Coupling strength induced BCS-BEC crossover on phase boundary of pion superfluid

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- **Background**
- **Order parameters**
- **Goldstone mode**
- **Equation of state**
- **Sarma & LOFF**
- **Summary**

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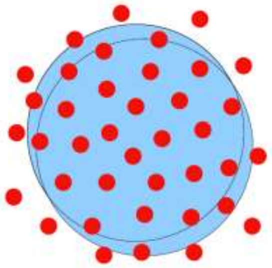
Background



BCS

Crossover →

BEC

BCS	BEC
weak coupling	strong coupling
large pair size **k**-space pairing	small pair size **r**-space pairing
strongly overlapping Cooper pairs	ideal gas of preformed pairs

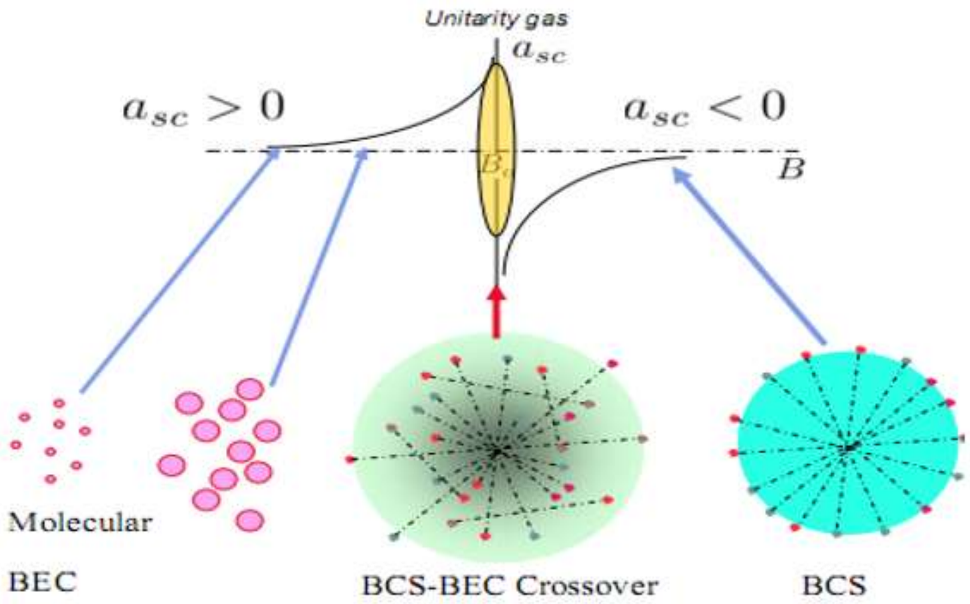
For theory

$$|\mathrm{BCS}\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow} \right) |0\rangle$$

$$v_{\mathbf{k}}^2 = N_B \phi_{\mathbf{k}}^2 / (1 + N_B \phi_{\mathbf{k}}^2) \qquad N_B \gg 1$$

$$|\mathrm{BEC}\rangle = \exp\left(\sqrt{N_B} \sum_{\mathbf{k}} \phi_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow} \right) |0\rangle$$

Background



For experiment

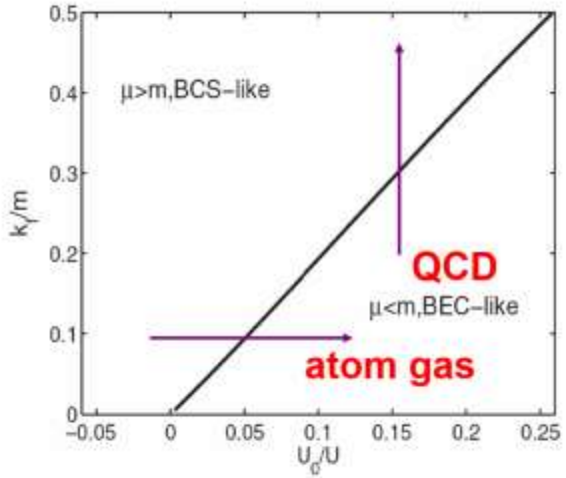
Feshbach Resonance

$a_{sc} > 0$ BEC

$a_{sc} < 0$ BCS

scattering length: sign reversal

Background



- **The increasing isospin (baryon) density leads to a phase transition from normal quark matter to a pion superfluid (color superconductor).**

M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B422, 247 (1998).
R. Rapp, T. Schaefer, E. V. Shuryak, and M. Veldovsky, Phys. Rev. Lett. 81, 53 (1998).
D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592 (2001); Phys. At. Nucl. 64, 834 (2001).

