

CEPC Physics/Detector Workshop, August 2023

New directions in kaon physics

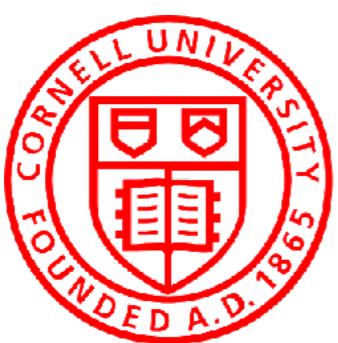
Avital Dery

AD, Ghosh, Grossman, Schacht, [\[arXiv: 2104.06427\]](https://arxiv.org/abs/2104.06427)

AD, Ghosh, [\[arXiv:2112.05801\]](https://arxiv.org/abs/2112.05801)

AD, Ghosh, Grossman, Kitahara, Schacht [\[arXiv:2211.03804\]](https://arxiv.org/abs/2211.03804)

Reviews: AD [\[arXiv:2211.06446\]](https://arxiv.org/abs/2211.06446), Schacht [\[arXiv:2305.06267\]](https://arxiv.org/abs/2305.06267)



Motivation -

Reminder: The Wolfenstein parameterization of the CKM

$$-\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} W^+_\mu \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Assuming SM: 4 physical parameters: A, λ, ρ, η

First 4 indep. measurements - calibration,
From 5th measurement on - test of the SM

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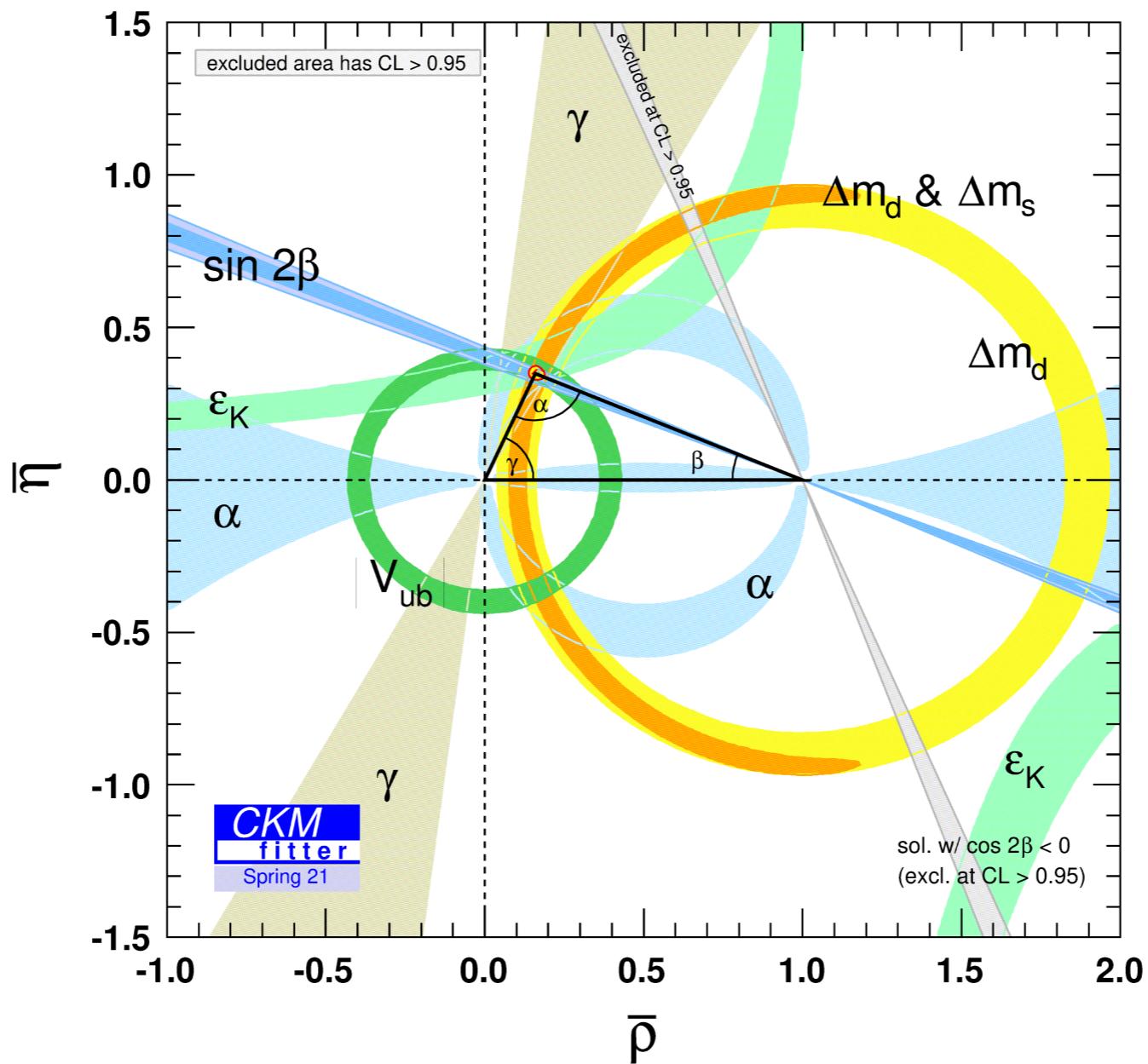
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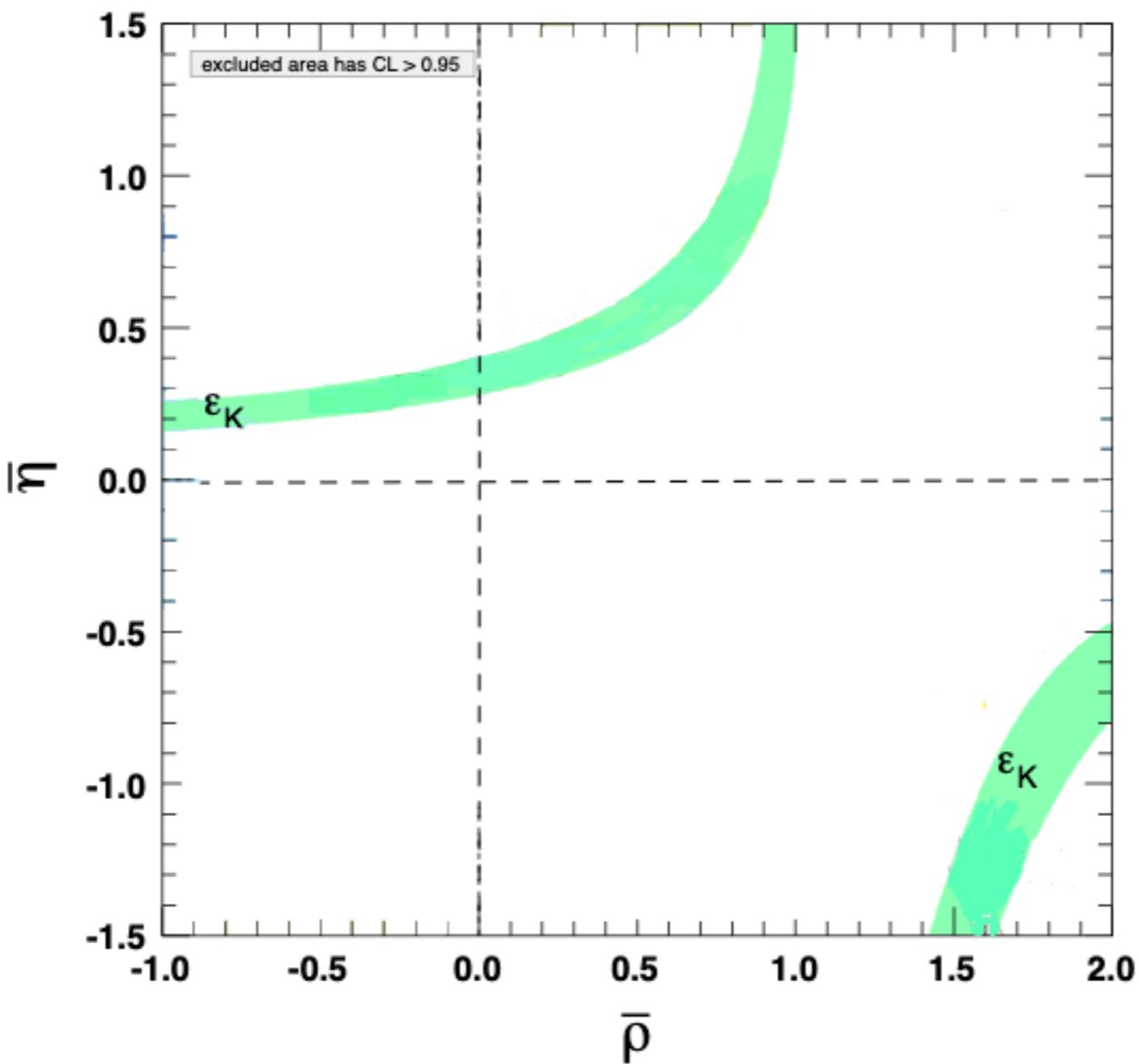
Current status: CKM picture verified in dozen of processes,

$$\frac{\delta A}{A} = 2\%, \quad \frac{\delta \lambda}{\lambda} = 0.2\%, \quad \frac{\delta \rho}{\rho} = 12\%, \quad \frac{\delta \eta}{\eta} = 3\%.$$

