A topological realization of spin polarization through vortex formation in collisions of BECs

------从BEC对撞产生涡旋到自旋极化的拓扑实现

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In collaboration with Qun Wang, Hong Zhang, Sen-Yue Lou, Hao-Hao Peng, Soeren Schlichting and Raju Venugopalan.

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Commun. Theory. Phys. 74 (2022) 9, 095002. [2207.13360]
Phys. Rev. A 97 (2018) 5, 053606. [1801.06260]

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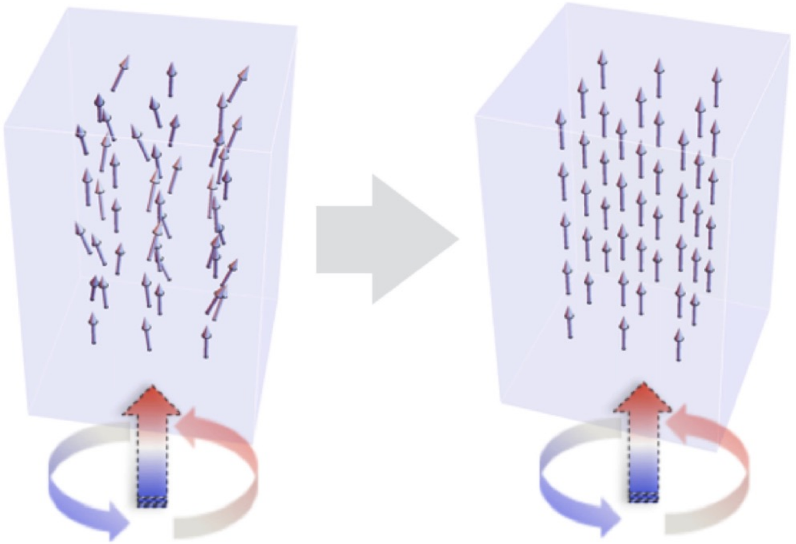
Outlines

1. Motivation and background
2. Global polarization in HICs
3. Vortex Dynamics in cold atoms
4. Vortex formation in BEC collisions
5. Summary and outlook

Polarization/Magnetization and Rotation

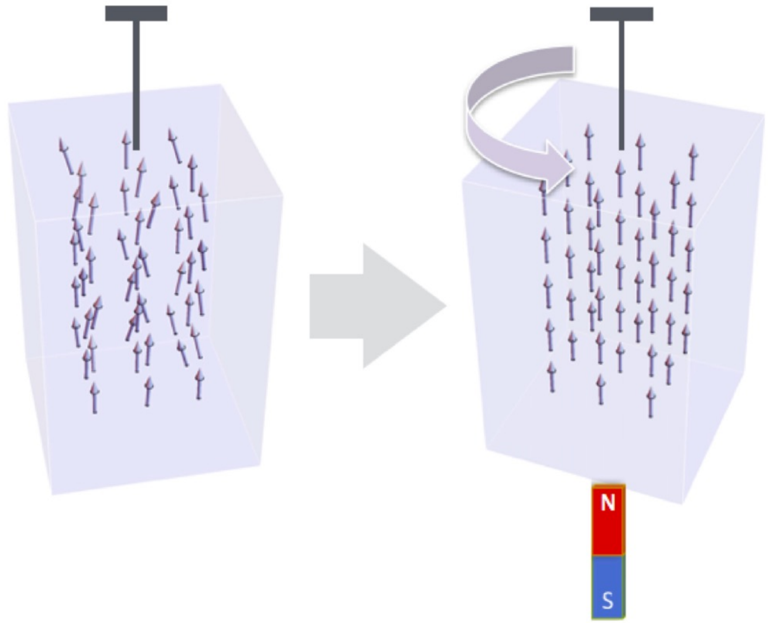
Barnett effect

spin-orbital coupling



Einstein-de Haas effect

Angular momentum conservation



Magnetization by Rotation,

S. J. Barnett, Phys. Rev. 6, 239 (1915)

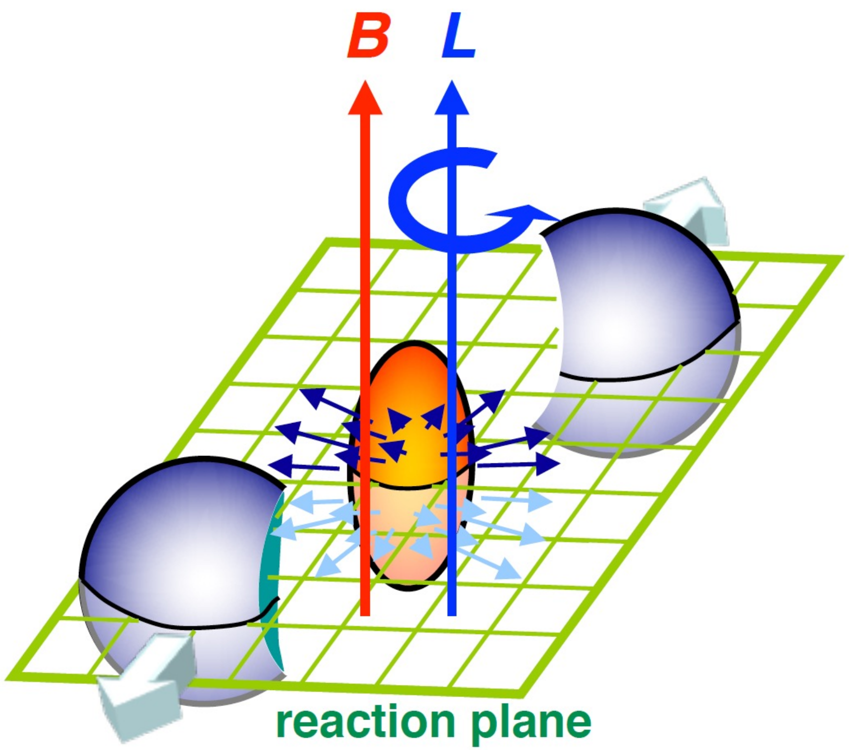
Gyromagnetic and Electron-Inertia Effect

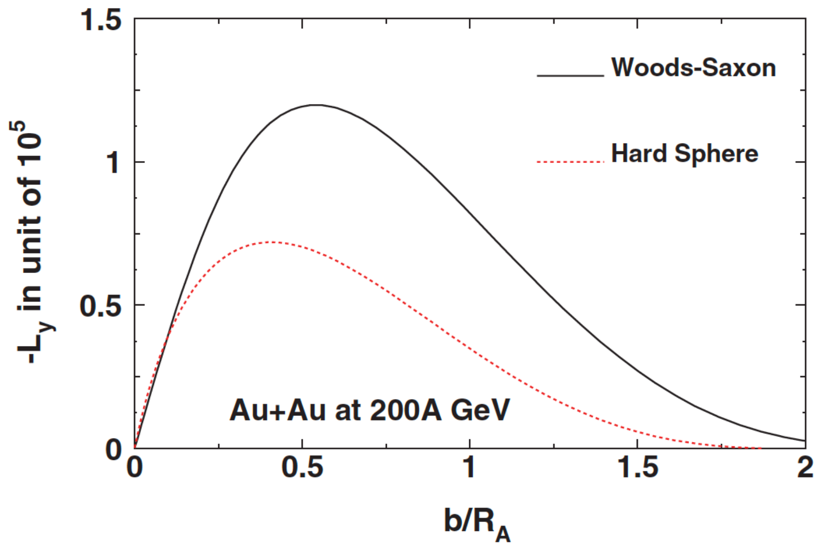
S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)