- **The BCS-BEC crossover happens with the change of isospin (baryon) density.**

M. Buballa, Phys. Rep. 407, 205 (2005).
L. Y. He, S. J. Mao and P. F. Zhuang, Inter. J. Mod. Phys. A 28, 1330054 (2013).
H Tajima, H Liang, Phys. Rev. A 106, 043308 (2022).

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Framework

2-flavor Nambu-Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \gamma^0\hat{\mu}\right)\psi + G\left[(\bar{\psi}\psi)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

The thermodynamic potential in the mean field approximation

$$\begin{aligned}\Omega(\Delta, m) &= \frac{1}{4G}\left[(m-m_0)^2 + \Delta^2\right] - \frac{T}{V}\mathrm{Tr}\ln\mathcal{S}_{\mathrm{mf}}^{-1}, \\ &= \frac{(m-m_0)^2+\Delta^2}{4G} - 2TN_c\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left[\ln\left(1+e^{-E_k^-/T}\right)\right. \\ &\left.+\ln\left(1+e^{E_k^-/T}\right) + \ln\left(1+e^{-E_k^+/T}\right) + \ln\left(1+e^{E_k^+/T}\right)\right].\end{aligned}$$

Order parameters:

$$m = m_0 - 2G\langle\bar{\psi}\psi\rangle$$

$$\Delta = -2G\langle\bar{\psi}i\gamma_5\tau_1\psi\rangle$$

Gap equations

$$\frac{\partial \Omega}{\partial m} = \frac{\partial \Omega}{\partial \Delta} = 0$$

Gap eq 1

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{m}{E_k} \left(\frac{E_k - \mu_I/2}{E_k^-} + \frac{E_k + \mu_I/2}{E_k^+} \right) = \frac{m - m_0}{4GN_c}$$

Gap eq 2

$$\Delta \left[1 - 4N_c G \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\frac{1}{E_k^-} + \frac{1}{E_k^+} \right) \right] = 0$$

Quasi-particle energy $E_k = \sqrt{|k|^2 + m^2}$ $E_k^{\pm} = \sqrt{(E_k \pm \mu_I/2)^2 + \Delta^2}$

Order parameters $m = m_0 - 2G\langle \bar{\psi}\psi \rangle$ $\Delta = -2G\langle \bar{\psi} i\gamma_5 \tau_1 \psi \rangle$

Pauli-Villars scheme

Range of application: Uniform and non-uniform superfluid

Pauli-Villars scheme

Normalize thermodynamic potential:

$$\Omega_{reg} = G(\sigma^2 + \pi^2) - 2N_c \sum_{j=0}^{N} \sum_{i=1}^{4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} c_j g(\omega_{ij}(\mathbf{p}, \mathbf{q}))$$

$$\omega_{1j}(\mathbf{p}, \mathbf{q}) = E_{+j} + \epsilon_{-j} + \frac{\mu_B}{3}, \ \omega_{2j}(\mathbf{p}, \mathbf{q}) = E_{+j} - \epsilon_{-j} - \frac{\mu_B}{3}$$

$$\omega_{3j}(\mathbf{p}, \mathbf{q}) = E_{-j} - \epsilon_{-j} + \frac{\mu_B}{3}, \ \omega_{4j}(\mathbf{p}, \mathbf{q}) = E_{-j} + \epsilon_{-j} - \frac{\mu_B}{3}$$

