



Heavy Flavor Physics

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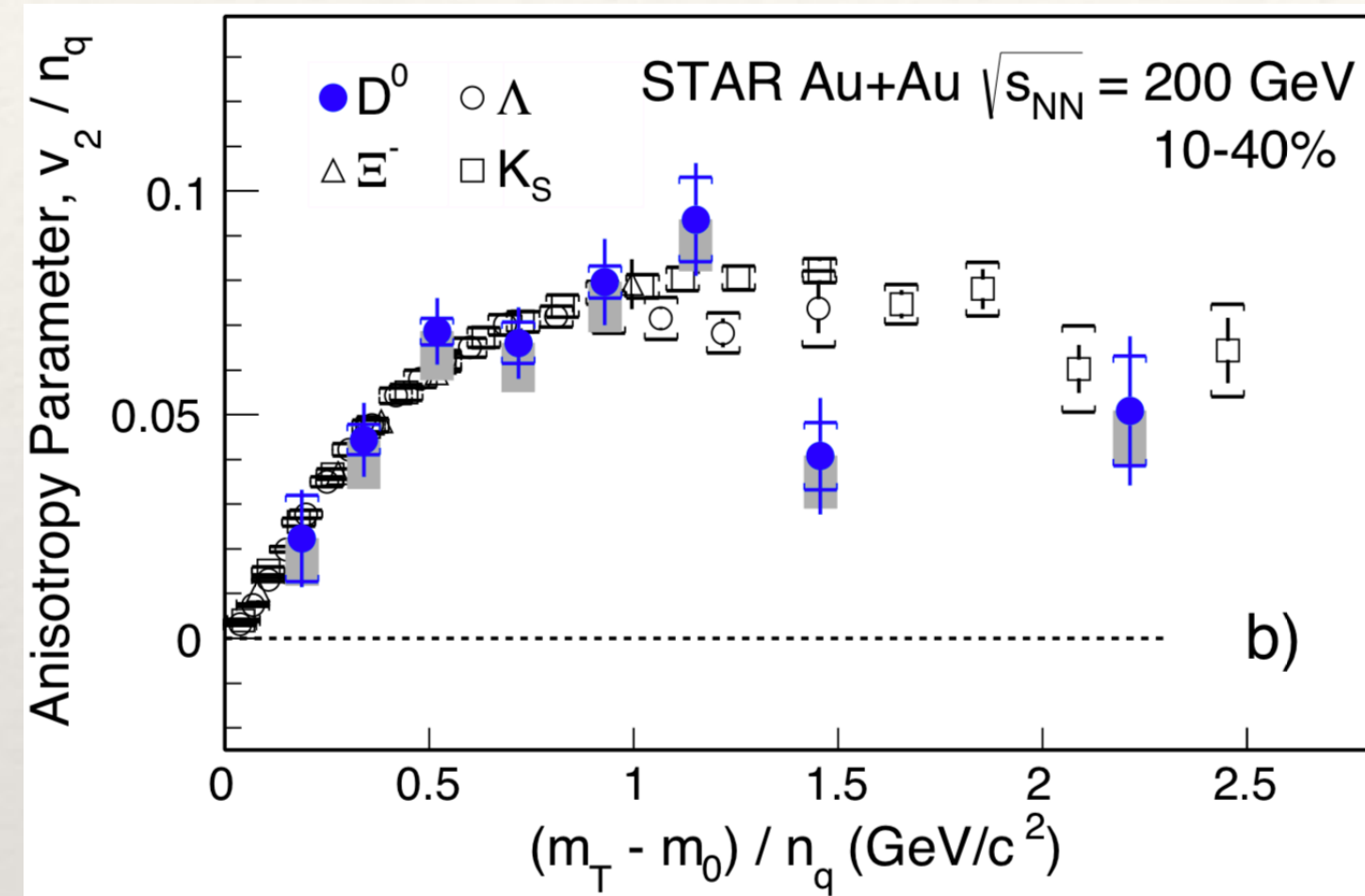
Shandong University

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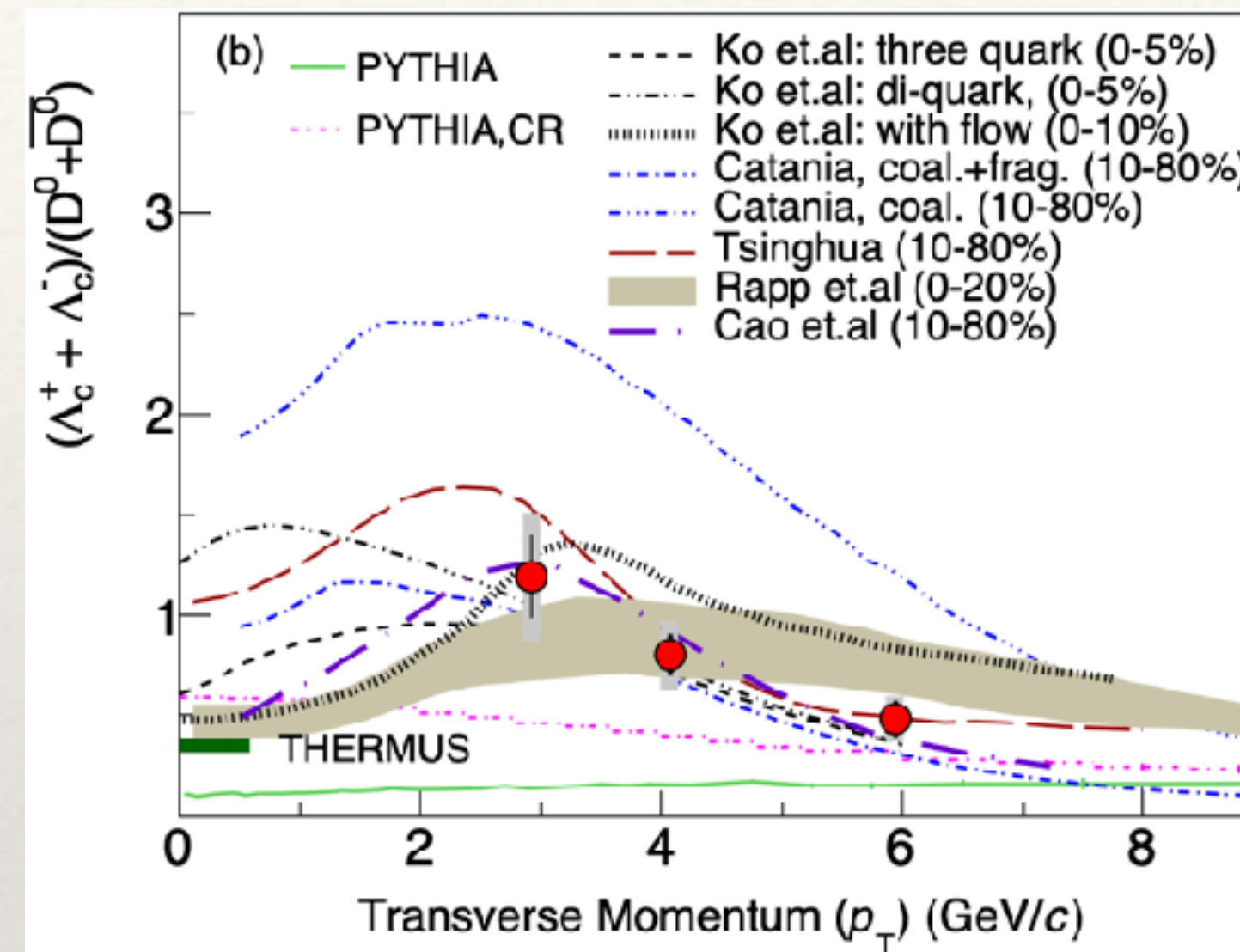
Heavy quark physics at different scales

low p_T



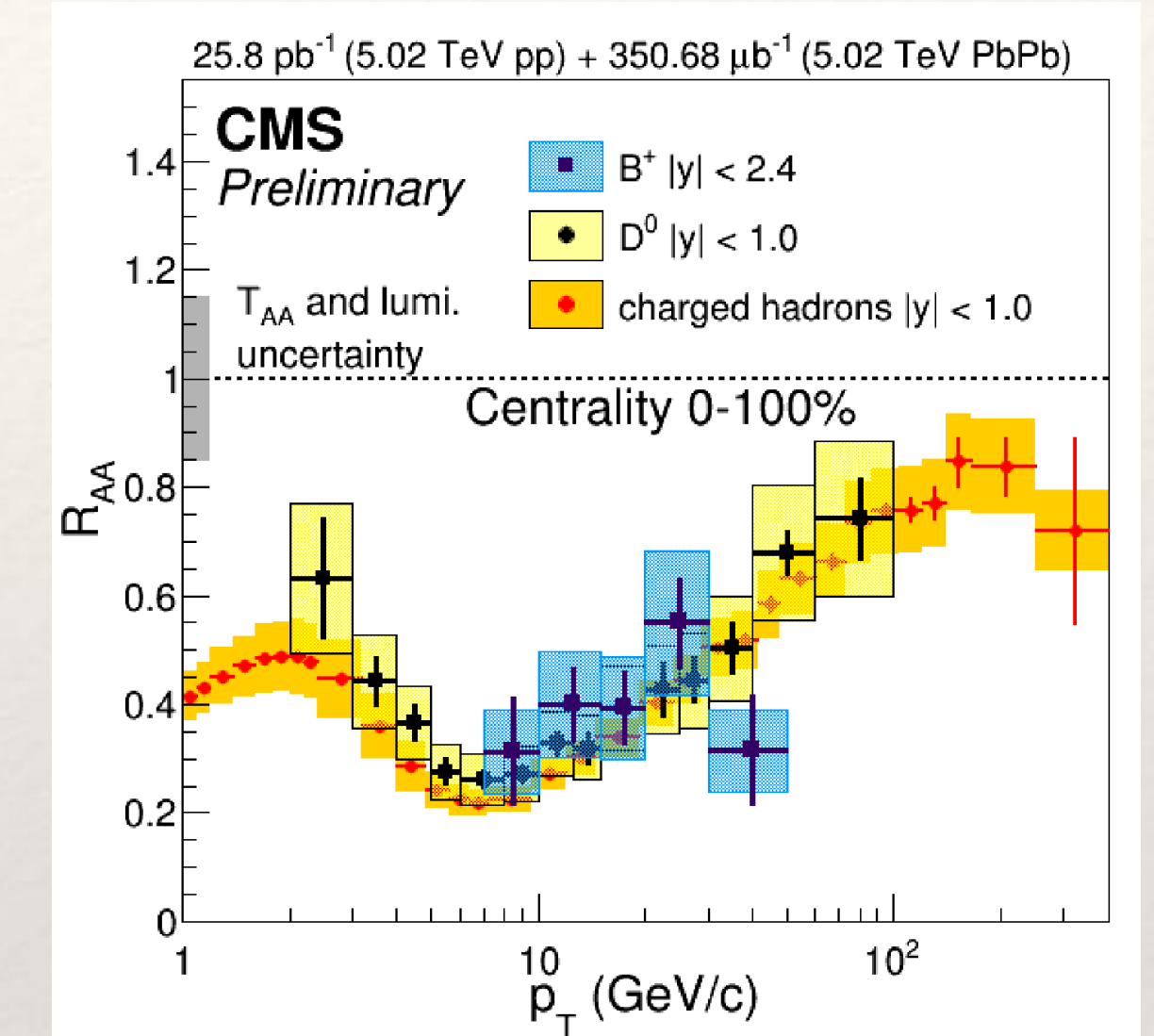
- Study the thermalization process of heavy quarks
- Constrain the color potential of quark interaction

medium p_T



- Study the hadronization process of heavy quarks
- Constrain the in-medium hadron wave-function

high p_T



- Study the energy loss process of heavy quarks
- Constrain the flavor hierarchy of parton energy loss

High p_T parton-medium interaction

Linear Boltzmann Transport (LBT)

$$p_a \cdot \partial f_a(x_a, p_a) = E_a (\mathcal{C}_a^{\text{el}} + \mathcal{C}_a^{\text{inel}})$$

High p_T parton-medium interaction

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Elastic energy loss ($ab \rightarrow cd$)

$$\mathcal{C}_a^{\text{el}} = \sum_{b,c,d} \int \prod_{i=b,c,d} \frac{d[p_i]}{2E_a} (\gamma_d f_c f_d - \gamma_b f_a f_b) \cdot (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) \left| \mathcal{M}_{ab \rightarrow cd} \right|^2$$

2 \rightarrow 2 scattering matrices

High p_T parton-medium interaction

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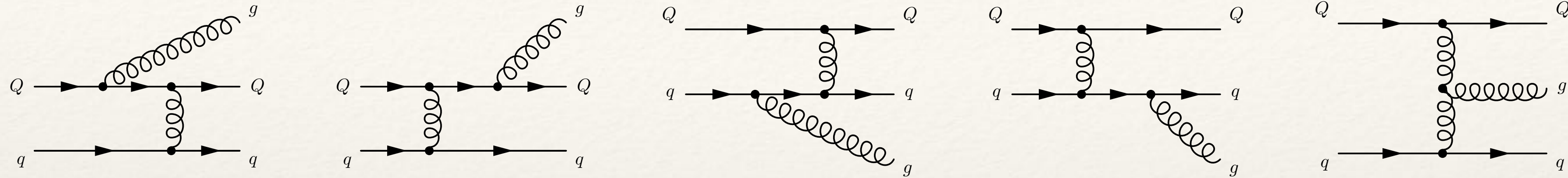
2 → 2 scattering matrices

loss term: **scattering rate**
(for Monte-Carlo simulation)

$$\Gamma_a^{\text{el}}(\mathbf{p}_a, T) = \sum_{b,c,d} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} d[p_i] f_b \cdot (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) \left| \mathcal{M}_{ab \rightarrow cd} \right|^2$$

Inelastic energy loss

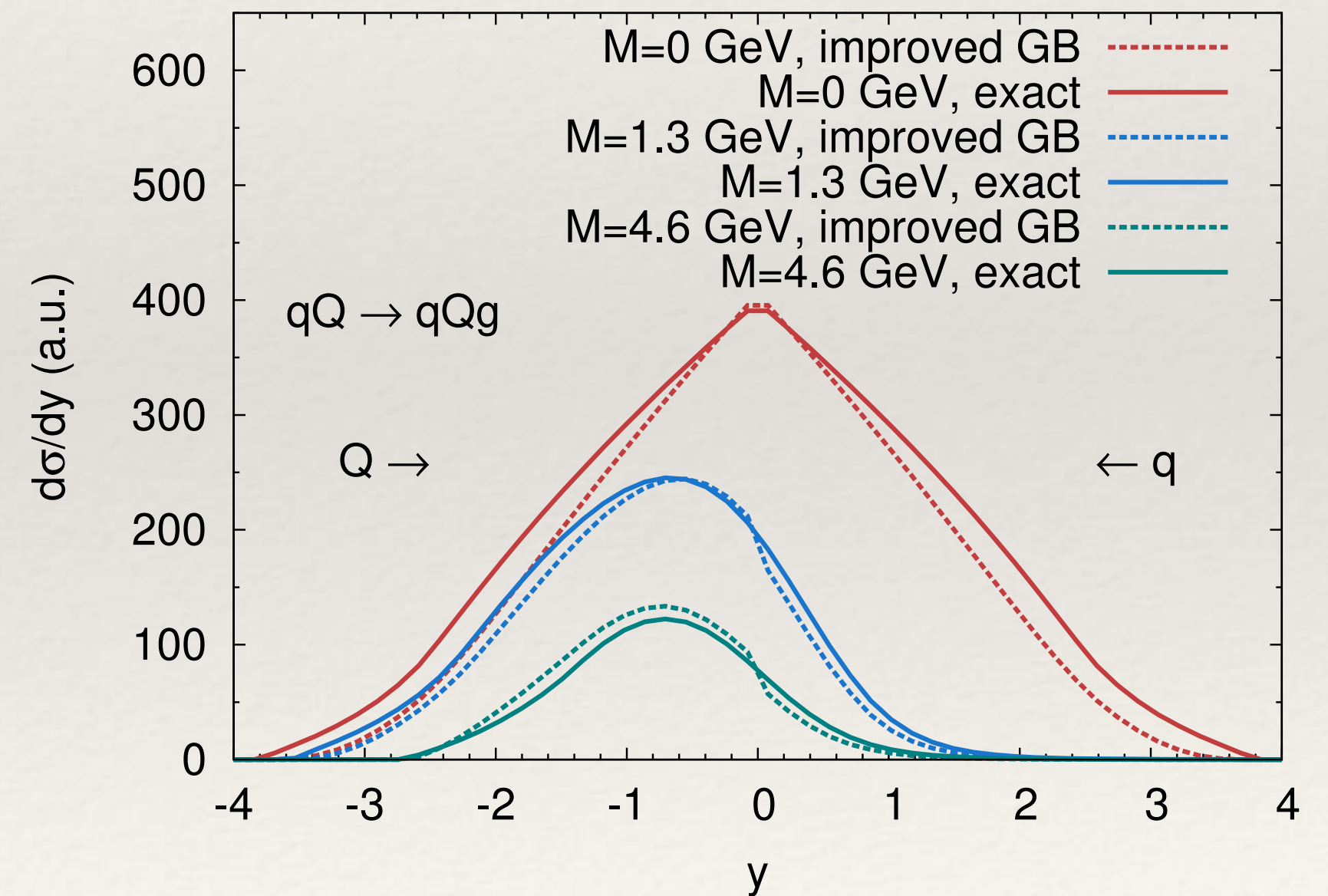
- $2 \rightarrow 3$ scattering with a quasi-particle



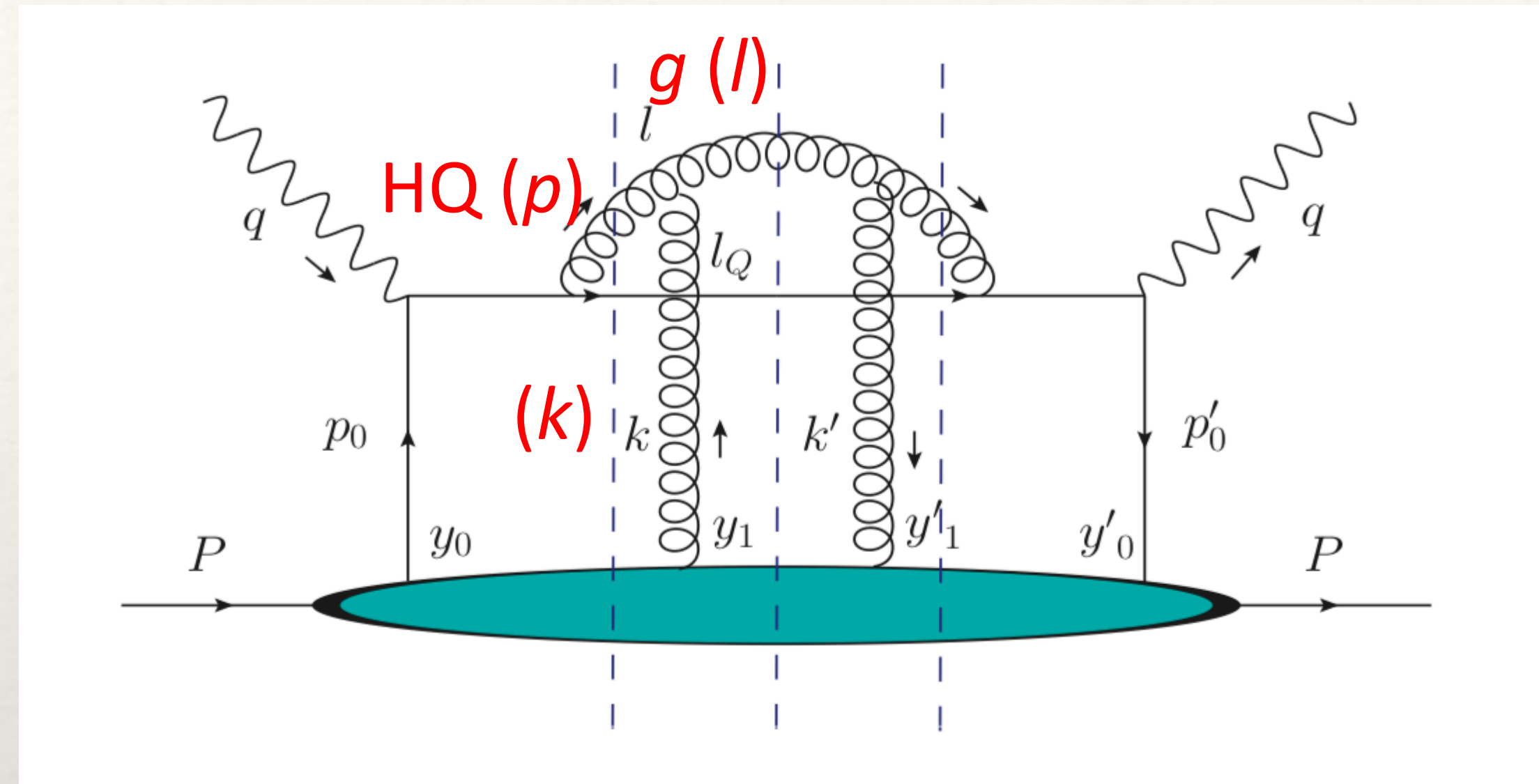
- Calculate LO diagrams [Kunszt et al., PRD21 (1980)]
- Gunion-Bertsch Approximation derived at high energy limit [Gossiaux et al., JPG 37 (2010); Fochler et al., PRD 88 (2013)]

$$\left| \overline{\mathcal{M}}_{qQ \rightarrow qQg} \right|^2 = 12g^2(1 - \bar{x})^2 \left| \overline{\mathcal{M}}_0^{qQ} \right|^2 \left[\frac{\vec{k}_\perp}{k_\perp^2 + x^2 M^2} + \frac{\vec{q}_\perp - \vec{k}_\perp}{(\vec{q}_\perp - \vec{k}_\perp)^2 + x^2 M^2} \right]^2$$

