西安交通大学
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Mass spectra of meson nonet (π, K, η, η') and the related QCD phase transitions under external magnetic field

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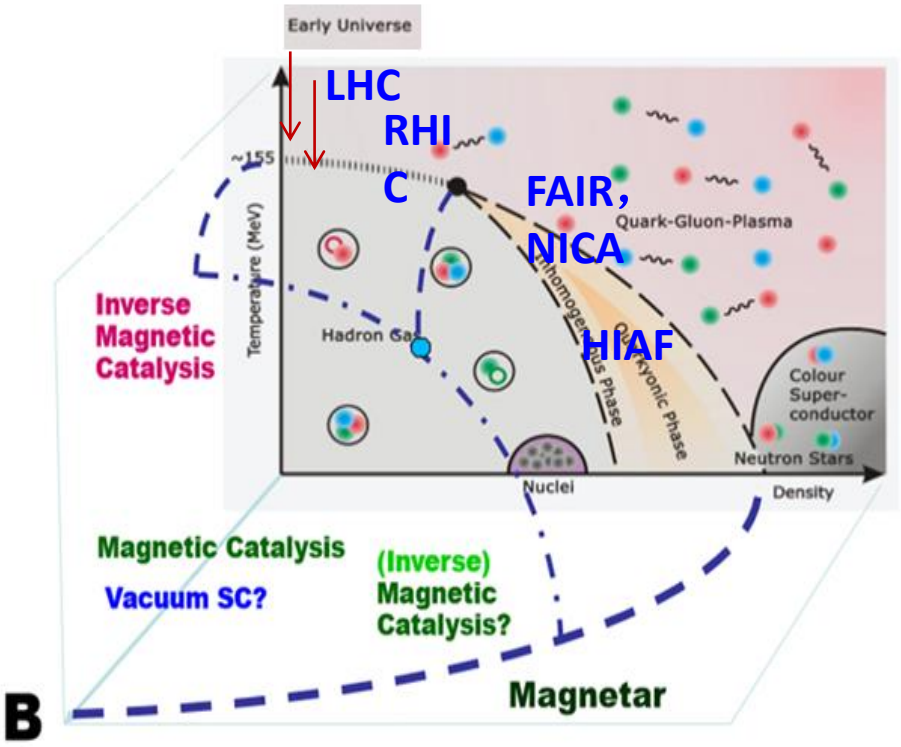
Outlines

1 QCD phase structure



- chiral restoration
  (chiral symmetry)
- deconfinement
  (center symmetry)
- QCD superconductor /superfluid
  (color/isospin symmetry)
- $U_A(1)$ restoration
  ……

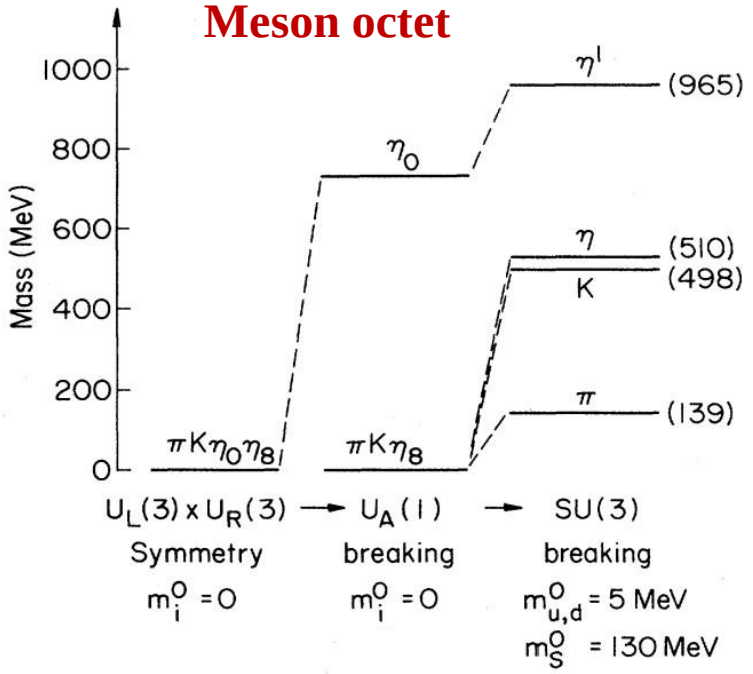
strong coupling & strong correlation

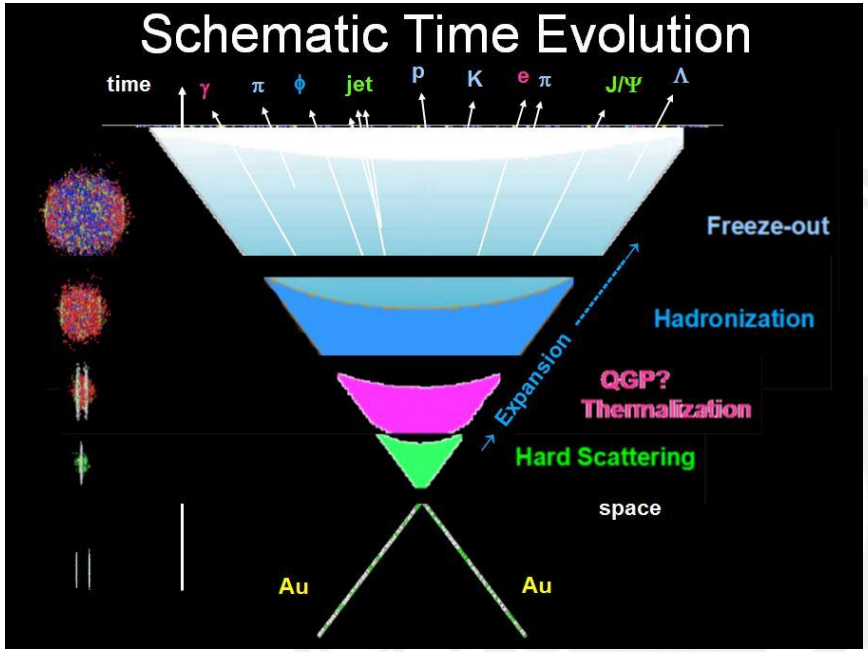
methods：LQCD;
effective models:(P)NJL,DS,(P)QM…

1 QCD phase structure

Study QCD phase transitions by mesons

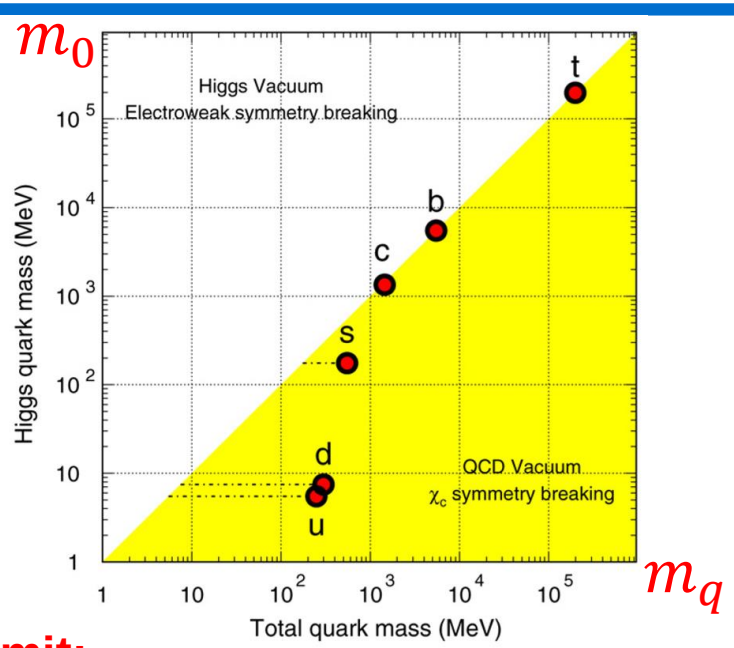
- Spontaneous breaking of symmetries: order parameters
- Goldstone modes: collective modes





1 Chiral symmetry

Light quark (hadron) mass is controlled by chiral symmetry.



- Order parameter: quark mass
- Goldstone modes;

X.Zhu, M.Bleicher, S.Huang, K.Schweda, H.Stoecker, N.Xu, P.Zhuang, PLB647, 366(2007)

● **chiral limit:**

$m_0 = 0 \rightarrow m_\pi = 0$(Goldstone boson)

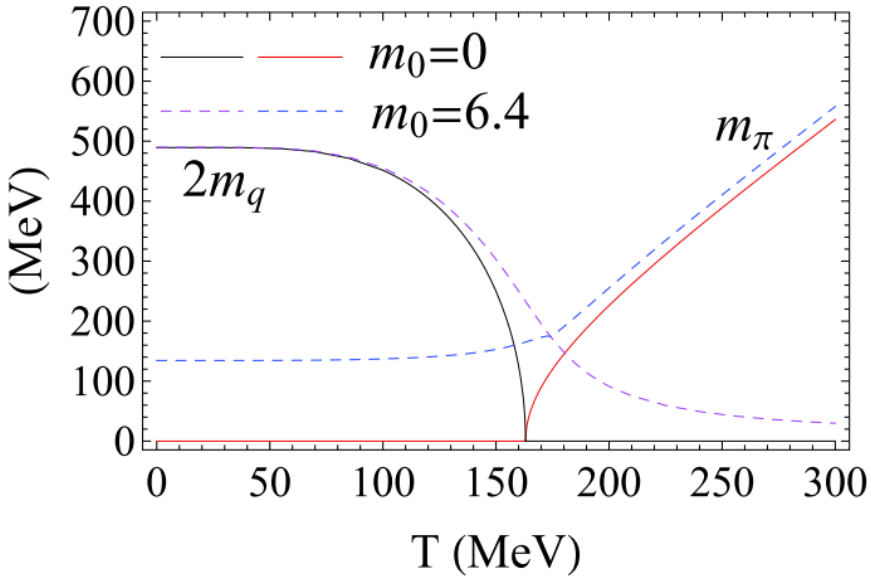
chiral phase transition

● **physical case**:

$m_0 \neq 0 \rightarrow m_\pi \neq 0(= 134\text{MeV})$

(pseudo-Goldstone boson)

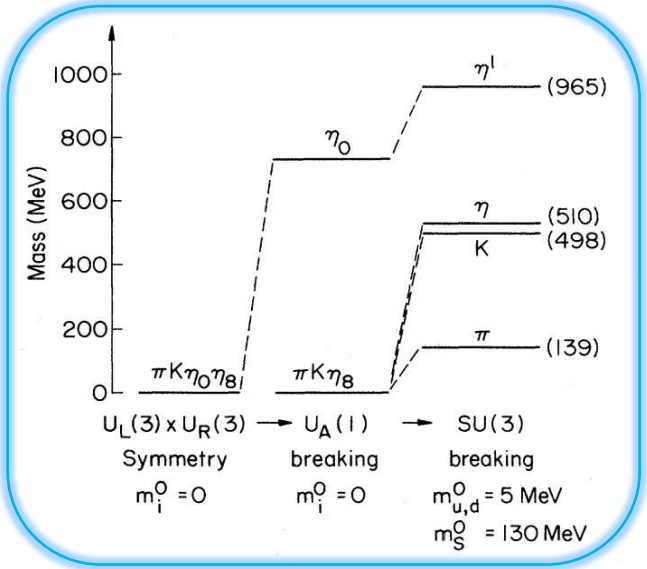
chiral crossover



1 Isospin and $U_A(1)$ symmetries

- Landau's theory
- Goldstone's theorem

Symmetry breaking	QCD phase transitions	Goldstone bosons
chiral condensates	chiral restoration	π^0, K^0
pion/kaon condensate	Pion/Kaon superfluid	$\pi^{\pm}, K^{\pm}$
topological susceptibility	$U_A(1)$ restoration	η, η'



methods： LQCD;
effective models

2 Framework: NJL model @ eB

Mesons and QCD phase transitions @eB

Quark level: LQCD, (P)NJL, DS equation

Meson level: (P)QM, chiral perturbation theory

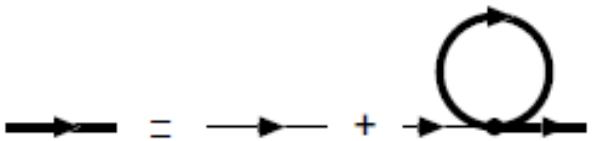
2 Nambu--Jona-Lasinio model

SU(2) NJL model

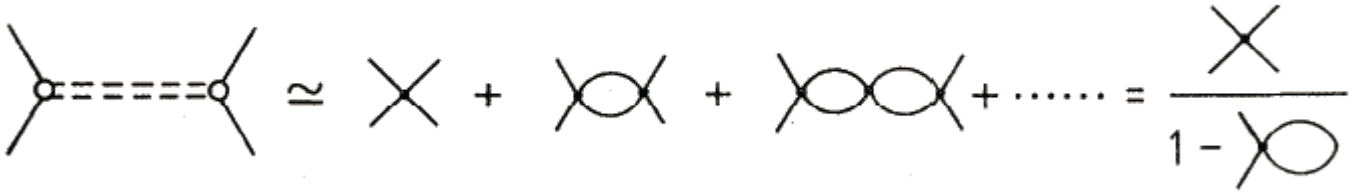
$$\mathcal{L}=\bar{\psi}\left(i\gamma_\nu D^\nu-m_0\right)\psi+\frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2+\left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \qquad \mathbf{B}=(0,0,B)$$

idea:

