第十五届 QCD相变与相对论重离子物理研讨会

QCD phase transition from effective models

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Outline

- Effective models
- Magnetic effect
- Rotational effect
- Summary

Effective models

- Nambu--Jona-Lasinio model
- chiral perturbation theory
- linear sigma model
- quark-meson model
- MIT bag model
- holographic QCD

Dyson-Schwinger equation

functional renormalization group

P. Zhuang, Y. Liu, M. Huang, D. Hou, X. Huang, L. He, W. Fu, Y. Jiang, S. Mao, G. Shao, K. Sun, H. Zhang, D. Li, GC

Why effective models?

1. no sign problem: more applications
2. analytic calculations: simpler and easier
3. numerical calculations: time-saving
4. discussions and intuitions: more physical
5. inspirations for experiments and LQCD

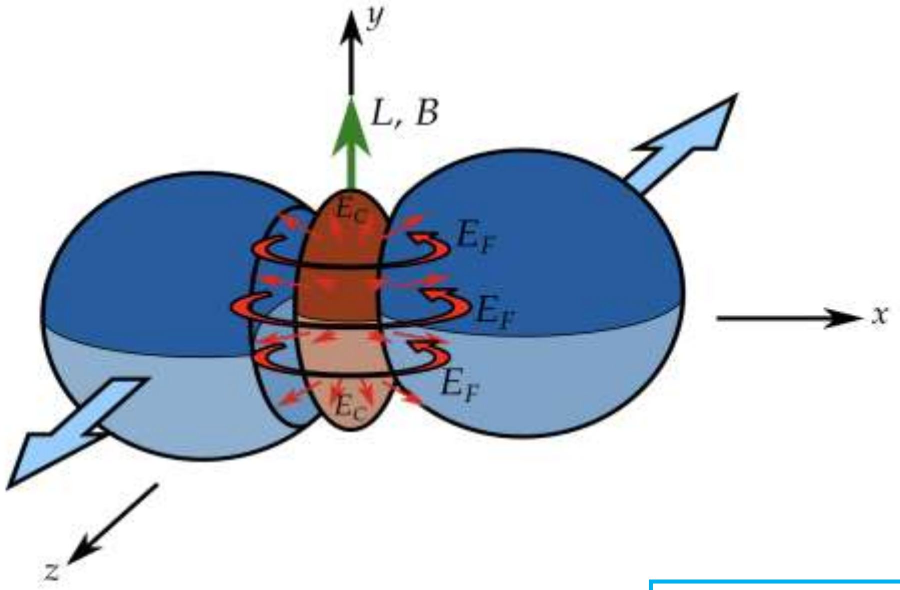
Peripheral heavy ion collisions

$eB, eE \sim (1 - 10^2)\, m_\pi^2$

HIJING W.-T. Deng et al. (PRC 2012)

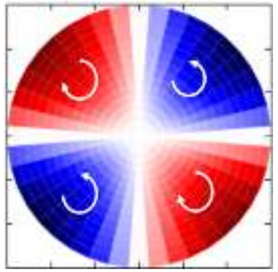
$\Omega \sim 10^{22} s^{-1} \sim 6\ MeV$

L. Adamczyk et al. (Nature 2017)

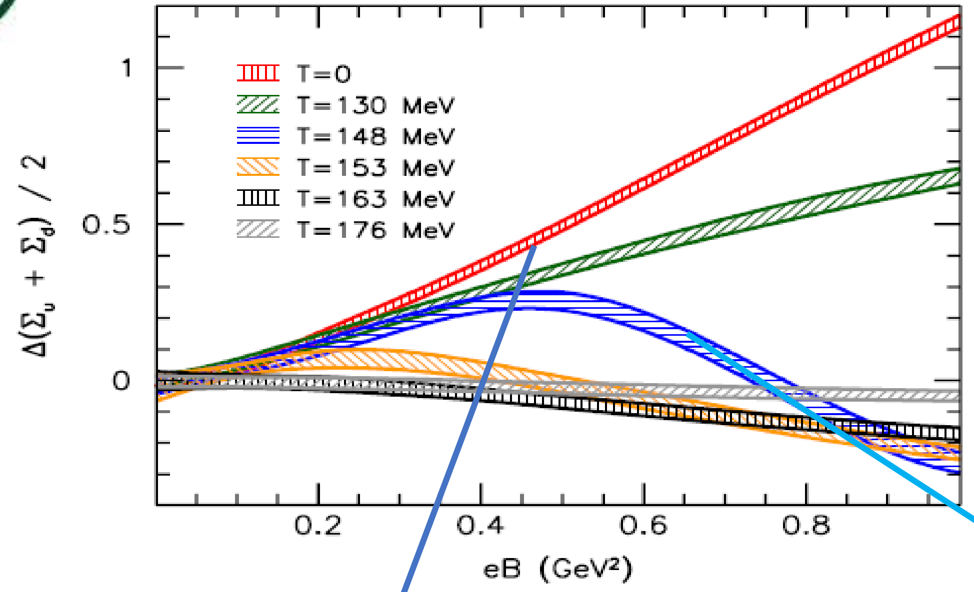


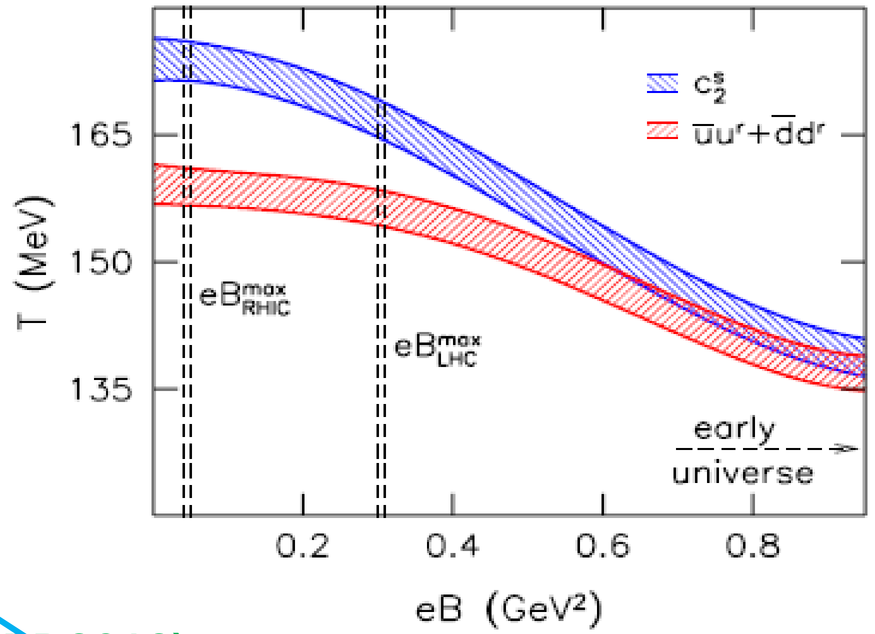
1. Chiral anomaly effects in HICs
   see Jie Zhao's talk
2. Spin polarization in HICs
   see Shuzhe Shi's talk
3. QCD phase transitions under eB or Ω