- Application: Frankfurt (BAMPS) [Uphoff et al., JPG 42 (2015)]
Nantes (EPOSHQ) [Gossiaux et al., JPG 37 (2010)]
Duke (Lido) [Ke et al, PRC 98 (2018)]



Inelastic energy loss



[Majumder PRD 85 (2012);
Zhang, Wang and Wang, PRL
93 (2004)]

- **Higher-twist formalism:** collinear expansion ($\langle k_{\perp}^2 \rangle \ll l_{\perp}^2 \ll Q^2$)

$$\frac{d\Gamma_a^{\text{inel}}}{dz dl_{\perp}^2} = \frac{dN_g}{dz dl_{\perp}^2 dt} = \frac{6\alpha_s P(z) l_{\perp}^4 \hat{q}}{\pi(l_{\perp}^2 + z^2 M^2)^4} \sin^2 \left(\frac{t - t_i}{2\tau_f} \right)$$

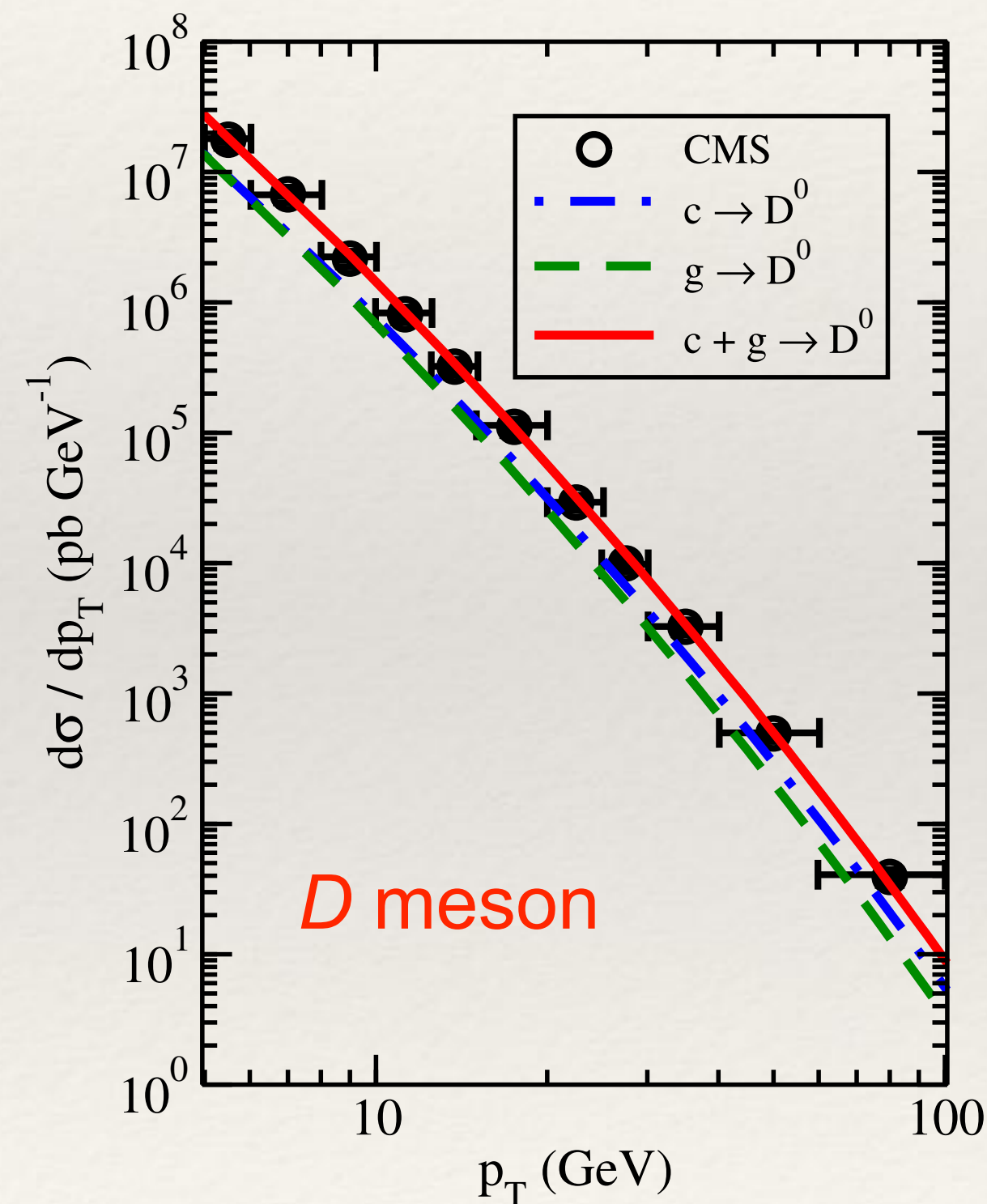
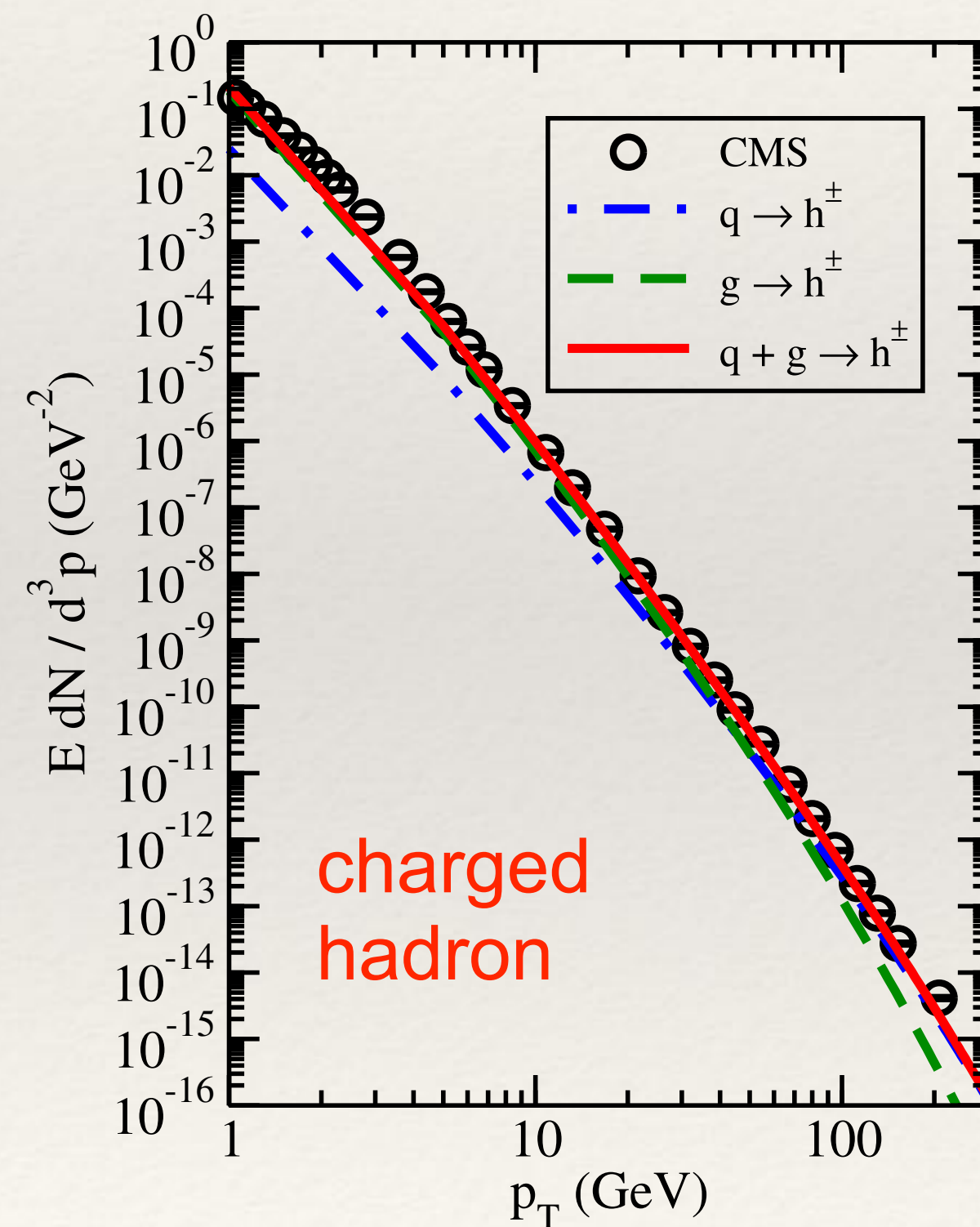
- Medium information absorbed in $\hat{q} \equiv d\langle p_{\perp}^2 \rangle / dt$

Flavor hierarchy of jet quenching

Clean perturbative calculation provides a good description of the flavor hierarchy at high p_T

[Xing, SC, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

pp baseline within the NLO production + fragmentation framework (NLO: including $g \rightarrow D$)



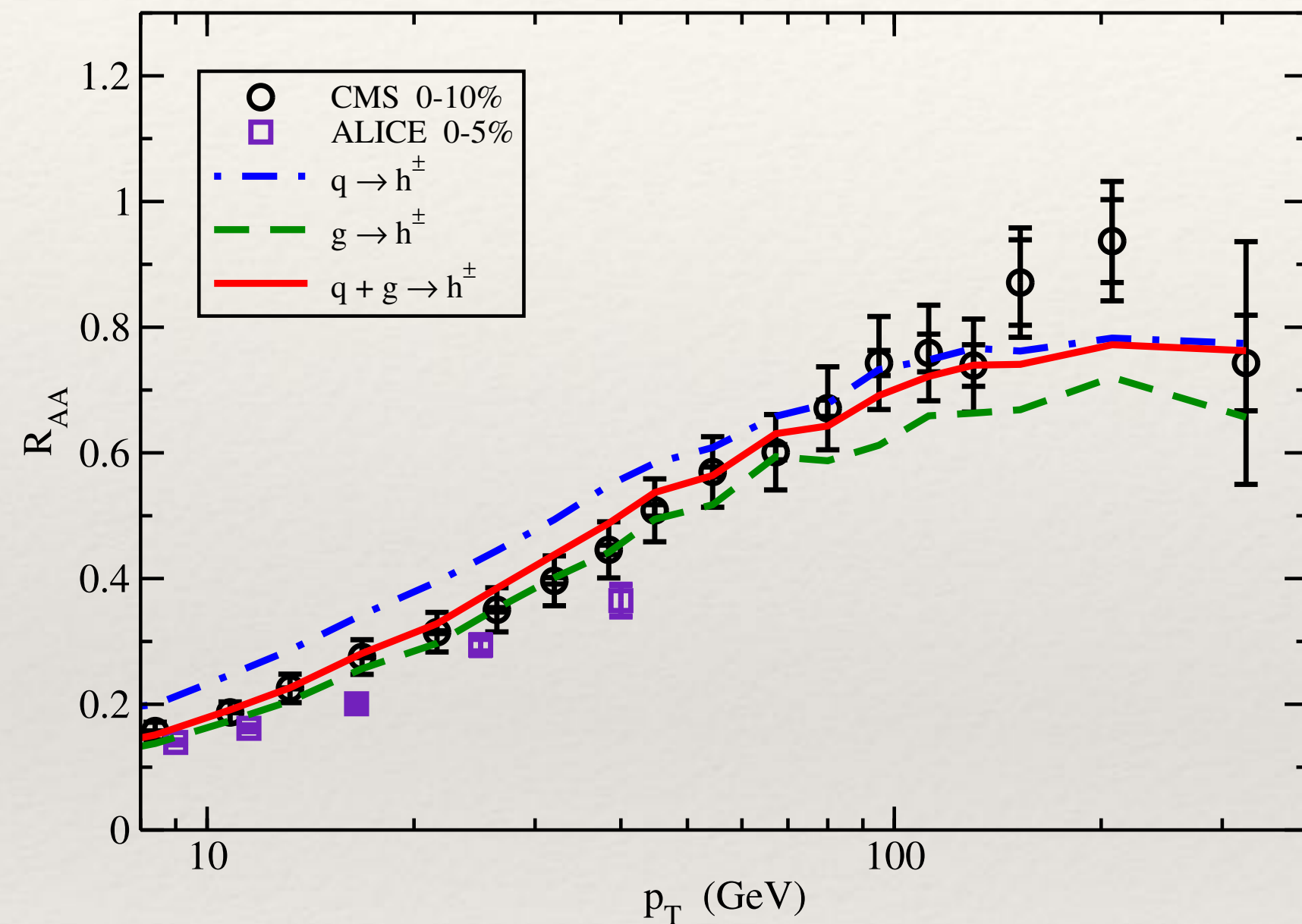
Gluon fragmentation

- dominates h^\pm production up to 50 GeV
- contributes to over 40% D up to 100 GeV

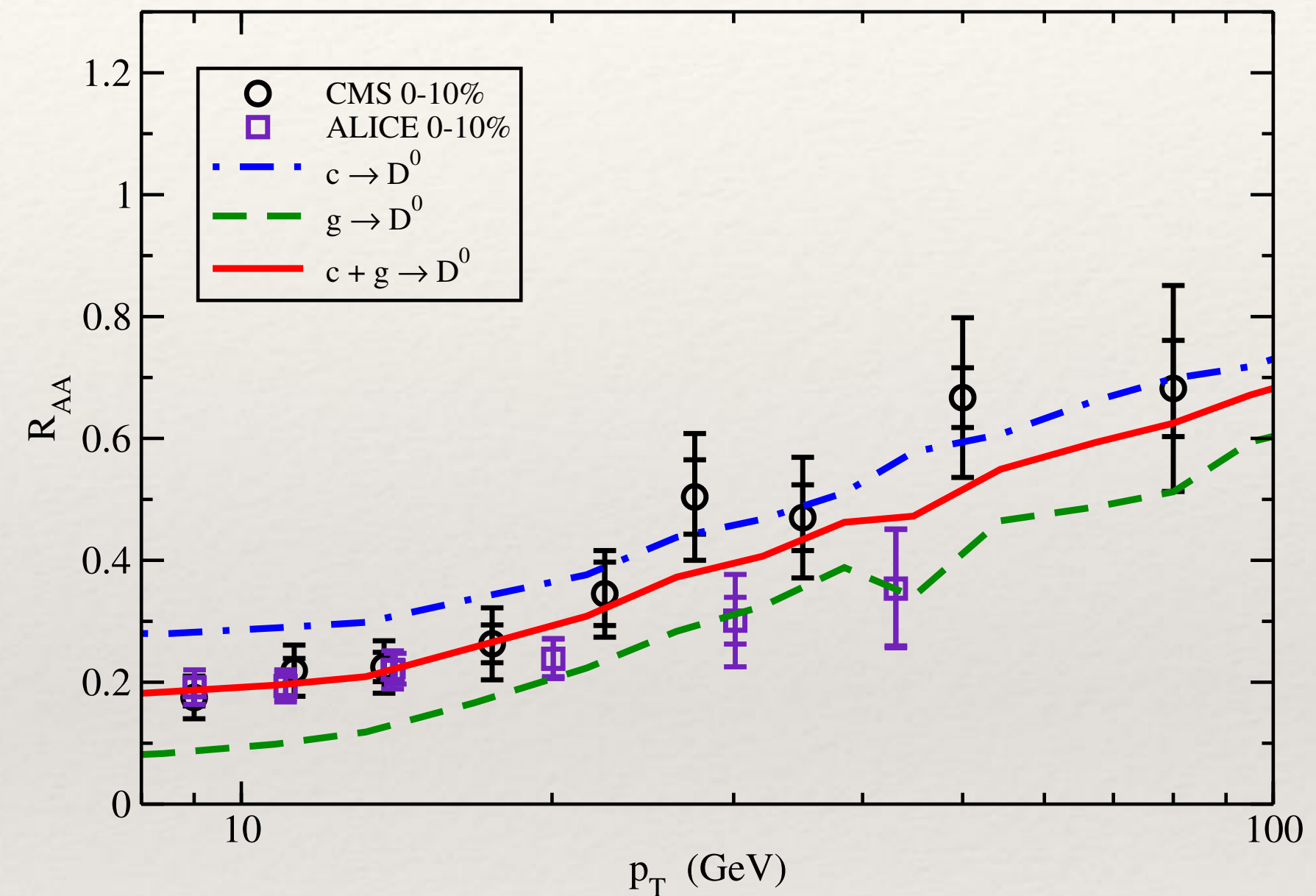
Flavor hierarchy of jet quenching

NLO initial production and fragmentation + Boltzmann transport (elastic and inelastic energy loss)
 + hydrodynamic medium for QGP

charged hadron



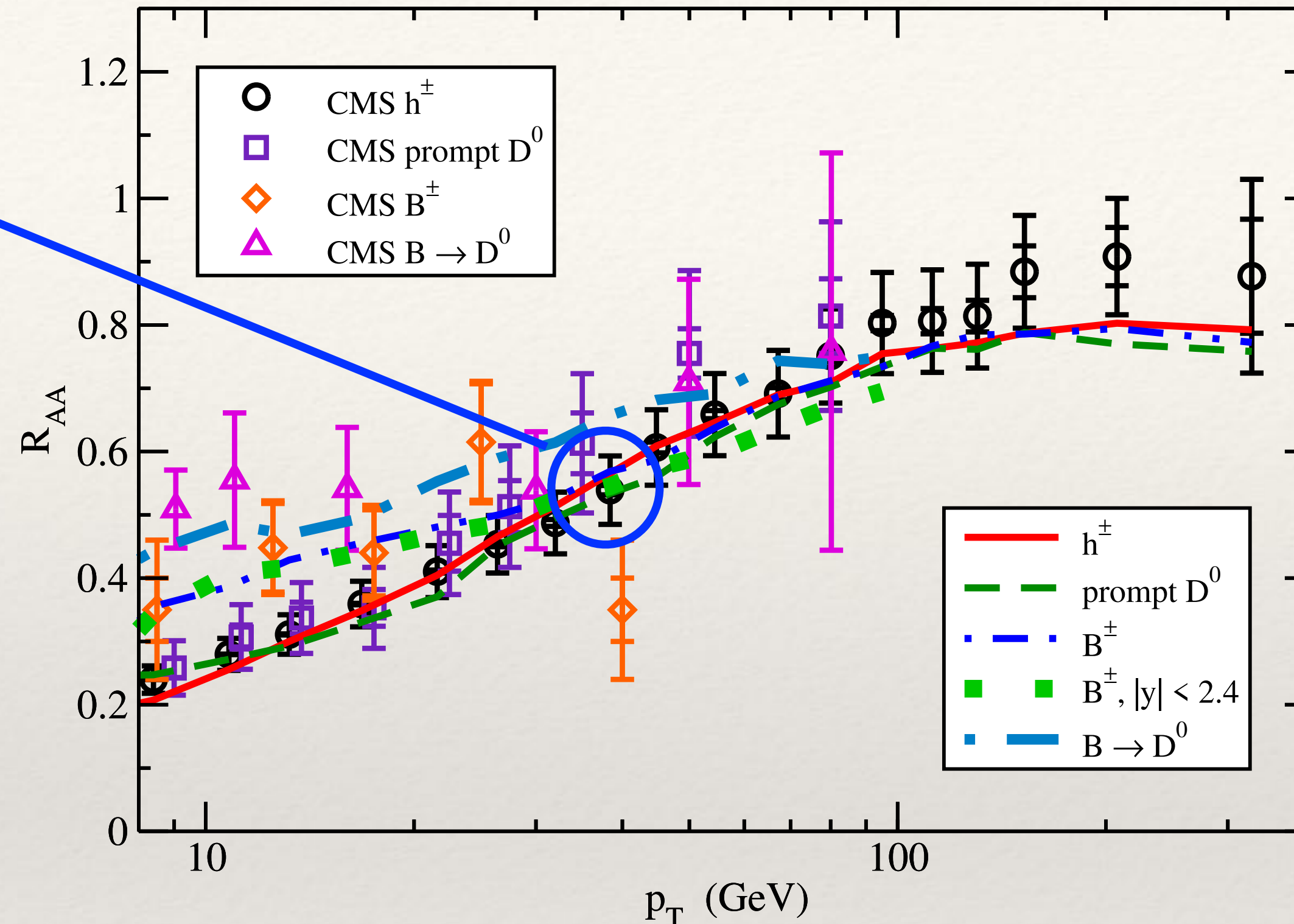
D meson



- g -initiated h & D $R_{AA} < q$ -initiated h & D $R_{AA} \Rightarrow \Delta E_g > \Delta E_{q/c}$
- Although $R_{AA}(c \rightarrow D) > R_{AA}(q \rightarrow h)$, $R_{AA}(g \rightarrow D) < R_{AA}(g \rightarrow h)$ due to different fragmentation functions $\Rightarrow R_{AA}(h) \approx R_{AA}(D)$

Flavor hierarchy of jet quenching

Merging of D and B R_{AA} at $p_T \sim 40$ GeV



[Xing, SC, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

- A simultaneous description of charged hadron, D meson, B meson, B -decay D meson R_{AA} 's starting from $p_T \sim 8$ GeV
- Predict R_{AA} separation between B and h / D below 40 GeV, but similar values above – **wait for confirmation from future precision measurement**

Low to medium p_T — effects of non-perturbative interaction

- Parametrization of the heavy-quark-QGP interaction potential:

$$V(r, T) = -\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r} - \frac{\sigma}{m_s} e^{-m_s r}$$

Yukawa (color coulomb) String

in which $m_d = a + b * T$ and $m_s = \sqrt{a_s + b_s * T}$ are the respective screening masses, α_s and σ are the respective Yukawa and confining interaction strength.