(1) Quarks： mean field



(2) Mesons： RPA resummation (quantum fluctuation)



✓ **Construct mesons in conserved momentum space**

Y.X.Wang, **S.J. Mao**, PRD96, 034004(2017);99, 056005(2019)

2 SU(3) Nambu-Jona-Lasinio model

$$\mathbf{B} = (0, 0, B)$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{8}[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2], \quad \mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi].$$

$$\begin{aligned}\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi &+ \sum_{a=0}^{8}[K_a^-(\bar{\psi}\lambda^a\psi)^2 + K_a^+(\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ &+ K_{30}^-(\bar{\psi}\lambda^3\psi)(\bar{\psi}\lambda^0\psi) + K_{30}^+(\bar{\psi}i\gamma_5\lambda^3\psi)(\bar{\psi}i\gamma_5\lambda^0\psi) \\ &+ K_{03}^-(\bar{\psi}\lambda^0\psi)(\bar{\psi}\lambda^3\psi) + K_{03}^+(\bar{\psi}i\gamma_5\lambda^0\psi)(\bar{\psi}i\gamma_5\lambda^3\psi) \\ &+ K_{80}^-(\bar{\psi}\lambda^8\psi)(\bar{\psi}\lambda^0\psi) + K_{80}^+(\bar{\psi}i\gamma_5\lambda^8\psi)(\bar{\psi}i\gamma_5\lambda^0\psi) \\ &+ K_{08}^-(\bar{\psi}\lambda^0\psi)(\bar{\psi}\lambda^8\psi) + K_{08}^+(\bar{\psi}i\gamma_5\lambda^0\psi)(\bar{\psi}i\gamma_5\lambda^8\psi) \\ &+ K_{83}^-(\bar{\psi}\lambda^8\psi)(\bar{\psi}\lambda^3\psi) + K_{83}^+(\bar{\psi}i\gamma_5\lambda^8\psi)(\bar{\psi}i\gamma_5\lambda^3\psi) \\ &+ K_{38}^-(\bar{\psi}\lambda^3\psi)(\bar{\psi}\lambda^8\psi) + K_{38}^+(\bar{\psi}i\gamma_5\lambda^3\psi)(\bar{\psi}i\gamma_5\lambda^8\psi)\end{aligned}$$

$$K_0^\pm = G \pm \frac{1}{3}K(\sigma_u + \sigma_d + \sigma_s),$$

$$K_1^\pm = K_2^\pm = K_3^\pm = G \mp \frac{1}{2}K\sigma_s,$$

$$K_4^\pm = K_5^\pm = G \mp \frac{1}{2}K\sigma_d,$$

$$K_6^\pm = K_7^\pm = G \mp \frac{1}{2}K\sigma_u,$$

$$K_8^\pm = G \mp \frac{1}{6}K(2\sigma_u + 2\sigma_d - \sigma_s),$$

$$K_{03}^\pm = K_{30}^\pm = \pm\frac{1}{2\sqrt{6}}K(\sigma_u - \sigma_d),$$

$$K_{08}^\pm = K_{80}^\pm = \mp\frac{\sqrt{2}}{12}K(\sigma_u + \sigma_d - 2\sigma_s),$$

$$K_{38}^\pm = K_{83}^\pm = \mp\frac{1}{2\sqrt{3}}K(\sigma_u - \sigma_d),$$

condensates $\sigma_u = \langle\bar{u}u\rangle, \quad \sigma_d = \langle\bar{d}d\rangle, \quad \sigma_s = \langle\bar{s}s\rangle.$

2 SU(3) Nambu-Jona-Lasinio model

$$\mathbf{B} = (0, 0, B)$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{8}[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2], \quad \mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi].$$

quark condensates: $\sigma_u = \langle\bar{u}u\rangle, \quad \sigma_d = \langle\bar{d}d\rangle, \quad \sigma_s = \langle\bar{s}s\rangle.$

gap eqs: $\partial\Omega_{\rm mf}/\partial\sigma_i = 0, \quad i = u, d, s,$

minimize $\Omega_{\rm mf}$

$$\Omega_{\rm mf} = 2G(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \Omega_q,$$

$$\Omega_q = -3\sum_{f=u,d,s}\frac{|Q_fB|}{2\pi}\sum_l\alpha_l\int\frac{dp_z}{2\pi}\left[E_f + T\ln\left(1+e^{-\frac{E_f+\mu}{T}}\right) + T\ln\left(+e^{-\frac{E_f-\mu}{T}}\right)\right],$$

$$E_f = \sqrt{p_z^2 + 2l|Q_fB| + m_f^2}$$

$$m_u = m_0^u - 4G\sigma_u + 2K\sigma_d\sigma_s,$$

$$m_d = m_0^d - 4G\sigma_d + 2K\sigma_u\sigma_s,$$

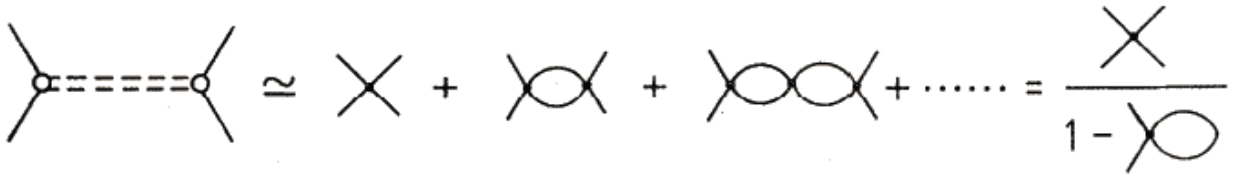
$$m_s = m_0^s - 4G\sigma_s + 2K\sigma_u\sigma_d$$

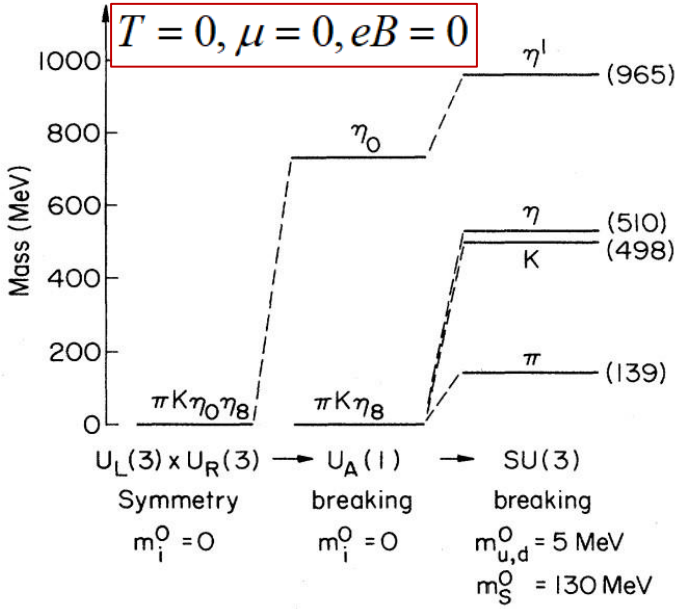
2 SU(3) Nambu-Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - \hat{m}_0)\psi + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{8}[(\bar{\psi}\lambda_\alpha\psi)^2 + (\bar{\psi}i\gamma_5\lambda_\alpha\psi)^2],\quad \mathcal{L}_{KMT} = -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi].$$

Mesons:





eB, T, μ

effect ???

J.Mei, T.Xia, **S.J.Mao**, PRD107, 074018(2023);
Y.M.Tian, **S.J.Mao,** arXiv:2023.XXXXX

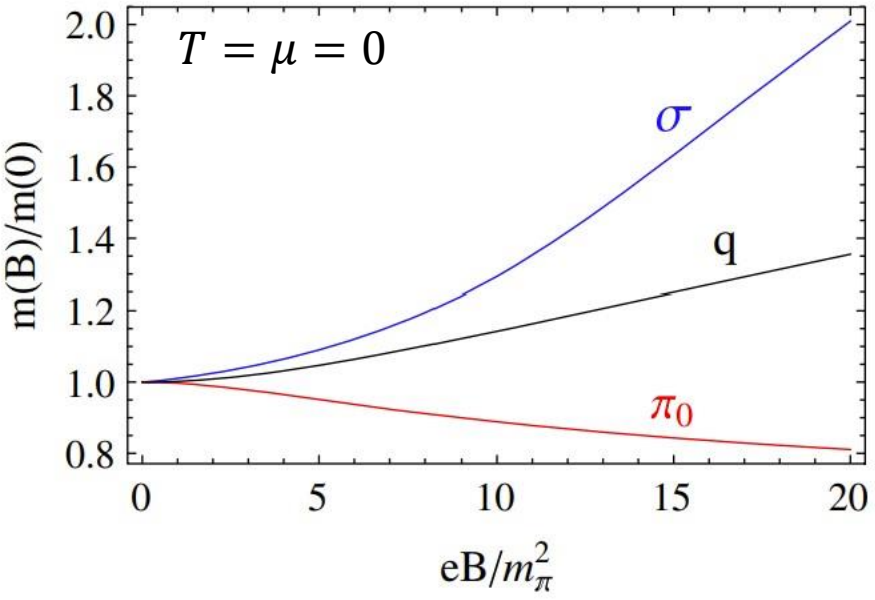
3 Results and discusssion

3.1 π_0 ---- chiral restoration phase transition

3.2 $\pi_\pm$ ---- pion superfluid phase transition

3.3 Kaons, η& η' ---- chiral, $U_A(1)$

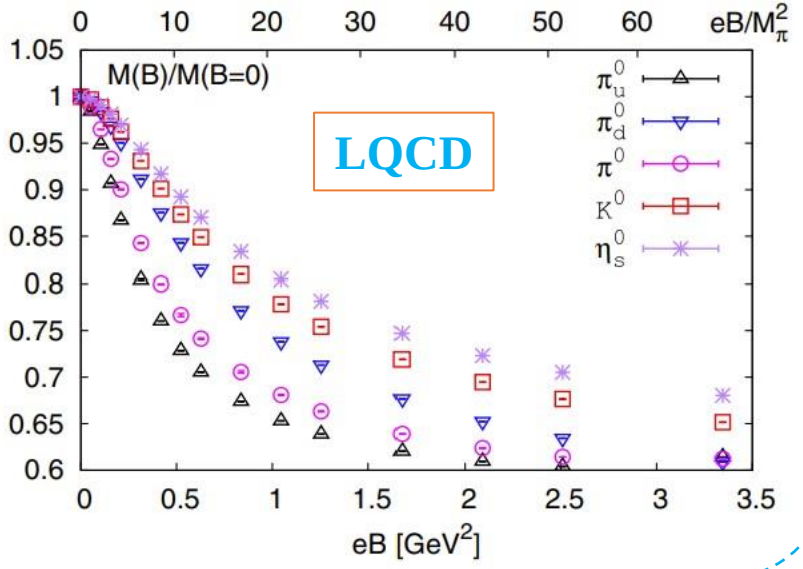
3.1 π_0 & chiral restoration phase transition



