Some progresses on Electroweak phase transition

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CEPC Flavor Physics/New Physics/Detector Technology Workshop (August 13-18, 2023)

Contents

- Motivation
- First-order phase transition & GW
- Implications&Constraints
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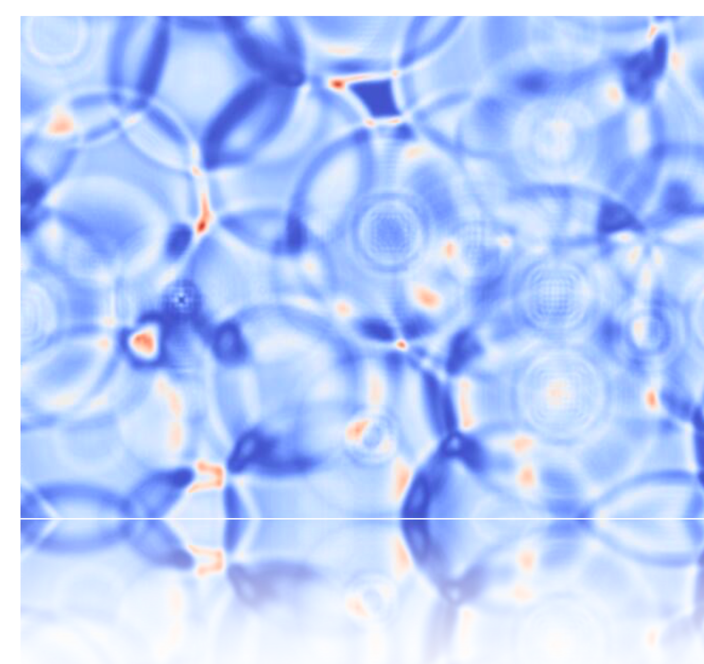
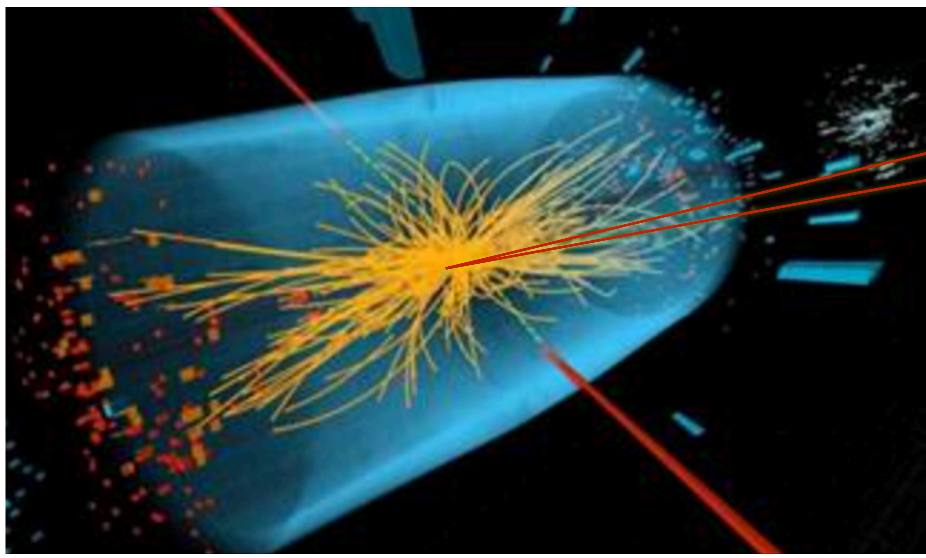
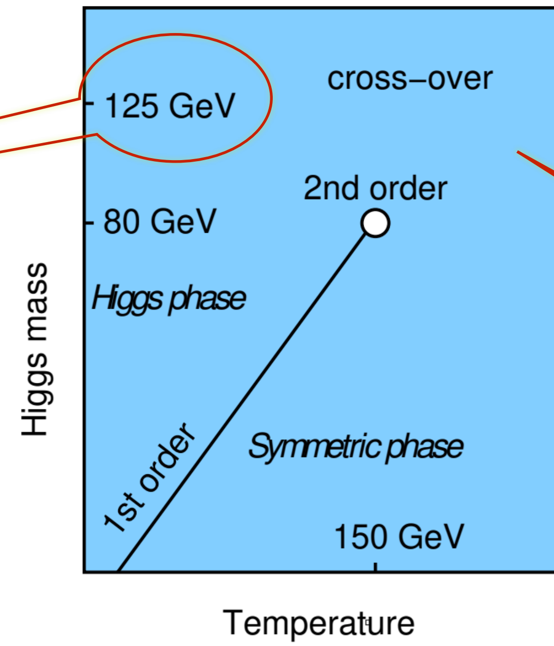
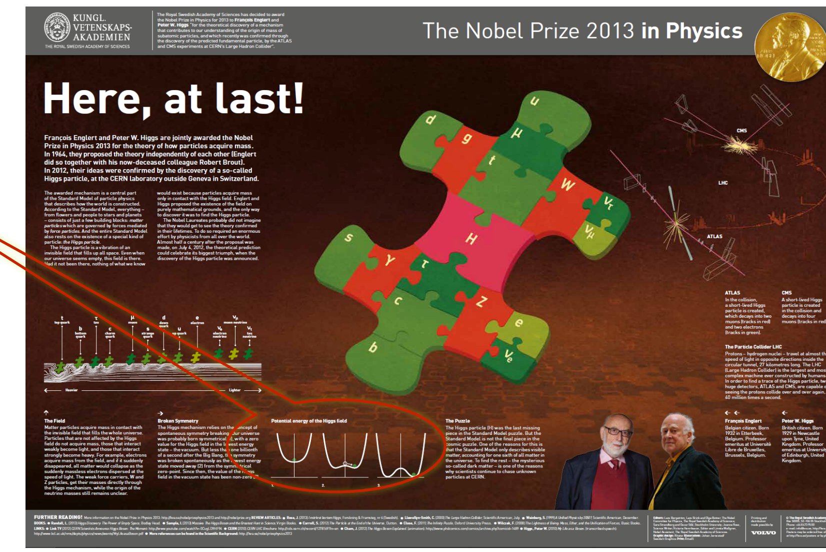