CPV



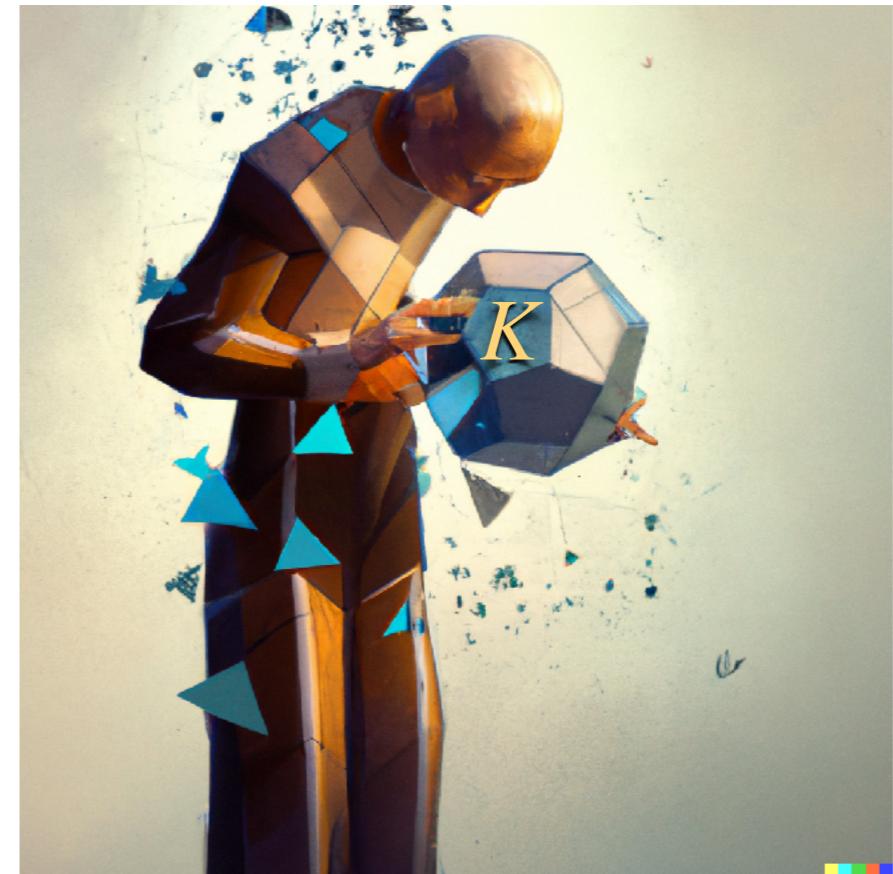
Cross-checks from **kaon physics** would be a unique and crucial test of the CKM paradigm.



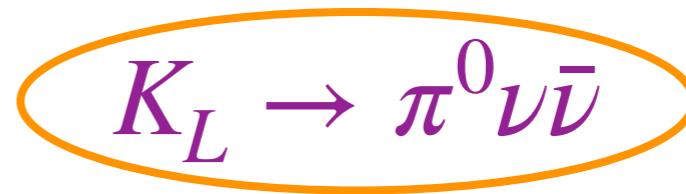
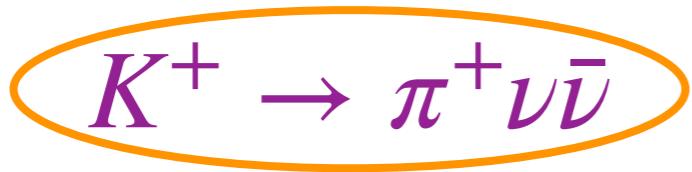
Cross-checks from **kaon physics** would be a
unique and crucial test of the CKM paradigm.

Kaon CKM measurements provide a valuable independent test.

- Employ a different set of experimental techniques
- Require a different set of theoretical tools
- Sensitive to different New Physics scenarios

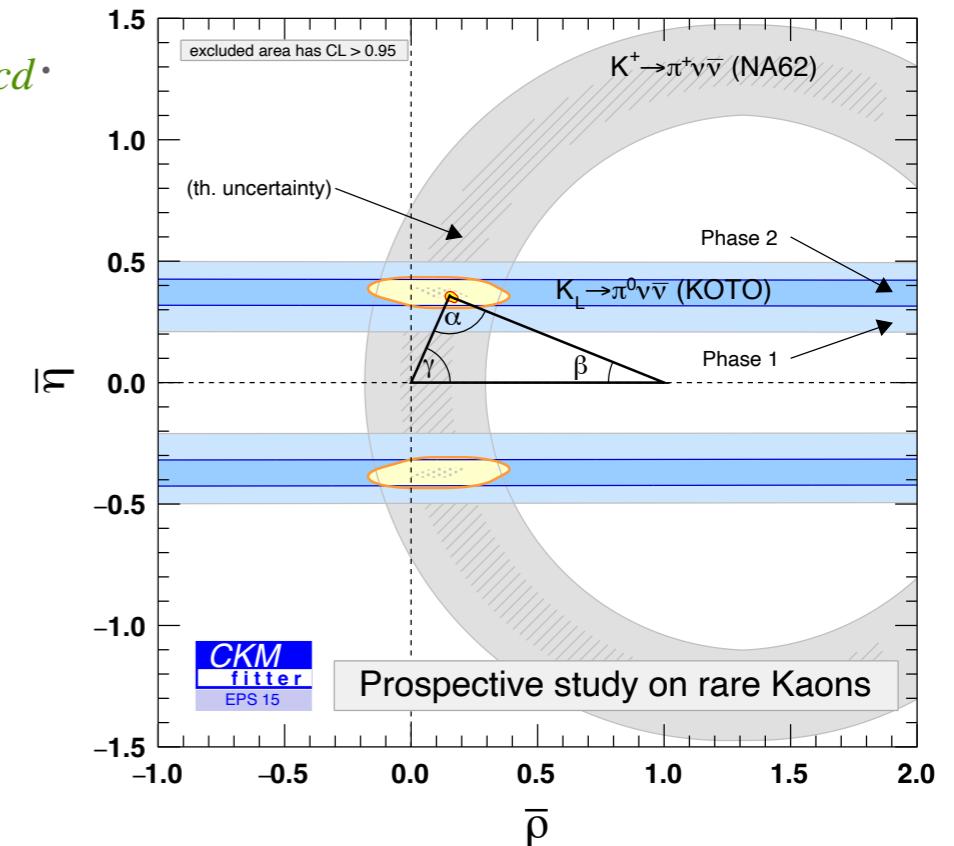


Two *golden modes* identified in the 1990's



Theoretically: Clean, sensitive to functions of $V_{ts}^* V_{td}$, $V_{cs}^* V_{cd}$.

The combination of both measurements would enable determination of $\{\rho, \eta\}$ from kaon physics.



Two *golden modes* identified in the 1990's

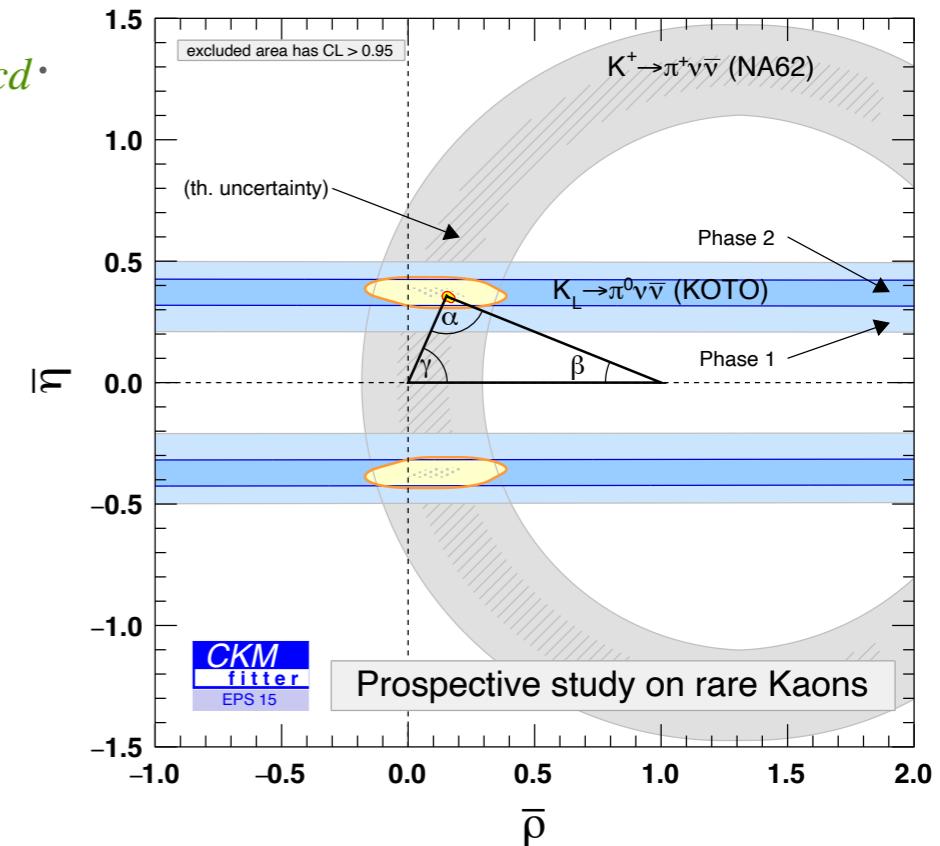
$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

Theoretically: Clean, sensitive to functions of $V_{ts}^* V_{td}$, $V_{cs}^* V_{cd}$.