q: order parameter

π_0: Goldstone mode

σ： Higgs mode

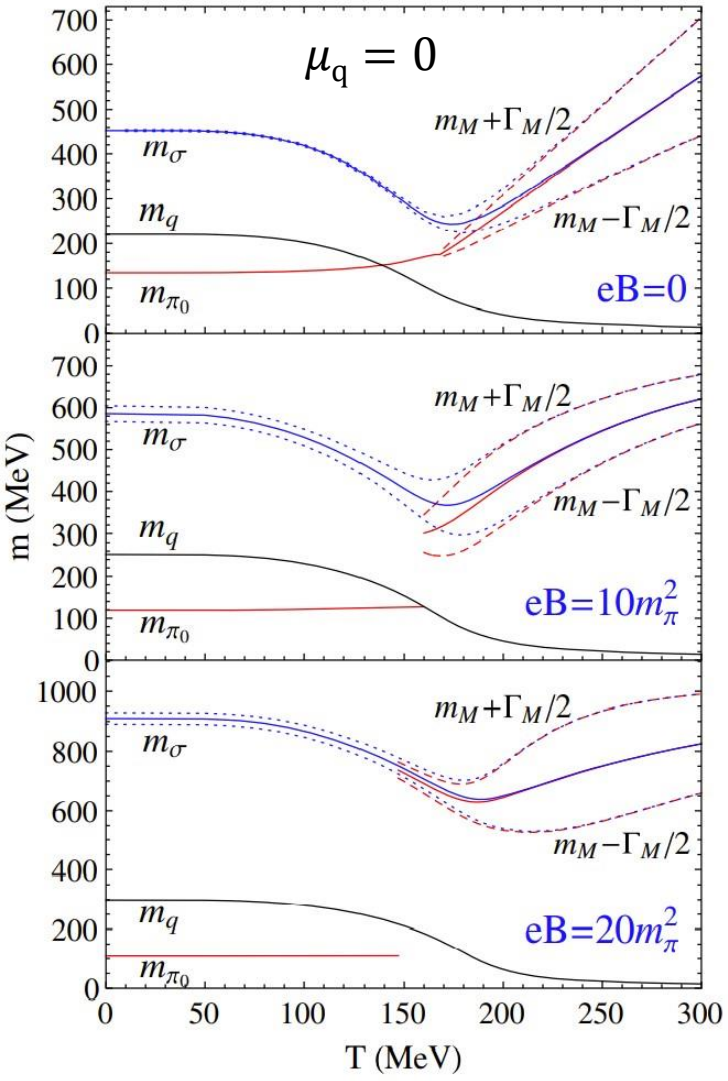


π_0 mass decreases with eB.

S.J. Mao, et al, PRD 96, 034004(2017)

H.T. Ding, et al, PRD 104, 014505(2021)

3.1 π_0 & chiral restoration phase transition

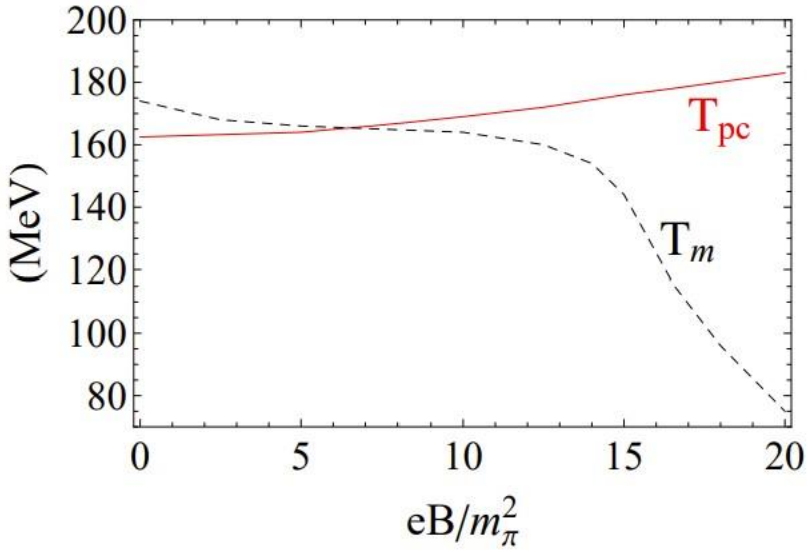


S.J. Mao, et al, PRD 96, 034004(2017)

π_0 mass jump at Mott transition

$$m_\pi = 2m_q \rightarrow T_m$$

Inverse magnetic catalysis

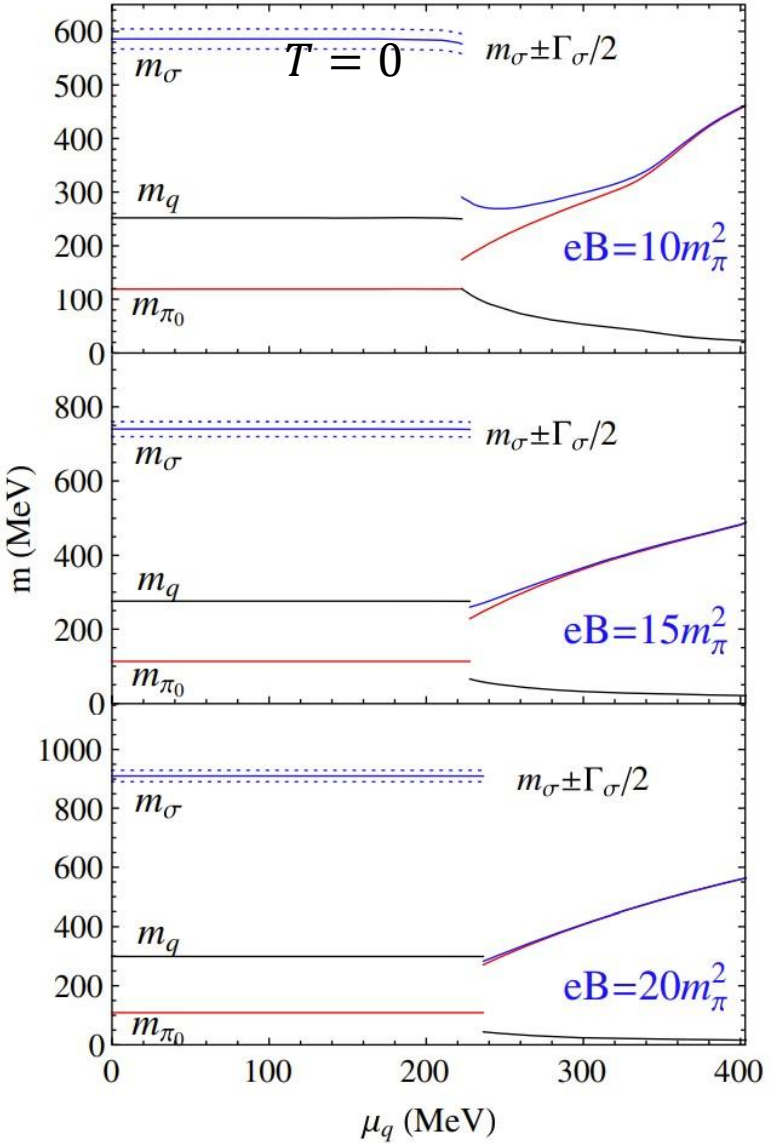


$$\frac{\partial^2 m_q}{\partial T^2} = 0 \rightarrow T_{pc}$$

magnetic catalysis

S.J. Mao, CPC45, 021004(R) (2021)

3.1 π_0 & chiral restoration phase transition



Mass jumps with 1st chiral restoration phase transition

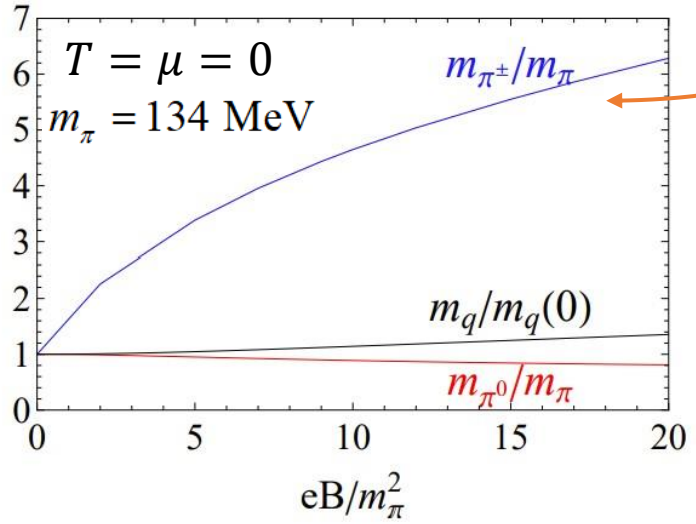
No Mott transition (Pauli blocking)

$$m_M > 2m_q \quad \text{and} \quad m_M > 2\mu_q = 2/3\mu_B.$$

S.J. Mao, et al, PRD 96, 034004(2017)

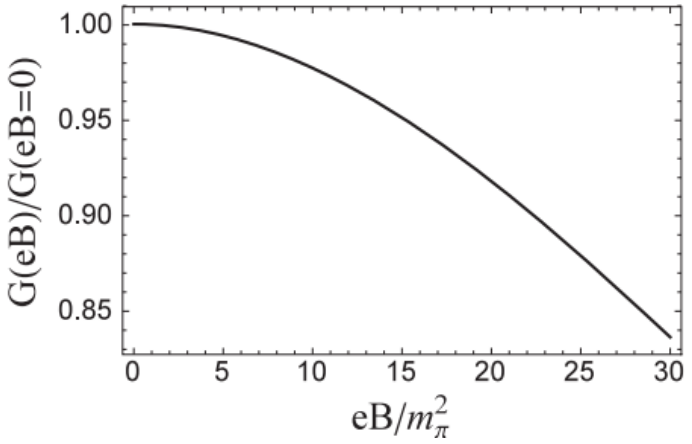
Charged pion @ eB

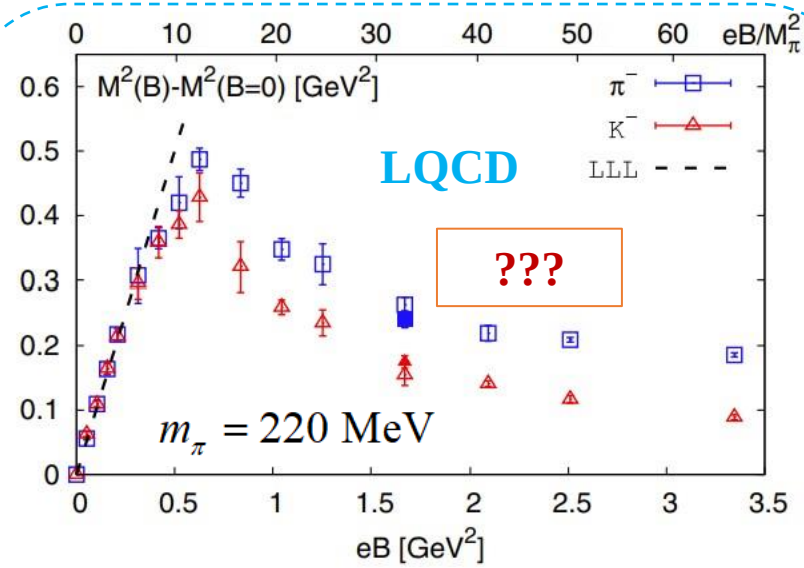
3.2 π_+ & pion superfluid phase transition



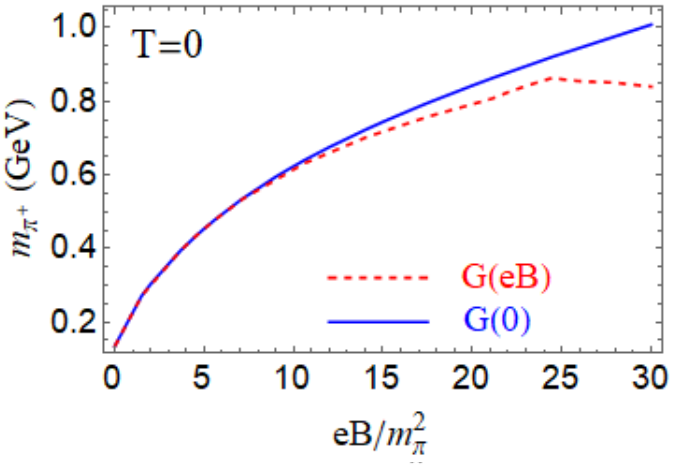
π_+ **mass increases** $(u\bar{d})$

S.J. Mao, PRD99, 056005 (2019)

