FRG STUDY OF QUARK-MESON MODEL UNDER ROTATION

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PhysRevD.108.054006

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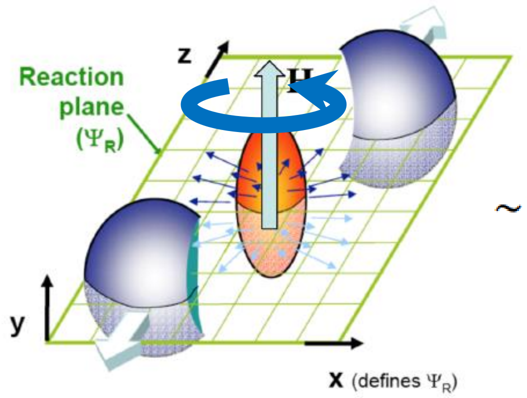
OUTLINE

- **Introduction**
- **fRG + QM + rotation**
- **Toward QCD under rotation**
- **Summary**

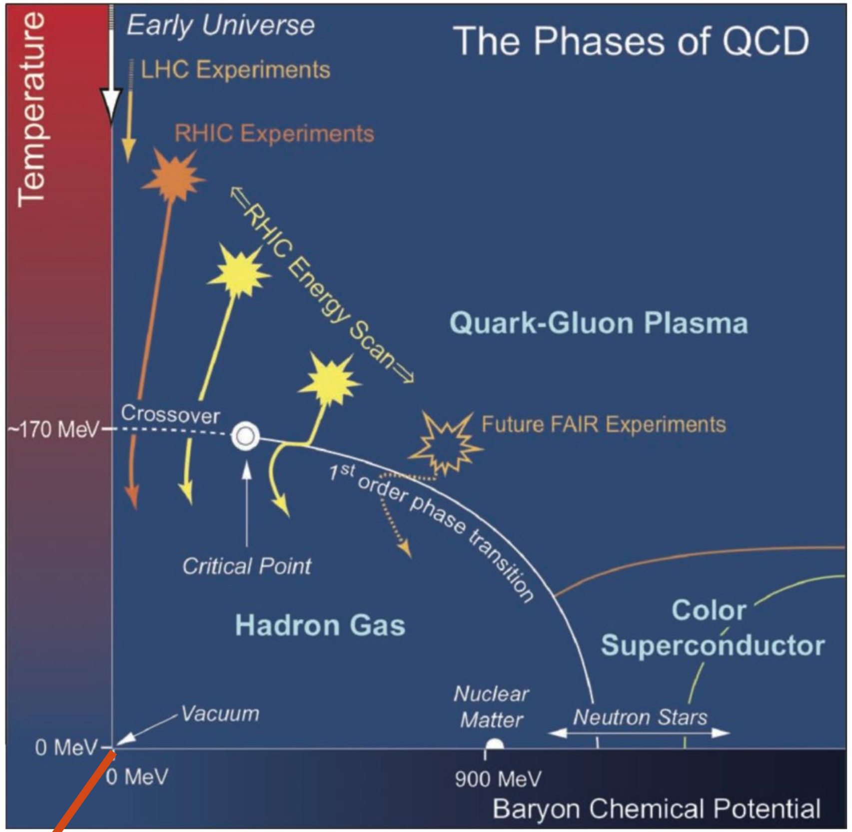
WHY ROTATION?



$\sim 10^{21}/s$ at RHIC

- Spin polarization/alignment

Phase transition



eB, Ω

Gaoqing's talk

Anomalous transport

CVE, Vilenkin (1979)

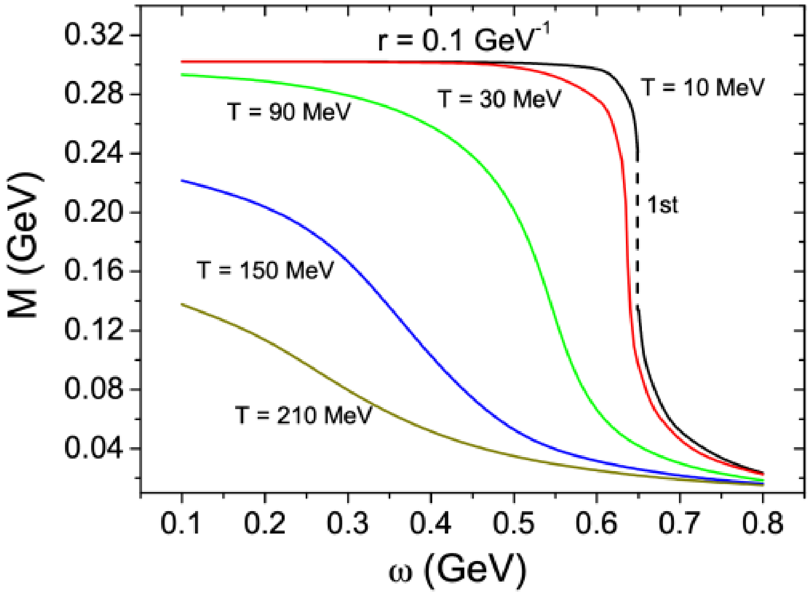
$$J = \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\omega$$

magneto-vortical transport, Hattori-Yin (2016)

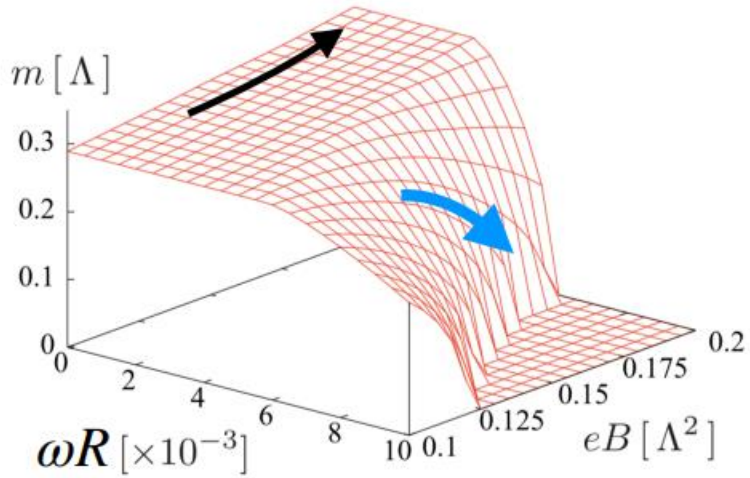
$$J^0 = \frac{eB\omega}{4\pi^2}$$

FIRST STUDIED BY NJL MODEL

- Analogy with chemical potential



Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),



HLC, et al, Phys. Rev. D 93, 104052 (2016)

