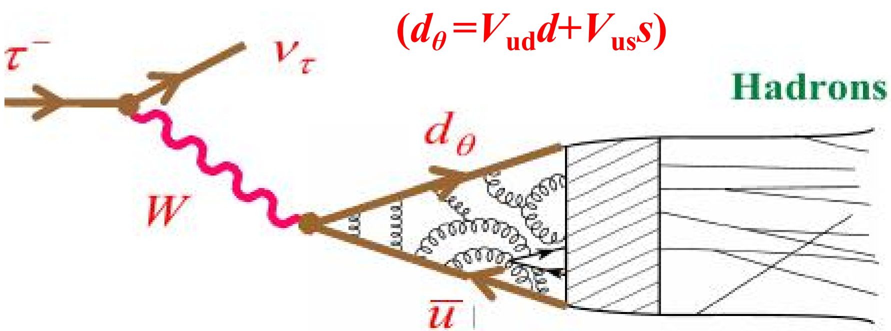
- **Chiral EFT** [Gasser,Leutwyler,'83 '84]

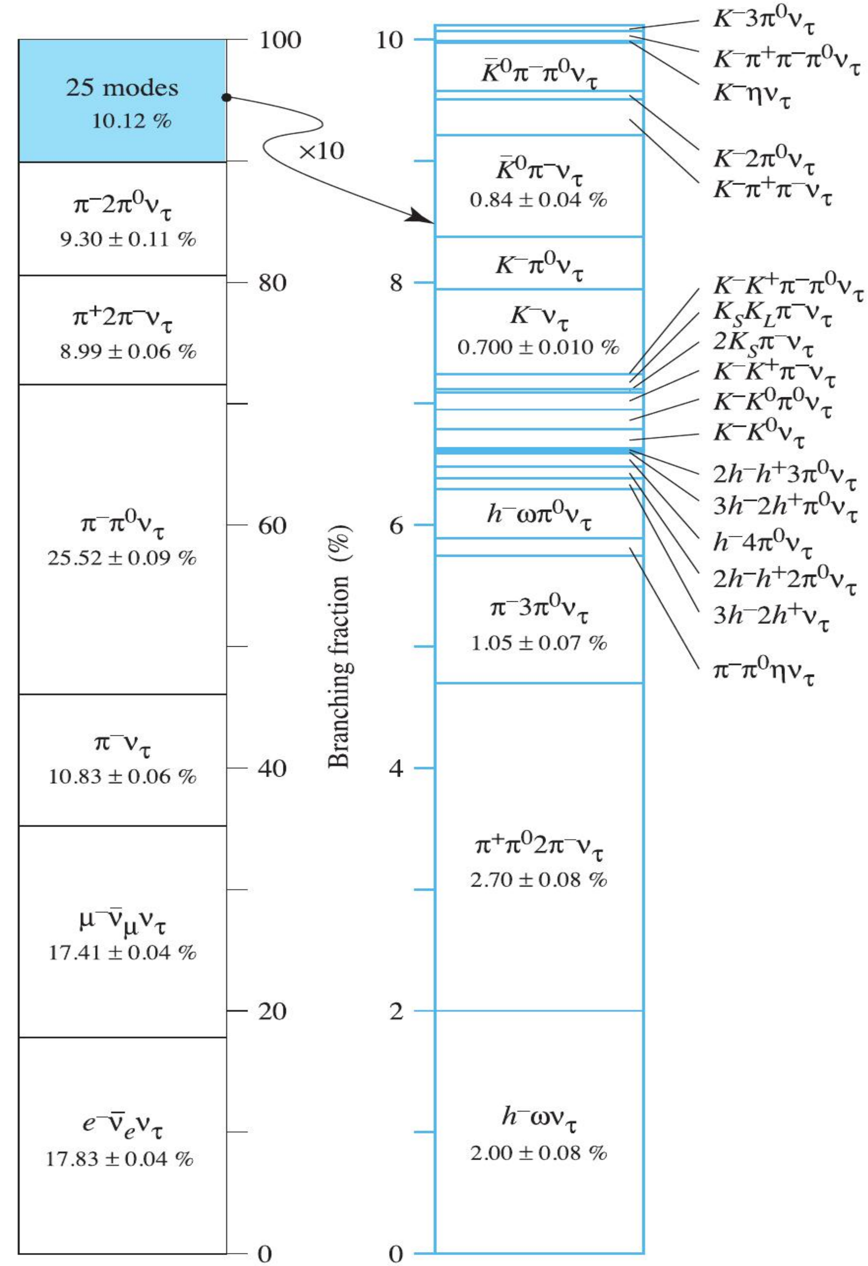
$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$\mathcal{O}(p^4)$:

$$\mathcal{L}_4^{\chi PT} = L_1\langle u_\mu u^\mu\rangle^2 + L_2\langle u_\mu u^\nu\rangle\langle u^\mu u_\nu\rangle + L_3\langle u_\mu u^\mu u_\nu u^\nu\rangle + L_4\langle u_\mu u^\mu\rangle\langle\chi_+\rangle$$
$$+L_5\langle u_\mu u^\mu \chi_+\rangle + L_6\langle\chi_+\rangle^2 + L_7\langle\chi_-\rangle^2 + \frac{L_8}{2}\langle\chi_+^2+\chi_-^2\rangle + \cdots$$

Hadronic decays: a unique feature for tau lepton



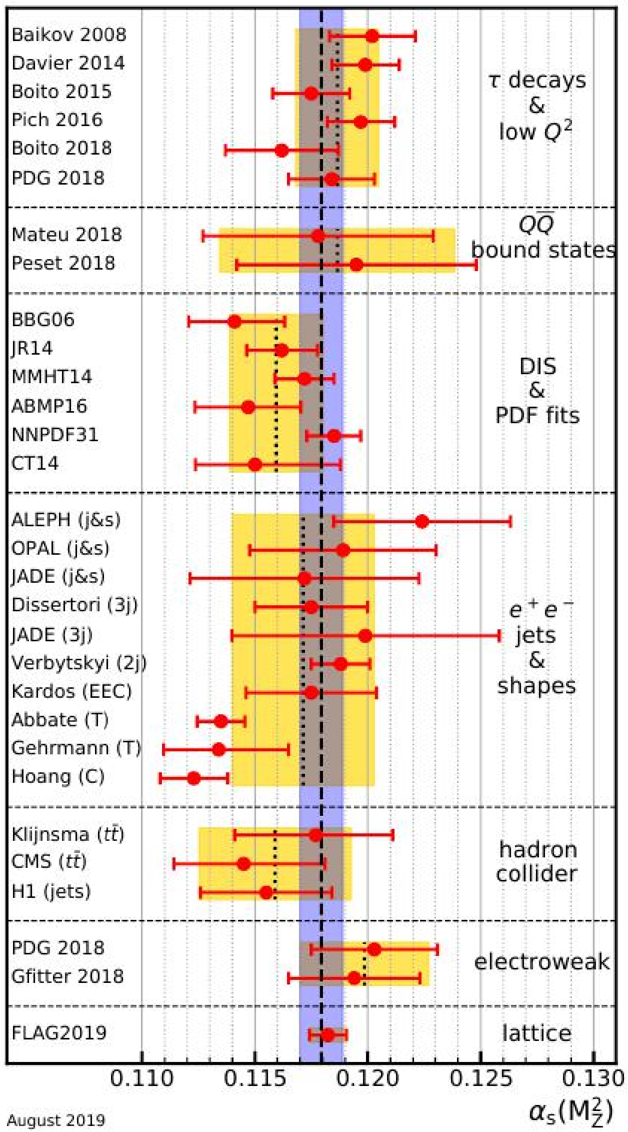


Valuable laboratory to study:

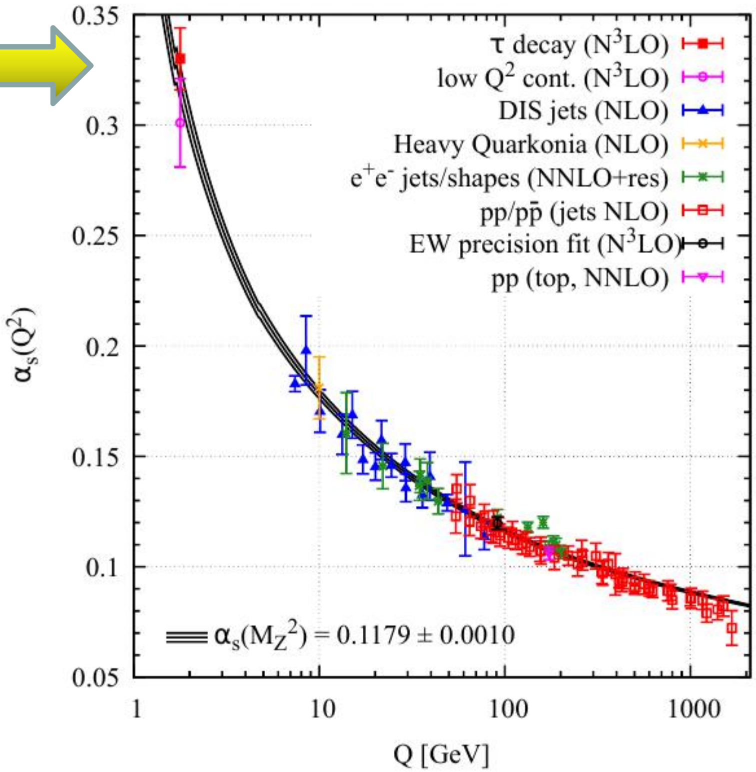
fundamental parameters &

rich hadron phenomenologies

Strong coupling of QCD: α_s



PDG

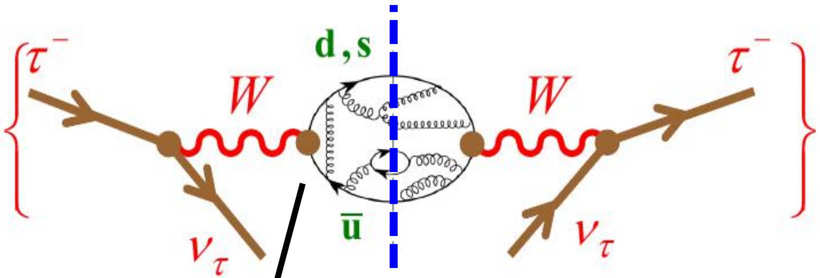


Hadronic tau decay: an invaluable source to test the QCD prediction of $\alpha_s(Q^2)$ below 2 GeV.

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}$$

[Bratten,Narison,Pich, '92]

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \quad \right\}$$



$$V^\mu_{ij} = \bar{\psi}_j \gamma^\mu \psi_i \qquad A^\mu_{ij} = \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i \int d^4x \; e^{iqx} \langle 0 | T[J^\mu_{ij}(x) J^\nu_{ij}(0)^\dagger] | 0 \rangle = \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{ij,J}(q^2)$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2 \left[\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right]$$

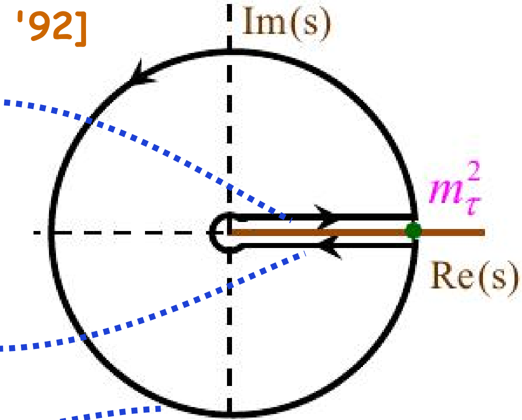
$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1)}(s) + \text{Im}\Pi^{(0)}(s)\right]$$

Exp observables

$\alpha_s(m_\tau)$ from spectral functions

[Bratten et al., '92]



$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \,\text{hadrons})}{\Gamma(\tau \to \nu_\tau e \nu_e)} = \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s)\, \text{Im}\Pi^J(s)$$

$$R_\tau = \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \omega_J(s)\, \Pi^J(s)$$

Along $|s| = m_\tau^2$,

$$\Pi^J(s) \overset{\text{OPE}}{=\!=} \sum_D \frac{C_D^J(s, \alpha_s(\mu), \mu)\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

$$R_\tau = N_C\, S_{\text{EW}} \left(1 + \delta_{\text{P}} + \delta_{\text{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\text{EW}} = 1.0201\ (3)$; $\delta_{\text{NP}} = -0.0064 \pm 0.0013$

Marciano-Sirlin, Braaten-Li, Erler — Fitted from data (Davier et al)

$\delta_{\text{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Baikov-Chetyrkin-Kühn

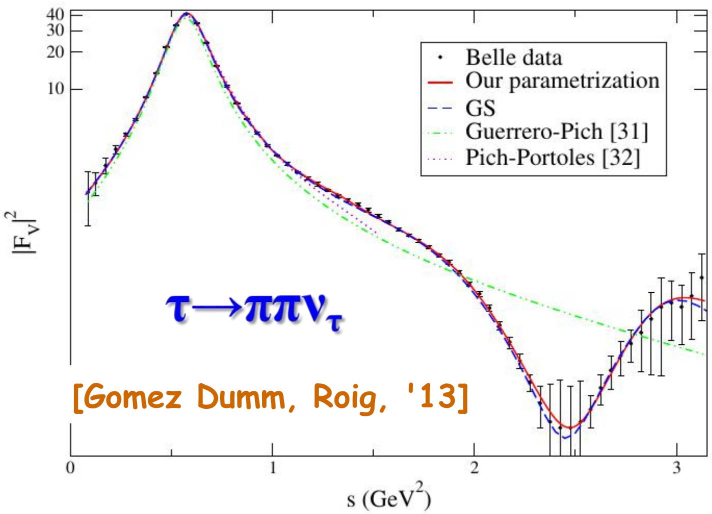
Precision limited by FOPT vs CIPT discrepancy:

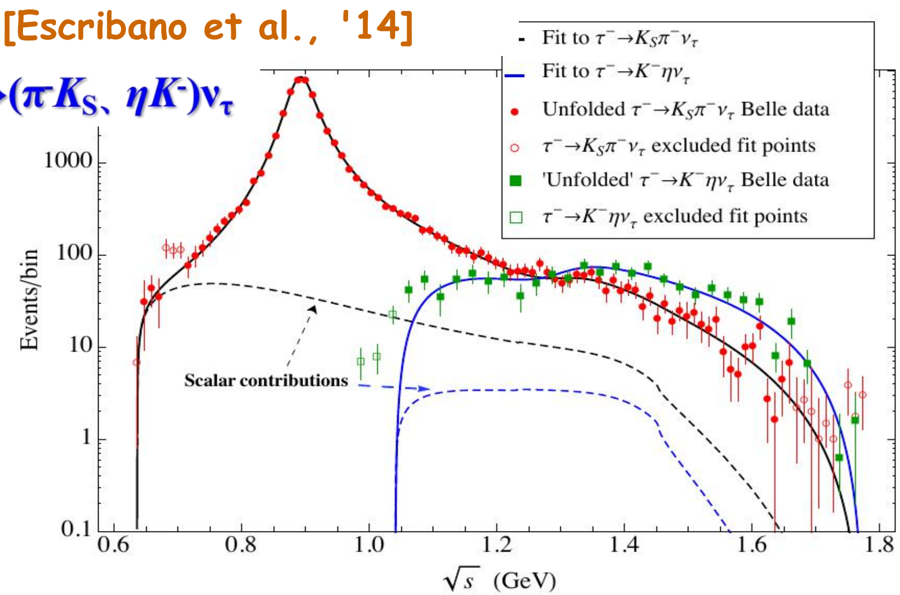
$\alpha_s(m_\tau^2) = 0.319 \pm 0.014$ $\quad \alpha_s(m_\tau^2) = 0.341 \pm 0.013$

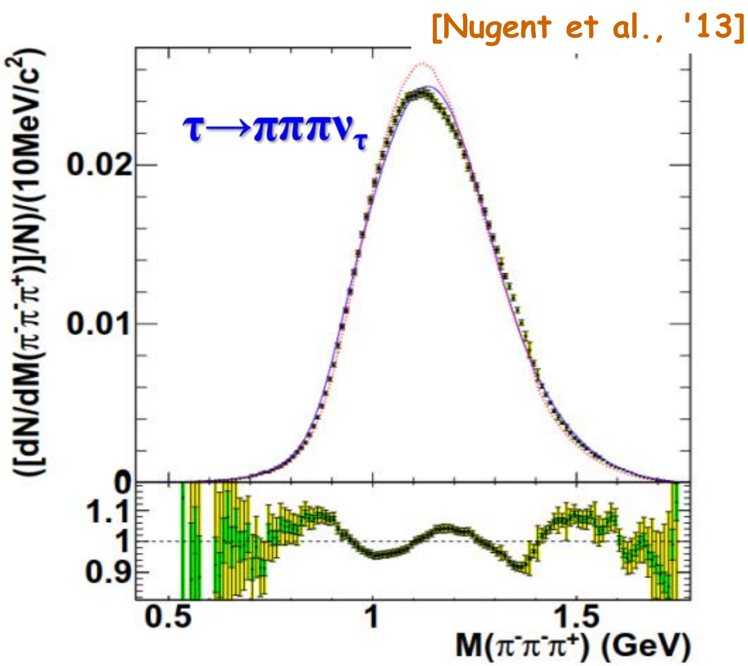
Duality violation issue:

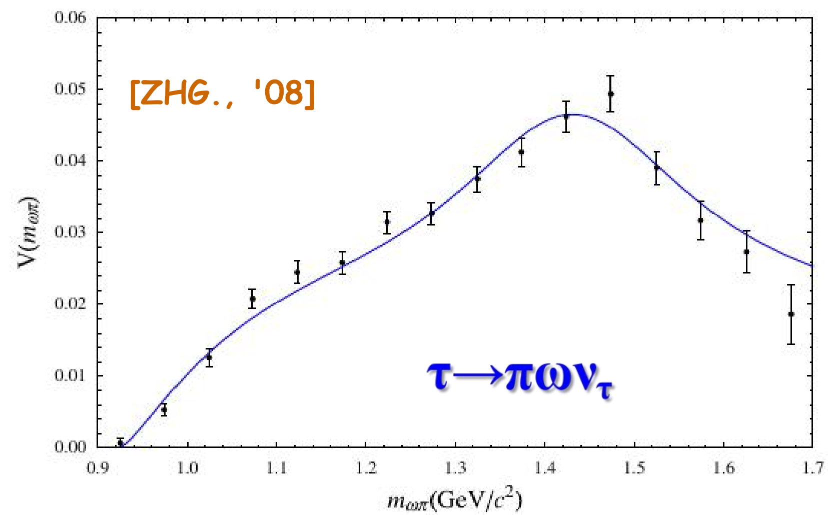
Boito et al., Pich et al., ...

Invariant-mass spectra for exclusive decays







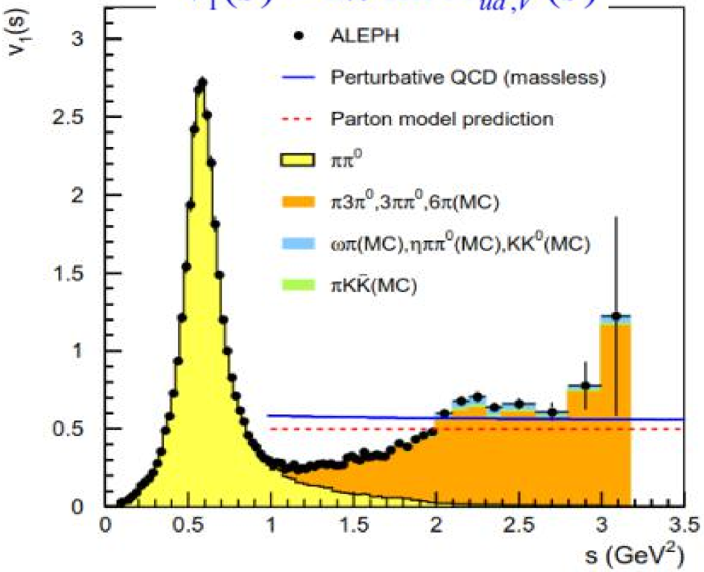


Hadron properties, Form factors, Chiral dynamics, ...

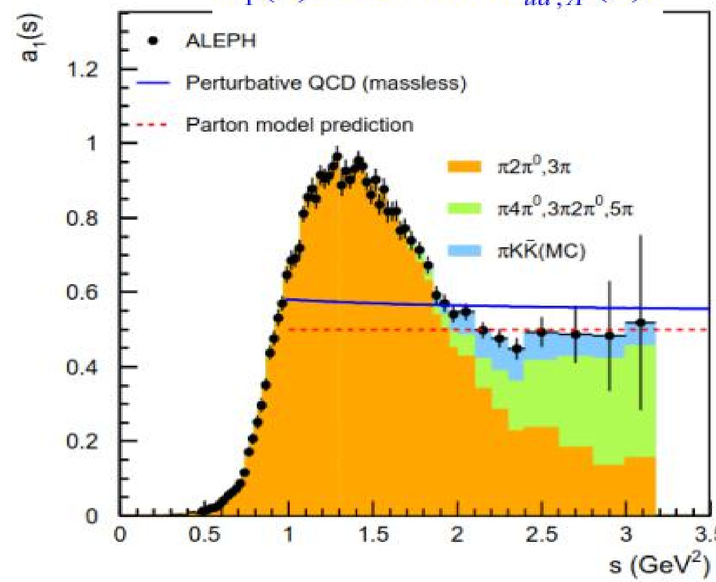
Inclusive decays: spectral functions

[Davier et al., '13]

$$\mathrm{v}_1(s) = 2\pi \,\mathrm{Im}\,\Pi^{(0+1)}_{ud,V}(s)$$

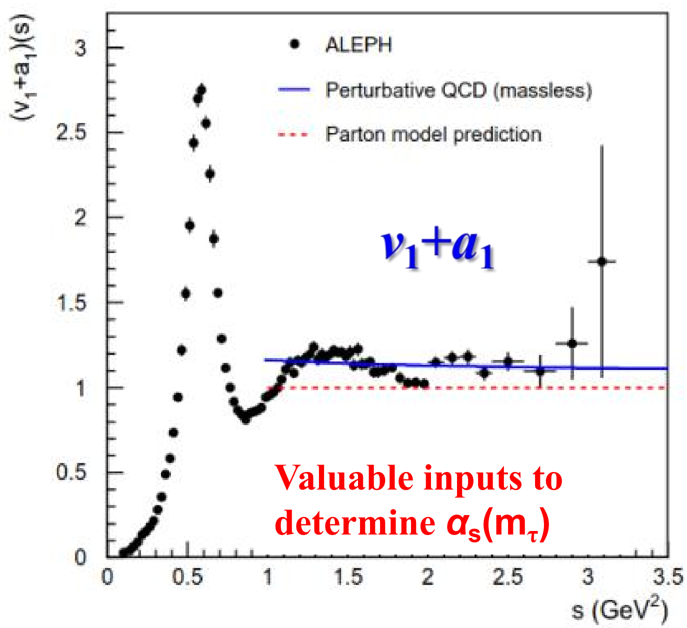


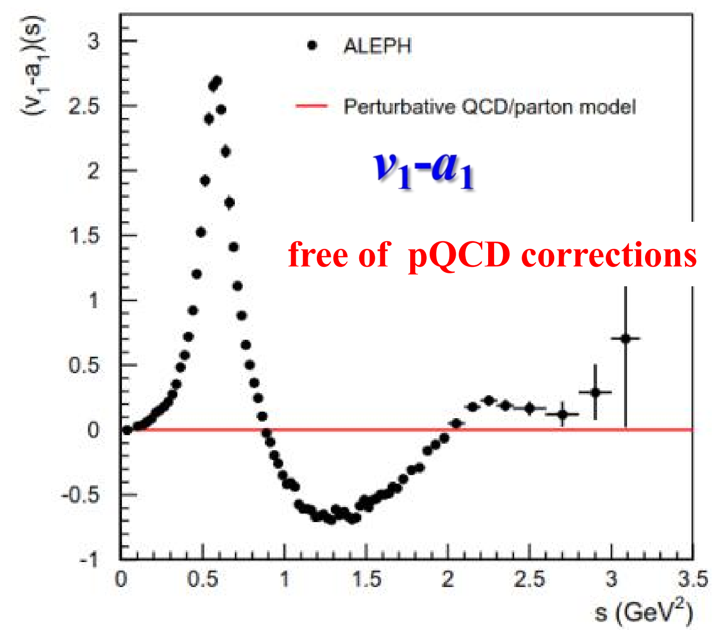
$$\mathrm{a}_1(s) = 2\pi \,\mathrm{Im}\,\Pi^{(0+1)}_{ud,A}(s)$$



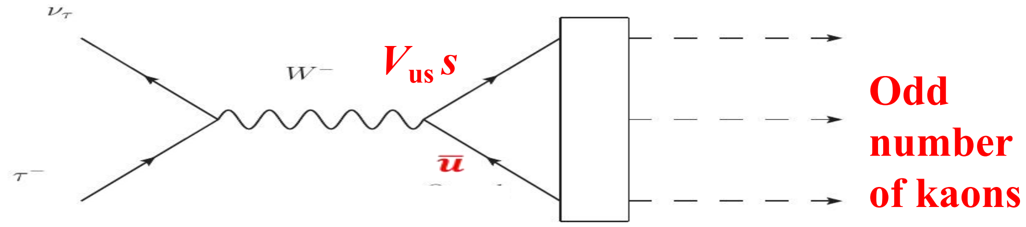
Most available data are from LEP.