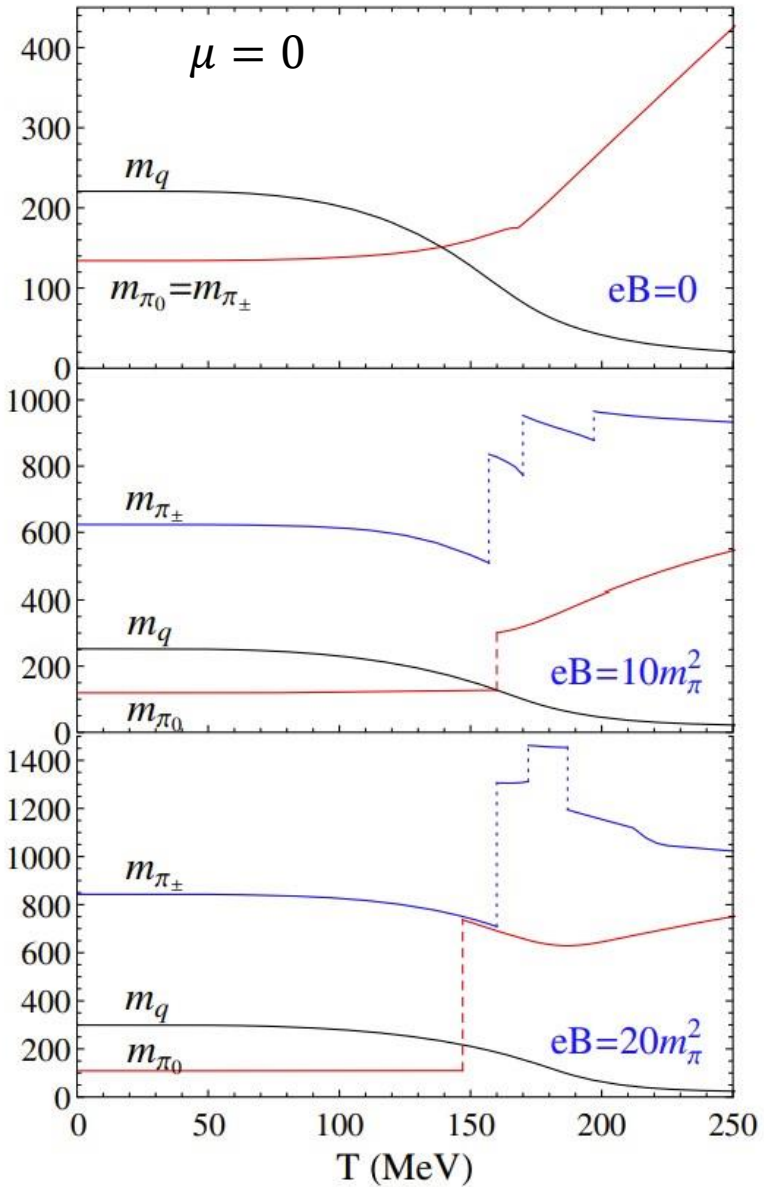


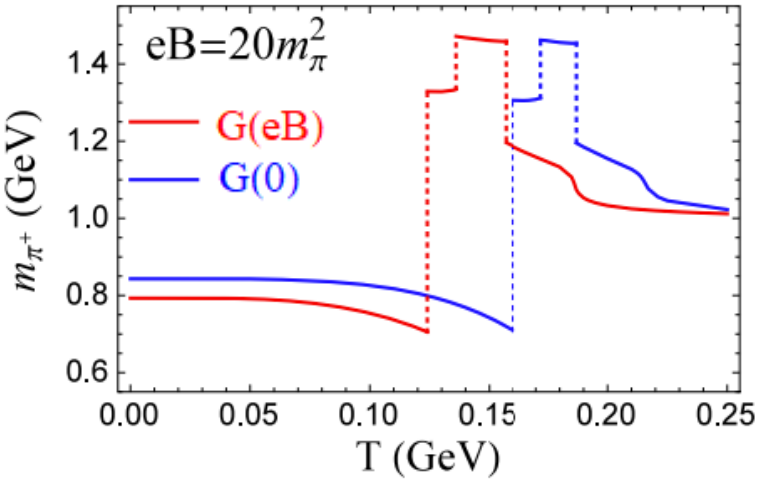


H.T. Ding, et al, PRD 104, 014505(2021)



3.2 π_+ & pion superfluid phase transition





Mass jumps @ Mott transitions

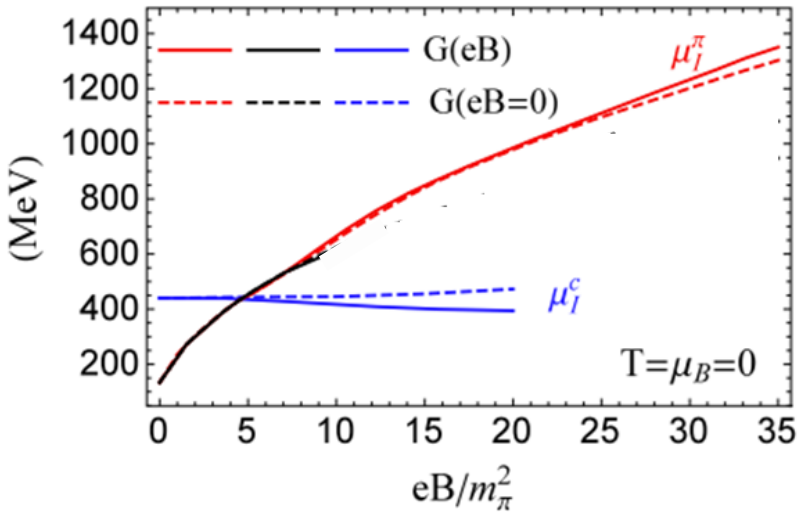
S.J. Mao, PRD99, 056005 (2019); PRD108. 054001(2023);arXiv:2023.XXXXX

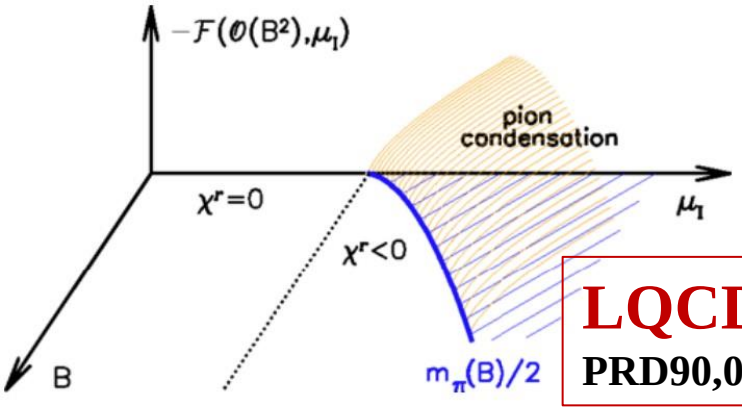
3.2 π_+ & pion superfluid phase transition

S.J. Mao, PRD102, 114006(2020);106.094017(2022).

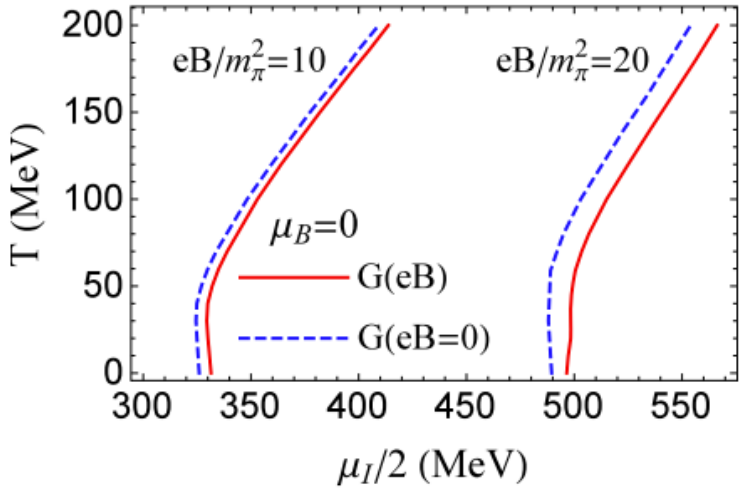
Goldstone's theorem: $M_{\pi^+} = 0 \Rightarrow$ **pion superfluid phase transition**

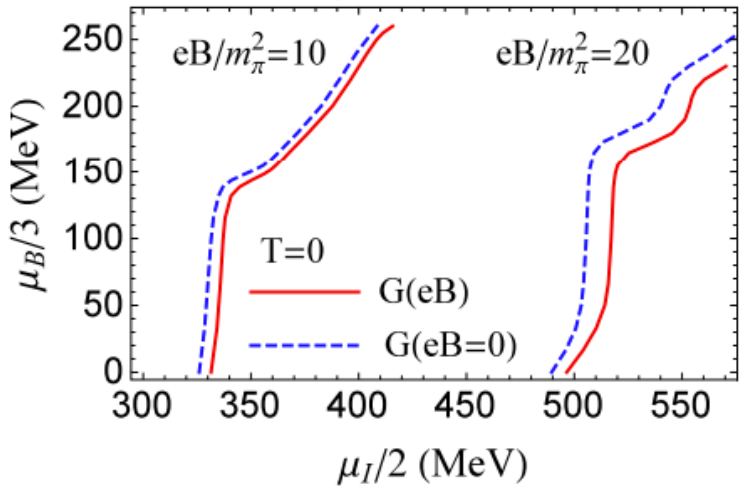
$T = 0$





LQCD
PRD90,094501(2014)





Kaon、η、 η’ @ eB

3.3 Kaon、η、η’@ eB

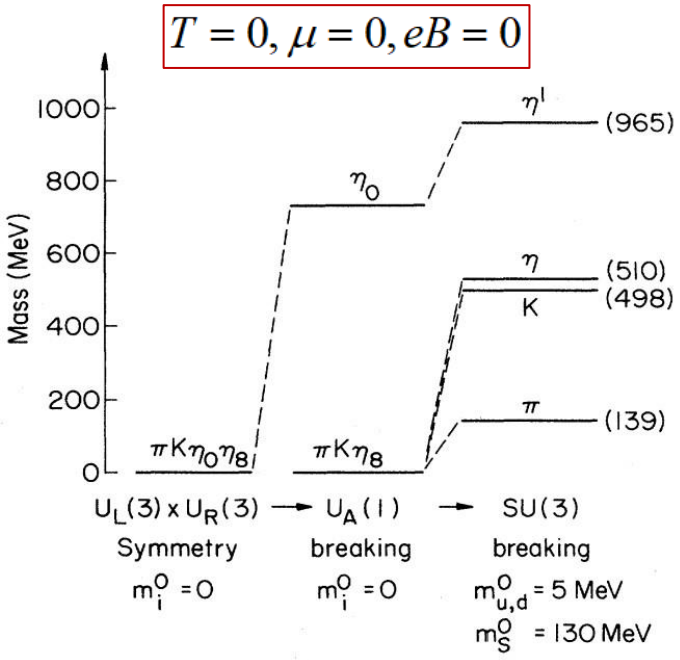
three-flavor NJL

$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^{\mu}D_{\mu} - \hat{m}_0\right)\psi(x) + \mathcal{L}_S + \mathcal{L}_{KMT},$$

$$\mathcal{L}_S = G\sum_{\alpha=0}^{N_f^2-1}\left[(\bar{\psi}\lambda_{\alpha}\psi)^2 + (\bar{\psi}i\gamma_5\lambda_{\alpha}\psi)^2\right],$$

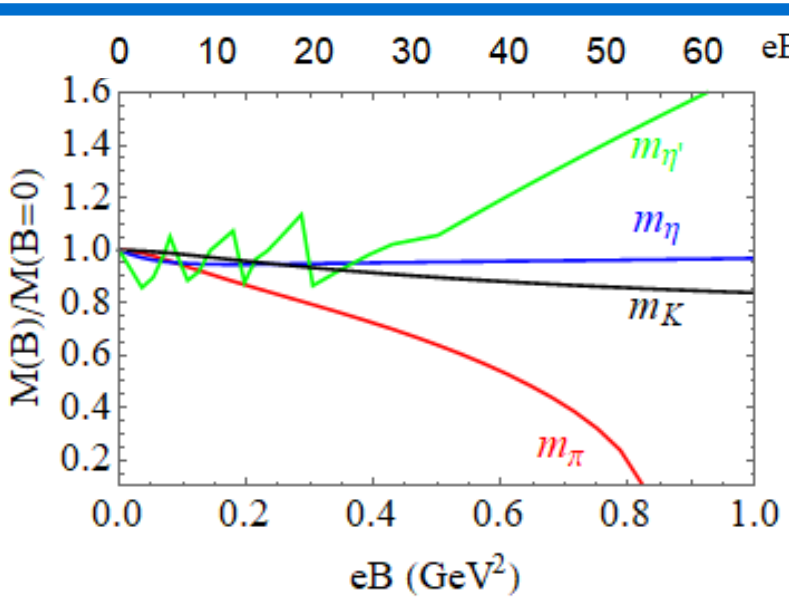
$$\mathcal{L}_{KMT} = -K\left[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi\right].$$

