Recent updates on spin polarization and spin alignment

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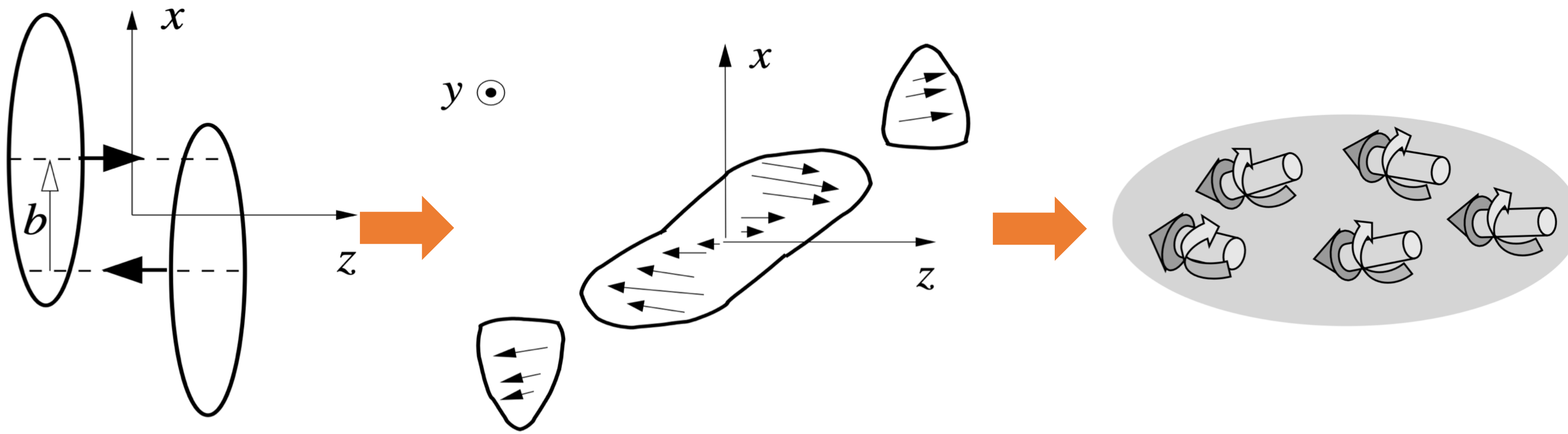
Xin-li Sheng, Shuo Fang, Shi-Zheng Yang, Xin-Qin Xie

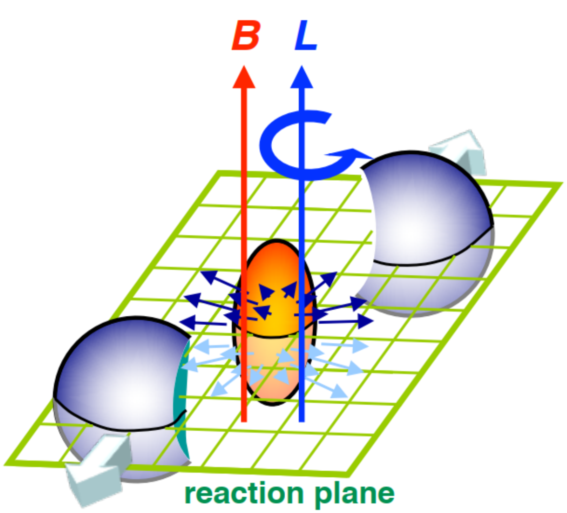
Outline

- **Introduction**
- **Spin polarization from quantum kinetic theory with self-energy corrections**
- **Recent updates on spin alignment**
  1) **Zubarev approaches at local equilibrium**
  2) **Discussion on K and D mesons**
- **Summary and outlook**

Spin polarization in HIC

OAM to polarization in HIC

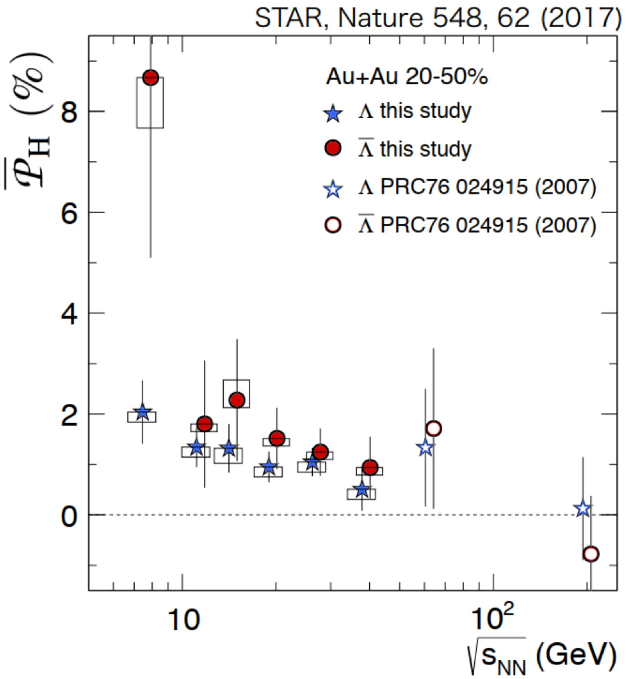




- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**
- **Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.**

  Liang, Wang, PRL (2005); PLB (2005);
  Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons

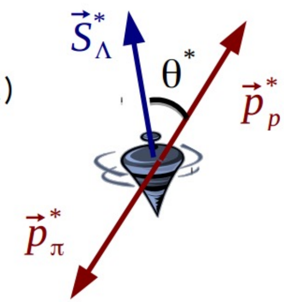


[parity-violating decay of hyperons]

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

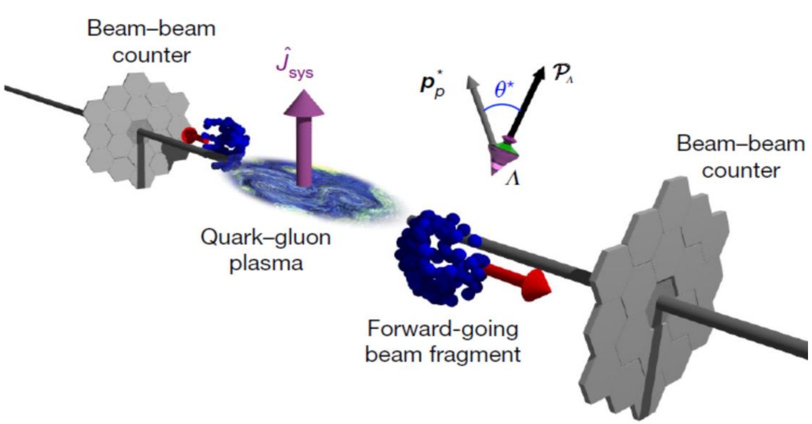
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \rightarrow p + \pi^+$$

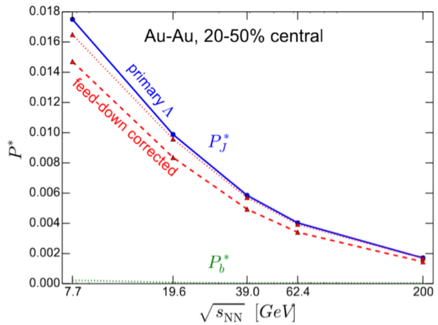
(BR: 63.9%, c τ ~7.9 cm)