$\mathbf{J} = \sigma_5 \mathbf{B}$



Magnetic effect





G. Bali et al. (JHEP 2012, PRD2012).

magnetic catalysis effect

inverse magnetic catalysis effect

Well established within NJL model

V. P. Gusynin et al. (PRL 1994, PRD 1995, PLB1996, NPB1996).

- mesonic inhibition (K. Fukushima, S. Mao),
- chiral chemical potential (M. Huang)
- running coupling (N. Su, R. Farias et al.),

NJL model

$$\mathcal{L} = \bar{\psi}(i\not{D} - m_0)\psi + G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

Hubbard-Stratonovich transformation

$$\mathcal{L} = \bar{\psi}[i\not{D} \underbrace{- m_0 - \sigma}_{-m} - i\gamma_5(\tau_3\pi^0 + \tau_\pm\pi^\pm)]\psi - \frac{\sigma^2 + (\pi^0)^2 + \pi^\mp\pi^\pm}{4G_S}$$

$\partial\Omega/\partial m = 0$ ➡ Gap equation:

$$\frac{m - m_0}{2G_S} - \frac{i}{V_4}\sum_{\mathrm{f=u,d}} \mathrm{Tr}G_\mathrm{f} = 0$$

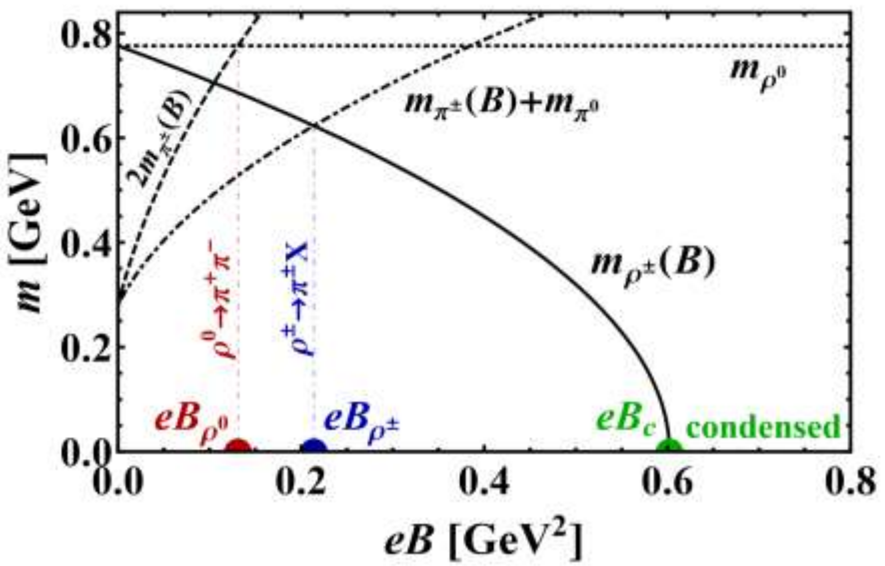
Full propagator: $G_\mathrm{f}(x, y) = e^{-iq_\mathrm{f}\int_y^x A_\mathrm{f}^\mu dx_\mu} S_\mathrm{f}(x - y),$

$$S_\mathrm{f}(x) = -i\int_0^\infty \frac{ds}{16(\pi s)^2} e^{-i[sm^2 + \frac{1}{4s}(x_0^2 - x_3^2 - (x_1^2 + x_2^2)B_\mathrm{f}^s \cot B_\mathrm{f}^s)]} B_\mathrm{f}^s[\cot B_\mathrm{f}^s + \gamma_1\gamma_2]$$

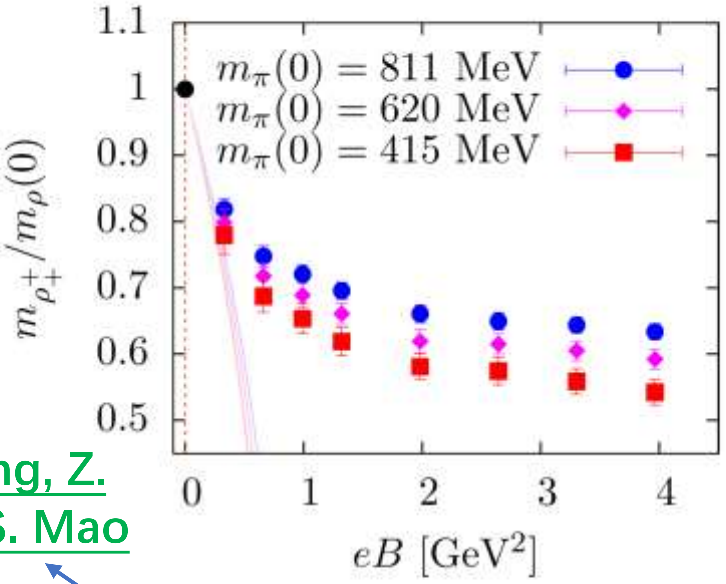
$$\times \left[m + \frac{1}{2s}(\not{x}_0 - \not{x}_3 - B_\mathrm{f}^s((\not{x}_1 + \not{x}_2)\cot B_\mathrm{f}^s - \not{x}_{21} + \not{x}_{12}))\right]$$