The combination of both measurements would enable determination of $\{\rho, \eta\}$ from kaon physics.

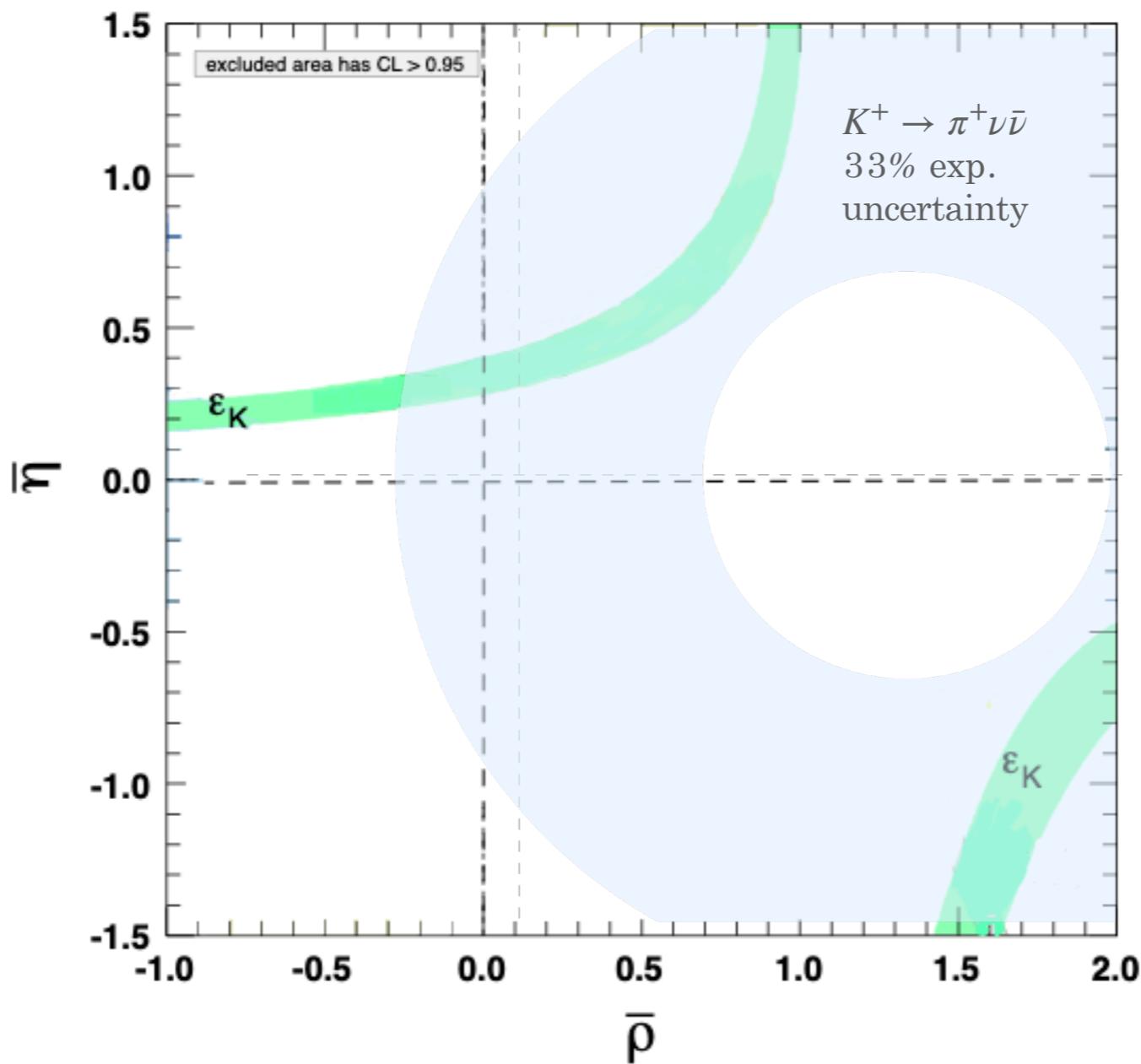
Experimentally: Extremely challenging.
Very rare decays, invisible particles in the final state.



Current status: $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.06^{+0.41}_{-0.34}) \cdot 10^{-10}$ 2021: First evidence observed by **NA62**!

$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.9 \cdot 10^{-9}$ 2021: Latest bound by **KOTO**

$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = 2.6 \cdot 10^{-11}$ Extremely challenging to approach SM sensitivity



Cross-checks from **kaon physics** would be a
unique and crucial test of the CKM paradigm.

What about

$K_{L,S} \rightarrow \mu^+ \mu^-$?

$K_{L,S} \rightarrow \mu^+ \mu^-$ - Measurement vs. SM prediction

$$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{LHCb 2020}} < 2.1 \cdot 10^{-10}$$

Hadronic uncertainty

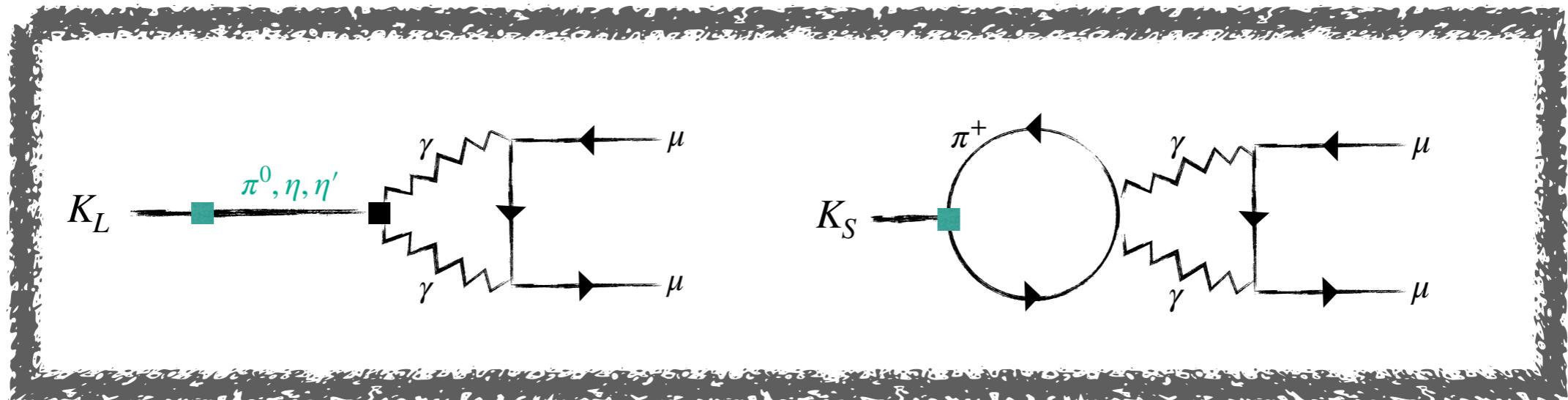
$$\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)_{\text{SM}} = \begin{cases} (6.85 \pm 0.80 \pm 0.06) \times 10^{-9} (+) \\ (8.11 \pm 1.49 \pm 0.13) \times 10^{-9} (-) \end{cases}$$

Hadronic uncertainty

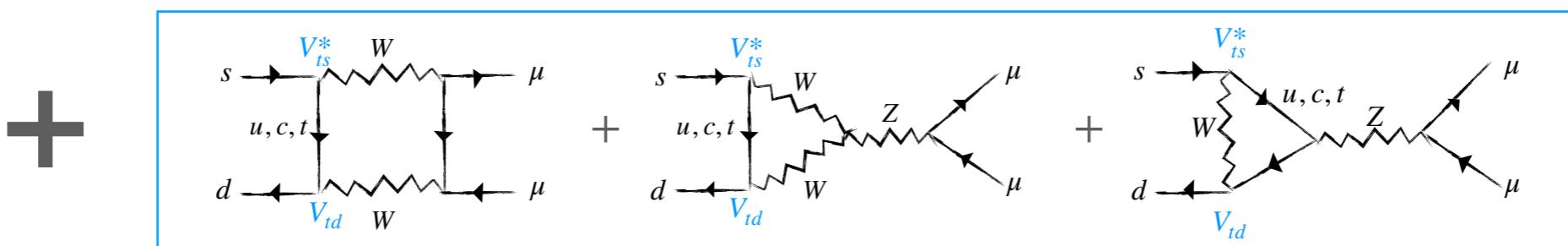
$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{SM}} = (5.18 \pm 1.50 \pm 0.02) \times 10^{-12}$$