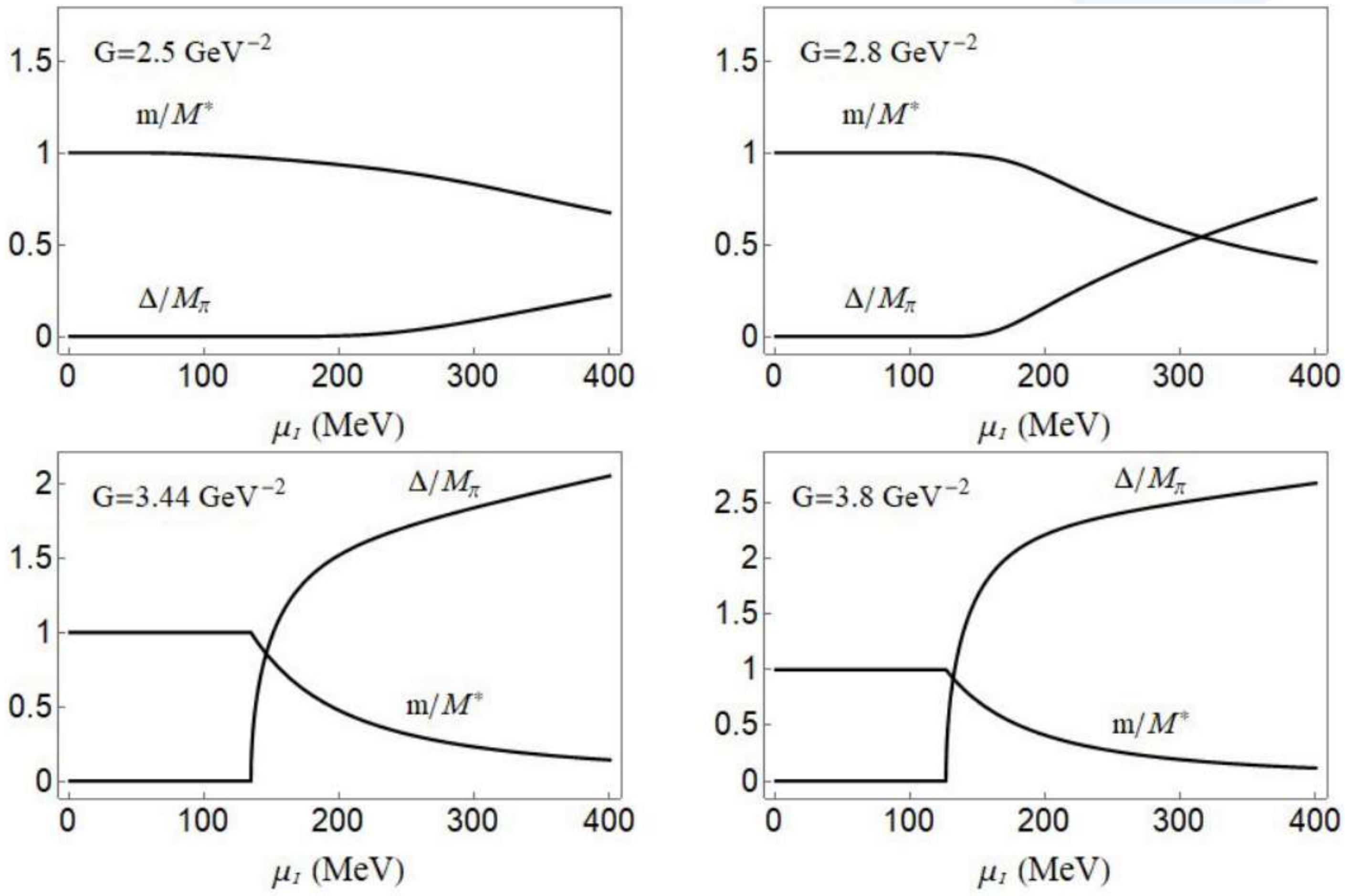
$$\epsilon_{\pm j} = \frac{1}{2}\left(\sqrt{|\mathbf{p} + \mathbf{q}|^2 + M_j^2} \pm \sqrt{|\mathbf{p} - \mathbf{q}|^2 + M_j^2}\right)$$

$$M_j^2 = m^2 + a_j\Lambda^2, \ E_{\pm j} = \sqrt{(\epsilon_{+j} \pm \mu_I/2)^2 + \Delta^2}$$

Normalize parameters:

$$a_0 = 0, \ c_0 = 1, \sum_{i=0}^{N} c_i = 0, \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2) = 0, \ldots \sum_{i=0}^{N} c_i(m^2 + a_i\Lambda^2)^{(N-1)} = 0$$

Condensates



At low isospin chemical potential region

$$\Delta = 0$$

When $m > \mu_I/2$

$$\frac{m_0 - m}{2G} + 4N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{m}{E_k} = 0$$