- By Fourier transformation,

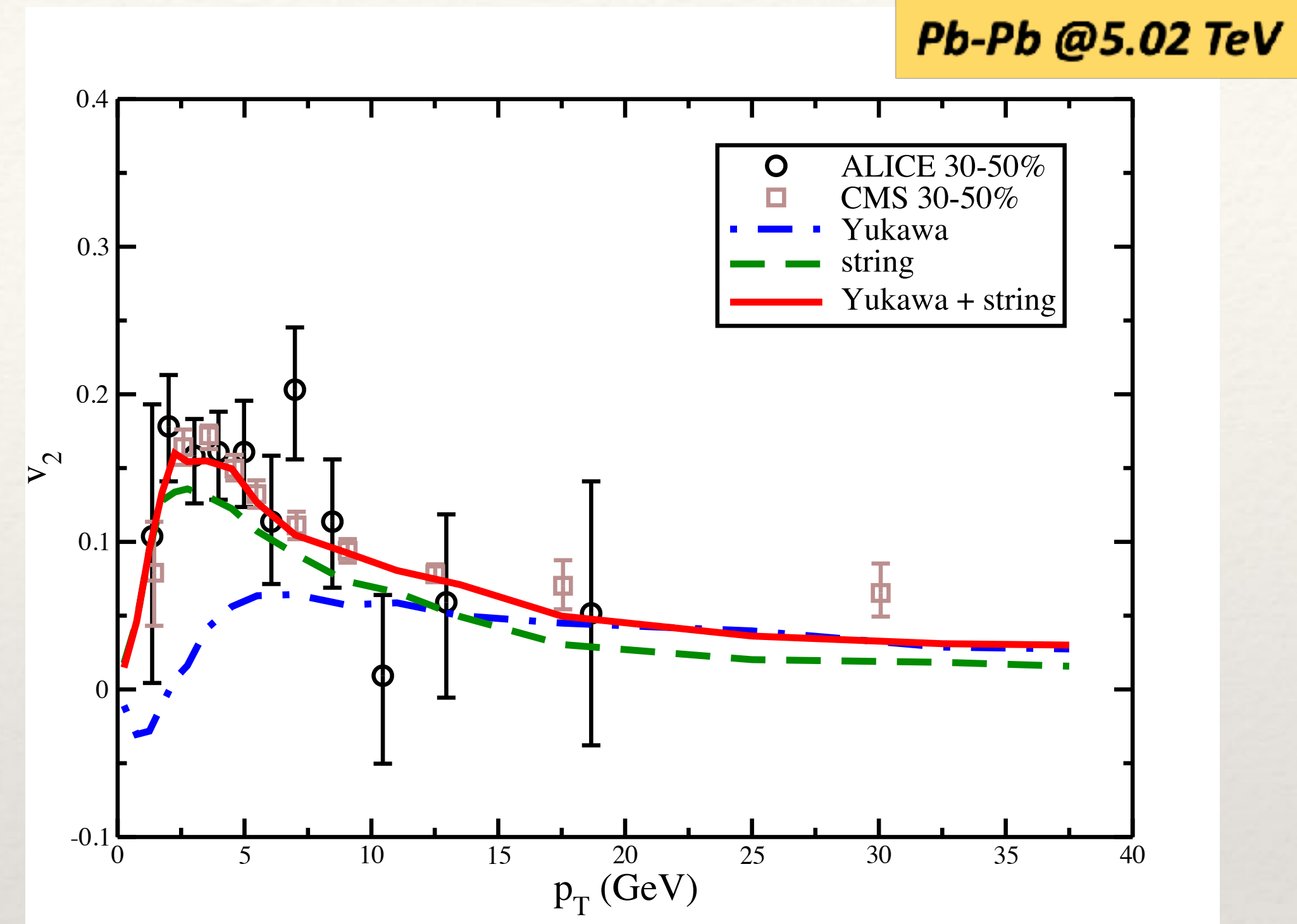
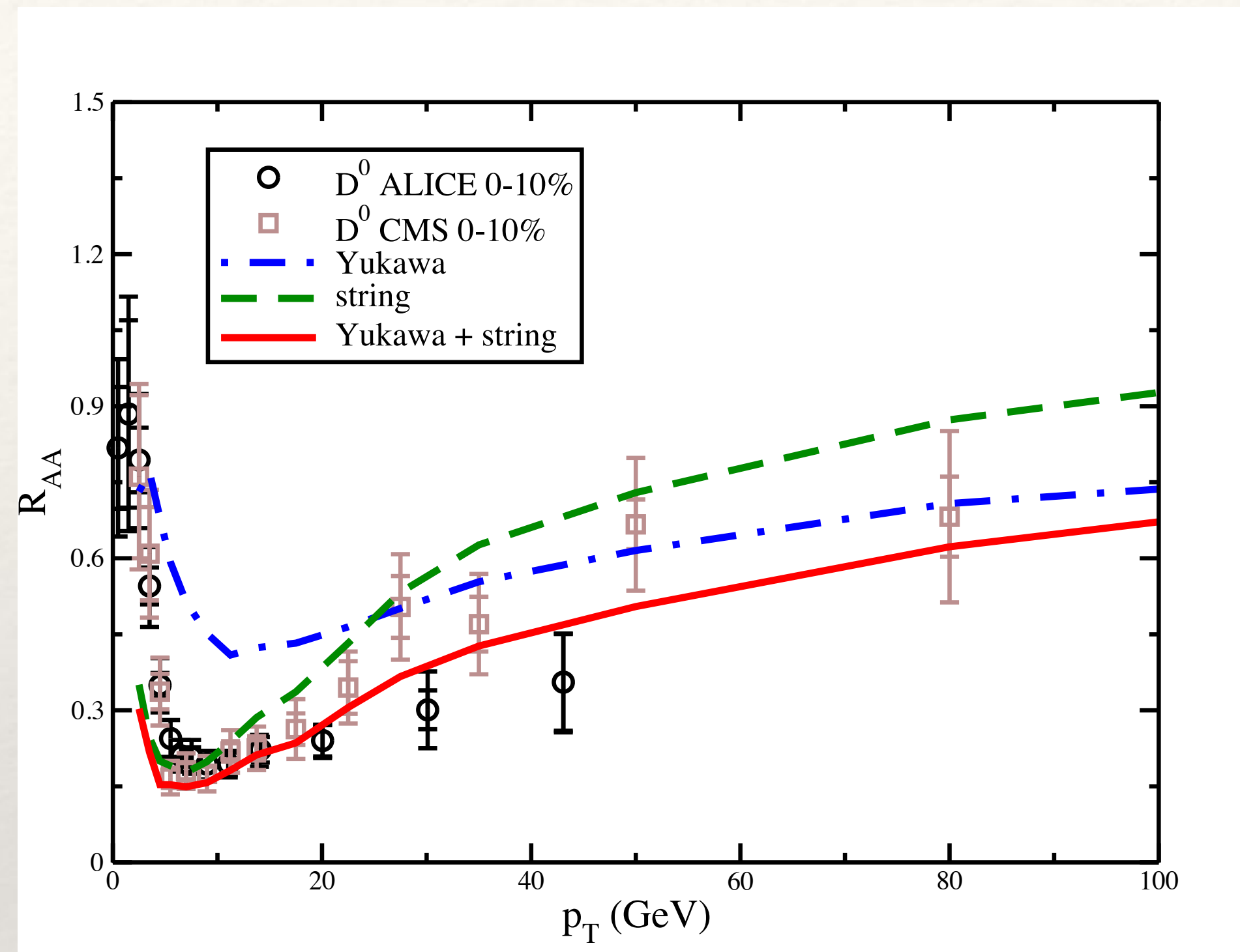
$$V(\vec{q}, T) = -\frac{4\pi\alpha_s C_F}{m_d^2 + |\vec{q}|^2} - \frac{8\pi\sigma}{(m_s^2 + |\vec{q}|^2)^2}$$

- For $Qq \rightarrow Qq$ process, we express the scattering amplitude with effective potential propagator,

Riek and Rapp, Phys. Rev. C 82 (2010) 035201

$$iM = iM_C + iM_S = \bar{u}\gamma^\mu u V_C \bar{u}\gamma^\nu u + \bar{u}u V_S \bar{u}u$$

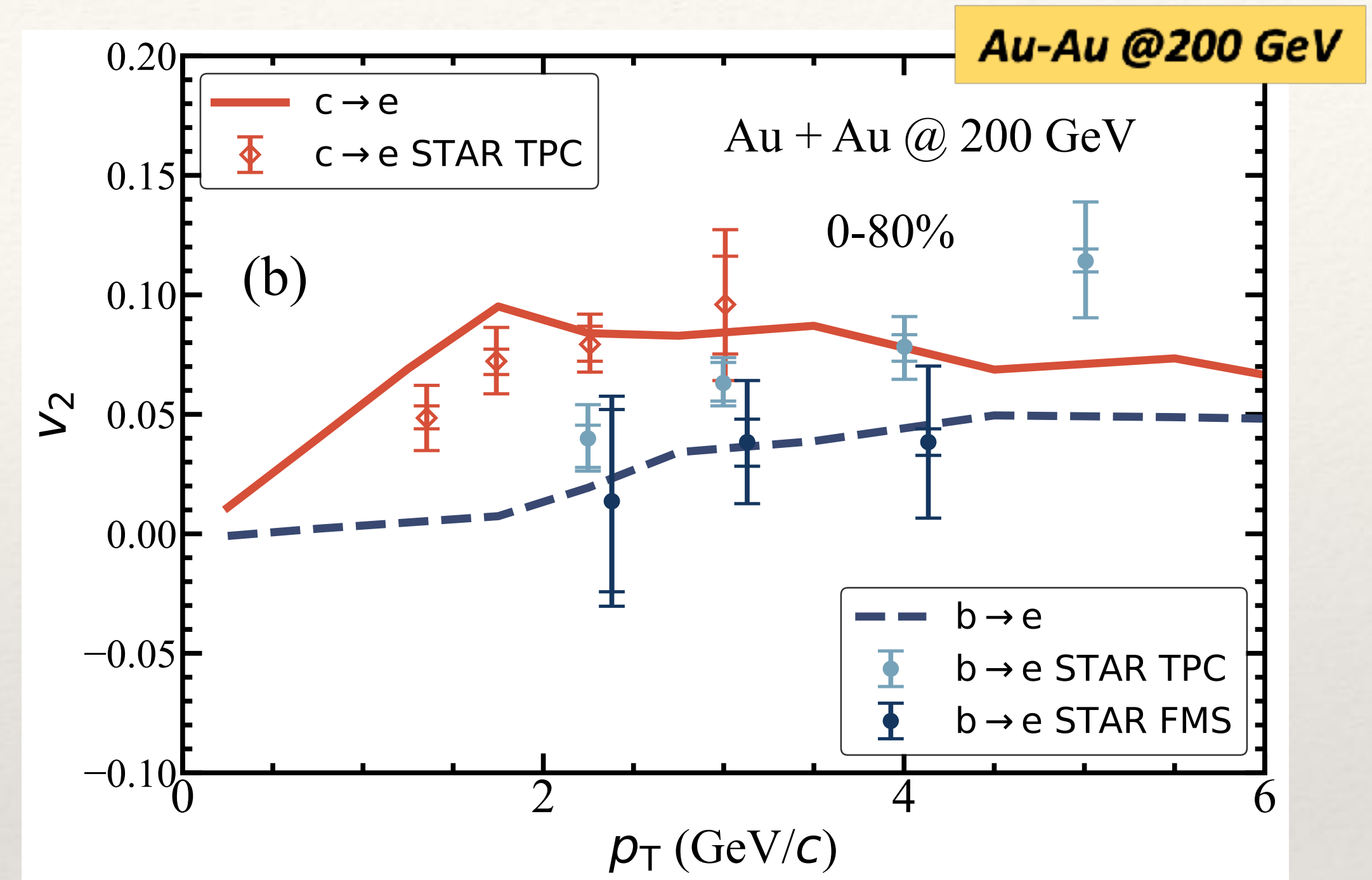
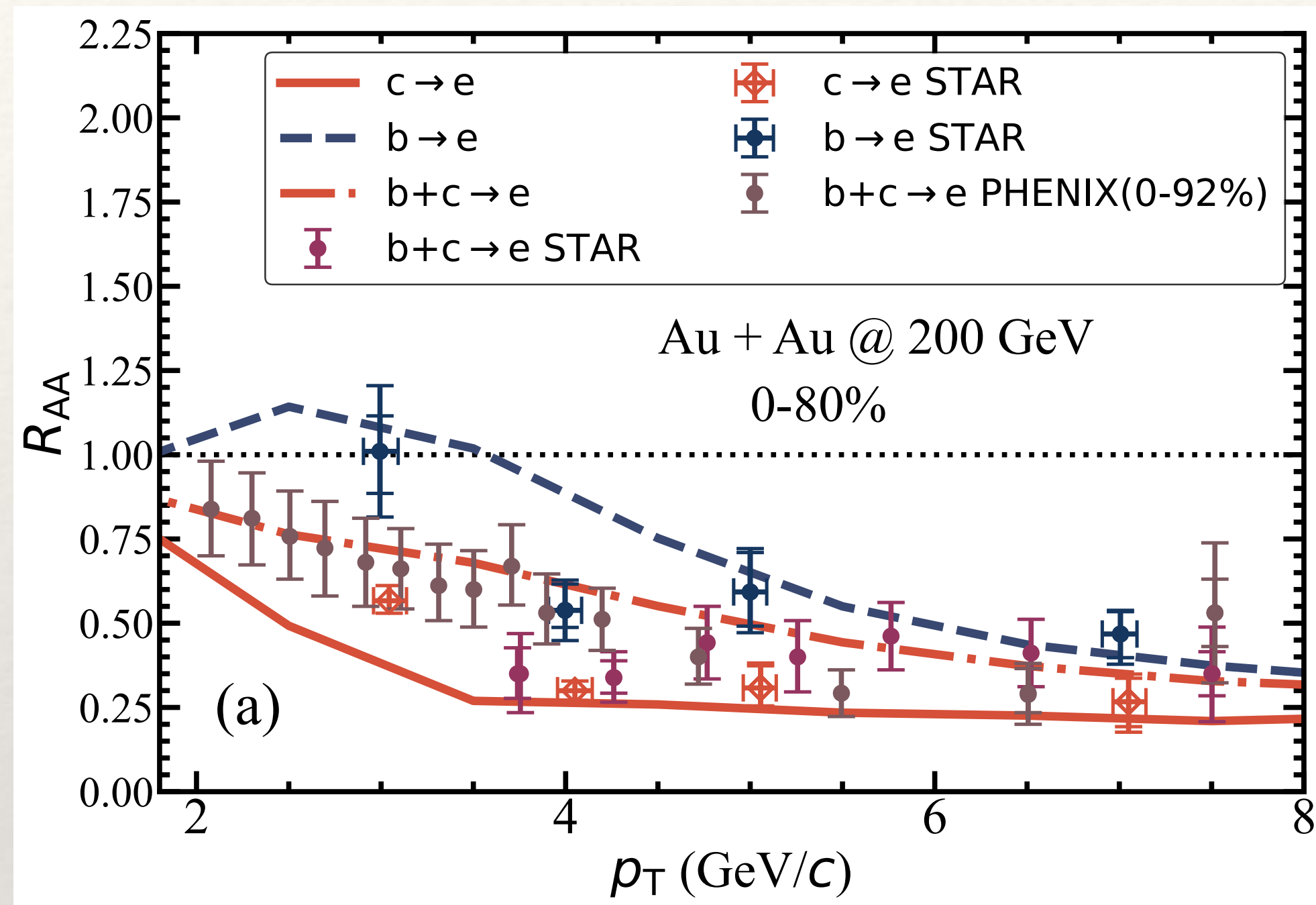
R_{AA} and v_2 of D mesons at LHC



[Xing, Qin, SC, Phys. Lett. B 838 (2023) 137733]

- At high p_T , the Yukawa interaction dominates heavy-quark-medium interaction
- At low to intermediate p_T , the string interaction dominates, stronger contribution at later evolution stage (near T_c)

R_{AA} and v_2 of heavy flavor decayed electrons at RHIC



[Dang, Xing, SC, Qin, arXiv:2307.14808]

- Combining short-range Yukawa and long-range string interactions provide a reasonable description of the current RHIC data of electrons decayed from charm and bottom quarks

Possible inverse hierarchy of c vs. b energy loss

[Dang, Xing, SC, Qin, arXiv:2307.14808]

- Enhanced non-perturbative interactions with larger mass

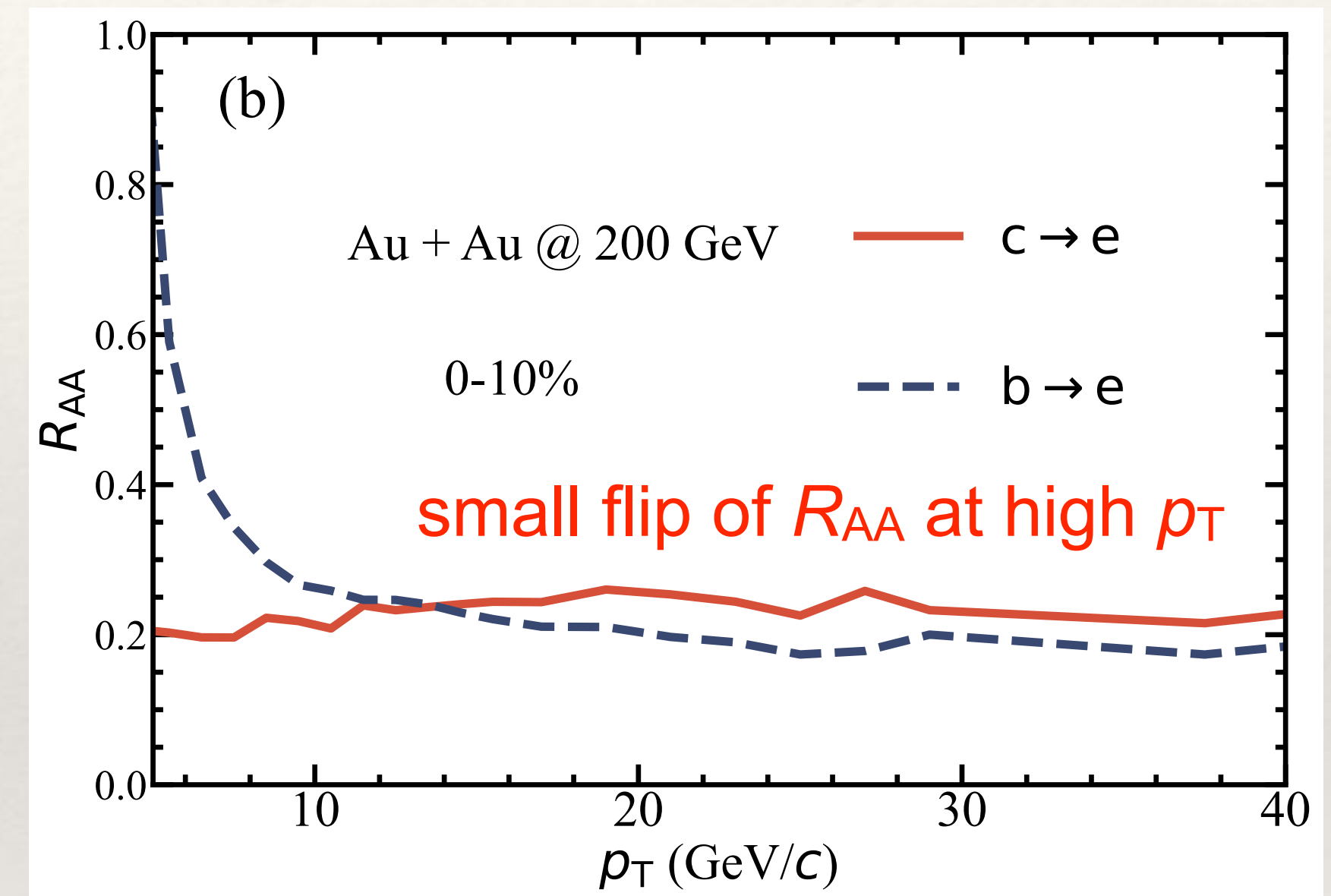
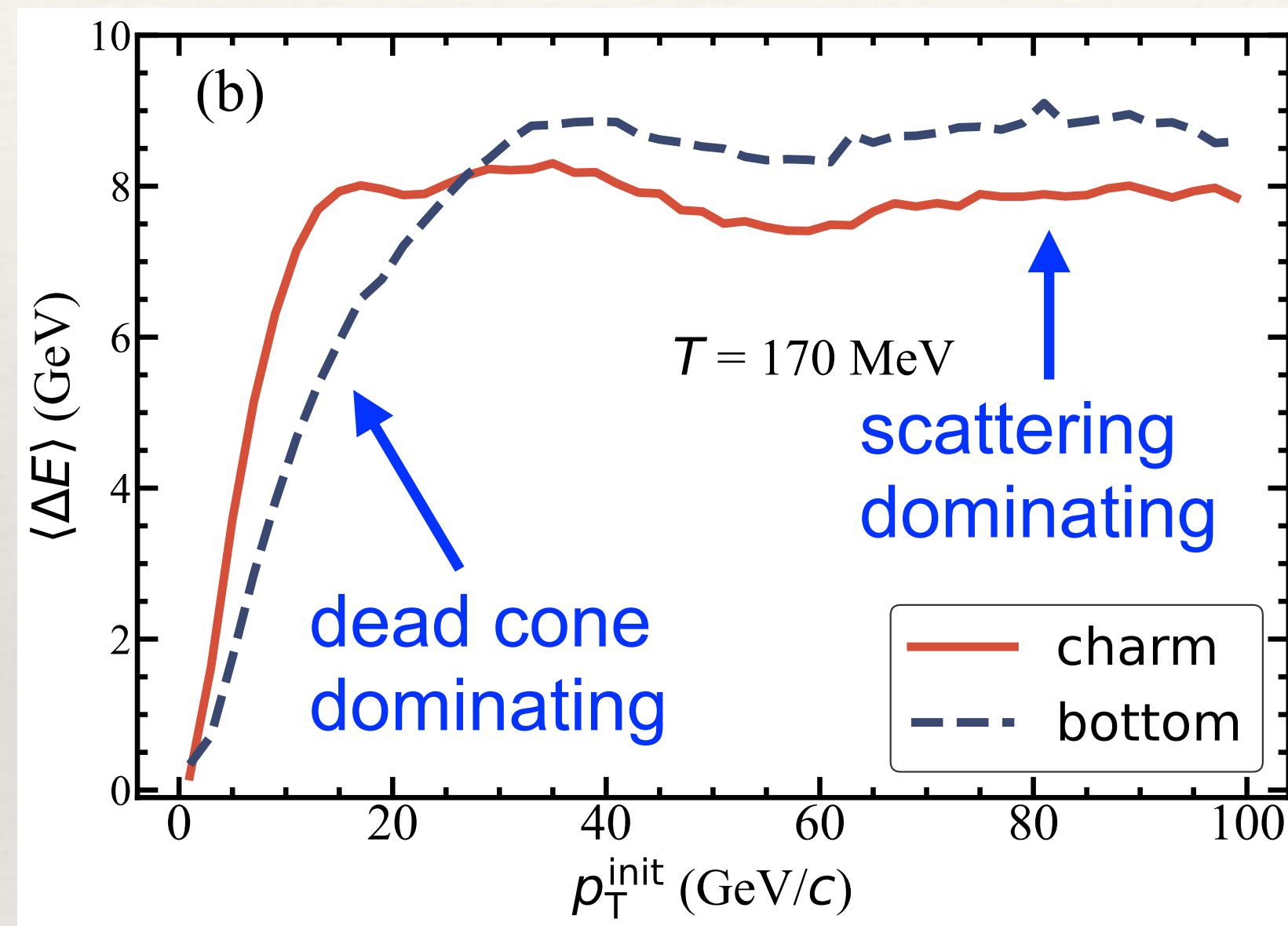
$$\left| \mathcal{M}_{Q+q/g} \right|_{\text{non-pert.}}^2 \sim \frac{t^2 - 4m_Q^2 t}{(t - m_s^2)^4}$$