Better data sets will be definitely welcome.





V_{us} from tau decays



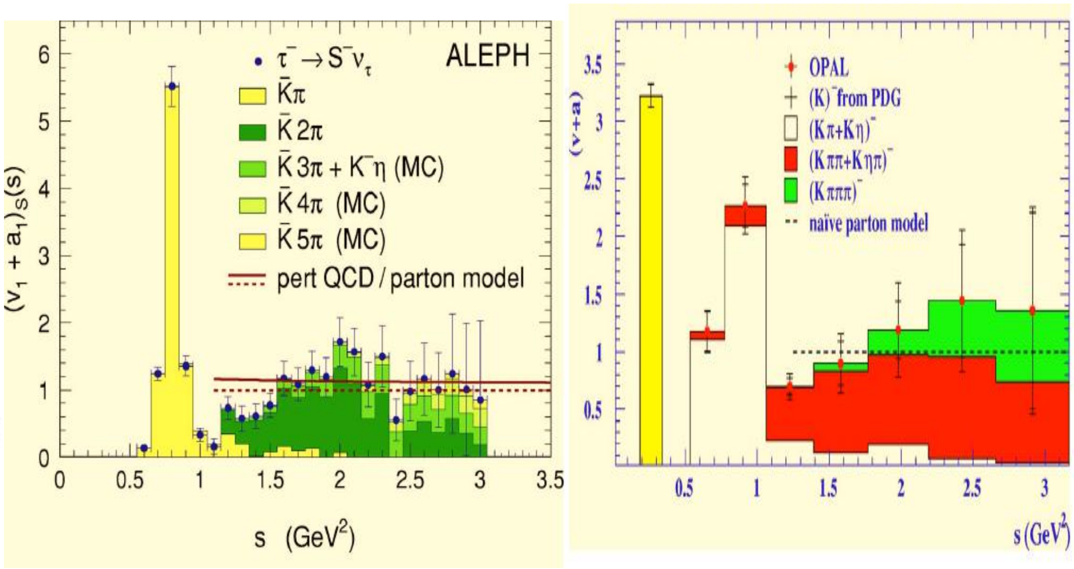
Inclusive case

Gámiz-Jamin-Pich-Prades-Schwab

$$|V_{us}| = \left(\frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}} \right)^{1/2}$$

$$\delta R_{\tau,\text{th}} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\; \Delta\,(\alpha_s)$$

$$\delta R_{\tau,\text{th}} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.084\,(33)}_{m_s(2\text{ GeV}) = 93.0\,(8.5)\ \text{MeV}} = 0.238\,(33)$$



[Davier et al.,'08]

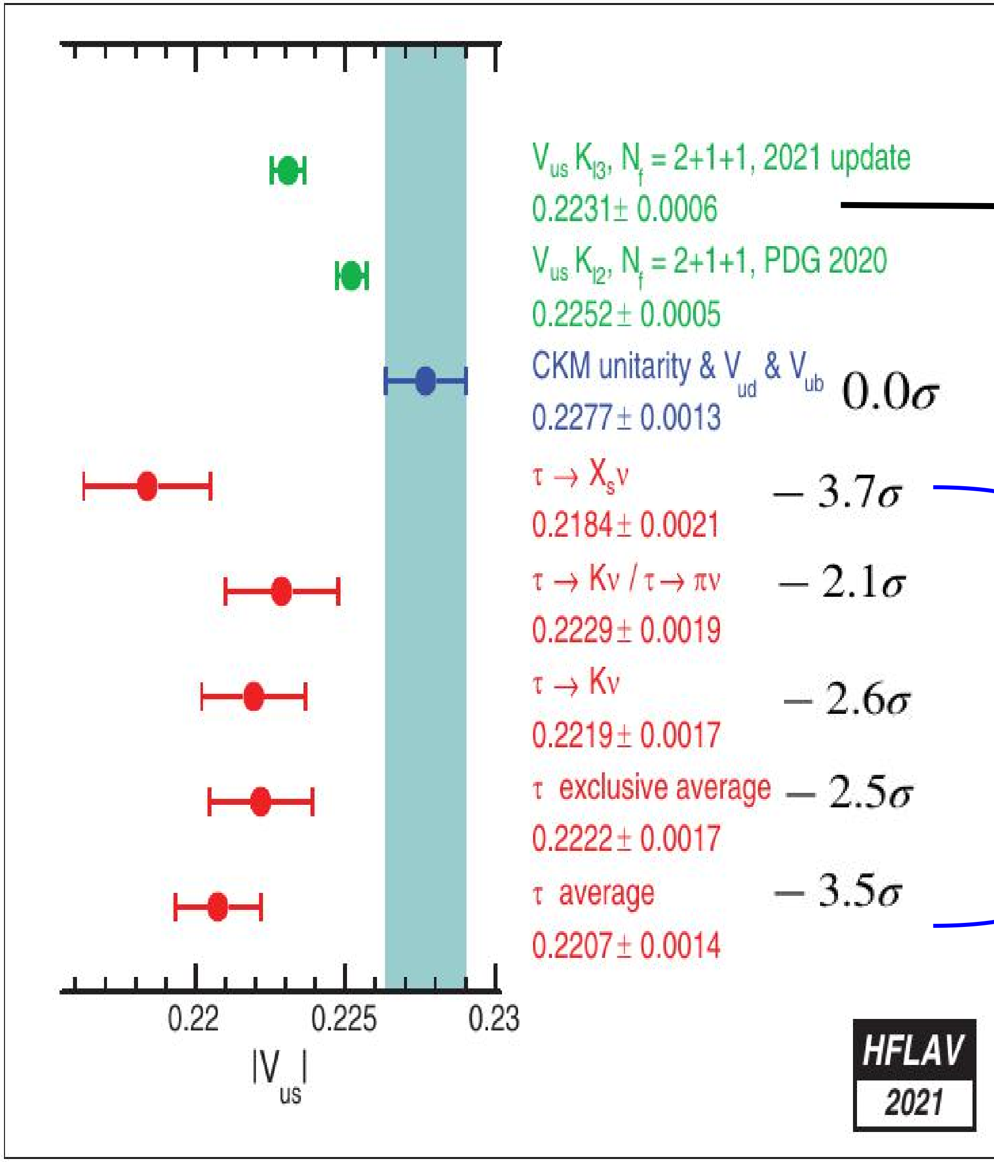
Exclusive case

$$\frac{\mathcal{B}(\tau^- \to K^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\nu_\tau)} = \frac{f_{K\pm}^2 |V_{us}|^2}{f_{\pi\pm}^2 |V_{ud}|^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} (1 + \delta R_{\tau K/\tau\pi})$$

$$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\text{EW}} (1 + \delta R_{\tau K})$$

[Arroyo-Urena et al.,'21 '22]

"Tension" of V_{us} from various determinations



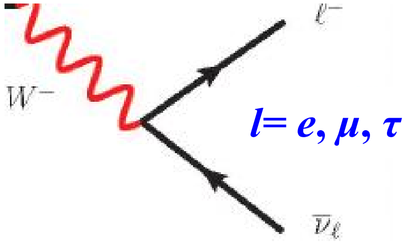
Precision limited by lattice inputs : $f_+^{K\pi}(0)$

$$\left[\sqrt{1-|V_{ud}|^2-|V_{ub}|^2}(\text{CKM unitarity})\right]$$

Precision limited mainly by BF uncertainties

Beyond SM tests in tau physics

- **Lepton Universality (LU) test**



$l = e, \mu, \tau$

LU assumption: g_l equal for all the three leptons

LU test via leptonic decays

[HFLAV,2021]

$$\frac{g_\tau}{g_e} = \sqrt{B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\text{corr}}(m_\mu, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}}$$

(m_τ^5 appears here !)