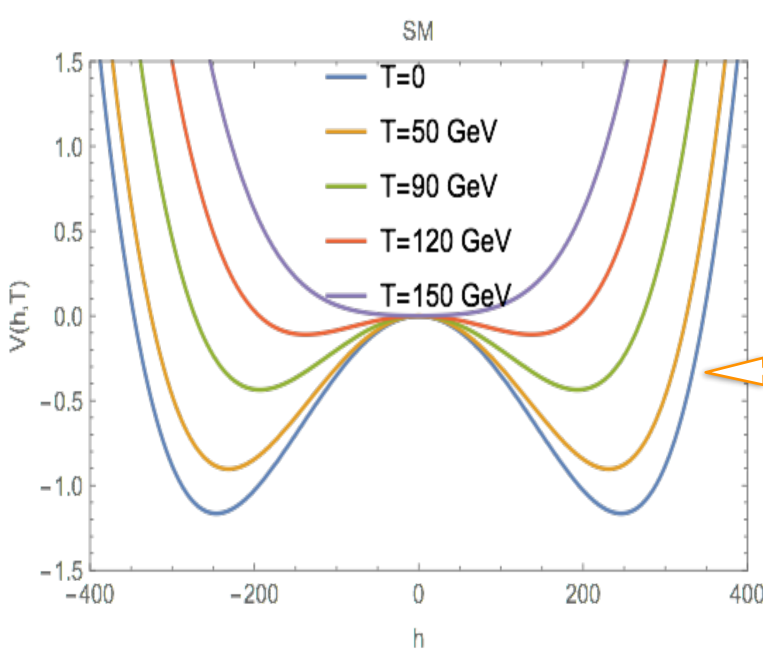
研究背景与动机

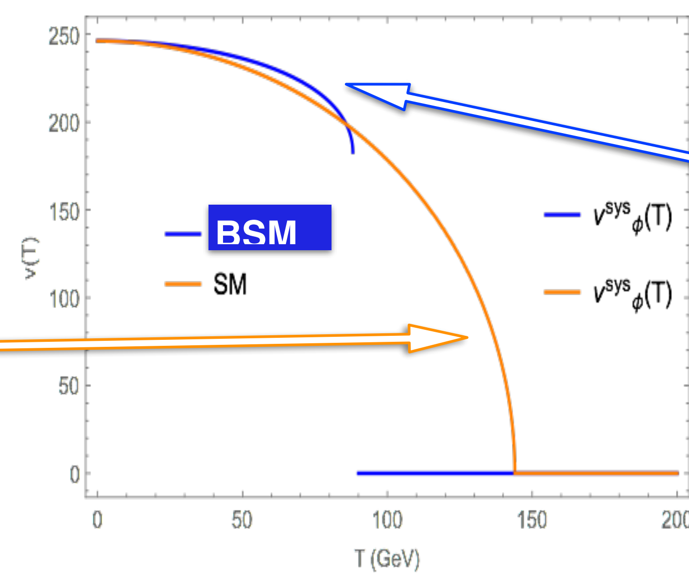
电弱对称性破缺的热历史是什么？

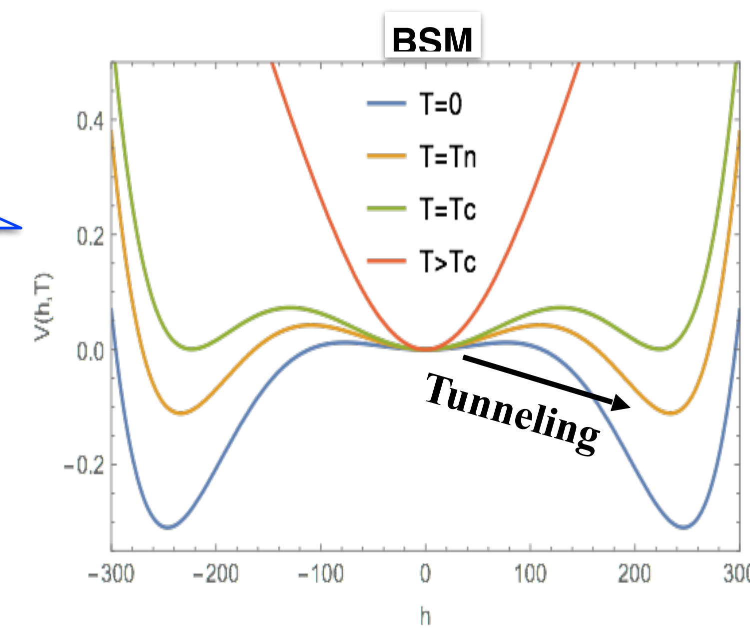










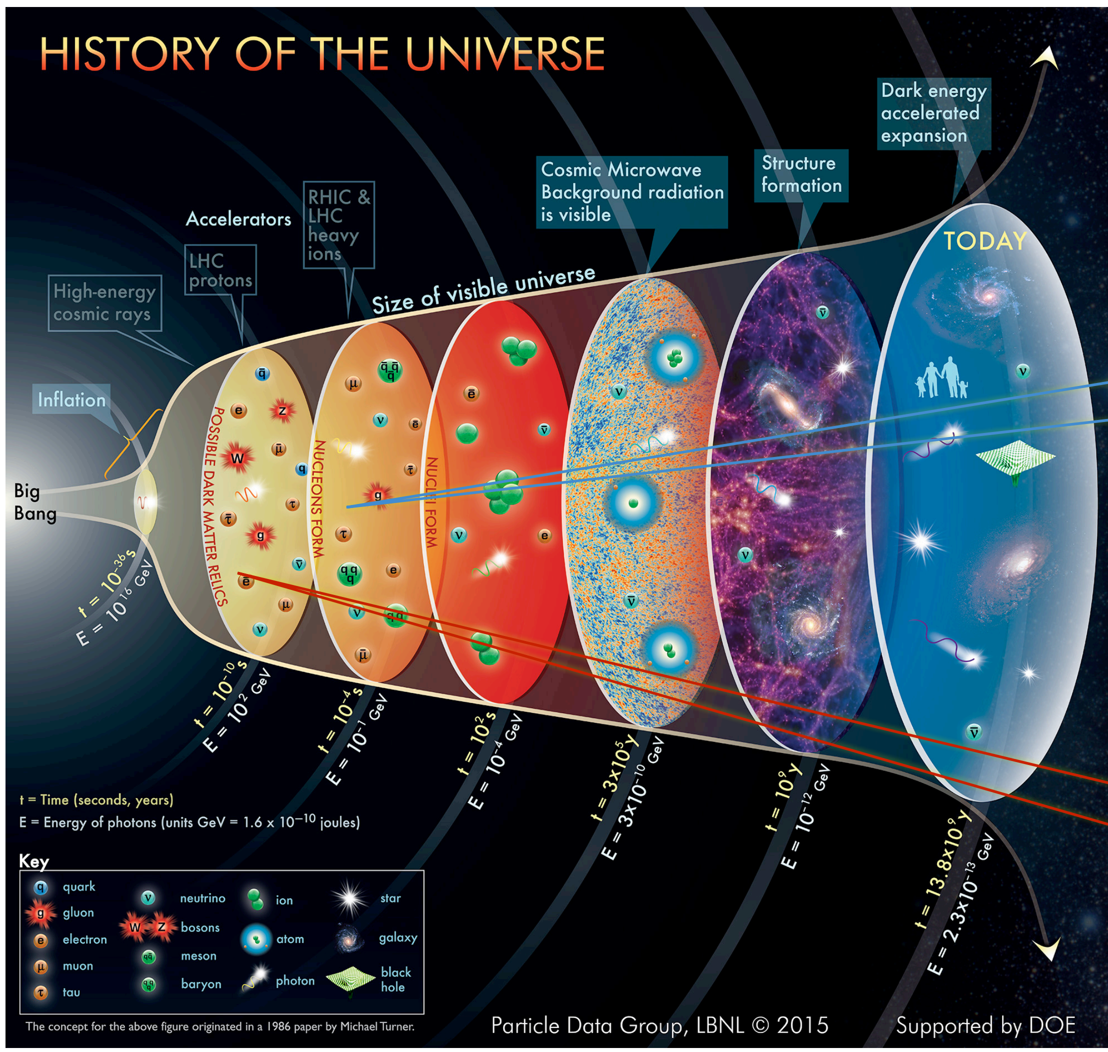


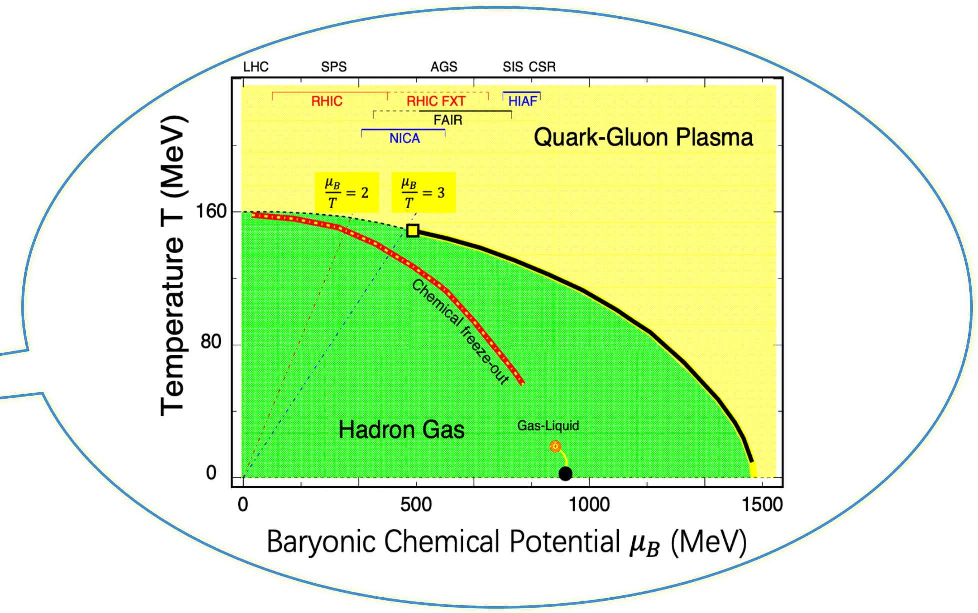
平滑过度

一阶相变

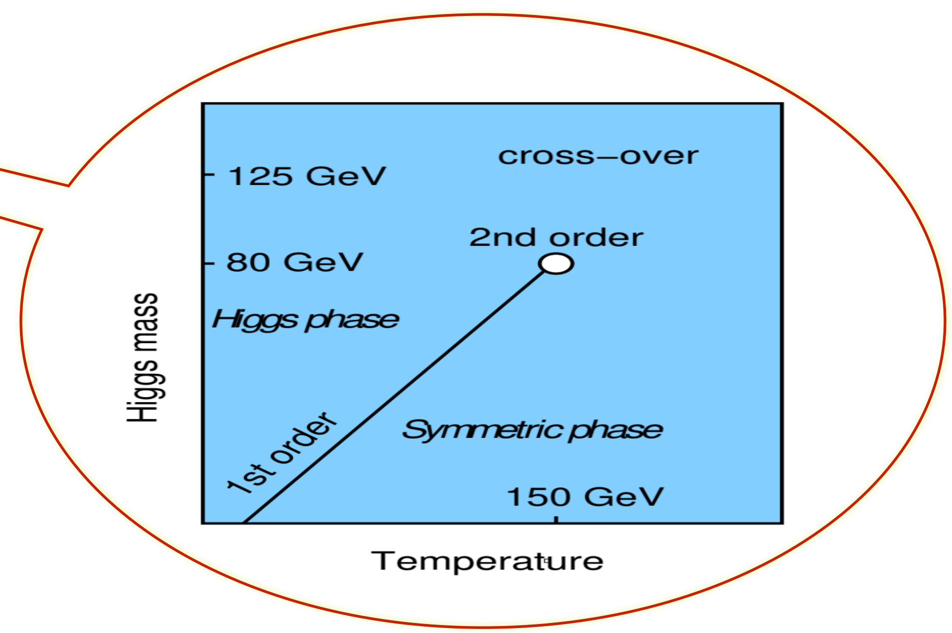
研究背景与动机

对称展示宇宙之美，不对称生成宇宙之实！—李政道





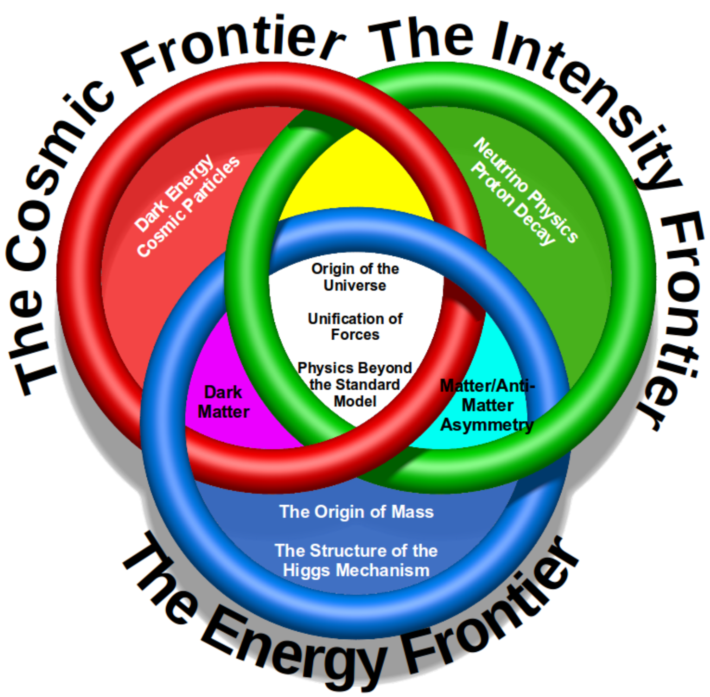
QCD
相图



电弱
相变

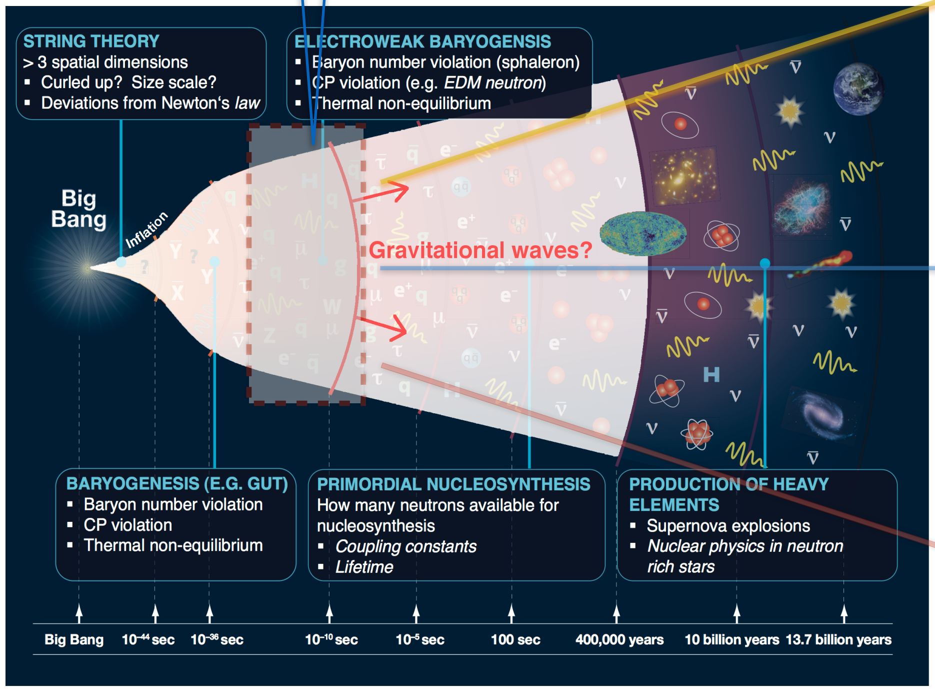
研究背景与动机

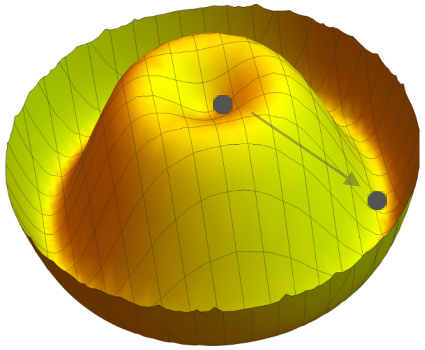
粒子物理三个前沿



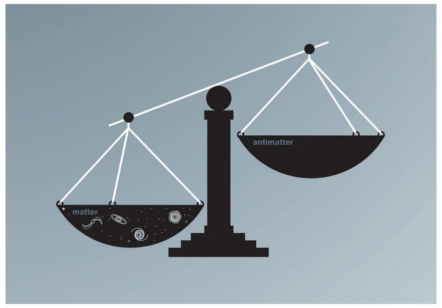
强一阶电弱相变

相关粒子物理

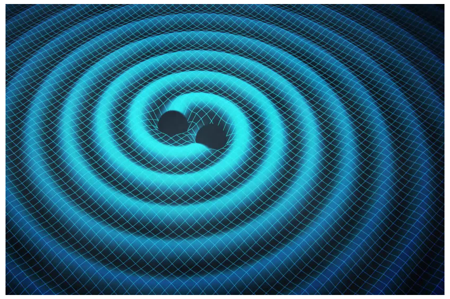




电弱对称性破缺，基本粒子质量起源



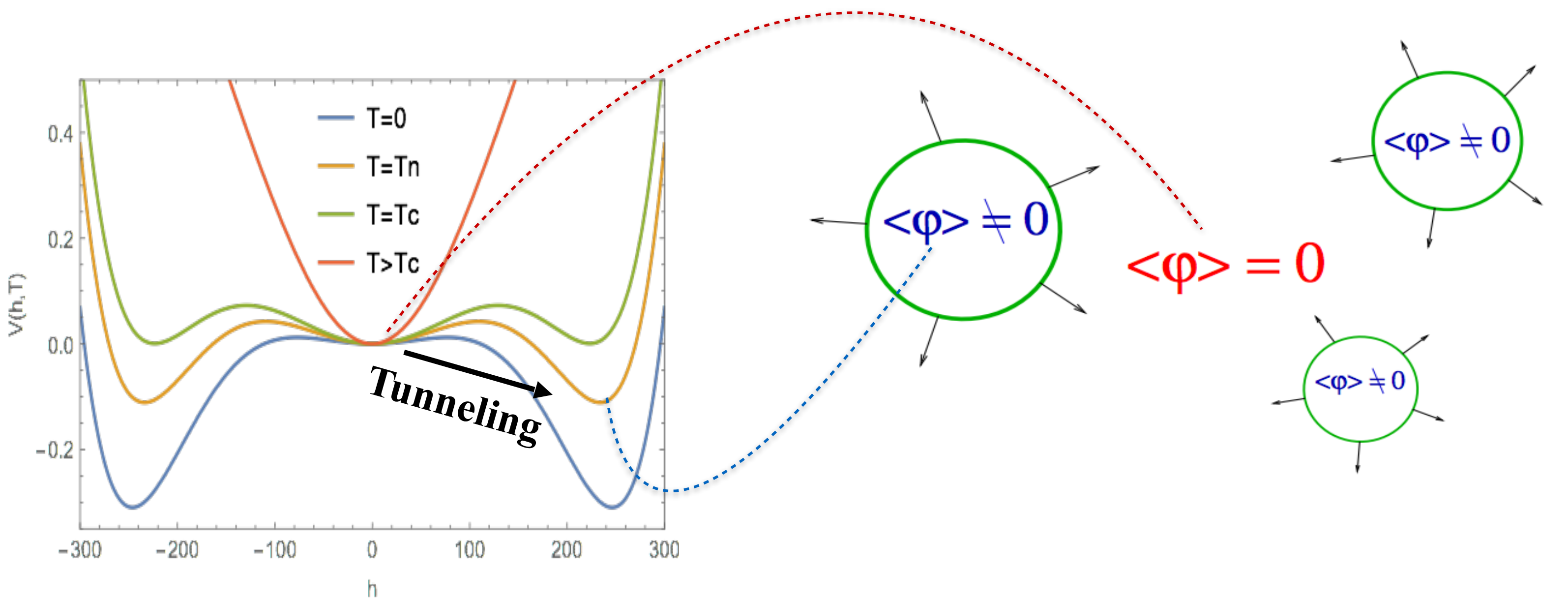
宇宙正反物质不对称

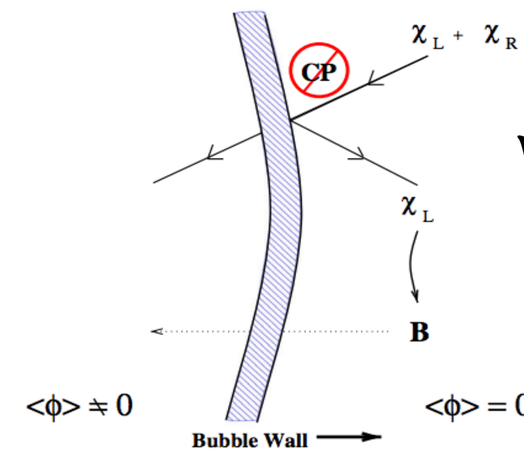


相变引力波，检验爱因斯坦广义相对论

粒子物理描述早期宇宙演化

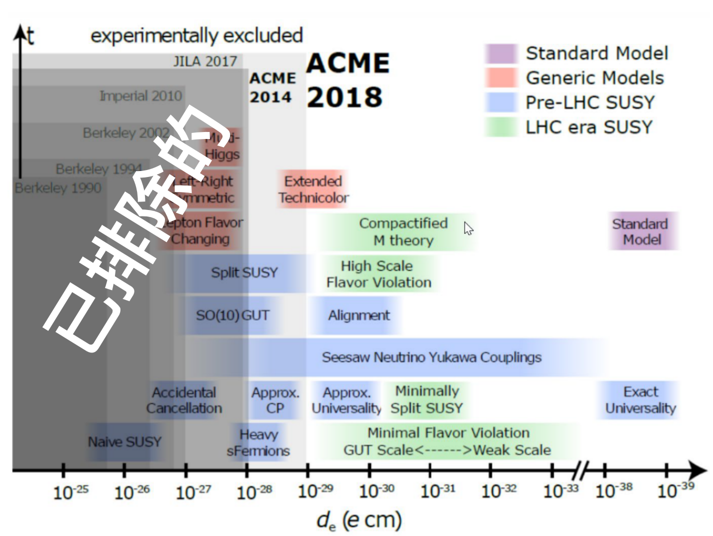
正反物质不对称&强一阶电弱相变

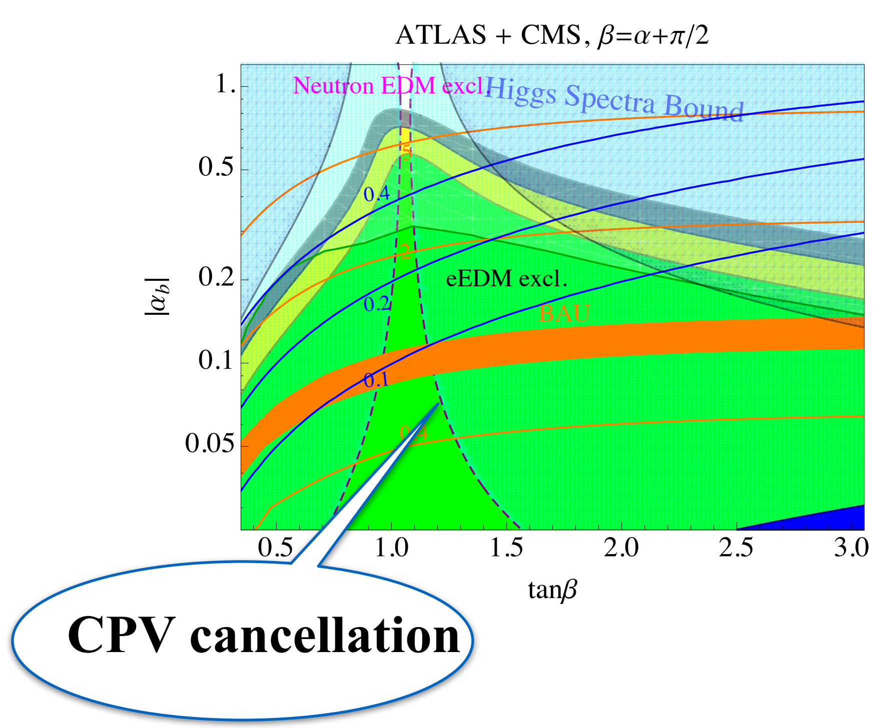


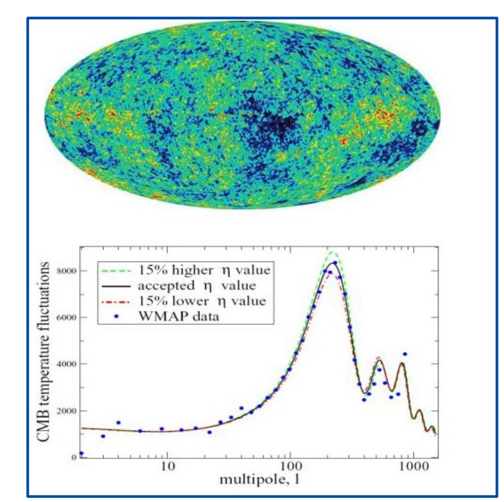


电弱相变中解释BAU

Chup etal，Rev Mod Phys.91.015001





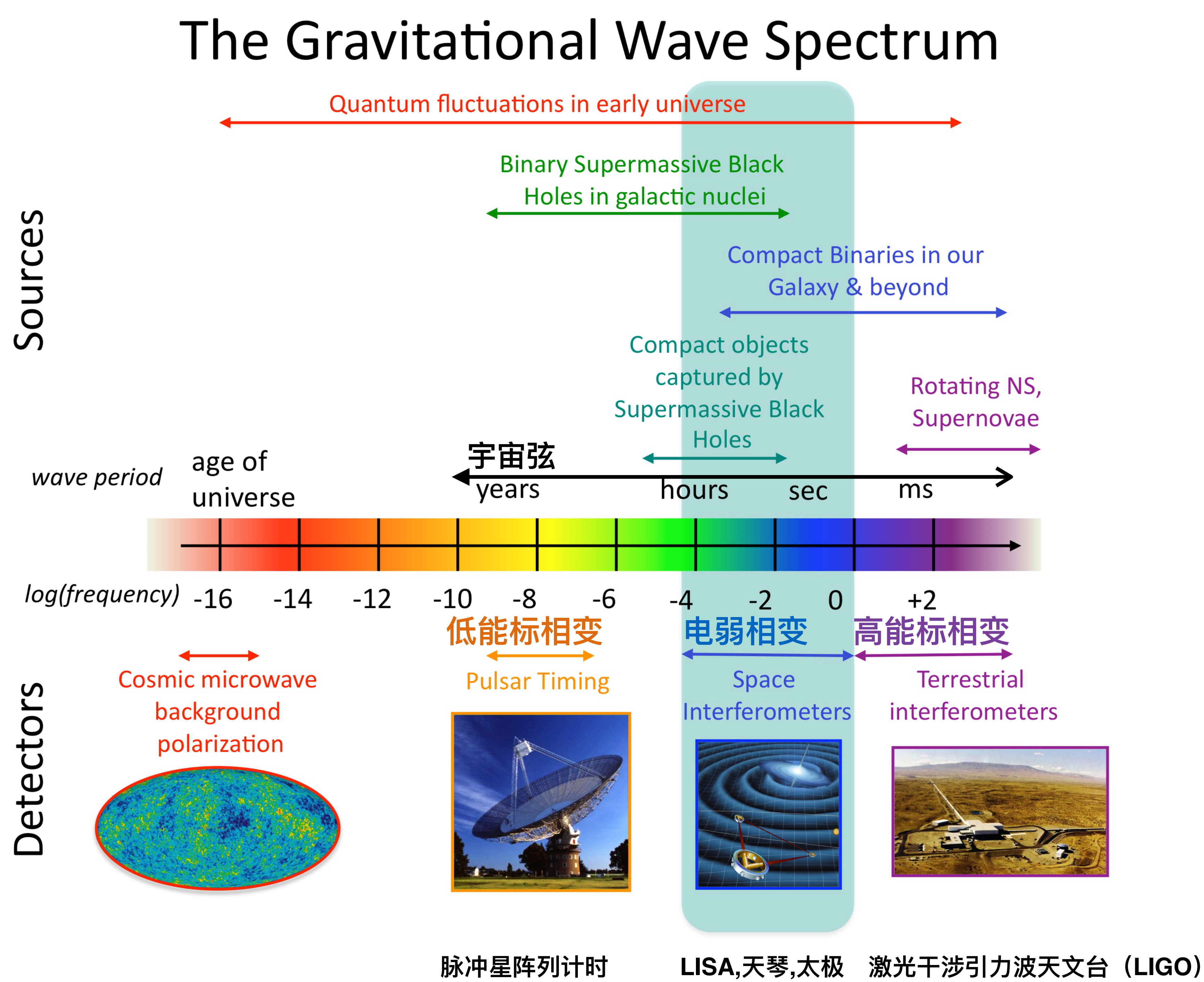


$$\frac{n_B}{s} = (8.579 \pm 0.109) \times 10^{-11}$$

Bian, Liu, Shu, PRL115 (2015) 021801

See also recent studies of FPH and YCWu

引力波探测



随机引力波探测开启了探索早期宇宙背后新物理的一个新的窗口

电弱相变相关粒子物理宇宙学

Beyond Standard Model for DM&Neutrino&etc

Collider phenomenology

Thermal QFT

Equilibrium thermodynamics

Perturbation
Or Lattice

Bubble nucleation &hydrodynamics

Lattice simulation

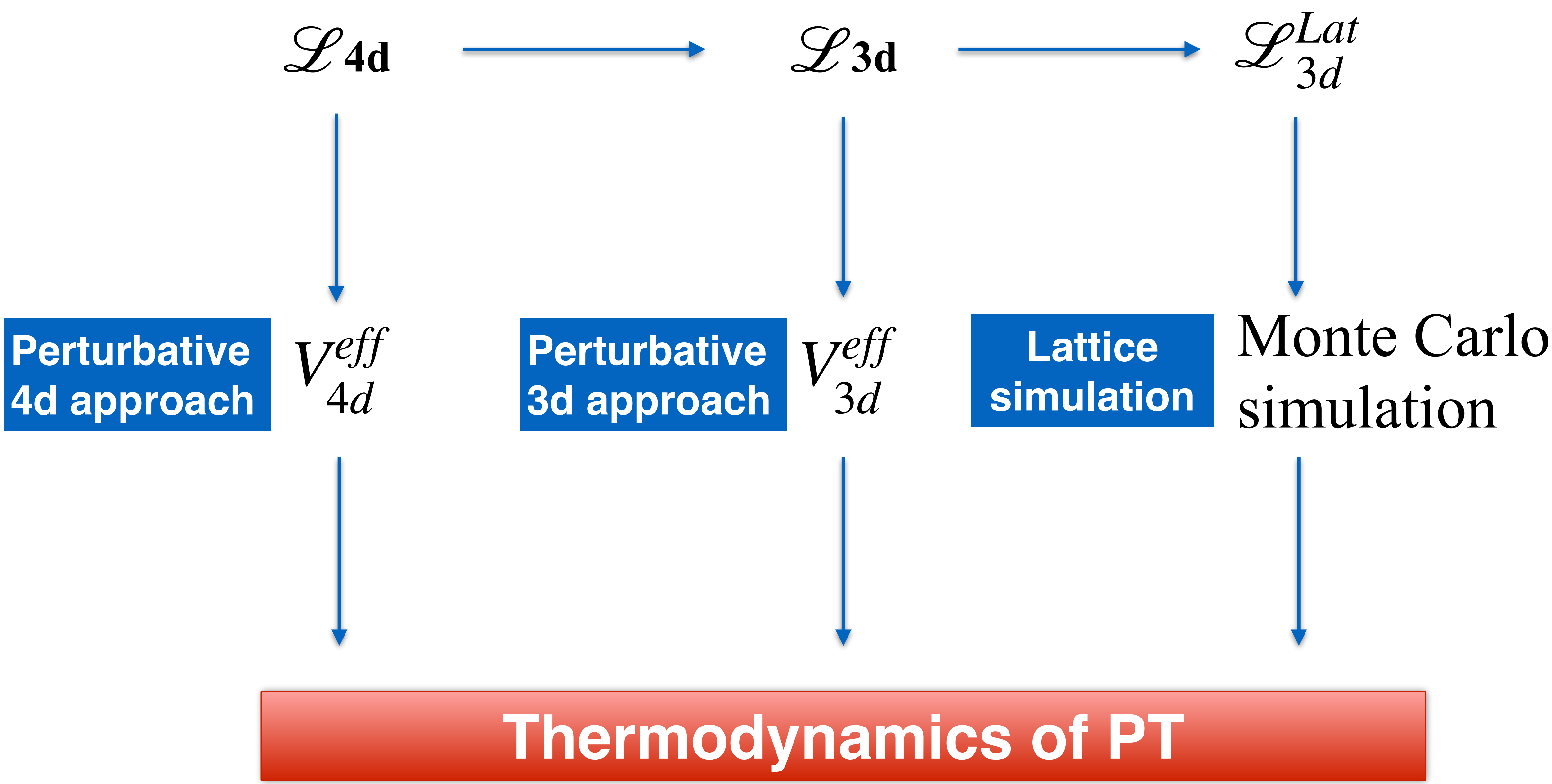
CPV
&Transport Eq.

Gravitational waves

Baryon asymmetry

For PT duration effect see
HK Guo's paper and talk

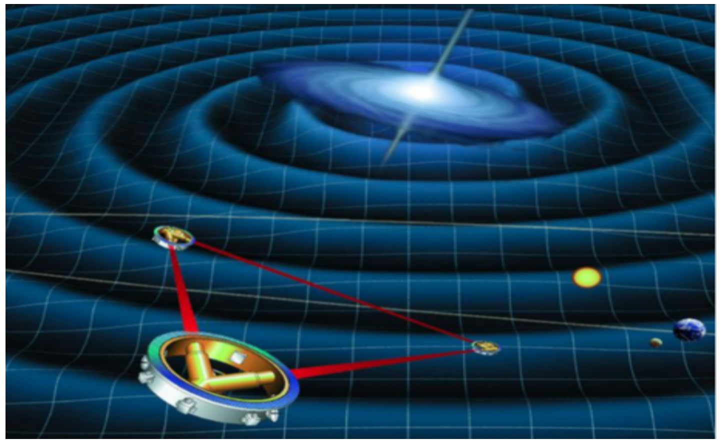
▸ Methods for PT dynamics study



粒子物理&相变引力波

重要的引力波源，主要科学目标之一

PTA,LIGO,LISA,天琴,太极,…



超出粒子物理模型
新物理模型

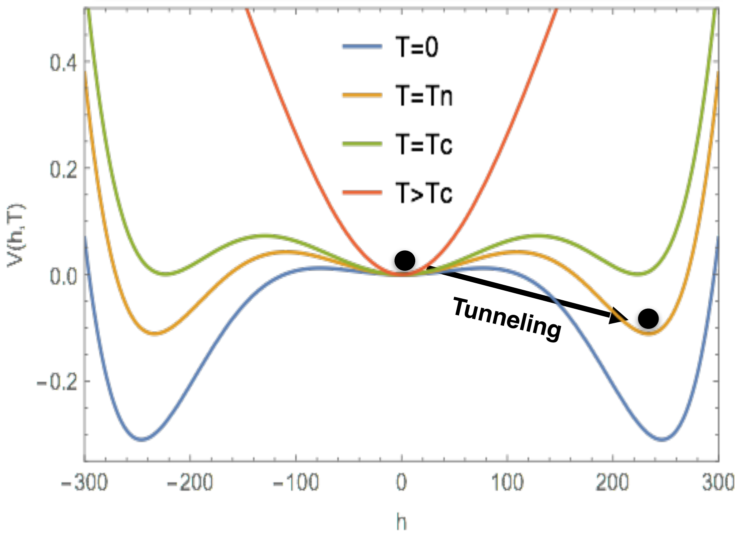
PT parameters

Effective action → β, H_*

Energy budget → α, $\kappa(\alpha, v_w)$

Bubble wall dynamics → v_w

有限温场论计算



GW power spectrum

Numerical simulations →
$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

格点场论模拟

LISA sensitivity

Configuration + noise level →
$h^2\Omega_{sens}(f)$

Signal-to-noise ratio

SNR

Chiara Caprini *et al* JCAP03(2020)024

真空泡碰撞、合并、流体演化产生引力波

有限温度有效势能

$$V(\phi,T)=\frac{1}{2}\gamma(T^2-T_0^2)\phi^2-\frac{1}{3}AT\phi^3+\frac{1}{4}\lambda\phi^4$$

新物理

标量场-相对论流体运动方程

$$-\ddot{\phi}+\nabla^2\phi-\frac{\partial V}{\partial\phi}=\eta W(\dot{\phi}+V^i\partial_i\phi)$$

η：粒子和真空泡壁相互作用

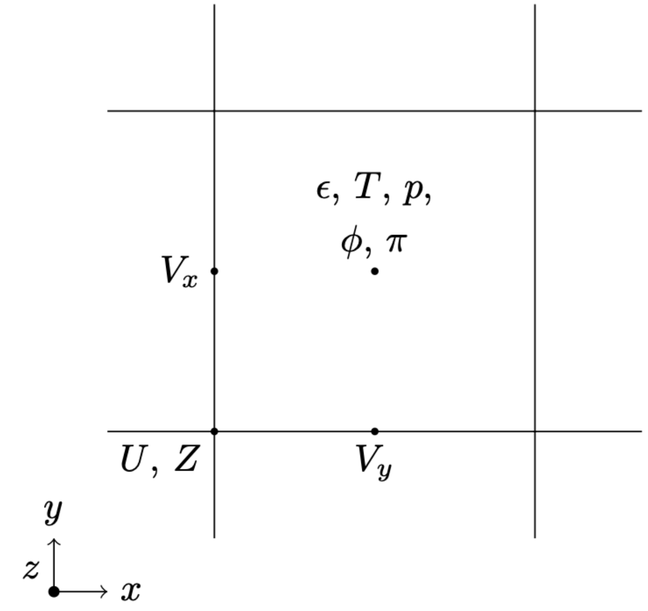
$$\dot{E}+\partial_i(EV^i)+p[\dot{W}+\partial_i(WV^i)]-\frac{\partial V}{\partial\phi}W(\dot{\phi}+V^i\partial_i\phi)=\eta W^2(\dot{\phi}+V^i\partial_i\phi)^2$$

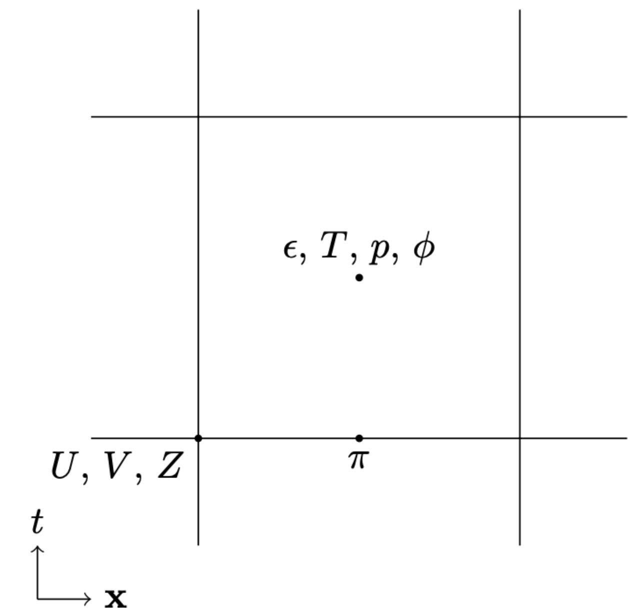
$$\dot{Z}_i+\partial_j(Z_iV^j)+\partial_i p+\frac{\partial V}{\partial\phi}\partial_i\phi=-\eta W(\dot{\phi}+V^j\partial_j\phi)\partial_i\phi$$

equation of state
$$\epsilon(T,\phi)=3aT^4+V(\phi,T)-T\frac{\partial V}{\partial T},$$
$$p(T,\phi)=aT^4-V(\phi,T)$$

fluid momentum density $Z_i=W(\epsilon+p)U_i$

fluid energy density $E=W\epsilon$





V^i is the fluid 3-velocity

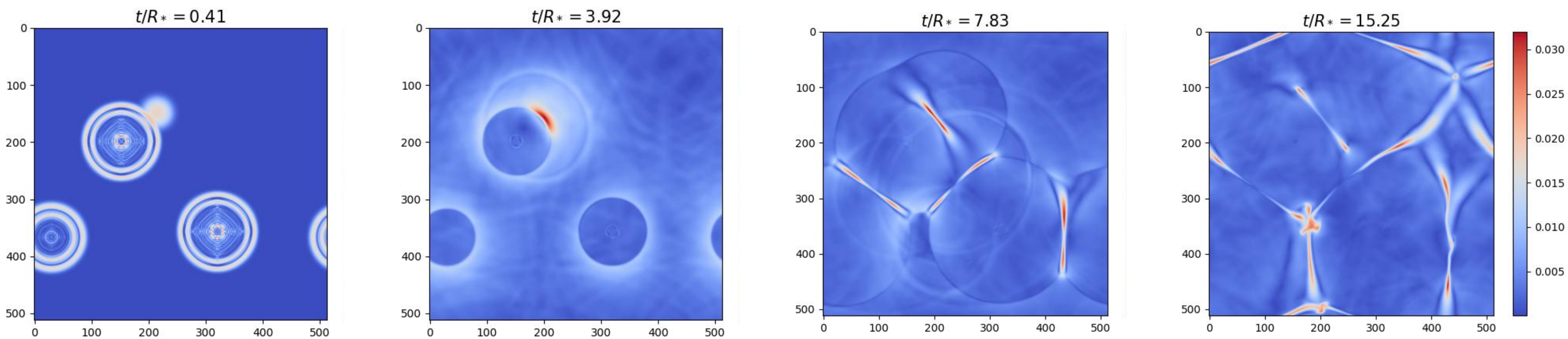
$U^i=WV^i$， W： relativistic γ-factor

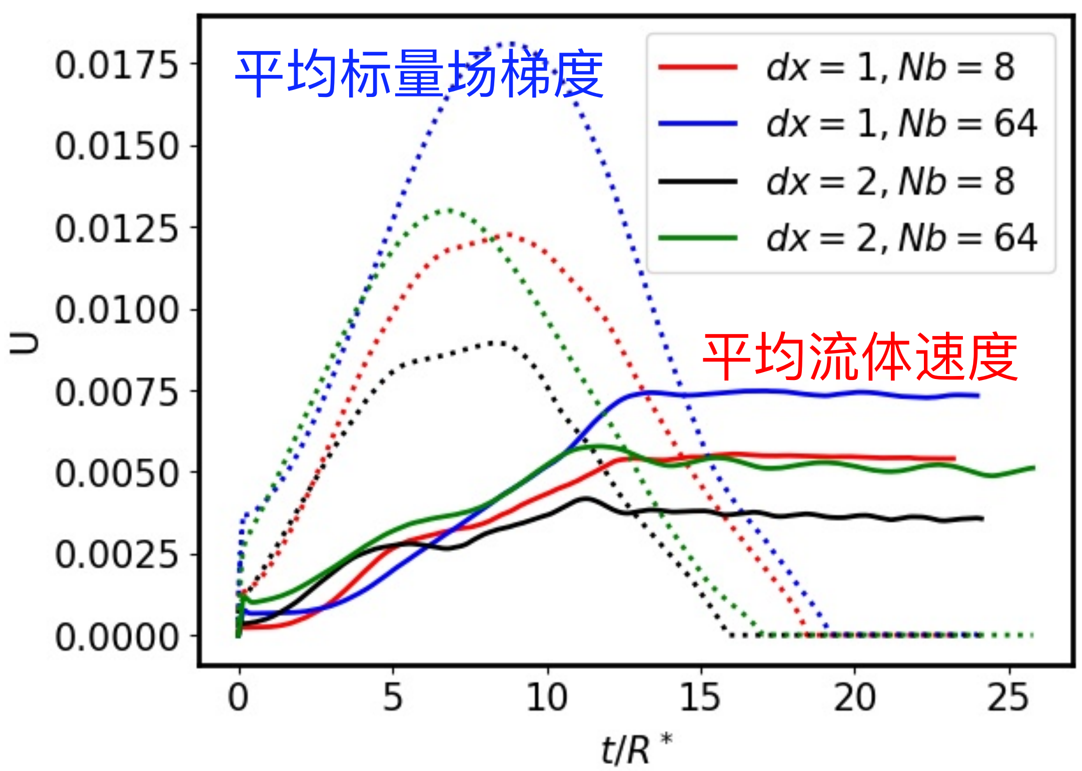
For single bubble and hydrodynamics, see JCAP06(2010)028 and FPH’s papers