J. S. Schwinger, Phys. Rev. 82, 664 (1951)

Meson masses in magnetic field



M. N. Chernodub (PRL 2011).



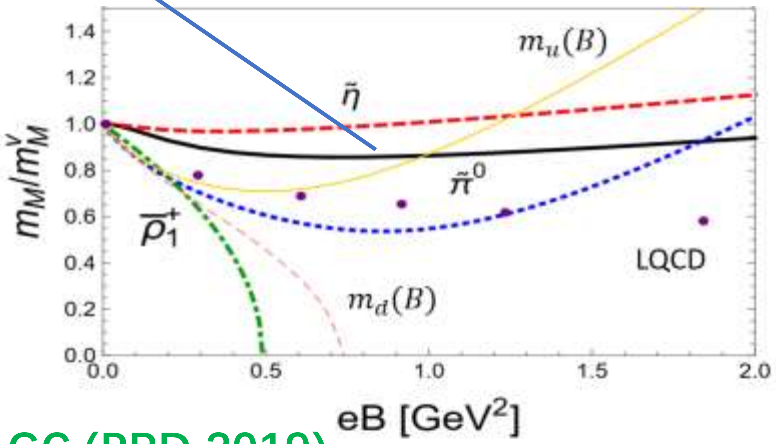
G. Bali et al., (PRD2018).

(Y. Hidaka et al., (PRD 2013))

M. Huang, Z. Wang, S. Mao

see S. Mao's talk

$$E_v^2(n, p_3) = (2n-1)eB + p_3^2 + m_v^2$$



mass splitting kills vacuum superconductor

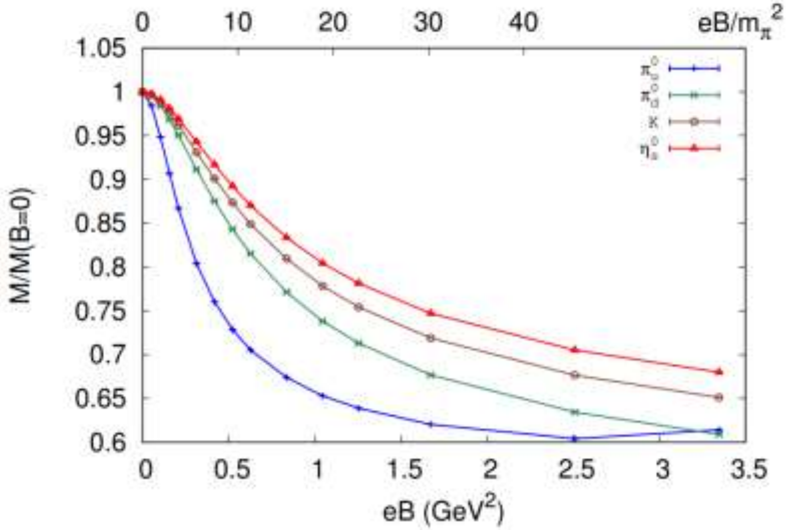
2f-NJL model: vacuum superconductor

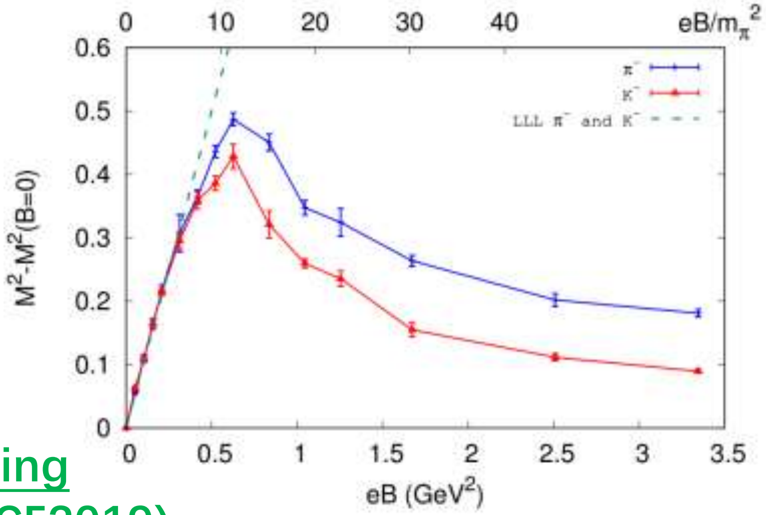
negative mass square ➡ new phase

GC (PRD 2019).

3f-NJL model

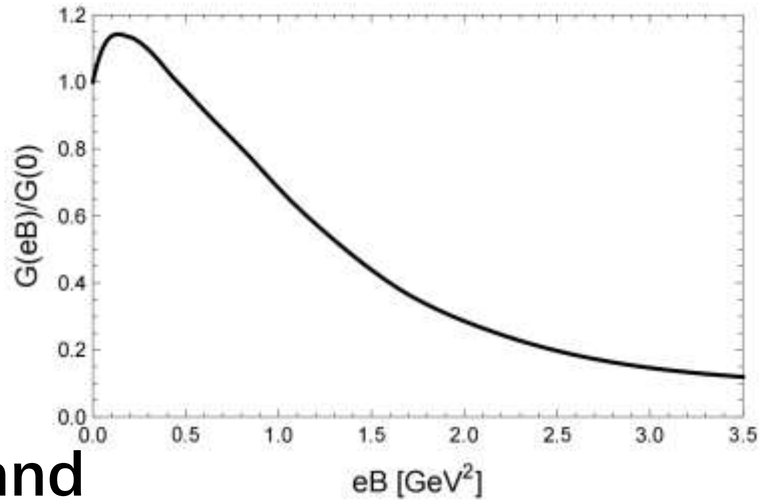
Meson masses in magnetic field





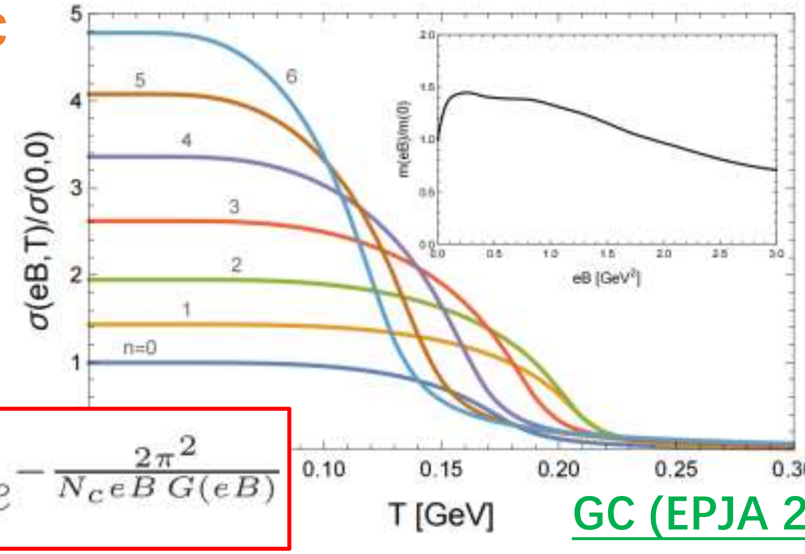
H. T. Ding (LATTICE2019)