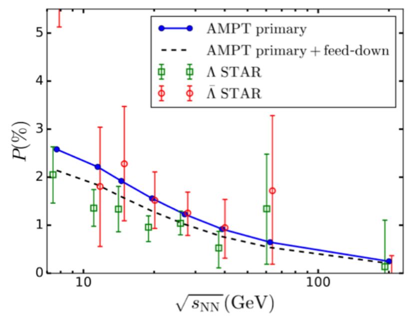
- **STAR实验结果可以反推出，QGP的涡旋可以高达ω = (9 ± 1)x10^{21}/s。**
- **这是迄今为止发现的旋转最快的系统。**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

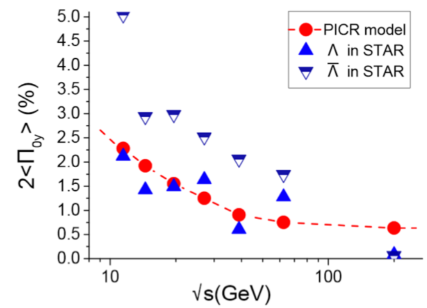
Phenomenological models for global polarization



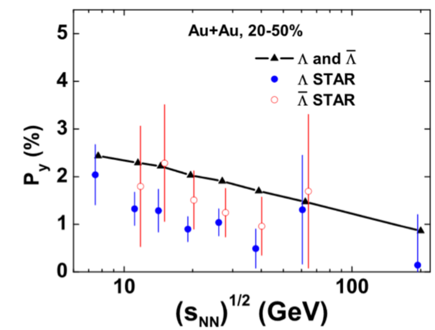
Karpenko, Becattini, EPJC(2017)



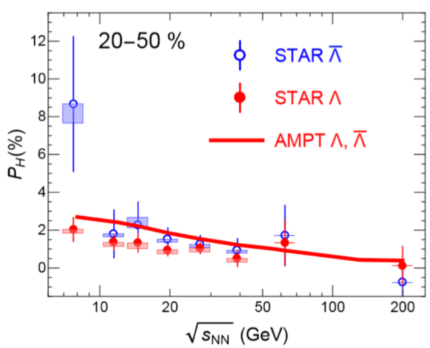
Li, Pang, Wang, Xia PRC(2017)



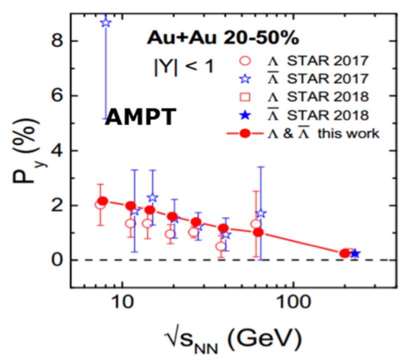
Xie, Wang, Csernai, PRC(2017)



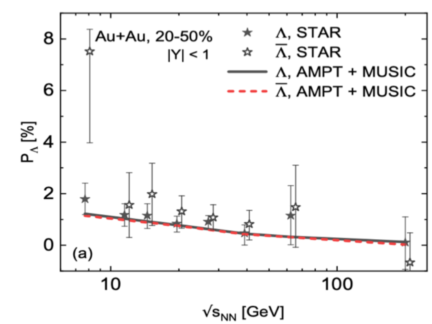
Sun, Ko, PRC(2017)



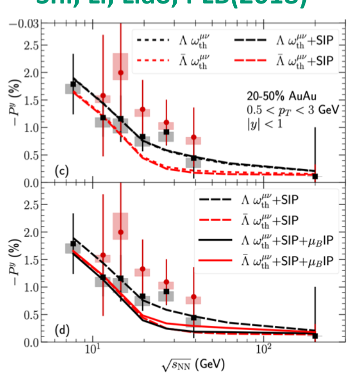
Shi, Li, Liao, PLB(2018)



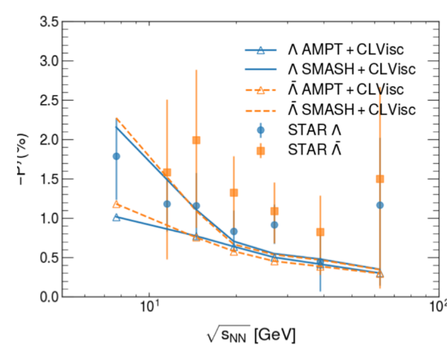
Wei, Deng, Huang, PRC(2019)



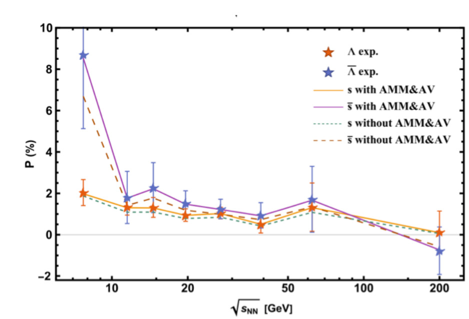
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



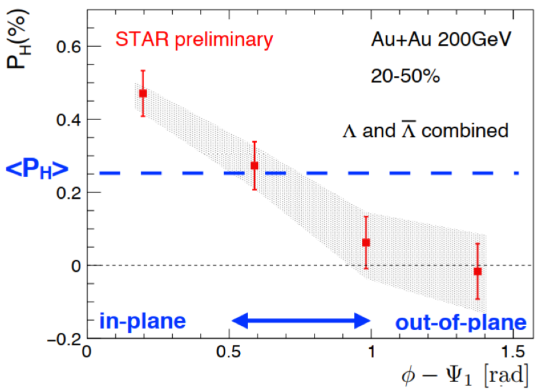
Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

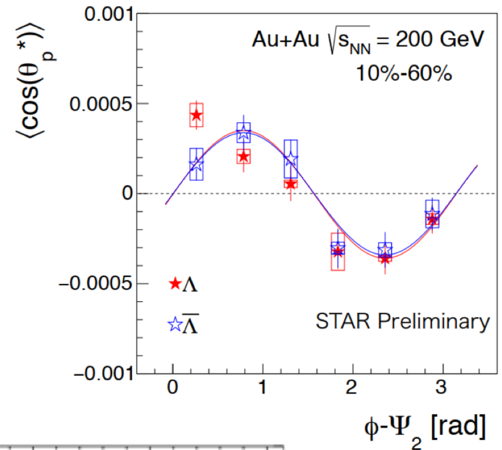


Xu, Lin, Huang, Huang, PRDL (2022)

Local polarization

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta}{(u\cdot p)T}\frac{1}{2}\{p^\sigma(\partial_\sigma u_\nu+\partial_\nu u_\sigma)-Du_\nu\}$$