Possible inverse hierarchy of c vs. b energy loss

[Dang, Xing, SC, Qin, arXiv:2307.14808]

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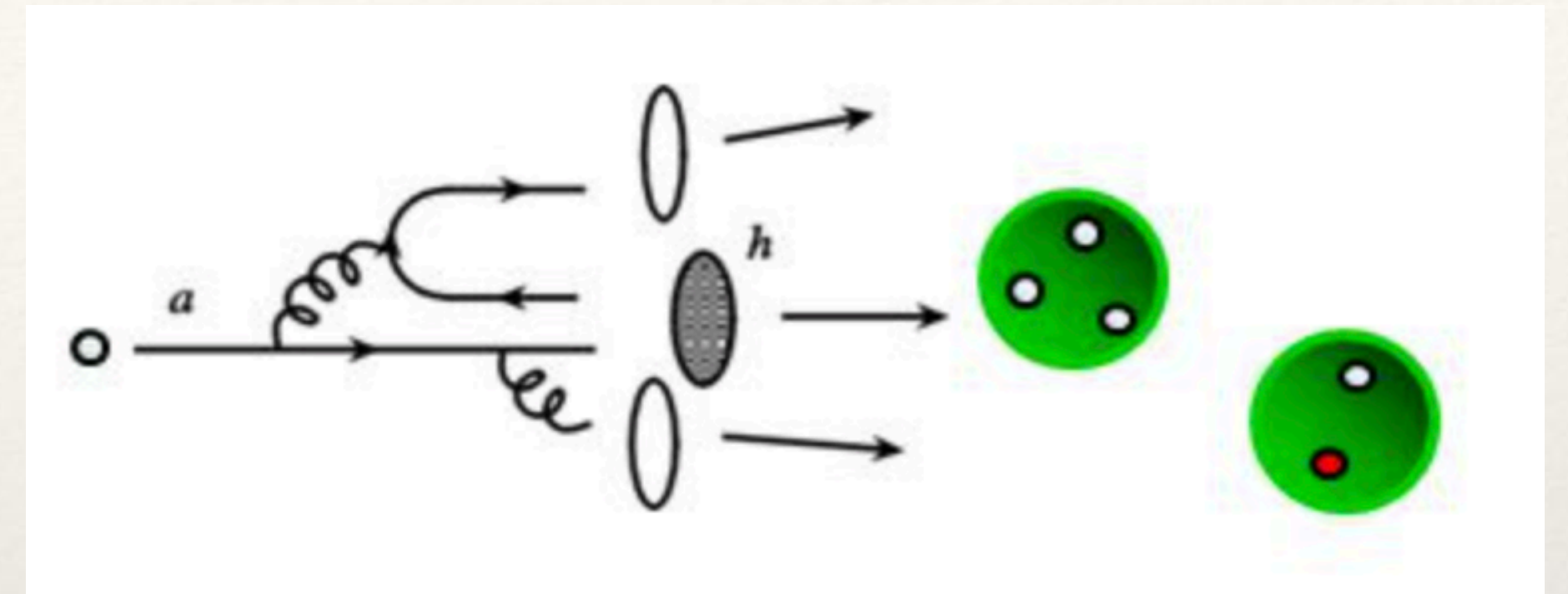


- Depend on competition between the dead cone effect and non-perturbative scattering
- At low T , heavier quarks may lose more energy at high p_T
- Need more precise data at high p_T to test this model prediction

Medium to low p_T hadrons — hadronization

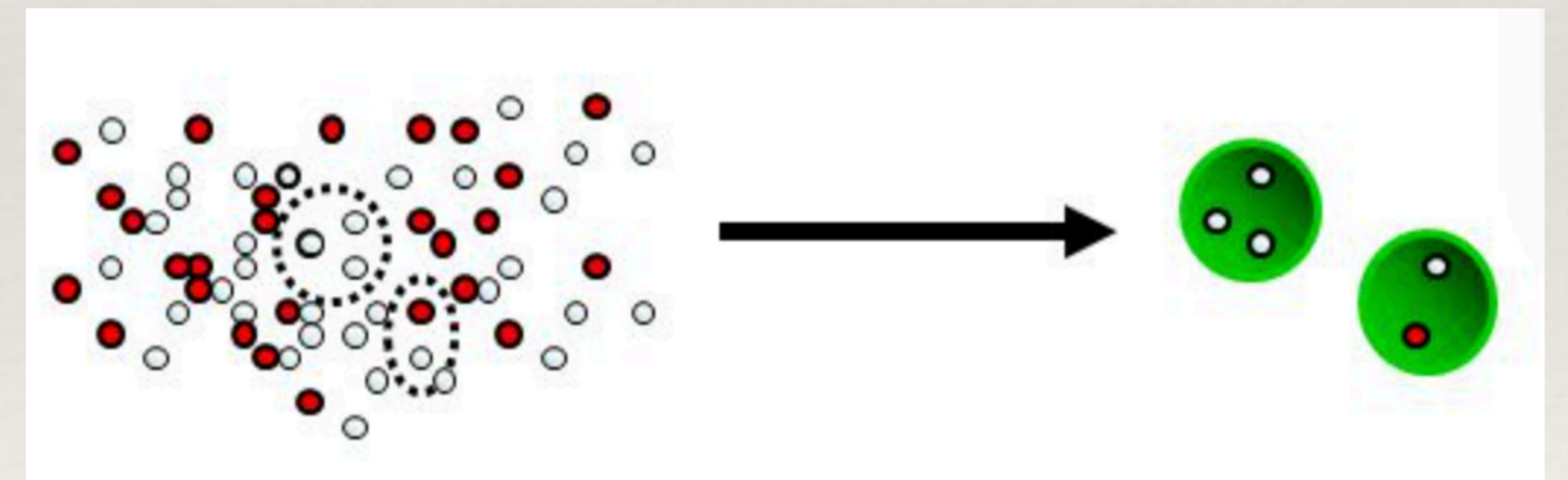
Fragmentation:

High momentum heavy quarks are more likely to fragment into hadrons
[Peterson, FONLL, NLO, Pythia, etc.]



Coalescence (recombination):

Low momentum heavy quarks are more likely to combine with thermal partons into hadrons



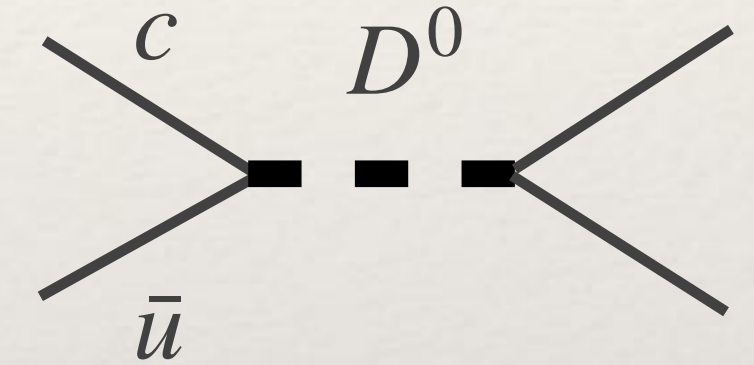
Coalescence models

- **Simplified models:** equal-velocity coalescence [Shao *et. al.*, e.g. EPJC 78 (2018) 344]
coalescence between neighboring particles [AMPT, e.g. PRC 101 (2020) 034905]

- **Resonance recombination:** coalescence probability \sim resonant scattering rate

$$P_{\text{coal}}(p) = \Delta\tau_{\text{res}} \Gamma_Q^{\text{res}}(p) \quad [\text{TAMU, e.g. PRL 124 (2020) 042301 }]$$

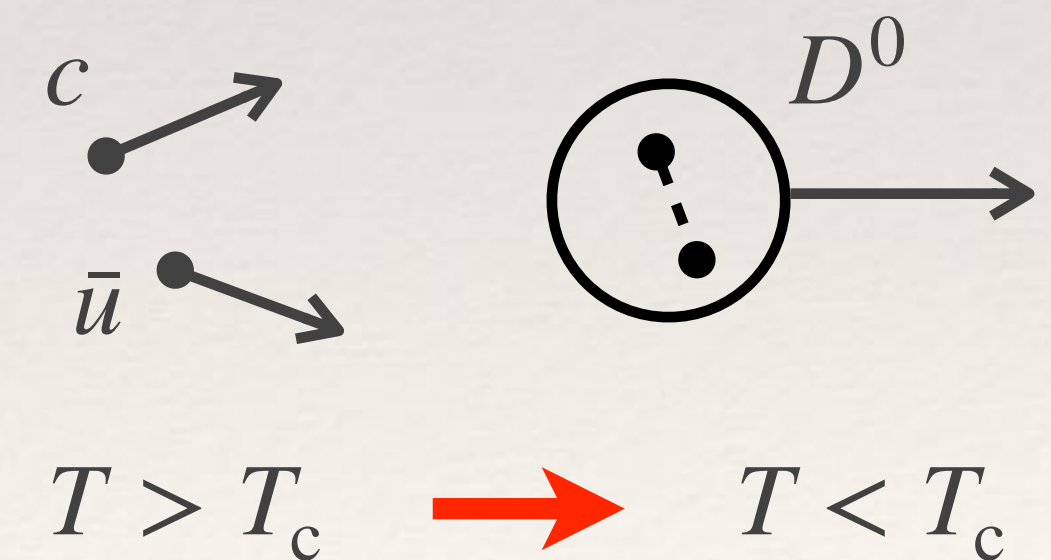
$\Delta\tau_{\text{res}}$: the time window for resonant state



- **Instantaneous coalescence:** coalescence probability \sim wavefunction overlap

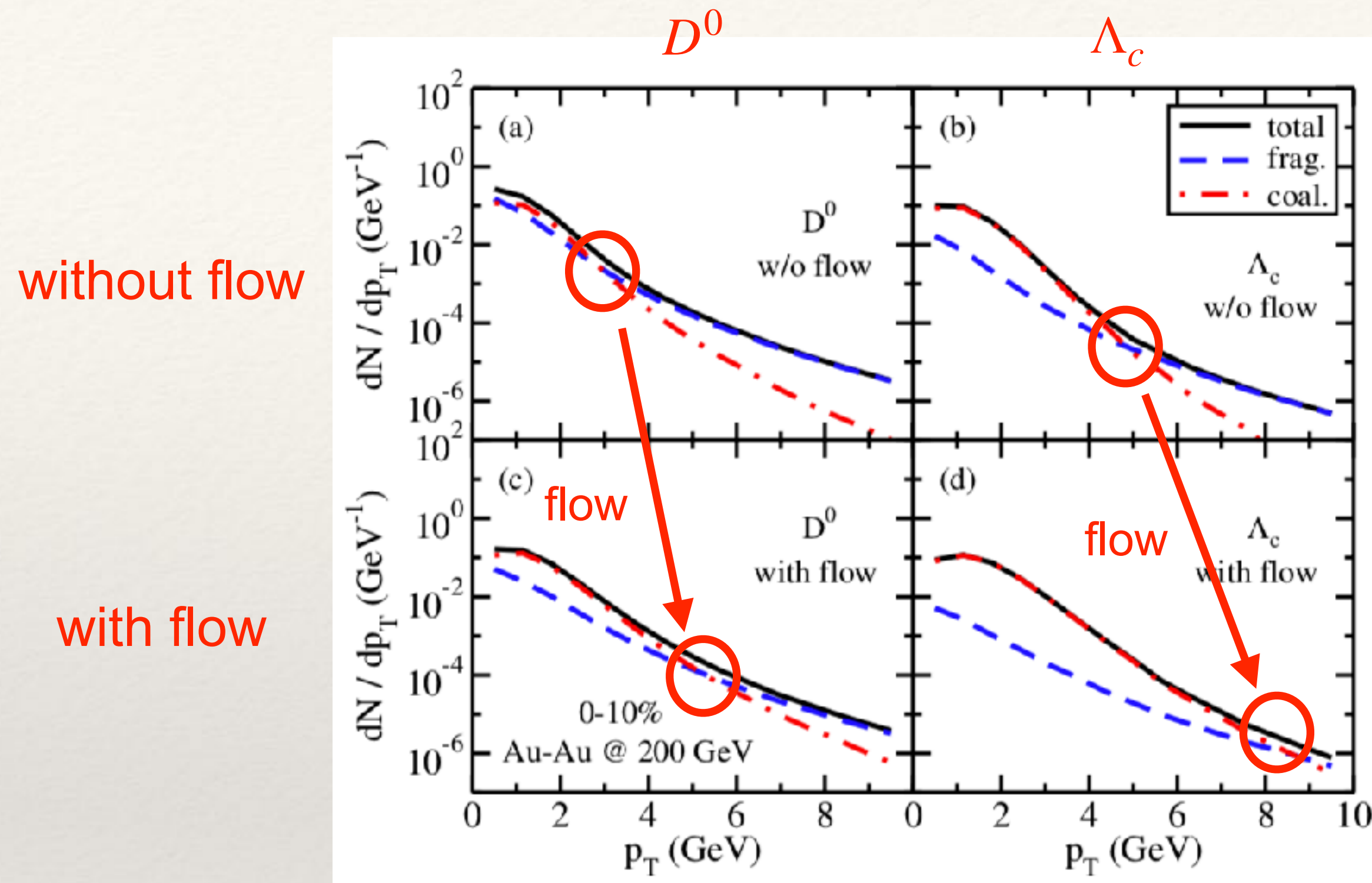
Probability: Wigner function $f_M^W \equiv |\langle M | q_1, q_2 \rangle|^2$ (for meson)

- Encodes information of microscopic hadron structures
- Wide application: Duke, LBL, Catania, Nantes, PHSD, Ko, Li, Zhuang, etc.



- **A recent comparison between different models:** [arXiv:2311.10621](https://arxiv.org/abs/2311.10621).

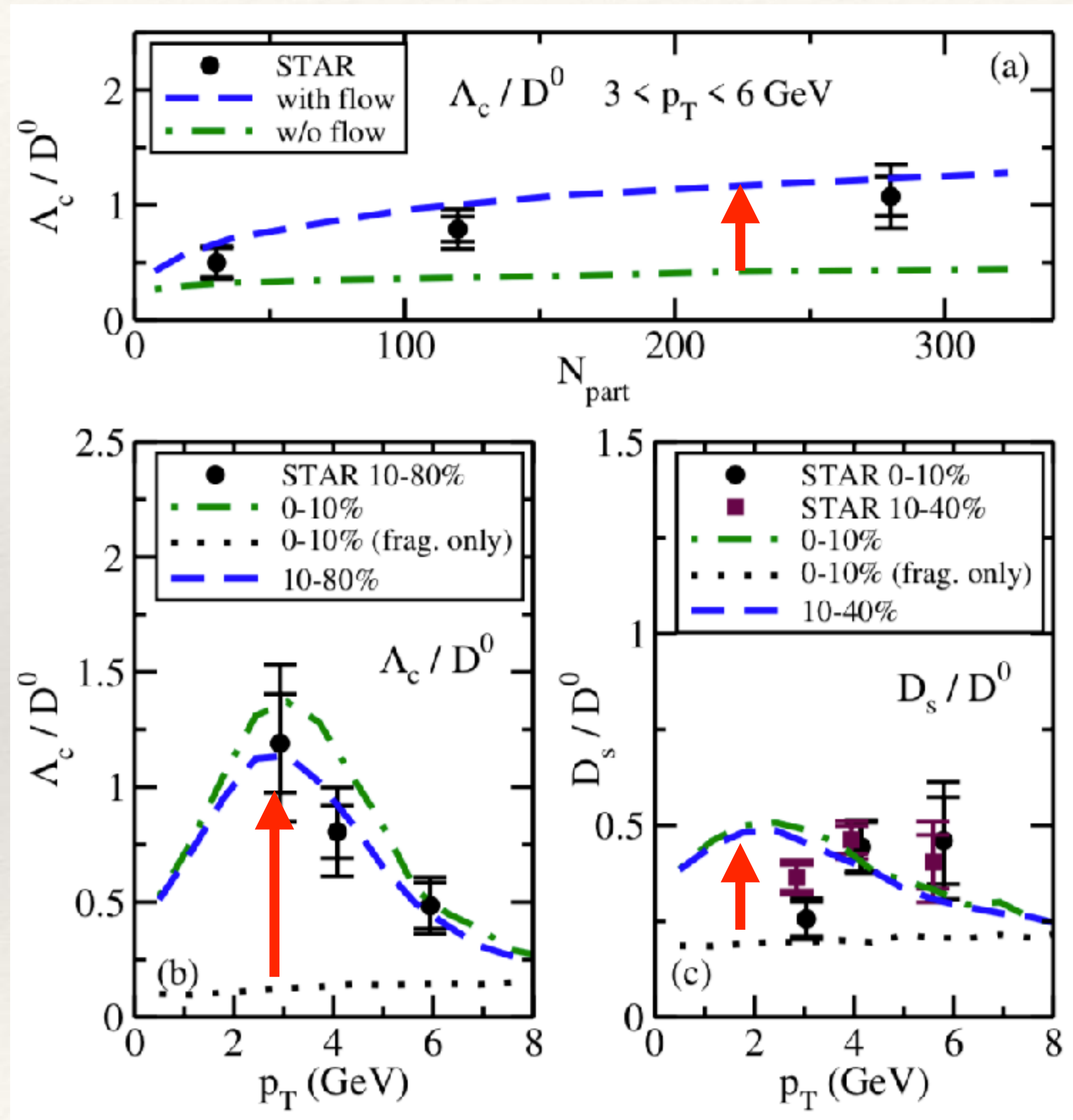
Charmed hadron spectra: QGP flow effect



[SC, Sun, Li, Liu, Xing, Qin, Ko, Phys. Lett. B 807 (2020) 135561]

- Coalescence dominates Λ_c production over a wider p_T region than D^0
- The QGP radial flow significantly enhances the coalescence contribution

Charmed hadron chemistry at RHIC



effects of the
QGP flow

- Stronger QGP flow boost on heavier hadrons => increasing Λ_c / D^0 with N_{part}