ansatz →



running coupling

explain π^0 mass and inverse magnetic catalysis effect consistently

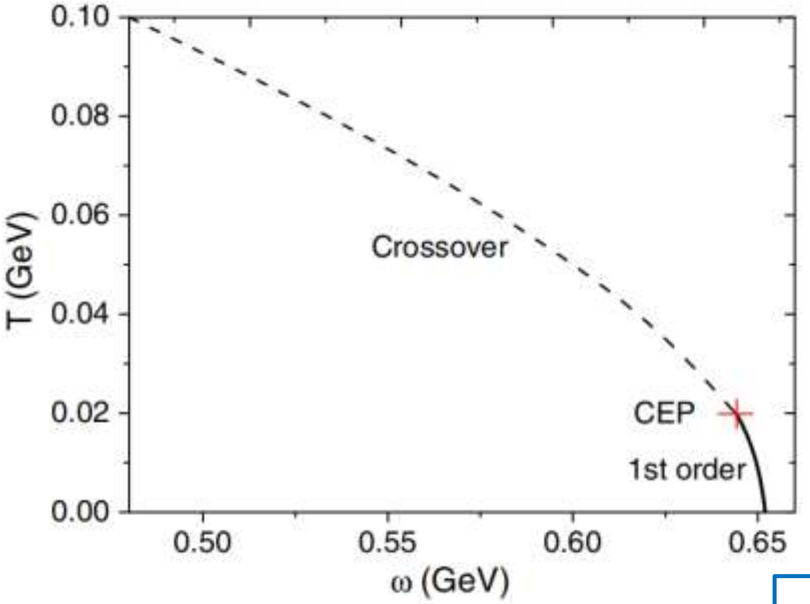


large B limit

$$T_c \approx \tilde{q}_u^{\tilde{q}_u}\tilde{q}_d^{\tilde{q}_d}(2eB)^{\frac{1}{2}}e^{-\frac{2\pi^2}{N_c eB\, G(eB)}}$$

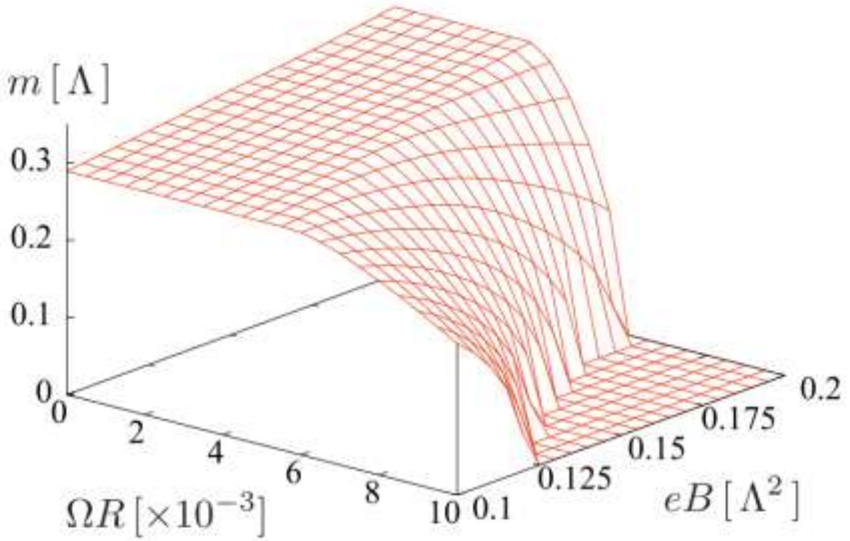
GC (EPJA 2021).

Rotational effect



Y. Jiang & J. Liao (PRL2016)

NJL model



H. Chen et al. (PRD2016)

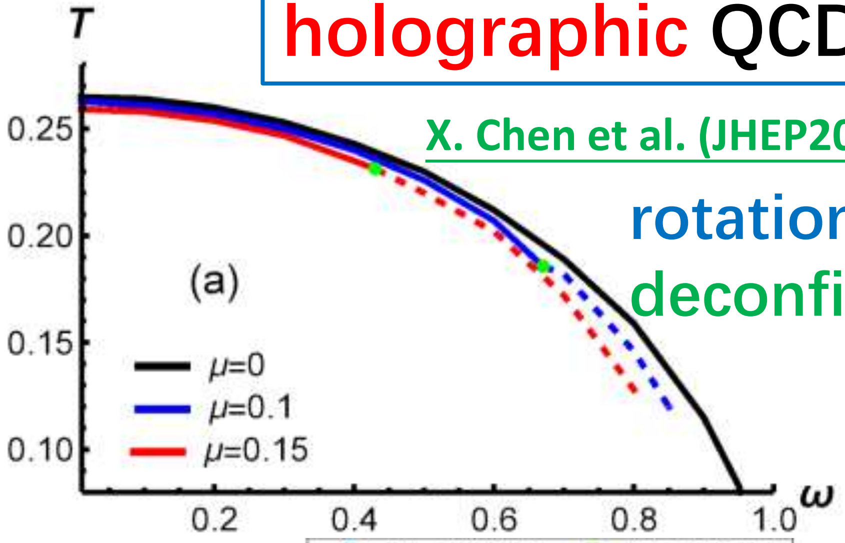
establish the basic formalism under constant rotation