$$\frac{g_\tau}{g_\mu} = \sqrt{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\text{corr}}(m_\mu, m_e)}{F_{\text{corr}}(m_\tau, m_e)}}$$

$$\left(\frac{g_\tau}{g_\mu}\right)_\tau = 1.0009 \pm 0.0014, \qquad \left(\frac{g_\tau}{g_e}\right)_\tau = 1.0027 \pm 0.0014,$$

LU test via hadronic decays

$$\left(\frac{g_\tau}{g_\mu}\right)_h^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2$$

$h = \pi, K$

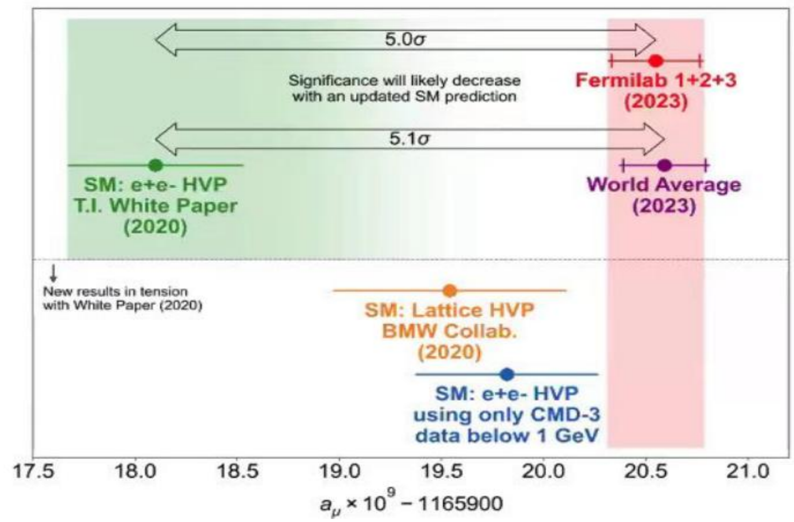
[HFLAV,'21] [Arroyo-Urena, et al.,'21'22]

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9959 \pm 0.0038 \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9855 \pm 0.0075$$

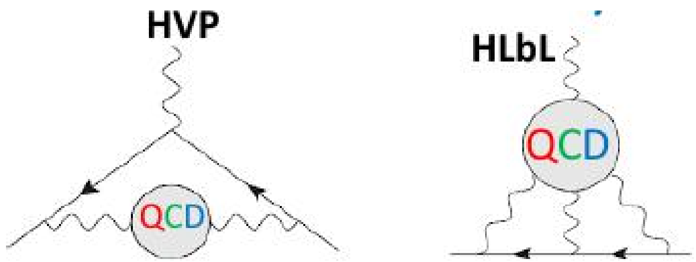
$\tau \to K\nu_\tau$ BF uncertainty dominated

➢ Relevance to precise determination of a_μ

[Muon g-2, '23]

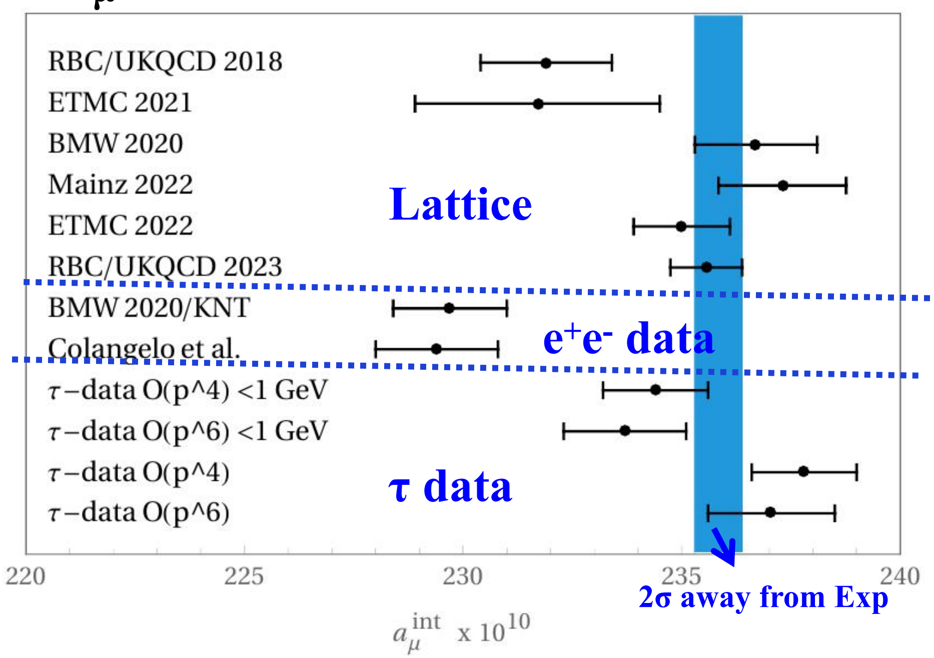


SM uncertainty dominated by

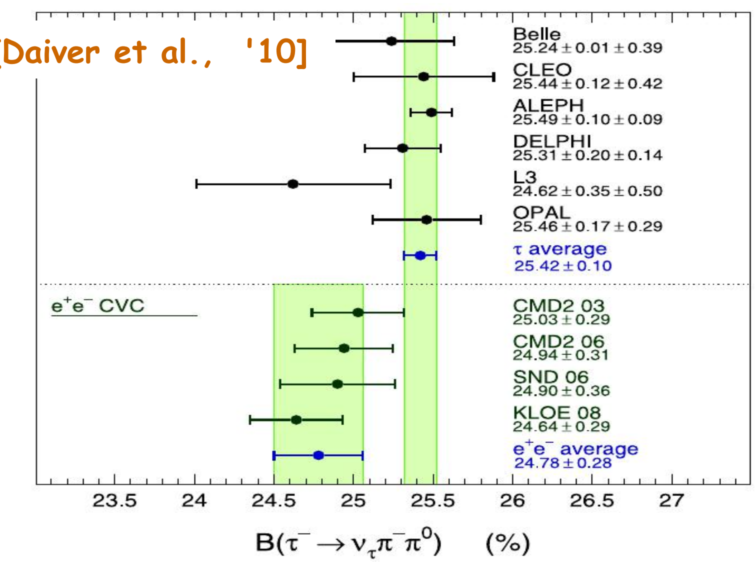


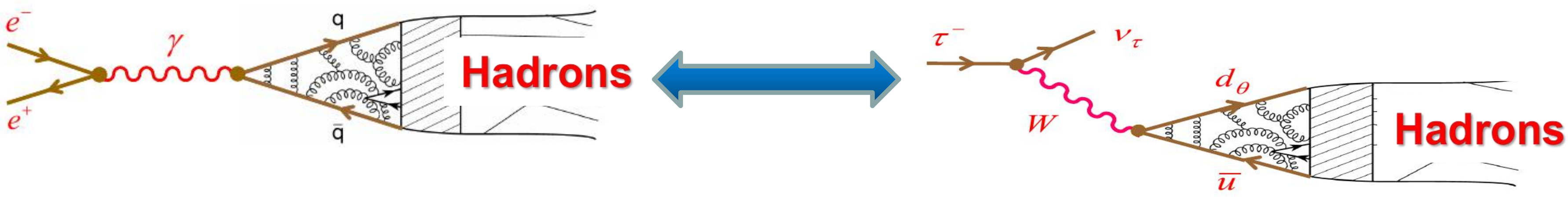
Dominated by ππ (> ~75%)

$a_\mu^{\mathrm{HVP,LO}}$ [Masjuan et al., '23]



[Daiver et al., '10]





❖ **Key problem in the matching:** **isospin breaking (IB) effects**

IB corrections to a_μ

[Cirigliano et al., JHEP'02]

$$\Delta a_\mu^{\rm vacpol} = \frac{1}{4\pi^3}\int_{4M_\pi^2}^{t_{\rm max}} dt K(t) \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma_{\pi\pi[\gamma]}}{dt}\right]\times\left(\frac{R_{\rm IB}(t)}{S_{\rm EW}}-1\right)$$

$$R_{\rm IB}(t) = \frac{1}{G_{\rm EM}(t)}\,\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^+\pi^0}}\,\left|\frac{F_V(t)}{f_+(t)}\right|^2$$

EM corrections | **Kinematics** | **IB effects in Form Factors**

$G_{\rm EM}(t) \sim$ **virtual photon + real photon**

Photon loops in $\tau \to \pi\pi\nu_\tau$

Radiative decays: $\tau \to \pi\pi\gamma\nu_\tau$

[Miranda,Roig, PRD'20]