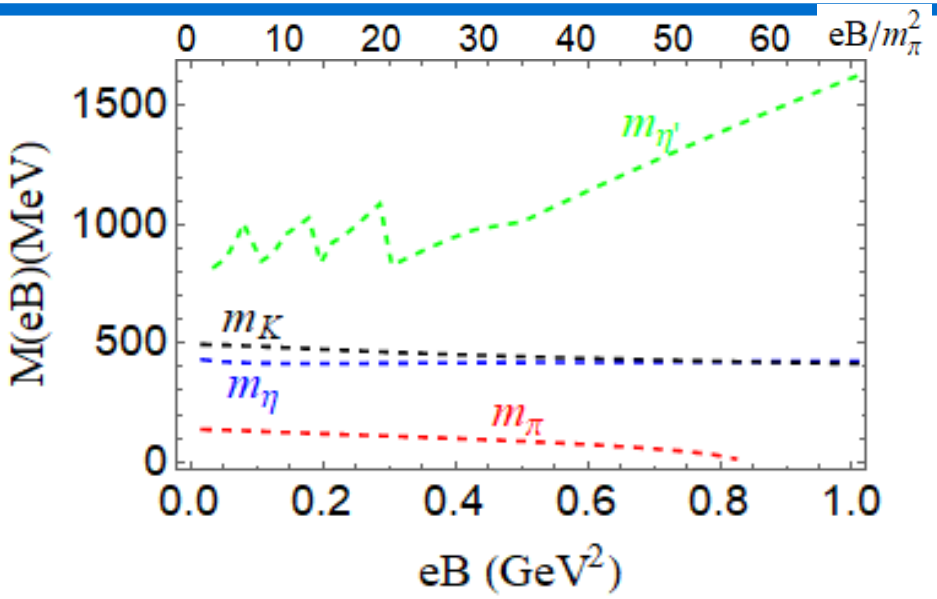
constant coupling strength G

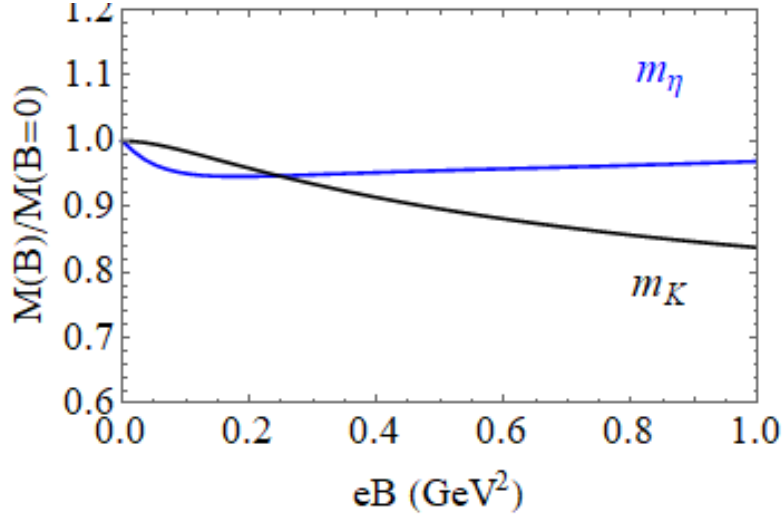


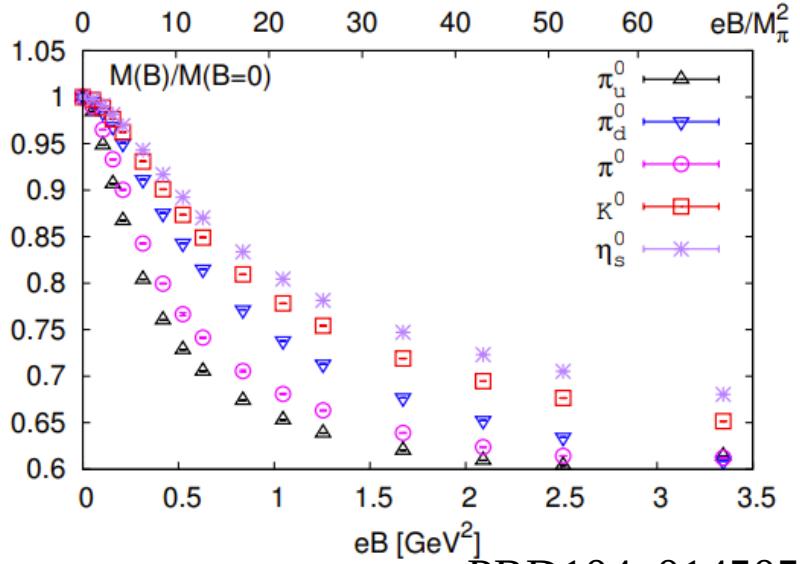
3.3.1 Neutral mesons (K_0, π_0, η, η')

3.3.1 Neutral meson mass spectra









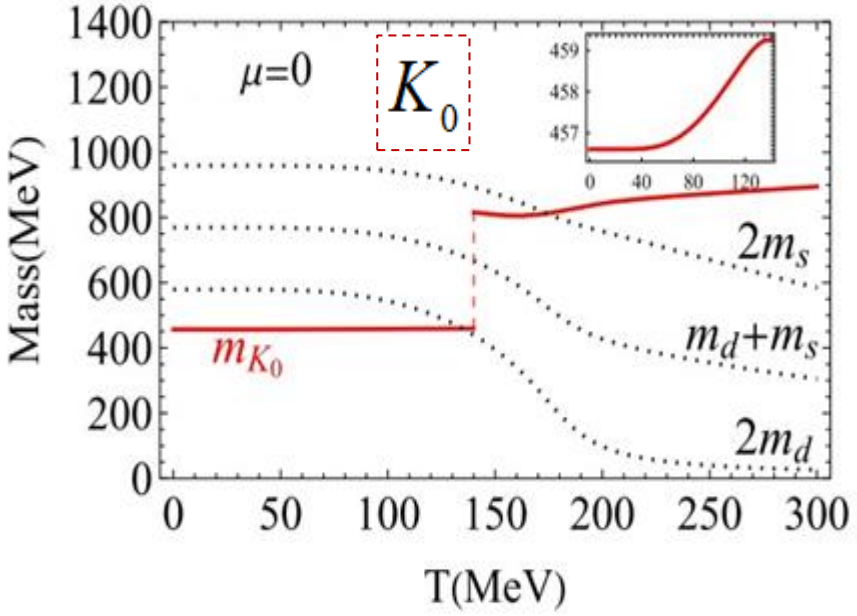
J.Mei, T.Xia, **S.J.Mao**, PRD107, 074018(2023)

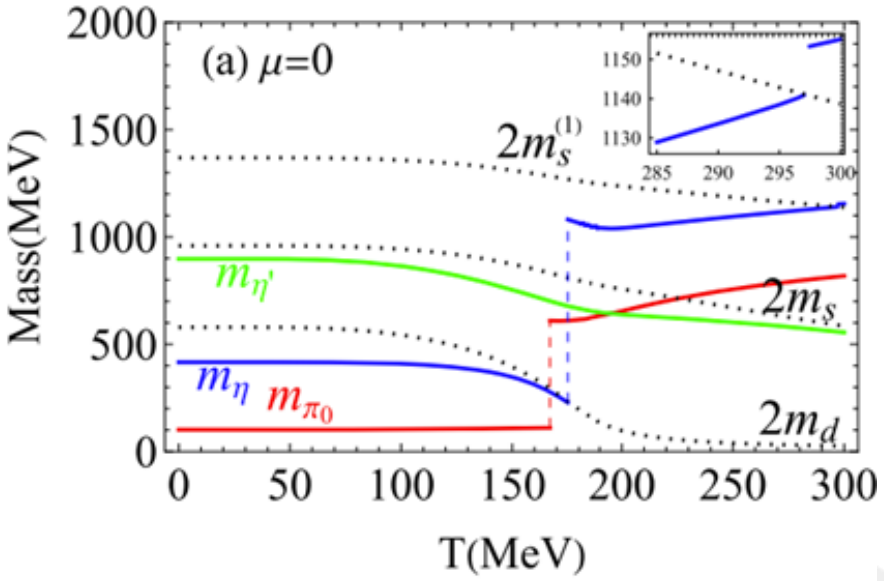
PRD104, 014505(2021

3.3.1

mass spectra @ T

$$eB = 20m_\pi^2$$





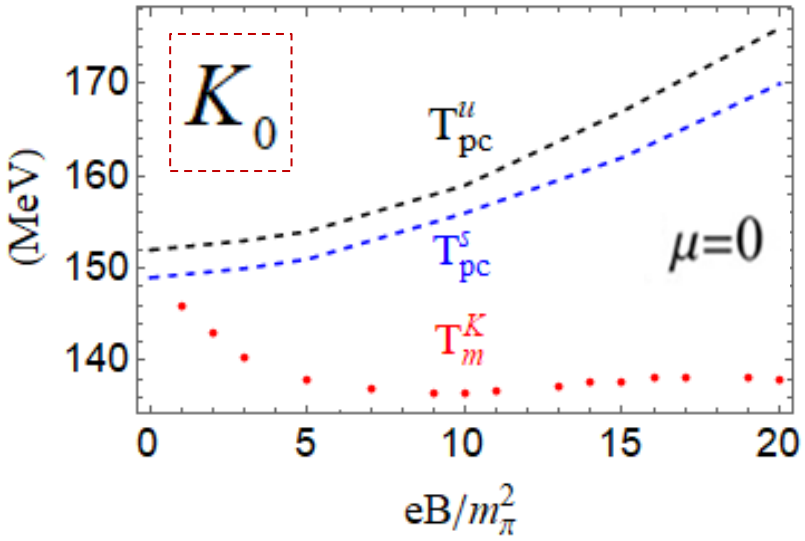
Mott transition

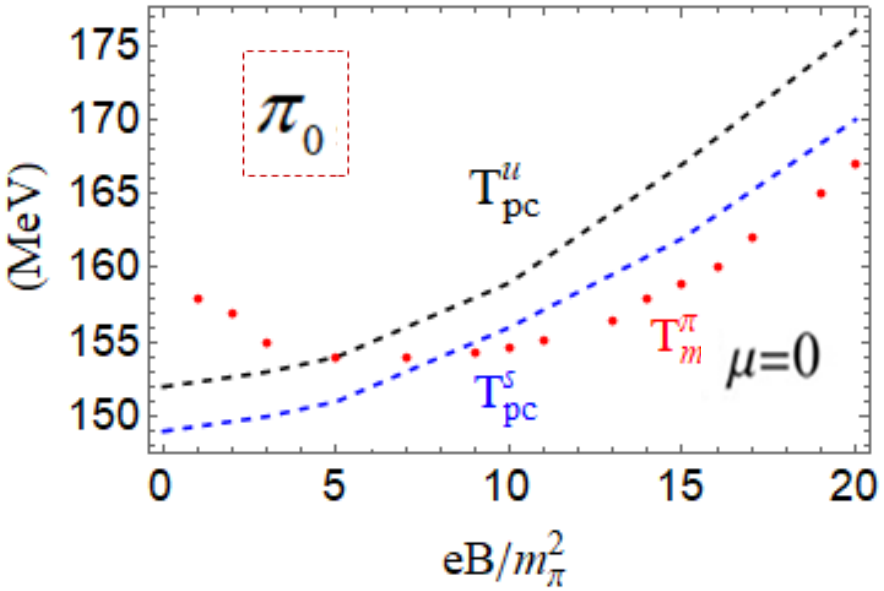
$$m_M = \sum quark\ mass$$

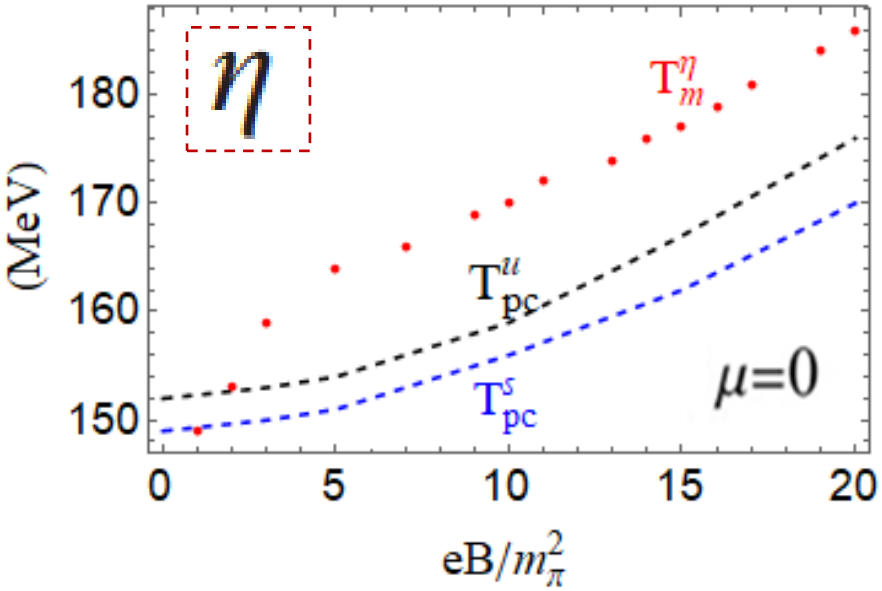
- **mass jumps @eB。**
- **observable?**

J.Mei, T.Xia, **S.J.Mao**, PRD107, 074018 (2023)