Experimental Proof of Ampere's Molecular Currents,

A. Einstein and W. J. de Haas, *Deutsche Physikalische Gesellschaft, Verhandlungen* (in German). **17**: 152–170 (1915)

Large OAM in noncentral HICs



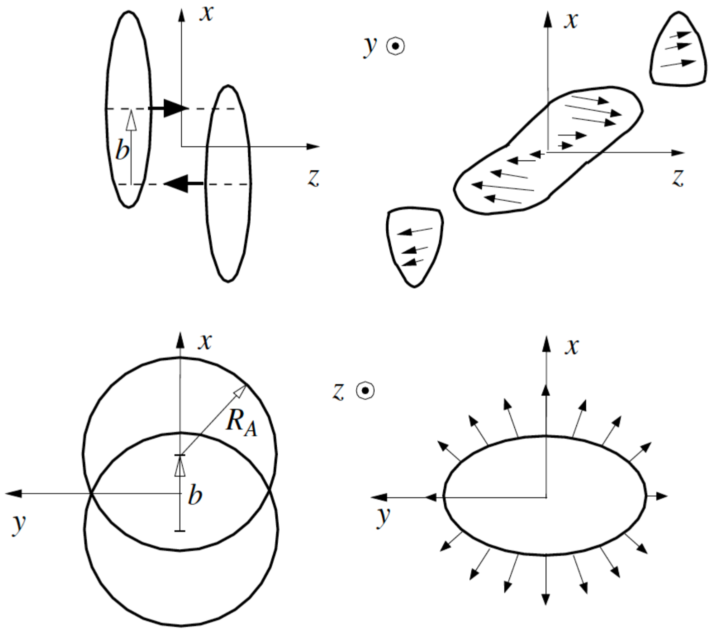


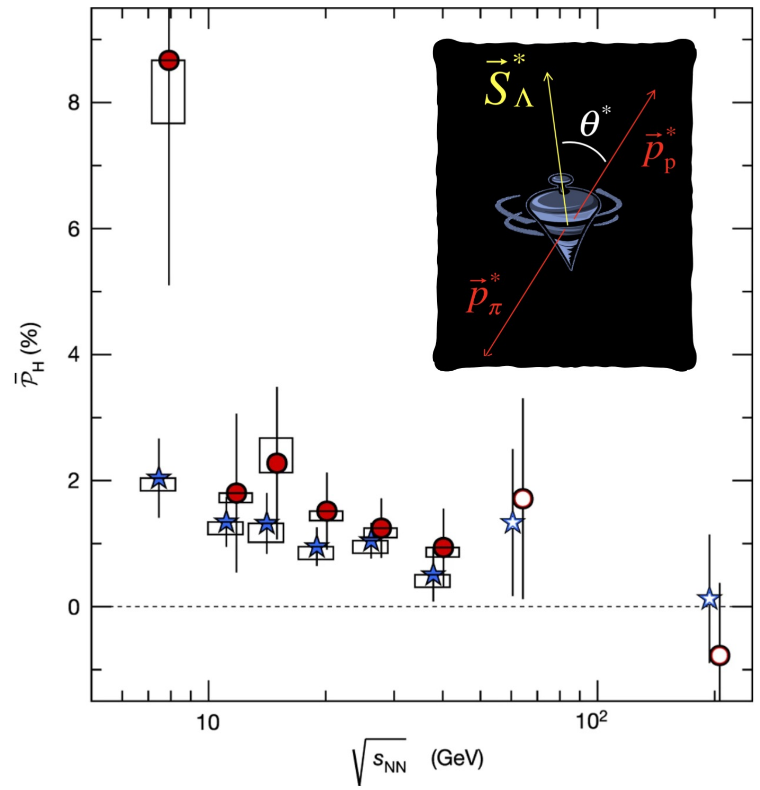
J.H. Gao, et. al, PRC 77, 044902 (2008)

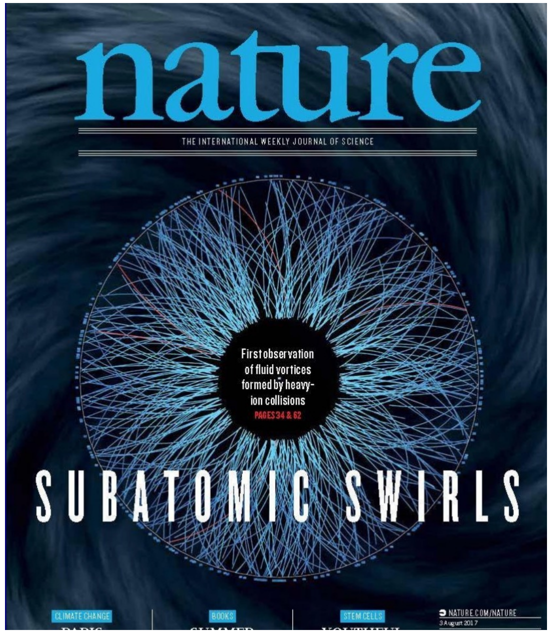
Global polarization: from prediction to measurements

Globally Polarized Quark-Gluon Plasma in noncentral A-A Collisions

Zuo-Tang Liang and Xin-Nian Wang, PRL 94, 102301(2005)







$\omega = (P_\Lambda + P_{\bar{\Lambda}}) k_B T/\hbar \sim 10^{22} s^{-1}$

Spin alignment of vector mesons

Zuo-Tang Liang and Xin-Nian Wang, PLB 629, 20 (2005)

Global Lambda Hyperon Polarization in Nuclear Collisions

STAR collaboration, Nature, 548,62 (2017).

How it happens?

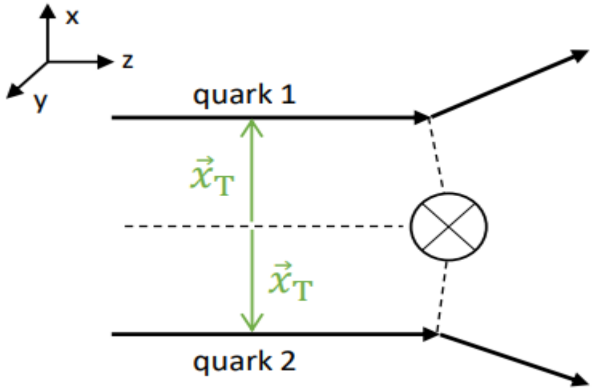
Spin-orbit coupling

$$\frac{d\sigma_\lambda}{d^2x_T} = C_T \int \frac{d^2q_T}{(2\pi)^2}\frac{d^2k_T}{(2\pi)^2} e^{i(\vec{k}_T - \vec{q}_T)\cdot\vec{x}_T} I_\lambda(\vec{q}_T, \vec{k}_T, E),$$

$$I_\lambda = \frac{g^2}{2(2E)^2}\bar{u}_\lambda(p_q)\not{A}(\vec{q}_T)(\not{p} + m_q)\not{A}(\vec{k}_T)u_\lambda(p_k)$$

$$P_q = -\frac{\pi\mu p}{4E(E+m_q)} \sim -0.3$$

Zuo-Tang Liang and Xin-Nian Wang, PRL 94, 102301(2005)



X-W Li, Z-F Jiang, S-S Cao, JD, EPJC 83(2023)1,96

Spin-vorticity coupling

$$f(x,p,S) \propto \exp\left[-\frac{p^2}{2mT} + \frac{\omega\cdot S}{T}\right]$$

$$P \approx \frac{S+1}{3}\frac{\hbar\omega}{kT}$$

$$\frac{\hbar\omega}{kT} \sim 0.08 \qquad \frac{m_q}{T} \sim 0.06$$

F. Becattini, V. Chandra, L. D. Zanna, and E. Grossi, Ann. Phys. 338,32 (2013)

Relativistic distribution function for particles with spin at local thermodynamical equilibrium

Imaging it in a non-relativistic way

Vortex, a topological realization of spin polarization

Spin ← → Vortex

quantized angular momentum

basic degree of freedom

How does the spin-orbit/vorticity coupling take place?

How does the orbit angular momentum transfer?

How does the energy of rotation and vorticity nucleate?

What is the dynamics in control?

HICs | BECs

Hydrodynamics

Transport models

Shear induced Polarization

Nonlocal collisions

…

Gross-Pitaevskii eqaution

Gross-Pitaevskii Equation(GPE)

$$\mathcal{H}_0 = \frac{1}{2}[\pi^2 + (\nabla\phi)^2 + m^2\phi^2],$$
$$\mathcal{H}_{\text{int}} = \frac{1}{24}\lambda\phi^4 + J\phi.$$

A non-relativistic effective field theory can be constructed systematically by canonical transformations

$$i\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(m - \frac{\nabla^2}{2m}\right)\psi(t,\mathbf{x}) + \frac{\lambda}{8m^2}|\psi(t,\mathbf{x})|^2\psi(t,\mathbf{x}),$$

JD, S. Schlichting, R. Venugopalan and Q. Wang Phys. Rev. A 97 (2018) 5, 053606.

$$\hat{H} = \int \frac{\hbar^2}{2m}\nabla\hat{\Psi}^\dagger(\mathbf{r},t)\cdot\nabla\hat{\Psi}(\mathbf{r},t)d^3\mathbf{r} + \int\frac{1}{2}\hat{\Psi}^\dagger(\mathbf{r},t)\hat{\Psi}^\dagger(\mathbf{r}',t)V(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r},t)\hat{\Psi}(\mathbf{r}',t)d^3\mathbf{r}d^3\mathbf{r}',$$

Large N

$$N = \int d\mathbf{r}\langle\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\rangle$$

s-wave

$$V(\mathbf{r}-\mathbf{r}') \approx \lambda\delta^3(\mathbf{r}-\mathbf{r}')$$

$$H = \int \mathcal{H}d\mathbf{r} = \int\left[\frac{\hbar^2}{2m}\nabla\psi(\mathbf{r},t)\cdot\nabla\psi^*(\mathbf{r},t) + \frac{\lambda}{2}|\psi(\mathbf{r},t)|^4\right]d^3\mathbf{r},$$

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \frac{\delta H}{\delta\psi^*(\mathbf{r},t)} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + \lambda|\psi(\mathbf{r},t)|^2\psi(\mathbf{r},t).$$

EOM of a classical field for BECs at zero T limit

JD, Q. Wang and H. Zhang, Phys. Fluids 34 (2022) 8, 087118.