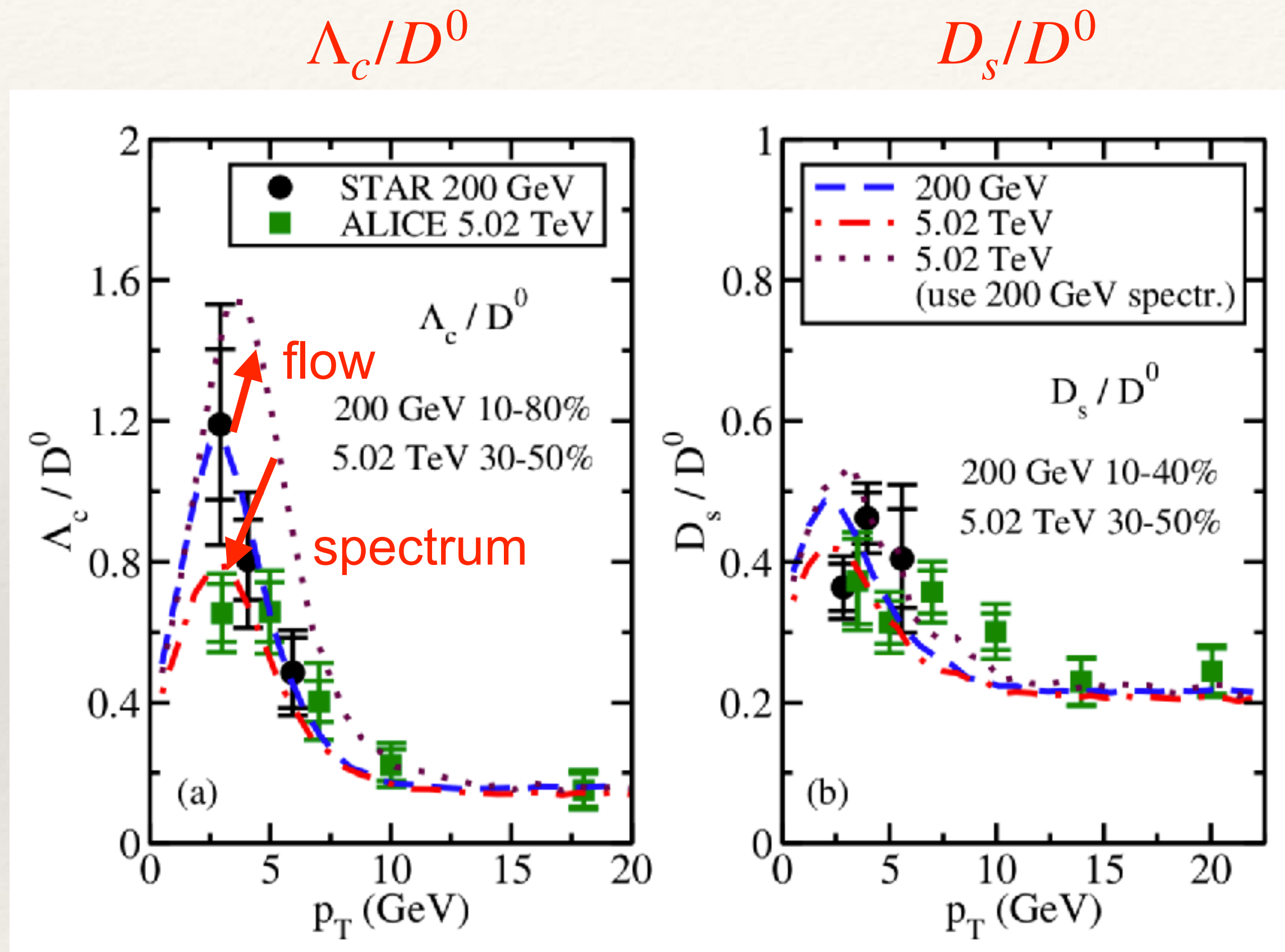
- Coalescence significantly increases Λ_c / D^0 , larger value in more central collisions (stronger QGP flow)

effects of
coalescence

- Enhanced D_s / D^0 due to strangeness enhancement in QGP and larger D_s mass than D^0

[SC, Sun, Li, Liu, Xing, Qin, Ko, Phys. Lett. B 807 (2020) 135561]

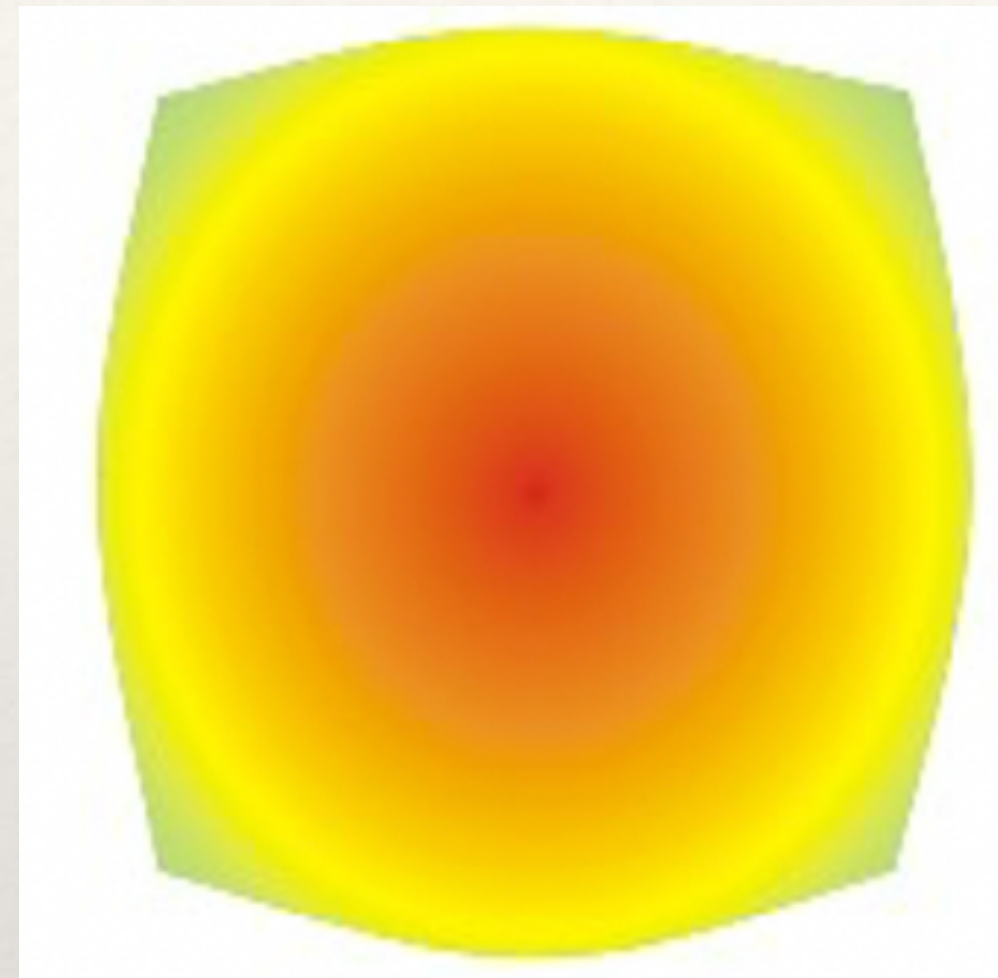
RHIC vs. LHC



- IF charm quarks have the same initial spectrum at RHIC and LHC, Λ_c/D^0 would be larger at LHC than RHIC due to the flow effect
- The harder initial charm quark spectra at LHC reduces Λ_c/D^0
- Similar theoretical prediction on D_s/D^0

[SC, Sun, Li, Liu, Xing, Qin, Ko, Phys. Lett. B 807 (2020) 135561]

Probing the EoS of QGP

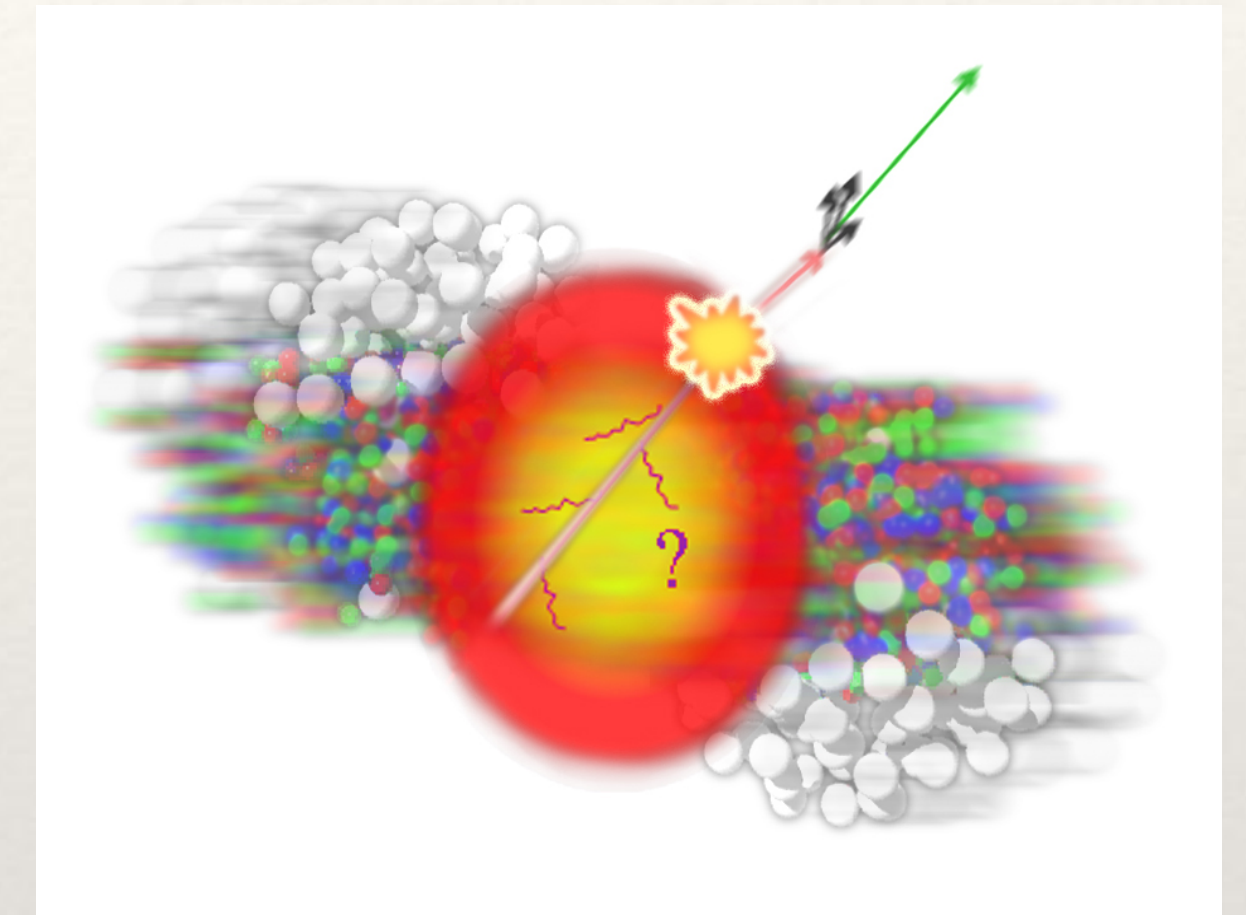


QGP

Usual conduct: fix QGP properties using soft hadron observables and study nuclear modification on hard particles



Inverse question: can we probe QGP properties using hard particle observables?



Hard probes through QGP

F.-L. Liu, X.-Y. Wu, SC, G.-Y. Qin, X.-N. Wang, Phys. Lett. B 848 (2024) 138355

Connection between transport and EoS

Transport

$$p_a \cdot \partial f_a(x_a, p_a) = E_a (\mathcal{C}_a^{\text{el}} + \mathcal{C}_a^{\text{inel}})$$

Strong coupling strength

$$g(E, T)$$

Thermal mass of partons

$$m_g^2 = \frac{1}{6} g^2 \left[(N_c + \frac{1}{2} n_f) T^2 + \frac{N_c}{2\pi^2} \sum_q \mu_q^2 \right]$$

$$m_{u,d}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left[T^2 + \frac{\mu_{u,d}^2}{\pi^2} \right]$$

$$m_s^2 - m_{0s}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left[T^2 + \frac{\mu_s^2}{\pi^2} \right]$$

Equation of state

$$\begin{aligned} P_{qp}(m_u, m_d, \dots, T) &= \sum_{i=u,d,s,g} d_i \int \frac{d^3 p}{(2\pi)^3} \frac{|\vec{p}^2|}{3E_i(p)} f_i(p) - B(T) \\ &= \sum_i P_{kin}^i(m_i, T) - B(T) \end{aligned}$$

$$\epsilon = TdP(T)/dT - P(T), \quad s = (\epsilon + P)/T$$

Strategy:

Fit g from comparing
transport model to data

Calculate EoS from g

Parametrization and Bayesian analysis

Strong coupling strength

Interaction between thermal partons (thermal scale):

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln \left[\frac{(aT/T_c + b)^2}{1 + ce^{-d(T/T_c)^2}} \right]}$$

Interaction with hard partons (parton energy scale):

$$g^2(E) = \frac{48\pi^2}{(11N_c - 2N_f) \ln [(AE/T_c + B)^2]}$$

Parameters: $\theta = (a, b, c, d, A, B)$

Bayes Theorem

$$P(\theta|\text{data}) \propto P(\text{data}|\theta)P(\theta)$$

posterior distribution

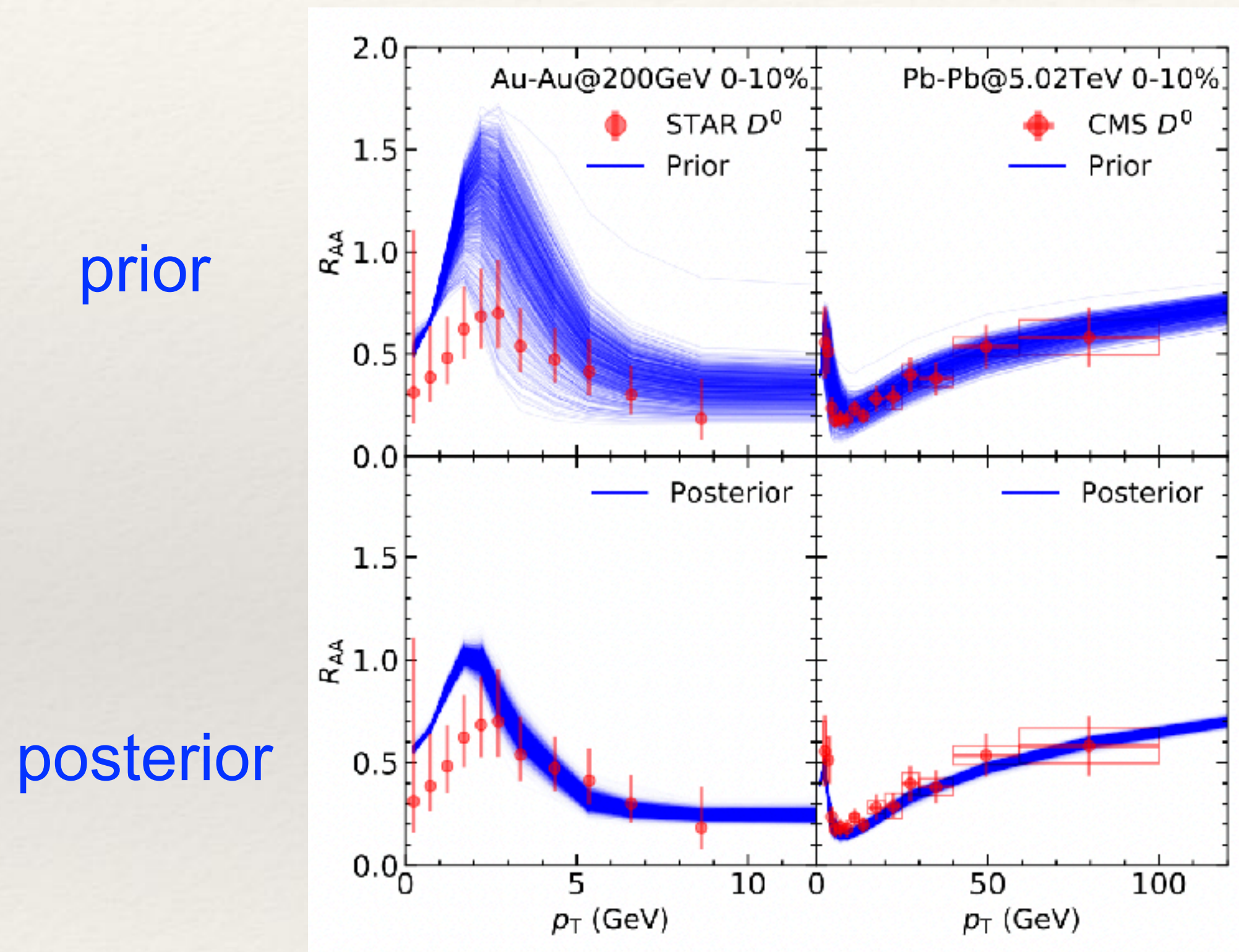
prior distribution

model-to-data comparison

$$P(\text{data}|\theta) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{[y_i(\theta) - y_i^{\text{exp}}]^2}{2\sigma_i^2}}$$

Model calibration and parameter extraction

Calibration against observables

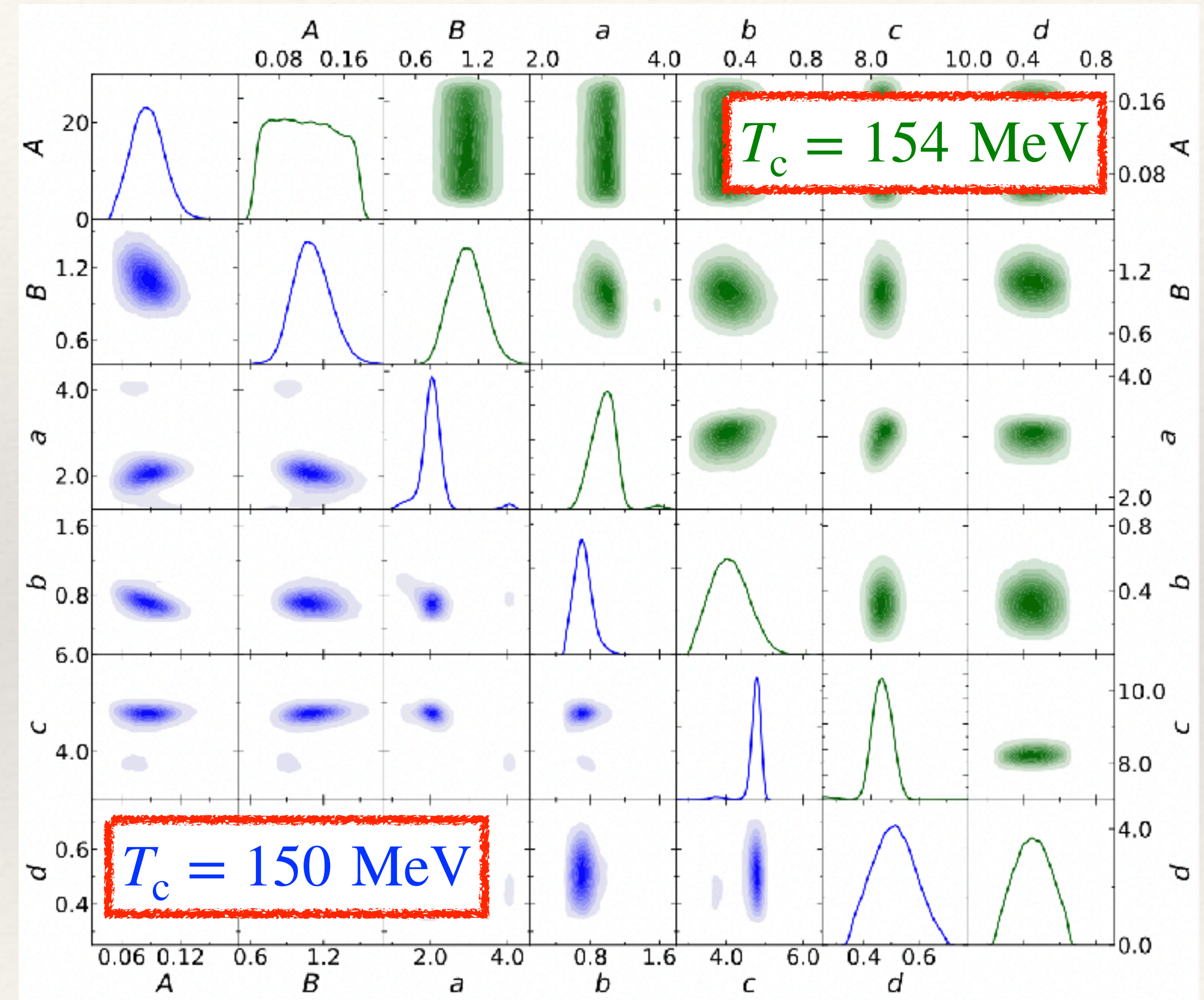


prior

posterior

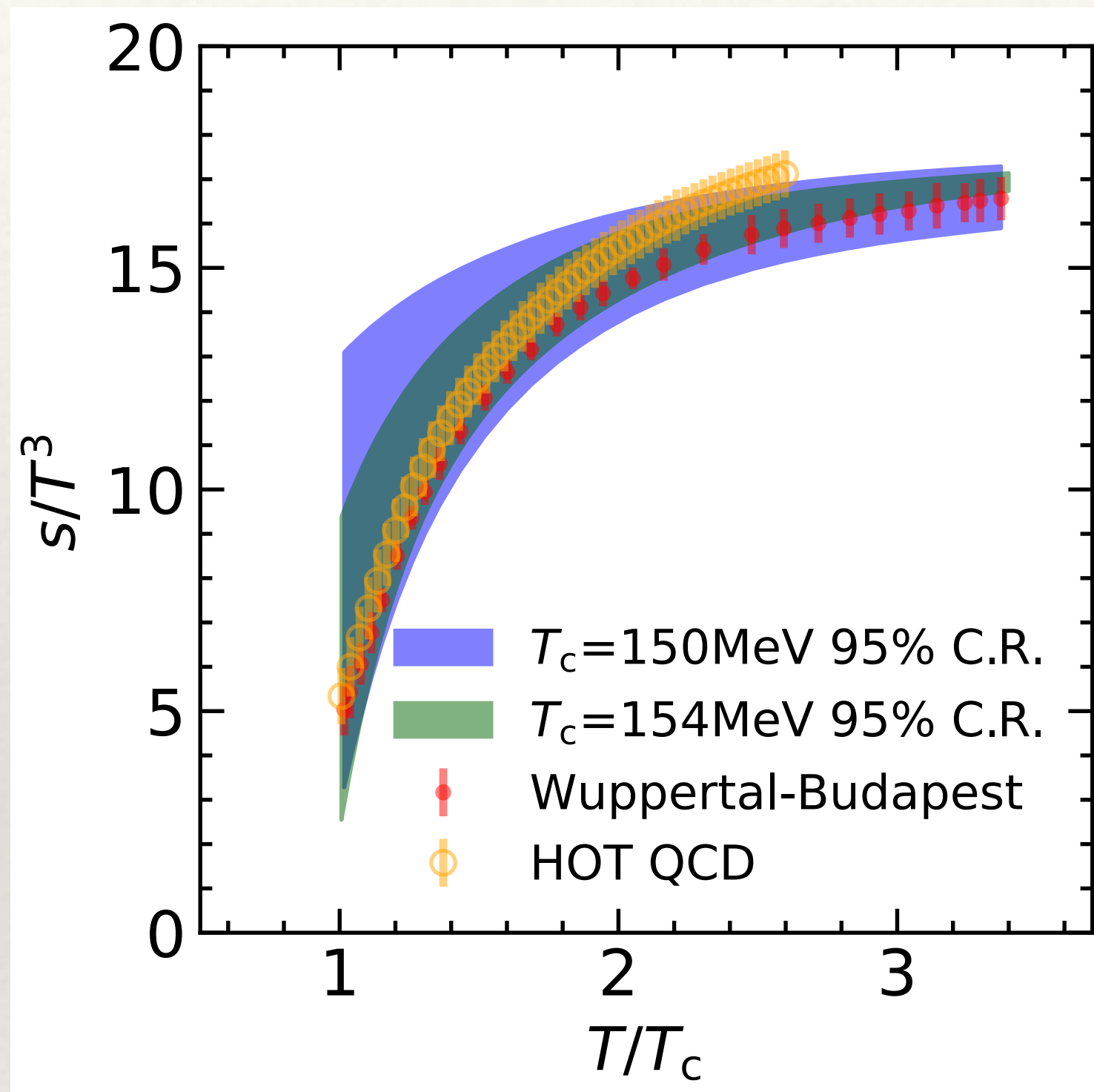
(Two examples from many observables)