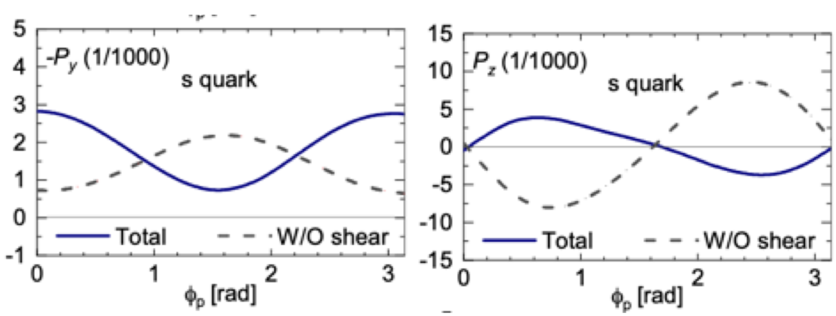
Early works:
(thermal vorticity only)

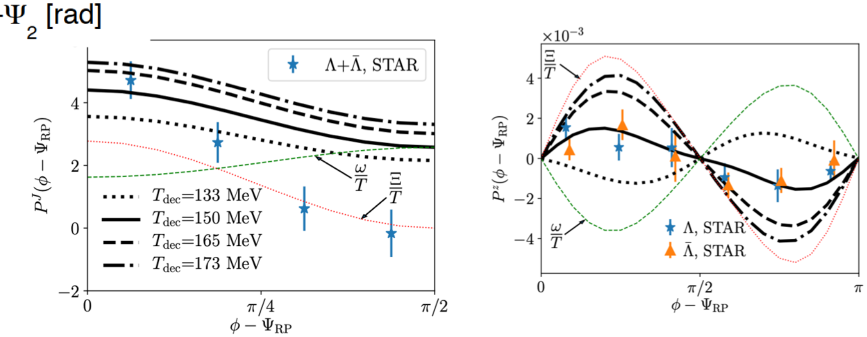
- **UrQMD :**

Becattini, Karpenko, PRL (2018)

- **AMPT:**

Xia, Li, Tang, Wang, PRC (2018)





s quark scenarios (Thermal vorticity + shear)

Fu, Liu, Pang, Song, Yin, PRL 2021

Isothermal equilibrium (Thermal vorticity + shear)

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

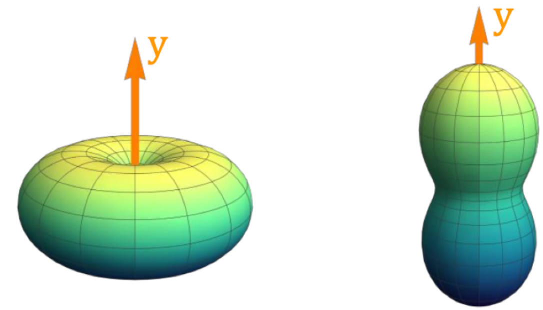
Also see:

Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022)
Ryu, Jupic, Shen, PRC (2021)

Spin alignment of vector mesons

- **Spin density matrix**

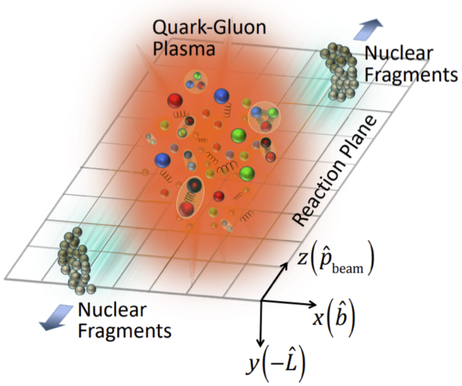
$$\rho = \begin{pmatrix} \rho_{11} & \rho_{1,0} & \rho_{1,-1} \\ \rho_{1,0}^{*} & \rho_{00} & \rho_{0,-1} \\ \rho_{1,-1}^{*} & \rho_{0,-1}^{*} & \rho_{-1,-1} \end{pmatrix}$$

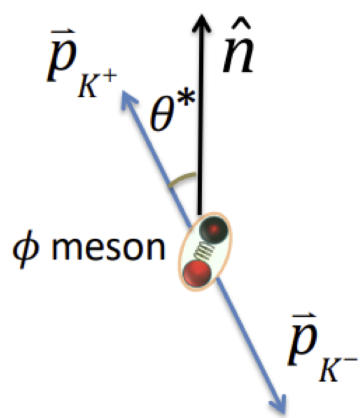


$\rho_{00} < 1/3$ 横向极化

$\rho_{00} > 1/3$ 纵向极化

- **Spin-1 meson decays to two spin-0 particles, the cross section is given by**

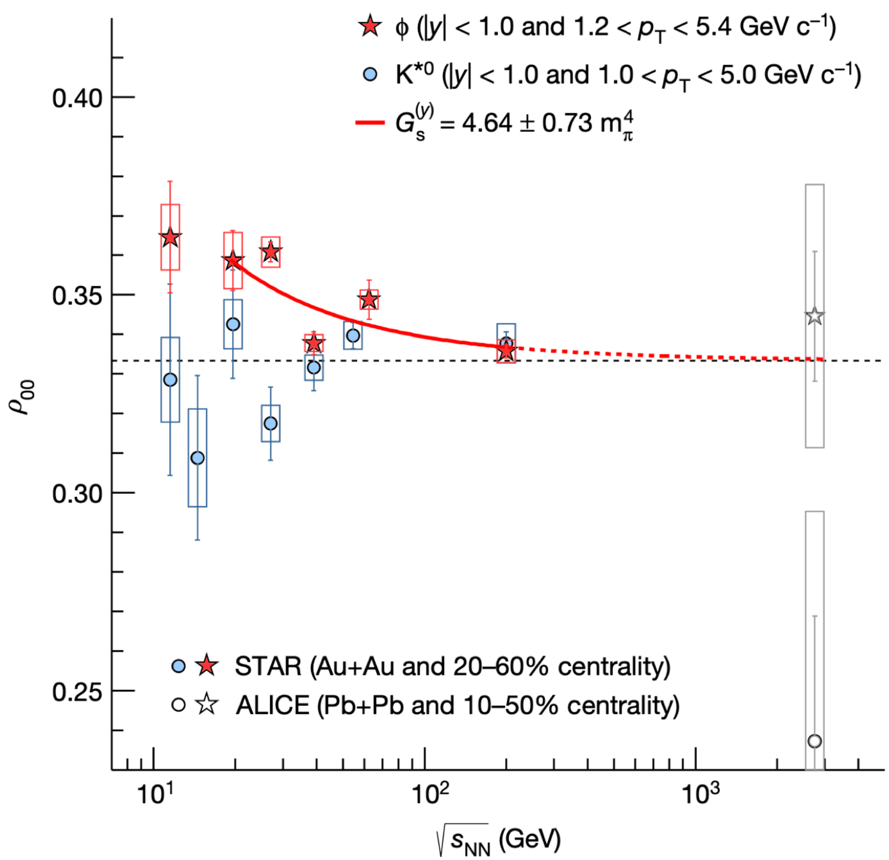




$$\frac{dN}{d(\cos\theta^*)} \propto (1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*,$$

Spin alignment for phi meson

STAR Collaboration, Nature 614, 244 (2023)