Property of GPE: Conservation laws and length scales

Madelung representation

$$\psi(\mathbf{r},t) = \sqrt{\rho(\mathbf{r},t)}e^{i\theta(\mathbf{r},t)}$$

$$\mathbf{v} = \frac{\hbar}{\rho m}\mathrm{Im}(\psi^*\nabla\psi) = \frac{\hbar}{m}\nabla\theta,$$

$$\mathbf{K} = \frac{\hbar}{\rho m}\mathrm{Re}(\psi^*\nabla\psi) = \frac{\hbar}{m}\nabla\ln\sqrt{\rho}.$$

Wave function: interference

Number: $\frac{\partial}{\partial t}\rho + \nabla\cdot\boldsymbol{j} = 0,$ $\rho = |\psi|^2$ $\boldsymbol{j} = \rho\mathbf{v}$

Energy: $\frac{\partial}{\partial t}\mathcal{H} + \nabla\cdot\vec{\mathcal{P}} = 0,$ $\mathcal{H} = \frac{1}{2}m\rho(|\mathbf{v}|^2 + |\mathbf{K}|^2) + \frac{\lambda}{2}\rho^2,$

Momentum: $\frac{\partial}{\partial t}m\boldsymbol{j}_i + \partial_k\Pi_{ki} = 0,$

$$\Pi_{ij} = m\rho K_iK_j + m\rho v_iv_j + \delta_{ij}\frac{\lambda}{2}\rho^2 - \delta_{ij}\frac{\hbar}{2}\nabla\cdot(\rho\mathbf{K}),$$

More complicated than a compressible hydrodynamics

OAM: $\frac{\partial}{\partial t}M_{0j} + \partial_i M_{ij} = 0,$

$$M_{0i} = (\mathbf{r}\times m\rho\mathbf{v})_i = m(\mathbf{r}\times\boldsymbol{j})_i,$$

Vorticity: $\frac{\partial}{\partial t}\mathcal{M}_{0j} + \partial_i\mathcal{M}_{ij} = 0,$

$$\mathcal{M}_{0j} = (\nabla\times\boldsymbol{j})_j,$$

key observables

Healing length: $\xi = \hbar/\sqrt{2m\lambda\rho_0}$ Interference length: $\lambda_D \sim 2\sqrt{2}\pi\xi.$ Speed of sound: $c_s = \hbar/(\sqrt{2}m\xi)$

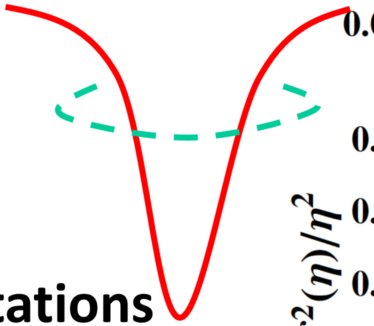
$\lambda_D \gg \xi \gg d \gg a_s$ for dilute Boson gas in weakly interacting limit

Vortex, a basic degree of freedom in rotating BECs

Stationary rotating solution with GPE

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + \lambda|\psi(\mathbf{r},t)|^2\psi(\mathbf{r},t)$$

$$\psi(r,\phi,t) = \sqrt{\rho(r)}e^{is\phi}e^{-i\mu t/\hbar},$$

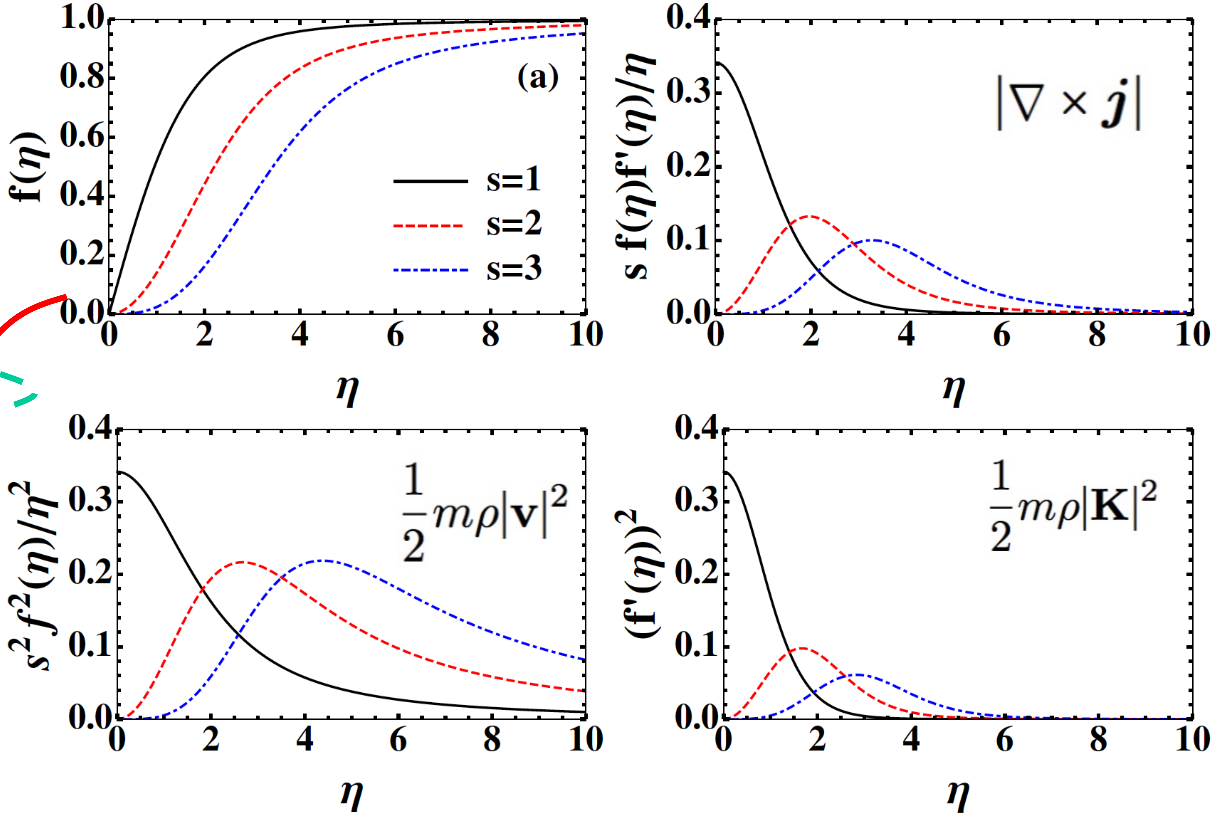


Winding number s = integer,
Vortices are qunatum soliton excitations

$$\mathbf{v} = (\hbar s)/(mr)\hat{\phi} \quad \boldsymbol{l} = \mathbf{r}\times\rho\mathbf{v} = \rho(r)s\hbar\hat{z}$$

$$f''(\eta) + \frac{1}{\eta}f'(\eta) + (1 - \frac{s^2}{\eta^2})f(\eta) - f(\eta)^3 = 0,$$