CURRENT STAGE

- Model studies
  (P)NJL, QM, Bag model, Holographic…
- Rotation + other backgrounds (magnetic field, chemical potential, etc)
- Lattice

  V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, and A. A. Roenko, Phys. Rev. D 103, 094515 (2021)
  J.-C. Yang and X.-G. Huang, arXiv:2307.05755 [hep-lat].
- Rotational effect on quark has been widely studied
- We don't know much about how rotation affects gluon sector

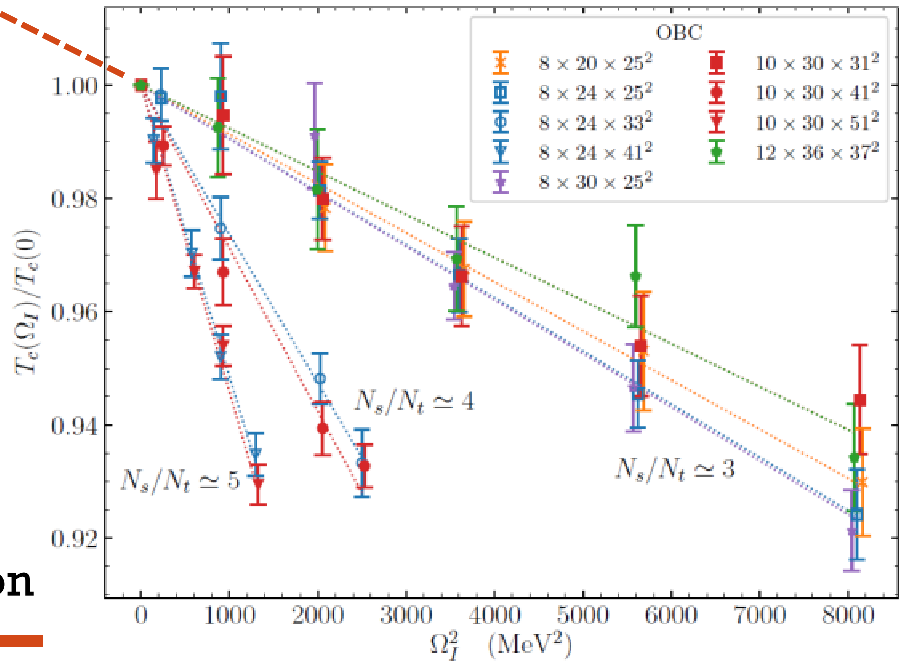
  S. Chen, K. Fukushima, and Y. Shimada, Phys. Rev. Lett. 129, 242002 (2022)
  M. N. Chernodub, Phys. Rev. D 103, 054027 (2021), Phys. Rev. D 107, 114502 (2023)
  Y-Q Zhao, S He, D Hou, L Li, and Z Li, JHEP04(2023)115
  Y. Jiang, 2312.06166

DISCREPANCY

- Lattice result with imaginary rotation

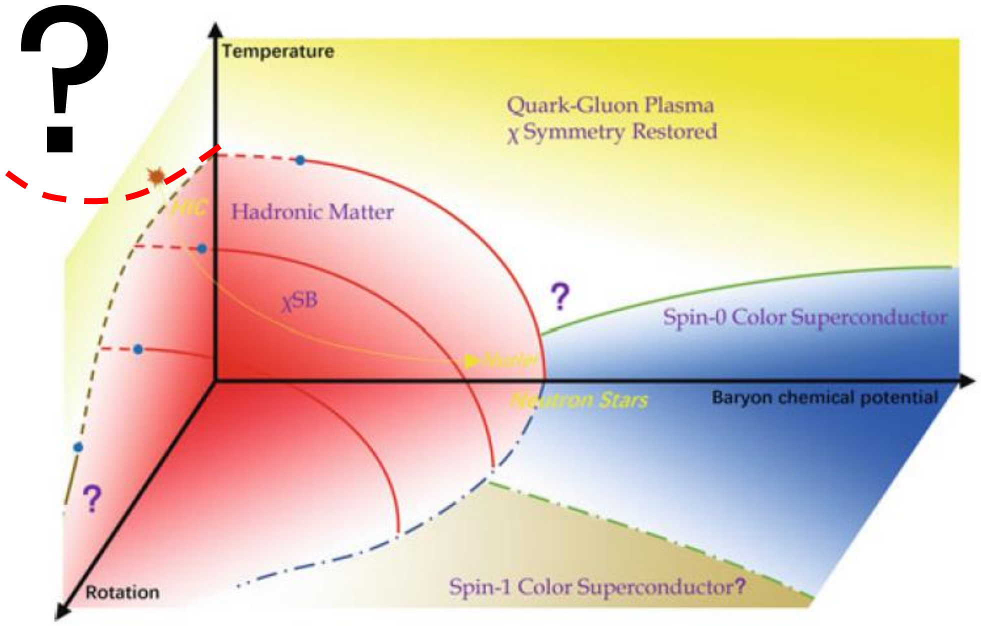
analytical continuation

$\Omega_I^2 \leftrightarrow -\Omega^2$



Real rotation

Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.



Tc decreases with increasing Ω_I

Tc increases with increasing real rotation

REAL & IMAGINARY ROTATION

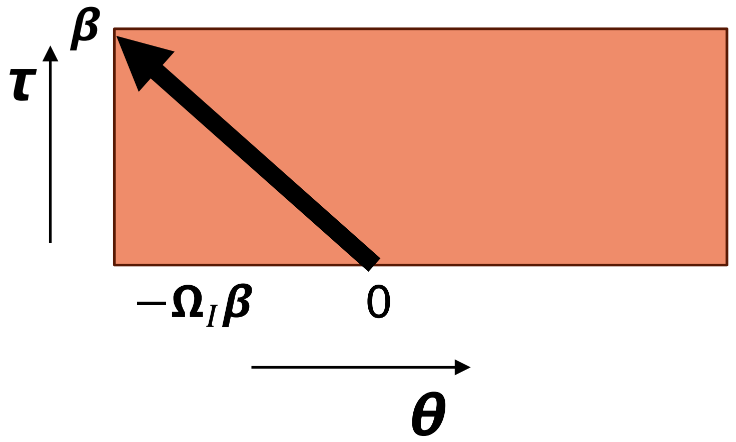
Real Rotation

- Boundary condition to preserve causality
- Inhomogeneity
- Sign problem
- Subtlety of a Euclidean field theory