Extraction of model parameters

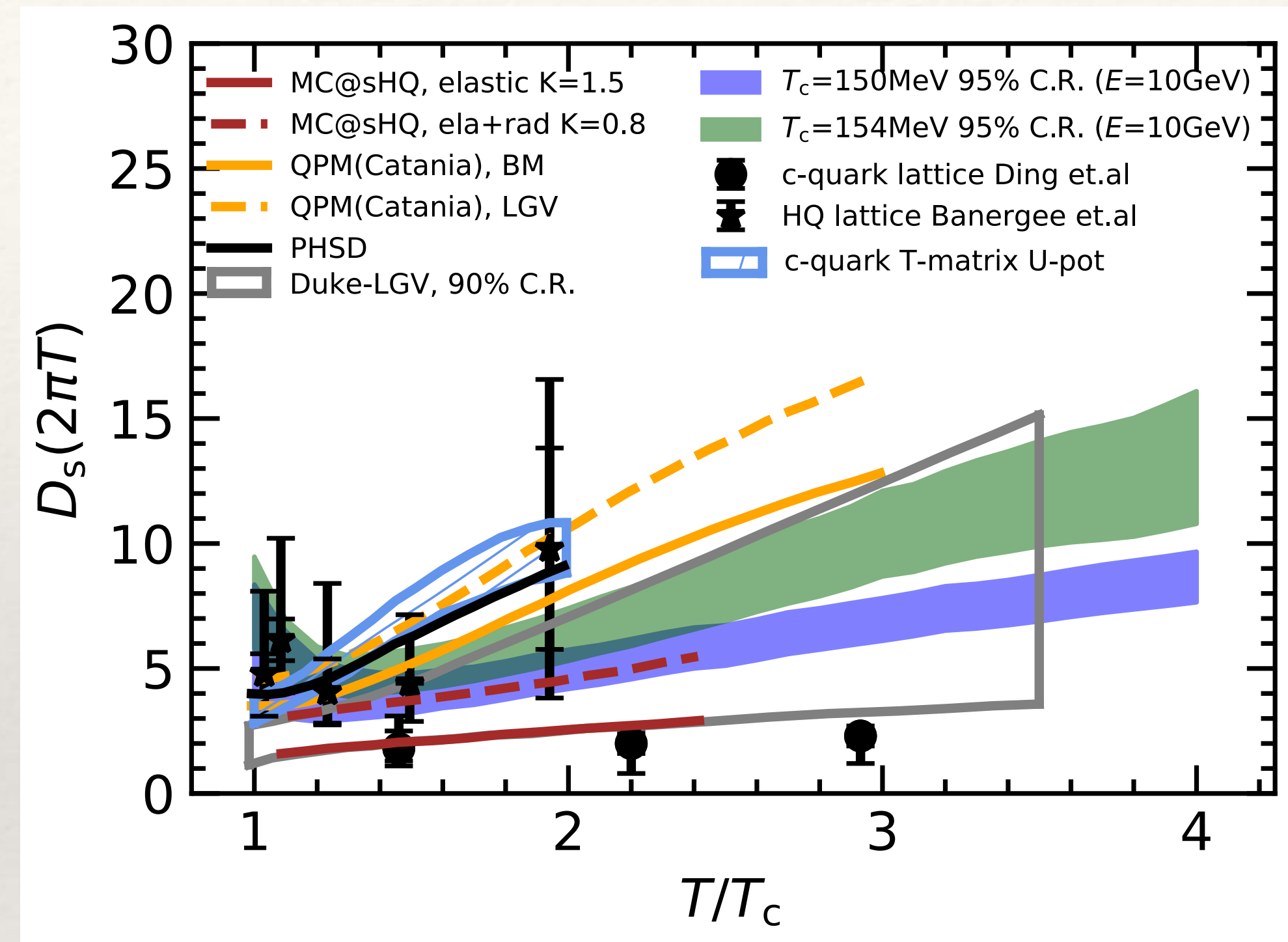


EoS of QGP and diffusion coefficient of heavy quarks

Equation of state



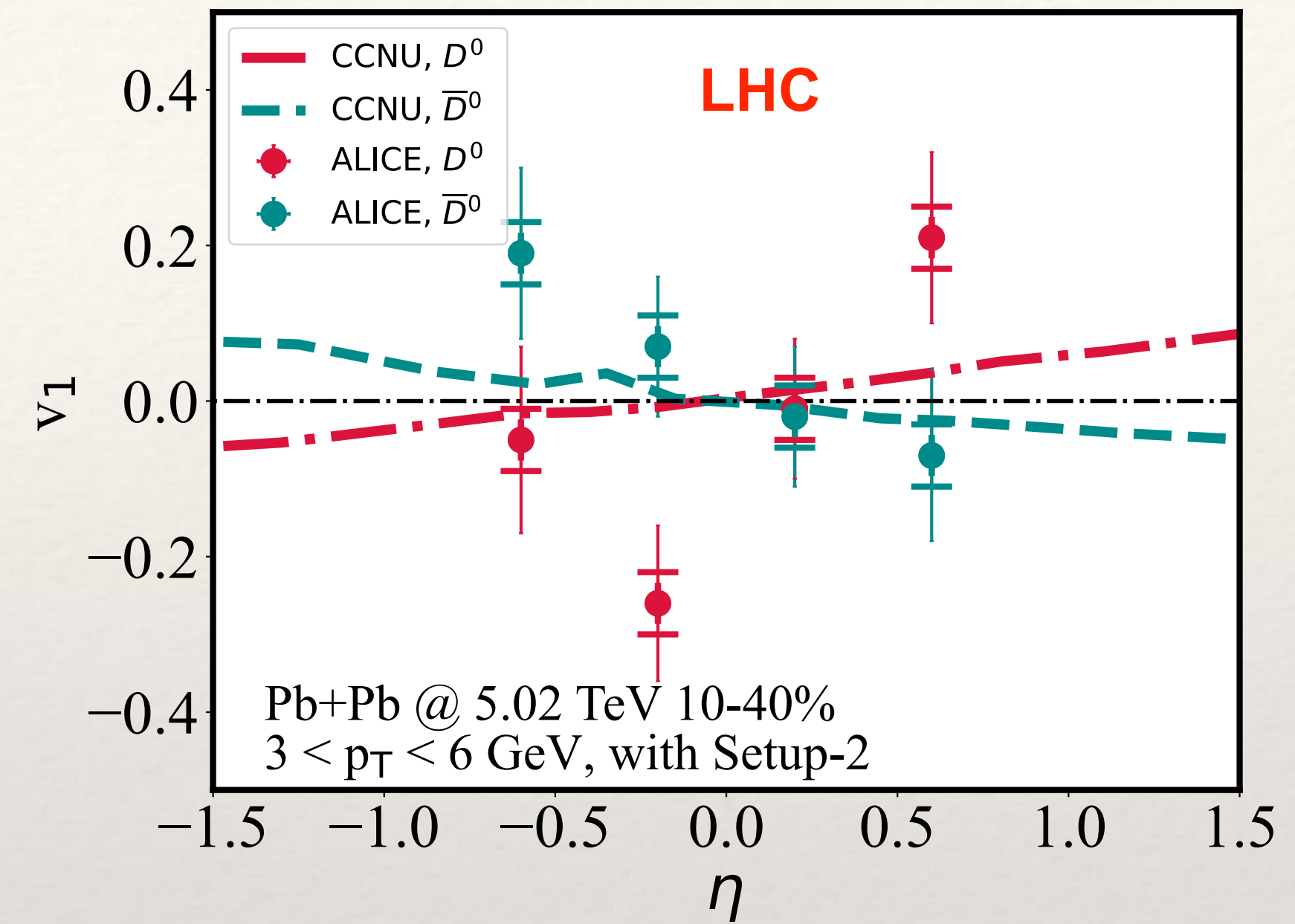
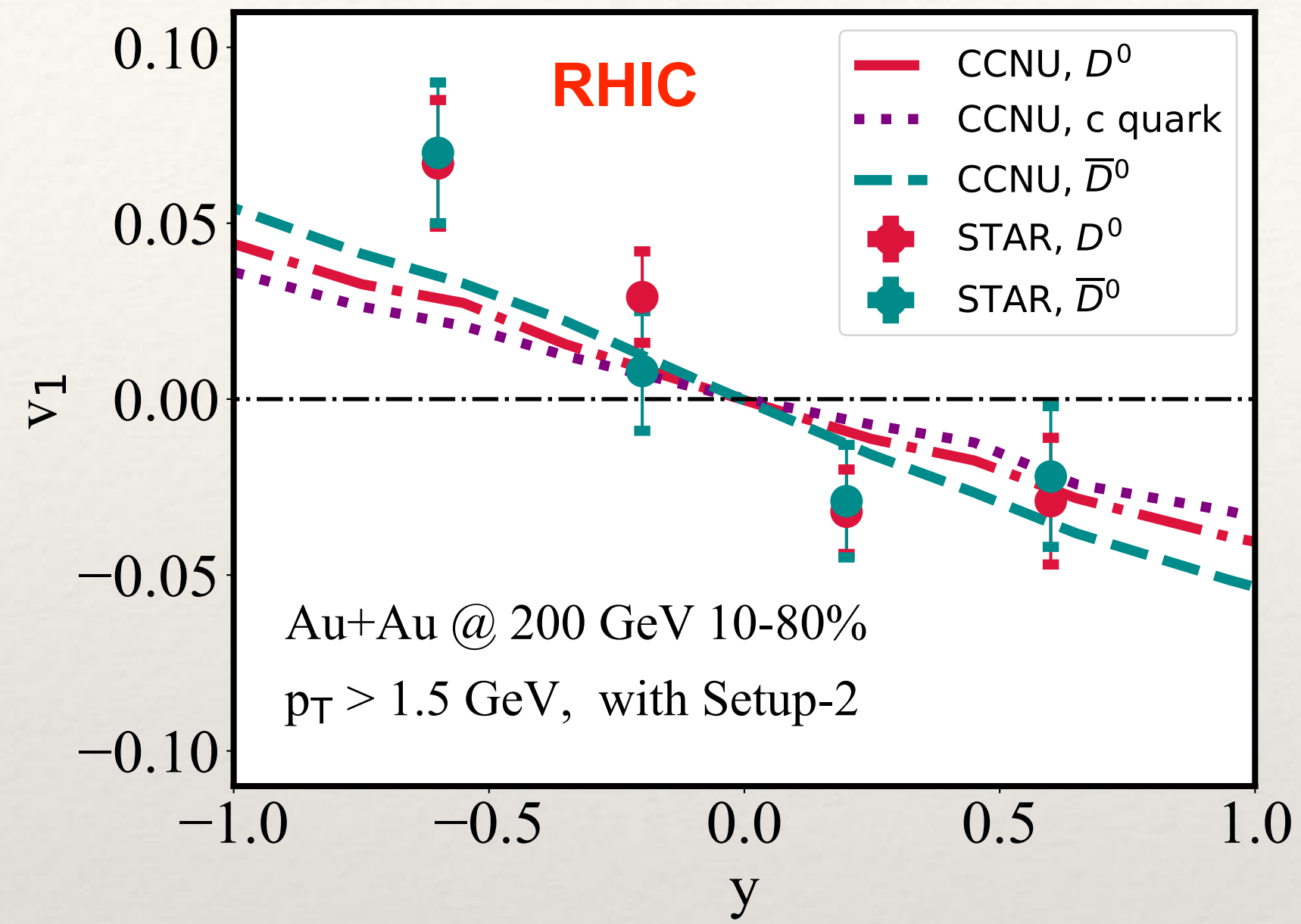
Diffusion coefficient



- Agreement with the lattice data
- Simultaneous constraint on QGP properties and transport properties of hard probes

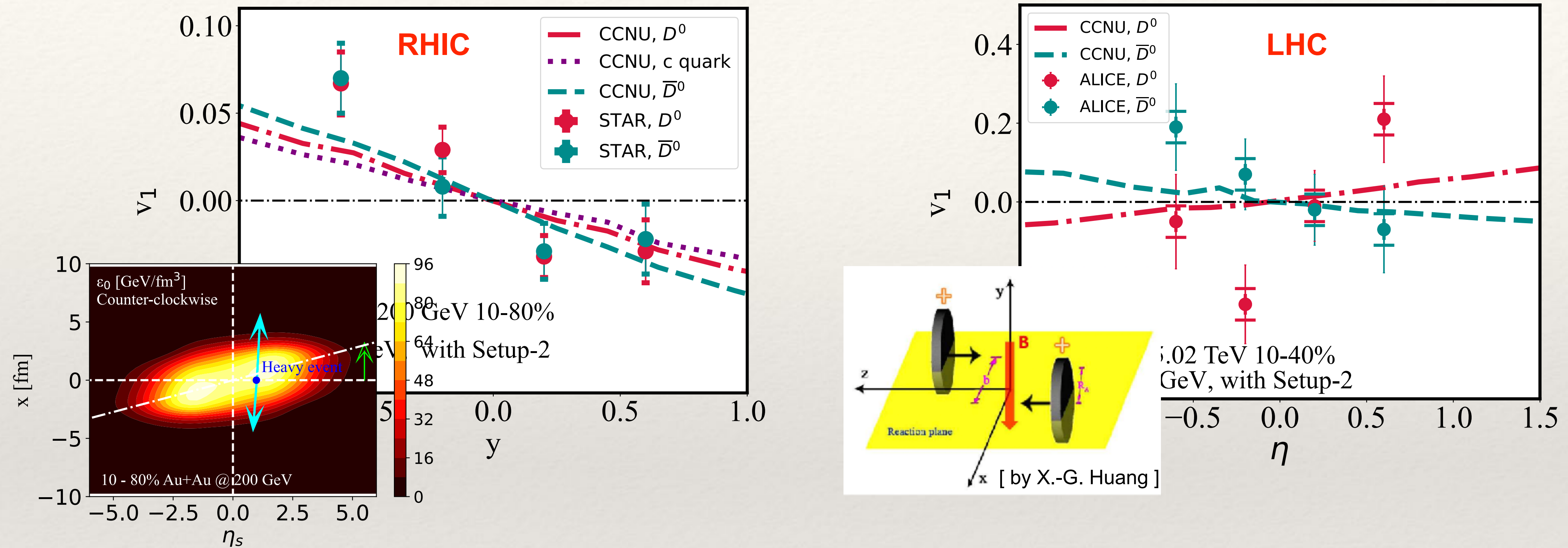
Probing medium geometry and E&M field with the D meson v_1

[Jiang, SC, Xing, Wu, Yang, Zhang, Phys. Rev. C 105 (2022) 5, 054907]



Probing medium geometry and E&M field with the D meson v_1

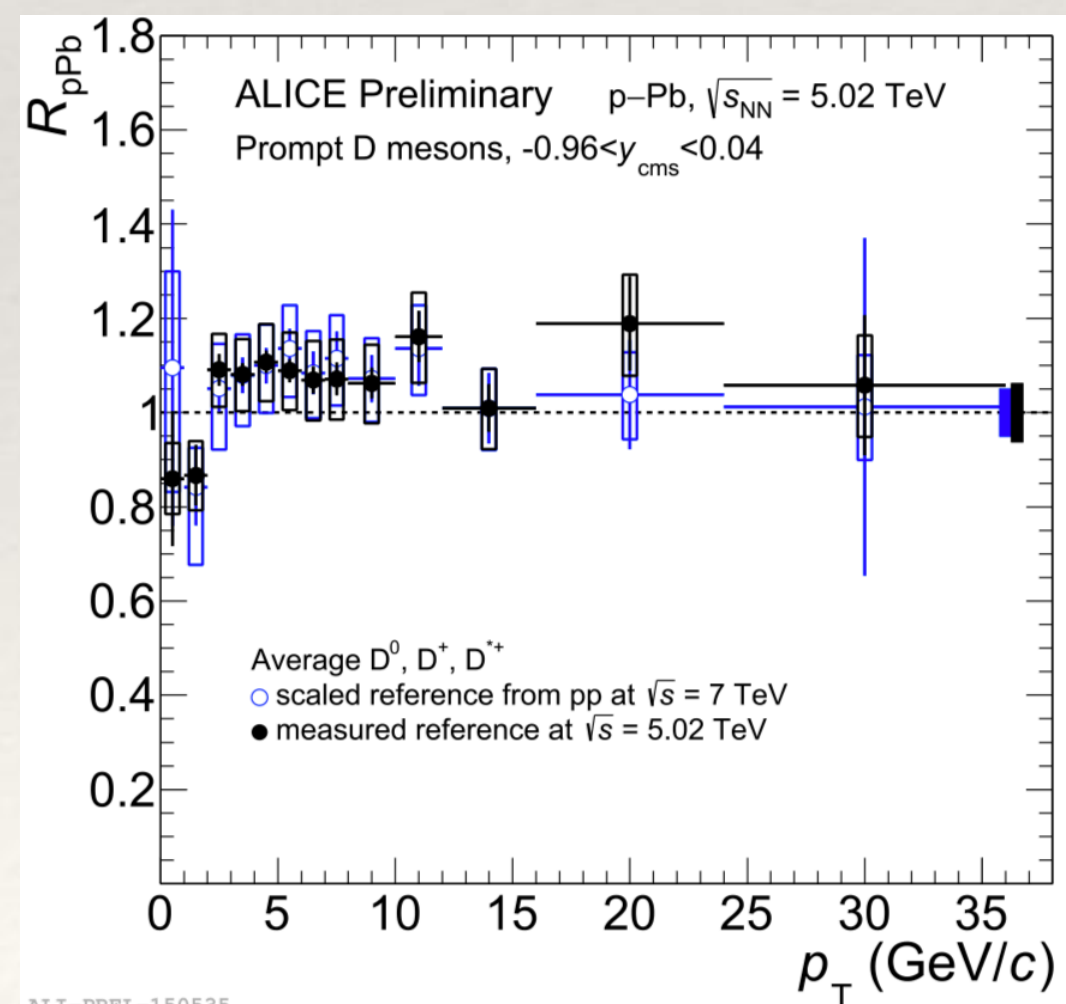
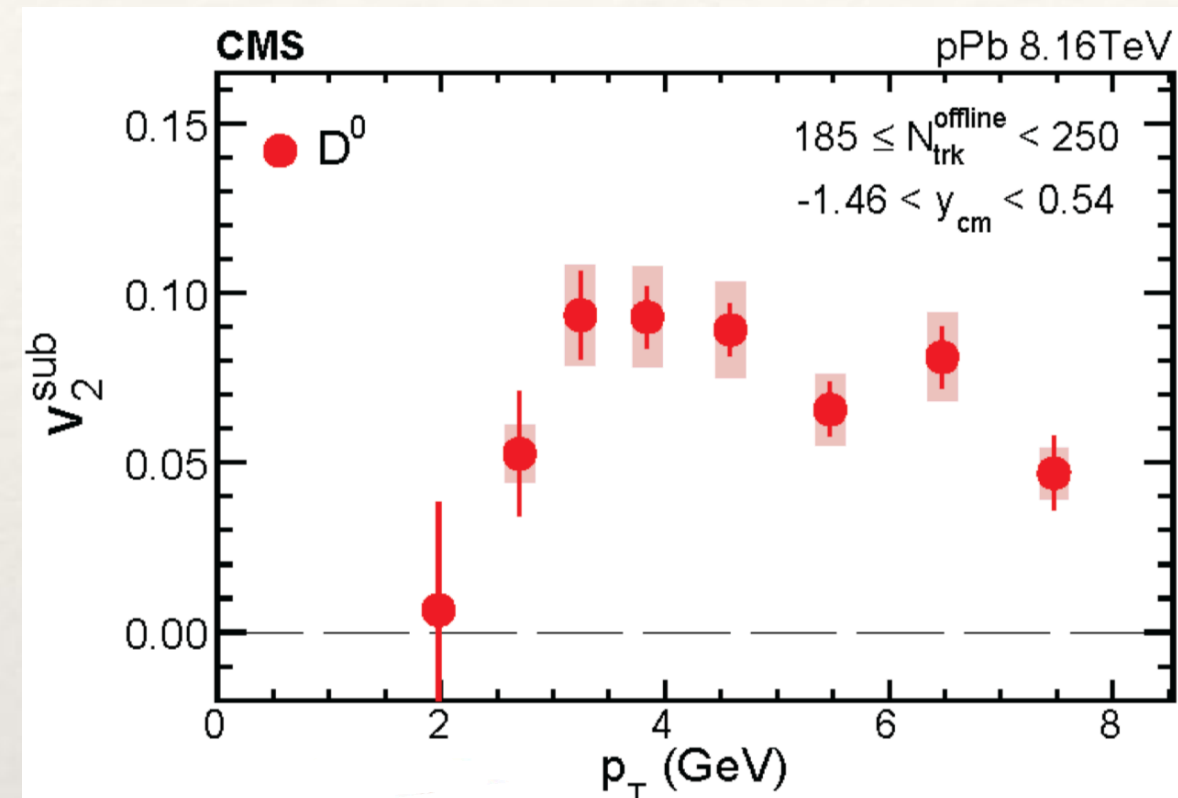
[Jiang, SC, Xing, Wu, Yang, Zhang, Phys. Rev. C 105 (2022) 5, 054907]