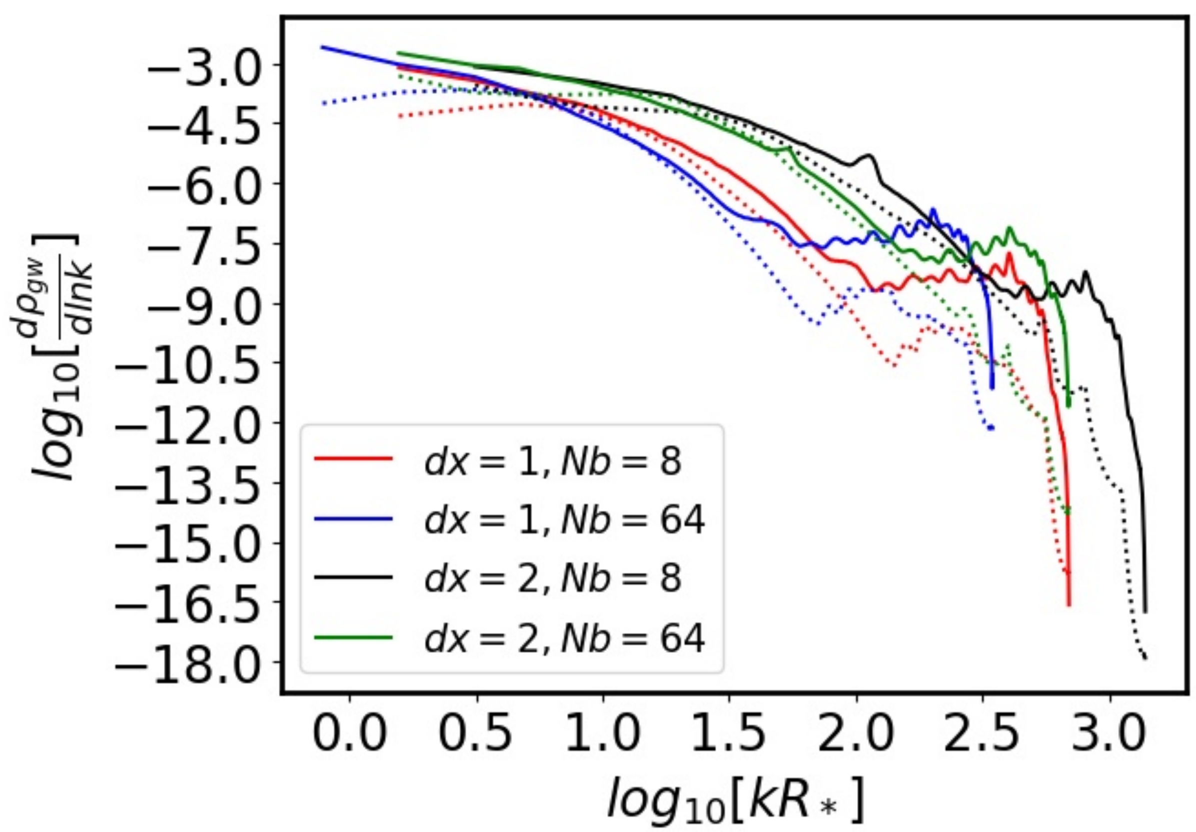
真空泡碰撞、合并、流体演化产生引力波

电弱相对论流体三速度大小的演化

格子大小 512^3　On GPUs







Bian, Cai, Jia, Zhao, Zhu, Paper In preparation

▶ One-step FOPT

PHYSICAL REVIEW LETTERS **126,** 251102 (2021)

Magnetic Field and Gravitational Waves from the First-Order Phase Transition

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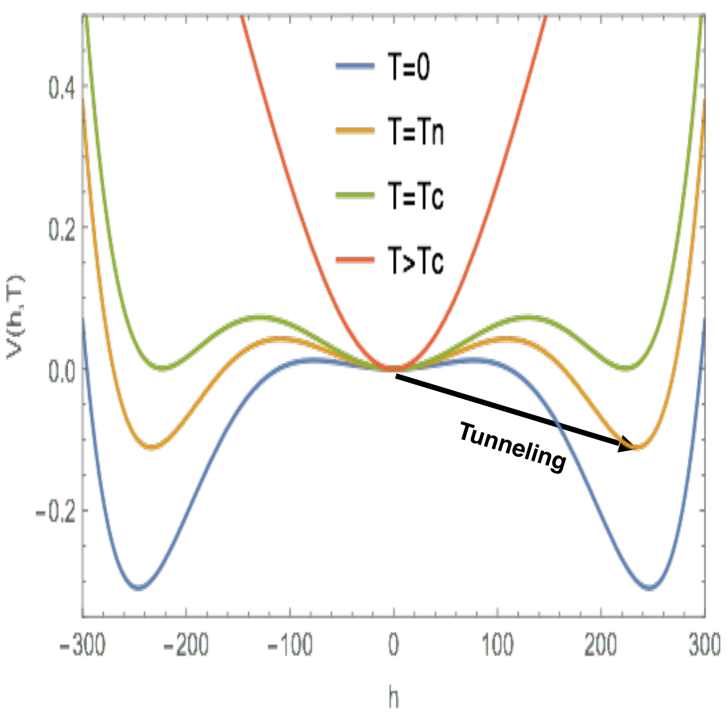
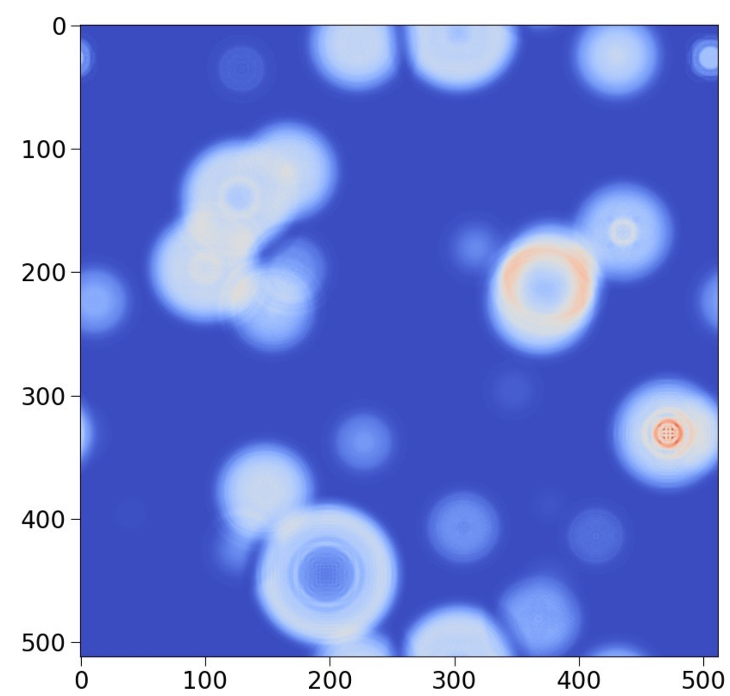
Finite-T Veff

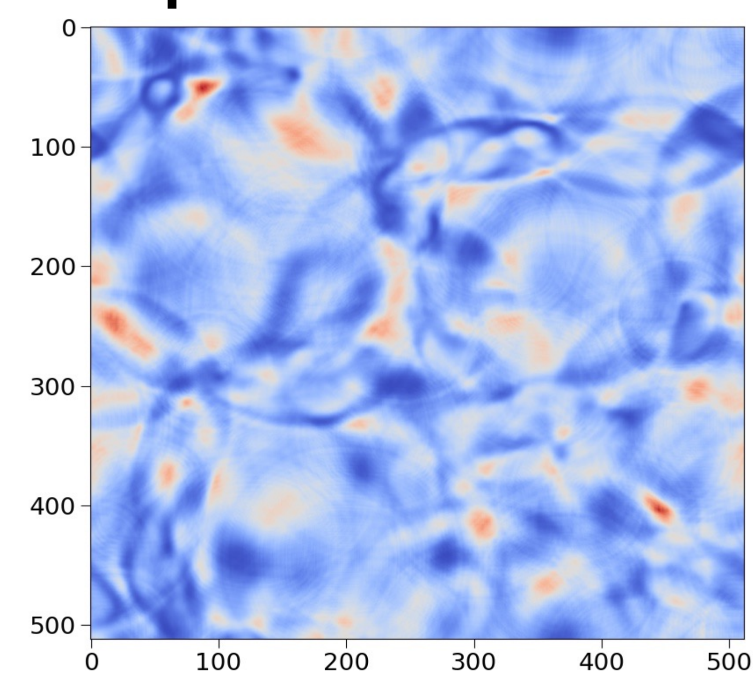


PT temperature（T_*）
duration（β）
strength（α）

Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

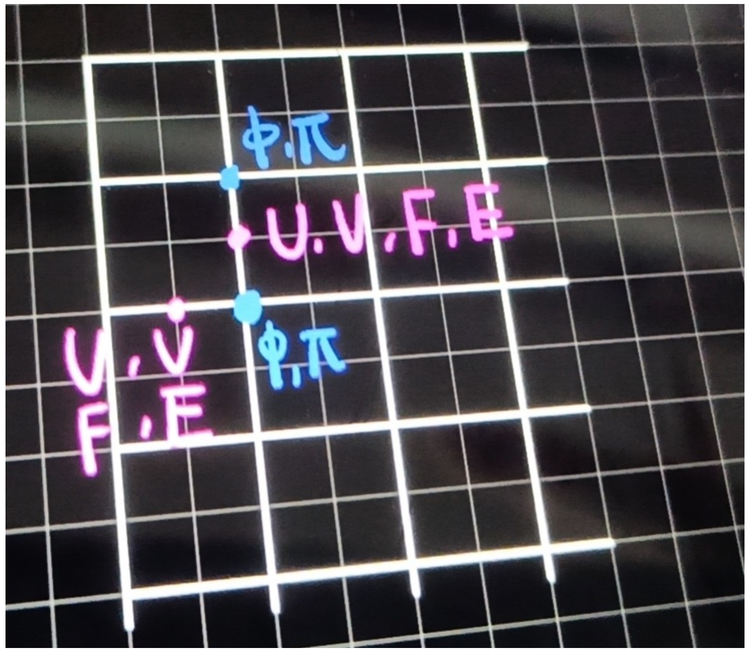
Lattice EW field foundation

$\Phi(t, x)$： Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$： SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i， $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields： Π(t+Δt/2, x), F (t+Δt/2, x) and E(t+ Δt/2, x), are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2,x) V_i(t,x)$$
$$U_i(t+\Delta t,x) = g \Delta x \Delta t F_i(t+\Delta t/2,x) U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

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PT process simulation

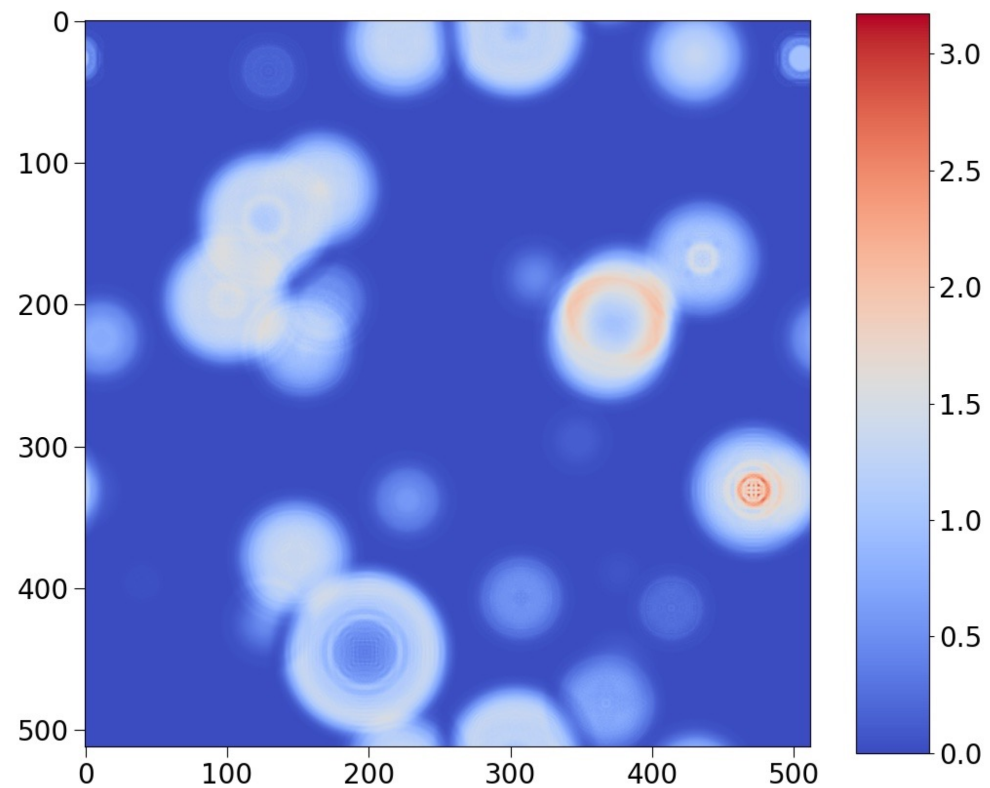
Field basis

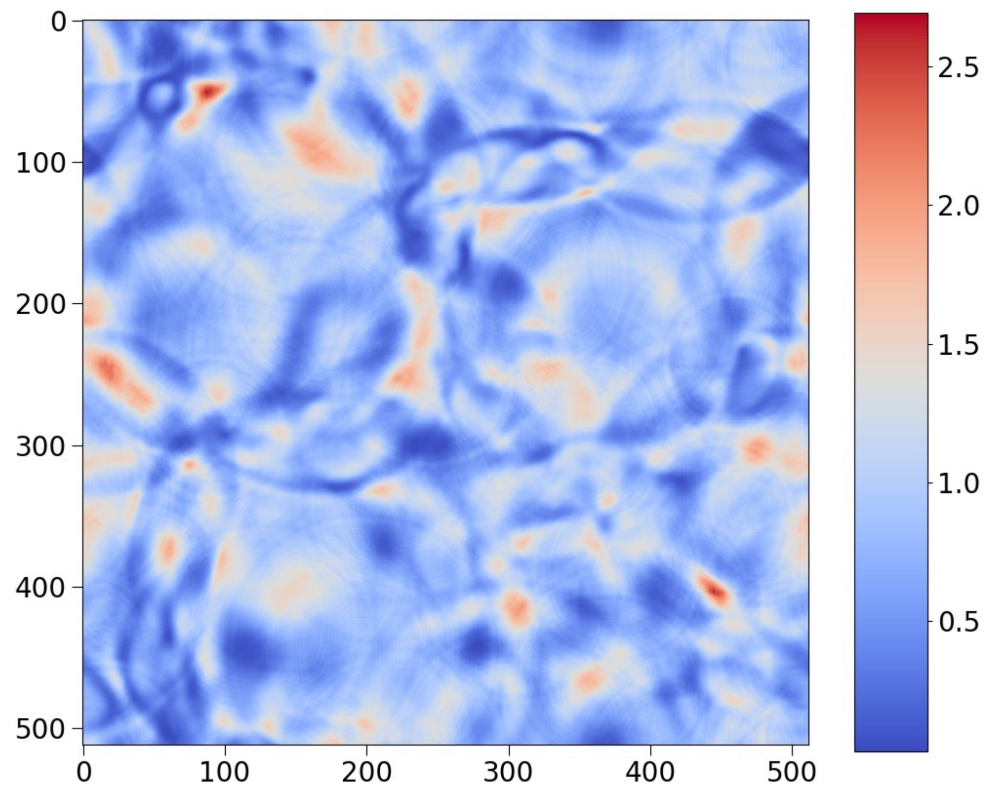
$$\begin{aligned}
\partial_0^2\Phi &= D_iD_i\Phi - 2\lambda(|\Phi|^2-\eta^2)\Phi - 3(\Phi^\dagger\Phi)^2\Phi/\Lambda^2,\\
\partial_0^2 B_i &= -\partial_j B_{ij} + g'\,\mathrm{Im}[\Phi^\dagger D_i\Phi],\\
\partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g\,\epsilon^{abc}W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger\sigma^a D_i\Phi].
\end{aligned}$$

$$\partial_0\partial_j B_j - g'\,\mathrm{Im}[\Phi^\dagger\partial_0\Phi] = 0,$$
$$\partial_0\partial_j W_j^a + g\,\epsilon^{abc}W_j^b\partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger\sigma^a\partial_0\Phi] = 0.$$

Lattice implementation

$$\begin{aligned}
\Pi(t+\Delta t/2,x) =& \Pi(t-\Delta t/2,x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i\big[U_i(t,x)V_i(t,x)\Phi(t,x+i)\\
&-2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)\big] - \frac{\partial U}{\partial\Phi^\dagger}\Big\}\\
\mathrm{Im}[E_k(t+\Delta t/2,x)] =& \mathrm{Im}[E_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)]\\
&-\frac{2}{g'\Delta x^3}\sum_i\mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x)\\
&+V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\}\\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2,x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)]\\
&-\frac{1}{g\Delta x^3}\sum_i\mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x)\\
&+i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}$$





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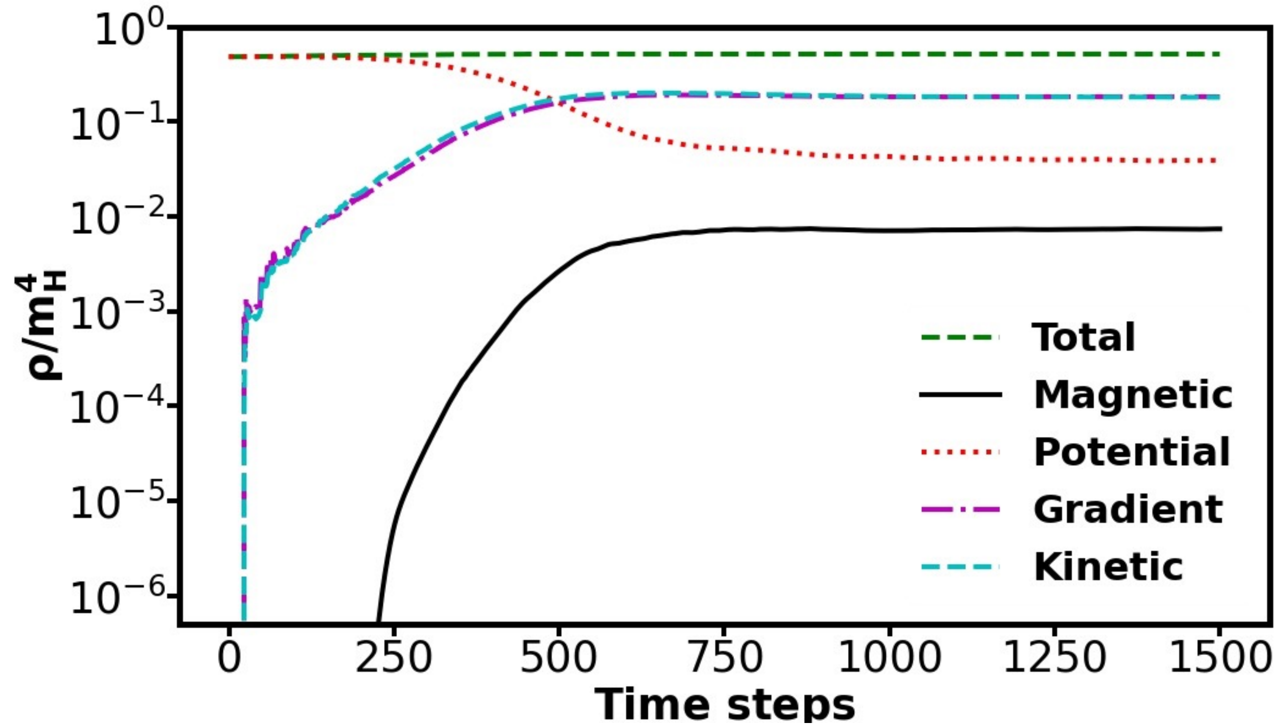
GW from Bubble collisions

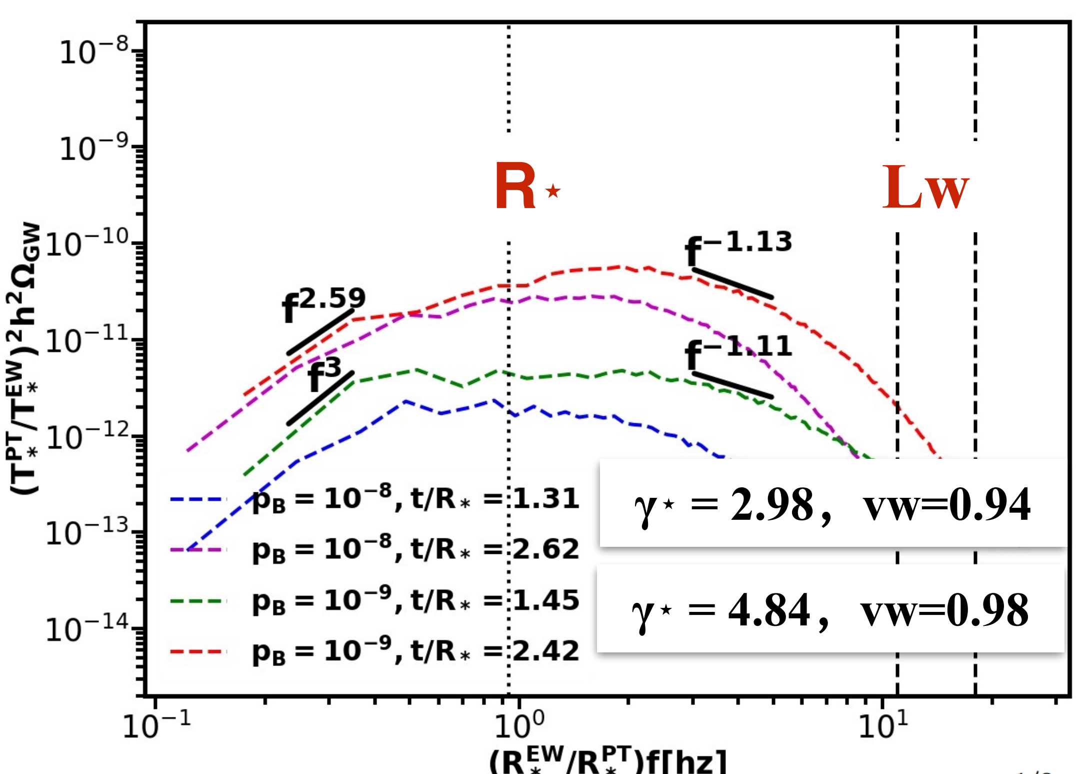
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\rm TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}{\rm Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

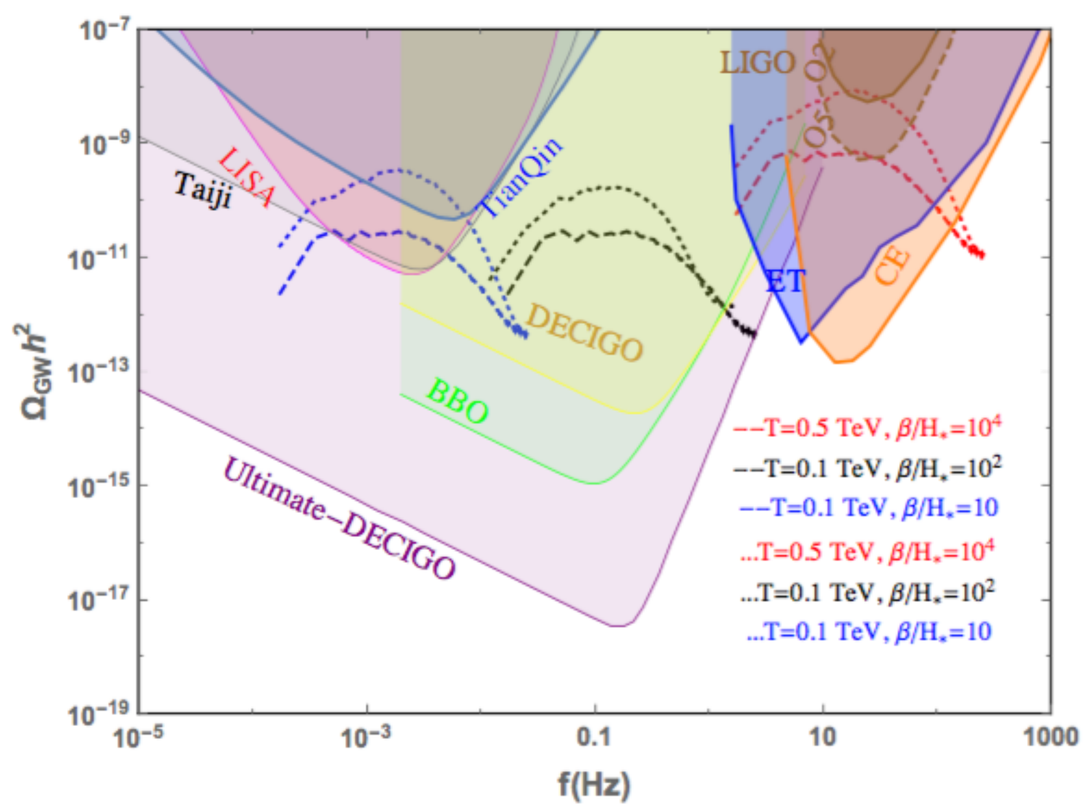
$$\frac{d\Omega_{\rm gw}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