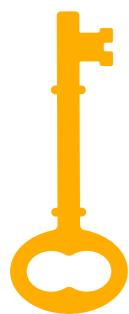
$K_{L,S} \rightarrow \mu^+ \mu^-$ - leading and sub-leading contributions

Total (time integrated) rates severely dominated by **non-perturbative effects**



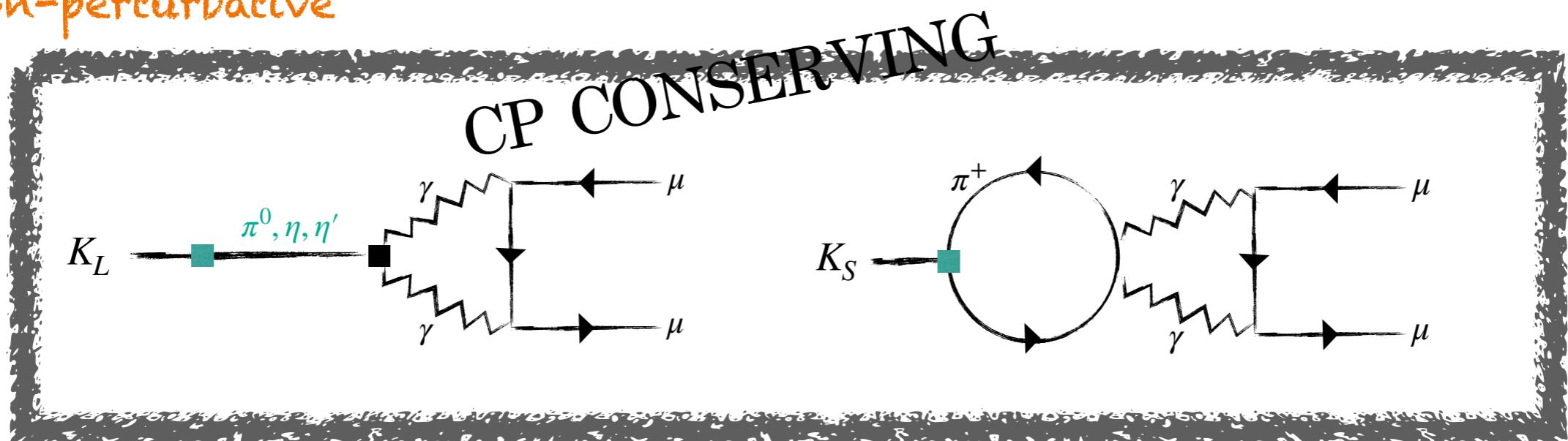
Sub-dominant contributions contain the desired clean CKM dependence





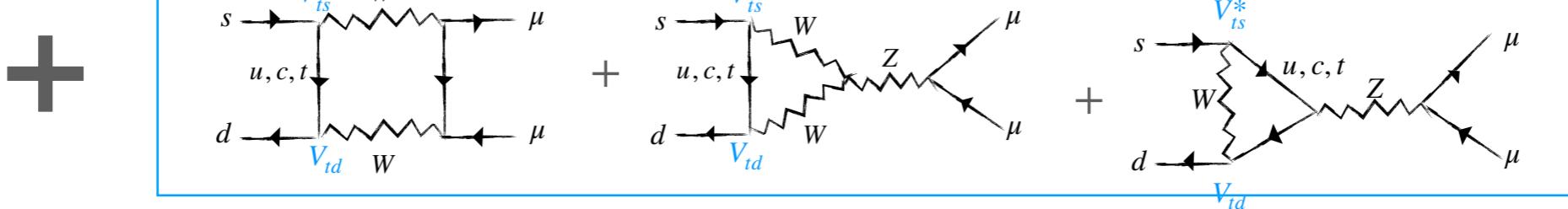
: CP Violation

Non-perturbative



Perturbative, CKM sensitive

Includes CP VIOLATION



Approximations used in the analysis

We work under the following 3 assumptions -

1. CPV in mixing is negligible, $\mathcal{O}(\varepsilon_K) \sim 10^{-3}$
2. No scalar operators are relevant, in the SM good to $\mathcal{O}(m_K^2/m_W^2) \sim 10^{-4}$
3. CPV in the non-perturbative contributions is negligible, of $\mathcal{O}(\lambda^4) \sim 10^{-3}$

All fulfilled in the SM to $\mathcal{O}(10^{-3})$

CP analysis of $K \rightarrow \mu^+ \mu^-$

Initial state: kaon mass eigenstates are CP eigenstates, (Up to $\mathcal{O}(10^{-3})$ corrections)

$$\begin{array}{cc} K_L & K_S \\ \text{CP-odd} & \text{CP-even} \end{array}$$

Final state: since the kaon has $J = 0$, the dimuon state can have either $S = 0, \ell = 0$ corresponding to final states: or $S = 1, \ell = 1$

$$\begin{array}{cc} (\bar{\mu}\mu)_{\ell=0} & (\bar{\mu}\mu)_{\ell=1} \\ \text{CP-odd} & \text{CP-even} \end{array}$$

In practice, we measure the incoherent sums,

$$\Gamma(K_S \rightarrow \mu^+ \mu^-)_{meas.} = \Gamma(K_S \rightarrow (\mu^+ \mu^-)_{\ell=0}) + \Gamma(K_S \rightarrow (\mu^+ \mu^-)_{\ell=1})$$

$$\Gamma(K_L \rightarrow \mu^+ \mu^-)_{meas.} = \Gamma(K_L \rightarrow (\mu^+ \mu^-)_{\ell=0}) + \Gamma(K_L \rightarrow (\mu^+ \mu^-)_{\ell=1})$$

If we could extract the CPV modes, we would have a similar situation (theoretically) to $K_L \rightarrow \pi^0 \nu \bar{\nu}$

Time dependent rate - naive parameter counting

$$\left(\frac{d\Gamma}{dt} \right) = N_f f(t), \quad f(t) = C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2 C_{Int.} \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$

4 Experimental parameters
 $\{C_L, C_S, C_{Int.}, \varphi_0\}$

Experimental

Theory parameters: 4 amplitudes and 2 phases

CPV $|A(K_S \rightarrow (\mu^+ \mu^-)_{\ell=0})|$

$|A(K_S \rightarrow (\mu^+ \mu^-)_{\ell=1})|$

$|A(K_L \rightarrow (\mu^+ \mu^-)_{\ell=0})|$

CPV $|A(K_L \rightarrow (\mu^+ \mu^-)_{\ell=1})|$

$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$

$\varphi_1 \equiv \arg(A(K_S)_1^* A(K_L)_1)$

Theoretical

A priori, 6 theory parameters



Time dependent rate - naive parameter counting

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Experimental

Theory parameters: 4 amplitudes and 2 phases

CPV $|A(K_S \rightarrow (\mu^+ \mu^-)_{\ell=0})|$

$|A(K_S \rightarrow (\mu^+ \mu^-)_{\ell=1})|$

$|A(K_L \rightarrow (\mu^+ \mu^-)_{\ell=0})|$

CPV $|A(K_L \rightarrow (\mu^+ \mu^-)_{\ell=1})|$

Under our approximations two are negligible

$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$

~~$\varphi_1 \equiv \arg(A(K_S)_1^* A(K_L)_1)$~~

Theoretical

A priori, ~~6~~ theory parameters

4, 1 of which is CP odd



Entire system can be solved.