$$f(\eta\to 0) = 0 \quad f(\eta\to\infty) = 1.$$



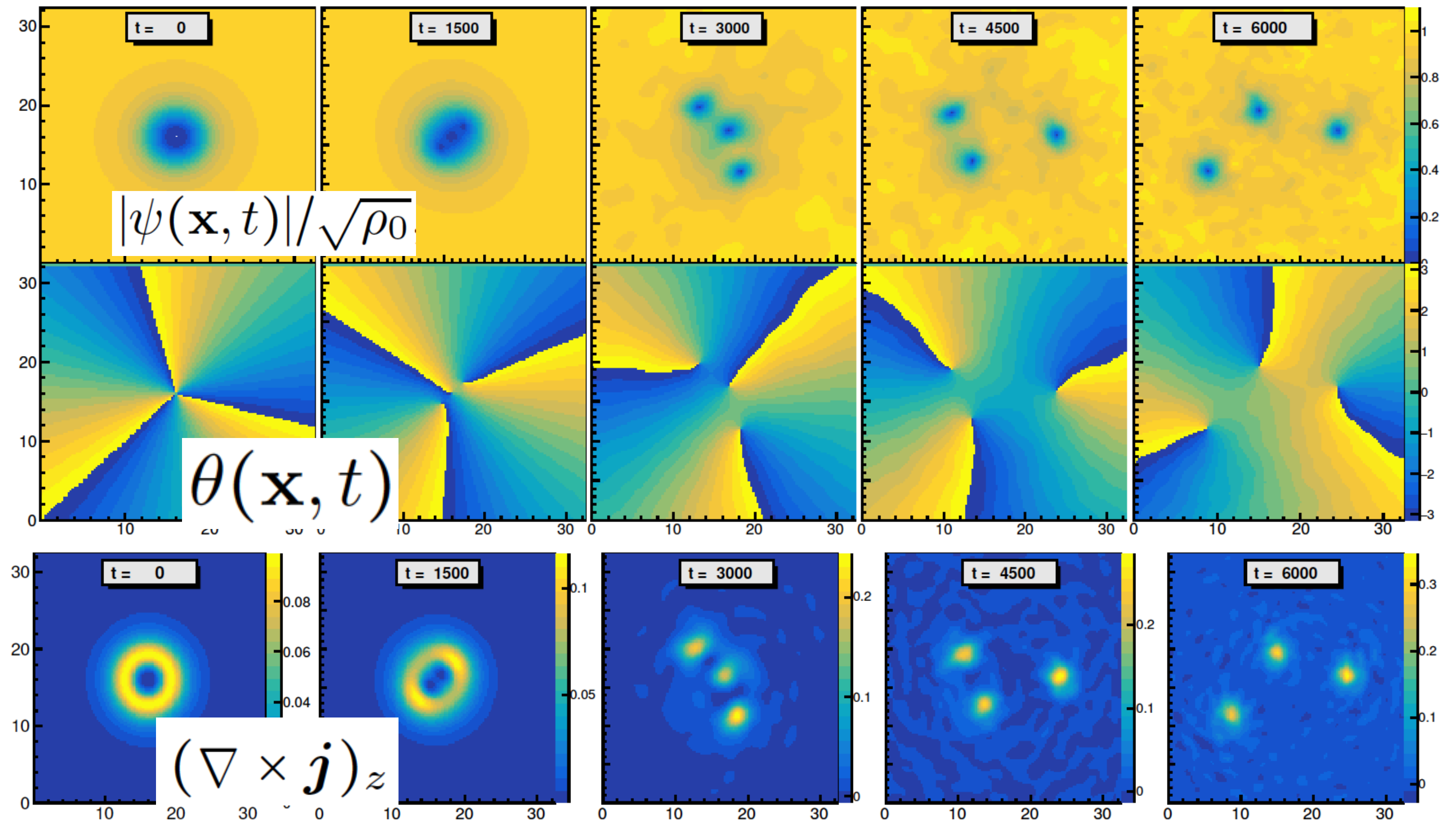
$$\Delta E \propto s^2 \qquad \Delta L \propto s$$

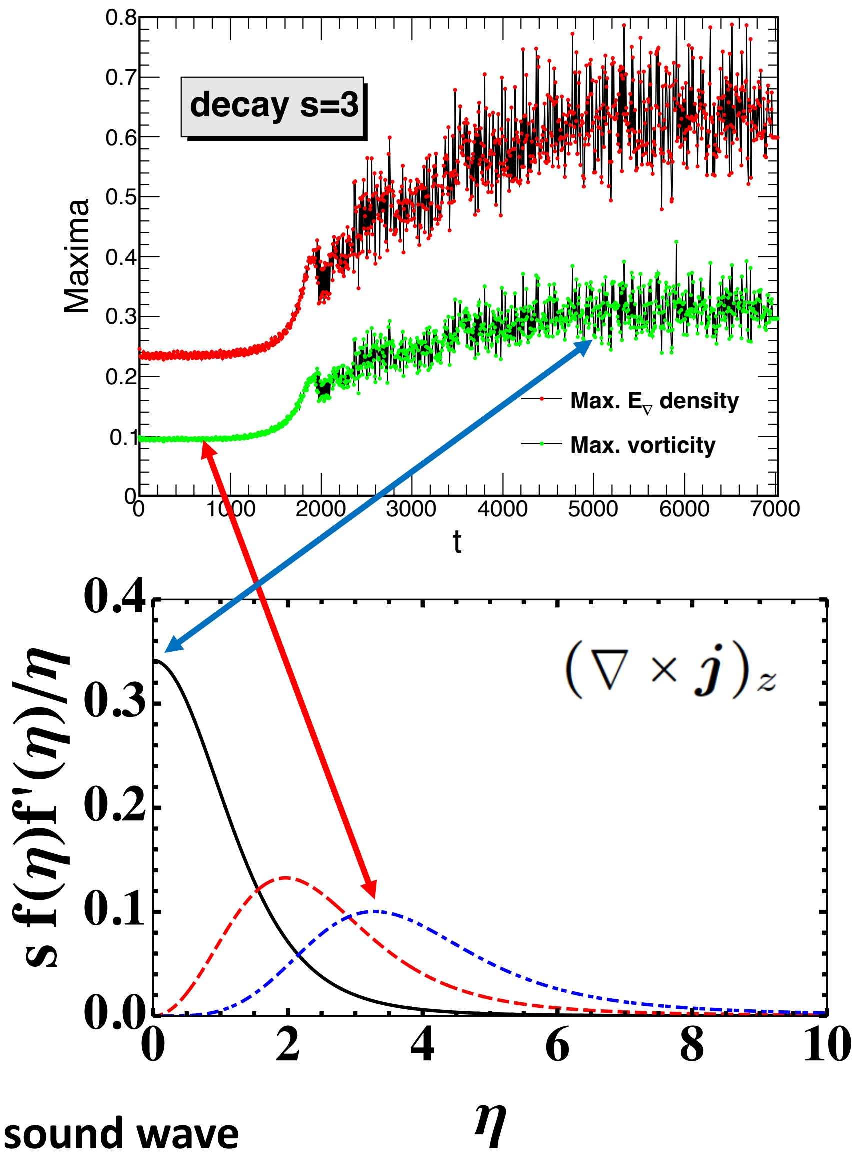
s > 1 unstable in energy,
s=1 basic dof in case of rotation

High precision solution to quantized vortices

H-H Peng, JD, S-Y Lou and Q Wang, Commun. Theory. Phys. 74 (2022) 9, 095002.

Decay to primary vortices, the basic DOF





Metastable states (s>1) decay process:

Singular point spliting➔ coupled vortices ➔ decoupled ➔ rotating + interacting + sound wave

Primary vortex (s=1) is stable, which is the fundamental DOF in rotating BECs

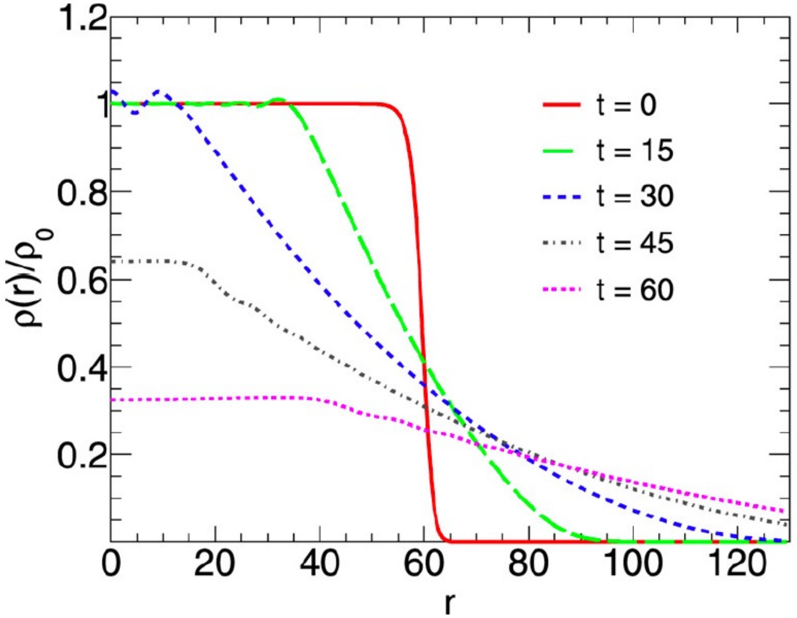
Prepare for the collision with BECs

$$\frac{\psi(t=0,\mathbf{r})}{\sqrt{\rho_0}} = w(r) = \frac{1}{\exp[(r-R_0)/a]+1},$$