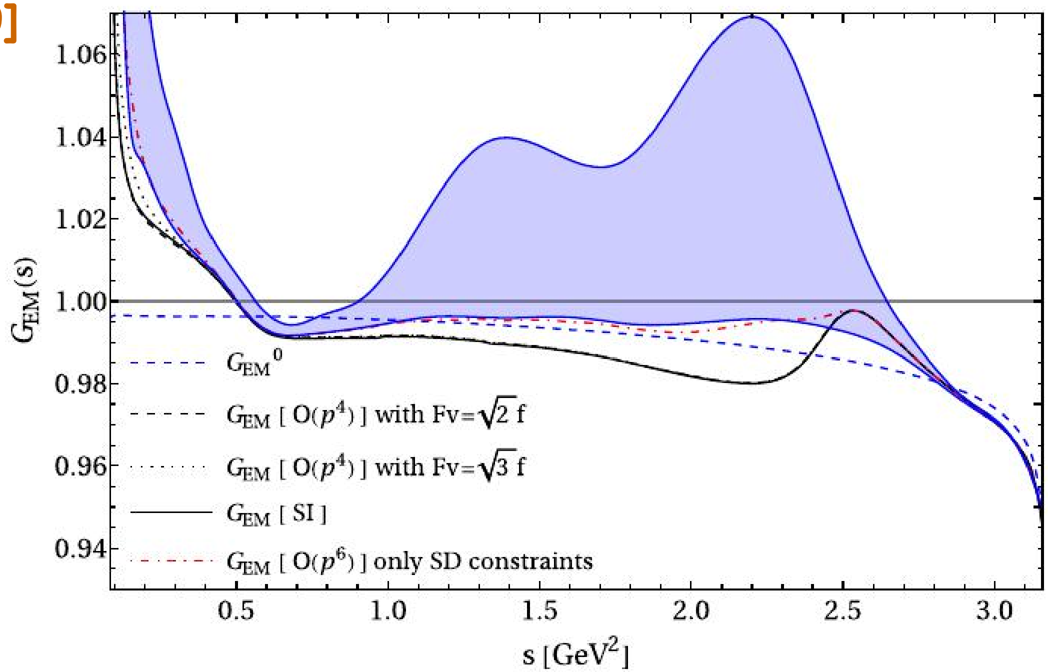


TABLE IV. Contributions to $\Delta a_\mu^{\mathrm{HVP,LO}}$ in units of 10^{-11} using the dispersive representation of the form factor. From the two evaluations labeled $\mathcal{O}(p^4)$, the left (right) one corresponds to $F_V = \sqrt{2}F$ ($F_V = \sqrt{3}F$).

$[s_1, s_2]$	$\Delta a_{\mu,\mathrm{G_{EM}^{(0)}}}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,\mathrm{SI}}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^4)]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^4)]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[SD]}^{\mathrm{HVP,LO}}$	$\Delta a_{\mu,[\mathcal{O}(p^6)]}^{\mathrm{HVP,LO}}$
$[4m_\pi^2, 1\ \mathrm{GeV}^2]$	+17.8	−11.0	−11.3	−17.0	−32.4	−74.8 ± 44.0
$[4m_\pi^2, 2\ \mathrm{GeV}^2]$	+18.3	−10.1	−10.3	−16.0	−31.9	−75.9 ± 45.5
$[4m_\pi^2, 3\ \mathrm{GeV}^2]$	+18.4	−10.0	−10.2	−15.9	−31.9	−75.9 ± 45.6
$[4m_\pi^2, m_\tau^2]$	+18.4	−10.0	−10.2	−15.9	−31.9	−75.9 ± 45.6

Referenced value using the tau data to calculate a_μ

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = (12.5 \pm 6.0) \times 10^{-10}$$

➢ CP violation in tau decays

$$A_{CP} = \frac{\Gamma(\tau^- \to \nu_\tau H) - \Gamma(\tau^+ \to \nu_\tau \bar{H})}{\Gamma(\tau^- \to \nu_\tau H) + \Gamma(\tau^+ \to \nu_\tau \bar{H})}$$

Intensive discussions on tau -> Ks pi nu

$$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$\approx (0.36 \pm 0.01)\%$$

SM prediction

$$\left(-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}}\right)\%$$

BaBar

[Bigi et al., PLB'05] [Grossman et al., JHEP'12] [Lees et al., PRD'12]

[Cirigliano et al., PRL'18] [Rendo et al., PRD'19] [Chen et al., PRD'19 JHEP'20]

Other types of CPV observables: T-odd triple-product asymmety

A typical T-odd kinematical variable:

$$\xi \equiv \varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma \underset{\text{of particle } a}{\overset{\text{rest frame}}{=\!=\!=}} \vec{b} \cdot (\vec{c} \times \vec{d})\, m_a / s_a$$

a, b, c, d*: either *momentum* or *spin

T transformation $(t \to -t, \vec{p} \to -\vec{p}, \cdots)$: $\xi \to -\xi$

❖ **When spin is involved, measurement of polarization is needed.**

[Nelson, et al., PRD'94] [Tsai, PRD'95] [Datta, PRD'07] ...

❖ **When focusing on the situation with four momenta, *i.e.***

$$\xi = \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \underset{\text{of particle } 1}{\overset{\text{rest frame}}{=\!=\!=}} \vec{p}_2 \cdot (\vec{p}_3 \times \vec{p}_4)\, m_1$$

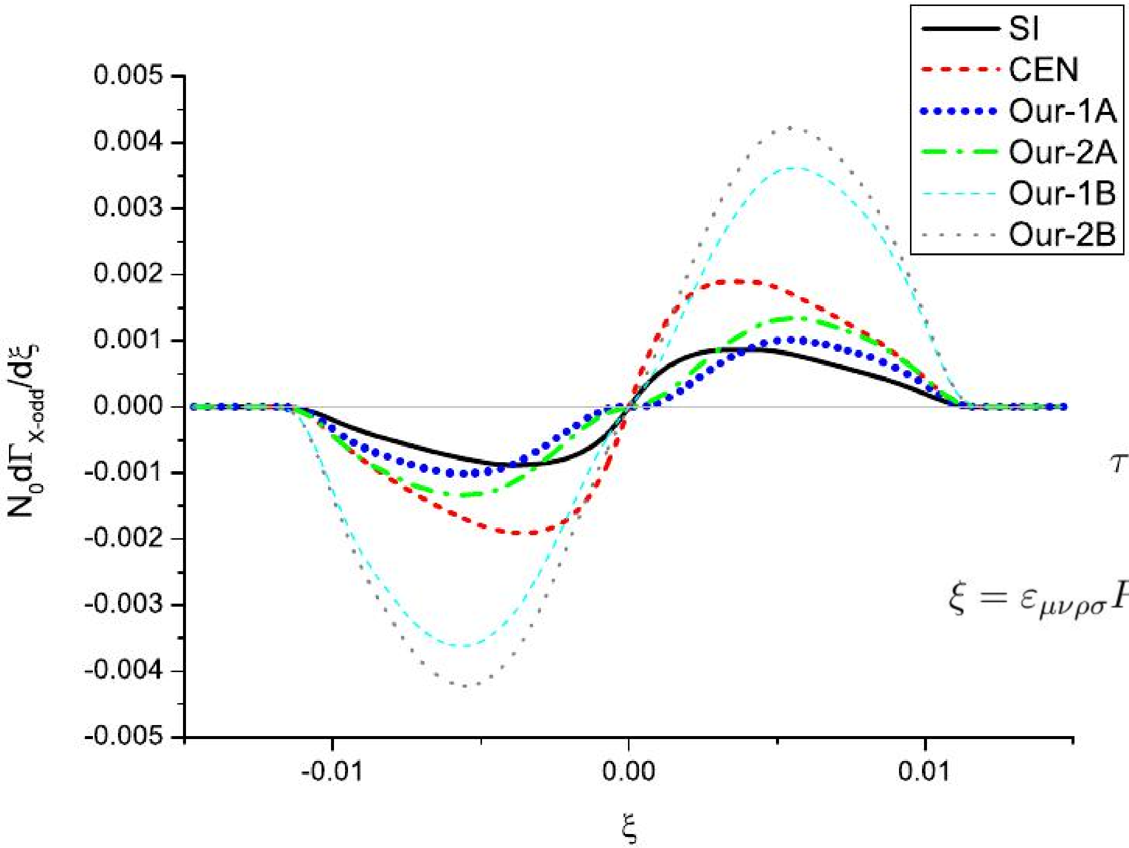
In this case, there should be at least four particles in the final state!