4 Experimental parameters

$$\{C_L, C_S, C_{Int.}, \varphi_0\}$$

4 theory parameters



$$\{ |A(K_S)_0|, |A(K_L)_0|, |A(K_S)_1|, \arg(A(K_S)_0^* A(K_L)_0) \}$$

$$C_L = |A(K_L)_0|^2$$

$$C_S = |A(K_S)_0|^2 + \beta_\mu^2 |A(K_S)_1|^2$$

$$C_{Int.} = D |A(K_S)_0 A(K_L)_0|$$

$$\varphi_0 = \arg(A(K_S)_0 A(K_L)_0)$$

$$D = \frac{N_{K^0} - N_{\bar{K}^0}}{N_{K^0} + N_{\bar{K}^0}}$$

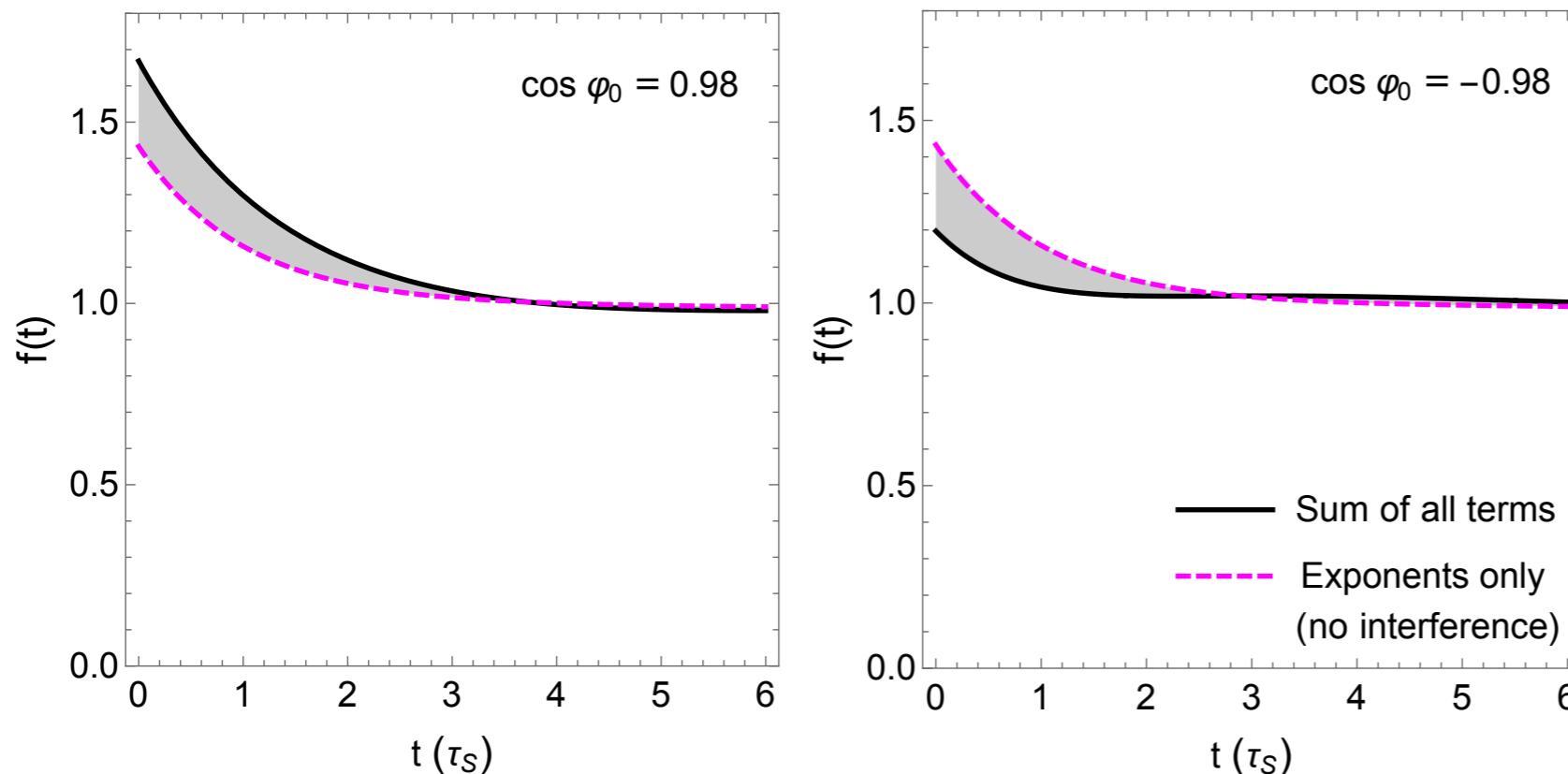
In particular, we can solve for
the CPV amplitude $|A(K_S)_0|$

$$\frac{1}{D^2} \frac{C_{Int.}^2}{C_L} = |A(K_S)_0|^2$$

Time dependence - measuring $C_{\text{Int.}}$.

$$f(t) = C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2 C_{\text{Int.}} \cos(\Delta m t - \varphi_0) e^{-\Gamma t},$$

Pure K^0 beam



$$\frac{1}{D^2} \frac{C_{\text{Int.}}^2}{C_L} = |A(K_S)_0|^2$$

SM prediction

[AD, M. Ghosh, Y. Grossman, S. Schacht [arXiv:2104.06427]

J. Brod and E. Stamou [arXiv:2209.07445]

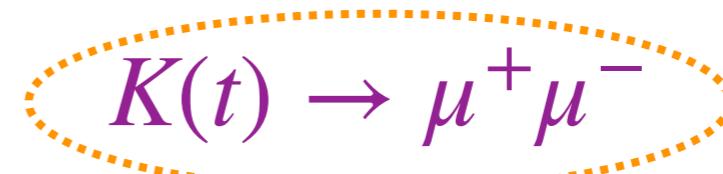
$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \frac{\tau_S \beta_\mu}{16\pi m_K} |A(K_S)_{\ell=0}|^2$$

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \frac{\tau_S \beta_\mu}{16\pi m_K} \left| \frac{2G_F^2 m_W^2}{\pi^2} m_K m_\mu Y_t \times f_K \times \underbrace{A^2 \lambda^5 \bar{\eta}}_{} \right|^2$$

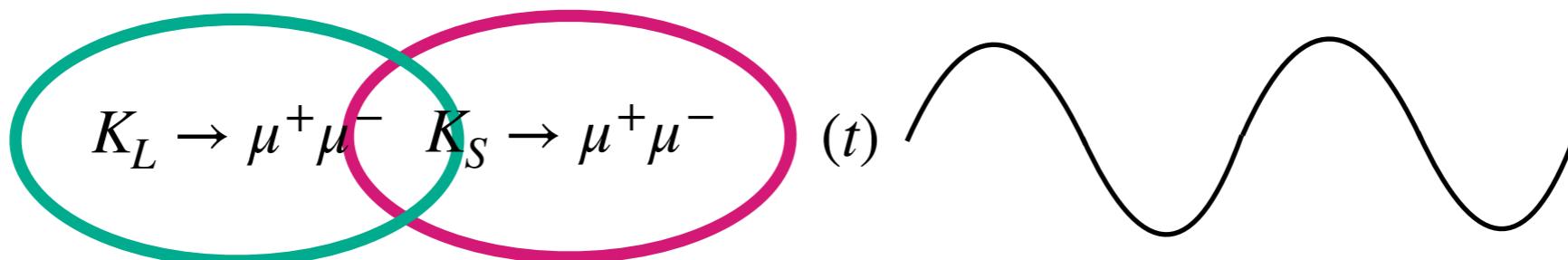
Only hadronic parameter,
 $\mathcal{O}(1\%)$ uncertainty from
isospin breaking