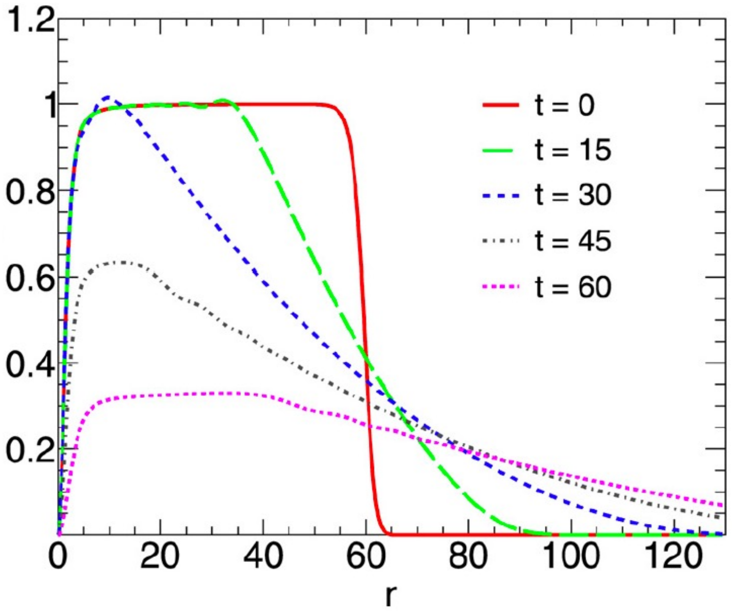
$$\frac{\psi(t=0,\mathbf{r})}{\sqrt{\rho_0}} = w(r) \times \psi_V(\mathbf{r}),$$

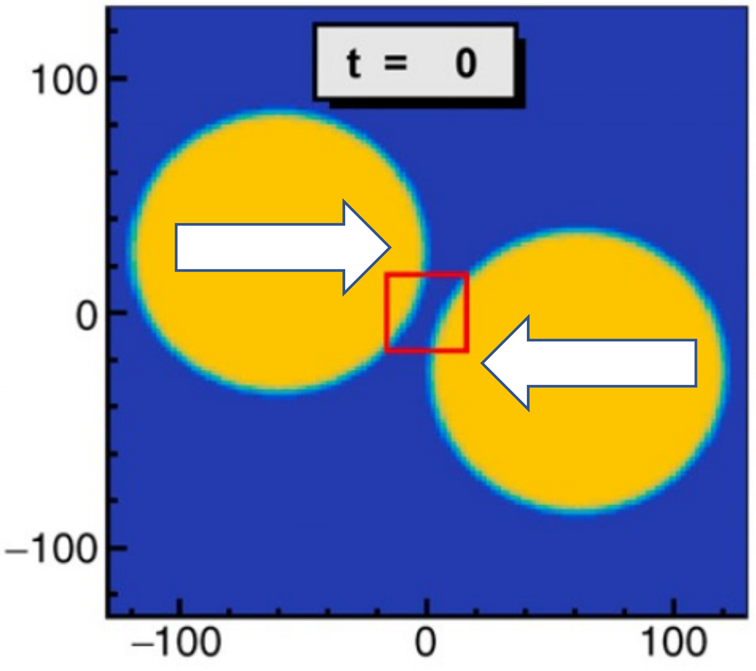
$$\begin{aligned}\frac{\psi(t=0,\mathbf{r})}{\sqrt{\rho_0}} &= w\left(|\mathbf{r}-\mathbf{r}_0|\right)\exp\left(\frac{i}{\sqrt{2}}\frac{v_0}{c_s}\frac{x+R_0}{\xi}\right) \\ &+ w\left(|\mathbf{r}+\mathbf{r}_0|\right)\exp\left(-\frac{i}{\sqrt{2}}\frac{v_0}{c_s}\frac{x-R_0}{\xi}\right)\end{aligned}$$

No internal structure



Rotating with a vortex



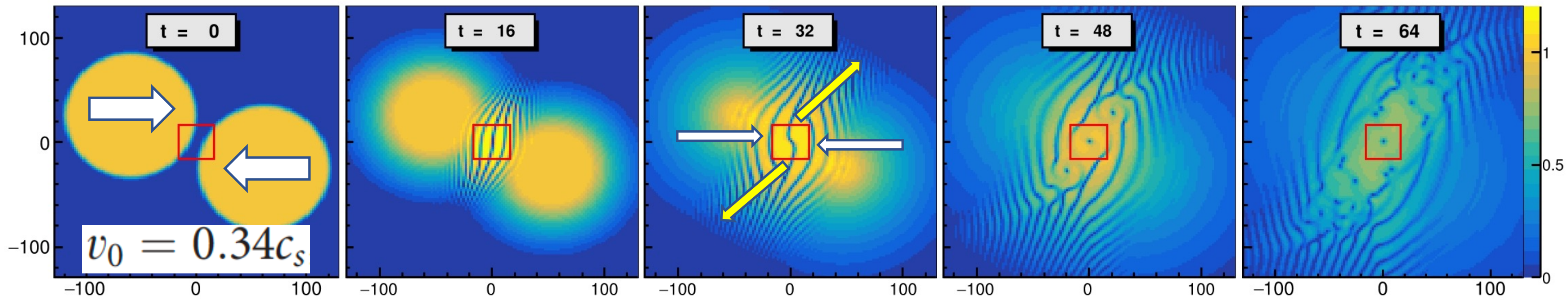


Isolated disk is not stable due to the repulsive quantum pressure and self-interaction. The inner region is inertial confined, it can maintain the density for a longer time with a larger size disk.

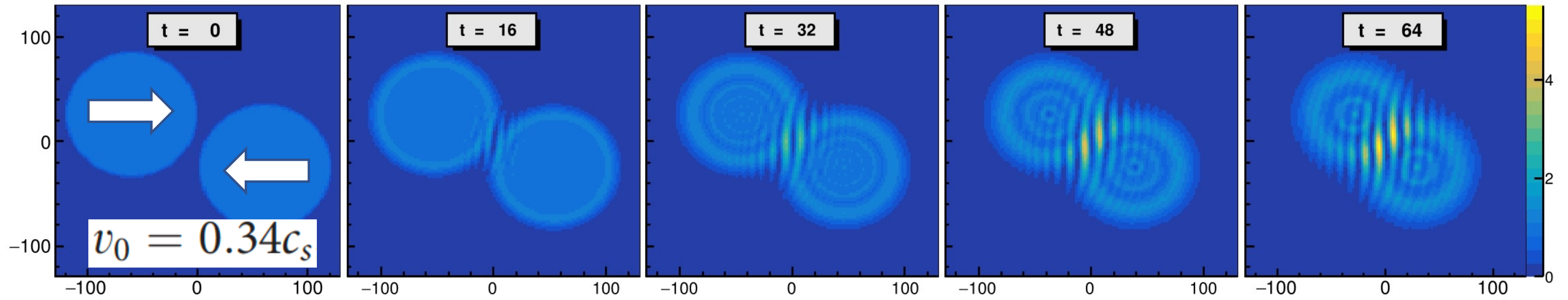
Larger size, closer distance, collision takes place before a significant expansion of the central region.