Imaginary Rotation

- Boundary condition for space is not necessary
- No sign problem
- Well defined Euclidean field theory
- twisted boundary condition

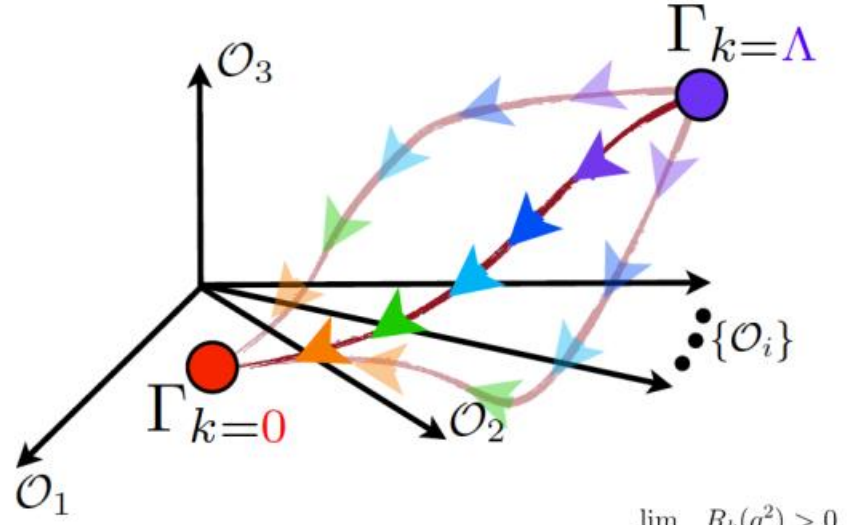
$$(\tau,\, \theta,\, r,\, z) \sim (\tau + \beta,\, \theta - \tilde{\Omega}_{\mathrm{I}},\, r,\, z)\,,$$



FUNCTIONAL RENORMALIZATION GROUP

Functional integral with an IR regulator

$$Z_k[J] = \int D\chi e^{-S[\chi]+\int_x \chi(x)J(x)-\Delta S_k[\chi]}$$

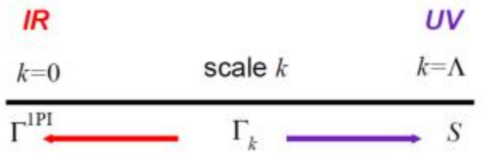


regulator

$$\Delta S_k[\chi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d}\chi^*(q)R_k(q)\chi(q)$$

Legendre transformation:

$$\Gamma_k[\phi] = -W_k[J] + \int_x \phi(x)J(x) + \Delta S_k[\phi]$$



$$\lim_{q^2/k^2\to 0} R_k(q^2) > 0$$

$$\lim_{k^2/q^2\to 0} R_k(q^2) = 0$$

$$\lim_{k\to\Lambda} R_k(q^2) \to \infty$$

When $\lim_{k\to\Lambda} R_k(q^2)\to\infty$ all fluctuation supressed

When $\lim_{k^2/q^2\to 0} R_k(q^2) = 0$ recover the full quatum theory

flow equation

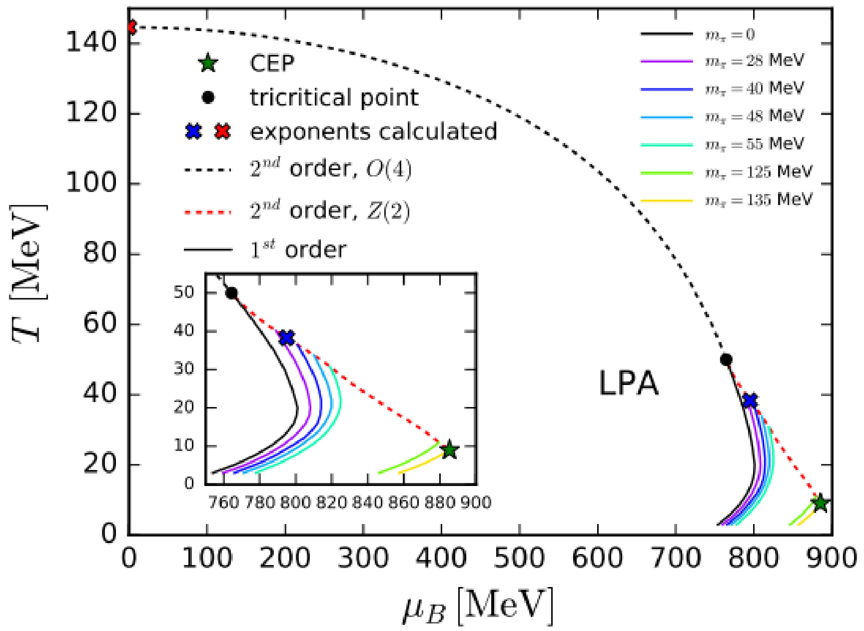
$$\partial_k \hat{\Gamma}_k[\Phi,\bar{\psi},\psi] = \frac{1}{2}\mathrm{STr}\left[\left(\hat{\Gamma}_k^{(2)}[\Phi,\bar{\psi},\psi] + \hat{R}_k\right)^{-1}\partial_k \hat{R}_k\right]$$

$$\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2,0)}+R_k^B}\partial_k R_k^B\right] - \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(0,2)}+R_k^F}\partial_k R_k^F\right]$$

C. Wetterich, PLB, 301 (1993) 90

QUARK MESON MODEL

Y. Chen, et al., Phys. Rev. D 104, 054009 (2021), arXiv:2101.08484 [hep-ph].



- Lagrangian

$$\mathcal{L} = \phi[-(-\partial_\tau + \Omega \hat{L}_z)^2 - \nabla^2]\phi + U(\phi) + \bar{q}[\gamma^0(\partial_\tau - \Omega \hat{J}_z) - i\gamma^i\partial_i + g(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma^5)]q$$

$$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 - c\sigma$$

- KG equation and Dirac equation can be easily solved

$$\phi = \frac{1}{N_{l,i}^2} e^{-i(\varepsilon - \Omega l)t + il\theta + ip_z z} J_l(p_{l,i} r),$$

$$u_+ = \frac{e^{-i(\varepsilon-\Omega j)+ip_z z}}{\sqrt{\varepsilon + m}} \begin{pmatrix} (\varepsilon+m)\phi_l \\ 0 \\ p_z\phi_l \\ i\tilde{p}_{l,i}\varphi_l \end{pmatrix}, \qquad u_- = \frac{e^{-i(\varepsilon-\Omega j)+ip_z z}}{\sqrt{\varepsilon + m}} \begin{pmatrix} 0 \\ (\varepsilon+m)\phi_l \\ -i\tilde{p}_{l,i}\varphi_l \\ -p_z\phi_l \end{pmatrix},$$

- Local potential approximation(LPA):
  the RG-scale dependence only enters the effective potential

BOUNDARY CONDITION IS IMPORTANT!