Desired CKM
dependence

Time dependent rate -

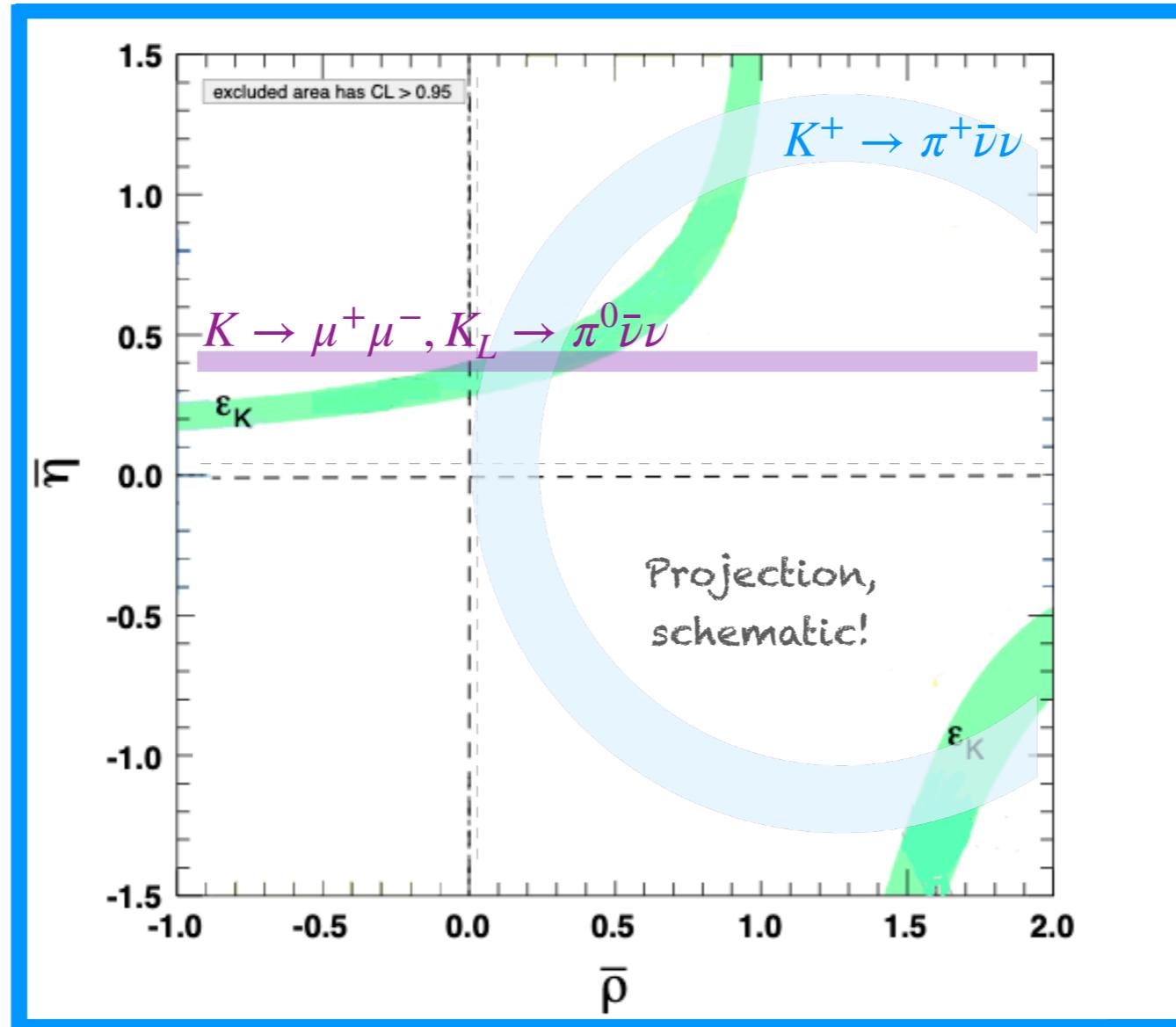


Third golden mode



$K_L - K_S$ interference in a measurement of the **time dependent rate** can be used to extract a **clean CPV observable**, measuring η from kaon physics with $\lesssim 1\%$ theory uncertainty

Third kaon golden mode.



The experimental challenge

Zeroth order requirements:

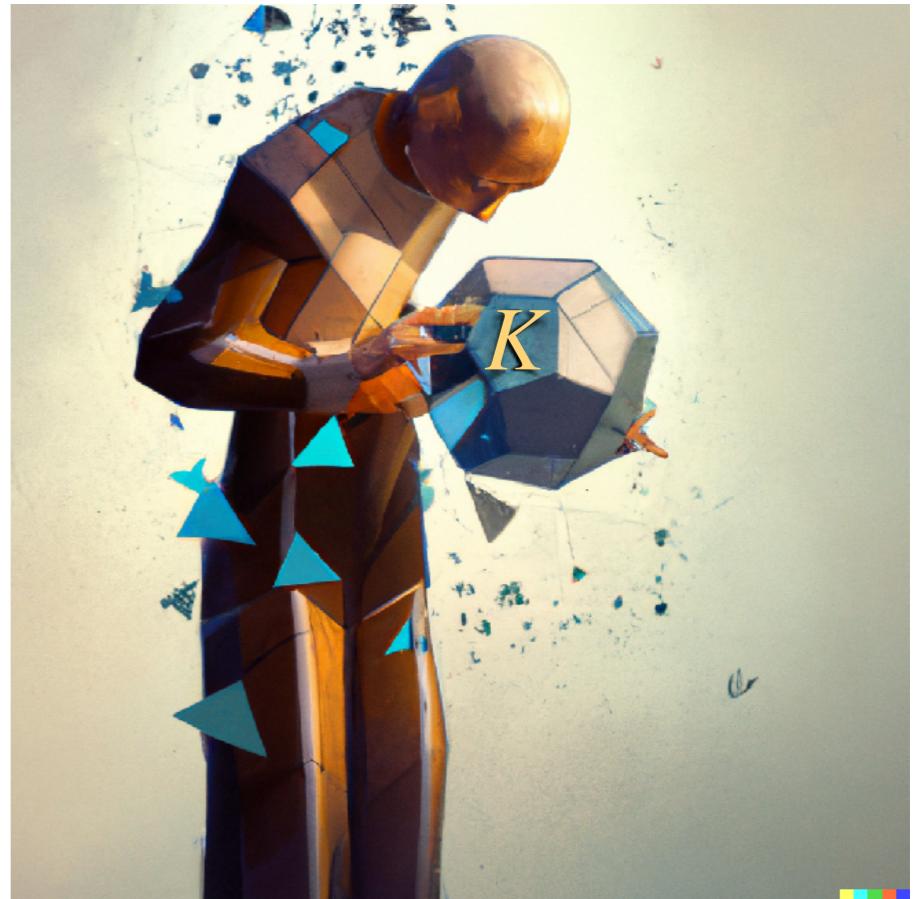
1. Neutral kaons with non-zero dilution factor, $D = \frac{N_{K^0} - N_{\bar{K}^0}}{N_{K^0} + N_{\bar{K}^0}}$
2. Sensitivity to K_S lifetimes

@ CEPC?

- A yield of $\mathcal{O}(10^{13})$ neutral kaons expected.
- Naively, QCD production => same number of K^0 and \bar{K}^0 , $D = 0$.
=> would need to explore creating an artificial non-zero D via post-selection.
- Time scales?

Summary

- ◆ Recent progress in kaon physics, in particular
third kaon golden mode: $K(t) \rightarrow \mu^+ \mu^-$
- ◆ Requires exploration of feasibility in relevant future experimental facilities.
- ◆ Theory avenues for future exploration: related decay modes (e.g., $K \rightarrow \pi \ell \ell, K \rightarrow \pi \pi \ell \ell$),



**Thank you for your
attention!**

Supplemental

A lot of recent attention in the literature

Theory developments: SM aspects -

D'Ambrosio and Kitahara [1707.06999]

AD, Ghosh, Grossman and Schacht [2104.06427]

Buras and Venturini [2109.11032]

Brod and Stamou [2209.07445]

AD, Ghosh, Grossman, Kithara and Schacht [2211.03804]

Beyond the SM -

Chobanova et al. [1711.11030]

Endo et al. [1712.04959]

AD and Ghosh [2112.05801]

D'Ambrosio et al. [2206.14748]

Lattice QCD:

Christ, Feng, Jin, Tu and Zhao (2020)

Zhao and Christ (2022)

Christ, Feng, Jin, Tu and Zhao [2208.03834]

Experimental:

LHCb [2001.10354]

LHCb $K \rightarrow 4\mu$

(talk at Kaon 2022)

Some remarks

- This CKM combination is almost identical to that appearing in $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$. We have

$$\left[\frac{\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})} \right]^{\text{SM}} = 1.55 \cdot 10^{-2} \left(\frac{\lambda}{0.225} \right)^2 \left(\frac{Y_t}{X_t} \right)^2$$

A. J. Buras and E. Venturini,
[arXiv:2109.11032]

- The current numeric SM prediction reads

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}^{\text{SM}} = 1.70(02)_{\text{QCD/EW}}(01)_{f_K}(19)_{\text{param.}} \times 10^{-13}$$

J. Brod and E. Stamou
[arXiv:2209.07445]

where the non-parametric uncertainties are of $\mathcal{O}(1\%)$.

- Including effects of CPV in mixing ($\varepsilon_K \neq 0$) results in an additional (parametric) uncertainty of order 3 % .

J. Brod and E. Stamou
[arXiv:2209.07445]

Additional SM test - the phase shift φ_0

[AD, Ghosh, Grossman, Kitahara,
Schacht [arXiv:2211.03804]]

One of the four parameters of $K \rightarrow \mu^+ \mu^-$,

$$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$$

How can we estimate it?

Additional SM test - the phase shift φ_0

[AD, Ghosh, Grossman, Kitahara,
Schacht [arXiv:2211.03804]]

One of the four parameters of $K \rightarrow \mu^+ \mu^-$,

$$\varphi_0 \equiv \arg(A(K_S)_0^* A(K_L)_0)$$

Closely related to the ratio of K_L decay rates, $R_{K_L} \equiv \frac{\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)}{\mathcal{B}(K_L \rightarrow \gamma\gamma)}$

To see this, we write

$$\cos^2 \varphi_0 = \frac{\text{Im}[A(K_S)_{\ell=0}^* A(K_L)_{\ell=0}]^2}{|A(K_S)_{\ell=0}|^2 |A(K_L)_{\ell=0}|^2} = \frac{\text{Im}[A(K_L)_{\ell=0}]^2}{|A(K_L)_{\ell=0}|^2}$$

absorptive (on-shell)

Additional SM test - the phase shift φ_0

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To see this, we write

$$\cos^2 \varphi_0 = \frac{\text{Im}[A(K_L)_{\ell=0}]^2}{|A(K_L)_{\ell=0}|^2} = \frac{\text{absorptive (on-shell)}}{\mathcal{B}(K_L \rightarrow \mu^+ \mu^-)} = \frac{C_{\text{QED}}}{R_{K_L}}$$

Model independent prediction -

$$\cos^2 \varphi_0 = 0.96 \pm 0.02_{\text{exp.}} \pm 0.02_{\text{th.}}$$

$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}$ beyond the SM

[AD, M. Ghosh [arXiv:2112.05801]]

Current upper bound leaves much room for possible NP,

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-) = \mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} + \mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=1} \leq 2.1 \cdot 10^{-10}$$

LHCb
[arXiv:2001.10354]