3.3.1 Mott transitions VS QCD phase transitions





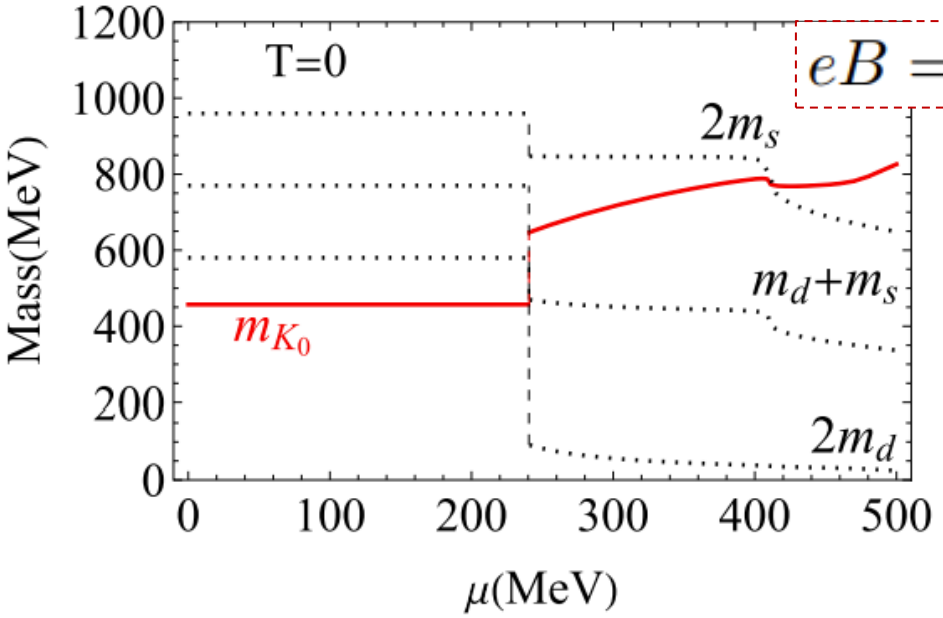


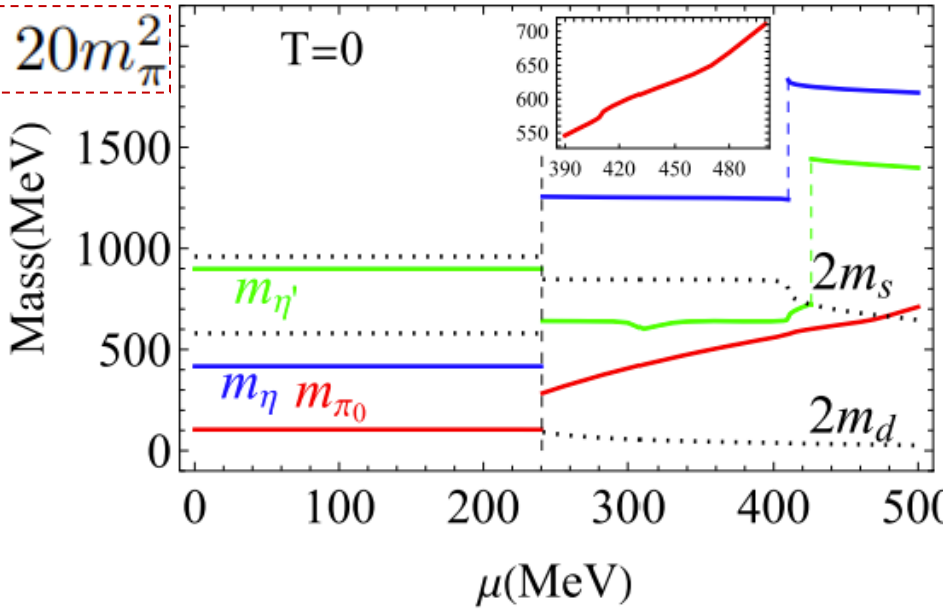
$K_0, \pi_0,$ **Mott transition VS chiral phase transition**