Vortex formation in collision of BECs

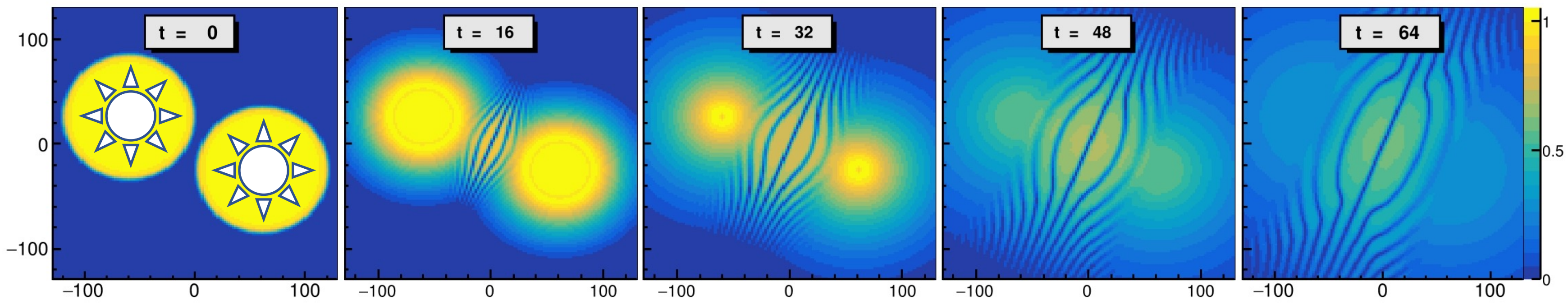
- **With OAM & interaction.**



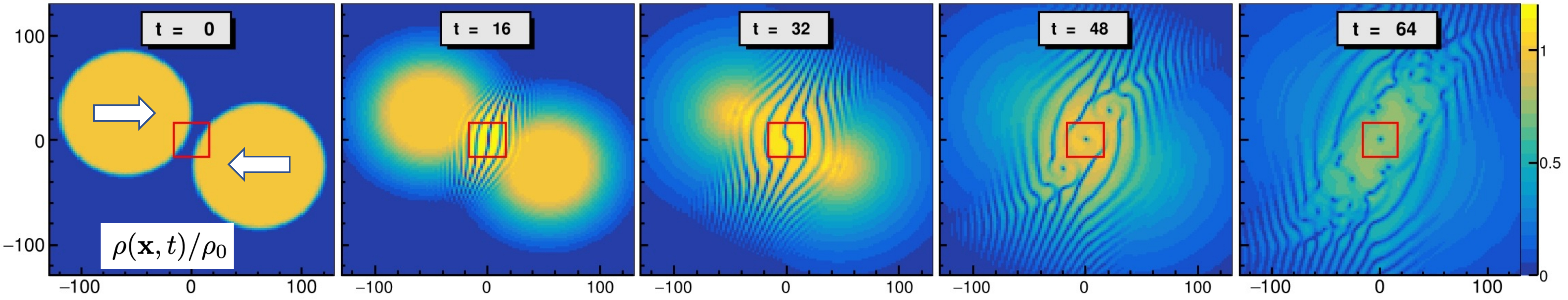
- **With OAM, no interaction.**



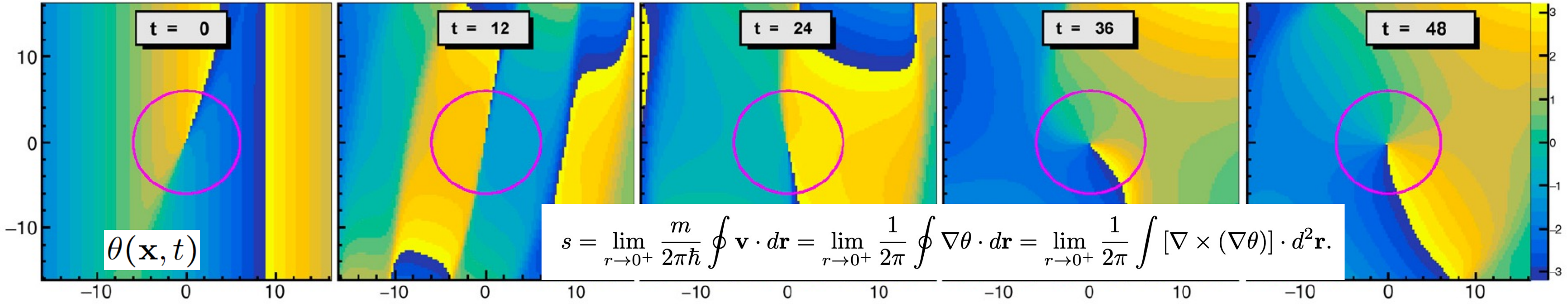
- **With interaction, no OAM.**



Wave function in collision of BECs



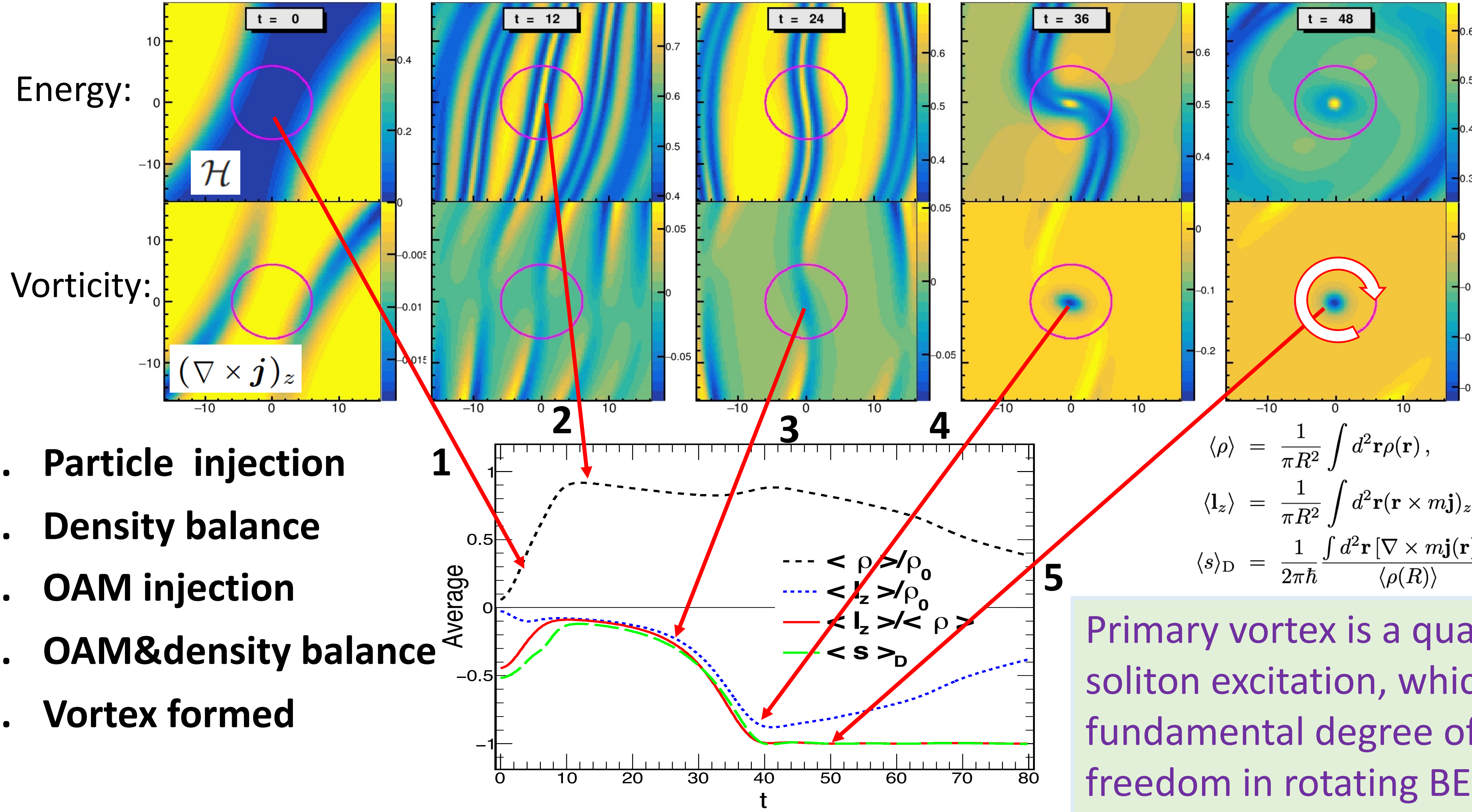
interference fringes spiral ➔ constructive fringes merge and destructive fringes breaks ➔ quantized vortices created



$$s = \lim_{r\to 0^+} \frac{m}{2\pi\hbar}\oint \mathbf{v}\cdot d\mathbf{r} = \lim_{r\to 0^+} \frac{1}{2\pi}\oint \nabla\theta\cdot d\mathbf{r} = \lim_{r\to 0^+} \frac{1}{2\pi}\int [\nabla\times(\nabla\theta)]\cdot d^2\mathbf{r}.$$

Winding number, Topological defect is conserved ➔ quantized vortices created around original defects
The smaller the initial OAM density ➔ the longer distance for OAM transfer ➔ the longer formation time

Energy density and vorticity evolution



1. **Particle injection**
2. **Density balance**
3. **OAM injection**
4. **OAM&density balance**
5. **Vortex formed**