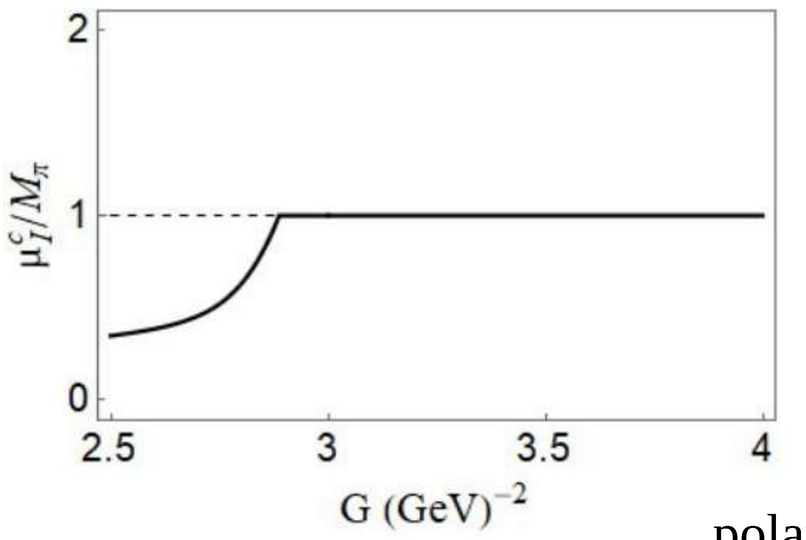
When $m < \mu_I/2$

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{m}{E_k} \left(\frac{E_k - \mu_I/2}{E_k^-} + \frac{E_k + \mu_I/2}{E_k^+} \right) = \frac{m - m_0}{4GN_c}$$

Condensates

The pion condensate begins to appear

$$1 - 4N_cG\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{E_k^-}+\frac{1}{E_k^+}\right)=0$$



$G_0 = 2.89\ \mathrm{GeV}^{-2}$

Strong coupling strength

Gap eq 2:

$$1 - 8N_cG\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{E_k}{E_k^2-(\mu_I^c)^2/4}=0$$

polarization function:

$$1 - 8N_cG\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{E_k}{E_k^2-(M_\pi)^2/4}=0$$

Critical isospin chemical potential

$\mu_I^c = M_\pi$

Weak coupling strength

Gap eq 2:

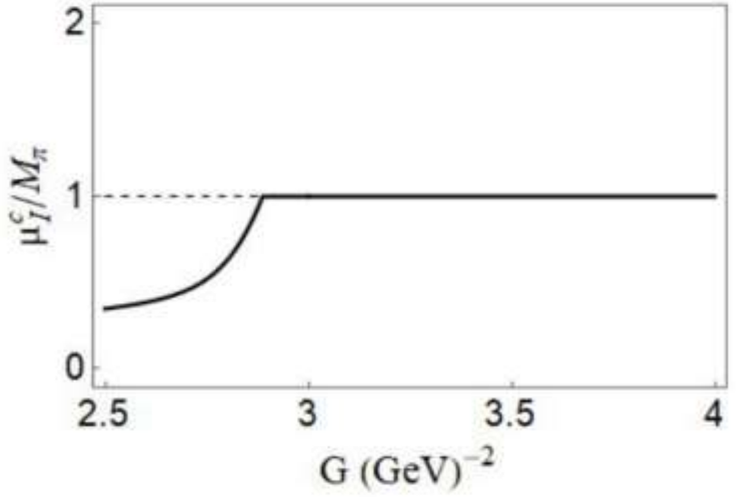
$$1 - 8N_cG\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\mu_I/2}{(\mu_I)^2/4-E_k^2} \quad E_k < \mu_I/2$$

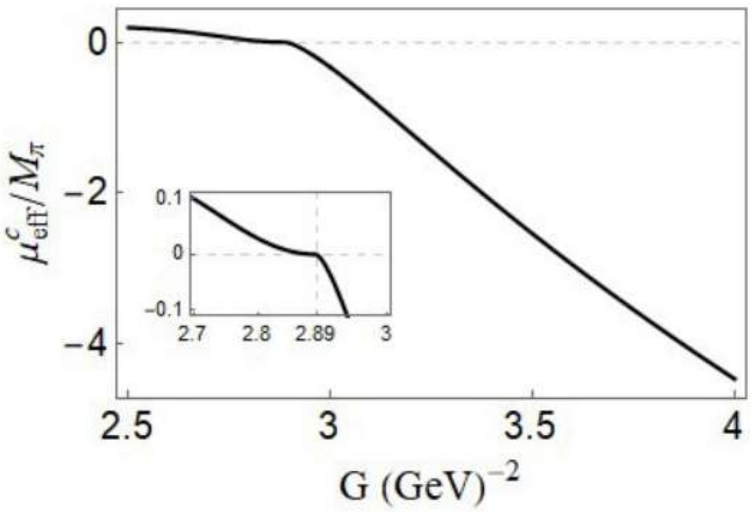
$$-8N_cG\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{E_k}{E_k^2-(\mu_I)^2/4}=0 \quad E_k > \mu_I/2$$