- Tilted geometry w.r.t. the beam direction dominates at the RHIC energy
- Strong E&M field dominates at the LHC energy
- Sensitivity of the D meson v_1 to different E&M evolution profiles at the LHC

Probing system size dependence of jet quenching

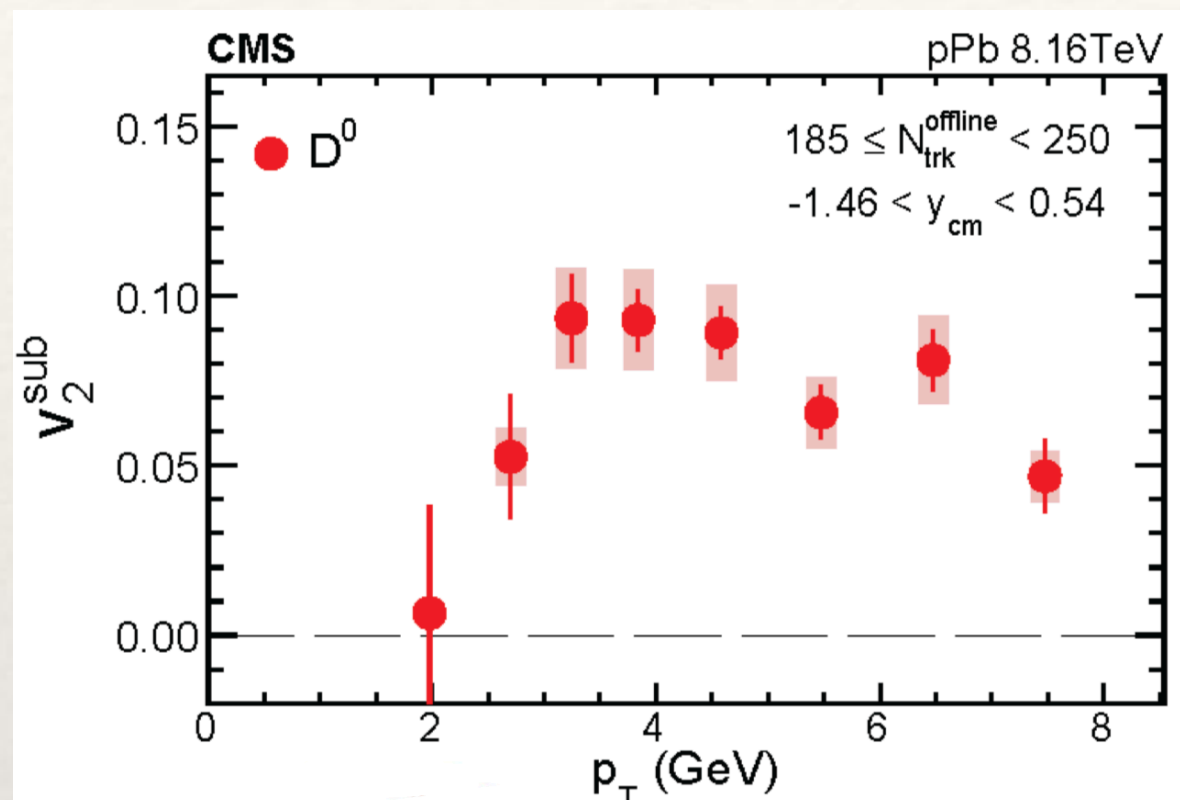
Small system (p-Pb) puzzle



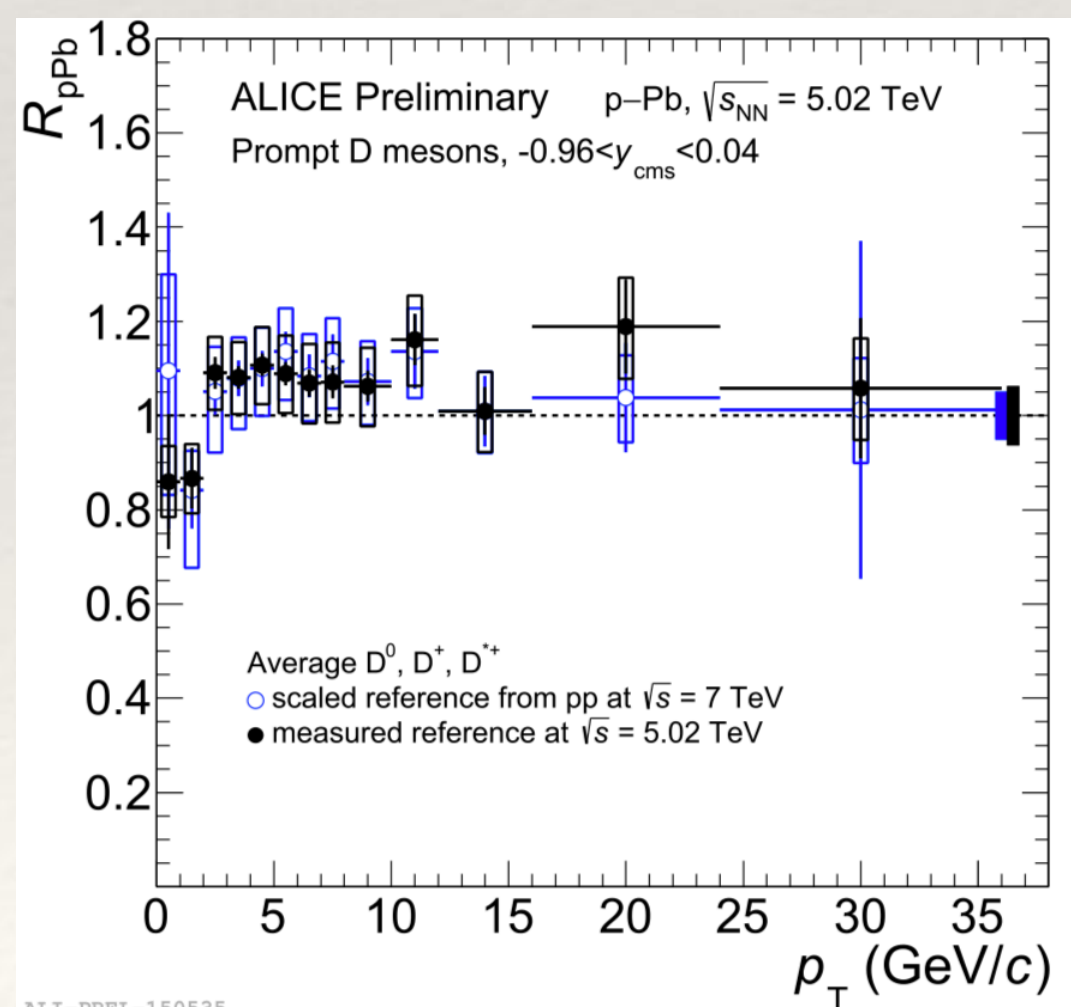
- Large D meson v_2 up to 8 GeV
- Almost no suppression
- Should not be QGP effects
- Could it be initial state effects?

Probing system size dependence of jet quenching

Small system (p-Pb) puzzle

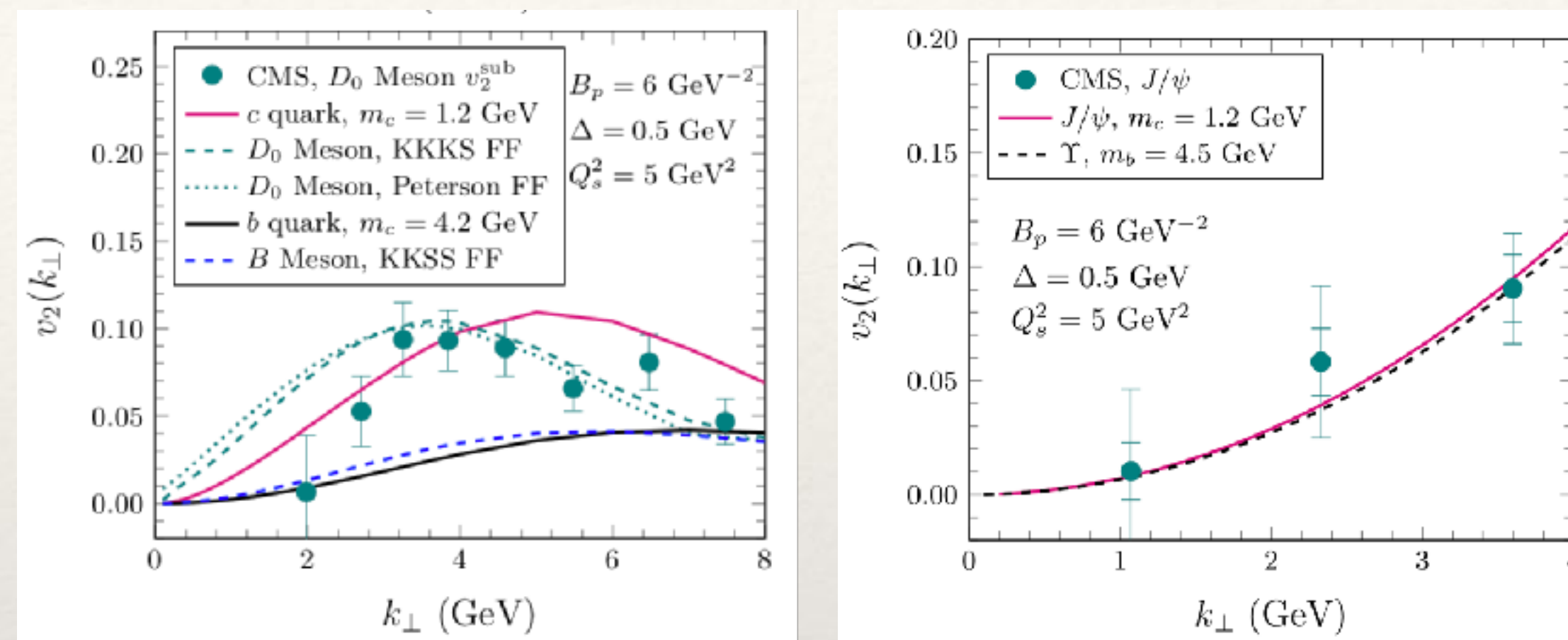


- Large D meson v_2 up to 8 GeV
- Almost no suppression



- Should not be QGP effects
- Could it be initial state effects?

Initial state effects

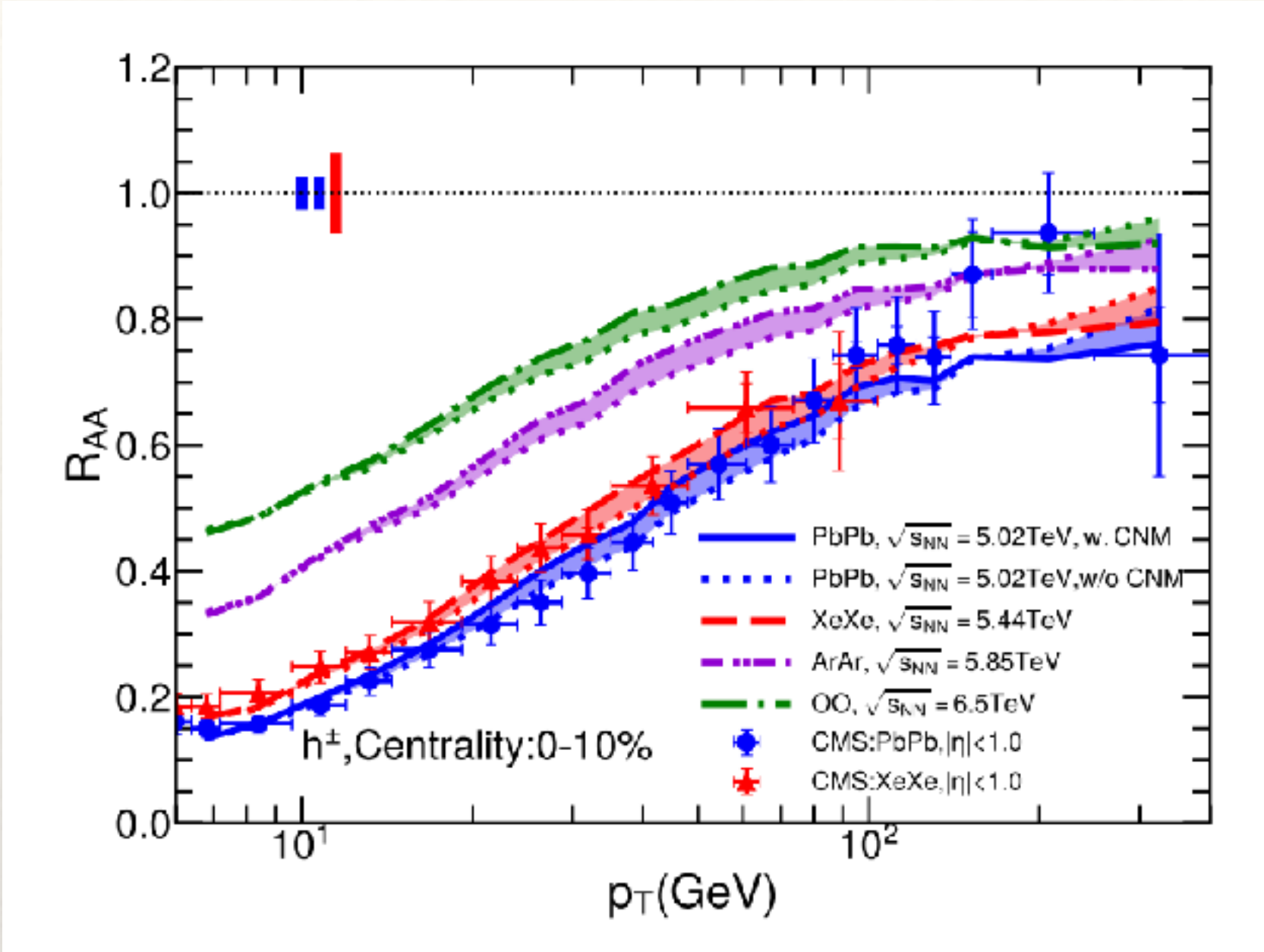


[Zhang, Marquet, Qin, Wei and Xiao, PRL 122 (2019)]

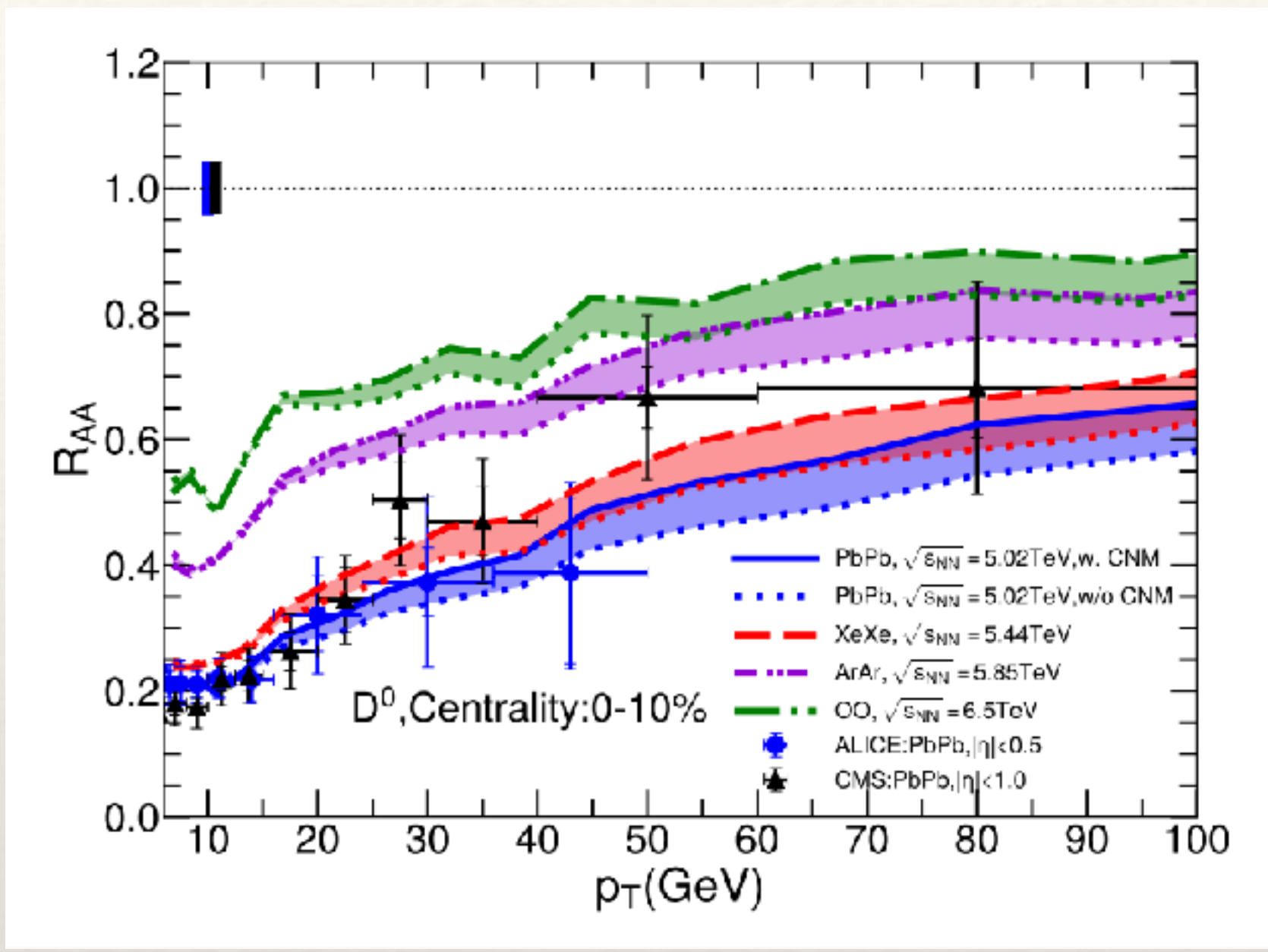
- Initial state interactions (CGC) successfully explain the large v_2 of both open charmed meson and charmonium in p-Pb collisions.
- How to separate initial state and QGP effect — a system size scan of jet quenching to bridge large and small systems