Physics Mechanisms	(ρ_{00})
c_Λ: Quark coalescence vorticity & magnetic field[1]	< 1/3 (Negative ~ 10^{-5})
c_ε: Vorticity tensor[1]	< 1/3 (Negative ~ 10^{-4})
c_E: Electric field[2]	> 1/3 (Positive ~ 10^{-5})
Fragmentation[3]	> or, < 1/3 (~ 10^{-5})
Local spin alignment and helicity[4]	< 1/3
Turbulent color field[5]	< 1/3
c_ϕ: Vector meson strong force field[6]	> 1/3

[1] Liang, Wang, PLB 629, 20 (2005); Yang et al., PRC 97, 034917 (2018); Xia et al., PLB 817, 136325 (2021); Beccattini et al., PRC 88, 034905 (2013)
[2] Sheng et al., PRC 101, 096005 (2020); Yang et al., PRC 97, 034917 (2018)
[3] Liang, Wang, PLB 629, 20 (2005);
[4] Xia et al., PLB 817, 136325 (2021); Gao, PRD 104, 076016 (2021)
[5] Muller, Yang, PRD 105, L011901 (2022)
[6] Sheng et al., PRD 101, 096005 (2020); PRD 102, 056013 (2020)

Cf. Jinfeng and Xu Sun's Talks in HENPIC200

Theoretical developments

- **Spin hydrodynamics (macroscopic approach)**

  Chapter 1

  Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  Montenegro, Tinti, Torrieri (2017-2019);
  Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); …
  S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
  Peng, Zhang, Sheng, Wang, CPL 2021

- **Quantum kinetic theory with collisions (microscopic approach)**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  Liu, Mameda, Huang, arXiv:2002.03753.
  Gao, Liang, PRD 2019
  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019) ; Z.Y. Wang, arXiv:2205.09334;
  Li ,Yee, PRD100, 056022 (2019)
  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  Fang, SP, Yang, PRD (2022)

  Chapter 2: Chinese QM

- **Other approaches:**

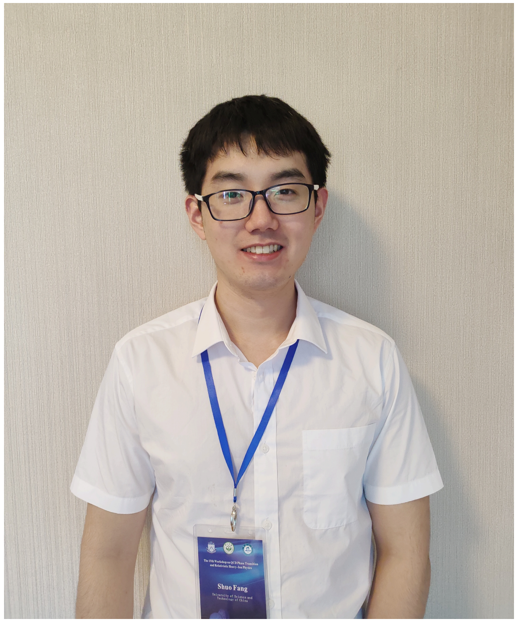
  Side-jump effect Liu, Sun, Ko PRL(2020)
  Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)

- **Recent reviews:**

  Gao, Ma, SP, Wang, NST (2020)
  Gao, Liang, Wang, IJMPA (2021)
  Hidaka, SP, Yang, Wang, PPNP (2022)

Spin polarization from quantum kinetic theory with self-energy corrections

Shuo Fang, Shi Pu, Di-Lun Yang, arXiv: 2311.15197



Cf. Shuo Fang's Talk 16:50 – 17:10,
Parallel II section: Spin polarization

Kadanoff-Baym (KB) equations

- **Given the Lagrangain in background gauge,**

$$\mathcal{L}_j = \overline{\psi}(i\hbar\gamma^\mu D_\mu - m)\psi + Qe\overline{\psi}\gamma^\mu a_\mu\psi + g\overline{\psi}\gamma^\mu \mathfrak{a}^a_\mu t^a\psi - \frac{1}{4}(\mathcal{F}^a_{\mu\nu}[\mathfrak{a}])^2 - \frac{1}{4}(F_{\mu\nu}[A+a])^2$$
$$+\mathcal{L}_{\mathrm{FP}} - \overline{\eta}\psi - \overline{\psi}\eta - j^\mu a_\mu - \mathfrak{j}^{a,\mu}\mathfrak{a}^a_\mu, \qquad \langle a_\mu\rangle = \langle \mathfrak{a}^a_\mu\rangle = 0$$

we define the connected lessor and greater Green's functions

$$S^<_{\alpha\beta}(x,y) = -\langle\overline{\psi}_\beta(y)U(y,x)\psi_\alpha(x)\rangle_{\mathrm{c}},$$
$$S^>_{\alpha\beta}(x,y) = \langle\psi_\alpha(x)U^\dagger(x,y)\overline{\psi}_\beta(y)\rangle_{\mathrm{c}},$$

$$U(A;y,x) = \mathcal{P}\exp\left[i\frac{Qe}{\hbar}\int_x^y \mathrm{d}z^\mu A_\mu(z)\right],$$

which satisfies the KB equation

$$\left\{i\hbar\gamma^\mu\left[\partial_{x,\mu} + \frac{iQe}{\hbar}(x^\nu - y^\nu)\int_0^1 \mathrm{d}s s F_{\mu\nu}(z(s))\right] - m + \Sigma^\delta(x)\right\} S^<(x,y)$$
$$= -\hbar\int_{-\infty}^{+\infty}\mathrm{d}^4z P(y,x,z)\left[\Sigma^{\mathrm{r}}_{\mathrm{g}}(x,z)S^<(z,y) + \Sigma^<_{\mathrm{g}}(x,z)S^{\mathrm{a}}(z,y)\right],$$

Master equation for Wigner functions

$$\left[\frac{i\hbar}{2}\gamma^{\mu}\nabla_{\mu}+\gamma^{\mu}\Pi_{\mu}-m+\underline{\overline{\Sigma}_{\mathrm{g}}\star}\right]S^{<}(q,X) = -\frac{i\hbar}{2}(\Sigma_{\mathrm{g}}^{>}\star S^{<}-\Sigma_{\mathrm{g}}^{<}\star S^{>}),$$

$$S^{<}\left(-\frac{i\hbar}{2}\gamma^{\mu}\overleftarrow{\nabla}_{\mu}+\gamma^{\mu}\overleftarrow{\Pi}_{\mu}-m\right)+S^{<}\underline{\star\overline{\Sigma}_{\mathrm{g}}} = -\frac{i\hbar}{2}(S^{>}\star\Sigma_{\mathrm{g}}^{<}-S^{<}\star\Sigma_{\mathrm{g}}^{>}),$$

$\star$ denotes the Moyal product

$$\overline{\Sigma}_{\mathrm{g}}(q,X) = \Sigma^{\delta}(X)+\mathrm{Re}\Sigma_{\mathrm{g}}^{\mathrm{r}}$$

For a long time, we always neglect the self-energy terms for simplicity. Now, we consider the contributions from them carefully.