Critical isospin chemical potential

$\mu_I^c < M_\pi$

BCS-BEC crossover





effective chemical potential

$$\frac{\mu_{\rm eff}}{2} = \frac{\mu_I}{2} - m,$$

BCS-BEC crossover

$\mu_{eff} > 0$ ⟶ $\mu_{eff} < 0$

effective chemical potential on phase boundary

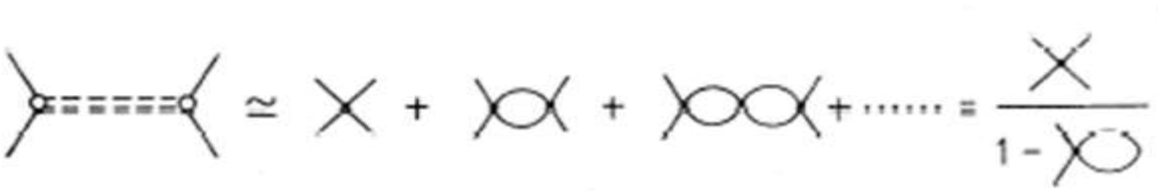
$$\mu_{eff}^c = 0$$

At $G_0 = 2.89\ \mathrm{GeV}^{-2}$

Mesons

S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
J. Hufner, S. P. Klevansky, P. Zhuang, and H. Voss, Ann. Phys. (N.Y.) 234, 225 (1994).
P. Zhuang, J. Hufner, and S. P. Klevansky, Nucl. Phys. A576, 525 (1994).

RPA resummation (quantum fluctuation)



Meson polarization functions

$$\Pi_{MM'}(p) = i\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\Gamma^*_M \mathcal{S}_{\mathrm{mf}}(p+k)\Gamma_{M'}\mathcal{S}_{\mathrm{mf}}(k)\right]$$

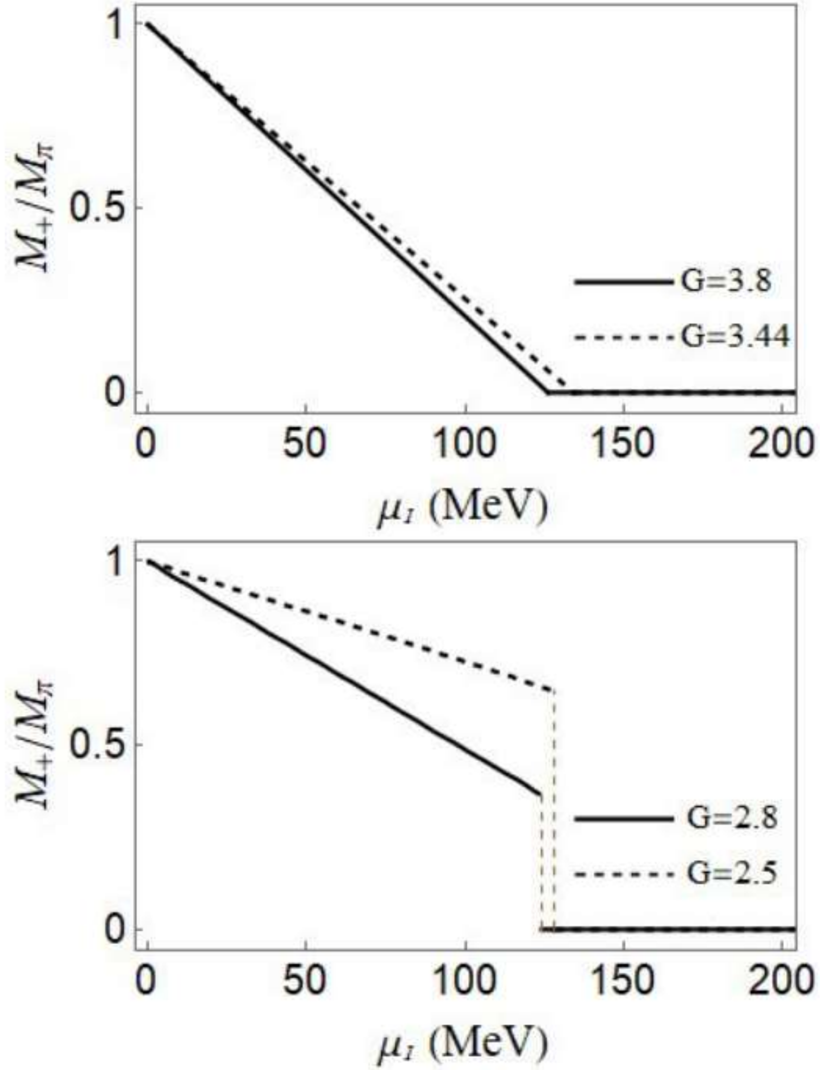
$$\Pi_{\pi_+\pi_+}(p_0) = 2N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{E_k + \frac{p_0+\mu_I}{2}} + \frac{Sign\left(E_k - \frac{\mu_I}{2}\right)}{E_k - \frac{p_0+\mu_I}{2}}\right)$$