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} \leq 2.1 \cdot 10^{-10}$$

$$\rightarrow \frac{\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}}{\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}^{\text{SM}}} \lesssim 10^3$$

Model independent effective operator analysis

[AD, M. Ghosh [arXiv:2112.05801]]

$$\mathcal{H}_{\text{eff.}}^{|\Delta S|=1} = \sum_i C_i O_i,$$

Vectorial	$O_{VLL} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{L}_L \gamma_\mu L_L);$	$O_{VLR} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{e}_R \gamma_\mu e_R),$
Scalar	$O_{VRL} = (\bar{d}_R \gamma^\mu d_R)(\bar{L}_L \gamma_\mu L_L);$	$O_{VRR} = (\bar{d}_R \gamma^\mu d_R)(\bar{e}_R \gamma_\mu e_R),$
	$O_{SLR} = (\bar{Q}_L d_R)(\bar{e}_R L_L),$	$O_{SRL} = (\bar{d}_R Q_L)(\bar{L}_L e_R)$

$$R(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} = \left(1 + \frac{1}{|C_{VLL}^{\text{SM}}| \sin \theta_{ct}} \left[A_S \left(|C_{SLR}^{\text{NP}}| \sin \Theta_{SLR} + |C_{SRL}^{\text{NP}}| \sin \Theta_{SRL} \right) \right. \right.$$

$$\left. \left. + |C_{VLL}^{\text{NP}}| \sin \Theta_{VLL} - |C_{VRL}^{\text{NP}}| \sin \Theta_{VRL} - |C_{VLR}^{\text{NP}}| \sin \Theta_{VLR} + |C_{VRR}^{\text{NP}}| \sin \Theta_{VRR} \right] \right)^2$$

$$R(K_L \rightarrow \pi^0 \bar{\nu}\nu) = \frac{1}{3} \sum_{i=e,\mu,\tau} \left(1 + \frac{|(C_{VLL}^{\text{NP}})_i| \sin \Theta_{VLL,i} + |(C_{VRL}^{\text{NP}})_i| \sin \Theta_{VRL,i}}{|C_{VLL}^{\text{SM}}| \sin \theta_{ct}} \right)^2$$

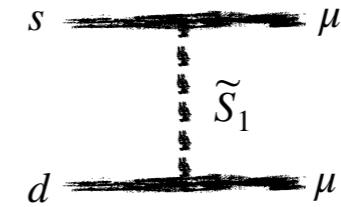
Additionally sensitive to RH currents and scalar operators

Example NP toy models

[AD, M. Ghosh [arXiv:2112.05801]]

A. Scalar Leptoquark $\tilde{S}_1 \sim (\bar{3}, 1)_{4/3}$

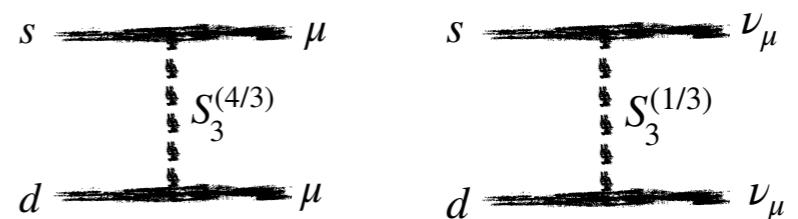
Generates the effective operator \mathcal{O}_{VRR}



Can saturate the current bound while satisfying all existing constraints, $R(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} \sim 10^3$

B. Scalar Leptoquark $S_3 \sim (\bar{3}, 3)_{1/3}$

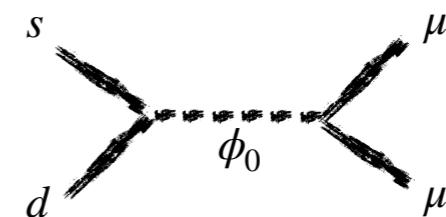
Generates the effective operator \mathcal{O}_{VLL}



Bounded by the GN bound on $R(K_L \rightarrow \pi^0 \bar{\nu}\nu)$, $R(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} \lesssim 26$

C. 2HDM $\Phi \sim (1,2)_{1/2} = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$

Generates the effective operators $\mathcal{O}_{SLR}, \mathcal{O}_{SRL}$



Can saturate the current bound while satisfying all existing constraints, $R(K_S \rightarrow \mu^+ \mu^-)_{\ell=0} \sim 10^3$

Relevant New Physics scales

[AD, M. Ghosh [arXiv:2112.05801]]

Assuming SM sensitivity -

Vectorial operators:

$$\Lambda_{\text{NP}} \sim 300 \text{ TeV}$$

Scalar operators:

$$\Lambda_{\text{NP}} \sim 1000 \text{ TeV}$$

[Direct searches, EW precision tests: $\Lambda_{\text{NP}} \sim 10 \text{ TeV}$]

The experimental challenge

Zeroth order requirements:

1. Neutral kaons with non-zero dilution factor, $D = \frac{N_{K^0} - N_{\bar{K}^0}}{N_{K^0} + N_{\bar{K}^0}}$
2. Sensitivity to K_S lifetimes

Current experiments:

LHCb -

1. Roughly the same number of K^0, \bar{K}^0 particles, $D = 0$.
Can induce an artificial non-zero D by tagging (post-selection).
2. YES

Bkgs such as $K_S \rightarrow \pi\pi$ may be difficult to control

KOTO -

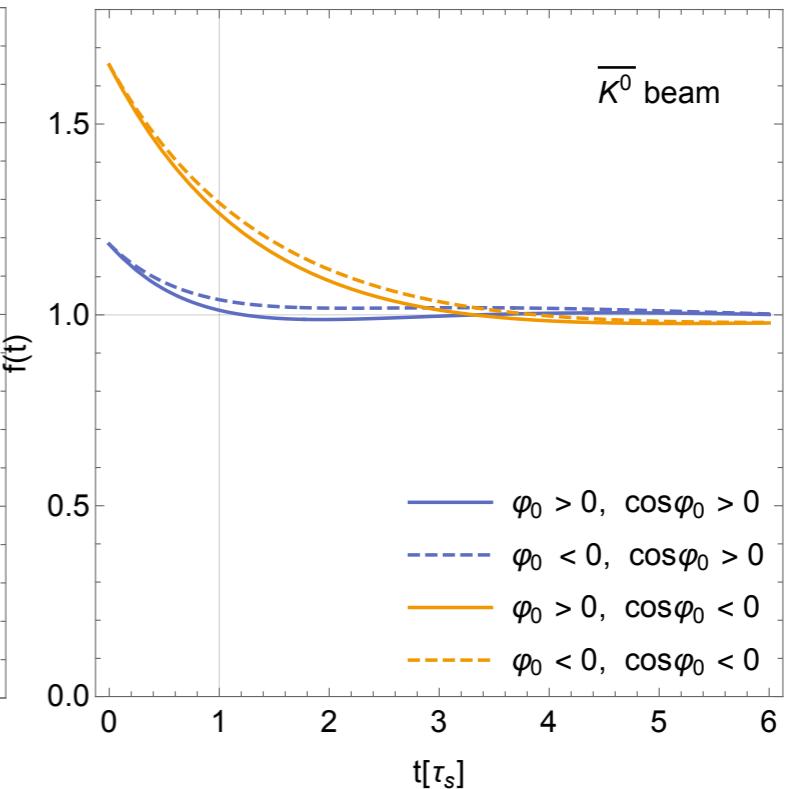
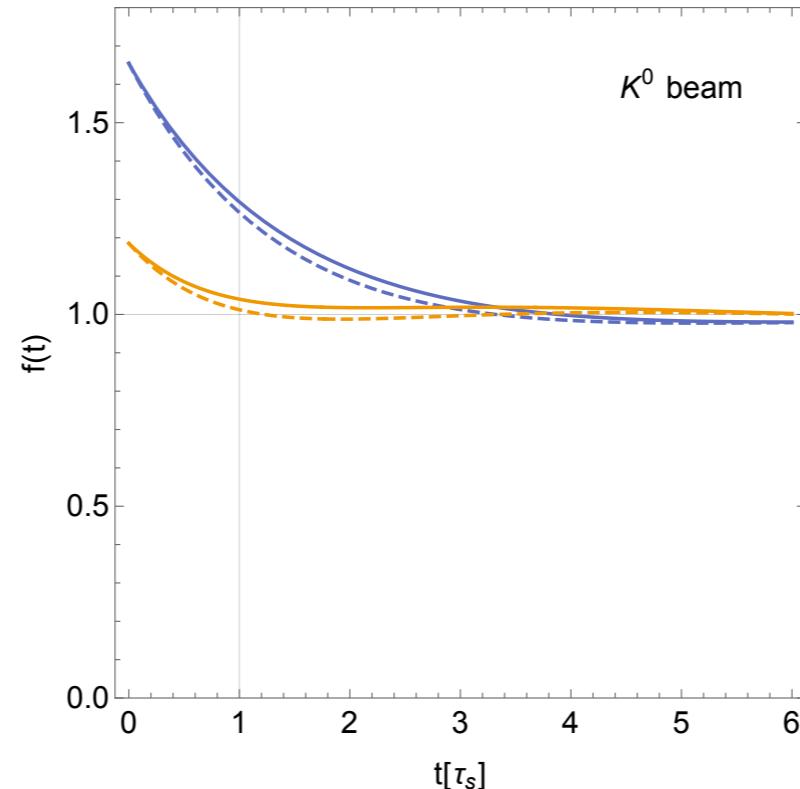
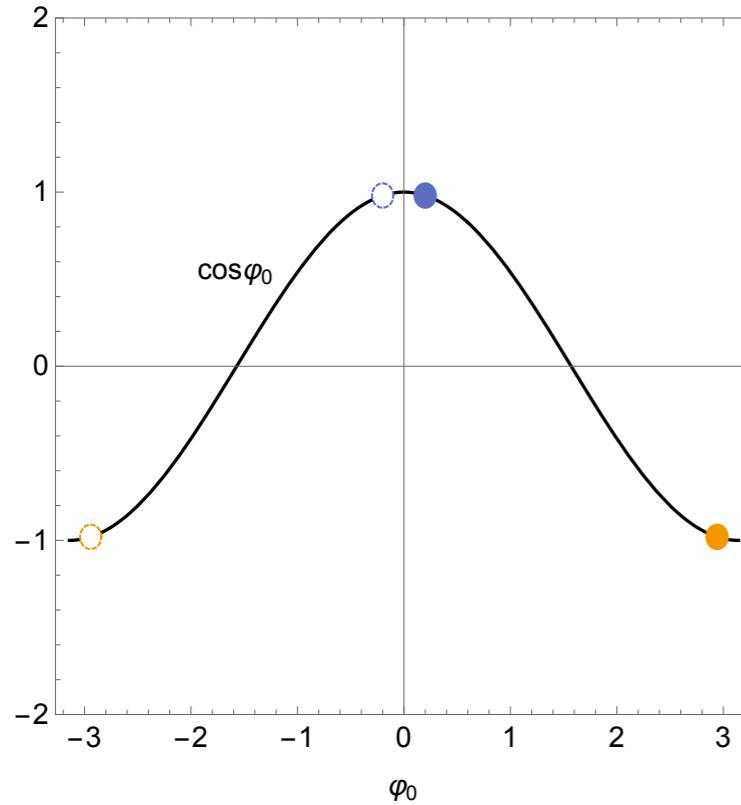
1. YES
2. Basically a K_L beam (detectors far from target)