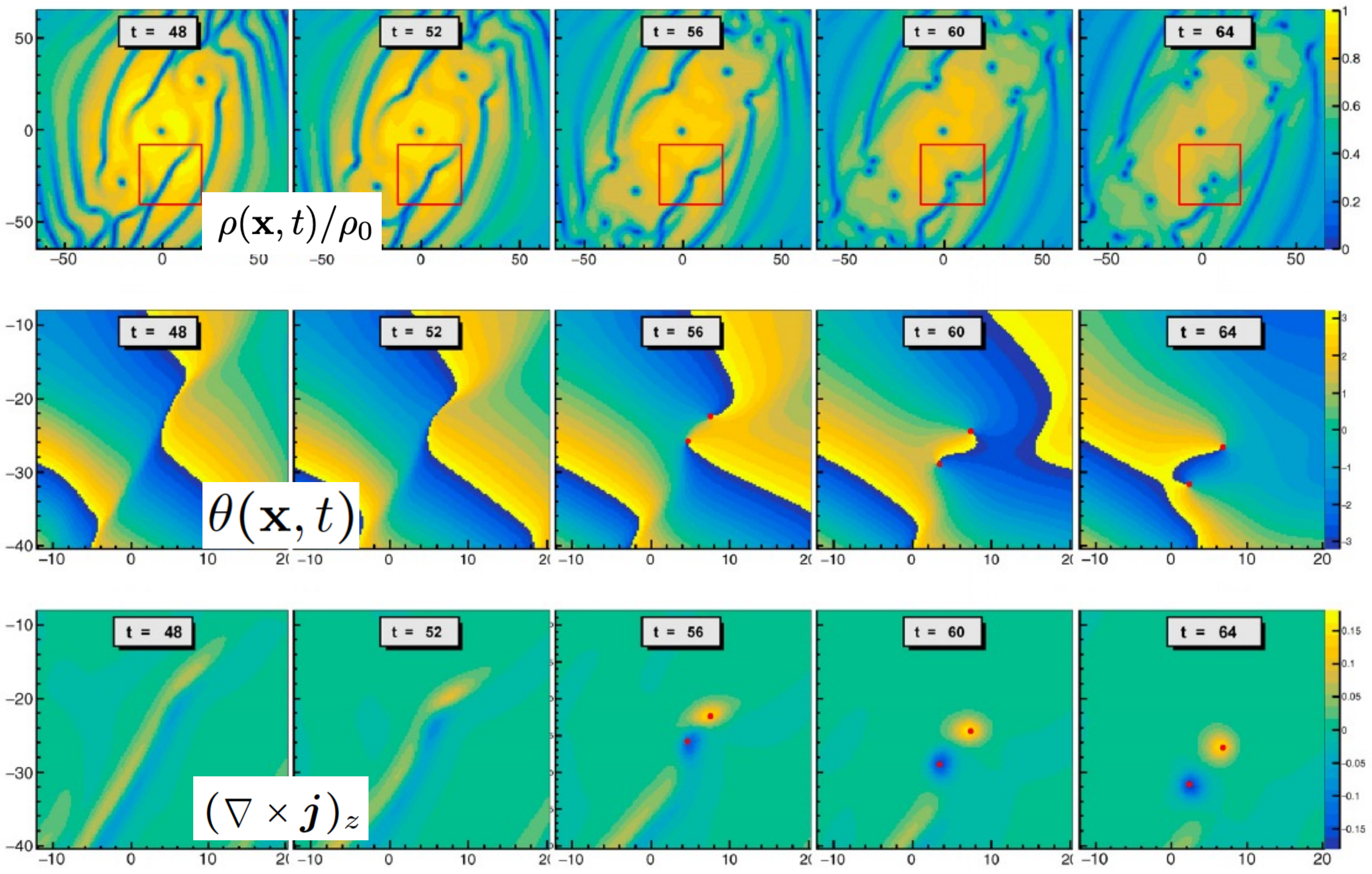
$$\langle \rho \rangle = \frac{1}{\pi R^2} \int d^2\mathbf{r}\, \rho(\mathbf{r})\,,$$

$$\langle \mathbf{l}_z \rangle = \frac{1}{\pi R^2} \int d^2\mathbf{r} (\mathbf{r} \times m\mathbf{j})_z\,,$$

$$\langle s \rangle_{\mathrm{D}} = \frac{1}{2\pi\hbar} \frac{\int d^2\mathbf{r}\, [\nabla \times m\mathbf{j}(\mathbf{r})]_z}{\langle \rho(R) \rangle},$$

Primary vortex is a quantum soliton excitation, which is a fundamental degree of freedom in rotating BECs.

Vortex-antivortex pair production



1. **Colliding between the internal flows, destructive fringes break further.**
2. **Positive and negative winding number created simultaneously.**
3. **Followed by the formation of vortex-antivertex pairs.**
4. **Pair production depends on the collision energy, not the initial OAM.**
5. **Pair production can take place in central collision.**

Summary and outlook

Stationary property and dynamical formation of vortices are investigated.

The primary vortices are demonstrated as the basic degrees of freedom with sizable OAM. The energy and vorticity density will nucleate around topological defects, leading to the formation of vortices .

A different approach to spin polarization through vortex formation.

Vortex is a topological excitation with local OAM. It is an analog of spin polarization. Vortex formation process may shed light on the nature of particle's spin as well as spin-orbit and spin-vorticity coupling.

outlook

Time reversibility vs. Chaos; Vortex dynamics; Relativistic simulation……

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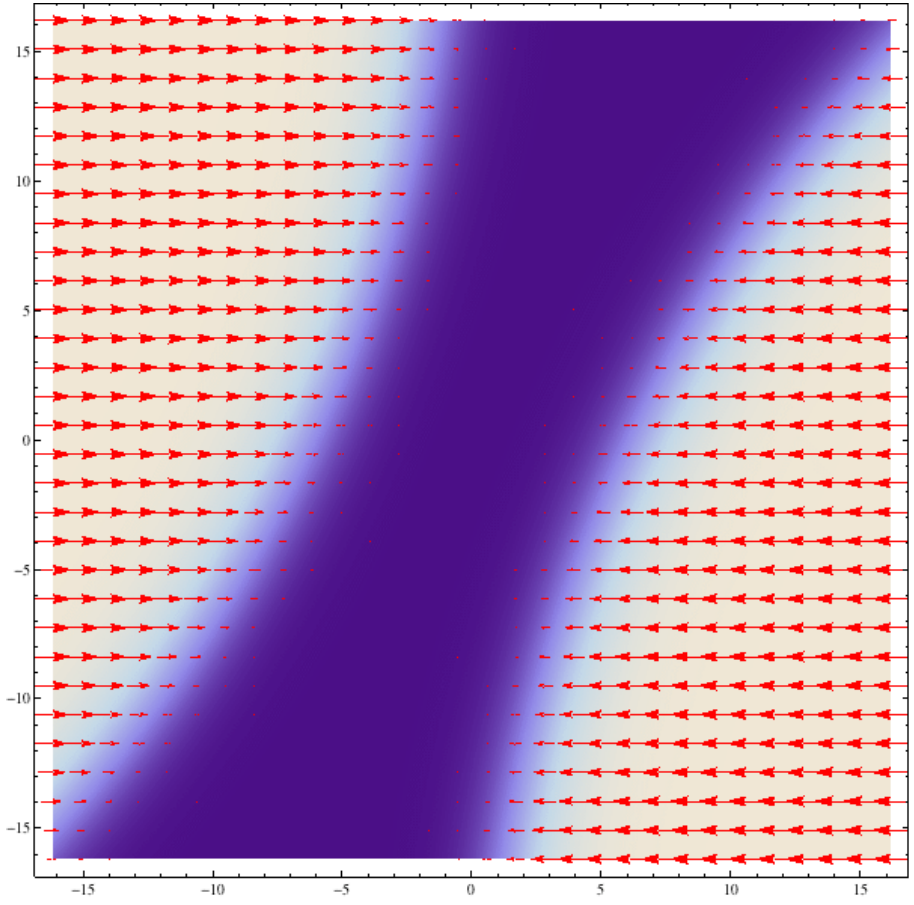
outlook

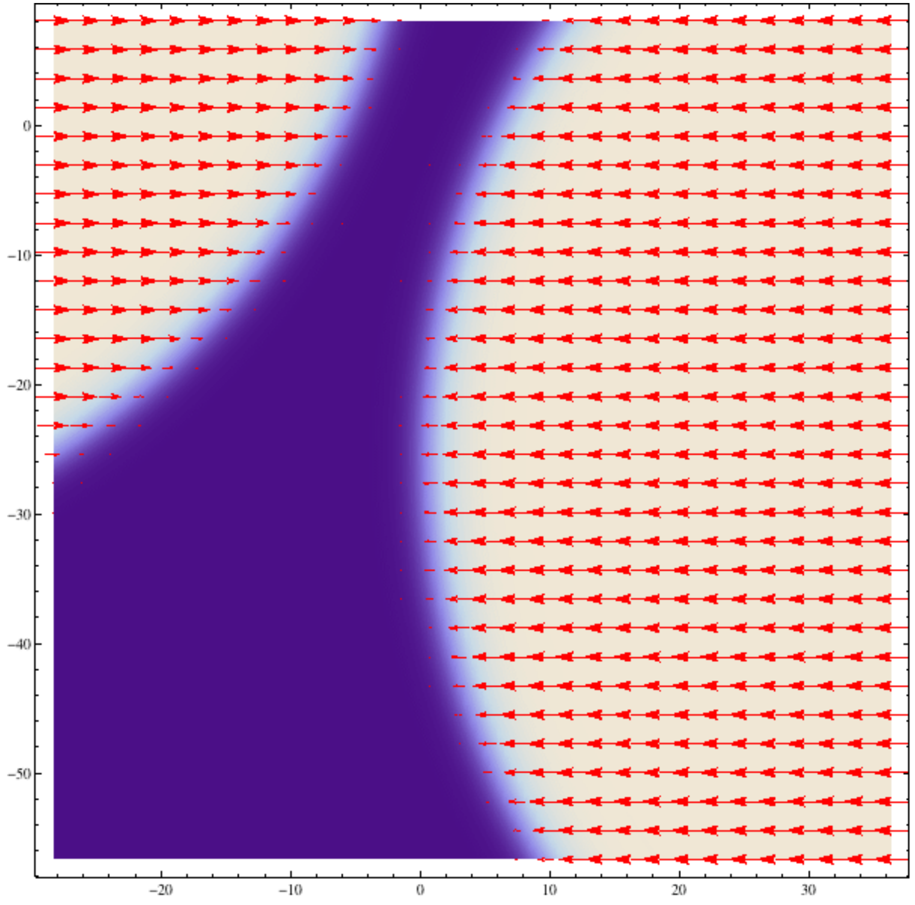
Thanks!