$$\mathcal{L} = \bar{\psi}\{\gamma^0[i\partial_t + \Omega(\hat{L}_z + \hat{S}_z)] + i\gamma^1 D_x + i\gamma^2 D_y + i\gamma^3\partial_z - m_0\}\psi + \mathcal{L}_{\rm int}$$

main conclusion: rotation catalysis chiral symmetry restoration

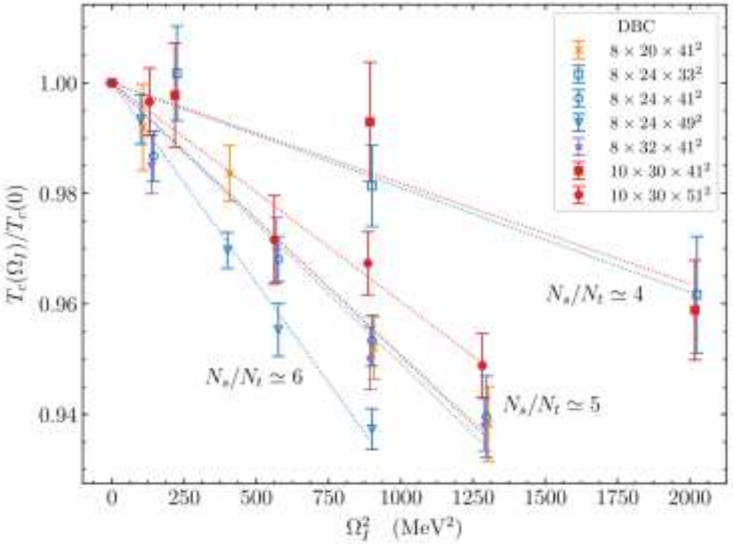
Deconfinement

holographic QCD



X. Chen et al. (JHEP2016)

rotation catalysis deconfinement

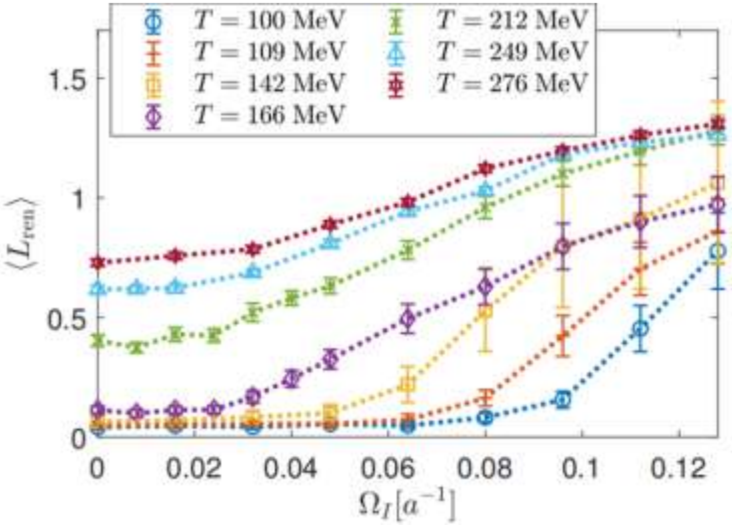


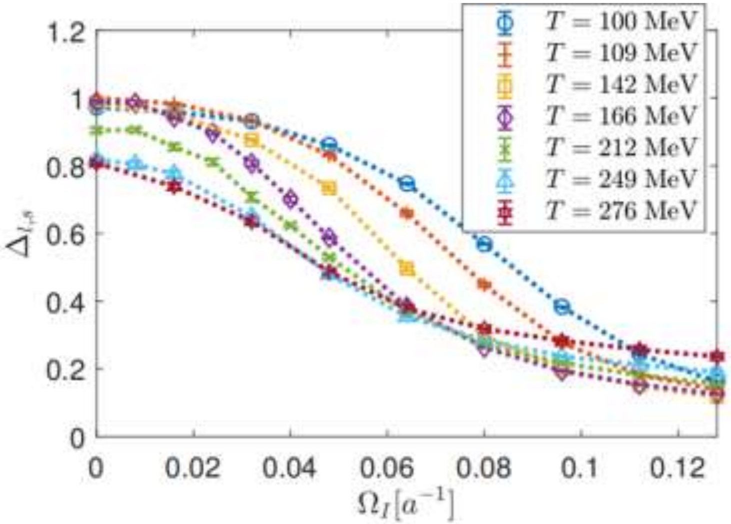
pure gauge theory

V. V. Braguta et.al. (PRD2021)

Two-flavor LQCD

J. Yang & X.G. Huang (2023)





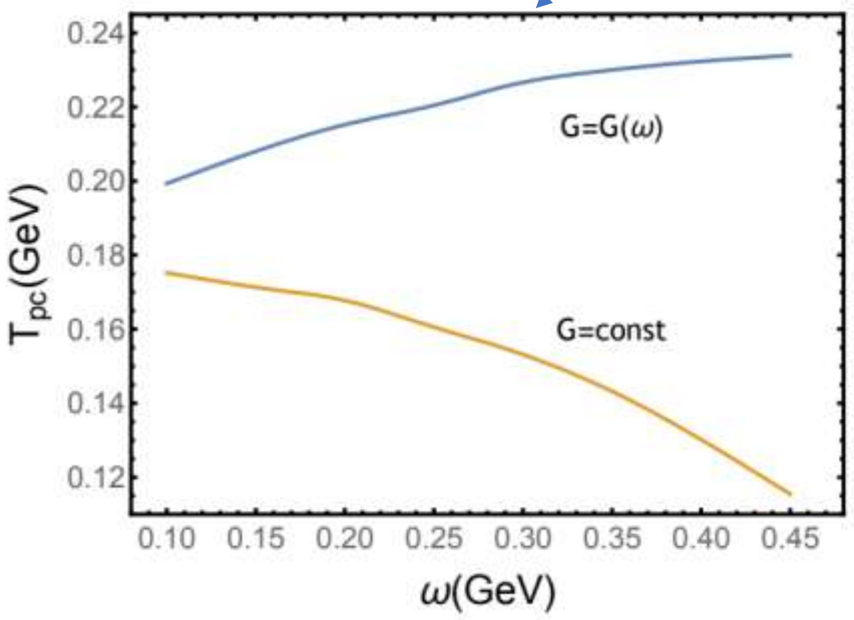
$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2$$