Applications to spin polarization

- **We consider effects from the thermal QCD background. After a heavy calculation, we get the corrections to polarization vectors from self-energies,**

Corrections to polarization induced by:

$$\delta\mathcal{P}^{\mu}_{\mathrm{therm}}(t,\mathbf{q}) = -\frac{\hbar^2}{2mN}\int_\Sigma q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{m_f^2 T}{E_q^3}\epsilon^{\mu\nu\alpha\beta}q_\nu\partial_\alpha\left(\frac{u_\beta}{T}\right),$$

↔ **Thermal vorticity**

$$\delta\mathcal{P}^{\mu}_{\mathrm{shear}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_\Sigma q\cdot d\sigma G_{\omega_1}(E_q,\mathbf{q})\frac{m_f^2}{E_q^3}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q}q^\gamma\sigma_{\nu\gamma},$$

↔ **Shear tensor**

$$\delta\mathcal{P}^{\mu}_{\mathrm{chem}}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_\Sigma q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{C_{\mathrm{F}}g^2\mu T}{4\pi^2 E_q^2}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q}\nabla_\nu\left(\frac{\mu}{T}\right),$$

↔ **Gradient of chemical Potential over temperature**

$$\delta\mathcal{P}^{\mu}_{\mathrm{acc}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_\Sigma q\cdot d\sigma G_{\mathrm{T}}(E_q,\mathbf{q})\frac{3m_f^2}{E_q^3}\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma D u_\nu,$$

↔ **Fluid acceleration**

$$\delta\mathcal{P}^{\mu}_{\mathrm{vor}}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_\Sigma q\cdot d\sigma\frac{m_f^2}{E_q^2}\left[\omega^\mu\left(4G_{\mathrm{T}}(E_q,\mathbf{q}) - \frac{|q_\perp|^2}{E_q^2}G_{\omega_1}(E_q,\mathbf{q}) + 2G_{\omega_2}(E_q,\mathbf{q})\right)\right.$$
$$\left. -\frac{(\omega\cdot q)}{E_q}\left(6u^\mu G_{\mathrm{T}}(E_q,\mathbf{q}) + \frac{q_\perp^\mu}{E_q}G_{\omega_1}(E_q,\mathbf{q})\right)\right],$$

↔ **Kinetic vorticity**

Shuo Fang, Shi Pu, Di-Lun Yang, arXiv: 2311.15197

Estimation

It can contribute 30% to the original polarization vector in the case of low momentum.

	$\|q_\perp\| = 0.5$ GeV	$\|q_\perp\| = 1.0$ GeV	$\|q_\perp\| = 2.0$ GeV
$\|\delta\mathcal{J}^{5,\mu}_{\text{therm}}/\mathcal{J}^{5,\mu}_{\text{therm,leq}}\|$	0.325	0.098	0.024
$\|\delta\mathcal{J}^{5,\mu}_{\text{shear}}/\mathcal{J}^{5,\mu}_{\text{shear,leq}}\|$	0.081	0.028	0.007
$\|\delta\mathcal{J}^{5,\mu}_{\text{vor}}/\mathcal{J}^{5,\mu}_{\text{therm,leq}}\|$	0.177	0.103	0.030

We have chosen temperature T = 0.165 GeV, chemical potential μ = 0.01 GeV and constituent s quark mass m = 0.3 GeV.

Cf. Shuo Fang's Talk 16:50 – 17:10, Parallel II section: Spin polarization

Spin alignment

Green function for spin-1 particles

- **From the Lagrangian of spin-1 particles,**

$$\mathcal{L} = -\frac{1}{4}F_V^2 + \frac{1}{2}m_V^2 A_V^2 - A_V \cdot j,$$

we can define the two point Green functions

$$G^{\mu\nu}(x_1, x_2) = \langle T_C A_V^{\mu}(x_1) A_V^{\nu\dagger}(x_2)\rangle = \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} = \begin{pmatrix} G^F & G^< \\ G^> & G^{\overline{F}} \end{pmatrix}.$$

- **We also know that**

$$A_V^{\mu}(x) = \sum_{\lambda=0,\pm 1} \int \frac{d^3k}{(2\pi\hbar)^3} \frac{1}{2E_k^V} \left[\epsilon^{\mu}(\lambda, \mathbf{k}) a_V(\lambda, \mathbf{k}) e^{-ik\cdot x/\hbar} + \epsilon^{\mu *}(\lambda, \mathbf{k}) a_V^{\dagger}(\lambda, \mathbf{k}) e^{+ik\cdot x/\hbar}\right],$$

Wigner function related to spin density matrix

- **Wigner function is defined as,**

$$G^{<}_{\mu\nu}(x,p)=\int d^4y e^{ip\cdot y/\hbar}G^{<}_{\mu\nu}(x_1,x_2)=\int d^4y e^{ip\cdot y/\hbar}\langle A^{V\dagger}_{\nu}(x_2)A^{V}_{\mu}(x_1)\rangle,$$

- **In gradient expansion, we get**

$$\begin{aligned}G^{<}_{\mu\nu}(x,p) &= \int d^4y e^{ip\cdot y/\hbar}\langle A^{V\dagger}_{\nu}(x_2)A^{V}_{\mu}(x_1)\rangle \\ &= (2\pi\hbar)\delta(p^2-m^2)\sum_{\lambda,\lambda'}\{\theta(p^0)\epsilon_\mu(\lambda,\mathbf{p})\epsilon^*_\nu(\lambda',\mathbf{p})\underline{f_{\lambda\lambda'}(\mathbf{p})} \\ &\quad +\theta(-p^0)\epsilon^*_\mu(\lambda,-\mathbf{p})\epsilon_\nu(\lambda',-\mathbf{p})[\delta_{\lambda\lambda'}+\underline{f_{\lambda'\lambda}(-\mathbf{p})}]\}.\end{aligned}$$

where matrix-valued spin dependent distribution (MVSD) in phase space is

$$f_{\lambda\lambda'}(\mathbf{p})=\int\frac{d^4u}{2(2\pi\hbar)^3}\delta(u\cdot p)e^{-iu\cdot x/\hbar}\langle a^\dagger_V(\lambda',\mathbf{p}-\mathbf{u}/2)a_V(\lambda,\mathbf{p}+\mathbf{u}/2)\rangle,$$

Spin density matrix $\rho_{\lambda\lambda'}$ $\sim$ normalized MVSD

Tensor decomposition

- **Let us say:**

$$G^{\mu\nu} \sim \epsilon^{\mu}(\lambda)\epsilon^{*\nu}(\lambda')\rho_{\lambda\lambda'}$$

- **We can also take a tensor decomposition to G,**

$$\begin{aligned} G^{\mu\nu} \sim\ & Au^{\mu}u^{\nu} + B(g^{\mu\nu} - u^{\mu}u^{\nu}) && \text{Leading order} \\ & + \underline{a\pi^{\mu\nu}} + b(u^{\mu}h^{\nu} + u^{\nu}h^{\mu}) + \ldots && \text{1st order} \\ & + \text{2nd order terms} && \text{2nd order} \end{aligned}$$

Shear tensor

$$h^{\mu} = \{\partial^{\mu}\mu, \partial^{\mu}T, \omega^{\mu}, Du^{\mu}, \ldots\}$$

All of these are allowed, but the coefficients can be zero or negligible!