Time reversibility vs. Chaos; Vortex dynamics; Relativistic simulation……

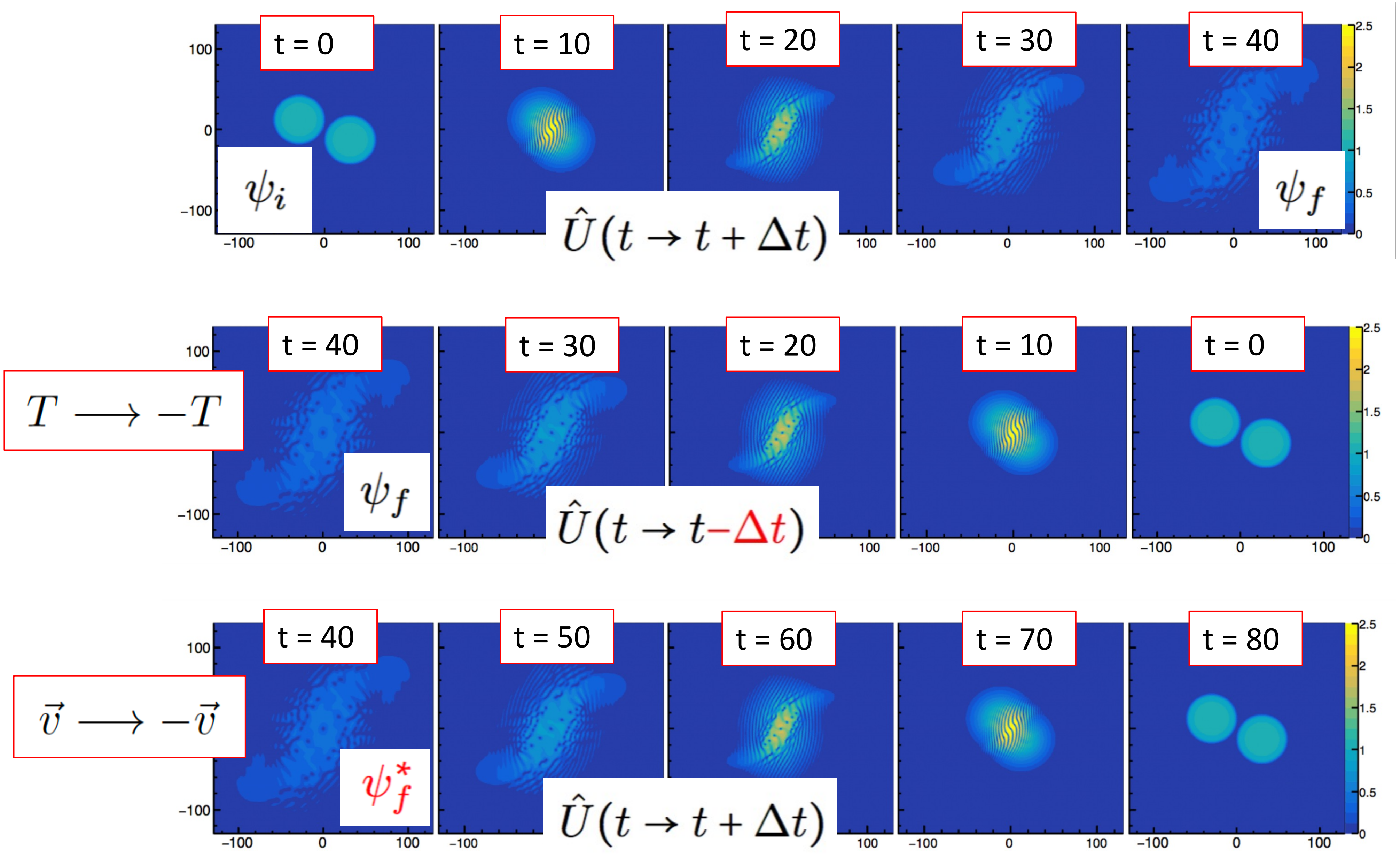
Back up

Vortex and pair production

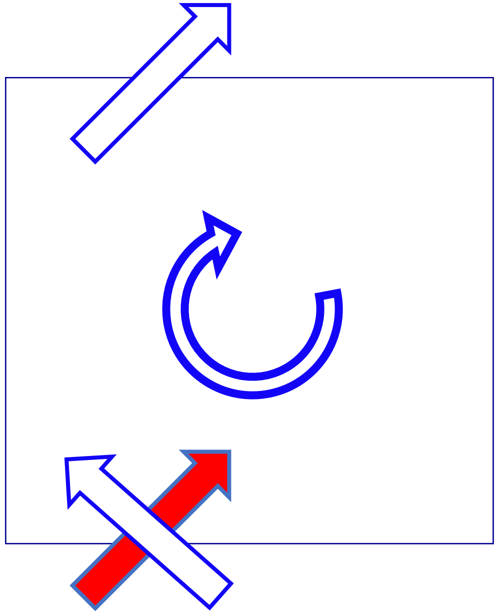


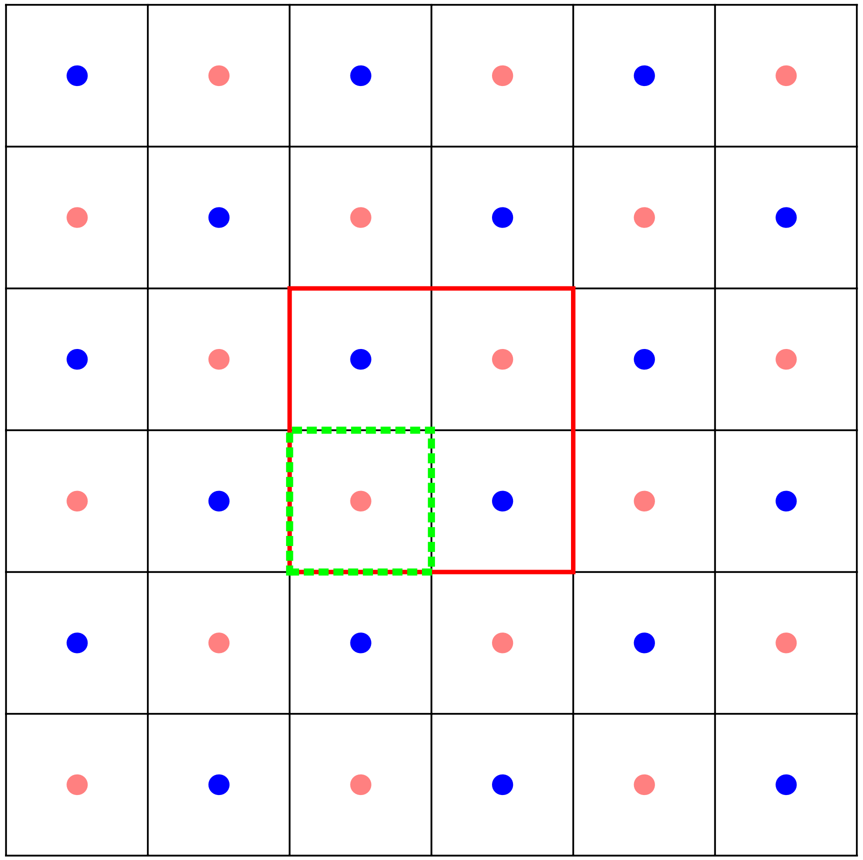


Time reversibility, any subtlety?



Periodic boundary condition VS. rotation





$$\psi(t=0,\mathbf{r}) = \sqrt{\rho_0}\prod_{n=-N_0}^{N_0-1}\prod_{m=-N_0}^{N_0-1} g\left(\frac{\left|\mathbf{r}-\mathbf{r}_{n+\frac{1}{2},m+\frac{1}{2}}\right|}{\xi}\right)\exp\left[is_{n+\frac{1}{2},m+\frac{1}{2}}\phi(\mathbf{r}-\mathbf{r}_{n+\frac{1}{2},m+\frac{1}{2}})\right].$$