$$\Gamma_M = \begin{cases} 1 & M=\sigma \\ i\tau_+\gamma_5 & M=\pi_+ \\ i\tau_-\gamma_5 & M=\pi_- \\ i\tau_3\gamma_5 & M=\pi_0 \end{cases}, \quad \Gamma^*_M = \begin{cases} 1 & M=\sigma \\ i\tau_-\gamma_5 & M=\pi_+ \\ i\tau_+\gamma_5 & M=\pi_- \\ i\tau_3\gamma_5 & M=\pi_0 \end{cases}.$$

The meson mass is determined through its own pole equation

$$1 - 2G\Pi_{MM}(p_0 = M_M, \mathbf{p} = \mathbf{0}) = 0$$

Goldstone mode



When $m > \mu_I/2$ Strong coupling

$$1 - 8N_cG \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{E_k}{E_k^2 - (p_0 + \mu_I)^2/4} = 0$$

$$\frac{M_{\pi_+}}{M_\pi} = 1 - \frac{\mu_I}{M_\pi}$$

When $m < \mu_I/2$ Weak coupling

$$1 - 8N_cG \int_A \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{E_k}{E_k^2 - (p_0 + \mu_I)^2/4} = 0$$

$$A = \sqrt{\left({}^{\mu_I}/_{2}\right)^2 - m^2}$$

$$\Pi_{\pi_+\pi_+}(p_0) = 2N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\frac{1}{E_k + \frac{p_0+\mu_I}{2}} + \frac{Sign\left(E_k - \frac{\mu_I}{2}\right)}{E_k - \frac{p_0+\mu_I}{2}} \right)$$

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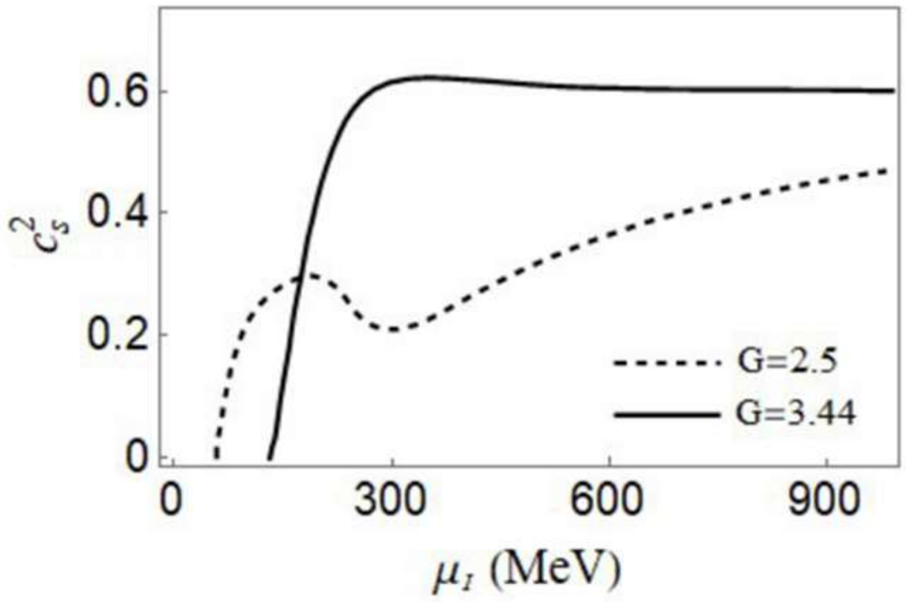
Equation of state