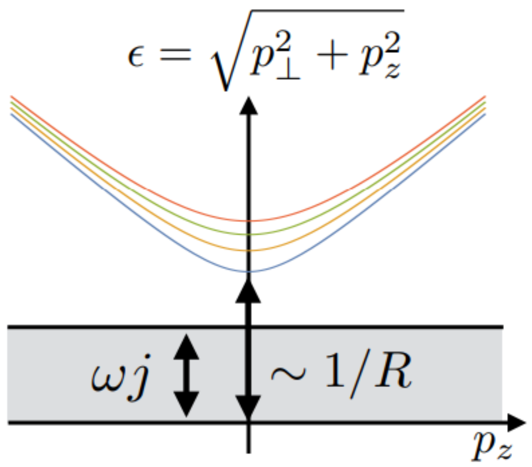
- To preserve causality $\Omega R \leq 1$
- For meson: $\phi(r=R)=0$

$$\frac{1}{e^{\beta(\varepsilon-\Omega l)}-1}$$

Without BC, when $\varepsilon-\Omega l<0$
Bose-Einstein condensate?

With BC, we are safe, since $p_{l,i} \pm \Omega l > 0$ with $\Omega R \leq 1$

- For quark: $\int_0^{2\pi} \mathrm{d}\theta \bar{\psi}\gamma^r\psi|_{r=R}=0;$
- No rotational effect in vacuum (perturbatively)



The same for gluon.
But does not hold if we add a magnetic field

FLOW EQUATION

- Regulator
$$R_{\phi,k} = (k^2 - p^2)\theta(k^2 - p^2),$$ for meson
$$\hat{R}_{q,k} = -i\gamma^i \partial_i \left(\frac{k}{\sqrt{-\nabla^2}} - 1 \right) \theta(k^2 + \nabla^2).$$ for quark

- Then with straightforward calculation, flow equation reads

$$\partial_k U_k(r) = \frac{1}{(2\pi)^2} \Big\{ \sum_{l,i} \frac{1}{N_{l,i}^2} \mathrm{tr} \frac{k\sqrt{k^2 - p_{l,i}^2}}{\varepsilon_\phi} \frac{1}{2} [\coth \frac{\beta(\varepsilon_\phi + \Omega l)}{2} + \coth \frac{\beta(\varepsilon_\phi - \Omega l)}{2}] J_l(p_{l,i} r)^2 \theta(k^2 - p_{l,i}^2)$$
$$- \sum_{l,i} \frac{1}{\tilde{N}_{l,i}^2} 2N_c N_f \frac{k\sqrt{k^2 - \tilde{p}_{l,i}^2}}{\varepsilon_q} \frac{1}{2} [\tanh \frac{\beta(\varepsilon_q + \Omega j)}{2} + \tanh \frac{\beta(\varepsilon_q - \Omega j)}{2}] [J_l(\tilde{p}_{l,i} r)^2 + J_{l+1}(\tilde{p}_{l,i} r)^2] \theta(k^2 - \tilde{p}_{l,i}^2) \Big\},$$

Non-rotating limit

$$p_{l,i} \to p_t$$
$$\sum_i \frac{1}{N_{l,i}^2} \to p_t dp_t$$
$$\sum_{l=-\infty}^{\infty} J_l(x)^2 = 1$$

- Numerical calculation: grid method, discretizing the effective potential in field space and numerically evolve all these grid with flow equation

NUMERICAL RESULTS

- Still contradict with lattice results

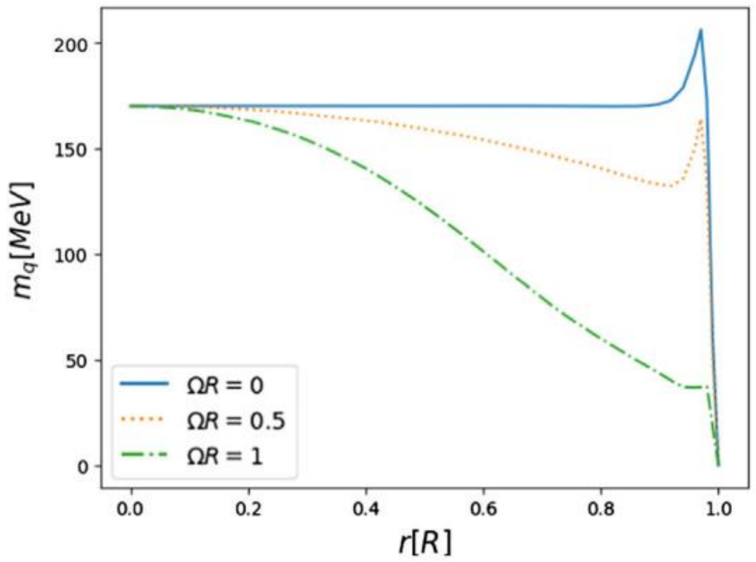


FIG. 5. The quark mass as a function of the radius r at different Ω at $T = 160$ MeV.