Can in principle use matter regeneration to induce a K_S component.

NA62 - Charged beam

Additional SM test - the phase shift φ_0

[AD, Ghosh, Grossman, Kitahara, Schacht [arXiv:2211.03804]]

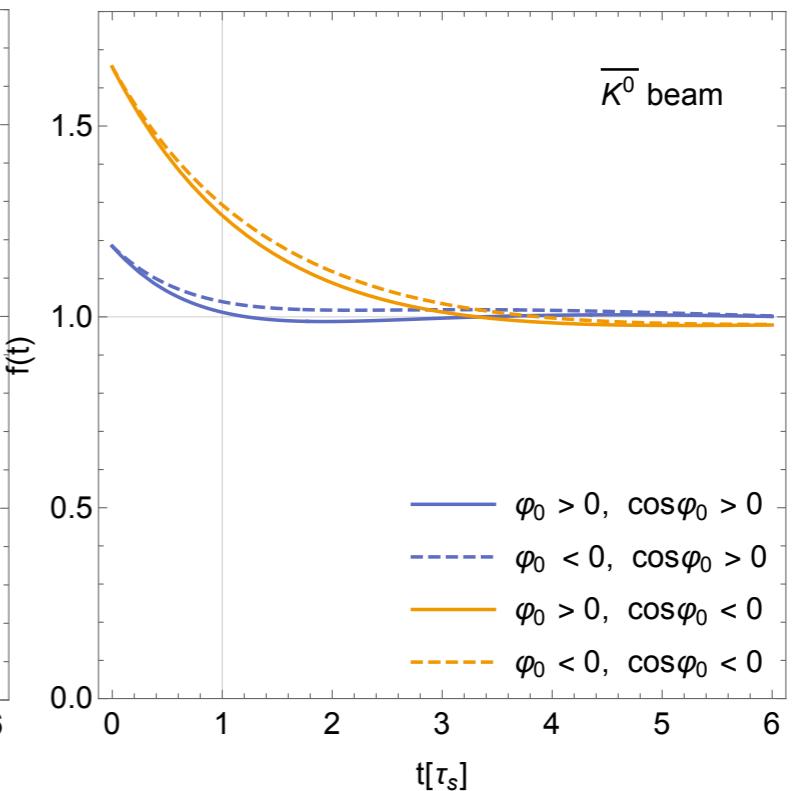
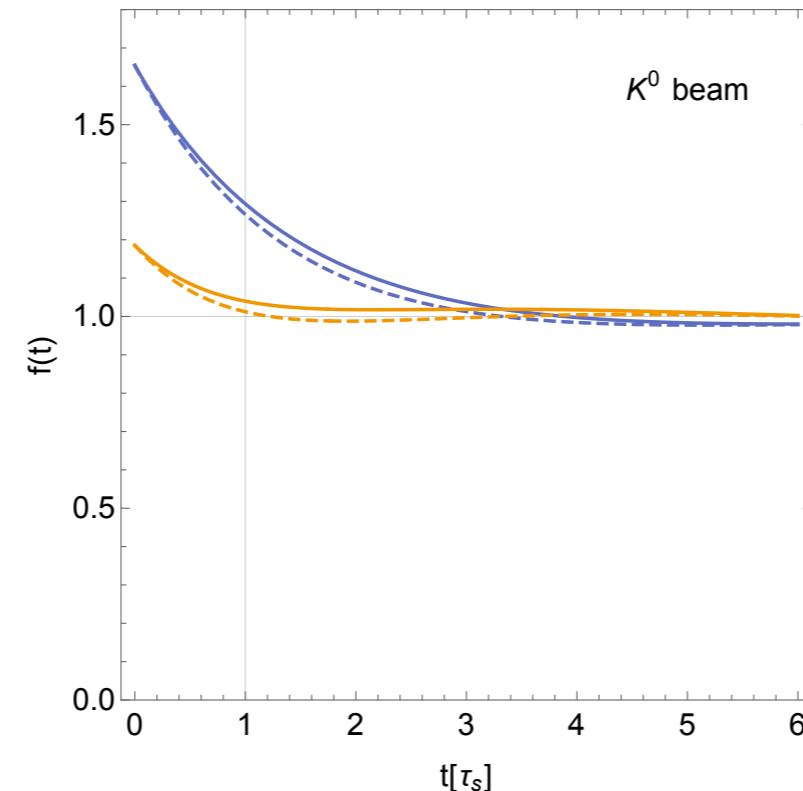
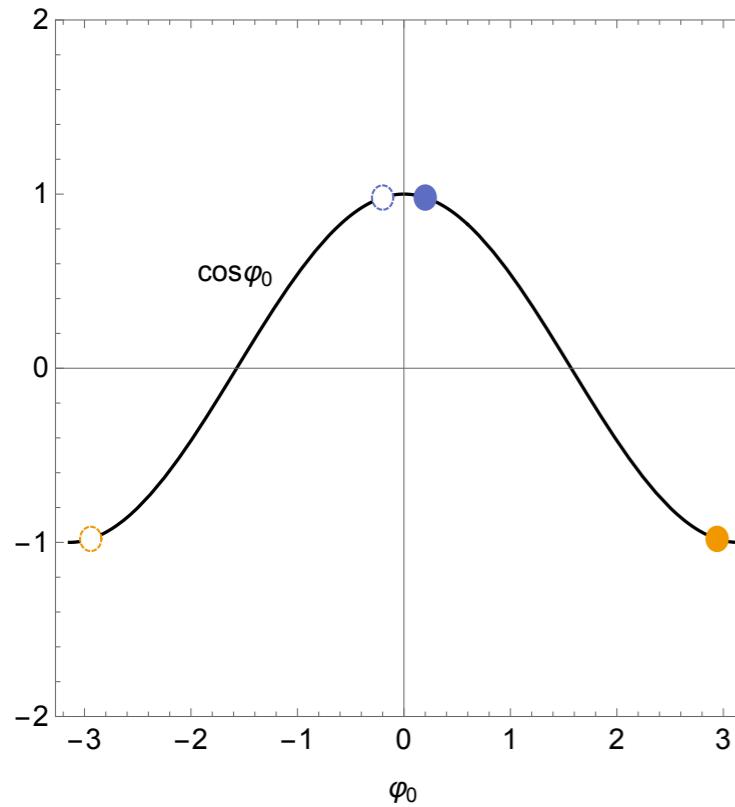


Model independent prediction -

$$\cos^2 \varphi_0 = 0.96 \pm 0.02_{\text{exp.}} \pm 0.02_{\text{th.}}$$

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ChPT, $\xrightarrow{\text{large } N_c}$ $\cos \varphi_0 < 0$