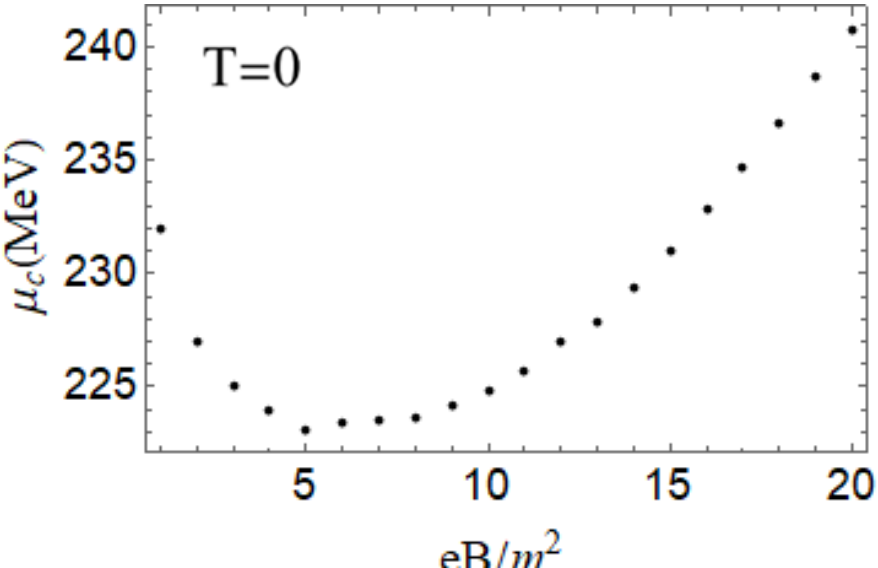
$U_A(1)$ restoration under progress

---- arXiv:2312.XXXXX, preparing

3.3.1 mass spectra@ baryon chemical potential μ



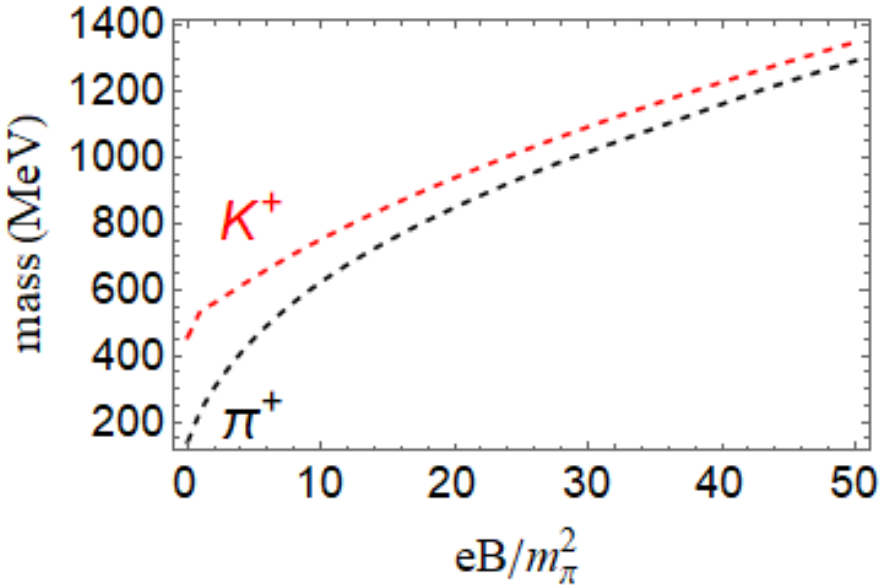


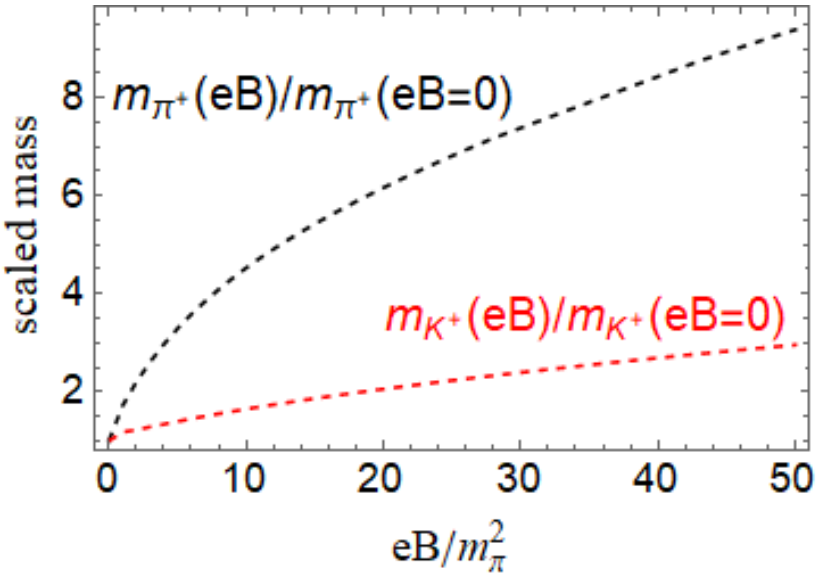


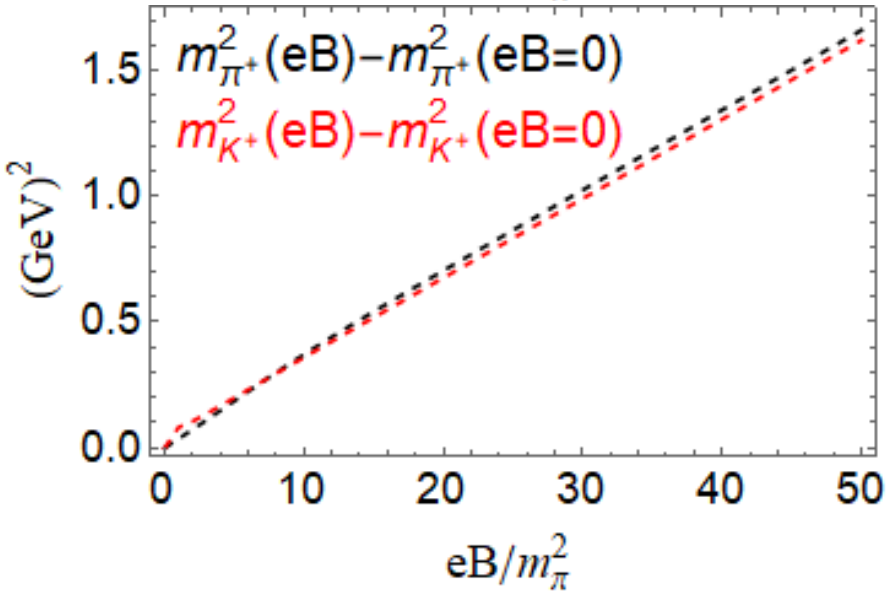
Charged mesons（$\pi^{\pm}$，$K^{\pm}$）

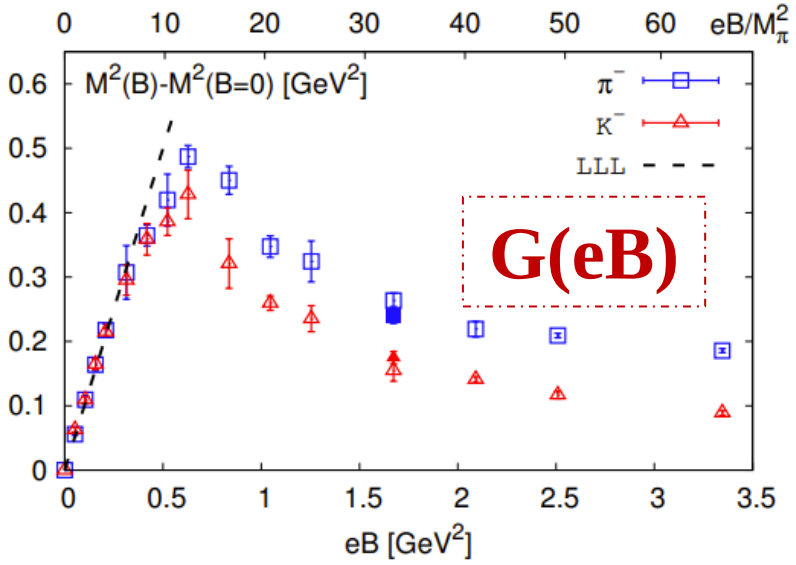
3.3.2

Mass spectra @ $T = \mu = 0$

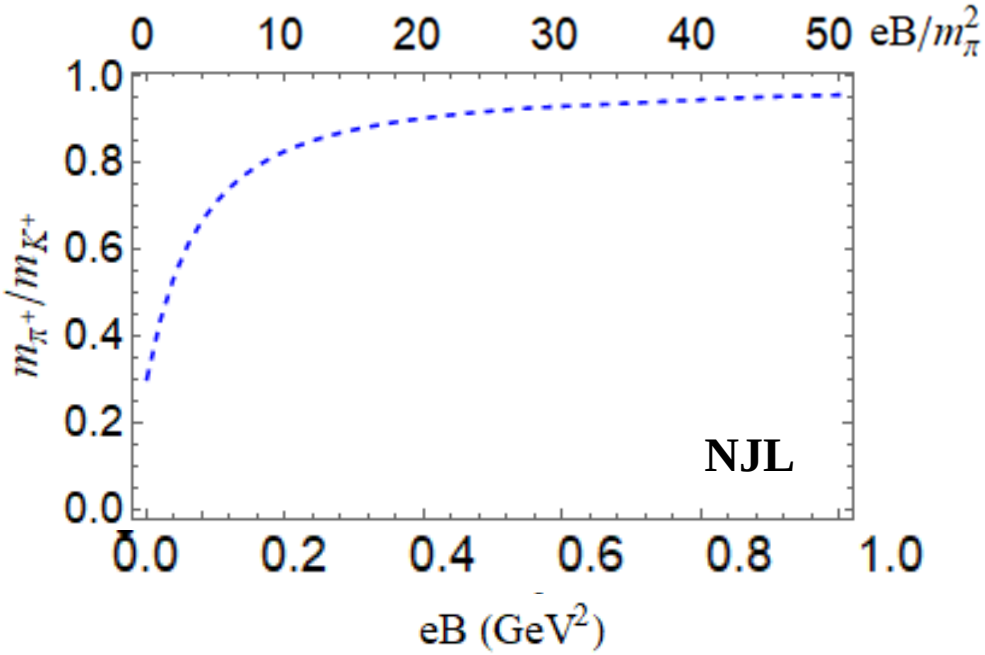


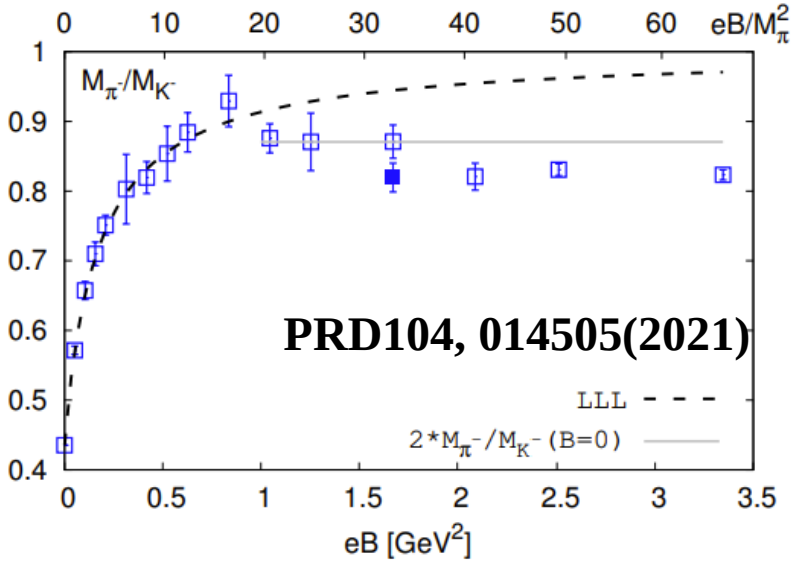






3.3.2 NJL VS LQCD

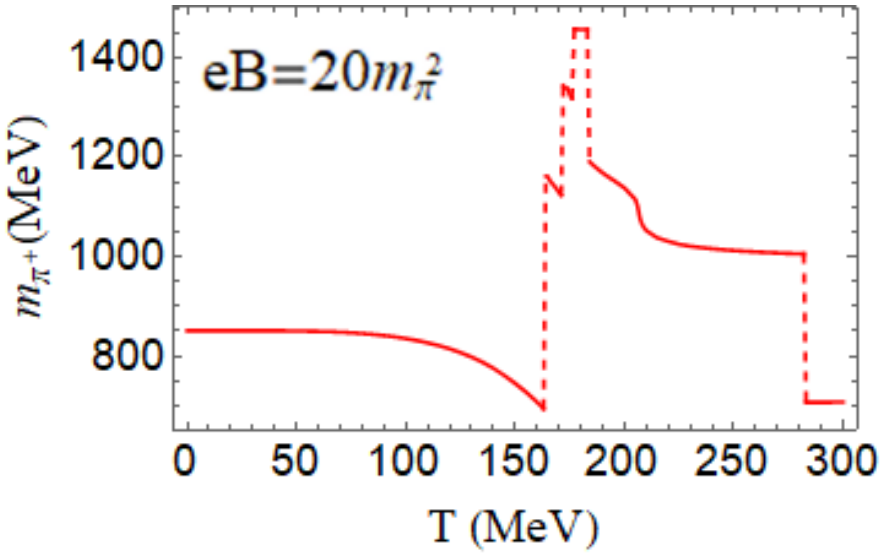


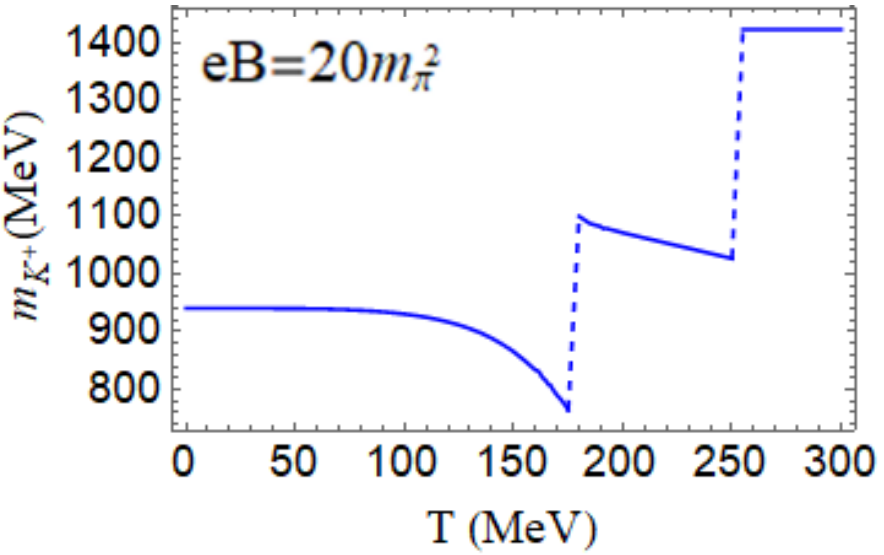


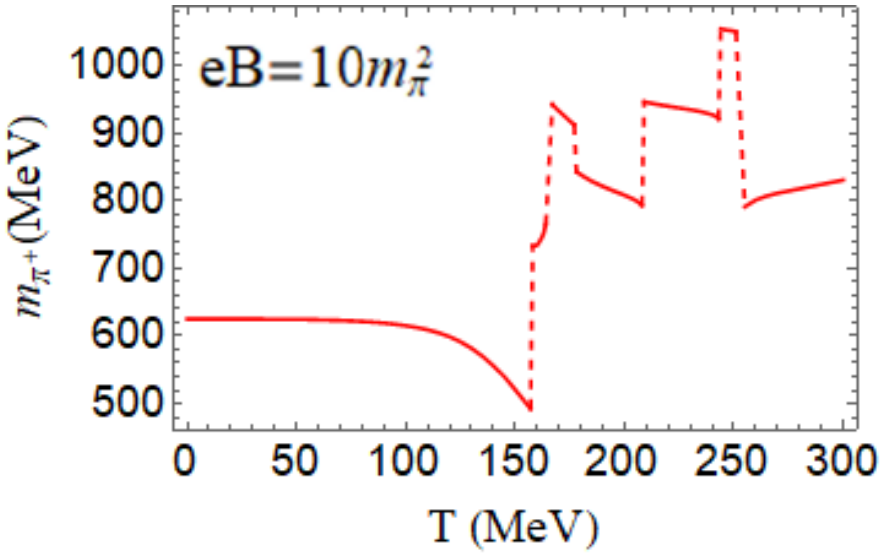
consider G(eB)