analytic continuation

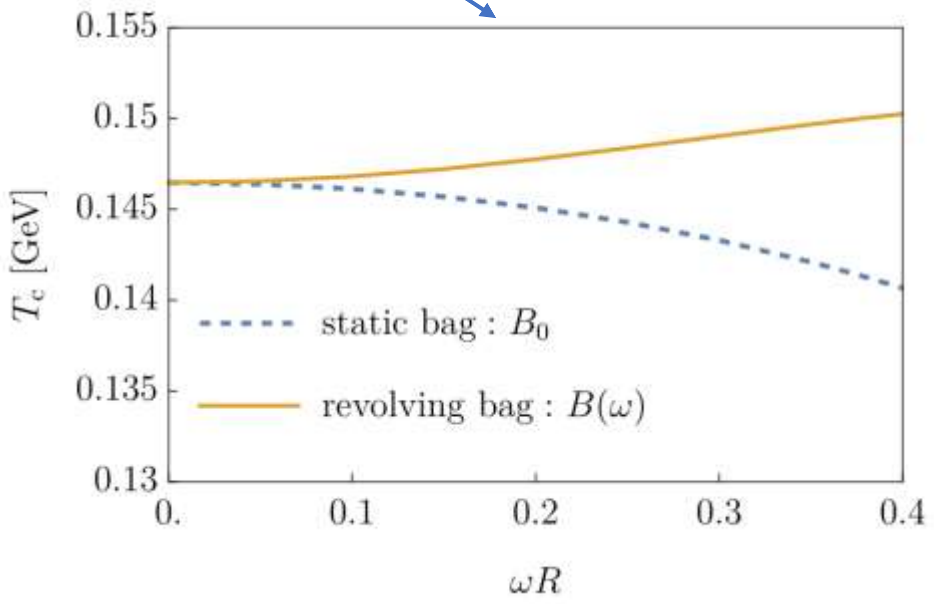
$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

Effective models

- Holographic QCD: M. Huang et.al. (PRD2023).
- QM model: H.L. Chen et.al. (PRD2023). → see H.L. Chen's talk
- NJL model: Y. Jiang (EPJC2022).
- Bag model: K. Mameda & K. Takizawa (2023).