➢ **Pro: Strong phase is not necessary for a CPV phenomenon using TPA.**

Con: TPA could also be caused by the final-state interactions!

Predicitons of the T-odd asymmetry distribution in $\tau \to \pi\pi\gamma\nu_\tau$

[Chen,Duan,ZHG,JHEP'22]



$$\tau^-(P) \to \pi^-(p_1)\pi^0(p_2)\nu_\tau(q)\gamma(k)$$

$$\xi = \varepsilon_{\mu\nu\rho\sigma} P^\mu k^\nu p_1^\rho p_2^\sigma / m_\tau^4 \xlongequal[\text{of } \tau]{\text{rest frame}} \vec{k} \cdot (\vec{p}_1 \times \vec{p}_2)/m_\tau^3$$

Variant predictions for the branching ratios (×10⁻⁴):

$E_\gamma^{\rm cut}$	SI	CEN	Our-1A	Our-2A	Our-1B	Our-2B
100MeV	7.9	8.3	8.7/9.6/8.6/9.4	9.5/10/9.2/9.7	13/9.6/12/9.4	14/10/13/9.7
300MeV	1.5	1.8	2.4/3.0/2.3/2.8	2.9/3.3/2.6/3.0	5.6/3.0/5.2/2.8	6.3/3.3/5.5/3.0
500MeV	0.26	0.40	0.73/1.0/0.68/0.90	0.93/1.1/0.81/0.91	2.6/1.0/2.4/0.90	2.9/1.1/2.4/0.91

It has the good chance to be measured in STCF.

Prospects of revealing the genuine CPV signals

- **CPV signals can be probed by taking the differences of A_ξ in $\tau^- \to \pi^-\pi^0\gamma\nu_\tau$ and $\tau^+ \to \pi^+\pi^0\gamma\nu_\tau$**

$$A_\xi = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \qquad \overline{A}_{\bar\xi} = \frac{\overline{\Gamma}_+ - \overline{\Gamma}_-}{\overline{\Gamma}_+ + \overline{\Gamma}_-}$$

$$\overline{\Gamma}_+ = \frac{(2\pi)^4}{2m_\tau}\int_{\bar\xi>0} \mathrm{d}\Phi\,(\overline{\hat{M}_0} + \bar\xi\overline{\hat{M}_1}), \qquad \overline{\Gamma}_- = \frac{(2\pi)^4}{2m_\tau}\int_{\bar\xi<0} \mathrm{d}\Phi\,(\overline{\hat{M}_0} + \bar\xi\overline{\hat{M}_1})$$

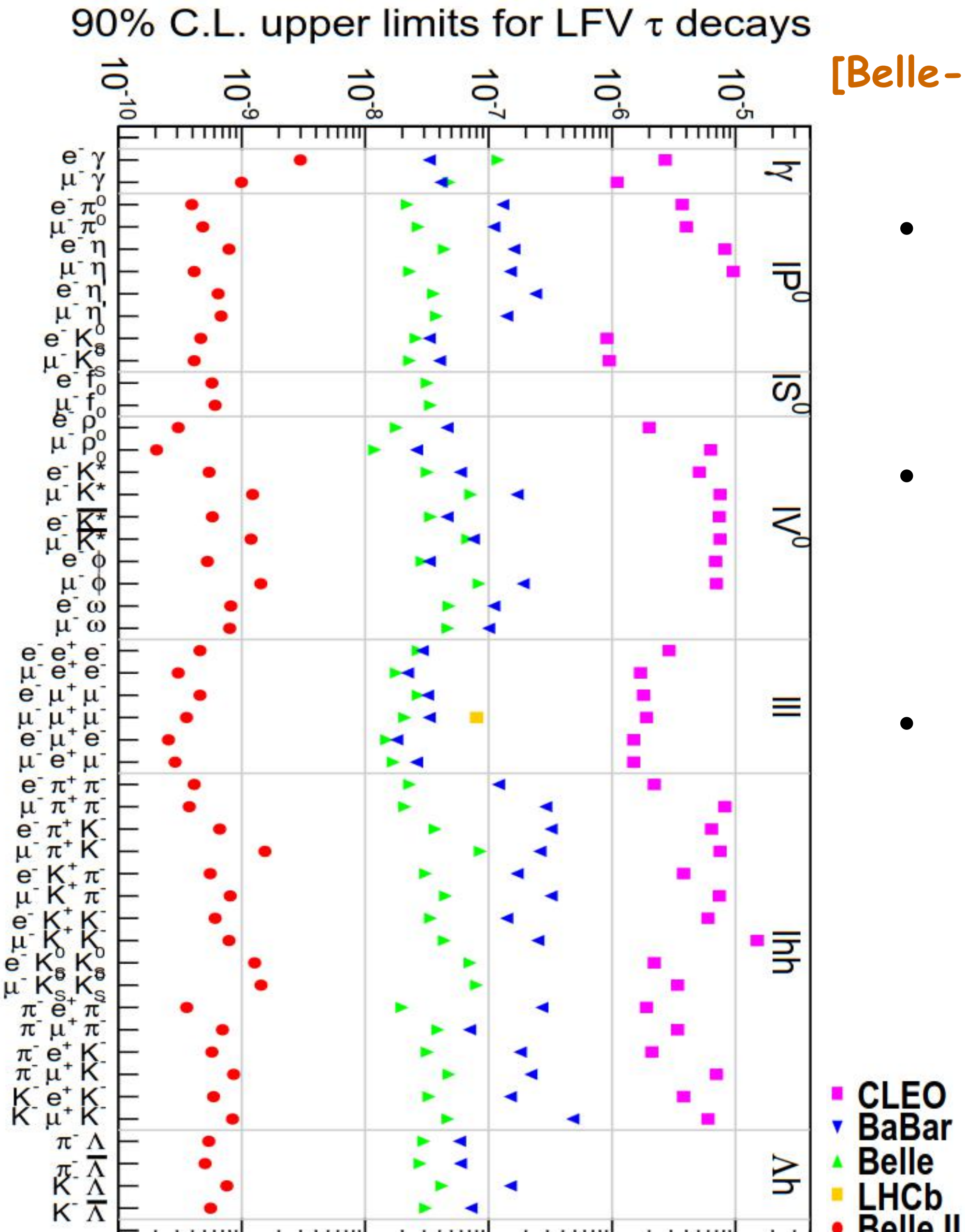
$$\mathcal{M} = e\,G_F\,V_{ud}^*\epsilon^{*\mu}(k)\,\Big\{(1+\mathbf{g_V})F_\nu\bar u(q)\gamma^\nu(1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u(P)$$
$$+\big[(1+\mathbf{g_V})V_{\mu\nu} - (1-\mathbf{g_A})A_{\mu\nu}\big]\bar u(q)\gamma^\nu(1-\gamma_5)u(P)\Big\}$$

$$\mathcal{A}_\xi = A_\xi - \overline{A}_{\bar\xi} \supset \mathrm{Im}(\mathbf{g_V^*g_A})\mathrm{Re}[F_V(t/u)^*A_i],\ \mathrm{Im}(\mathbf{g_V^*g_A})\mathrm{Re}(V_j^*A_i)$$

- **Generally speaking, sizable hadronic contributions are also expected to enhance the CPV signals in $\tau \to \pi\pi\gamma\nu_\tau$.**

- **TPA in other types of τ decays could be also possible.**

➢ Charged lepton flavor violation in tau decays



[Belle-II, '22]

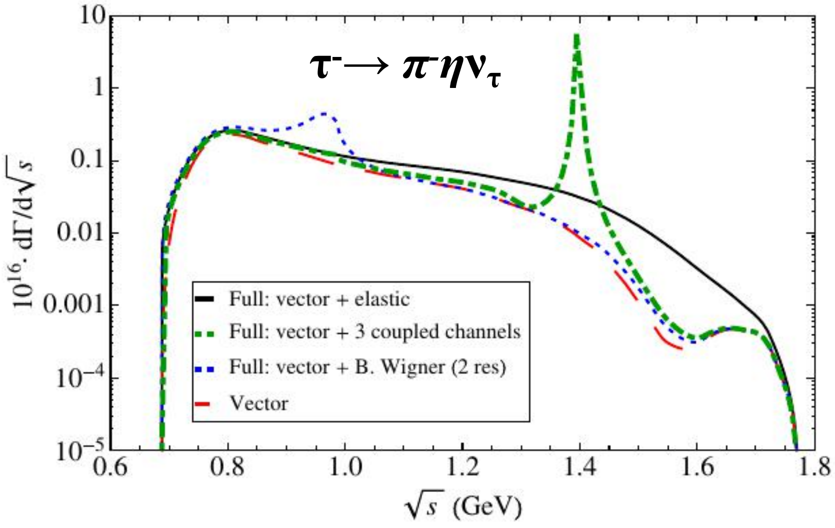
- **Not only statistic but also systematic uncertainties are important in $\tau \rightarrow l\,\gamma$**