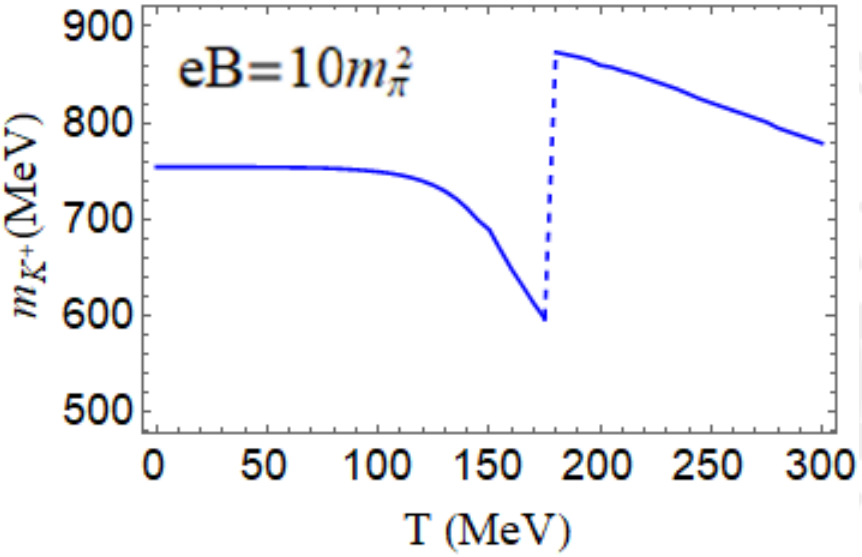
3.3.2

Mass spectra@T $\mu = 0$





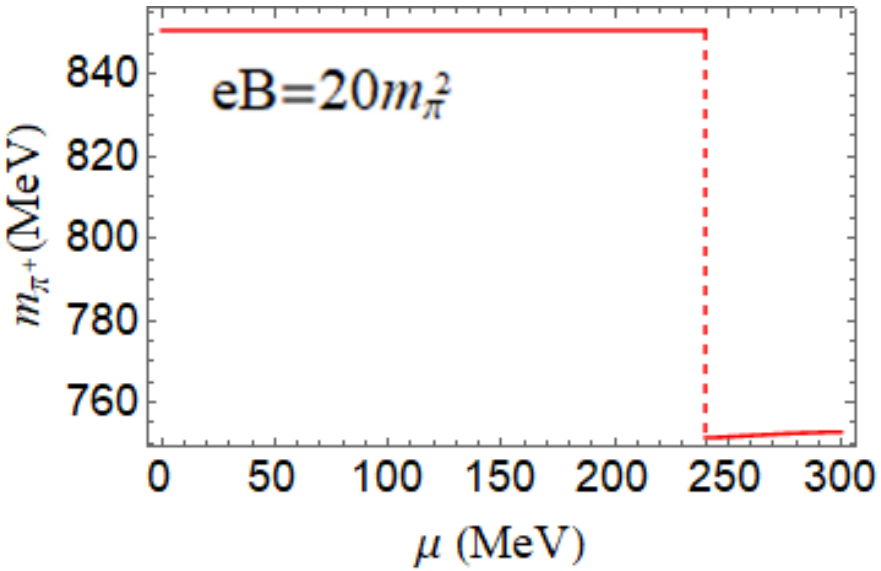


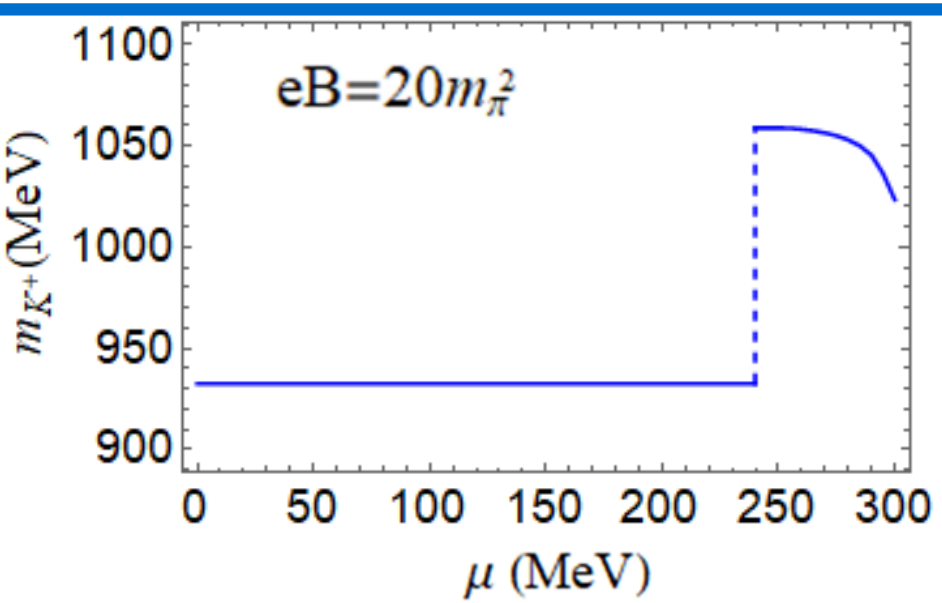


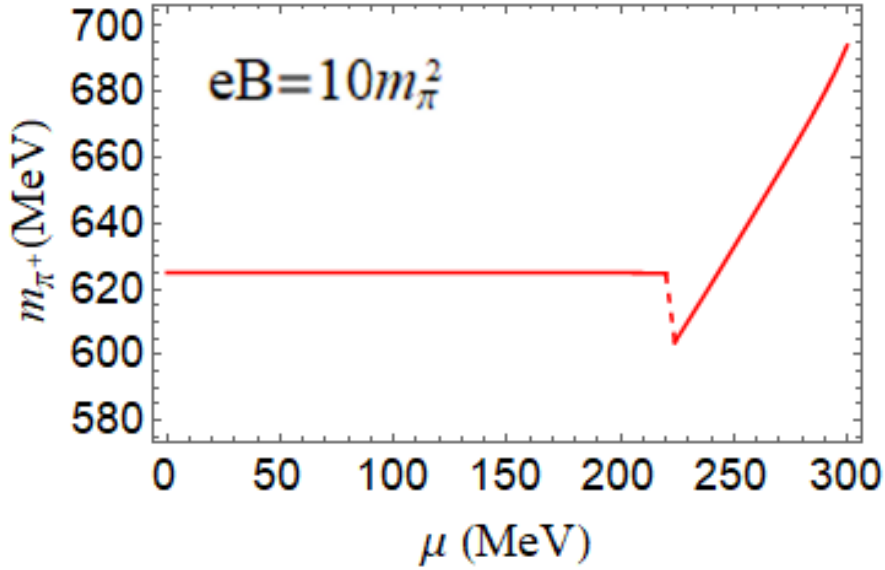
3.3.2

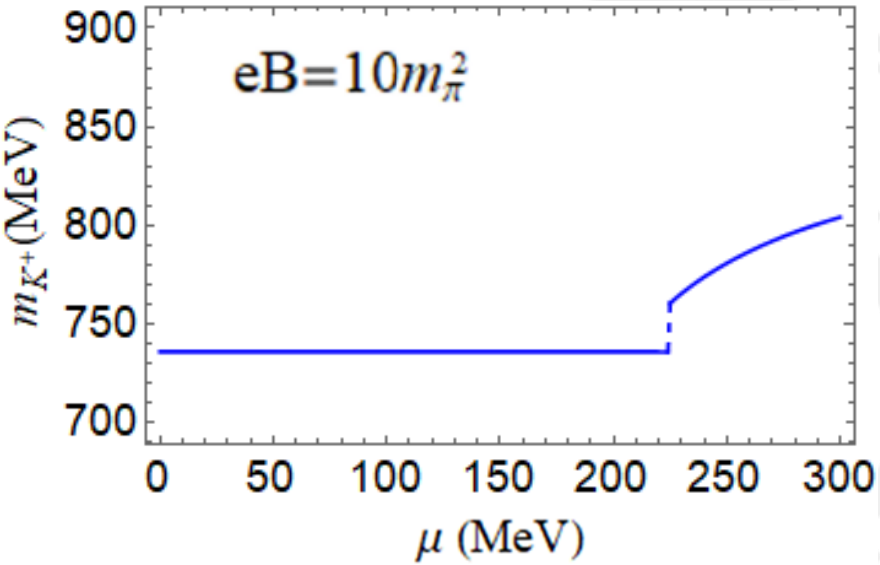
Mass spectra @ μ

$T = 0$









3.3.2 Mass spectra @ μ

$$m_{K^+}(\mu_s = \mu_s^c) = 0 \Rightarrow \text{Kaon superfluid @eB}$$

under progress…

4 Summary and outlook

Summary：

QCD phase transition @ meson level

① π^0,K^0 mass jump： chiral restoration

② Massless π^+ ,K^+ ： pion/Kaon superfluid

③ η, η’： $U_A(1)$ restoration

Outlook：

① Experimental observables

② Baryon decuplet @eB

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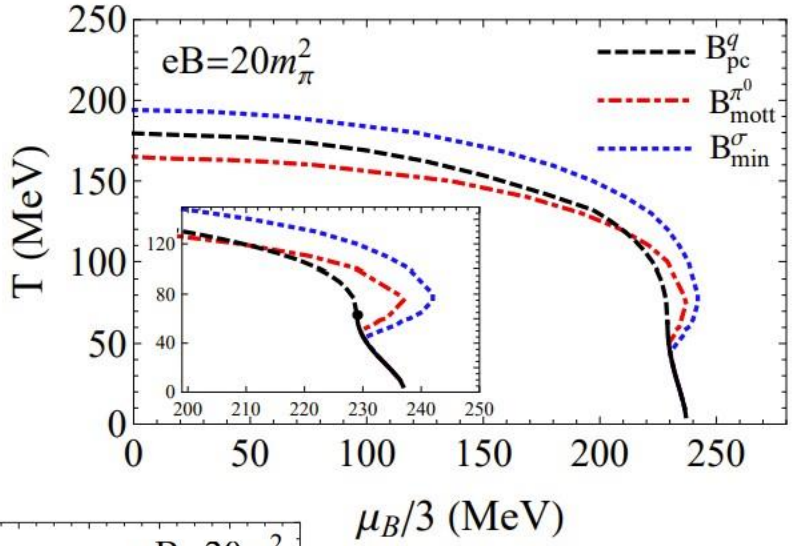
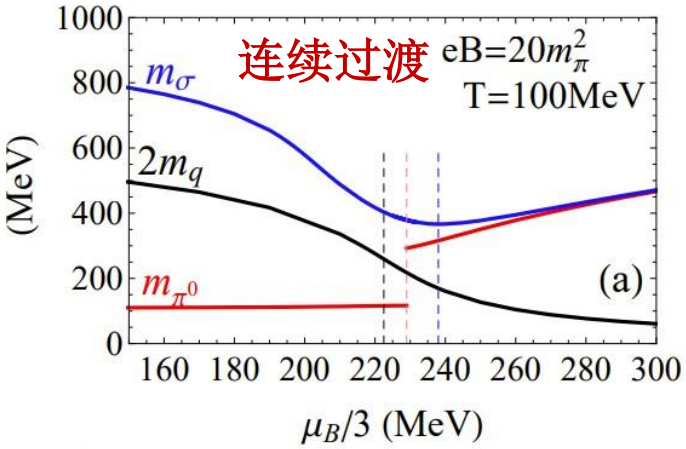
T – μ平面

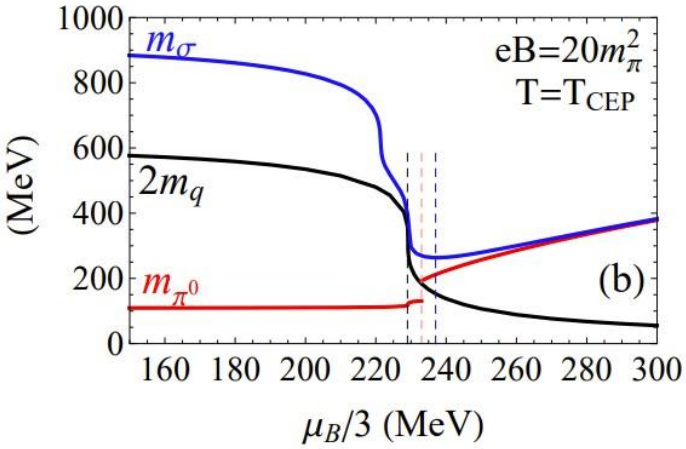
S.J. Mao, et al, Chin. Phys. C (2022) accepted

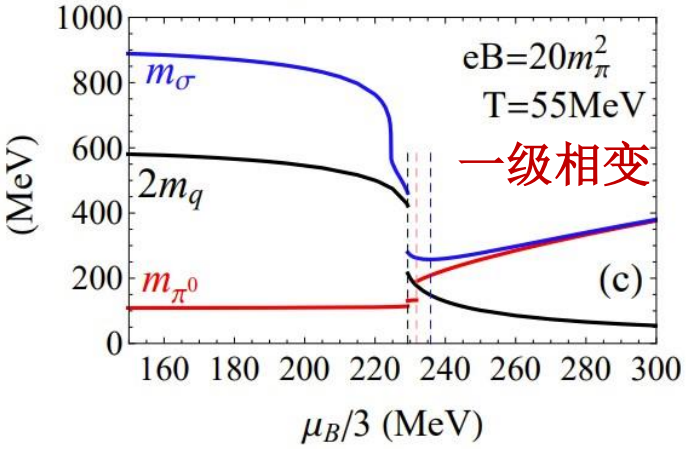
介子谱多次跳变：
寻找CEP和手征相边界

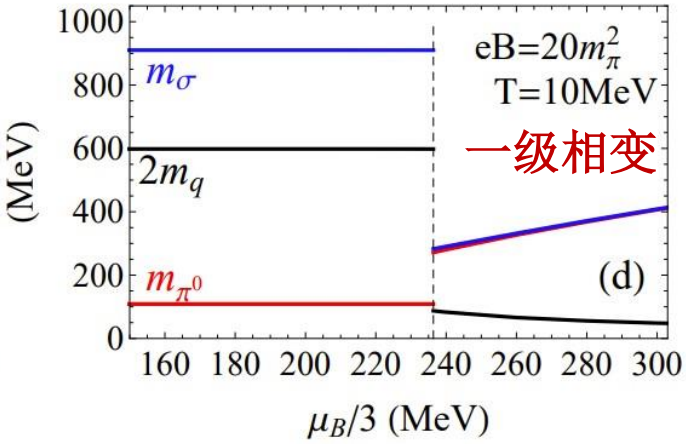
相图：











2.3 mass spectra @CEP

$$eB = 20m_\pi^2$$