In principle, any rank-2 symmetric tensor can undergo decomposition as presented here. However, it does NOT mean that the shear tensor can induce effects. The crucial factor lies in the magnitude of the coefficient "a."

1^{st} order results

1) Zubarev approaches at local equilibrium

In Collaboration with:

Shi-Zheng Yang (杨诗正)，Xin-Qin Xie (谢心晴)，

Jian-Hua Gao, Qun Wang

Zubarev approaches

- **Spin density matrix**

$$\rho(x,f) \equiv \frac{f(x,k)}{\mathrm{Tr}\ f(x,k)}$$

spin dependent distribution

$$f^{sr}(x,k) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}e^{-iq\cdot x}\langle k+q/2, r|\hat{\rho}|k-q/2, s\rangle$$

$$= \frac{1}{(2\pi)^3}\int d^3\mathbf{q}e^{-iq\cdot x}\mathrm{Tr}(\hat{\varrho}a^{s\dagger}_{\mathbf{k}-\mathbf{q}/2}a^{r}_{\mathbf{k}+\mathbf{q}/\mathbf{2}})$$

density operator

- **In Zubarev approaches, the density operator in local equilibrium can be written as,**

Belinfante form

$$\hat{\varrho}_B = \frac{1}{Z_B}\exp\left[-\int_\Sigma d\Sigma_\mu \hat{T}^{\mu\nu}_B(y)\beta_\nu(y)\right],$$

Canonical form

$$\hat{\varrho}_C = \frac{1}{Z_C}\exp\left[-\int_\Sigma d\Sigma_\mu \hat{T}^{\mu\nu}_C(y)\beta_\nu(y) - \frac{1}{2}\Omega_{\lambda\nu}(y)\hat{S}^{\mu\lambda\nu}(y)\right]$$

Zubarev 1979; Becattini 2015

Diagonal and off-diagonal components

- **We expand all the operators in terms of creation and annihilate operators. After a straightforward and tedious calculations, we get**

$$\rho_{sr}(x,k) = \frac{f_{sr}^{(0)} + f_i^{(1)sr}}{f_{++}^{(0)} + f_{00}^{(0)} + f_{--}^{(0)}} = \frac{1}{3}\delta^{sr} + \frac{f_i^{(1)sr}}{3f_B}.$$

Belinfante form

$$f_B^{(1)sr}(x,k) = -i\omega_{\lambda\nu} f_0' \left(u^\nu \frac{m}{E_k+m}\Psi_{sr}^{\lambda} + \frac{1}{2}\Psi_{sr}^{\nu\lambda} \right) + i\xi_{\lambda\nu} f_0' k^\nu \frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\lambda}$$
$$+ i\xi_{\lambda\nu} f_0' \sigma_{0\gamma}' \left(k^\lambda u^\nu \frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\gamma} + k^\lambda \bar{k}^\gamma \frac{1}{E_k^2}\frac{m}{E_k+m}\Psi_{sr}^{\nu} - k^\lambda \frac{1}{E_k}\Psi_{sr}^{\gamma\nu} \right).$$

vorticial tensor $\omega_{\lambda\nu} = -\frac{1}{2}(\partial_\lambda \beta_\nu - \partial_\nu \beta_\lambda),$

shear tensor $\xi_{\lambda\nu} = \frac{1}{2}(\partial_\lambda \beta_\nu + \partial_\nu \beta_\lambda),$

$$\Psi_{sr}^{\lambda} = \frac{k\cdot\epsilon^s}{m}\epsilon^{r\lambda *} - \frac{k\cdot\epsilon^{r*}}{m}\epsilon^{s\lambda}$$

$$\Psi_{sr}^{\nu\lambda} = \epsilon^{r\nu *}\epsilon^{s\lambda} - \epsilon^{r\lambda *}\epsilon^{s\nu}$$

Can we observe it?

- **But!**

Cross section

$$\rho_{00} = 0,$$
$$\rho_{+0} - \rho_{0-} = 0,$$
$$\rho_{+-} = 0,$$

$$\frac{dN}{d\cos\theta^* d\phi^*} = \frac{3}{8\pi}\left[1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta^* - \sqrt{2}\mathrm{Re}\,(\rho_{+0} - \rho_{0-})\sin\theta^*\cos\phi^* + \sqrt{2}\mathrm{Im}\,(\rho_{+0} - \rho_{0-})\sin\theta^*\sin\phi^* - 2\mathrm{Re}\rho_{+-}\sin^2\theta^*\cos(2\phi^*) - 2\mathrm{Im}\rho_{+-}\sin^2\theta^*\sin(2\phi^*)\right]$$

There are no contributions in the local equilibrium!

- **Parity + Time reversal (PT) - Symmetry analysis:**

$$f_i^{(1)sr}(x,k) = -(-1)^{r+s} f_i^{(1)-r-s}(x,k), (i = B, C)$$
$$f_i^{(1)++}(x,k) + f_i^{(1)00}(x,k) + f_i^{(1)--}(x,k) = 0,$$
$$f_i^{(1)00}(x,k) = f_i^{(1)+-}(x,k) = f_i^{(1)-+}(x,k) = 0.$$

Private discussion with Zhong-hua Zhang in Xuguang's group

Beyond the local equilibrium

Linear response theory for spin alignment of vector mesons in thermal media

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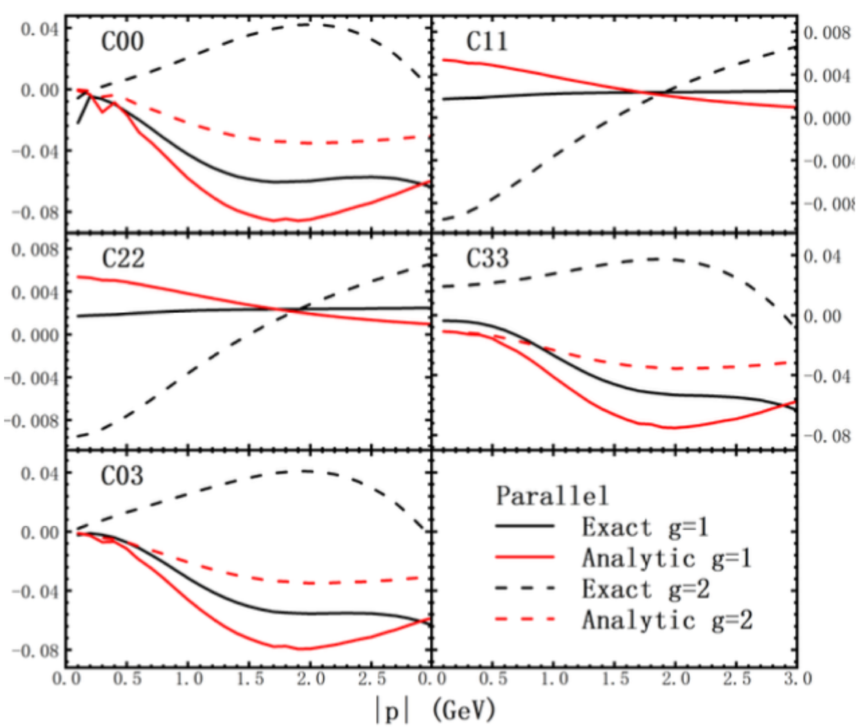
(Based on: arXiv:2311.18400v1)

Cf. Wen-Bo Dong's poster



$$\rho_{00} = C^{\mu\nu}\pi_{\mu\nu}$$

$C^{\mu\nu} \sim 10^{-2} - 10^{-3}$

Shear tensor $\pi_{\mu\nu} \sim 10^{-2}$ from hydro simulations

Total contribution is $10^{-4} - 10^{-5}$.