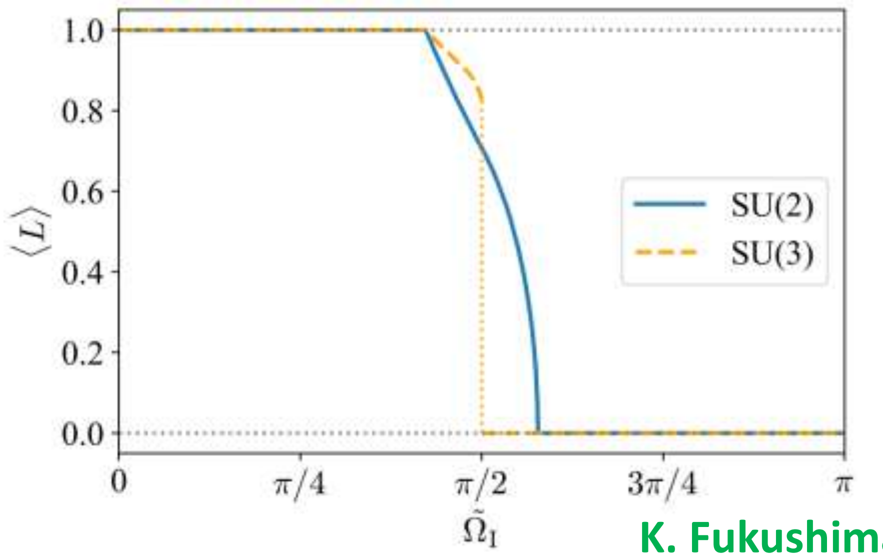
chiral vortical catalysis

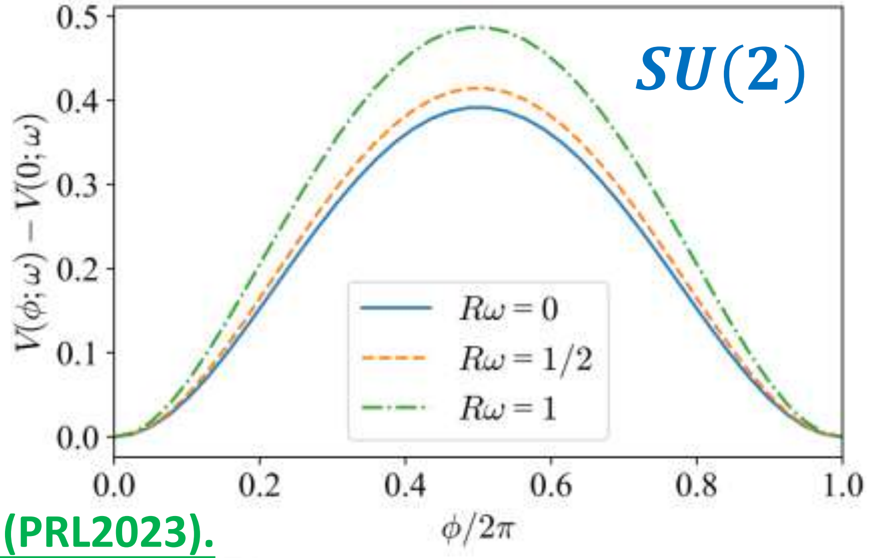


revolving bag constant

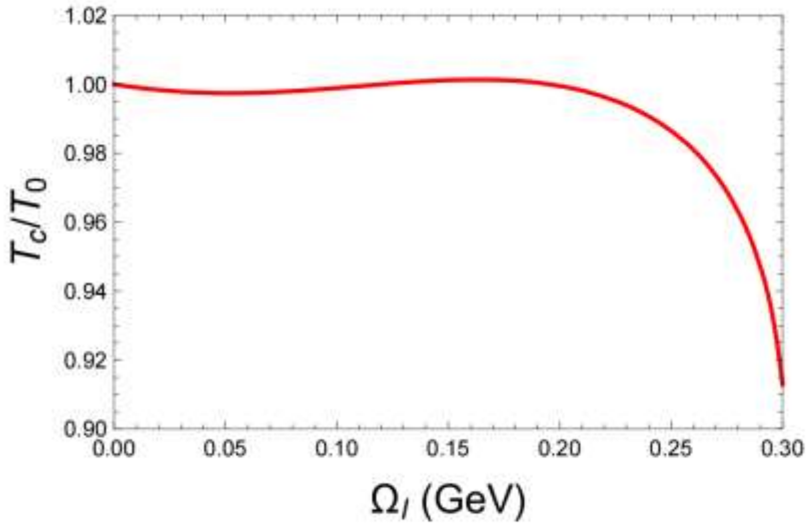
perturbative study

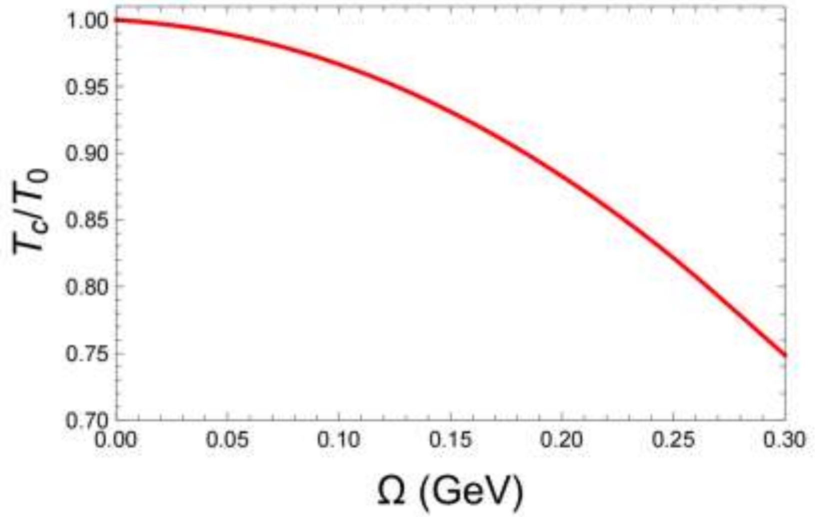
opposite to LQCD





K. Fukushima et.al. (PRL2023).



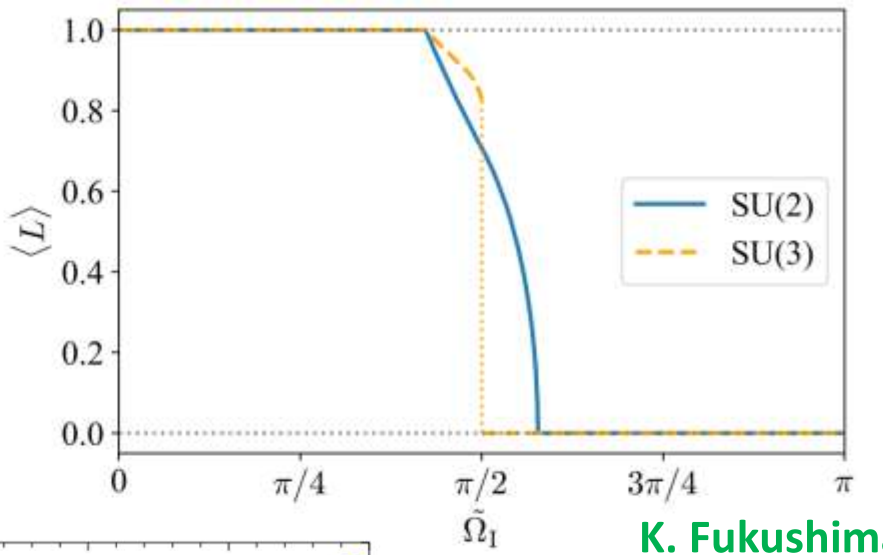


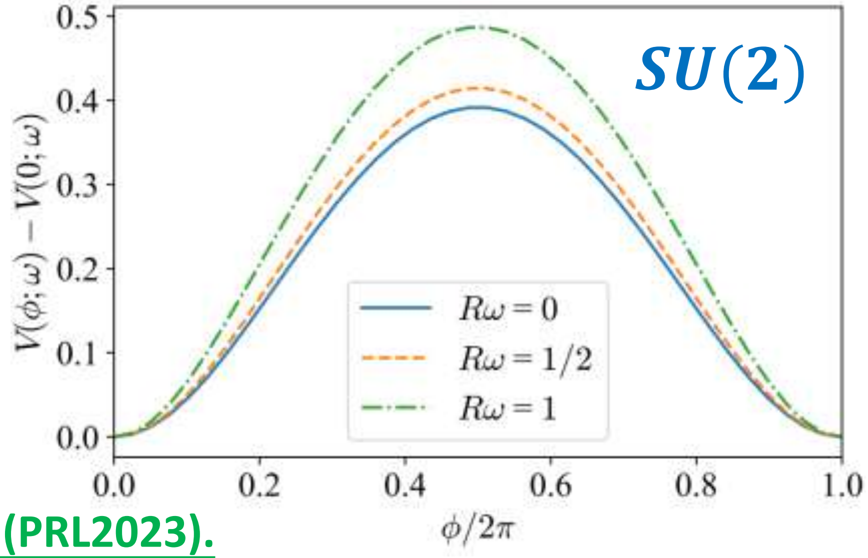
Modified PNJL model

$$\bar{V}_2(q_{ij}, \Omega_I) = \frac{1}{2} \sum_{s=\pm} V_2(q_{ij} + s\,\Omega_I)$$