$\gamma_\star = R_\star/(2Rc)$

$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



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Beyond SM models for FOPT

Higgs&GWs

- ☑ **SM+Scalar Singlet**

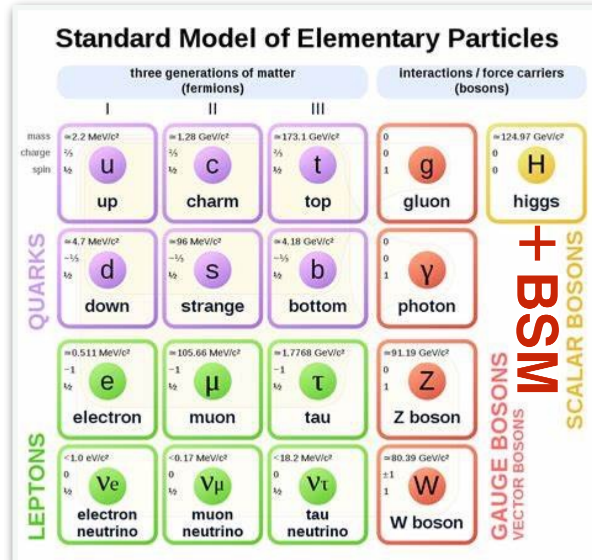
  Bian, Huang, Shu 15， Cheng, **Bian** 17, **Bian**, Tang 18,Chen, Li, Wu, **Bian**,19

- ☑ **SM+Scalar Doublet**

  Bernon, **Bian,** Jiang 17, **Bian**, Liu 18

- ☑ **SM + Scalar Triplet**

  Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19， Zhou, **Bian**,Du,22



- ☑ **Composite Higgs**

  Bian,Wu,Xie 19， **Bian**,Wu,Xie 20

- ☑ **NMSSM**

  Bi, **Bian**, Huang, Shu, Yin 15, **Bian**, Guo, Shu 17

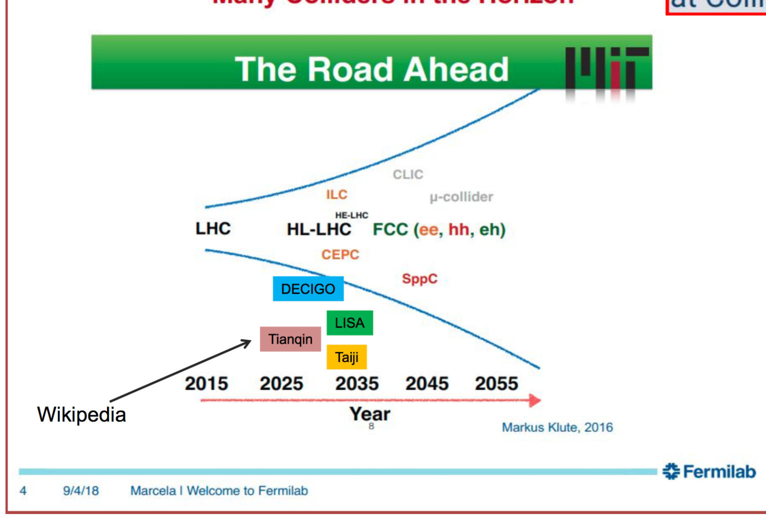
- ☑ **SMEFT**

  Zhou, **Bian**, Guo 19

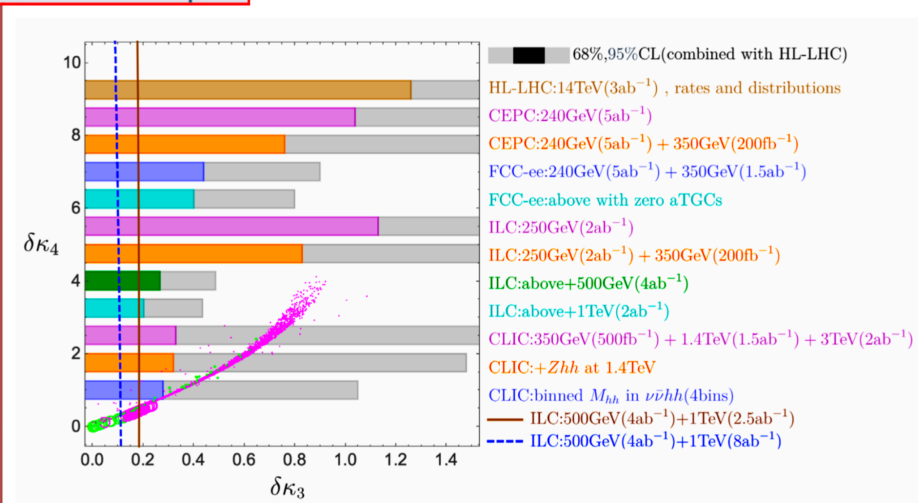
Symmetry breakdown

Symmetry breaking process

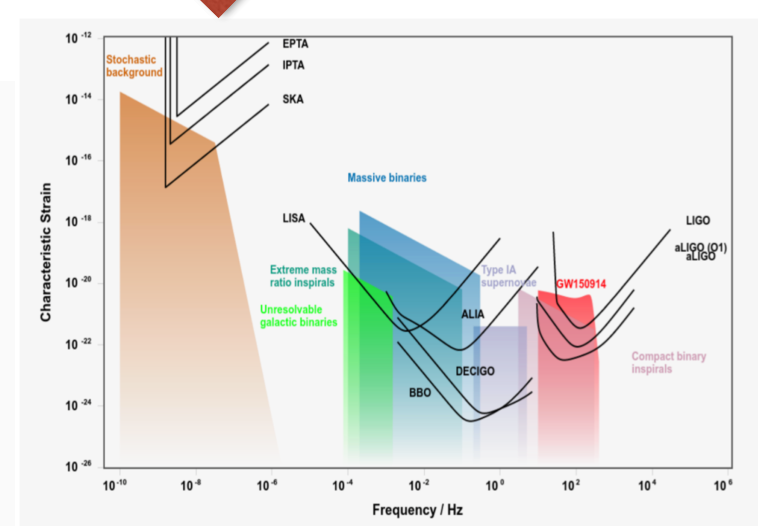
Double Higgs Production at Colliders Workshop



$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$



SNR > 10 for two-step and one-step SFOEWPT



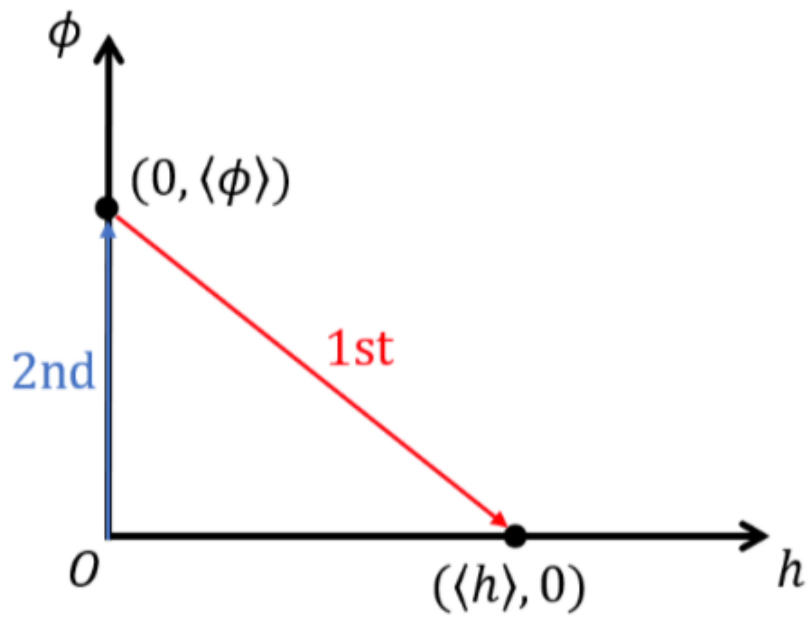
C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

PTA,LISA,TianQin,Taiji,LIGO,…

See HK Guo, Yun Jiang, Wei Liu, and Wei Su's talk

▸ Two-step FOPT potential

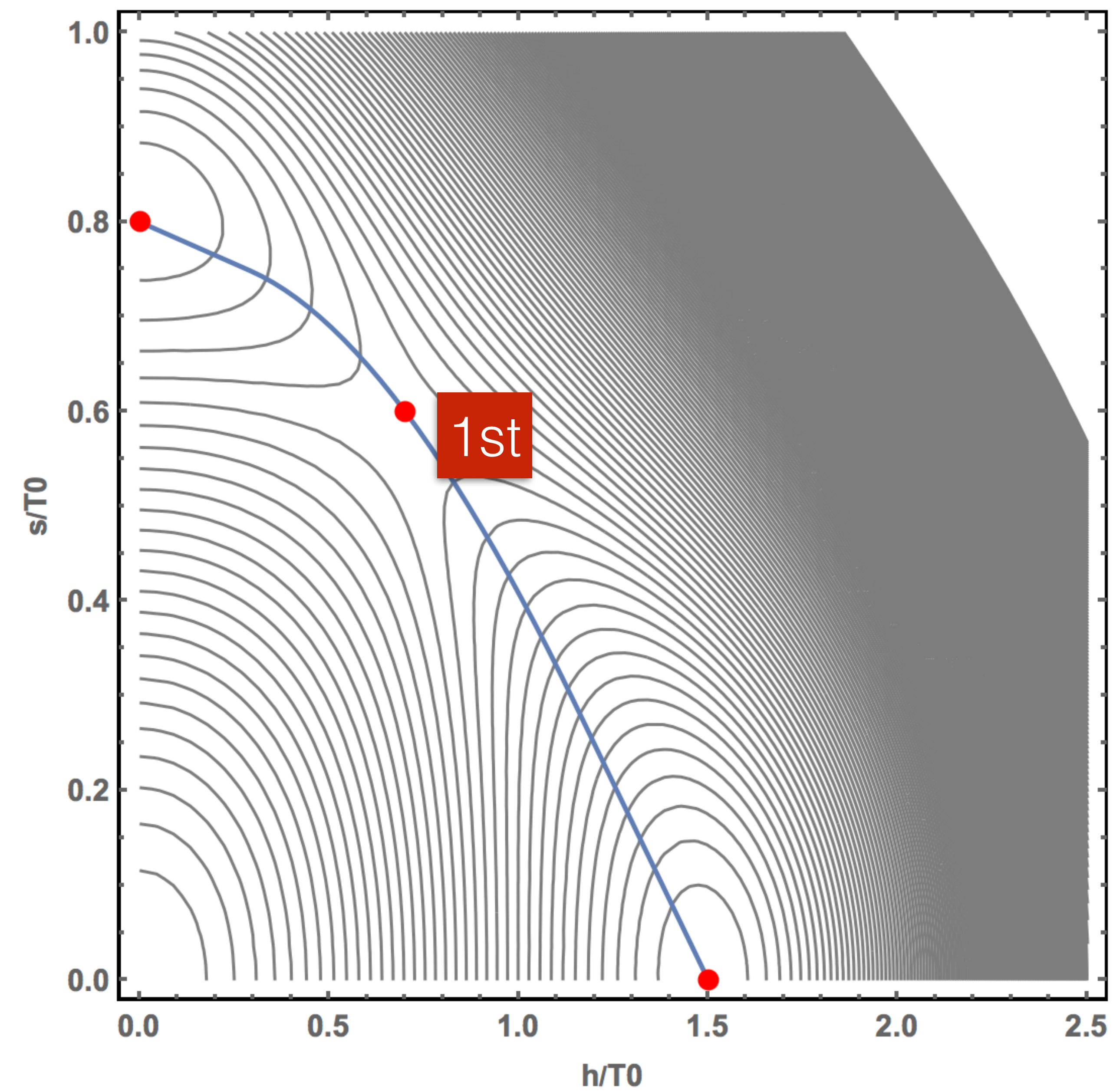
Type-a



$$V_a(\phi,h,T)=\frac{1}{2}(\mu_\phi^2+c_\phi T^2)\phi^2+\frac{1}{2}\lambda_{h\phi}h^2\phi^2+\frac{1}{4}\lambda_\phi\phi^4$$
$$+\frac{1}{2}(-\mu_h^2+c_hT^2)h^2+\frac{1}{4}\lambda_h h^4$$

$c_\phi=\lambda_\phi/4+\lambda_{h\phi}/3$

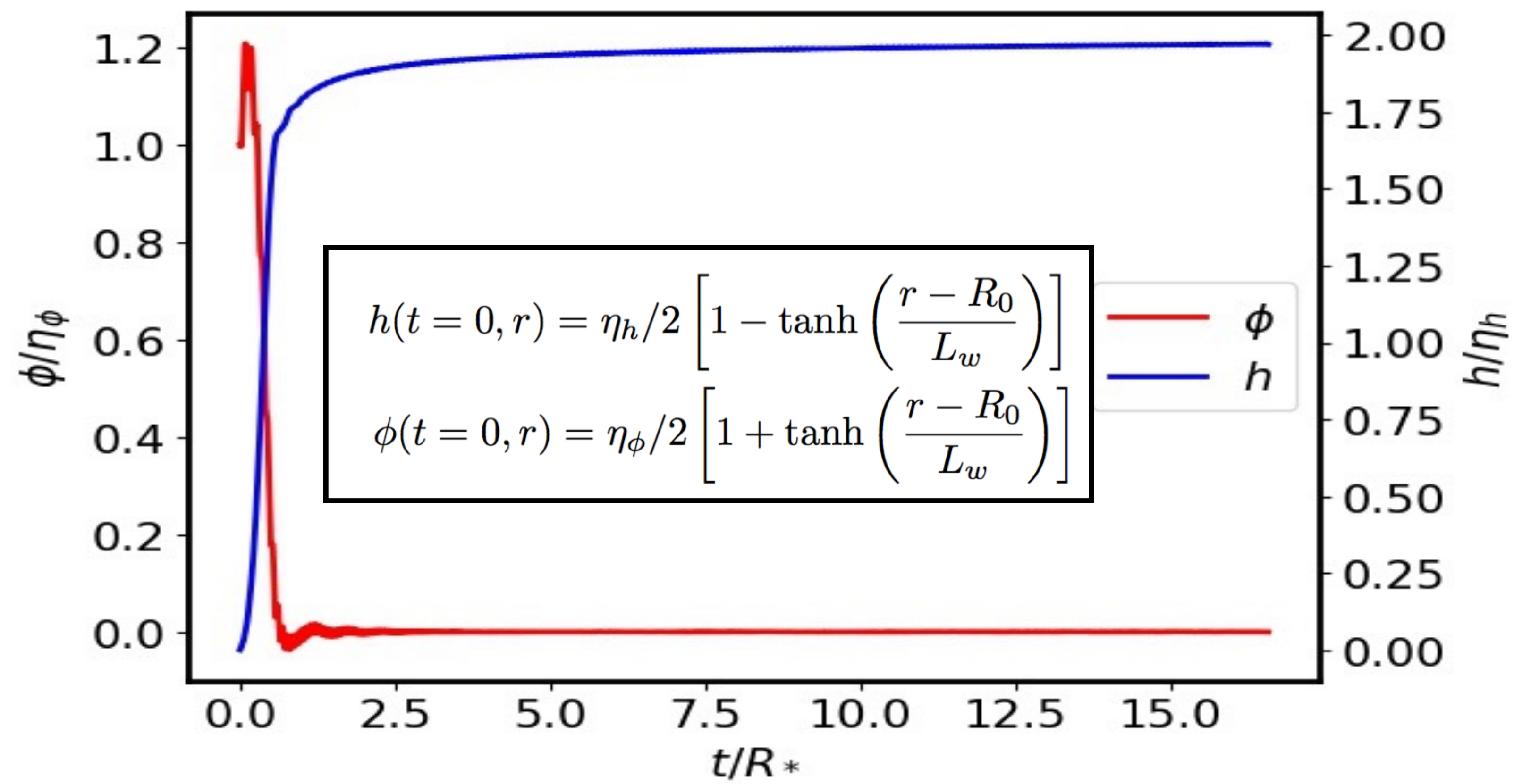
$c_h=(2m_W^2+m_Z^2+2m_t^2)/(4v^2)+\lambda_h/2+\lambda_{h\phi}/12$

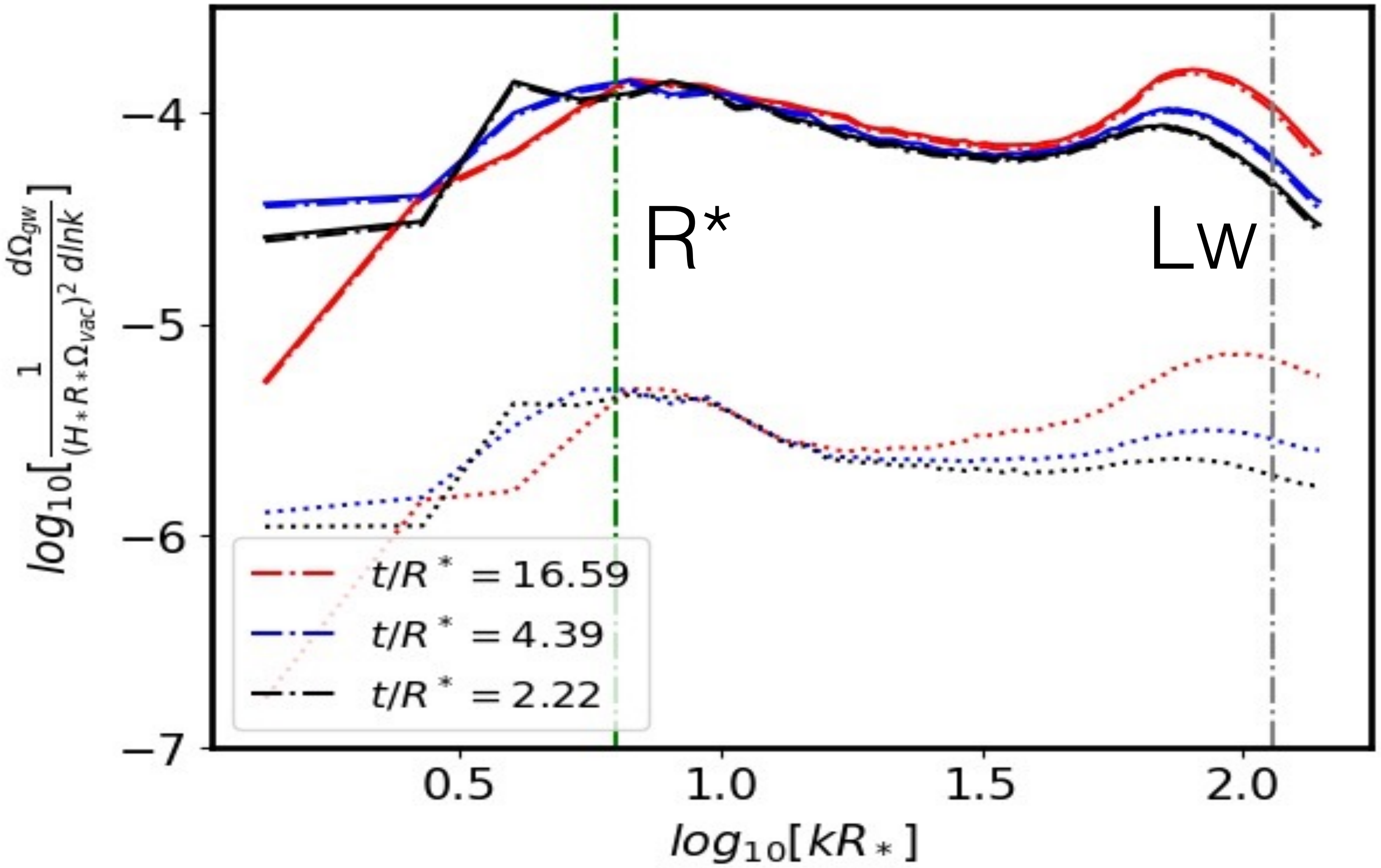


Motivated for DM&EWBG, see:1804.06813,1702.06124,1609.07143, 1605.08663, 1605.08663,etc

▸ Two-step PT with the second-step being FOPT

Type-a





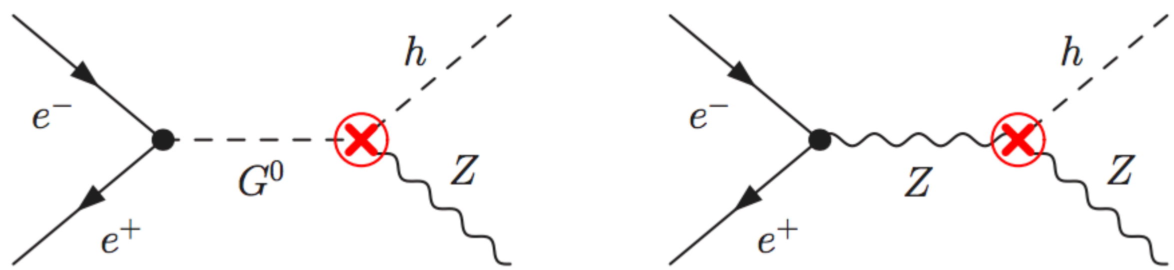
Zhao, Di, **Bian**, Cai,2204.04427

▸ Collider search for 2step FOPT

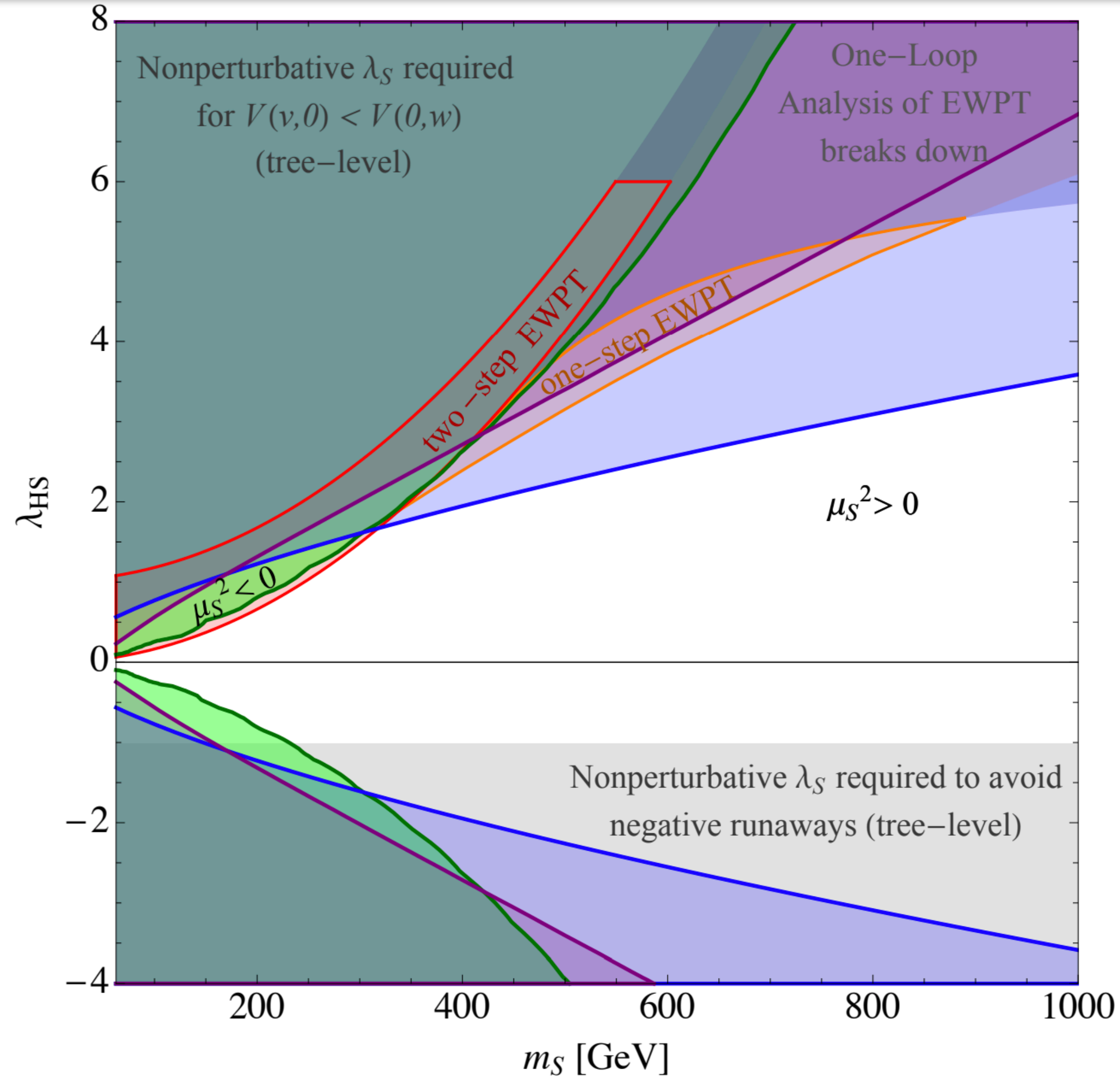
◉ Zh@ILC/CEPC

$$V_0 = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}\mu_S^2 S^2 + \lambda_{HS}|H|^2 S^2 + \frac{1}{4}\lambda_S S^4$$