W.B. Dong, Y.L. Yin, X.L Sheng, S.Z. Yang, Q. Wang, arXiv:2311.18400

Puzzle : T-odd/T-even VS dissipative/non-dissipative

Are shear induced polarization or spin alignment non-dissipative?

Q1: If the coefficient is T-even, it is non-dissipative.

CME $\mathbf{j} \sim C\mathbf{B}$ — Taking T transformation → $\mathbf{j} \to -\mathbf{j}$, $\mathbf{B} \to -\mathbf{B}$ — C: T-even ✓

Spin polarization vector $\mathcal{S}^i \sim C^{ijk}(\partial_j u_k + \partial_k u_j)$ → $\mathcal{S}^i \to -\mathcal{S}^i$, $\partial_j u_k \to -\partial_j u_k$ — C: T-even. Non-dissipative?

Spin alignment $\epsilon^i(\lambda)\epsilon^{*j}(\lambda')\rho_{\lambda\lambda'} \sim a\pi^{ij}$ →

$$\rho_{\lambda\lambda'} \to (-1)^{\lambda+\lambda'}\rho_{\lambda\lambda'}$$

$$\epsilon_\mu^{s*}(\mathbf{p}) \to -(-1)^s \tilde{\delta}_\mu^\alpha \epsilon_\alpha^{-s*}(-\mathbf{p})$$

$$\pi^{ij} \to -\pi^{ij}$$

For ρ_{00}, the coefficient "a" is T-odd. Dissipative?

In Zubarev approach, the non-dissipative means the results does NOT depend on hypersurface. But, shear tensor comes from local equilibrium operators and should always depends on hypersurface. So, shear induced something is always dissipative?

Estimation corrections in 2nd order

$$G^{\mu\nu} \sim Au^\mu u^\nu + B(g^{\mu\nu} - u^\mu u^\nu)$$ **Leading order**

$$+a\pi^{\mu\nu} + b(u^\mu h^\nu + u^\nu h^\mu) + ...$$ **1st order**

$$+\text{2nd order terms}$$ **2nd order**

$$h^\mu = \{\partial^\mu \mu, \partial^\mu T, \omega^\mu, Du^\mu, ...\}$$

Possible sources:

1) **Coupling between the hydrodynamic quantities,**
   e.g. square of thermal vorticity
   Hydrodynamic quantities 10^{-2}, totally, it gives 10^{-4}
   C. Yi, X.Y Wu, SP, G.Y. Qin, in preparation

2) **Vector meson strong force**
   Sheng et al., PRD 101, 096005 (2020); PRD 102, 056013 (2020); PRC 2023

Other sources beyond 2nd order:
Early glamsa, scattering, self-energies corrections …

2) Discussion on K and D mesons

In Collaboration with:

Shuo Fang, Xu-Guang Huang, Qun Wang

Spin alignment from vector meson strong force

- **Spin density matrix for ϕ mesons given by spin Boltzmann equation:**

$$s+\bar{s} \rightleftharpoons \phi$$

$$\begin{aligned}\rho^{\phi}_{\lambda_1\lambda_2}(x,\mathbf{p}) \propto & \frac{\Delta t}{32}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{s}}_{p'}E^{s}_{\mathbf{p}-\mathbf{p}'}E^{\phi}_{p}}f_{\bar{s}}(x,\mathbf{p}')f_{s}(x,\mathbf{p}-\mathbf{p}') \\ & \times 2\pi\hbar\delta\left(E^{\phi}_{p}-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'}\right)\epsilon^{*}_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p}) \\ & \times \mathrm{Tr}\left\{\Gamma^{\beta}(p'\cdot\gamma-m_{\bar{s}})\left[1+\gamma_5\gamma\cdot P^{\bar{s}}(x,\mathbf{p}')\right]\Gamma^{\alpha}\right. \\ & \left.\times\left[(p-p')\cdot\gamma+m_s\right]\left[1+\gamma_5\gamma\cdot P^{s}(x,\mathbf{p}-\mathbf{p}')\right]\right\},\end{aligned}$$

Spin polarization of s and $\bar{s}$ quarks

Sheng, Oliva, Liang, Q. Wang, and X.-N. Wang, PRD (2022); PRL (2023)

Chiral quark model

$$\mathcal{L} = \overline{\psi}(i\gamma \cdot \partial - \gamma \cdot g_v \phi_v - M)\psi, \qquad \psi = (u, d, s)^T$$

$$\phi_{v,\mu} = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*,+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*,0} \\ K^{*,-} & \overline{K}^{*,0} & \phi \end{pmatrix}_\mu \quad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

A. Manohar, H. Georgi, Nucl. Phys. B 234, 189 (1984).

Effective Hamiltonian

- **By using the effective field theories, we derive the effective Hamiltonian for SU(3) case,**

LO + NLO

$$H''_{\mathrm{fd}} = H_{\mathrm{mass}} + H_{\mathrm{Coulomb}} + H_{\mathrm{meson\text{-}couplings}} + H_{\mathrm{spin\text{-}magnetic}} + H_{\mathrm{electric\text{-}dipole}} + H_{\mathrm{spin\text{-}orbtial}} + H_{\mathrm{Darwin}}$$

It contains dozens of terms.

$$H_{\mathrm{spin\text{-}magnetic}} = -\frac{\hbar}{2}\begin{pmatrix} \frac{1}{\sqrt{2}\hat{m}}\beta\mathbf{\Sigma}\cdot(\mathbf{B}_{\rho^0}+\mathbf{B}_{\omega}) & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}\hat{m}}\beta\mathbf{\Sigma}\cdot(-\mathbf{B}_{\rho^0}+\mathbf{B}_{\omega}) & 0 \\ 0 & 0 & \frac{1}{m_s}\beta\mathbf{\Sigma}\cdot\mathbf{B}_{\phi} \end{pmatrix}$$

NNLO

It contains 200 - 300 terms.