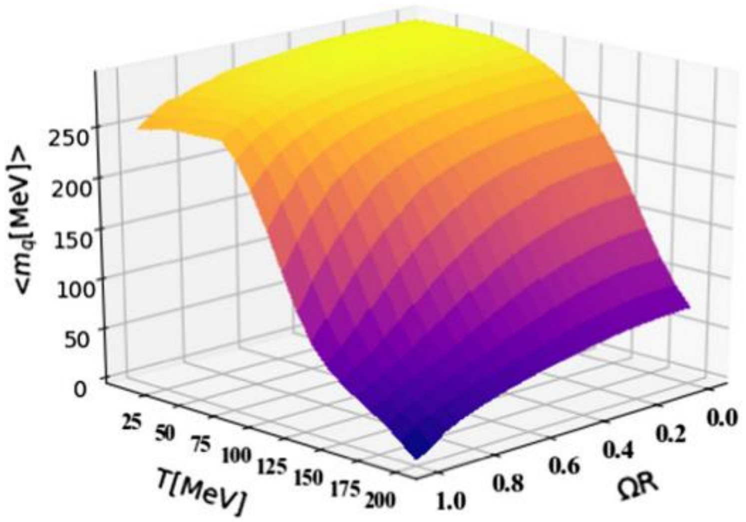


FIG. 1. The quark mass m_q as a function of Ω and T at $r = 0.9R$ in the QM model.

NUMERICAL RESULTS

Due to causality, we can not reach here

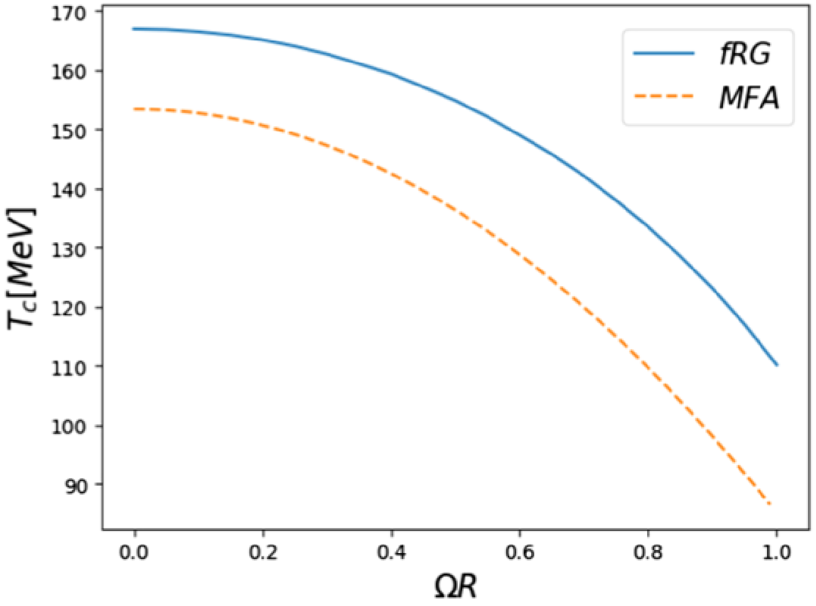


FIG. 2. The pesudocritical temperature T_c as a function of Ω at $r = 0.9R$ from fRG and MFA in the QM model.

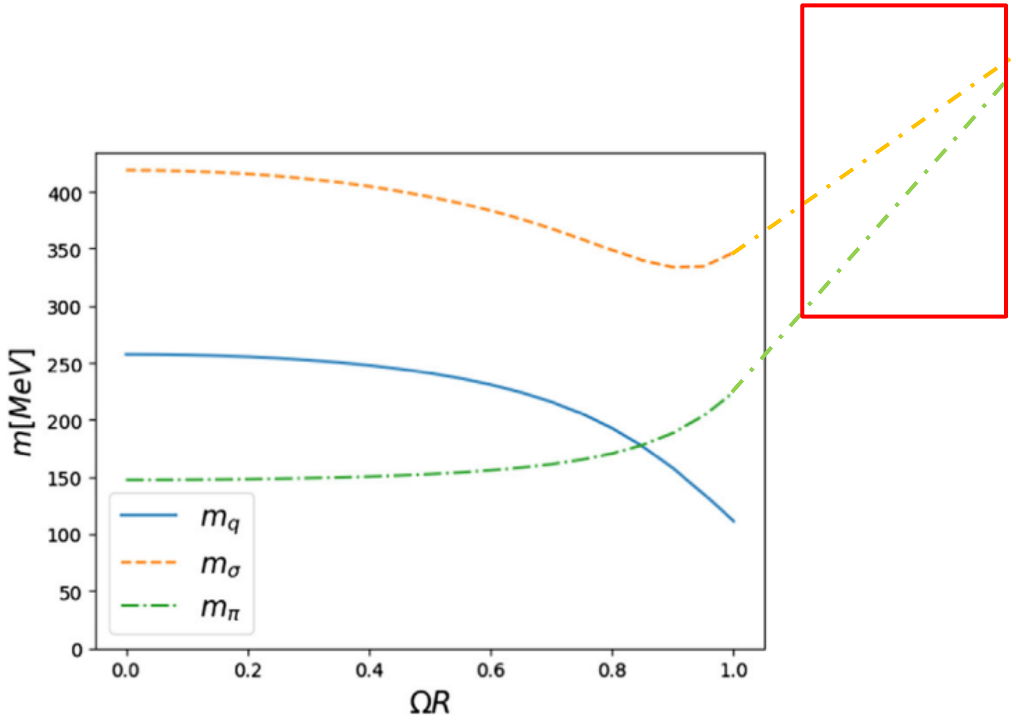


FIG. 3. Meson masses and quark mass as functions of Ω at $T = 120$ MeV from QM model.