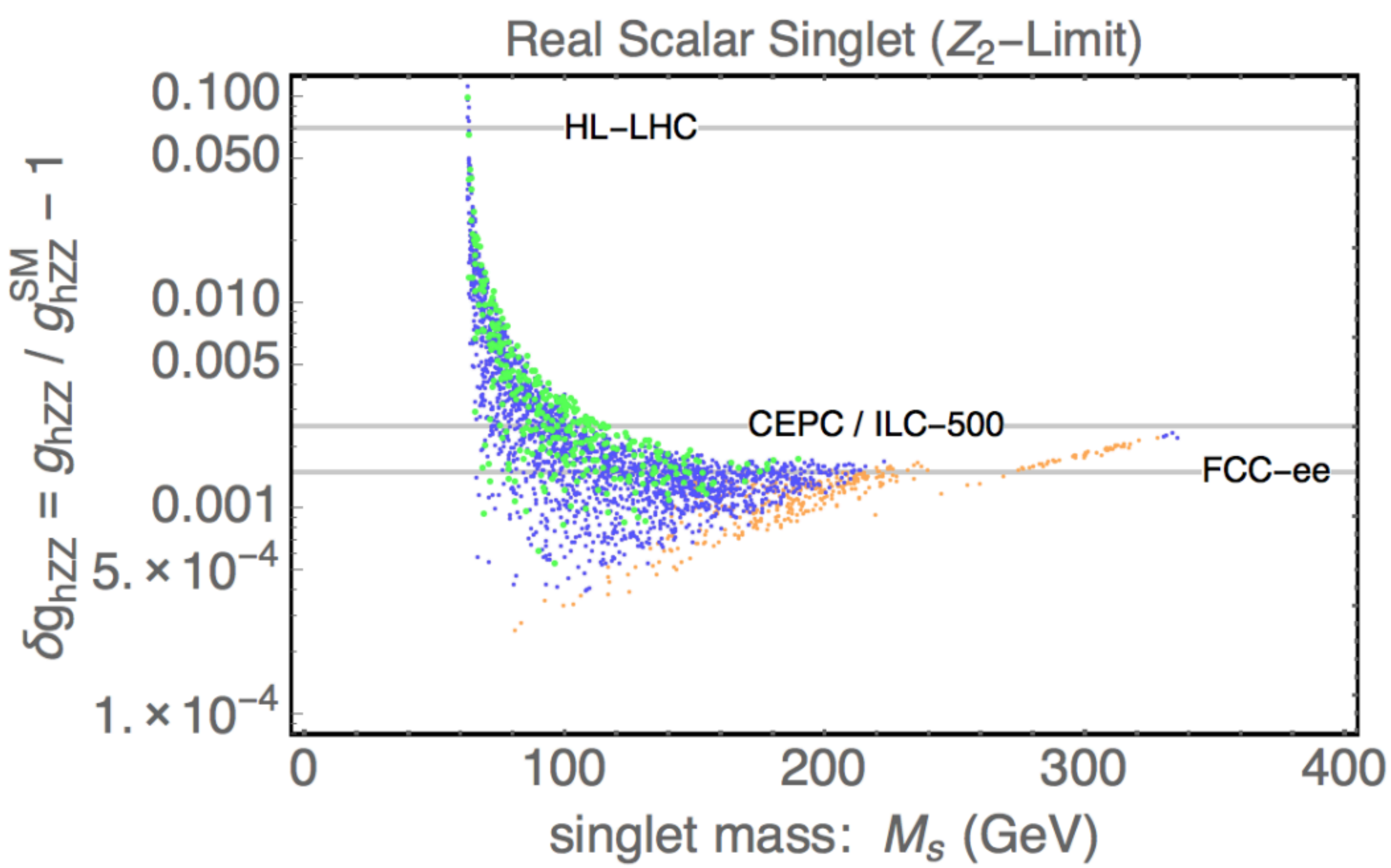
$$V_{\rm eff}(h,T) = V_0(h) + V_0^{CW}(h) + V_T(h,T) + V_r(h,T)$$



Craig,Englert, and McCullough,1305.5251



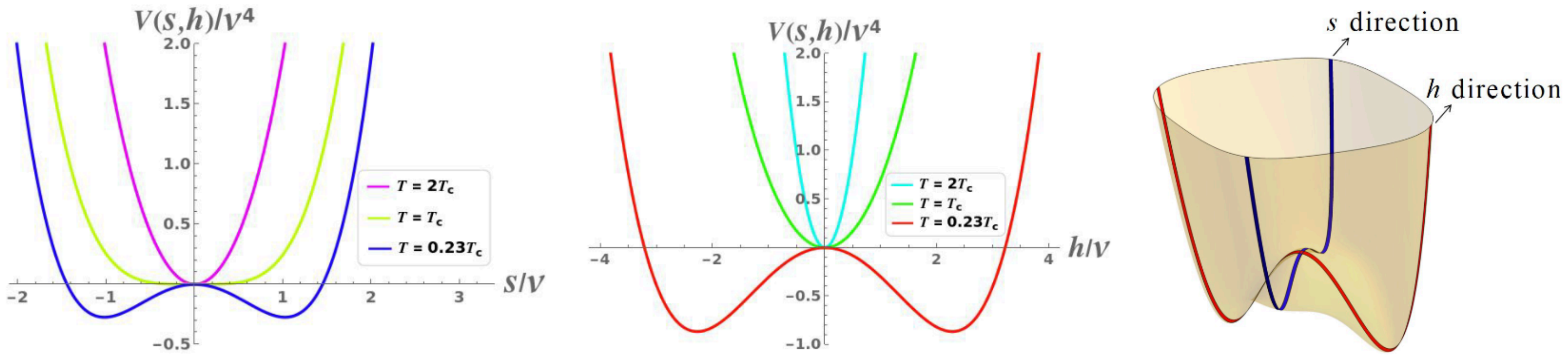
Curtin, Meade, Yu,1409.0005

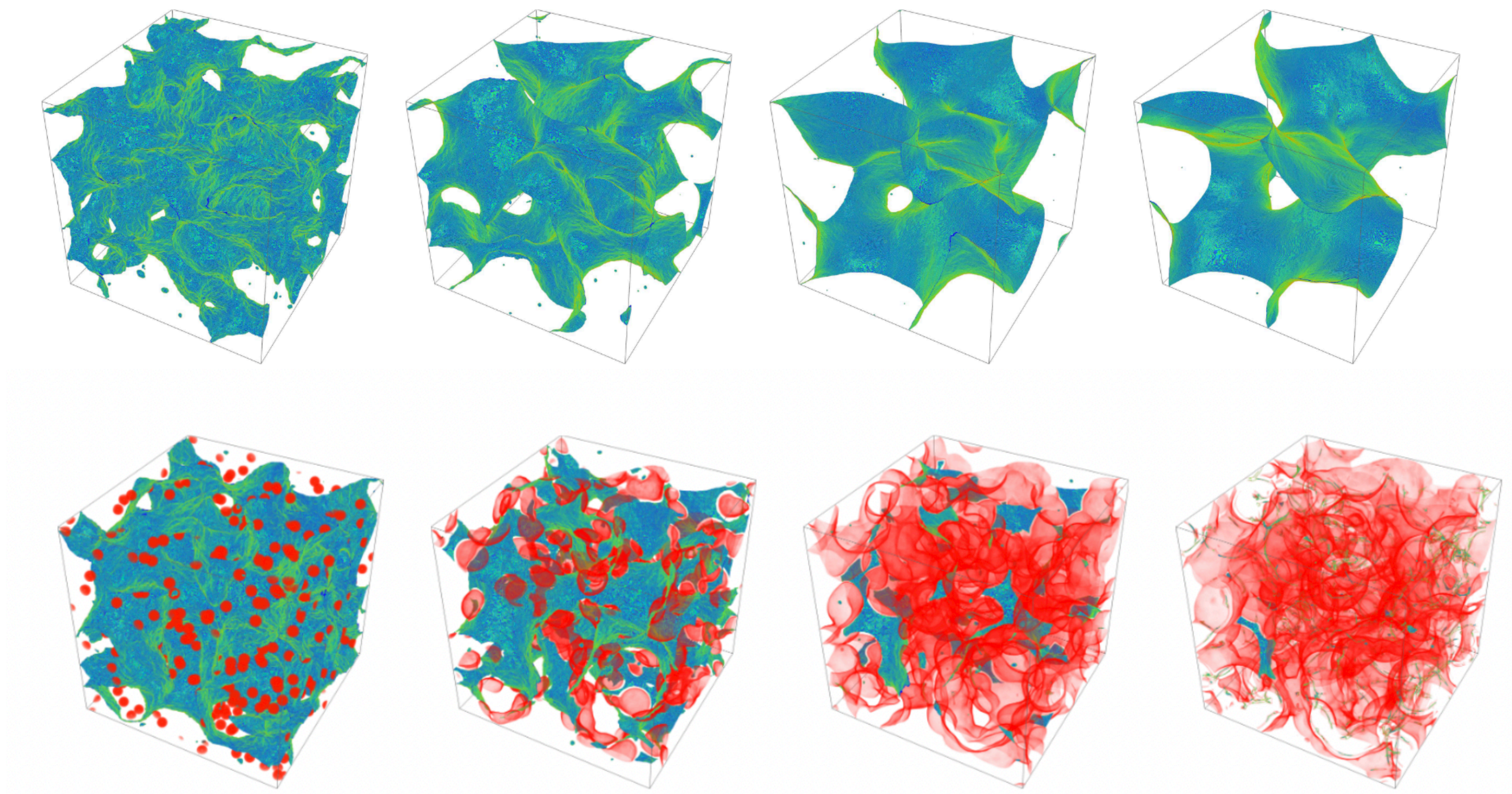


Huang, Long, and Wang,1608.06619

▸ 畴壁问题（宇宙学上的 Domain wall 问题）

3D snapshots of the
pure DW networks

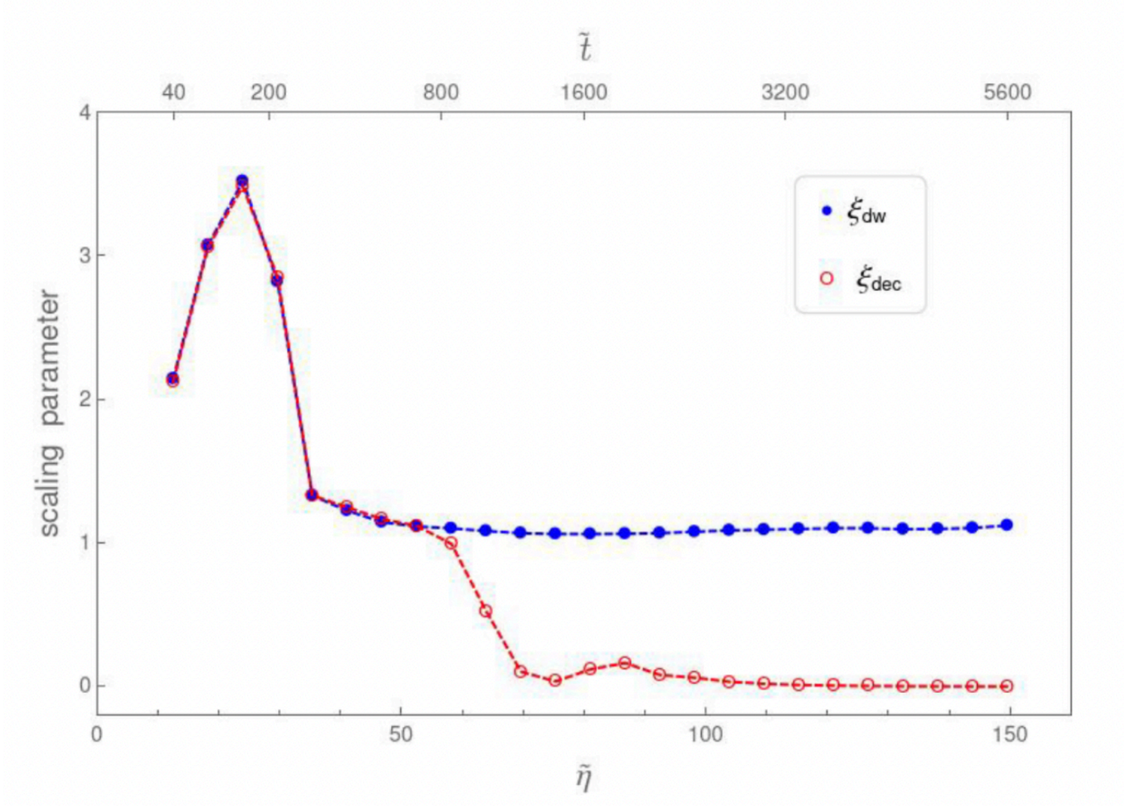


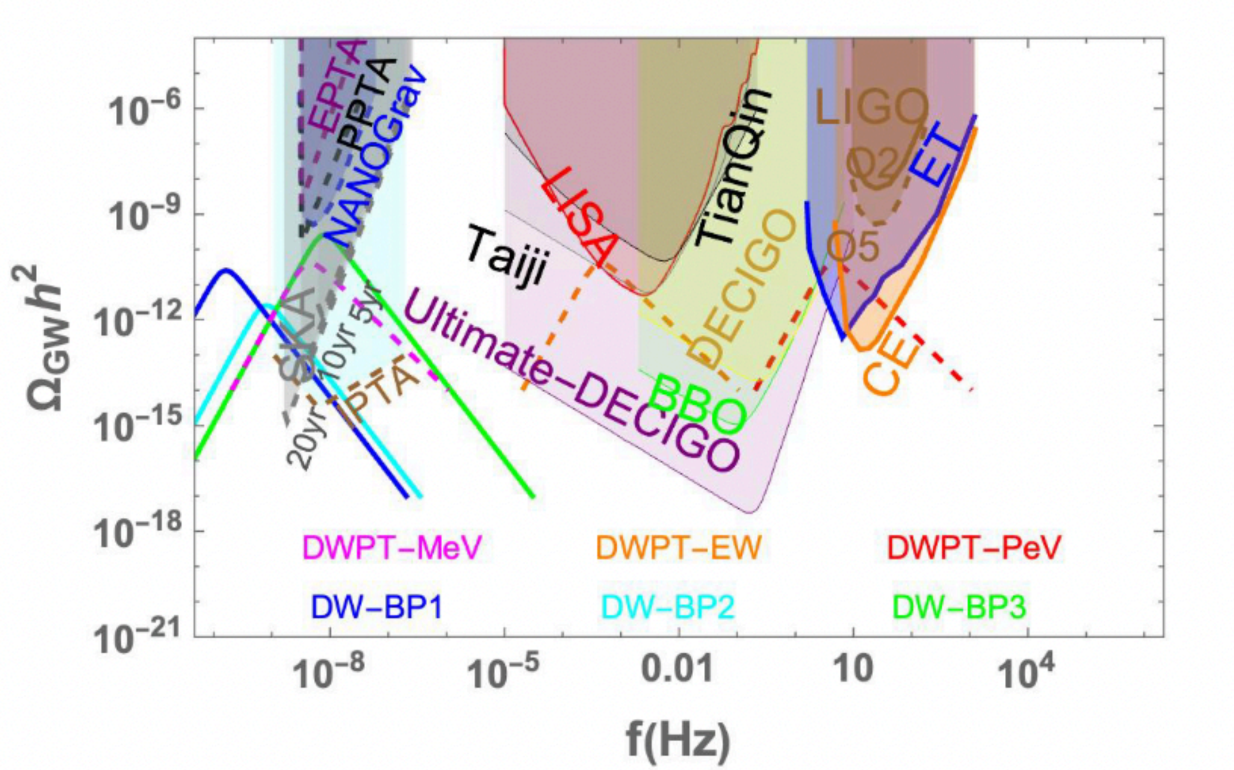


▸ 畴壁问题（宇宙学上的 Domain wall 问题）

Second order PT followed by a first-order PT that leads to EWSB

量子隧穿耗散经典的畴壁



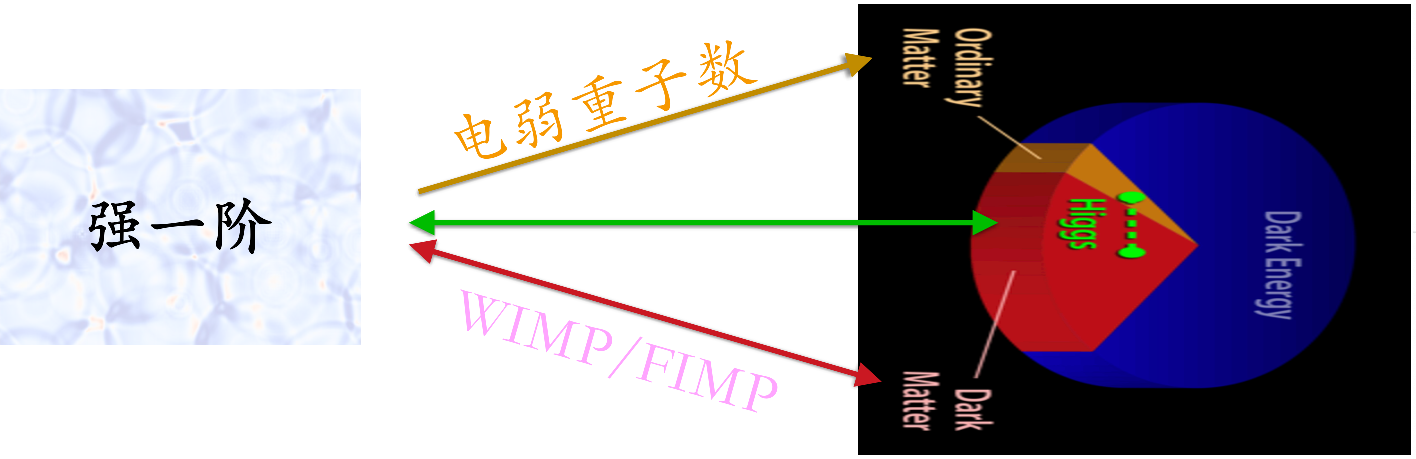


On CPUs

格子大小 1024^3

Bian, Jia, Li, 2304.05220

一阶相变效应



Impact of a complex singlet: Electroweak baryogenesis and dark matter #2

Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC) (Feb 26, 2015)

Published in: *Phys.Rev.D* 93 (2016) 6, 065032 • e-Print: 1502.07574 [hep-ph]

pdf DOI cite claim reference search 112 citations

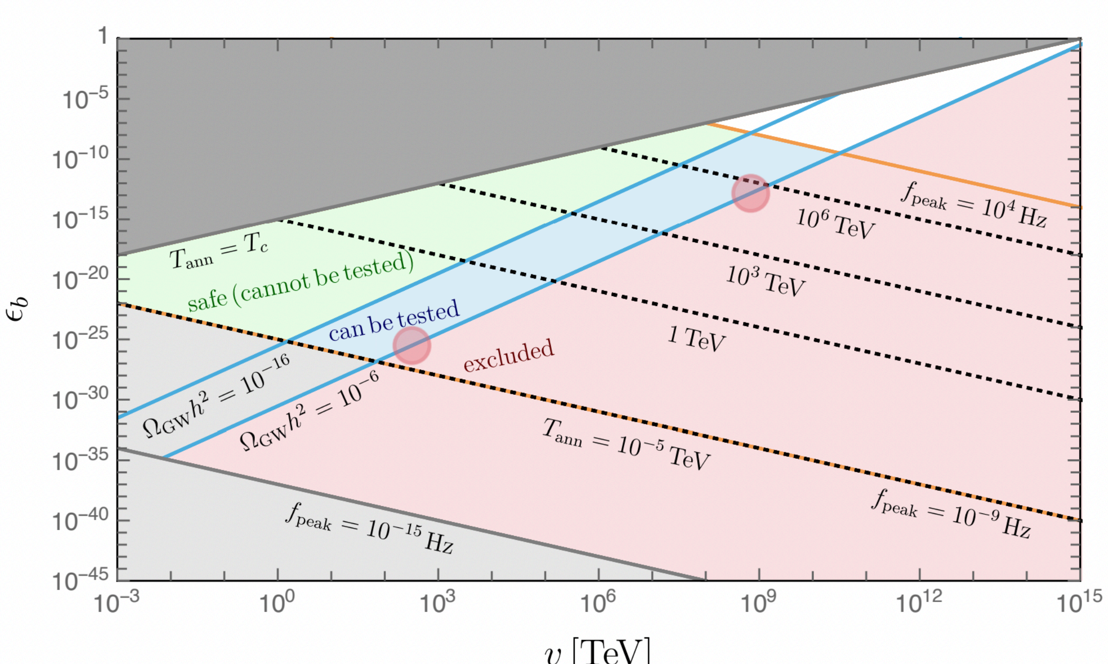
Thermally modified sterile neutrino portal dark matter and gravitational waves from phase transition: The Freeze-in case #3

Ligong Bian (Chongqing U. and Chung-Ang U.), Yi-Lei Tang (Korea Inst. Advanced Study, Seoul) (Oct 7, 2018)

Published in: *JHEP* 12 (2018) 006 • e-Print: 1810.03172 [hep-ph]

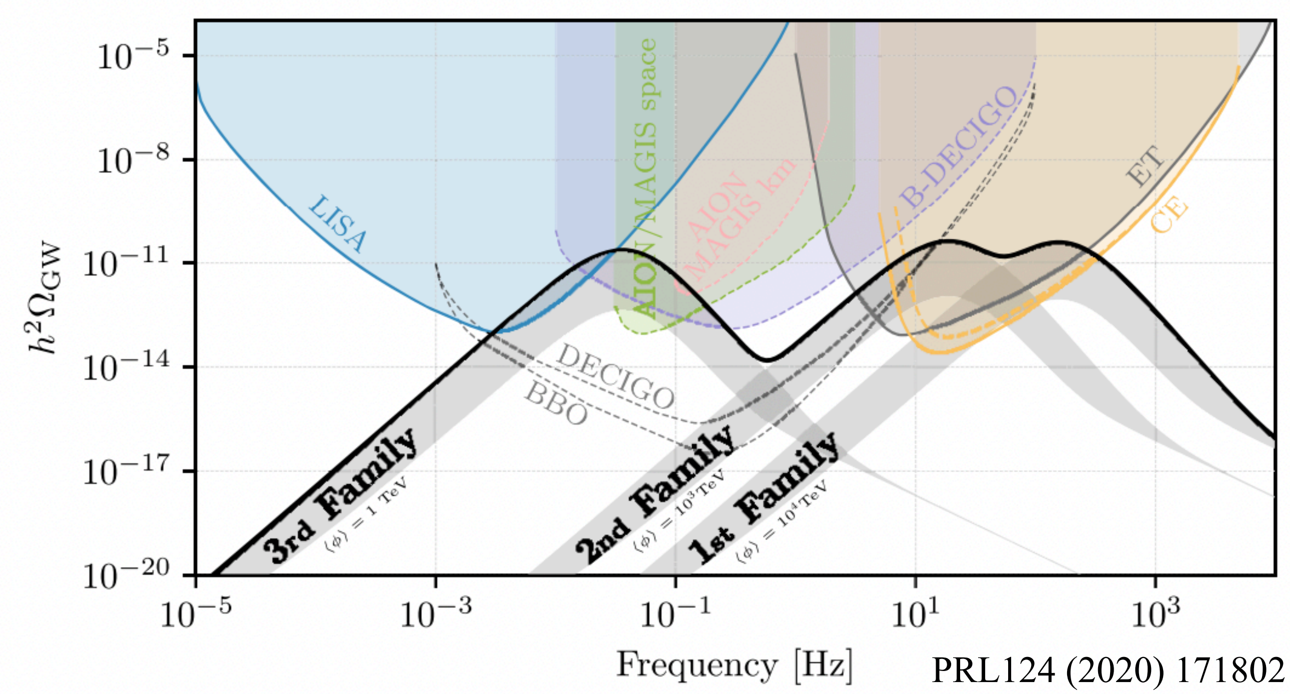
pdf DOI cite claim reference search 52 citations

离散的味对称性A4 与畴壁（DW）



JCAP 02 (2021) 032

三代夸克和轻子质量等级问题与FOPT



PRL124 (2020) 171802

一阶相变与Seesaw scale

Gravitational waves from first-order phase transitions in Majoron models of neutrino mass #9

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U. and HIAS, UCAS, Hangzhou and ICTP-AP, Beijing) (May 31, 2021)