Roughly discussion on ρ meson

- **If we only consider the contributions from quark coalescence, and we assume** $F^{\mu\nu}_{\rho/\omega} \sim F^{\mu\nu}_{\phi}$

$$
\begin{aligned}
\bar{\rho}^{\rho}_{00} = \frac{1}{3} + \hbar^2 C_1 \Bigg[-\frac{1}{3}\left(\omega^2 - \frac{2g_V^2}{M_V^2 T^2}\left(B_\rho^2 + B_\omega^2\right)\right) - |\epsilon\cdot\omega|^2 \\
+ \frac{g_V^2}{M_V^2 T^2}\left(|\epsilon\cdot B_\rho + \epsilon\cdot B_\omega|^2 + |\epsilon\cdot B_\rho - \epsilon\cdot B_\omega|^2\right)\Bigg] \\
+ \hbar^2 C_2 \Bigg[-\frac{1}{3}\left(\varepsilon^2 - \frac{2g_V^2}{M_V^2 T^2}(E_\rho^2 + E_\omega^2)\right) - |\epsilon\cdot\varepsilon|^2 \\
+ \frac{g_V^2}{M_V^2 T^2}\left(|\epsilon\cdot E_\rho + \epsilon\cdot E_\omega|^2 + |\epsilon\cdot E_\rho - \epsilon\cdot E_\omega|^2\right)\Bigg]
\end{aligned}
$$

then, spin alignment for ρ meson is close to ϕ meson.

$$\delta\rho_{00}(\rho) \sim \delta\rho_{00}(\phi) \qquad \delta\rho_{00} = |\rho_{00} - 1/3|$$

But, scattering during the evolution may decrease it!

For scattering effects,
Cf. Yi-Liang Yin's poster

Suppressed spin alignment for K* and D* mesons

- In LO+NLO, the coupling between (u,d,s) and K* does not exist. The spin alignment for K* is highly suppressed.

- In NNLO, we can estimate the at 62.4 GeV

$$\frac{\text{s quark polarization induced by } K^*}{\text{s quark polarization induced by } \phi} \sim \left|\frac{\mathcal{P}^{\mu}_{s,K^{*0}}}{\mathcal{P}^{\mu}_{s,\phi}}\right| \sim \quad v_s^3 \sim 0.187 \ll 1,$$

- Eventually, we get

$$\delta\rho_{00}(K^*) \ll \delta\rho_{00}(\phi) \qquad \delta\rho_{00} = |\rho_{00} - 1/3|$$

Similarly, if we extend the discussion to SU(4) case (with c quark), we can also comment on D*

$$\delta\rho_{00}(D^*) \ll \delta\rho_{00}(J/\psi)$$

Summary and outlook

Summary (1)

- **We calculate the corrections from self-energies to the spin polarization.**

Cf. Shuo Fang's Talk 16:50 – 17:10, Parallel II section: Spin polarization

	$\vert q_\perp\vert = 0.5$ GeV	$\vert q_\perp\vert = 1.0$ GeV	$\vert q_\perp\vert = 2.0$ GeV
$\vert\delta\mathcal{J}^{5,\mu}_{\text{therm}}/\mathcal{J}^{5,\mu}_{\text{therm,leq}}\vert$	0.325	0.098	0.024
$\vert\delta\mathcal{J}^{5,\mu}_{\text{shear}}/\mathcal{J}^{5,\mu}_{\text{shear,leq}}\vert$	0.081	0.028	0.007
$\vert\delta\mathcal{J}^{5,\mu}_{\text{vor}}/\mathcal{J}^{5,\mu}_{\text{therm,leq}}\vert$	0.177	0.103	0.030

Summary (2)

- We revisited the spin density matrix by tensor decomposition:

$$G^{\mu\nu} \sim \epsilon^{\mu}(\lambda)\epsilon^{*\nu}(\lambda')\rho_{\lambda\lambda'}$$

$$\begin{aligned} G^{\mu\nu} \sim\ & Au^{\mu}u^{\nu} + B(g^{\mu\nu} - u^{\mu}u^{\nu}) \\ & + a\pi^{\mu\nu} + b(u^{\mu}h^{\nu} + u^{\nu}h^{\mu}) + ... \\ & + \text{2nd order terms} \end{aligned}$$

Leading order

1st order

2nd order

$$h^{\mu} = \{\partial^{\mu}\mu, \partial^{\mu}T, \omega^{\mu}, Du^{\mu}, ...\}$$

- By using Zubarev approaches, we get the 1st order corrections to ρ_{00} vanishes at local equilibrium.

Summary (3)

- **We analyze the spin alignment of vector meson based on the chiral quark model.**
- **In quark coalescence case and assuming** $F^{\mu\nu}_{\rho/\omega} \sim F^{\mu\nu}_{\phi}$

$$\delta\rho_{00}(\rho) \sim \delta\rho_{00}(\phi) \qquad \delta\rho_{00} = |\rho_{00} - 1/3|$$

- **The spin alignment of K* are suppressed,**

$$\delta\rho_{00}(K^*) \ll \delta\rho_{00}(\phi) \qquad \delta\rho_{00} = |\rho_{00} - 1/3|$$

Thank you!

Backup

Results in canonical form

$$
\begin{aligned}
f_C^{(1)sr}(x,k) = & \frac{i}{2}\omega_{\lambda\nu} f_0' i k^\nu \frac{1}{E_k}\frac{E_k - m}{E_k+m}\Psi_{sr}^{\lambda} - \frac{i}{2}\xi_{\lambda\nu} f_0' i k^\nu \frac{1}{E_k}\frac{E_k - m}{E_k+m}\Psi_{sr}^{\lambda} \\
& -\frac{i}{2}\Omega_{\lambda\nu} f_0' \left(\Psi_{sr}^{\nu\lambda} + u^\nu \frac{2m}{E_k+m}\Psi_{sr}^{\lambda} + k^v \frac{1}{E_k}\frac{E_k-m}{E_k+m}\Psi_{sr}^{\lambda}\right) \\
& +\frac{i}{2}(\Omega_{\lambda\nu} - \omega_{\lambda\nu}) f_0' \sigma_{0\gamma}' \left(k^\nu u^\lambda \frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\gamma} + \bar{k}^\gamma k^\nu \frac{1}{E_k}\frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\lambda} - \frac{1}{E_k} k^\nu \Psi_{sr}^{\gamma\lambda}\right) \\
& +\frac{i}{2}\xi_{\lambda\nu} f_0' \sigma_{0\gamma}' \left(k^\lambda u^\nu \frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\gamma} + k^\lambda \bar{k}^\gamma \frac{1}{E_k}\frac{1}{E_k}\frac{m}{E_k+m}\Psi_{sr}^{\nu} - \frac{1}{E_k} k^\lambda \Psi_{sr}^{\gamma\nu}\right).
\end{aligned} \tag{70}
$$

ϕ meson VS ρ meson

$\rho(770)$ WIDTH

We no longer list S-wave Breit-Wigner fits, or data with high combinatorial background.

NEUTRAL ONLY, e^+e^-

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
147.4 ±0.8	**OUR AVERAGE**	Error includes scale factor of 2.0.		See the ideogram below.

$\rho(770)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\pi\pi$	$\sim$ 100 %	

$\phi(1020)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
4.249±0.013 OUR AVERAGE		Error includes scale factor of 1.1.		

$\phi(1020)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	K^+K^-	(49.1 ±0.5) %	S=1.3
Γ_2	$K^0_L K^0_S$	(33.9 ±0.4) %	S=1.2

PDG book