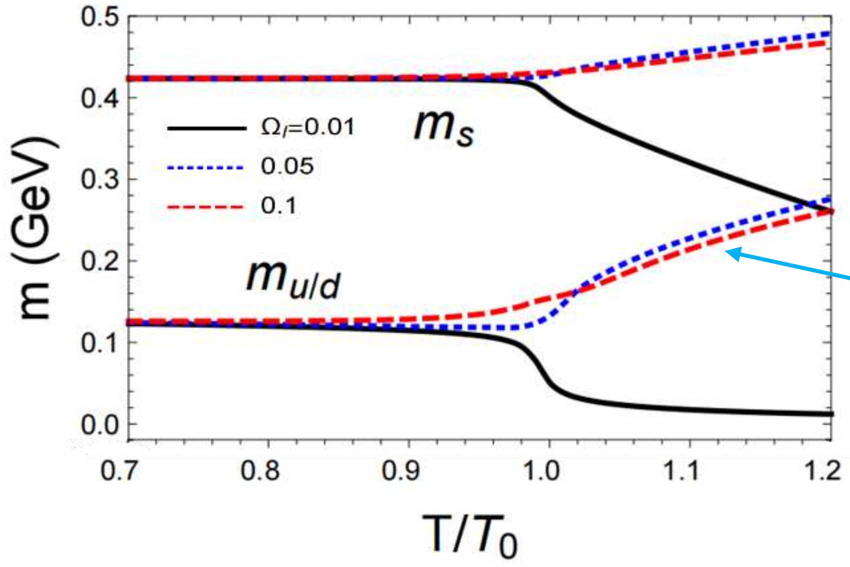
analytic continuation invalid GC (PRD2023) perturbative study

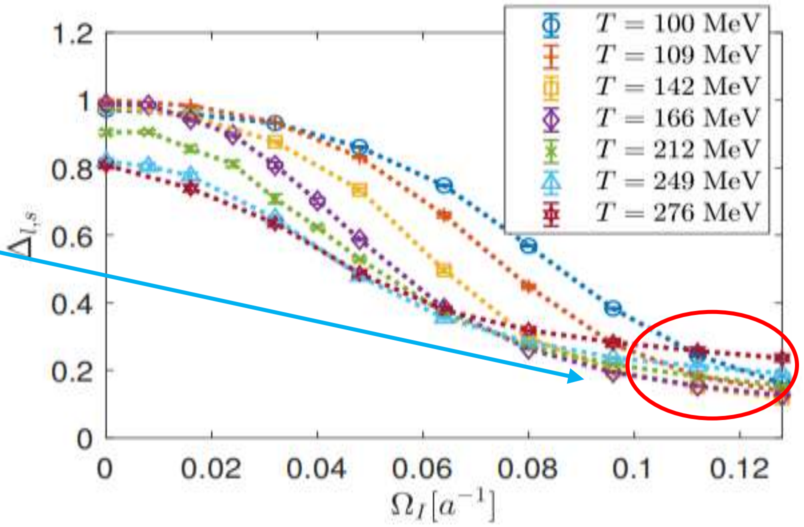
opposite to LQCD





K. Fukushima et.al. (PRL2023).



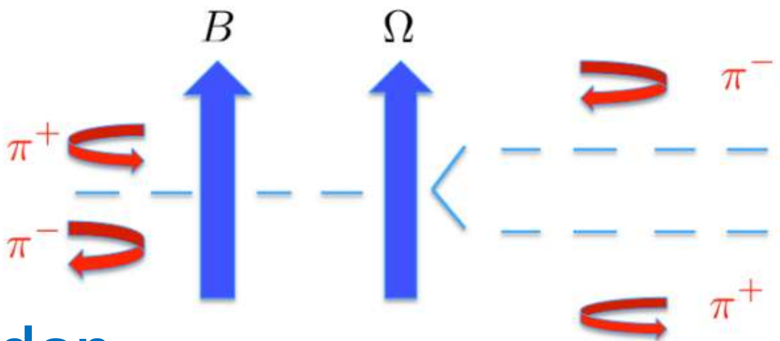


Modified PNJL model

$$\bar{V}_2(q_{ij}, \Omega_I) = \frac{1}{2} \sum_{s=\pm} V_2(q_{ij} + s\,\Omega_I)$$

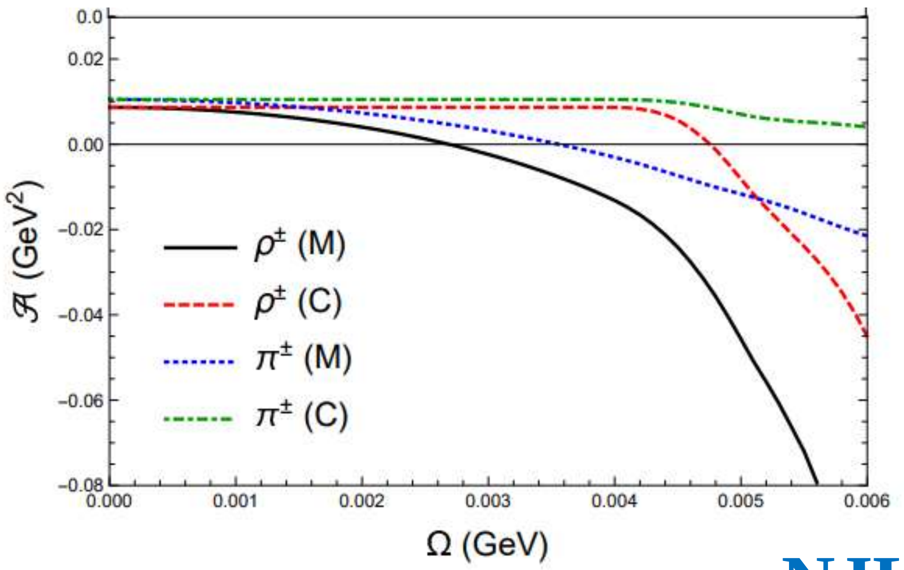
analytic continuation invalid GC (PRD2023)

Combined magnetic and rotation effect



Klein-Gordon theory

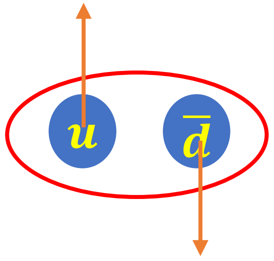
Y. Liu & I. Zahed (PRL2017)



GC & L. He (PRD2019, 2020) **NJL model**

charge conservation: π^- and π^+ condensation in different region

not only pion superfluid, but also rho superconductor



unstable with both Ω and B

reason: increasing isospin density

more possibility → H. Zhang (CPC 2020)

Summary

- Inverse magnetic catalysis effect and meson masses are widely studied

  A good explanation should reproduce many aspects consistently

- The puzzle with rotational effect?

  Need direct first-principle calculations

- Complicated QCD phase diagram in $\vec{B} \parallel \vec{\Omega}$

Thanks