$$P = -\Omega, \quad s = -\frac{\partial \Omega}{\partial T}, \quad n_I = -\frac{\partial \Omega}{\partial \mu_I},$$
$$\epsilon = -P + Ts + \mu_I n_I.$$

Sound velocity

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

degree of softness or hardness



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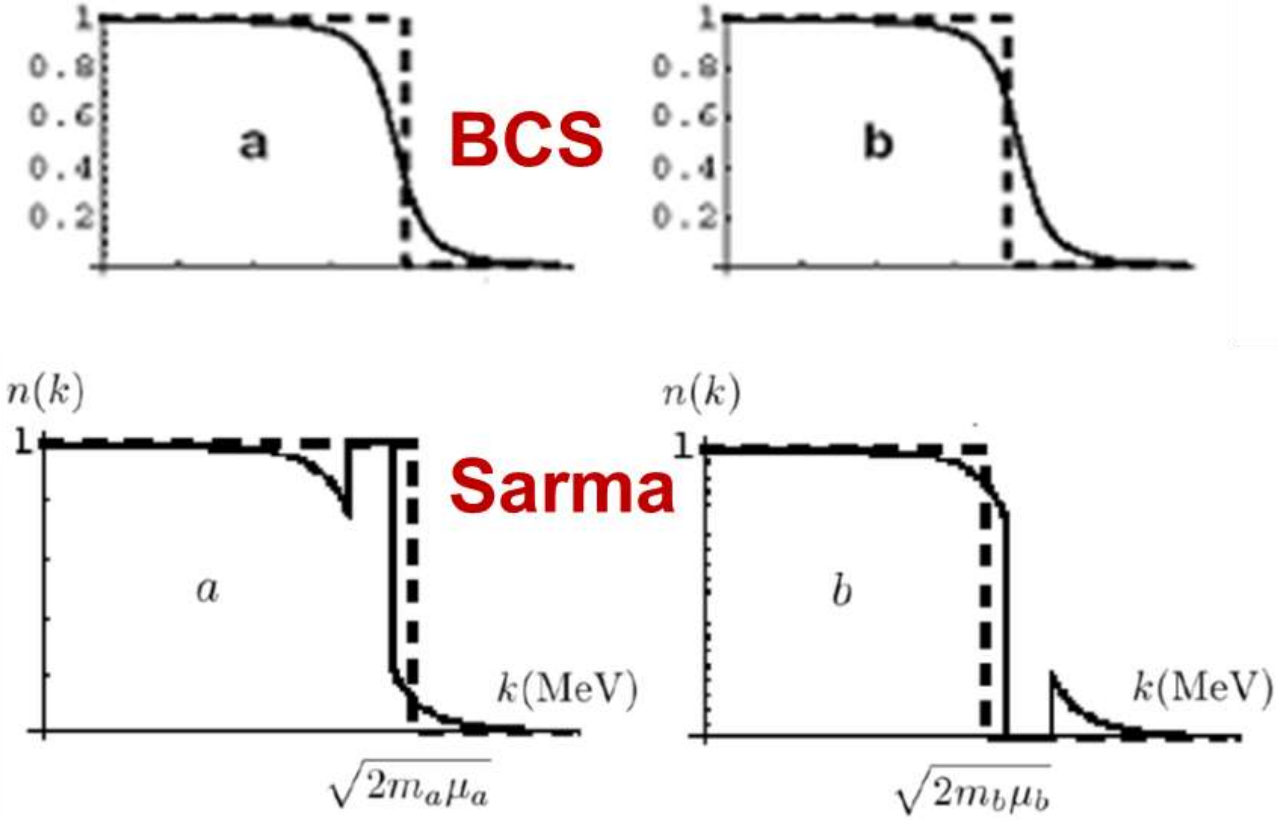
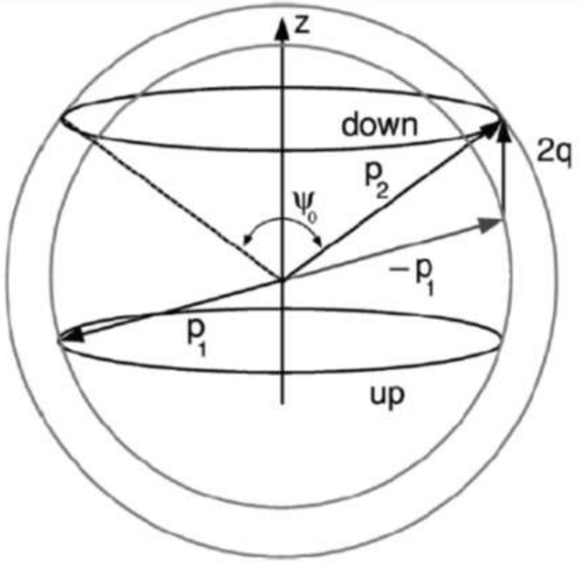
Sarma & LOFF