Charged hadron and D meson R_{AA} in different systems

charged hadron



D meson

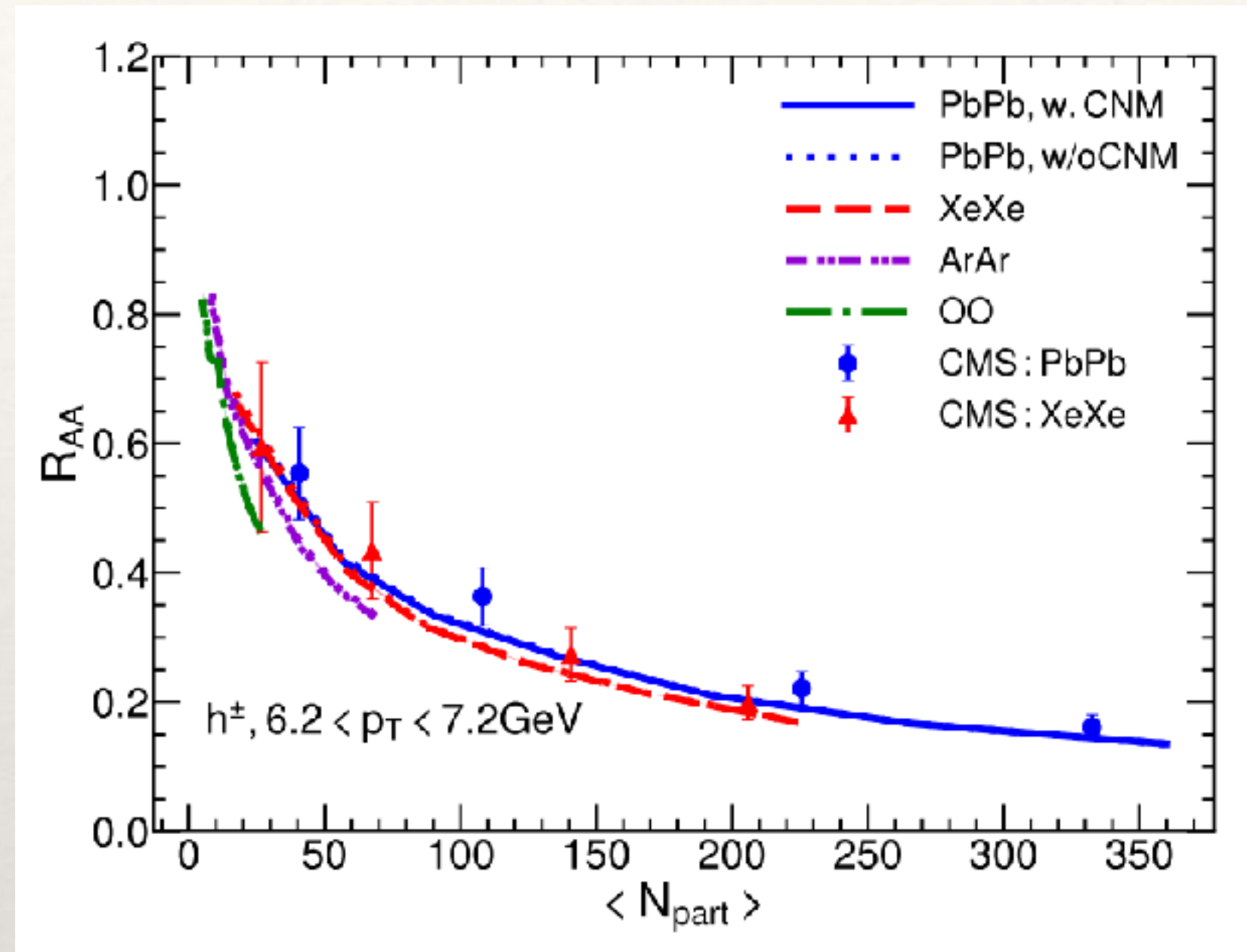


Liu, Xing, Wu, Qin, SC, Xing, Phys. Rev. C 105 (2022) 044904

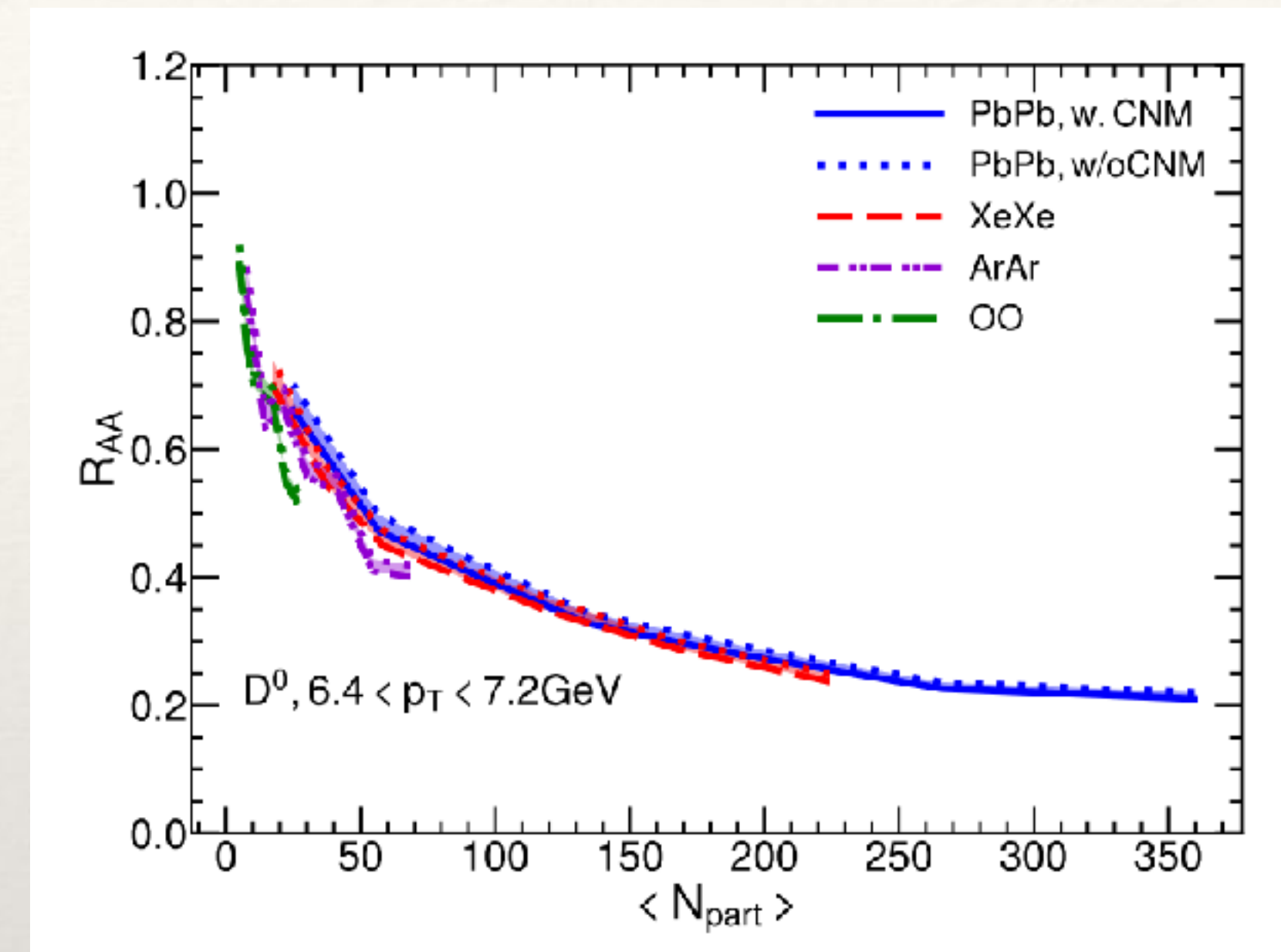
- Clear hierarchy of hadron R_{AA} with respect to the system size
- Significant hadron R_{AA} in the small O-O system, existence of QGP

Scaling of R_{AA} with respect to N_{part}

charged hadron



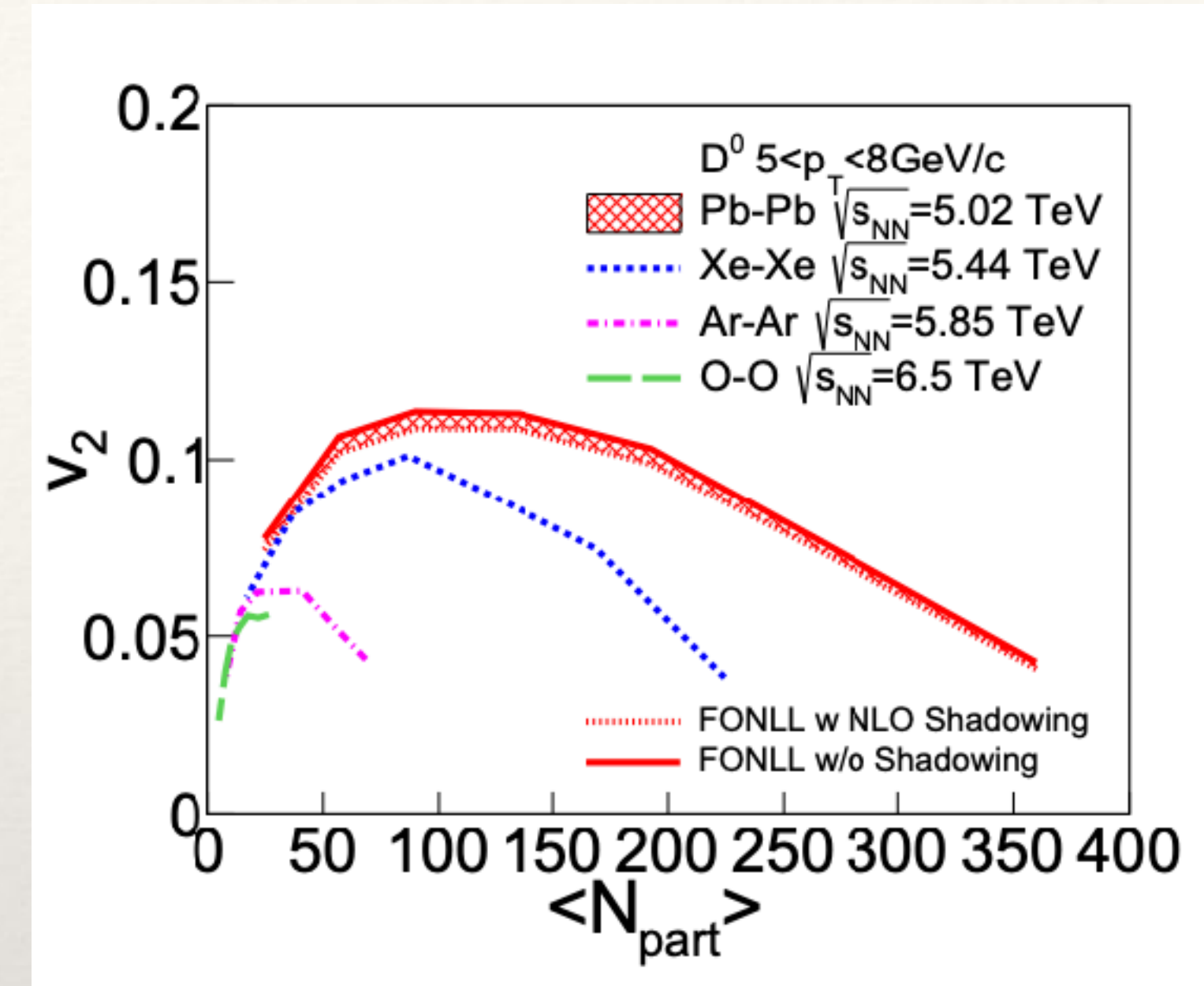
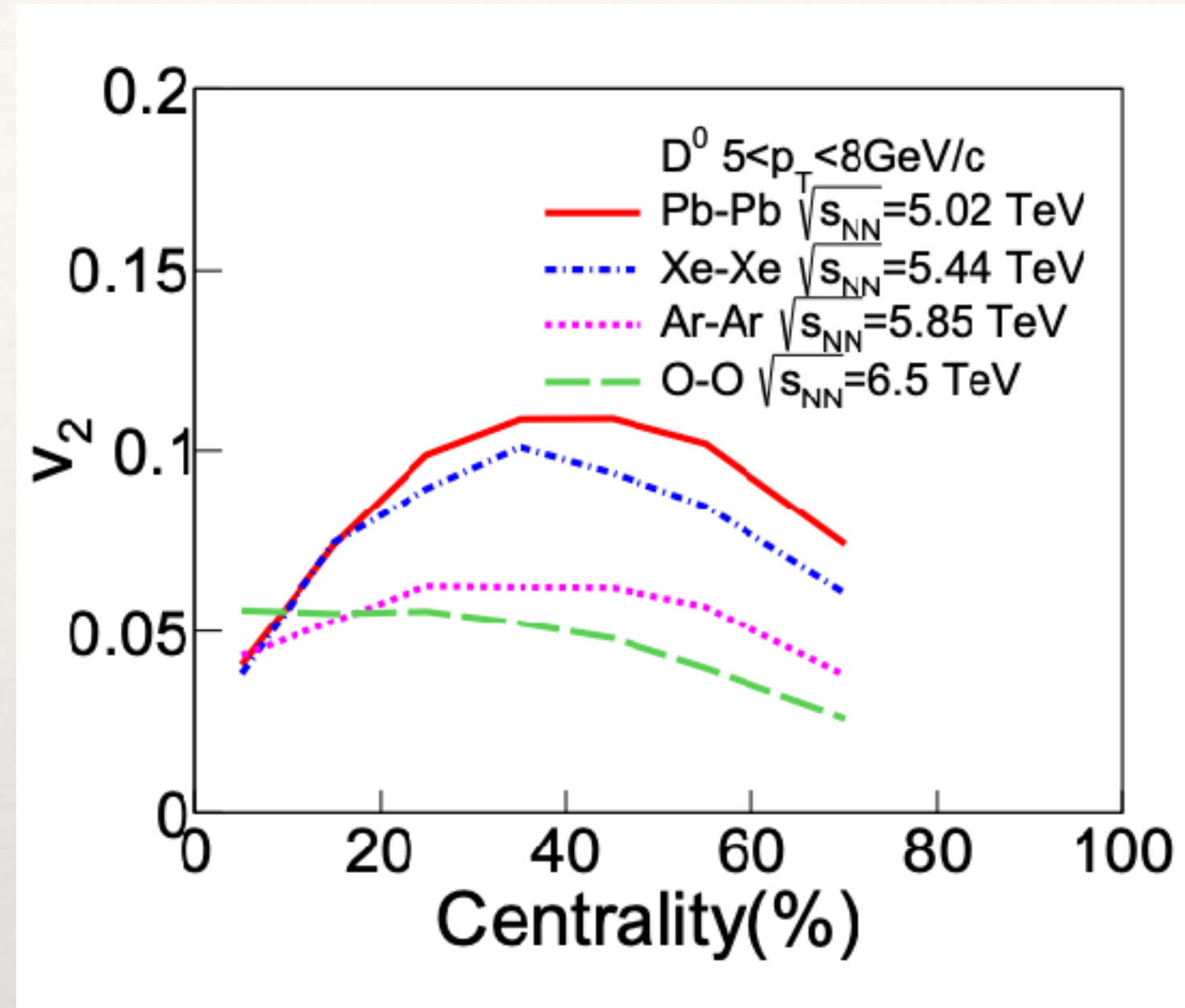
D meson



Liu, Xing, Wu, Qin, SC, Xing, Phys. Rev. C 105 (2022) 044904

- Scaling of the hadron R_{AA} with the system size (quantified by N_{part}) across different collision systems
- $R_{pA} \sim 1$ in proton-nucleus collisions is mainly due to the small size of the medium

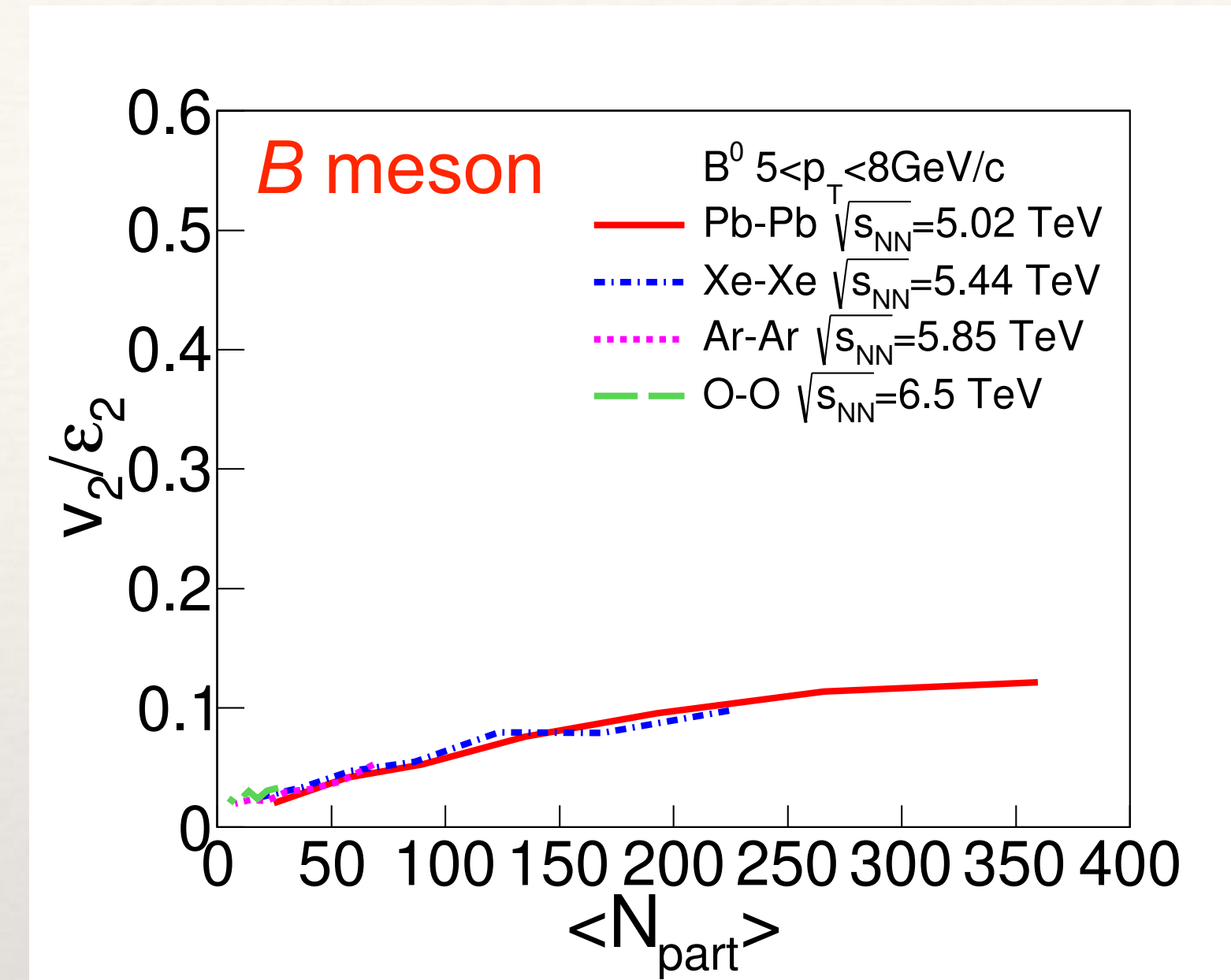
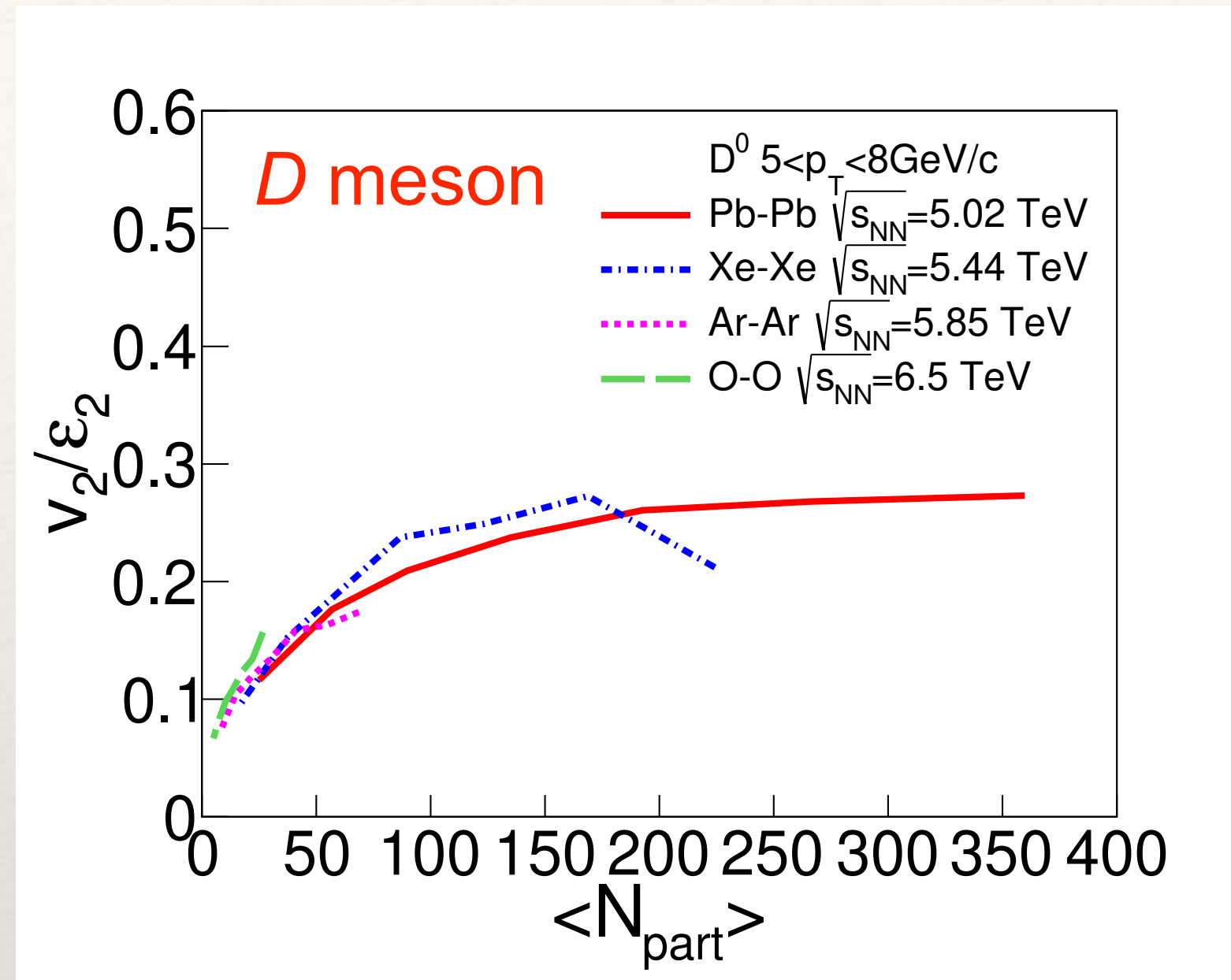
D meson v_2 in different systems



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- Energy loss effect: for a given centrality, v_2 increases with the system size
- Geometry effect: for a given N_{part} , v_2 increases from O-O, Ar-Ar, Xe-Xe to Pb-Pb

Scaling of v_2/ε_2 with respect to N_{part}



Li, Xing, Wu, SC, Qin, EPJC 81 (2021) 11, 1035

- Separate energy loss and geometry effects by rescaling heavy quark v_2 with bulk ε_2
- v_2/ε_2 scales with the system size across different collision systems
- Search for the breaking of the scaling with future experiments — initial state effect overwhelms QGP effect

Summary

Heavy-quark-QGP interaction at different p_T and in different collision systems

- pQCD provides a good description of flavor hierarchy of jet quenching at high p_T
- Color potential interaction improves model calculation at low to medium p_T
- Coalescence + fragmentation hadronization is crucial for understanding the hadron chemistry at low to medium p_T
- Heavy flavor observables can be used to constrain the EoS of QGP
- Heavy quark v_1 probes medium deformation at RHIC, while $E\&M$ field at LHC
- System size scan of HQ observables bridges large and small collision systems