Published in: *JHEP* 10 (2021) 193 • e-Print: 2106.00025 [hep-ph]

pdf DOI cite claim reference search 23 citations

Gravitational Waves from First-Order Phase Transitions: LIGO as a Window to Unexplored Seesaw Scales #1

Vedran Brdar (Heidelberg, Max Planck Inst.), Alexander J. Helmboldt (Heidelberg, Max Planck Inst.), Jisuke Kubo (Heidelberg, Max Planck Inst. and Toyama U.) (Oct 29, 2018)

Published in: *JCAP* 02 (2019) 021 • e-Print: 1810.12306 [hep-ph]

pdf DOI cite claim reference search 93 citations

Gravitational wave imprints of left-right symmetric model with minimal Higgs sector #1

Lukáš Gráf (Heidelberg, Max Planck Inst. and UC, Berkeley and UC, San Diego), Sudip Jana (Heidelberg, Max Planck Inst.), Ajay Kaladharan (Oklahoma State U.), Shaikh Saad (Basel U.) (Dec 22, 2021)

Published in: *JCAP* 05 (2022) 05, 003 • e-Print: 2112.12041 [hep-ph]

pdf DOI cite claim reference search 7 citations

Prospects of gravitational waves in the minimal left-right symmetric model #19

Mingqiu Li (Beijing, GUCAS), Qi-Shu Yan (Beijing, GUCAS and Beijing, Inst. High Energy Phys.), Yongchao Zhang (Southeast U., Nanjing and Washington U., St. Louis), Zhijie Zhao (Beijing, Inst. High Energy Phys.) (Dec 26, 2020)

Published in: *JHEP* 03 (2021) 267 • e-Print: 2012.13686 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational waves from neutrino mass and dark matter genesis #16

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U.) (Jan 21, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 095017 • e-Print: 2001.07637 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational wave pathway to testable leptogenesis #2

Arnab Dasgupta (Pittsburgh U.), P.S. Bhupal Dev (Washington U., St. Louis and McDonnell Ctr. Space Sci.), Anish Ghoshal (Warsaw U.), Anupam Mazumdar (U. Groningen, VSI) (Jun 14, 2022)

Published in: *Phys.Rev.D* 106 (2022) 7, 075027 • e-Print: 2206.07032 [hep-ph]

pdf DOI cite claim reference search 23 citations

Cosmological implications of a B − L charged hidden scalar: leptogenesis and gravitational waves #5

Ligong Bian (Chongqing U.), Wei Cheng (Beijing, Inst. Theor. Phys.), Huai-Ke Guo (Oklahoma U.), Yongchao Zhang (Washington U., St. Louis and Peking U., CHEP) (Jul 31, 2019)

Published in: *Chin.Phys.C* 45 (2021) 11, 113104 • e-Print: 1907.13589 [hep-ph]

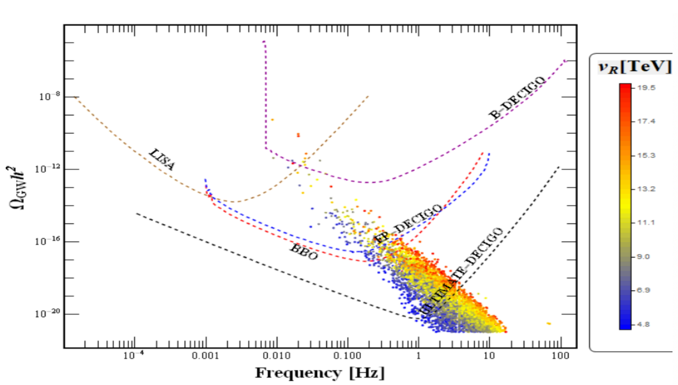
pdf DOI cite claim reference search 27 citations

Electroweak phase transition and gravitational waves in the type-II seesaw model #12

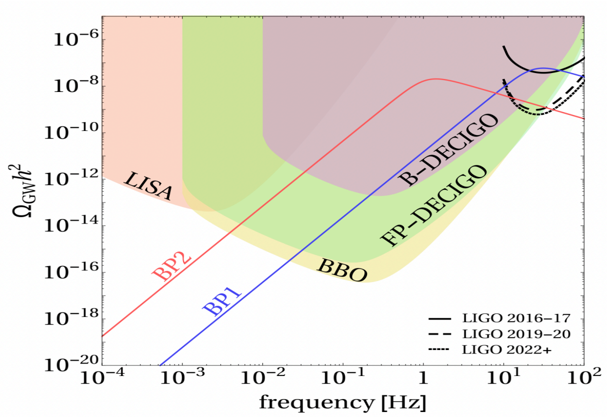
Ruiyu Zhou (CUPT, Chongqing), Ligong Bian (Chongqing U. and Peking U., CHEP), Yong Du (Beijing, Inst. Theor. Phys.) (Mar 3, 2022)

Published in: *JHEP* 08 (2022) 205 • e-Print: 2203.01561 [hep-ph]

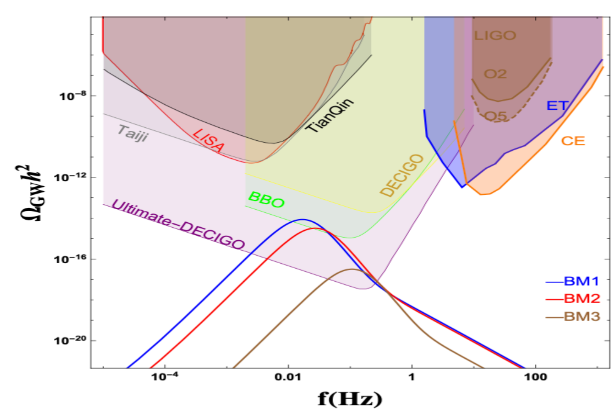
pdf DOI cite claim reference search 15 citations



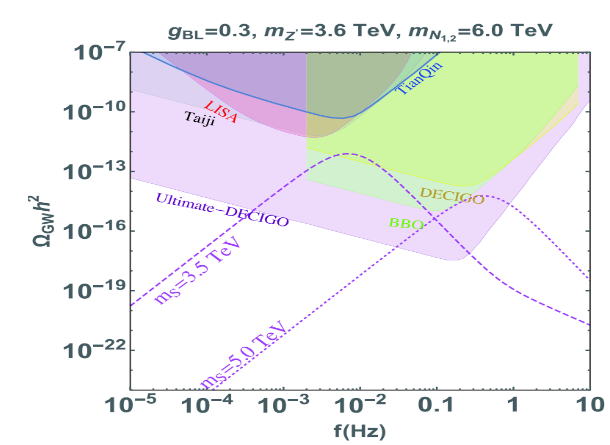
L-R



CSB

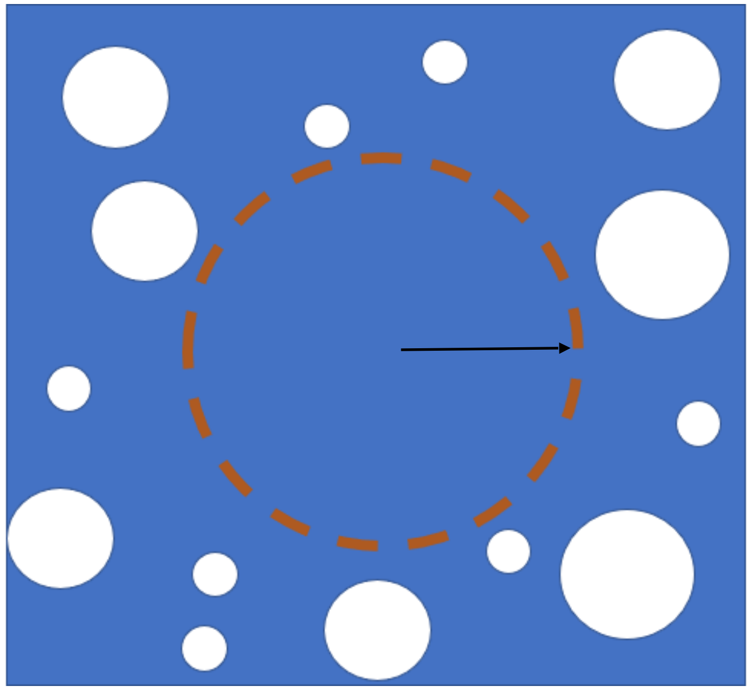


Type-II

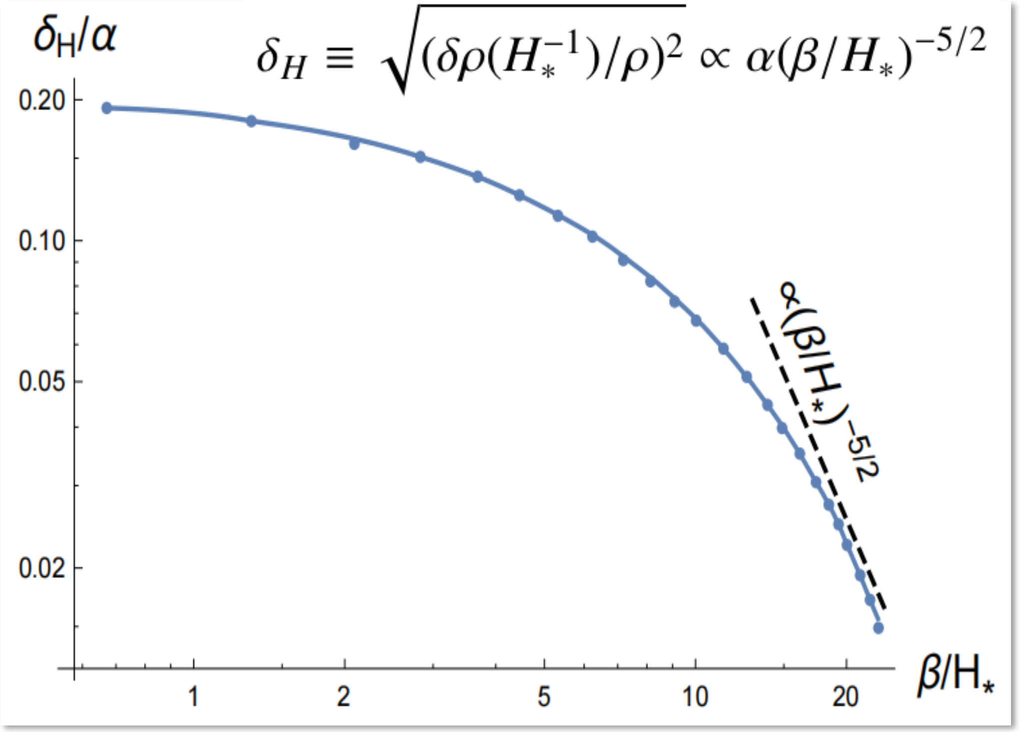


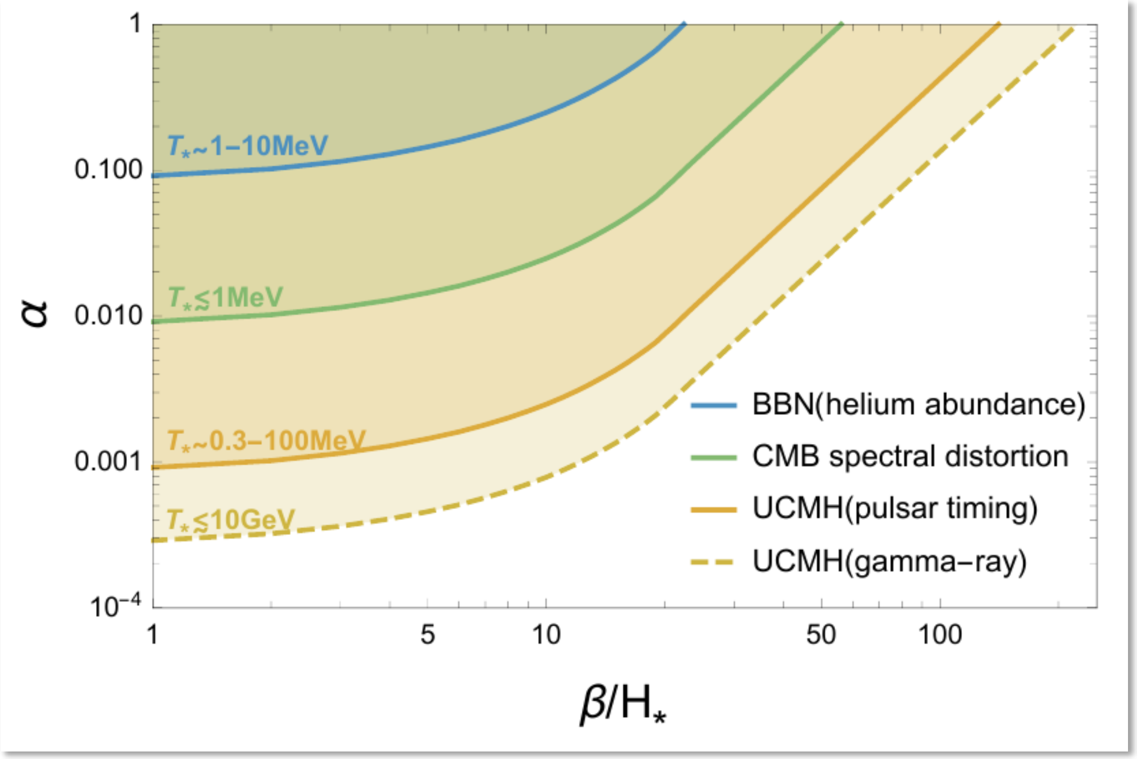
Type-I

真空延迟衰变与曲率扰动限制相变



Hubble-sized perturbations

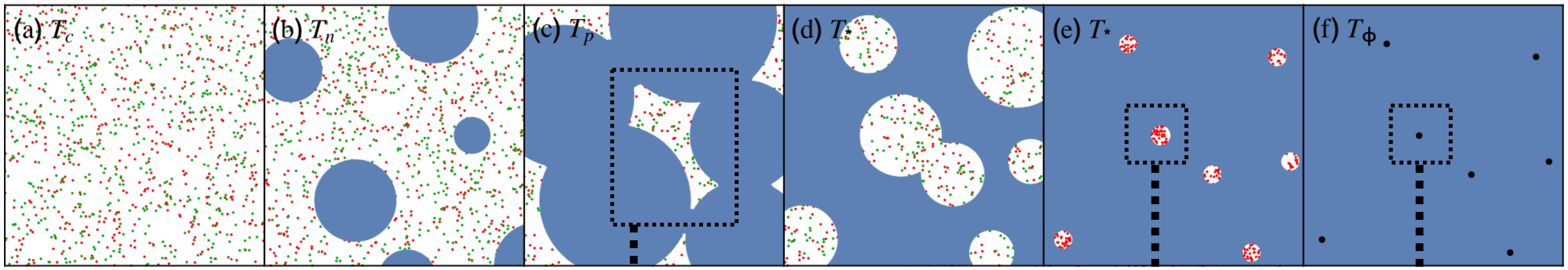




low-scale and slow 1st PTs
motived for dark PT and BAU

Liu, **Bian**, Cai, Guo, Wang, PRL130 (2023) 051001

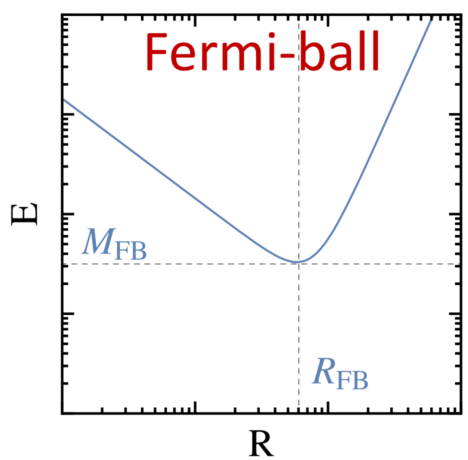
Solitons and black holes from a FOPT



Trapping fermions in the false vacuum by mass gap $\Delta m_\chi \gg T_*$

Forming Fermi-ball solitons by the Pauli exclusion principle

Fermi-balls collapse to primordial black holes



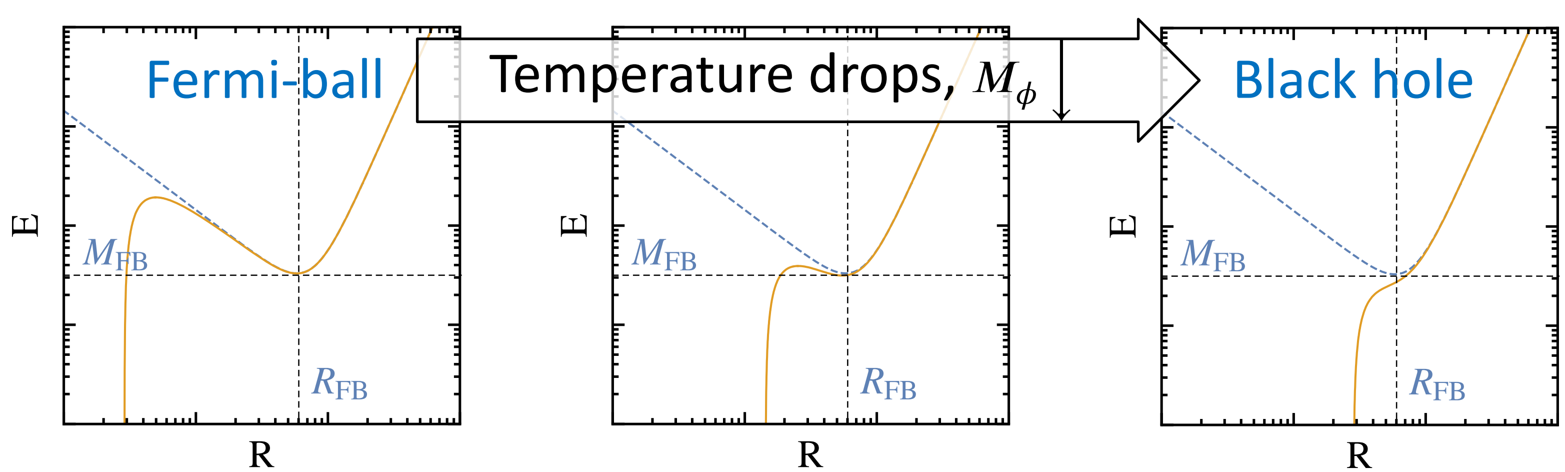
$$E_{\rm FB} = \frac{3\pi}{4}\left(\frac{3}{2\pi}\right)^{2/3}\frac{Q_{\rm FB}^{4/3}}{R} + \frac{4\pi}{3}R^3 U_0$$

Fermi-gas ↑ Volume ↑

Hong, Jung and Xie, PRD 102 (2020) 7, 075028

$$E_{\rm Yuk} \approx -\frac{15 y_\chi^2}{40\pi}\frac{Q_{\rm FB}^2}{R}\left(\frac{1}{R M_\phi}\right)^2$$

Internal attractive Yukawa force mediated by ϕ



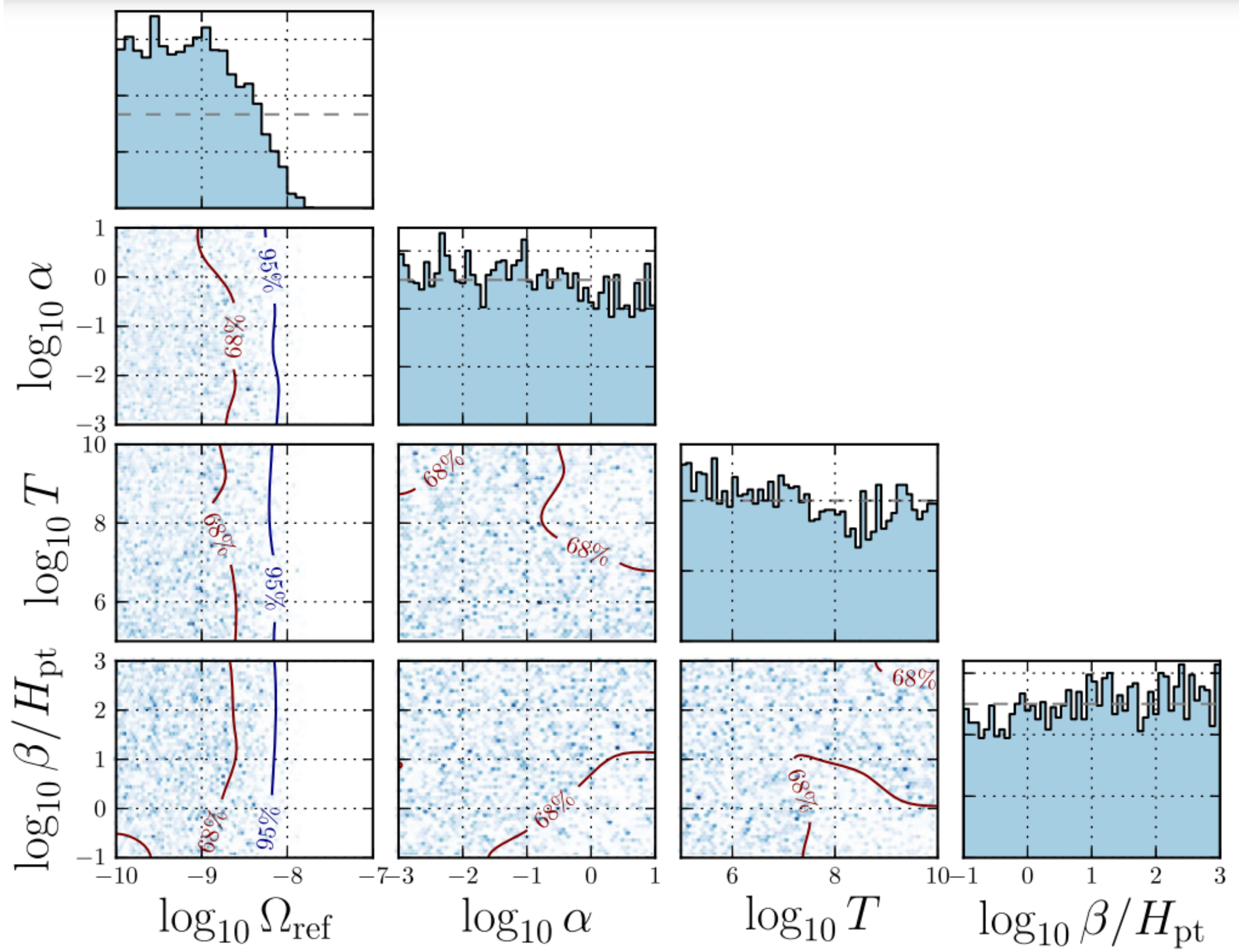
Kawana and Xie, PLB 824 (2022) 136791

▸ LIGO-Virgo search for FOPT

High-scale PT

Romero, Martinovic,Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3



See Huai-Ke Guo's talk

▸ PPTA search for FOPT

- **PPTA DR2 dataset constrain low-scale phase transition，dark sector and QCD scale FOPT**

PHYSICAL REVIEW LETTERS **127,** 251303 (2021)

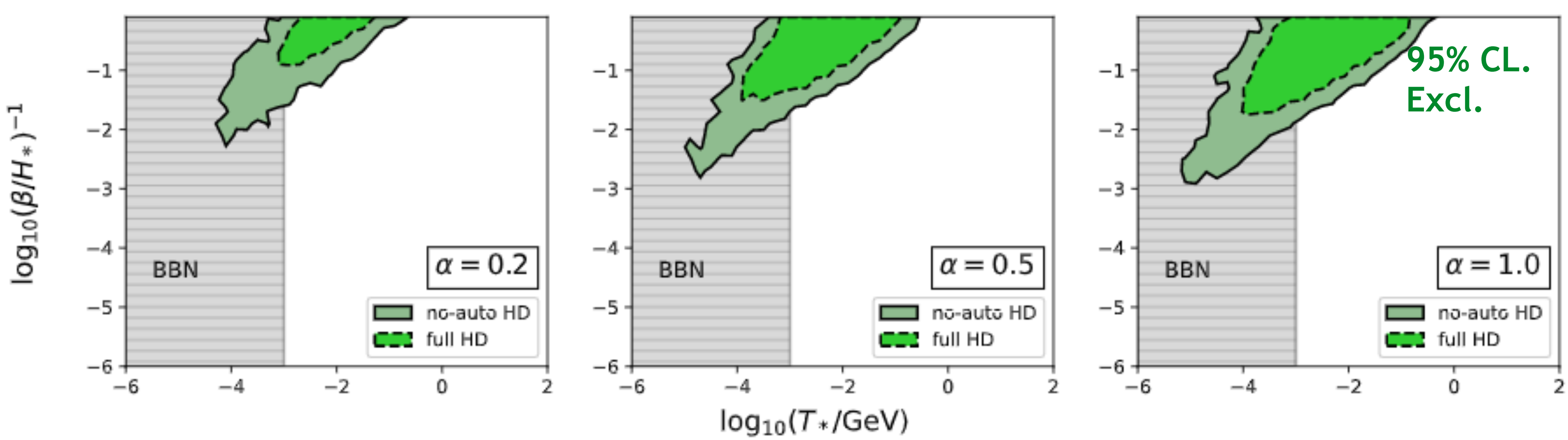
Editors' Suggestion | Featured in Physics

Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

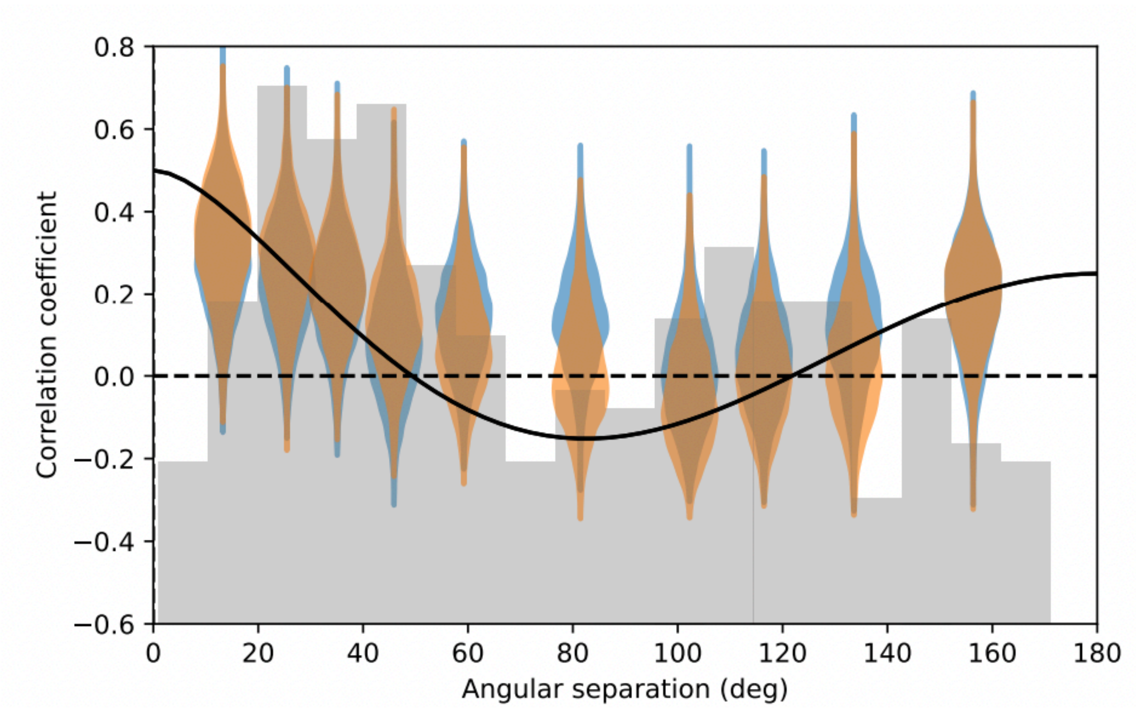
Xiao Xue,[1,2,3] Ligong Bian,[4,5,*] Jing Shu,[1,2,6,7,8,†] Qiang Yuan,[9,10,7,‡] Xingjiang Zhu,[11,12,13,§] N. D. Ramesh Bhat,[14] Shi Dai,[15] Yi Feng,[16] Boris Goncharov,[11,12] George Hobbs,[17] Eric Howard,[17,18] Richard N. Manchester,[17] Christopher J. Russell,[19] Daniel J. Reardon,[12,20] Ryan M. Shannon,[12,20] Renée Spiewak,[21,20] Nithyanandan Thyagarajan,[22] and Jingbo Wang[23]

TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

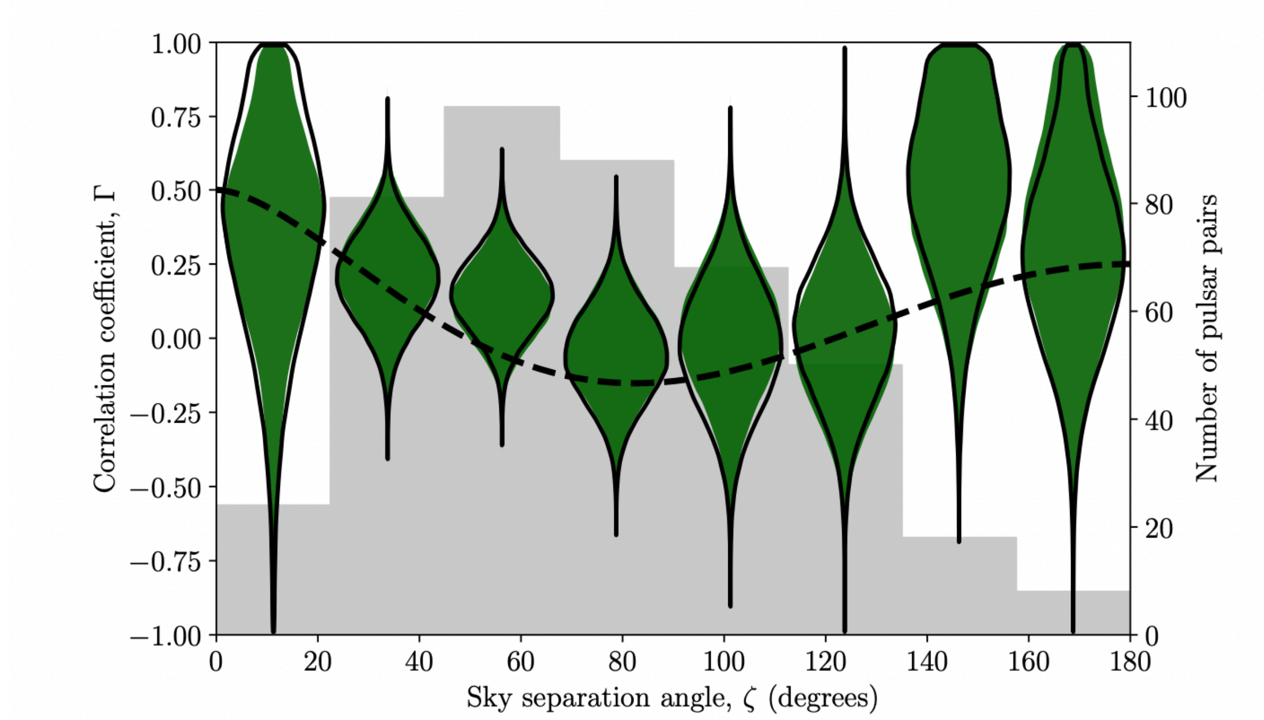
Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1-σ interval)	
					T_*/MeV, $\alpha \times 10^3$, β/H_*	$A_{\rm comred}$, $\gamma_{\rm comred}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}$, $3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}$, 271^{+165}_{-92}, $9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}$, $3.8^{+27.9}_{-3.4}$, 854^{+9622}_{-782}	$-14.51^{+0.64}_{-0.68}$, $3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}$, $3.2^{+19.9}_{-2.8}$, 1053^{+11256}_{-962}	$-14.45^{+0.62}_{-0.64}$, $3.27^{+1.37}_{-1.54}$



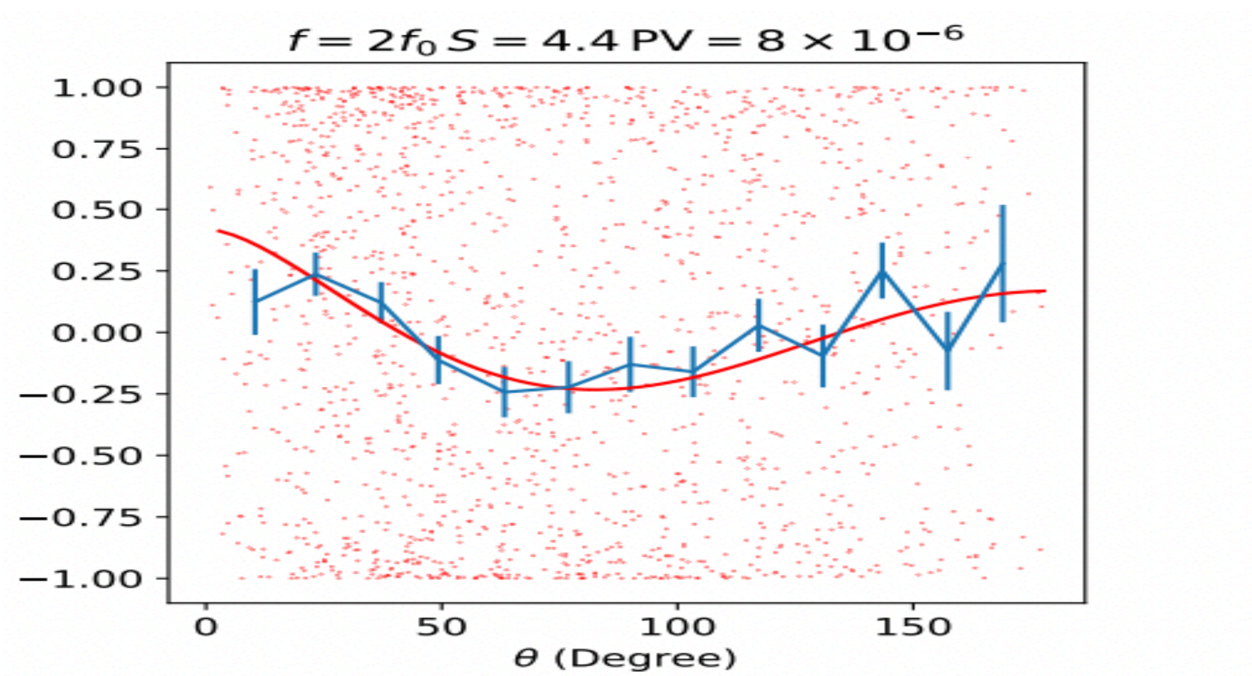
New dataset from PTAs



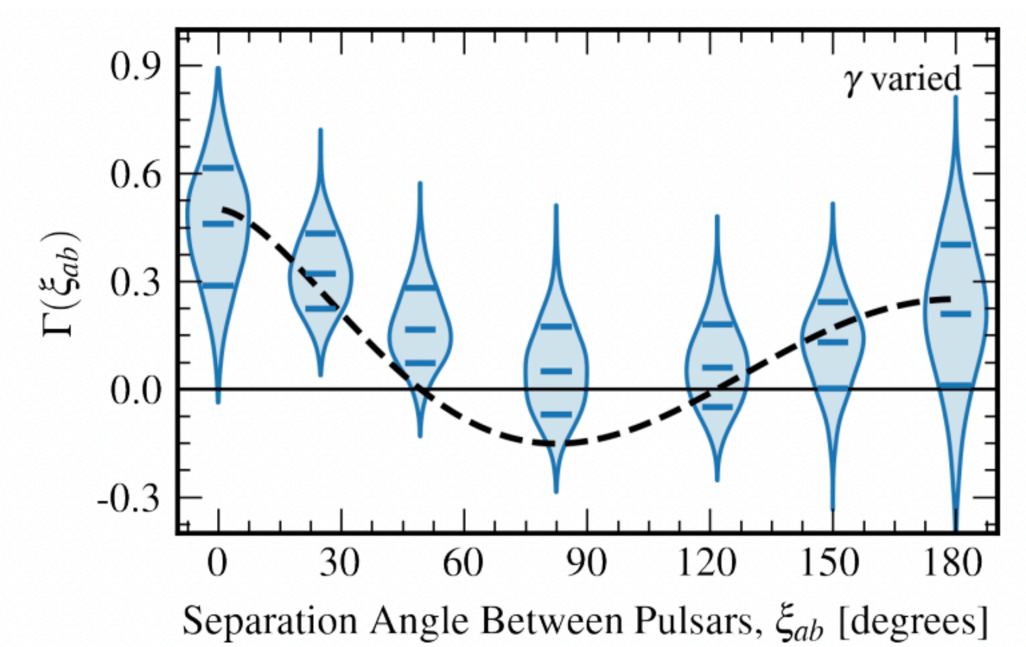
EPTA,2306.16214



PPTA,2306.16215

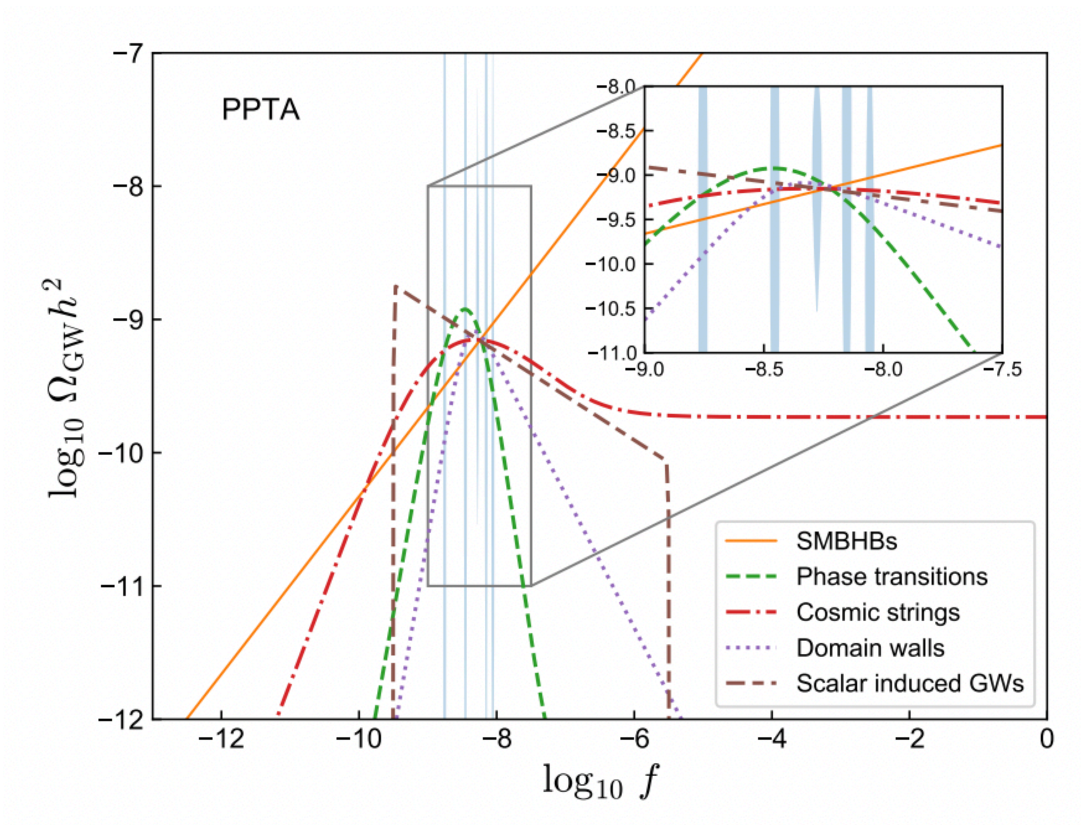


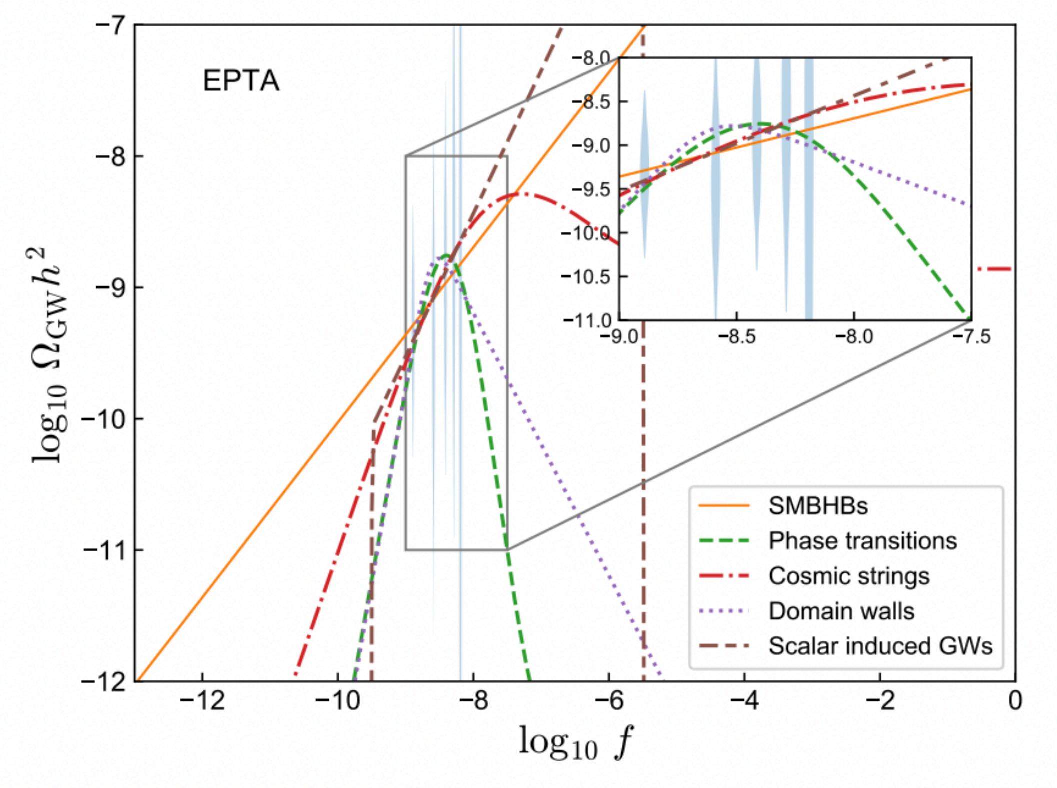
CPTA ,2306.16216

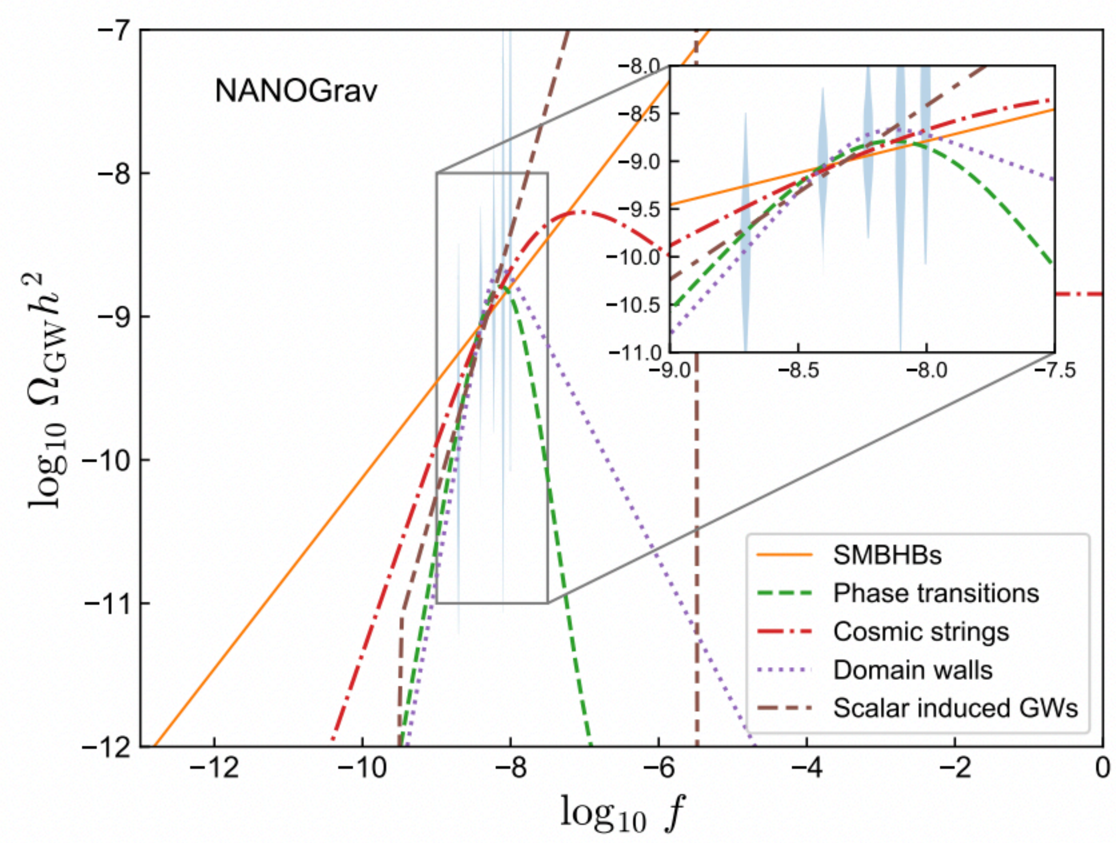


NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays

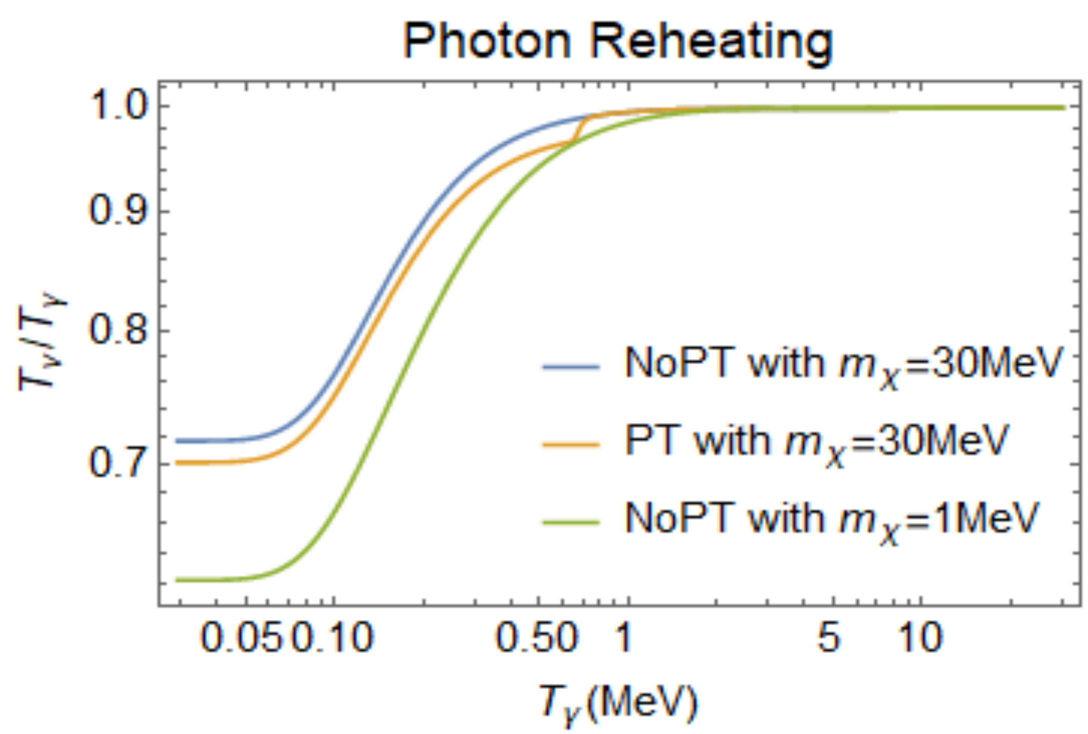


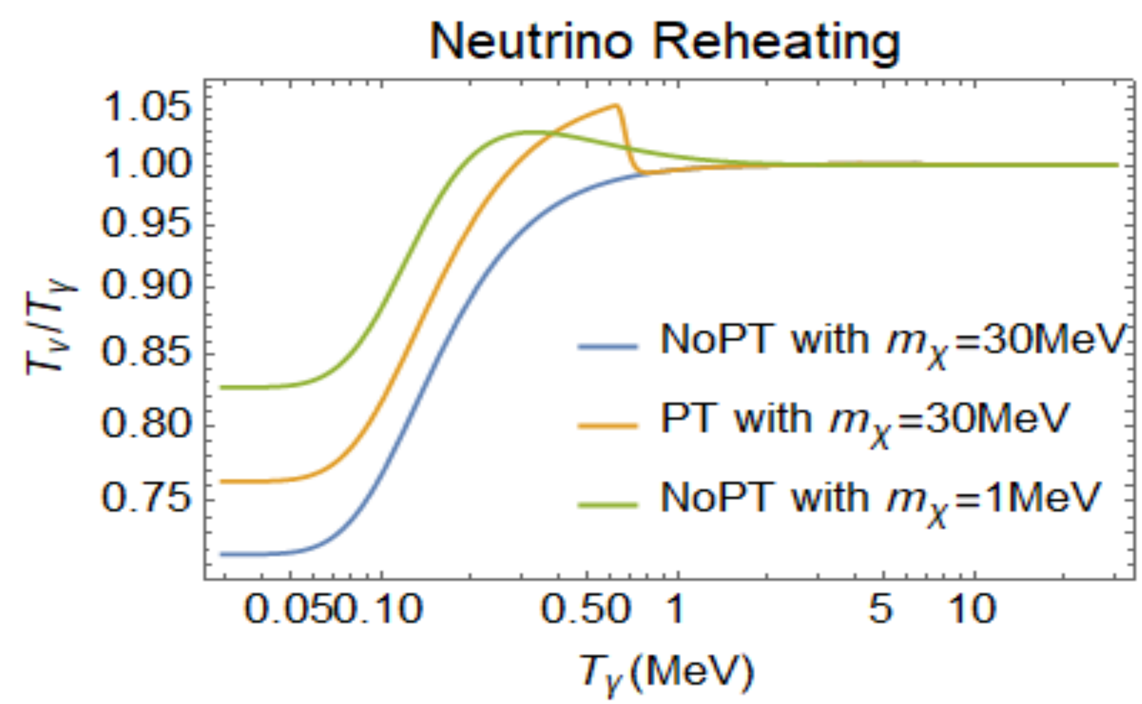


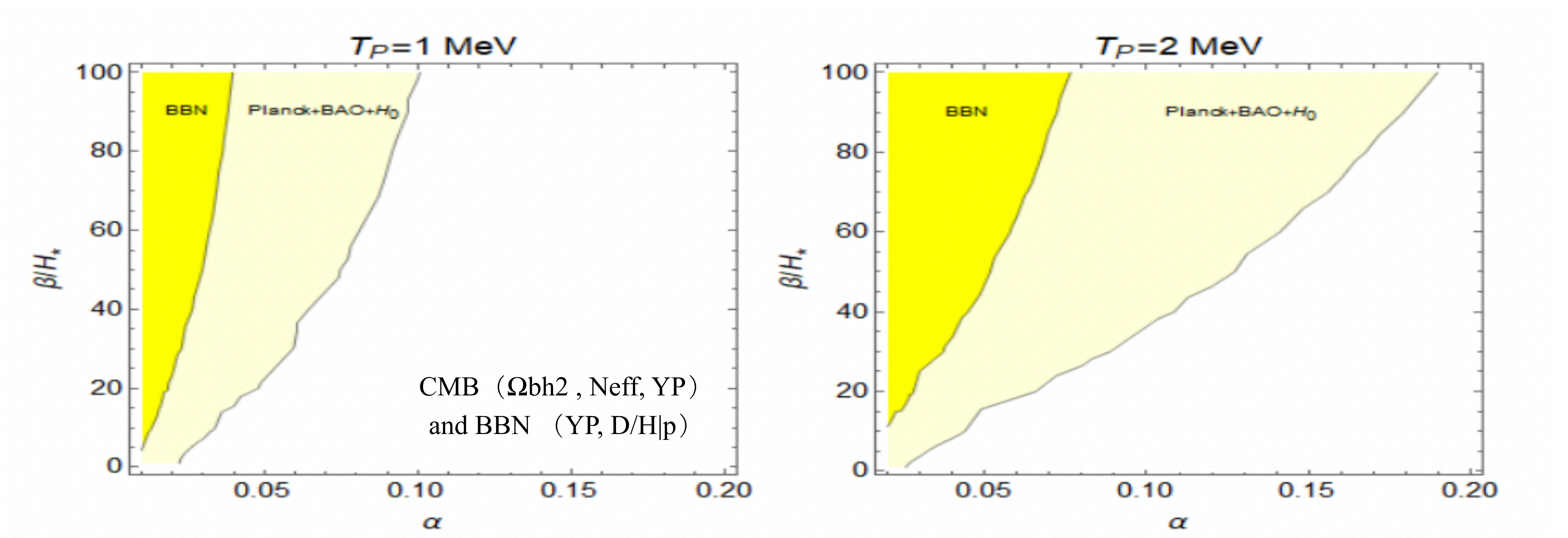


Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

宇宙学观测限制低能标一阶相变







Deng, **Bian**,2304.06576

▸ Related topics

❖Lattice simulation

- **PT GW simulation, Electroweak sphaleron, PT dynamics**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**