IMAGINARY ROTATION

- Analytical continuation seems OK

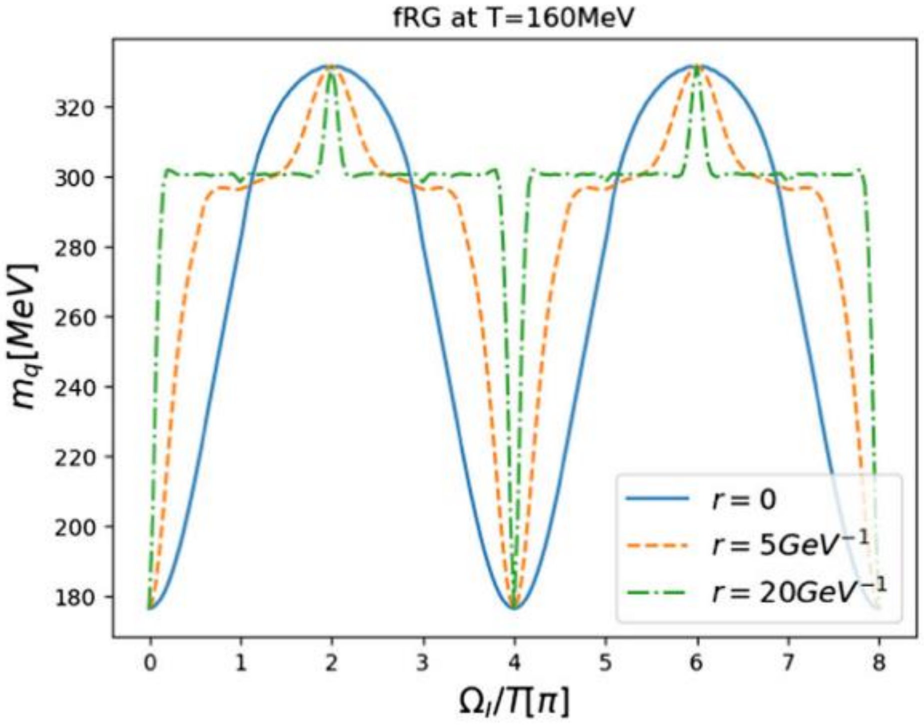


FIG. 9. The quark mass as a function of the imaginary rotating angular velocity at $T = 160$ MeV from fRG.

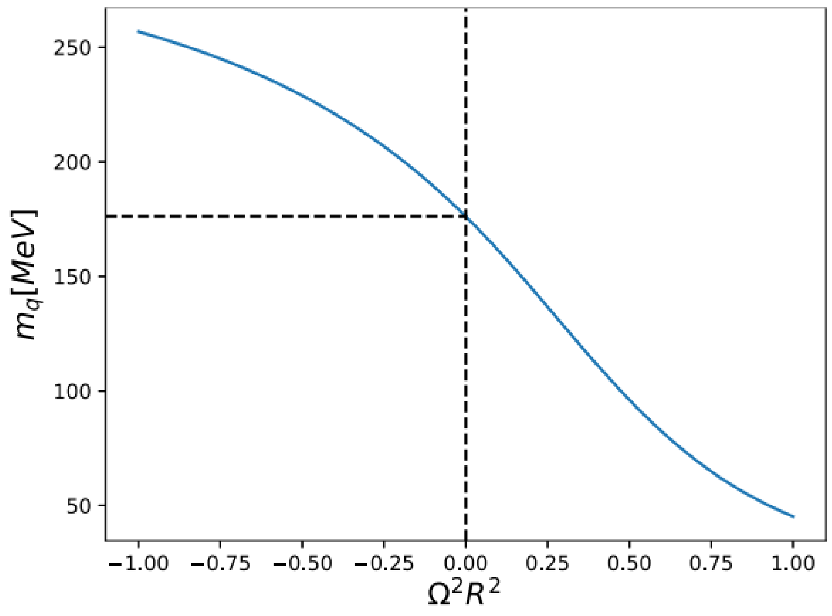


FIG. 10. The quark mass as a function of the square of the rotating angular velocity at $T = 160$ MeV and $r = 0.9R$ from fRG calculation.

TOWARD QCD UNDER ROTATION

- With the help of vierbein

$$A_{\hat{\mu}} = e_{\hat{\mu}}{}^{\mu} A_{\mu}$$

- Simple replacement in Lagrangian

$$\partial_\tau \to \partial_\tau - i\Omega_I \hat{J}_z$$

$$\hat{J}_z A_{\hat{\mu}} = (\hat{L}_z \delta_\mu^\nu + \hat{S}_{z\mu}{}^{\nu}) A_{\hat{\nu}}$$

$$\hat{J}_z c = \hat{L}_z c$$

$$\hat{S}_z = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- At one loop level, the Lagrangian can be diagonalized
- One can further add some background fields

A0 CONDENSATE

S. Chen, K. Fukushima, and Y. Shimada
Phys.Rev.Lett. 129 (2022) 24, 242002

- Weiss potential

$$V(\phi;\tilde{\Omega}_{\rm I})|_{\tilde{r}=0} = \frac{\pi^2 T^4}{3}\sum_{\boldsymbol{\alpha}}\sum_{s=\pm 1} B_4\left(\left(\frac{\boldsymbol{\phi}\cdot\boldsymbol{\alpha}+s\tilde{\Omega}_{\rm I}}{2\pi}\right)_{\rm mod\ 1}\right).$$

- Simple replacement

$$\phi\cdot\alpha \to \phi\cdot\alpha \pm \beta\Omega_I$$

- Still true at higher loop?
  Rotation contribution to 3-gluon interaction and gluon-ghost vertex differently

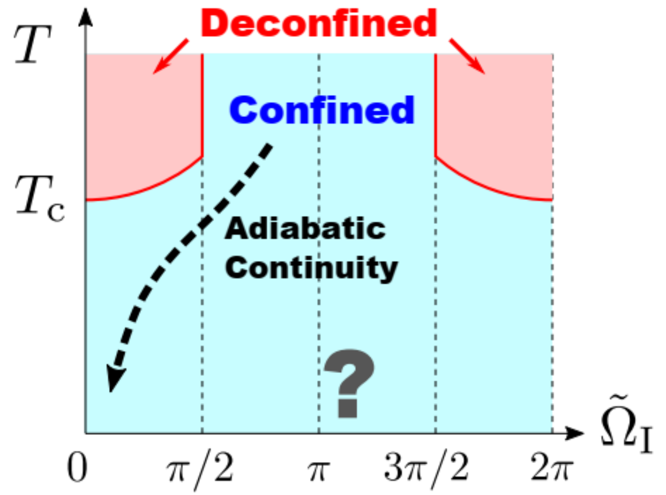
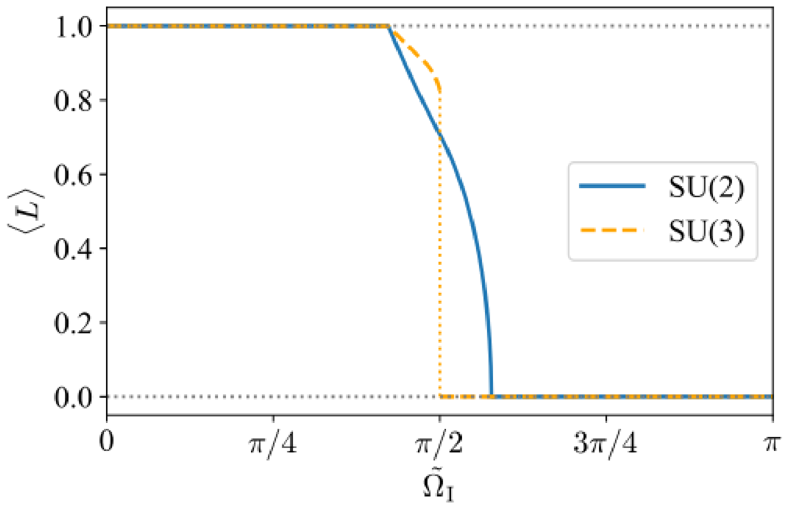


FIG. 4. Conjectured phase diagram on the $\tilde{\Omega}_{\rm I}$-T plane around the rotation axis, $\tilde{r}=0$, for the SU(3) case. Solid curves represent the phase transition.