- **Clean backgroud makes $\tau \rightarrow l\, l'\, l''$ one of the best channels to search for LFV signals.**

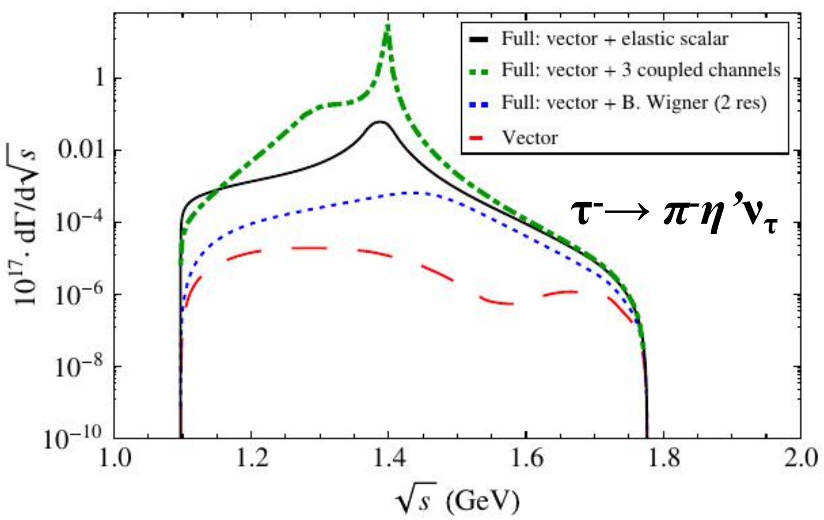
- **$\tau \rightarrow l + hadrons$ provides a different laboratory to probe different LFV origins, comparing with the pure leptonic processes.**

➢ Proposals to search for second-class currents in tau decays

- First class of hadron currents : $J^{PG} = 0^{++}, 0^{--}, 1^{+-}, 1^{-+}$
- Second class of hadron currents : $J^{PG} = 0^{+-}, 0^{-+}, 1^{++}, 1^{--}$, which are usually suppressed at the level of $10^{-6} \sim 10^{-5}$ and are not observed yet in EXP.



BR: $(0.3\sim2.0) \times 10^{-5}$



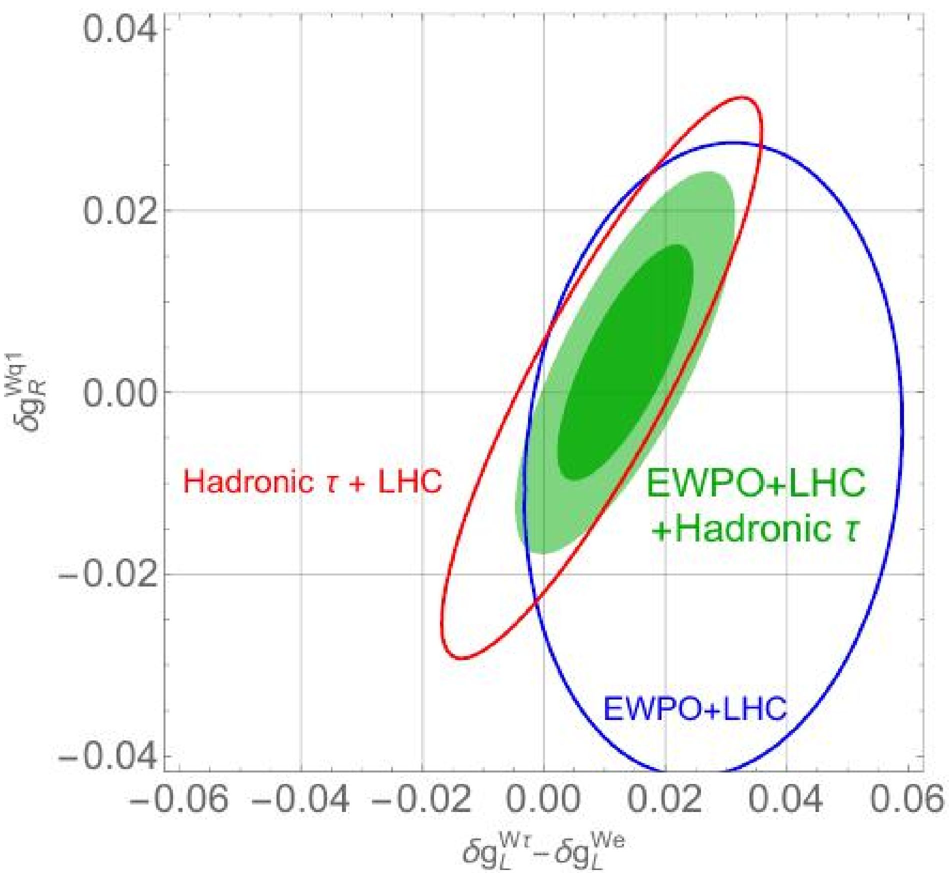
BR: $10^{-9} \sim 10^{-6}$

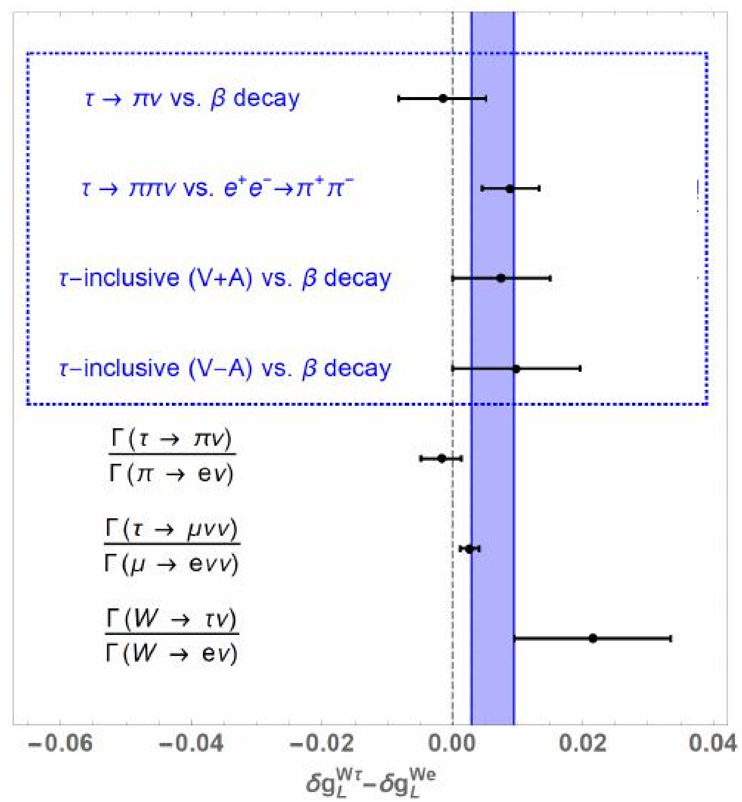
[Escribano et al., PRD'16]

- **Powerful tool to constrain new physics: combination of hadronic tau data + LHC data**

$$\mathcal{L}_{\rm eff} = -\frac{G_\mu V_{uD}}{\sqrt{2}}\Big[\Big(1+\epsilon_L^{D\ell}\Big)\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1-\gamma_5)D+\epsilon_R^{D\ell}\;\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar{u}\gamma^\mu(1+\gamma_5)D$$
$$+\bar{\ell}(1-\gamma_5)\nu_\ell\cdot\bar{u}\Big[\epsilon_S^{D\ell}-\epsilon_P^{D\ell}\gamma_5\Big]D+\frac{1}{4}\hat{\epsilon}_T^{D\ell}\,\bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Big]+{\rm h.c.},$$

[Cirigliano et al., PRL'18]





Summary

Tau offers a laboratory for a broad range of interesting topics:

- **Precision tests of SM: CKM, α_s, m_τ, lepton universality, ...**
- **Hadron interactions: light-flavor resonances, chiral symmetry, form factors, second-class currents, ...**
- **BSM tests:**

  CPV (rate asym., triple-product asym.)

  LFV (lepton/radiative decays, hadron decays)

  ... ...

Thanks for your patience!