❖Pheno

1. EWSB and GW from FOPT

- **Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily**

2. BAU and GW from FOPT

- **Sphaleron process, bubble dynamics**

3. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

谢谢！

Finite temperature potential and free energy

The grand canonical partition function

$$\mathcal{Z}(T) \equiv \mathrm{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}], \quad \text{where} \quad \beta \equiv \frac{1}{T} \qquad \mu_B/T \ll 1$$

$$\phi(x) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{ik\cdot x}\phi(k) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{i(\omega_k\tau-\mathbf{k}\cdot\mathbf{x})}\phi(k)$$

$$\omega_n = 2n\pi T \qquad\qquad k = (\omega_n, \mathbf{k})$$

$$\mathcal{Z}(T) = \int \mathcal{D}\phi \exp\left(-T\sum\int_{\mathbf{k}}\frac{1}{2}\left(\mathbf{k}^2+\omega_n^2+m^2\right)|\phi(k)|^2\right)$$

$$= \exp\left[-\frac{V}{T}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)\right]$$

The free energy

$$F = -T\ln\mathcal{Z}$$

$$\lim_{V\to\infty}\frac{F}{V} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)$$

$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$

$$\equiv J_0(m)+\tilde{J}_B(m,T) \qquad \tilde{J}_i = T^4/2\pi^2 J_i.$$

$$\left(\lim_{V\to\infty}\frac{F}{V}\right)_{\text{fermions}} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1+e^{-\omega/T}\right)\right)$$

$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1+e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$

$$\equiv J_0(m)+\tilde{J}_F(m,T)$$

$$\tilde{J}_B(m,T) = T\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$

$$= \frac{T}{2\pi^2}\int d|\mathbf{k}|\,\mathbf{k}^2\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$

$$= \frac{T^4}{2\pi^2}\int dx\,x^2\ln\left(1-e^{-\sqrt{(m/T)^2+x^2}}\right)$$

$$\tilde{J}_F = \frac{T^4}{2\pi^2}\left(-\frac{7\pi^4}{360}+\frac{\pi^2m^2}{24T^2}-\frac{m^4}{32T^4}\left[\ln\left(\frac{e^{\gamma_E}}{\pi^2}\frac{m^2}{T^2}\right)-\frac{3}{2}\right]+\mathcal{O}\left(\frac{m^6}{T^6}\right)\right)$$

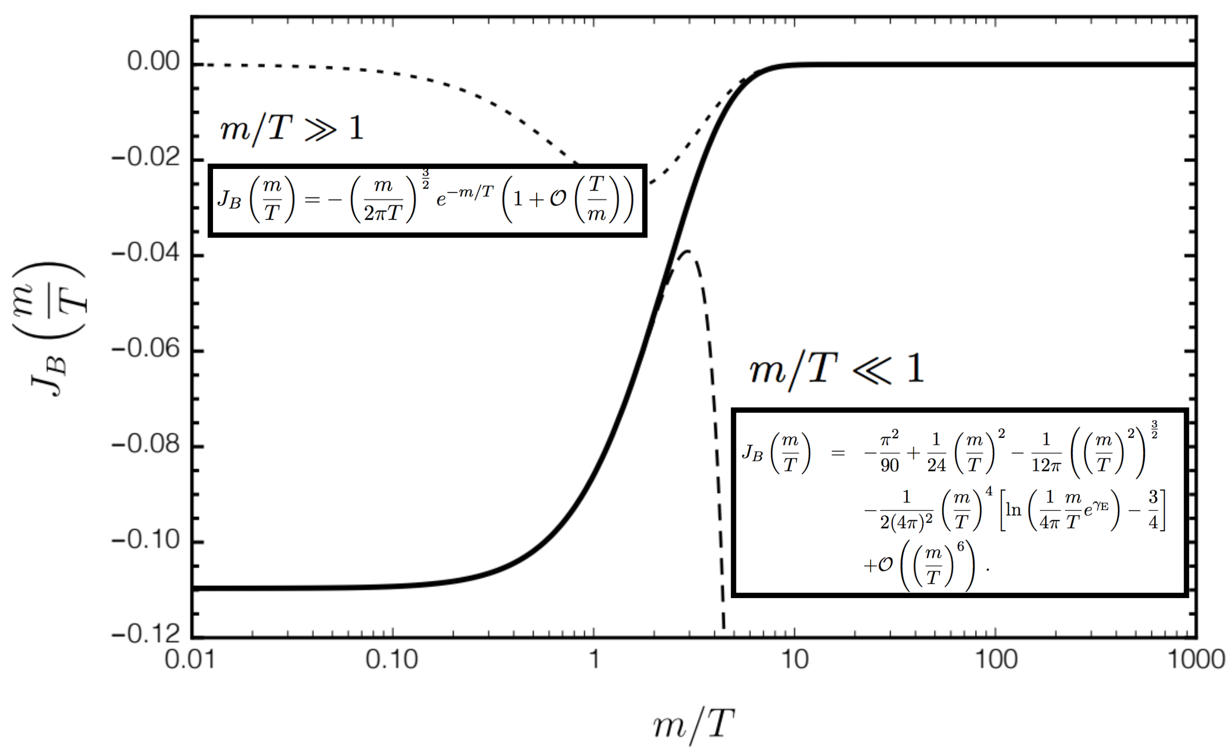
high-T expansion m ≪ T

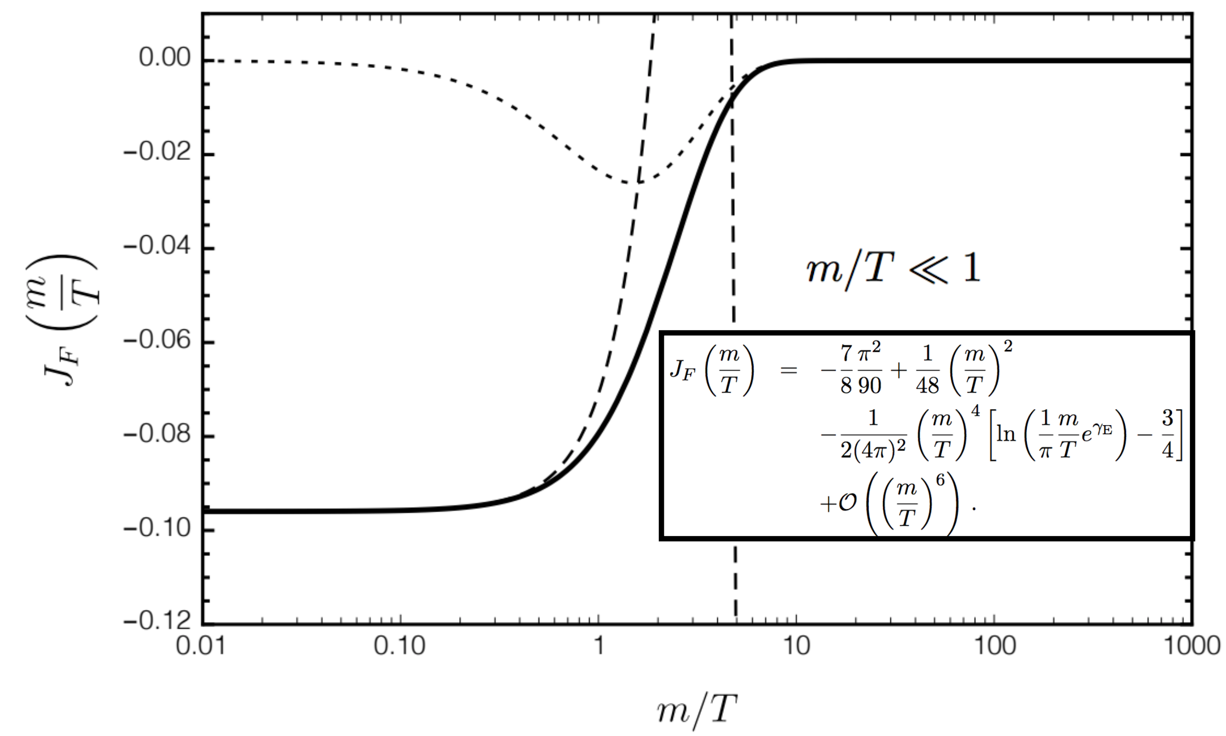
2307.00068

Thermal effective scalar potential for PT study

$$V_T(\phi,T) = V_0(\phi) + T^4\left[\sum_B J_B\left(\frac{M_B}{T}\right) + \sum_F J_F\left(\frac{M_F}{T}\right)\right]$$

all fermions F and bosons B that are relativistic at temperature T





High-T expansion

$m/T \ll 1$

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24}\left(\sum_S M_S^2(\phi) + 3\sum_V M_V^2(\phi) + 2\sum_F M_F^2(\phi)\right) - \frac{T}{12\pi}\left(\sum_S \left(M_S^2(\phi)\right)^{\frac{3}{2}} + \sum_V \left(M_V^2(\phi)\right)^{\frac{3}{2}}\right) + \text{higher order terms}.$$

MS, MV , MF are the masses of the scalar fields S, vector fields V and fermonic fields F

1-loop thermal mass at Finite temperature

Euclidean four-momenta $P \equiv (\omega_n, \mathbf{p})$

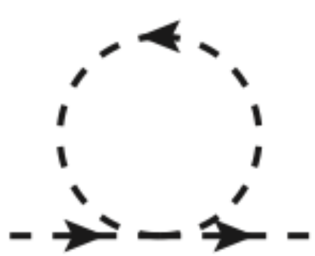
bosonic Matsubara frequency is $\omega_n = 2\pi\, n\, T$

Fermionic… $\omega_n = 2\pi\,(n+1)\, T$

$$\sum\!\!\!\!\!\!\int_P \equiv T \sum_{\omega_n} \int_p , \qquad \int_p \equiv \left(\frac{\bar{\mu}^2 e^{\gamma_{\rm E}}}{4\pi}\right)^{\epsilon} \int \frac{{\rm d}^d p}{(2\pi)^d} ,$$

$$\sum\!\!\!\!\!\!\int_P{}' \equiv T \sum_{\omega_n \neq 0} \int_p , \qquad n_{\rm B/F}(E_p, T) \equiv \frac{1}{e^{E_p/T} \mp 1} .$$

$D = d + 1 = 4 - 2\epsilon$



$$\equiv \sum\!\!\!\!\!\!\int_P \frac{1}{P^2 + m^2} = \left(\frac{\bar{\mu}^2 e^{\gamma_{\rm E}}}{4\pi}\right)^{\epsilon} \int \frac{{\rm d}^D p}{(2\pi)^D} \frac{1}{p^2 + m^2} + \int_p \frac{n_{\rm B}(E_p, T)}{E_p}$$

T =0, UV-div. T $\neq$ 0, UV-finite, IR-sensitive

$$= \int_p \frac{T}{p^2 + m^2} + \sum\!\!\!\!\!\!\int_P{}' \frac{1}{P^2 + m^2}$$

UV-finite, IR-sensitive UV-div., IR-safe

$$m_T^2 = m^2 + \# g^2 T^2$$

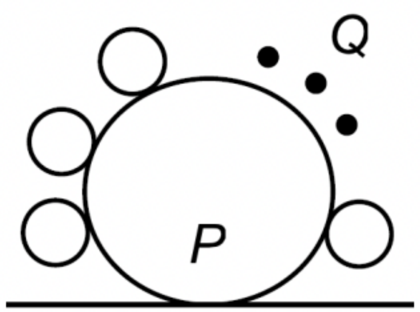
thermal correction

effective thermal mass for zero-mode/thermally corrected mass

Linde's IR problem and Resummation

$$g^2 \to g^2 n_{\rm B}(E,T) = \frac{g^2}{e^{E/T}-1} \approx \frac{g^2 T}{E} \geq \frac{g^2 T}{m}$$

the perturbative expansion breaks down for the bosonic zero mode when m $\ll g^2$T （Linde's IR problem）



$$\propto g^{2N}\left[\int_p \frac{T}{(p^2+m_T^2)^N}\right]\left[\sum\!\!\!\!\!\!\int_Q{}' \frac{1}{Q^2}\right]^N \propto m_T^3 T\left(\frac{gT}{m_T}\right)^{2N}$$

$$m_T^2 = m^2 + \#g^2T^2$$

thermal correction

$$\int_p \equiv \left(\frac{\bar{\mu}^2 e_E^{\gamma}}{4\pi}\right)^{\epsilon} \int \frac{d^d p}{(2\pi)^d}, \qquad \text{and} \qquad \sum\!\!\!\!\!\!\int_P{}' \equiv T \sum_{\omega_n \neq 0} \int_p$$

$\sim\mathcal{O}(g^3)$, when P = (0, **p**) , Q=(ω_n,**q**) with $\omega_n \neq 0$

IR-divergent for N ≥ 2 for m=0 when thermal correction are not included

Phys. Rev. D 9 (1974) 3320
arXiv:hep-ph/9204216
Phys.Lett.B 96 (1980) 289-292

▸ 1-loop Effective potential at finite temperature

1-loop finite-T thermal effective potential

$$V_{\rm eff}(\phi, T) = V_{\rm tree} + V_{\text{1-loop}}$$

1-loop

$$V_{\text{1-loop}} = \frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left(P^2 + m^2\right)$$

$$= \frac{1}{2}\left(\frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon} \int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m^2) - \int_p T \ln\left(1 \mp n_{B/F}(E_p, T)\right)$$

$V_{CW}(m)$ $\qquad V_T \sim J_{T,b/f}\left(\frac{m^2}{T^2}\right)$

$$= \frac{T}{2}\int_p \ln(p^2 + m^2) + \frac{1}{2}\sum\!\!\!\!\!\!\!\int'_{P/\{P\}} \ln(P^2 + m^2)$$

$V_{\rm soft}(m)$ $\qquad V_{\rm hard}(m)$

Daisy/ring resummation $\quad V_{daisy} = V_{soft}^{resummed} - V_{soft}$

$$V_{\rm soft}(m) = -\frac{T}{12\pi}(m^2)^{\frac{3}{2}} \longrightarrow V_{soft}^{resummed} = -\frac{T}{12\pi}(m^2 + \Pi_T)^{\frac{3}{2}}$$

Arnold-Espinosa eff potential $\quad V_{\rm eff}^{\text{A-E res.}}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm CW} + V_T + V_{\rm daisy}$

$$V_{\rm eff}^{\rm resummed}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm soft}^{\rm resummed} + V_{\rm hard}$$

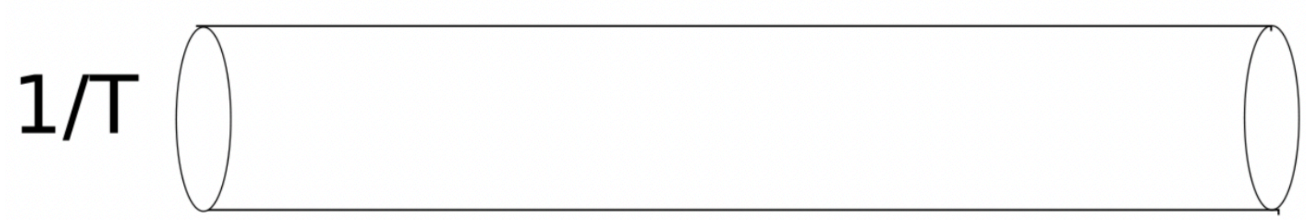
Phys. Rev. D47 (1993) 3546 [hep-ph/9212235]
See also Parwani method in Phys. Rev. D45 (1992) 4695 [hep-ph/9204216]

Finite temperature EFT for the 3d Phase transition study

Matsubara decomposition

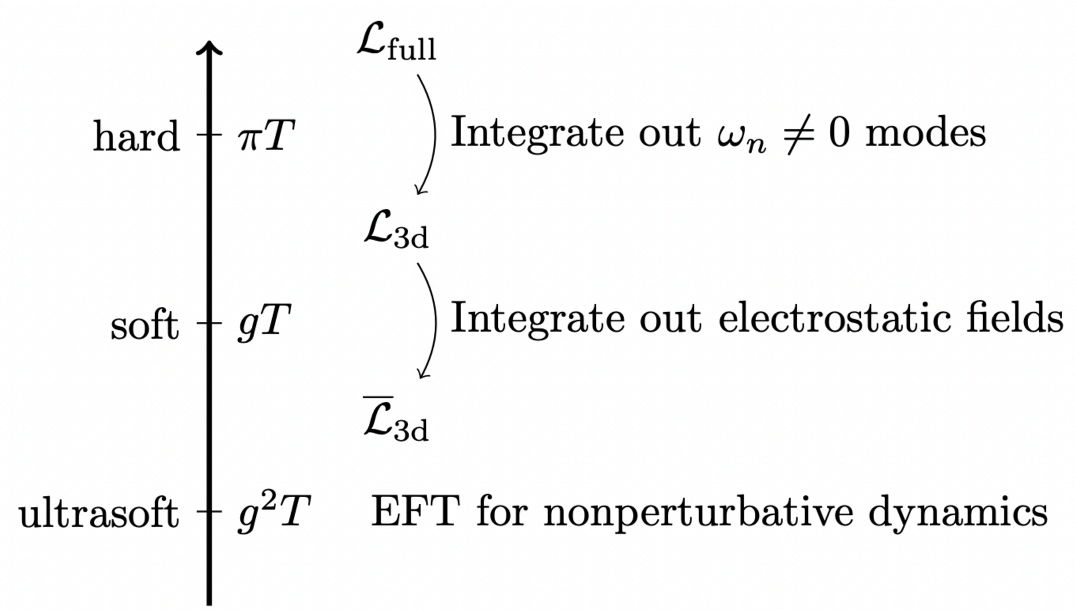
$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\mathbf{p}) e^{i\omega_n \tau}, \quad \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

$\omega_n \neq 0$ modes are heavy and decouple at distances » 1/T, and can be integrated out



$$S = \int d^4x \, [\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}]$$

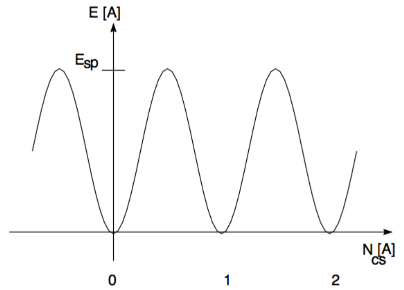
Procedure



$$S_{\text{3d}} = \int d^3x \, \Big[\frac{1}{4} F^a_{ij} F^a_{ij} + (D_i\phi)^\dagger (D_i\phi) + \bar{m}^2 \phi^\dagger \phi + \bar{\lambda}(\phi^\dagger\phi)^2 + \mathcal{L}_{\text{BSM}}$$

$$+ \text{higher-order operators}\Big]$$

BNPC & EW sphaleron



$$\partial_\mu J^\mu_B = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}),$$

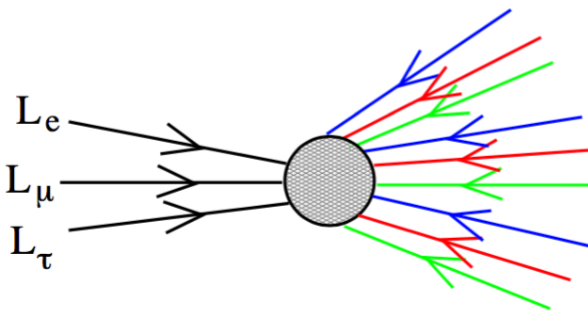
$$N_{\rm CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,\mathrm{Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right],$$

$$n_{\rm CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\,\epsilon^{ijk}\,\partial_i B_j B_k,$$

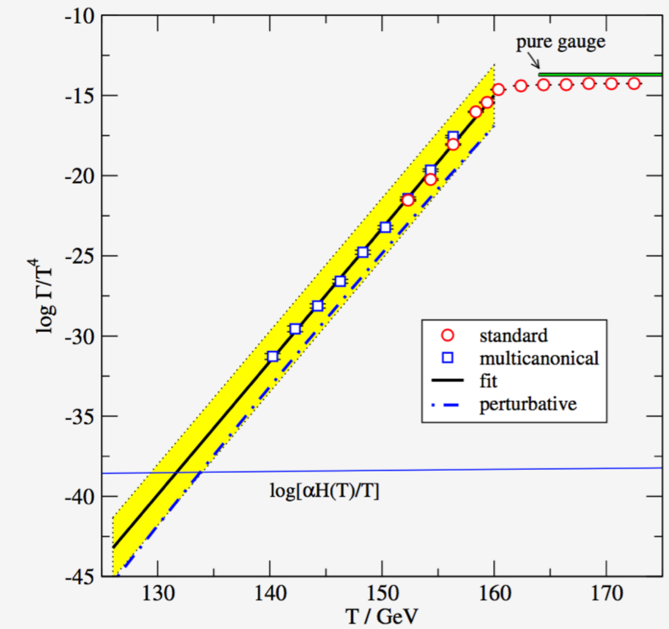
$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1},$$

$$\delta N_{\rm CS} = \frac{1}{24\pi^2}\int d^3x\,\mathrm{Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}.$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleros" is Greek for "ready to fall").



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



Lattice result, $T_C = (159.5 \pm 1.5)\mathrm{GeV}$, Phys.Rev.Lett,113, 141602 (2014).

$$\Gamma^{\rm sym} \approx 6\times(18\pm 3)\alpha_W^5 T^4, \qquad \Gamma^{\rm brok} \sim T^4 \exp\left(-\frac{E_{\rm sph}}{T}\right)$$

Washout avoidance,BNPC

$$\Gamma_{\rm sph} = A_{\rm sph}(T)\exp[-E_{\rm sph}(T)/T] < \mathrm{H(T)}$$

$$E_{\rm sph}(T) \approx E_{\rm sph,0}\frac{v(T)}{v} \qquad \frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\rm sph,0}}{1.916\times 4\pi v/g}\right)^{-1}$$

$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ \mathrm{GeV}} \qquad PT_{sph} > (35.9 - 42.8)$$

Hiren H. Patel and Michael J. Ramsey-Musolf, 15’

Xucheng Gan, Andrew J. Long, Lian-Tao Wang, 17’