CHROMO-MAGNETIC CONDENSATE

Ω=0, T=0: Savvidy Vacuum

G.K. Savvidy, Phys. Lett. B, 71:133, 1977

- SU(2) Effective potential

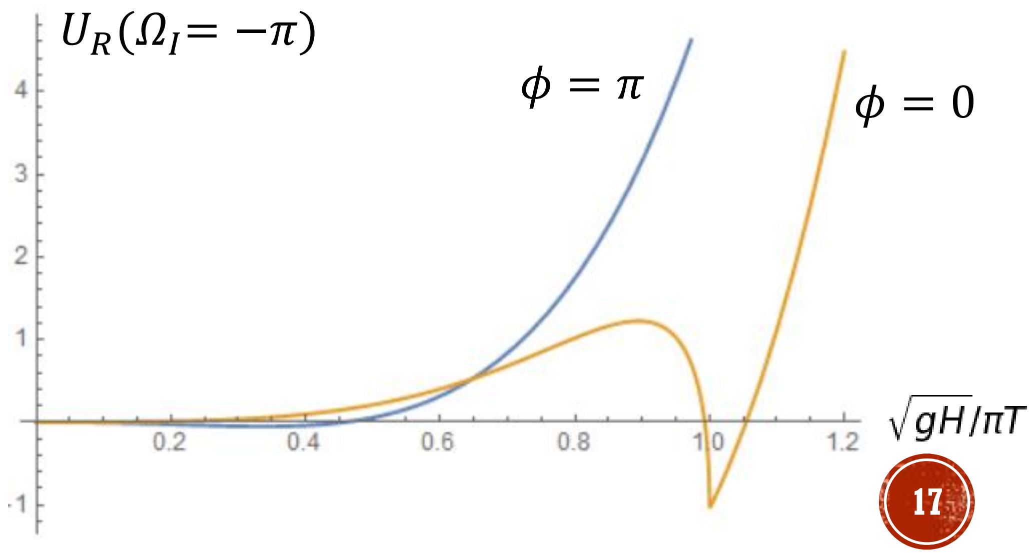
$$U_R = \frac{11 g^2 H^2}{48\pi^2}\ln(\frac{gH}{\mu_0^2}) - \frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\frac{1}{n}[K_1(n\beta\sqrt{gH}) - \frac{\pi}{2}Y_1(n\beta\sqrt{gH}))]\cos n\beta(g\phi - \Omega_I)$$

$$- 2\frac{(gH)^{\frac{3}{2}}}{\pi^2\beta}\sum_{n=1}^{\infty}\sum_{\lambda=0}^{\infty}\frac{1}{n}\sqrt{2\lambda+3}K_1(n\beta\sqrt{gH(2\lambda+3)})\cos n\beta g\phi\cos n\beta\Omega_I$$

$$U_I = -\frac{(gH)^2}{8\pi} - \frac{(gH)^{\frac{3}{2}}}{2\pi^2\beta}\sum_{n=1}\frac{1}{n}J_1(n\beta\sqrt{gH})\cos n\beta(g\phi - \Omega_I)$$

- No replacement $\phi\cdot\alpha \to \phi\cdot\alpha \pm \beta\Omega_I$
- Stable at 2-loop level

M. Bordag, V. Skalozub, Eur.Phys.J.C 82 (2022) 5, 390



PERTURBATIVE CALCULATION

- Too many Bessel functions

$$\int r\mathrm{d}r J_{l_1}(p_{1t}r)J_{l_2}(p_{2t}r)J_{l_3}(p_{3t}r)J_{l_4}(p_{4t}r)\delta(l_1+l_2+l_3+l_4)$$

- Small rotation expansion

Fermion: $\ln Z_2 = 2\int \mathrm{d}^3x \int \mathrm{đ}^3p \frac{\mathrm{e}^{\beta\varepsilon}}{(1+\mathrm{e}^{\beta\varepsilon})^2}\beta^2\Omega^2(\frac{1}{2}r^2p_\perp^2+\frac{1}{4})$ **YM :** $\ln Z_2 = 2N_g\int \mathrm{d}^3x \int \mathrm{đ}^3p \frac{\mathrm{e}^{\beta\varepsilon}}{(1-\mathrm{e}^{\beta\varepsilon})^2}\beta^2\Omega^2(\frac{1}{2}r^2p_\perp^2+1)$

- r=0 or spin contribution only

A. Ayala et al., Phys. Rev. D 103, no.7, 076021 (2021), [erratum: Phys. Rev. D 104, no.3, 039901 (2021)],

$$S(p) = \frac{(p_0+\Omega/2-p_z+ip_\perp)(\gamma_0+\gamma_3)+m(1+\gamma_5)}{(p_0+\Omega/2)^2-\bar{p}^2-m^2+i\epsilon}\mathcal{O}^+ + \frac{(p_0-\Omega/2+p_z-ip_\perp)(\gamma_0-\gamma_3)+m(1+\gamma_5)}{(p_o-\Omega/2)^2-\bar{p}^2-m^2+i\epsilon}\mathcal{O}^-.$$

SUMMARY & OUTLOOK

- Our fRG study of QM model under rotation agree with previous model studies
- More careful study of gluon under rotation is needed
- QM model beyond LPA
- Chromo-magnetic induced by rotation?
- Some non-perturbative aspects? Rotating QCD vacuum?

THANKS !