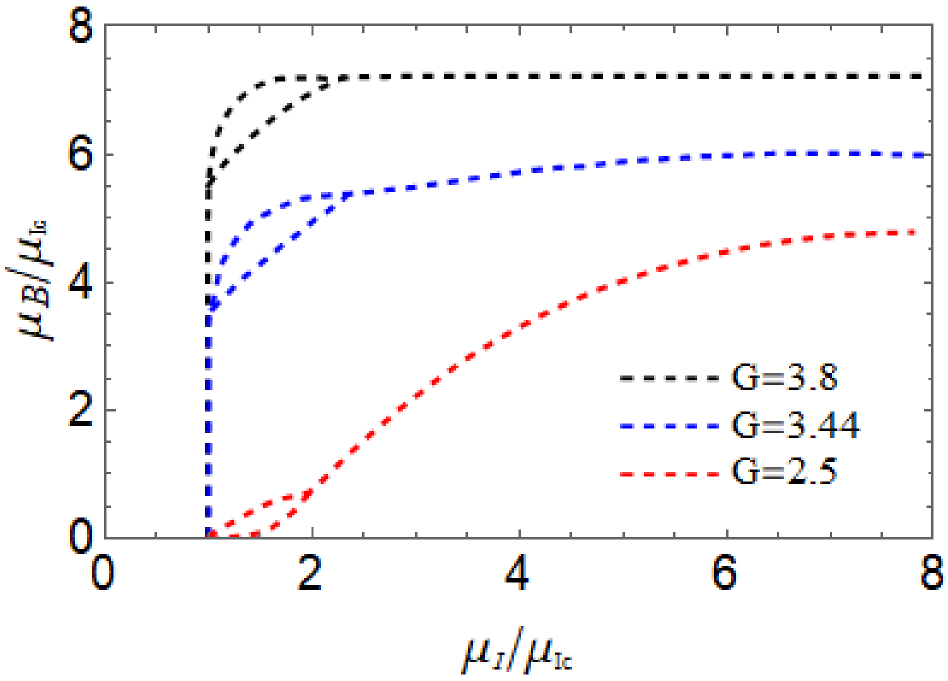


LOFF(q≠0)



$$g\langle\psi_\downarrow\psi_\uparrow\rangle = \Delta\, e^{2i\mathbf{q}\cdot\mathbf{r}},$$
$$g\langle\psi_\uparrow^\dagger\psi_\downarrow^\dagger\rangle = \Delta\, e^{-2i\mathbf{q}\cdot\mathbf{r}}$$

Sarma & LOFF



$$Min(E_k^- = \sqrt{(E_k - \mu_I/2)^2 + \Delta^2}) < \mu_B/3$$

When $m < \mu_I/2$

$$Min(E_k^- = \sqrt{(E_k - \mu_I/2)^2 + \Delta^2}) = \Delta$$

Summary

	Strong Coupling	**Weak Coupling**
Pion superfluid	BEC , Fast	BCS , Slow
Critical point	$\mu_I^c = M_\pi$	$\mu_I^c \neq M_\pi$
Goldstone mode	Continuous zero	Jump to zero
Equation of state	c_s^2 monotonous and high	c_s^2 Non-monotonic and low
Sarma	High μ_B	Low μ_B

Thank You!