BACKUP

CEP

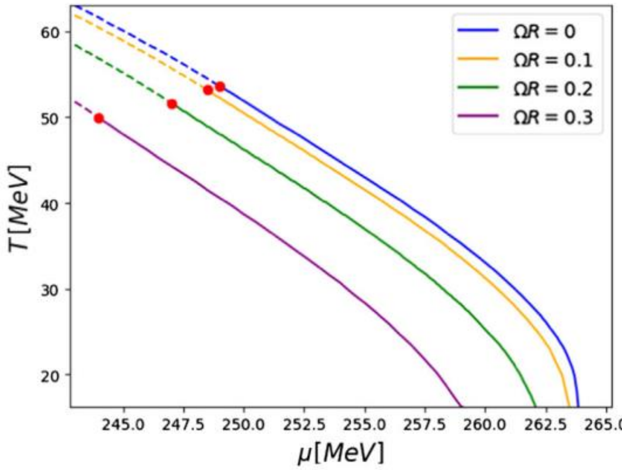


FIG. 6. $T - \mu$ phase diagram near the critical end point at different angular velocity.

MFA

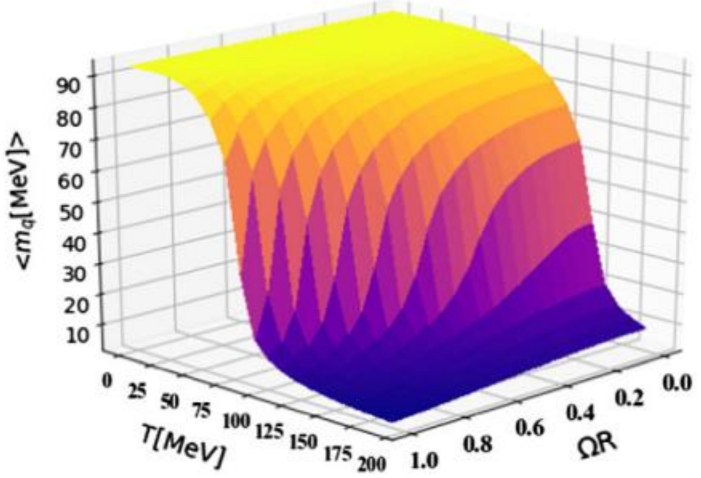


FIG. 7. The quark mass m_q as a function of Ω and T at $r = 0.9R$ under MFA of QM model.

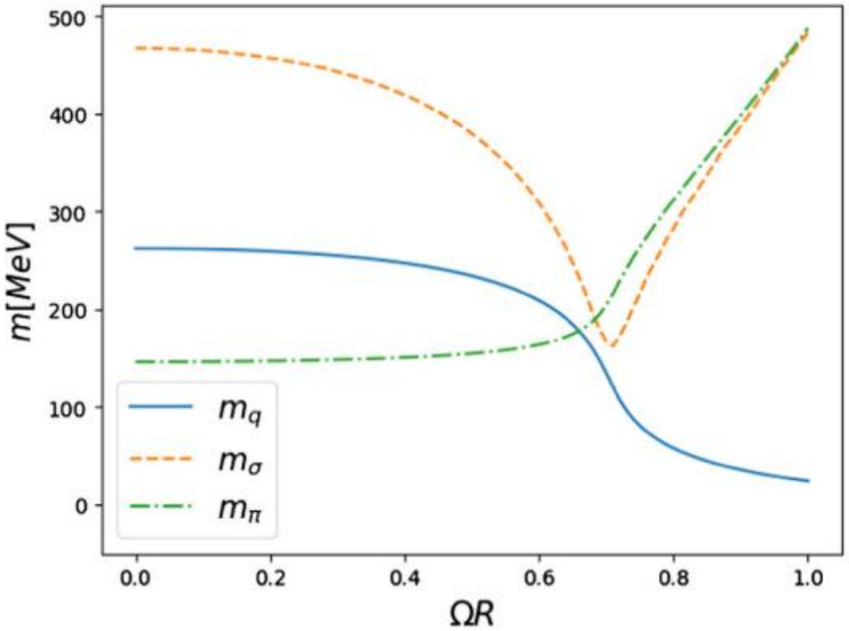


FIG. 8. Meson masses and quark mass vs Ω at $T = 120$ MeV and $r = 0.9R$ under MFA of QM model.

2+1D NJL

$$m(m-m_0)-\sum_{n\neq 0}(-1)^n \frac{m e^{-m\sqrt{(n\beta)^2+2r^2[1-\cos(n\beta\Omega_I)]}}}{\sqrt{(n\beta)^2+2r^2[1-\cos(n\beta\Omega_I)]}}\cos\left(n\beta\frac{\Omega_I}{2}\right)=m(m-m_0)-\mathrm{tr}(S_\beta-S_0)=0,$$

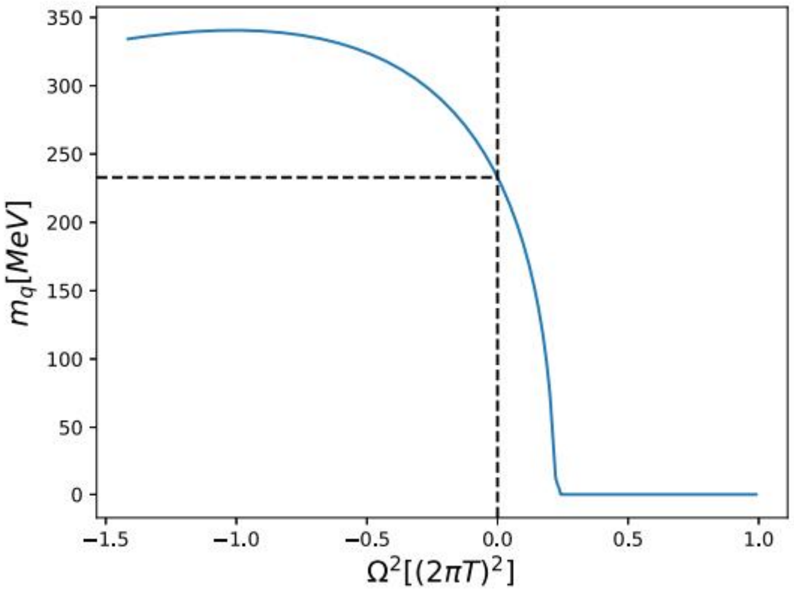


FIG. 12. The quark mass as a function of the square of rotating angular velocity at $T = 160$ MeV and $r = 0$ with 2D NJL model.